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Dual Signal Optical Bistability in a Semiconductor Optical Amplifier

Aiswarya Kannan

A thesis presented for the degree of

Master of Science in Telecommunications Engineering Technology

Thesis Advisor

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Dual Signal Optical Bistability in a Semiconductor

Optical Amplifier

Aiswarya Kannan

Abstract

Future all-optical signal processing applications may require the use of multiple optical signals passing through a single, optically bistable device. My thesis investigates an improved model for modeling two optical signals passing through an optically bistable Fabry-Perot semiconductor optical amplifier (FP-SOA). The optical power and phase of these signals are both modeled, as well as the optical gain of the FP-SOA. My improved model is based on an improved model used to study optical bistability of just a single optical signal, in which the internal power is related to the output power with an expression accounting for the Fabry-Perot structure. Work has been performed to put all models into a consistent notation. The resulting, improved model for dual-signal operation indicates lower critical powers to trigger bistable action.

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CHAPTER 1

Light Propagation in a Resonator-Less Semiconductor Optical Amplifier

My thesis begins with an introduction to light propagation in a semiconductor optical amplifier (SOA). Chapter 1 deals with the power, phase, gain equations which govern the light propagation in an SOA. Chapter 2 deals with the optical bistability of a single light signal within an SOA that is within a resonator. This chapter studies the behavior of output gain, power and phase for the light signal. Chapter 3 deals with the same system of Chapter 2, but uses a simpler model. The simpler approach is compared with the results obtained in chapter 2. Chapter 4 also deals with optical bistability, but now using 2 different input signals. Chapter 5 also deals with the 2 signal case but the equations are obtained using the simpler method followed in Chapter 2. We compare and quantify the two different

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modeling approaches.

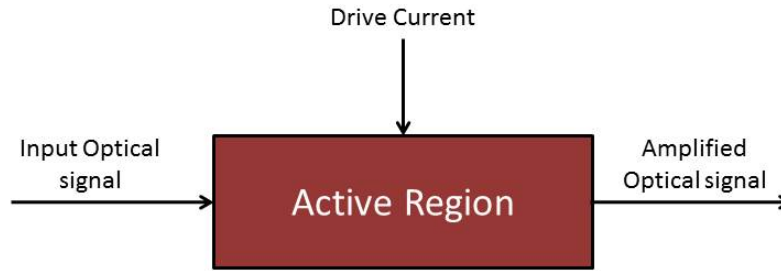


Figure 1.0.1: Semiconductor Optical Amplifier

Semiconductor optical amplifiers are based on the semiconductor gain medium in the middle called as active region. As shown in Fig.1 , a weak optical signal is provided as input and it comes out amplified as an amplified optical signal. The electrical current is applied while the signal travels through the semiconductor cavity. This injection current creates large number of electron and holes. When the carrier density exceeds carrier transparency, the material is capable of optical gain, thus behaving as an amplifier [1].

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This chapter introduces semiconductor optical amplifiers and derives equations for power, gain and phase of an optical pulse inside a semi-conducting optical amplifier. These quantities and expressions serve as a foundation for the rest of the chapters in my thesis.

1.1 Carrier density equation

The carrier density in the SOA is given by a rate equation [2]

$$\frac{dN}{dt} = \frac{J}{qd} - \frac{N}{\tau_c} - \frac{a(N - N_T)|\tilde{E}|^2}{hf}, \quad (1.1.1)$$

where N is the carrier density [$\frac{1}{\text{cm}^3}$], J is the current density [$\frac{\text{A}}{\text{cm}^2}$], q is the electron charge [C], d is the active layer thickness [μm], τ_c is the carrier lifetime [ps], a is the differential gain factor [cm^2], hf is the photon energy[eV], and \tilde{E} is the electric field. This rate equation ignores carrier diffusion.

N is the carrier density and N_T is the carrier density at transparency which is the state when the system experiences no loss. We assume that the carrier density does not vary much in the transverse dimension across the active region, and therefore average over the transverse dimension. The carrier density equation is re-written as [3]

$$\frac{dN}{dt} = \frac{J}{qd} - \frac{N}{\tau_c} - \frac{\Gamma a(N - N_T)}{hf} \frac{\sigma}{wd} |A|^2, \quad (1.1.2)$$

where $|A|^2\sigma$ represents the optical power P and Γ is the confinement factor. More-

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over, a saturation energy can be defined as [3]:

$$E_{sat} = \frac{hf\omega d}{a\Gamma},$$

and a saturation power can be defined as

$$P_{sat} = \frac{E_{sat}}{\tau_c} = \frac{hf\omega d}{\tau_c a\Gamma}.$$

Thus, the rate equation for N can be written as:

$$\frac{dN}{dt} = \frac{J}{qd} - \frac{N}{\tau_c} - \frac{(N - N_T)P}{E_{sat}},$$

$$\frac{dN}{dt} = \frac{J}{qd} - \frac{N}{\tau_c} - \frac{(N - N_T)P}{\tau_c P_{sat}},$$

$$\frac{dN}{dt} = \frac{J}{qd} - \frac{N}{\tau_c} - \frac{(N - N_T)\bar{P}}{\tau_c},$$

where

$$\bar{P} = \frac{P}{P_{sat}}.$$

Further simplification can occur by multiplying throughout by τ_c [2] :

$$\tau_c \frac{dN}{dt} = \frac{J\tau_c}{qd} - N - (N - N_T)\bar{P},$$

$$\tau_c \frac{dN}{dt} = N_0 - N - (N - N_T)\bar{P},$$

where $\frac{J\tau_c}{qd} = N_0$ and the expression for steady-state can be found below.

1.1.1 Steady state and small signal carrier density

To find the steady state equation, the time derivative is equated to zero.

$$0 = \frac{J\tau_c}{qd} - N - (N - N_T)\bar{P},$$

$$N = \frac{J\tau_c}{qd} - (N - N_T)\bar{P}.$$

For the small signal case, $P_a = 0$ which yields the steady state small signal carrier density N_0 :

$$N_0 = \frac{J\tau_c}{qd}. \quad (1.1.3)$$

Similarly, the carrier density at transparency N_T is given by

$$N_T = \frac{J_T\tau_c}{qd}. \quad (1.1.4)$$

Thus [3],

$$N = N_0 - (N - N_T)\bar{P},$$

$$N + N\bar{P} = N_0 + N_T\bar{P},$$

$$N = \frac{N_0 + N_T\bar{P}}{1 + \bar{P}}. \quad (1.1.5)$$

1.1.2 Steady state and small signal gain

The general expression for the gain coefficient is given by [7]

$$g = \Gamma a(N - N_T), \quad (1.1.6)$$

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where N_T is the carrier density at transparency and N represents the initial carrier density. The small signal steady state gain is therefore given by

$$g = \Gamma a(N_0 - N_T),$$
$$g = \Gamma a \left[\frac{J\tau_c}{qd} - \frac{J_T\tau_c}{qd} \right].$$

Re-arranging the equation gives

$$g = \frac{\Gamma a\tau_c}{qd}(J - J_T),$$
$$g = \frac{\Gamma a\tau_c J_T}{qd} \left(\frac{J}{J_T} - 1 \right).$$

Thus we obtain an expression for small-signal steady state gain as

$$g = \Gamma a N_T \left(\frac{J}{J_T} - 1 \right).$$

At small signal, the gain coefficient is given by

$$g_0 = \Gamma a(N_0 - N_T).$$

Note that using the expression for g , the carrier density rate equation can be written as

$$\frac{dN}{dt} = \frac{J}{qd} - \frac{N}{\tau_c} - \frac{g}{hf} \frac{\sigma}{wd} |A|^2.$$

1.1.3 Gain rate equation

Differentiating the gain equation with respect to time yields,

$$\frac{dg}{dt} = \Gamma a \frac{dN}{dt}.$$

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Substituting the carrier density rate equation in the gain equation gives,

$$\frac{dg}{dt} = \Gamma a \left(\frac{N_0 - N}{\tau_c} - \frac{(N - N_T)\bar{P}}{\tau_c} \right),$$

$$\tau_c \frac{dg}{dt} = \Gamma a(N_0 - N) - \Gamma a(N - N_T)\bar{P}.$$

Note that $(N_0 - N)$ can be expanded as $N_0 - N_T - (N - N_T)$. Thus,

$$\tau_c \frac{dg}{dt} = \Gamma a(N_0 - N_T) - a\Gamma(N - N_T) - a\Gamma(N - N_T)\bar{P},$$

$$\tau_c \frac{dg}{dt} = g_0 - g - g\bar{P},$$

$$\tau_c \frac{dg}{dt} = g_0 - g(1 + \bar{P}).$$

At steady state, [3]

$$0 = g_0 - g(1 + \bar{P}),$$

$$g = \frac{g_0}{1 + \bar{P}}.$$

1.2 Optical field

The complex field is given by [5]

$$\tilde{E}(x, y, z, t) = F(x, y)\tilde{A}(z, t)e^{i\beta z - i\omega t},$$

where F is the transverse distribution [-], \tilde{A} is the slowly varying envelope $[\sqrt{W}]$, β is the wavenumber [Rad/m] and ω is the angular frequency [Rad/Hz]. The rate

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equation for the envelope is [5]

$$\frac{\partial \tilde{A}}{\partial z} + \frac{1}{v_g} \frac{\partial \tilde{A}}{\partial t} = \frac{-i\Gamma}{2}(\alpha + i)a(N - N_T)\tilde{A} - \frac{1}{2}\alpha_{int}\tilde{A},$$

where v_g is the group velocity [m/s], α is the line-width enhancement factor [-], and α_{int} is the scattering loss. Using the gain coefficient g , the rate equation becomes

$$\begin{aligned} \frac{\partial \tilde{A}}{\partial z} + \frac{1}{v_g} \frac{\partial \tilde{A}}{\partial t} &= -i(\alpha + i)\frac{g\tilde{A}}{2} - \frac{1}{2}\alpha_{int}\tilde{A}, \\ \frac{\partial \tilde{A}}{\partial z} + \frac{1}{v_g} \frac{\partial \tilde{A}}{\partial t} &= (1 - i\alpha)\frac{g\tilde{A}}{2} - \frac{1}{2}\alpha_{int}\tilde{A}. \end{aligned} \quad (1.2.1)$$

1.3 Optical Power

The amplitude \tilde{A} of the electric field can be written in terms of power and phase as

$$\tilde{A} = \sqrt{P}e^{i\phi}. \quad (1.3.1)$$

Substituting this equation into the rate equation yields

$$\begin{aligned} \frac{\partial(\sqrt{P}e^{i\phi})}{\partial z} + \frac{1}{v_g} \frac{\partial(\sqrt{P}e^{i\phi})}{\partial t} &= \frac{g\sqrt{P}e^{i\phi}(1 - i\alpha)}{2} - \frac{1}{2}\alpha_{int}\sqrt{P}e^{i\phi}, \\ \frac{\partial\sqrt{P}}{\partial z}e^{i\phi} + \sqrt{P}\frac{\partial e^{i\phi}}{\partial z} + \frac{1}{v_g} \frac{\partial\sqrt{P}}{\partial t}e^{i\phi} + \frac{1}{v_g}\sqrt{P}\frac{\partial e^{i\phi}}{\partial t} &= \frac{g\sqrt{P}e^{i\phi}}{2} - \frac{1}{2}\alpha_{int}\sqrt{P}e^{i\phi} - \frac{i\alpha g\sqrt{P}e^{i\phi}}{2}, \\ \frac{\partial\sqrt{P}}{\partial z}e^{i\phi} + \sqrt{P}i\frac{\partial\phi}{\partial z}e^{i\phi} + \frac{1}{v_g} \frac{\partial\sqrt{P}}{\partial t}e^{i\phi} + \frac{1}{v_g}\sqrt{P}\frac{\partial\phi}{\partial t}e^{i\phi} &= \frac{(g - \alpha_{int})\sqrt{P}e^{i\phi}}{2} - \frac{i\alpha g\sqrt{P}e^{i\phi}}{2}. \end{aligned}$$

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$e^{i\phi}$ can be dropped since it is a common term. The expression is then separated into real and imaginary part as follows. The real part is [5]

$$\frac{\partial \sqrt{P}}{\partial z} + \frac{1}{v_g} \frac{\partial \sqrt{P}}{\partial t} = \frac{(g - \alpha_{int})\sqrt{P}}{2}. \quad (1.3.2)$$

The equation can be re-written in terms of derivatives of P by noting

$$\begin{aligned} \frac{\partial P}{\partial X} &= \frac{\partial(\sqrt{P}\sqrt{P})}{\partial X} = \sqrt{P} \frac{\partial(\sqrt{P})}{\partial X} + \frac{\partial(\sqrt{P})}{\partial X} \sqrt{P}, \\ \frac{\partial P}{\partial X} &= \frac{\partial \sqrt{P}}{\partial X} 2\sqrt{P}. \end{aligned}$$

To use this equation, multiply the real part equation by $2\sqrt{P}$

$$\begin{aligned} 2\sqrt{P} \frac{\partial \sqrt{P}}{\partial z} + \frac{2\sqrt{P}}{v_g} \frac{\partial \sqrt{P}}{\partial t} &= (g - \alpha_{int})P, \\ \frac{\partial P}{\partial z} + \frac{1}{v_g} \frac{\partial P}{\partial t} &= (g - \alpha_{int})P. \end{aligned} \quad (1.3.3)$$

The imaginary part is:

$$\begin{aligned} \sqrt{P}i \frac{\partial \phi}{\partial z} + \frac{1}{v_g} \sqrt{P}i \frac{\partial \phi}{\partial t} &= \sqrt{P} \frac{(-i\alpha_g)}{2}, \\ \frac{\partial \phi}{\partial z} + \frac{1}{v_g} \frac{\partial \phi}{\partial t} &= \frac{-i\alpha_g}{2}. \end{aligned} \quad (1.3.4)$$

1.4 Linewidth enhancement factor

The linewidth enhancement factor was introduced above. This section describes its origin [4].

$$n = n_b + n_a,$$

where n_a is written as

$$n_a = \text{Re}\{n_a\} + i\text{Im}\{n_a\}.$$

The linewidth enhancement factor is defines as

$$\alpha = \frac{\text{Re}\{n_a\}}{\text{Im}\{n_a\}},$$

where

$$\text{Im}\{n_a\} = \frac{g}{-2\beta_0},$$

$$\beta_0 = \frac{2\pi}{\lambda_0}.$$

Therefore,

$$\alpha = \frac{-\text{Re}\{n_a\}}{\Delta g} 2\beta_0,$$

$$\alpha = \frac{-4\pi}{\lambda_0} \frac{\Delta n}{\Delta g},$$

$$\alpha = \frac{-4\pi}{\lambda_0} \frac{\frac{dn}{dN}}{\frac{dg}{dN}},$$

$$\alpha = \frac{-4\pi}{\lambda_0} \frac{\frac{dn}{dN}}{a}.$$

Quite often, α is introduced to link changes in n to changes in g .

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The refractive index n can be written as

$$n = n_T + (n - n_T),$$

where n_T is the value of n at transparency. The changes n_x from n_y can be represented by a change in the carrier density N :

$$n_x = n_y + \Gamma \frac{dn}{dN} (N_x - N_y).$$

The refractive index can therefore be given by

$$n = n_T + \Gamma \frac{dn}{dN} (N - N_T),$$

where N_T is the value at transparency.

The equation for n becomes, after replacing $\frac{dn}{dN}$,

$$n = n_T - \frac{\Gamma \alpha a \lambda_0 (N - N_T)}{4\pi}.$$

We know that

$$g = \Gamma a (N - N_T).$$

So, the equation for n can be re-written as

$$\begin{aligned} n &= n_T - \frac{\alpha g \lambda_0}{4\pi}, \\ n &= n_T - \frac{\alpha g_0 \lambda_0}{4\pi} + \frac{\alpha (g_0 - g) \lambda_0}{4\pi}. \end{aligned} \tag{1.4.1}$$

There is another way to breakdown n :

$$n = n_T - \frac{\Gamma \alpha a \lambda_0}{4\pi} (N - N_T),$$

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$$n = n_T - \frac{\Gamma a a \lambda_0}{4\pi} (N - N_0 + N_0 - N_T),$$

where N_0 is the small-signal carrier density.

$$n = n_T - \frac{\Gamma a a \lambda_0}{4\pi} (N_0 - N_T) - \frac{\Gamma a a \lambda_0}{4\pi} (N - N_0). \quad (1.4.2)$$

The second term represents the change in n as N_0 varies from N_T due to injection current. The third term represents the change in n as N varies from N_0 due to gain saturation.

It is instructive to explicitly show the changes in gain from its small-signal value. When $N = N_T$, the gain co-efficient $g = \Gamma a (N - N_T) = 0$. When $N = N_0$, the gain co-efficient is $g = \Gamma a (N_0 - N_T) = g_0$, the small signal gain coefficient. Thus,

$$g = \Gamma a (N - N_T),$$

$$g = \Gamma a (N - N_0 + N_0 - N_T),$$

$$g = \Gamma a (N - N_0) + \Gamma a (N_0 - N_T),$$

$$g = \Gamma a (N - N_0) + g_0,$$

$$g - g_0 = \Gamma a (N - N_0),$$

$$g_0 - g = \Delta = \Gamma a (N_0 - N).$$

Thus, $\Gamma a (N_0 - N)$ is the difference Δ of the gain coefficient from its small-signal value. In steady state,

$$g = \frac{g_0}{(1 + P)}.$$

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Thus,

$$\begin{aligned}
 g - g_0 &= \frac{g_0}{(1 + \bar{P})} - g_0, \\
 g - g_0 &= \frac{g_0}{1 + \bar{P}} - \frac{g_0(1 + \bar{P})}{1 + \bar{P}}, \\
 g - g_0 &= \frac{g_0 - g_0(1 + \bar{P})}{1 + \bar{P}}, \\
 g - g_0 &= \frac{g_0 \bar{P}}{1 + \bar{P}}, \\
 g_0 - g &= -\frac{g_0 \bar{P}}{1 + \bar{P}}.
 \end{aligned}$$

Thus, three equivalent expression in steady state are

$$g_0 - g = -\frac{g_0 \bar{P}}{1 + \bar{P}} = \Gamma a(N_0 - N).$$

Using these gain expressions, the refractive index can be understood as

$$\begin{aligned}
 n &= n_T - \frac{\alpha \lambda_0 g_0}{4\pi} + \frac{\alpha \lambda_0 (g_0 - g)}{4\pi}, \\
 n &= n_T - \frac{\alpha \lambda_0 g_0}{4\pi} - \frac{\alpha \lambda_0}{4\pi} \frac{g_0 \bar{P}}{(1 + \bar{P})}.
 \end{aligned}$$

Above, we derived the following equation for the optical phase:

$$\frac{\partial \phi}{\partial z} + \frac{1}{v_g} \frac{\partial \phi}{\partial t} = -\frac{i\alpha g}{2}.$$

We can now understand the final term in terms of refractive index as follows:

$$\begin{aligned}
 n &= n_T - \frac{\alpha g \lambda_0}{4\pi}, \\
 \alpha g &= -(n - n_T) \frac{4\pi}{\lambda_0},
 \end{aligned}$$

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$$\begin{aligned}\frac{\partial \phi}{\partial z} + \frac{1}{v_g} \frac{\partial \phi}{\partial t} &= i(n - n_T) \frac{2\pi}{\lambda_0}, \\ \frac{\partial \phi}{\partial z} + \frac{1}{v_g} \frac{\partial \phi}{\partial t} &= i\beta, \\ \beta &= \frac{2\pi}{\lambda_0}(n - n_T).\end{aligned}$$

These expressions can be represented in terms of phase ϕ , where

$$\phi = \int_0^L \frac{2\pi n(z) dz}{\lambda_0}.$$

For the moment, we will assume n does not vary with z . In this case,

$$\begin{aligned}\phi &= \frac{2\pi n L}{\lambda_0} = \frac{2\pi n_T L}{\lambda_0} + \frac{2\pi(n - n_T)L}{\lambda_0}, \\ \phi &= \frac{2\pi n_T L}{\lambda_0} + \frac{2\pi(n_0 - n_T)L}{\lambda_0} + \frac{2\pi(n - n_0)L}{\lambda_0}, \\ \phi &= \frac{2\pi n_T L}{\lambda_0} - \frac{2\pi L \Gamma \alpha a (N_0 - N_T) \lambda_0}{\lambda_0 4\pi} - \frac{2\pi(N - N_0) L \Gamma \alpha a \lambda_0}{\lambda_0 4\pi}, \\ \phi &= \frac{2\pi n_T L}{\lambda_0} - \frac{L \Gamma \alpha a (N_0 - N_T)}{2} - \frac{(N - N_0) L \Gamma \alpha a}{2}.\end{aligned}$$

In terms of gain,

$$\phi = \frac{2\pi n_T L}{\lambda_0} - \frac{\alpha L g_0}{2} - \frac{\alpha L}{2} \frac{g_0 \bar{P}}{(1 + \bar{P})}, \quad (1.4.3)$$

or similarly

$$\phi = \frac{2\pi n_T L}{\lambda_0} - \frac{\alpha g L}{2}.$$

The first term on the right of the equation is the phase at transparency. The second term is the change in phase due to the small-signal gain current injection. The third term is the change in the phase due to gain saturation.

1.5 Summary of SOA Equations

In summary, the basic equations to describe optical pulse propagation in SOAs are

[5]:

$$\begin{aligned}\frac{\partial g}{\partial t} &= \frac{g_0 - g}{\tau_c} - \frac{gP}{E_{sat}}, \\ \frac{\partial P}{\partial z} + \frac{1}{v_g} \frac{\partial P}{\partial t} &= (g - \alpha_{int})P,\end{aligned}$$

and

$$\frac{\partial \phi}{\partial z} + \frac{1}{v_g} \frac{\partial \phi}{\partial t} = \frac{-i\alpha g}{2}.$$

CHAPTER 2

Single Optical Input Signal into a Bistable Semiconductor Optical Amplifier

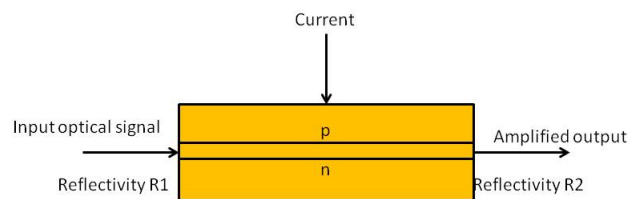


Figure 2.0.1: Schematic of a Fabry-Perot SOA

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A schematic of a Fabry-Perot SOA is given. It consists of p and n junction. A current applied across the junction creates electrons and holes causing an amplification of light entering the device. The SOA is surrounded by two reflective surfaces with reflectivity R1 and R2 producing an optical resonator [6].

A model for optical bistability in Fabry-Perot-Type semiconductor optical amplifiers (FP-SOAs) was considered in Reference [2]. We adapt this model in this chapter to be notationally consistent with the previous chapter. The rate equation for the carrier density N in the amplifier is [2]

$$\frac{dN}{dt} = \frac{J}{qd} - \frac{N}{\tau_c} - \frac{\Gamma a(N - N_T)}{hf} \frac{\sigma}{wd} |A|^2, \quad (2.0.1)$$

where N = carrier density [$\frac{1}{\text{cm}^3}$], J= current density [$\frac{\text{A}}{\text{cm}^2}$], q = electron charge [C], d = active layer thickness [μm], τ_c = carrier lifetime [ps], Γ = confinement factor [–], $\sigma|A|^2 = P$ = the optical power averaged over the length of the SOA, hf = photon energy [eV]. Just like as in Chapter 1, this expression can be simplified to

$$\frac{dN}{dt} = \frac{J}{qd} - \frac{N}{\tau_c} - \frac{(N - N_T)\bar{P}}{\tau_c},$$

where N_T is the carrier density at transparency.

2.1 Internal phase

The phase change ϕ experienced by an optical signal after traversing a single pass along the FP-SOA is given by

$$\begin{aligned}\phi &= \frac{2\pi}{\lambda_0} n L, \\ n &= n_0 + \frac{dn}{dN} (N - N_0) \Gamma, \\ \phi &= \frac{2\pi}{\lambda_0} n_0 L + \frac{2\pi}{\lambda_0} L \frac{dn}{dN} (N - N_0) \Gamma,\end{aligned}$$

where n is the refractive index, n_0 = small-signal refractive index and N_0 = carrier density when the input signal is absent.

$$\phi = \phi_0 + \frac{2\pi L (N - N_0)}{\lambda_0} \frac{dn}{dN} \Gamma, \quad (2.1.1)$$

$$\phi_0 = \frac{2\pi}{\lambda_0} n_0 L,$$

where ϕ_0 = small-signal change in phase, independent of the optical power [rad], λ_0 = free-space wavelength [μm], and L = amplifier length [μm]. Knowing

$$\begin{aligned}\alpha &= -\frac{4\pi}{\lambda_0} \frac{dn}{dN} \frac{1}{a}, \\ \phi &= \phi_0 - \frac{2\pi L (N - N_0) \Gamma}{\lambda_0} \frac{\alpha a \lambda_0}{4\pi} = \phi_0 - \frac{\alpha a \Gamma (N - N_0) L}{2}, \\ \phi &= \phi_0 - \frac{\alpha g L}{2}.\end{aligned}$$

2.2 Internal Phase Rate Equation

Eqn.(1.0.3) can be represented in dynamic version by taking the time derivative of the phase leading to

$$\frac{d\phi}{dt} = \frac{2\pi L}{\lambda_0} \left(\frac{dN}{dt} \right) \frac{dn}{dN} \Gamma.$$

The $\frac{dN}{dt}$ term is substituted from rate Eqn.(1.0.1)

$$\frac{d\phi}{dt} = \frac{2\pi L}{\lambda_0} \frac{dn}{dN} \left[\frac{J}{ed} - \frac{N}{\tau_c} - \frac{(N - N_T)\bar{P}}{\tau_c} \right] \Gamma.$$

Multiplying on either side with τ_c , we get

$$\tau_c \frac{d\phi}{dt} = \frac{2\pi L}{\lambda_0} \frac{dn}{dN} \left[\frac{\tau_c J}{ed} - N - (N - N_T)\bar{P} \right] \Gamma.$$

Simplifying by substituting $\frac{I\tau_c}{ed}$ as N_0 the small-signal carrier density ,

$$\begin{aligned} \tau_c \frac{d\phi}{dt} &= \frac{2\pi L}{\lambda_0} \frac{dn}{dN} \left[N_0 - N - (N - N_T)\bar{P} \right] \Gamma, \\ \frac{\lambda_0}{\Gamma 2\pi L} \frac{dn}{dN} \tau_c \frac{d\phi}{dt} &= N_0 - N - (N - N_T)\bar{P}, \\ \frac{\lambda_0}{\Gamma 2\pi L} \frac{dn}{dN} \tau_c \frac{d\phi}{dt} + (N - N_T)\bar{P} &= N_0 - N. \end{aligned}$$

Note that Eqn.(1.1.1) can be re-written as

$$\frac{\lambda_0}{\Gamma 2\pi} \frac{\phi - \phi_0}{\frac{\partial n}{\partial N} L} = (N - N_0). \quad (2.2.1)$$

Combining these two previous equations yields

$$\frac{\lambda_0}{\Gamma 2\pi} \frac{1}{\frac{dn}{dN}} \tau_c \frac{d\phi}{dt} \frac{1}{L} = \frac{(\phi_0 - \phi)\lambda_0}{\Gamma \frac{dn}{dN} L 2\pi} - (N - N_T)\bar{P},$$

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$$\tau_c \frac{d\phi}{dt} = (\phi_0 - \phi) - (N - N_T) \bar{P} \frac{dn}{dN} L \frac{2\pi}{\lambda_0} \Gamma.$$

Adding and subtracting with N_0 in the R.H.S, we get

$$\tau_c \frac{d\phi}{dt} = (\phi_0 - \phi) - [(N - N_0) + (N_0 - N_T)] \bar{P} \frac{dn}{dN} L \frac{2\pi}{\lambda_0} \Gamma.$$

Expanding the above equation yields

$$\tau_c \frac{d\phi}{dt} = (\phi_0 - \phi) - (N - N_0) \bar{P} \Gamma \frac{dn}{dN} \frac{2\pi}{\lambda_0} L - (N_0 - N_T) \bar{P} \frac{dn}{dN} \frac{2\pi}{\lambda_0} L \Gamma.$$

Substituting Eqn.(1.0.5) in the above equation , we get

$$\begin{aligned} \tau_c \frac{d\phi}{dt} &= (\phi_0 - \phi) - (\phi - \phi_0) \bar{P} - (N_0 - N_T) \frac{dn}{dN} \frac{2\pi}{\lambda_0} \bar{P} \Gamma. \\ \tau_c \frac{d\phi}{dt} &= (\phi_0 - \phi)(1 + \bar{P}) + (N_T - N_0) \frac{dn}{dN} \frac{2\pi}{\lambda_0} \bar{P} \Gamma. \end{aligned} \quad (2.2.2)$$

The phase rate equation can be expanded as

$$\tau_c \frac{d\phi}{dt} = (\phi_0 - \phi)(1 + \bar{P}) - \frac{2\pi L}{\lambda_0} (N_0 - N_T) \bar{P} \frac{dn}{dN} \Gamma. \quad (2.2.3)$$

We can re-write the phase equation in terms of linewidth enhancement factor α as

$$\alpha = - \frac{4\pi}{\lambda_0 a} \frac{dn}{dN}. \quad (2.2.4)$$

Rearranging the terms yields

$$\frac{-2\pi}{\lambda_0} \frac{dn}{dN} = \frac{\alpha a}{2}.$$

We define the small-signal gain g_0 to be

$$g_0 = \Gamma a (N_0 - N_T). \quad (2.2.5)$$

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Substituting Eqn.(1.0.8) and Eqn.(1.0.9) in the phase equation yields

$$\begin{aligned}\tau_c \frac{d\phi}{dt} &= (\phi_0 - \phi)(1 + \bar{P}) + \frac{\alpha a}{2} L(N_0 - N_T) \bar{P} \Gamma, \\ \tau_c \frac{d\phi}{dt} &= (\phi_0 - \phi)(1 + \bar{P}) + \frac{\alpha}{2} g_0 L \bar{P}.\end{aligned}\tag{2.2.6}$$

2.3 Steady state internal phase

Eqn.(1.2.6) is solved in steady state by assuming $\frac{d}{dt}=0$

$$\begin{aligned}0 &= (\phi_0 - \phi)(1 + \bar{P}) + \frac{\alpha}{2} g_0 L \bar{P}, \\ \frac{\alpha}{2} g_0 L \bar{P} &= (\phi - \phi_0)(1 + \bar{P}), \\ \phi - \phi_0 &= \alpha \frac{g_0 L}{2} \left(\frac{\bar{P}}{1 + \bar{P}} \right), \\ \phi &= \phi_0 + \alpha \frac{g_0 L}{2} \left(\frac{\bar{P}}{1 + \bar{P}} \right).\end{aligned}\tag{2.3.1}$$

Note that in steady state ,

$$\Delta = g_0 - g = g_0 - \frac{g}{1 + \bar{P}} = \frac{(1 + \bar{P})g_0 - g_0}{1 + \bar{P}} = \frac{g_0 \bar{P}}{1 + \bar{P}}.$$

Thus,

$$\begin{aligned}\phi &= \phi_0 + \alpha \frac{\Delta L}{2}, \\ \phi - \phi_0 &= \frac{\alpha \Delta L}{2} = \frac{\alpha g_0 L \bar{P}}{2(1 + \bar{P})}.\end{aligned}\tag{2.3.2}$$

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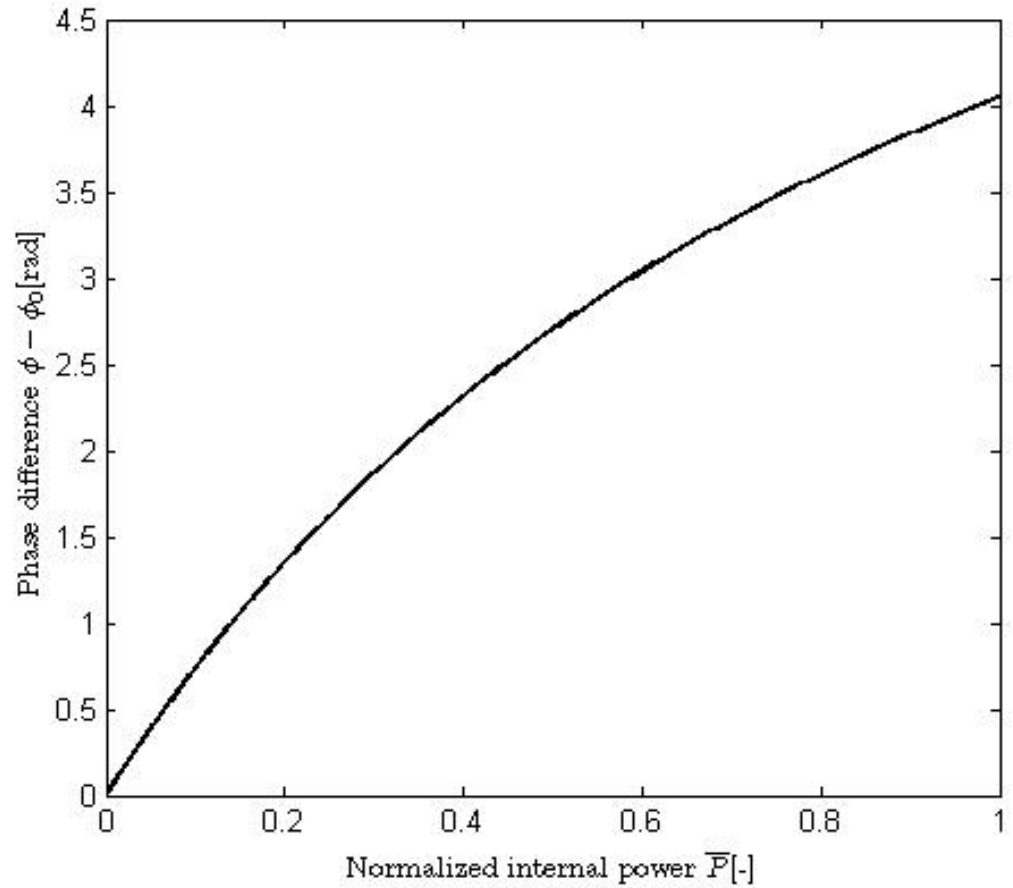


Figure 2.3.1: Phase difference versus normalized internal power

Fig (2.3.1) [2] is plotted using \bar{P} as given in Eqn. (2.5.1) with $\phi_0 = -\pi/4$ [rad] and α of 5 [-] and g_0L of 3.24 [-] with $R_1=R_2=0.3$ [-]. The loss factor $\alpha_{int}L$ is 0.5 [-] and confinement factor Γ is 0.5 [-]. Unless specified otherwise, these parameter values are used for all graphs in this chapter.

From the figure we can see that $\phi - \phi_0$ has a linear and then saturated increase with increase in internal power. Since the values are normalized, the value of $\bar{P}=1$

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is when the optical power matches the saturated power P_{sat} .

2.4 Steady state gain coefficient

In the steady state equation for phase Eqn.(1.3.1)

$$\phi = \phi_0 + \frac{2\pi L}{\lambda_0}(N - N_0)\frac{dn}{dN}\Gamma,$$

$$\phi - \phi_0 = \frac{2\pi L}{\lambda_0}(N - N_0)\frac{dn}{dN}\Gamma.$$

Simplifying the equation using

$$\frac{-2\pi}{\lambda_0} \frac{\partial n}{\partial N} = \frac{\alpha a}{2},$$

we get

$$\phi - \phi_0 = -\frac{\alpha a L}{2}(N - N_0)\Gamma.$$

$$\frac{2(\phi_0 - \phi)}{\alpha L} = a(N - N_0)\Gamma,$$

$$\Gamma a N = \Gamma a N_0 + \frac{2}{\alpha L}(\phi_0 - \phi).$$

Subtracting either side of the equation by $-aN_T\Gamma$

$$\Gamma a N - \Gamma a N_T = \Gamma a N_0 - \Gamma a N_T + \frac{2}{\alpha L}(\phi_0 - \phi),$$

$$\Gamma a (N - N_T) = \Gamma a (N_0 - N_T) + \frac{2}{\alpha L}(\phi_0 - \phi),$$

$$g = g_0 + \frac{2}{\alpha L}(\phi_0 - \phi). \quad (2.4.1)$$

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The net modal gain g^- is given by

$$g^- = g - \alpha_{int}. \quad (2.4.2)$$

where α_{int} = effective loss coefficient $[\frac{1}{\text{cm}}]$. The relation between the net gain g^- and small-signal gain g_0 is given by combining Eqns.(1.4.1) and (1.4.2)

$$g^- = g_0 + (\phi_0 - \phi) \frac{2}{\alpha L} - \alpha_{int}. \quad (2.4.3)$$

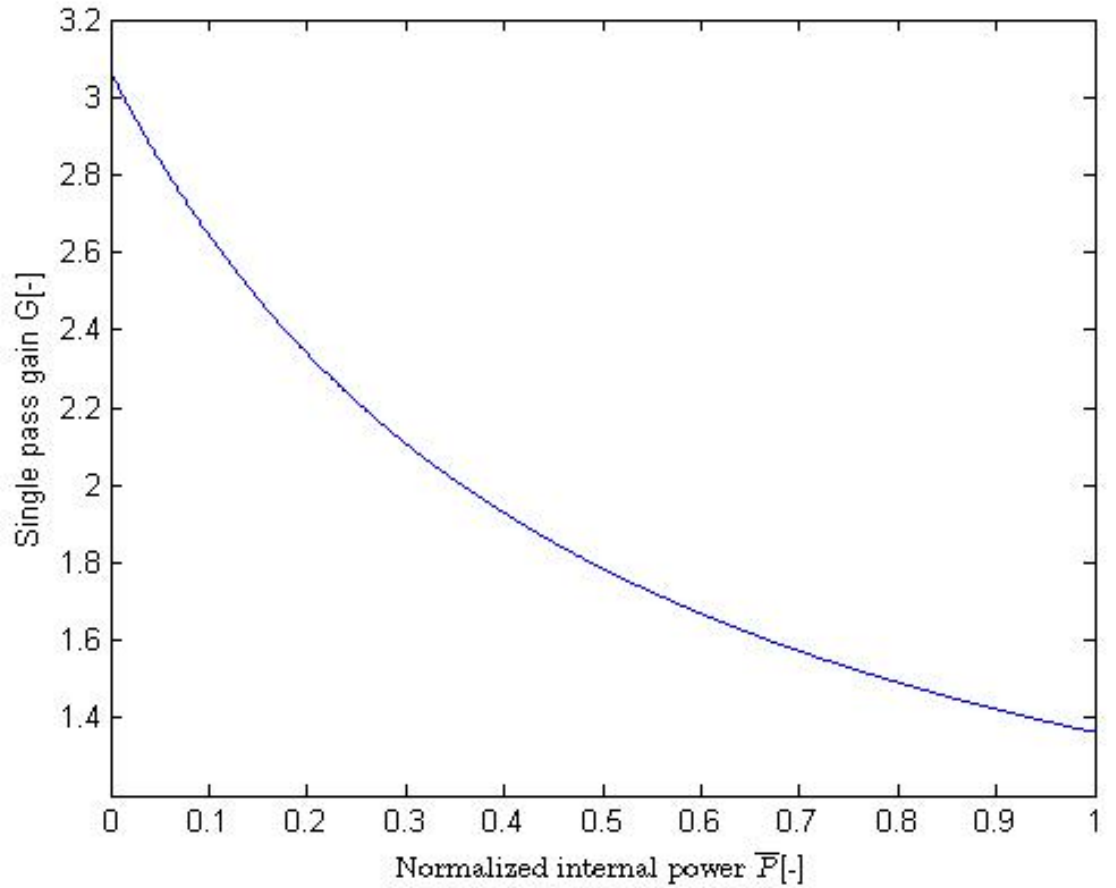


Figure 2.4.1: Single pass gain versus normalized internal power

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Fig (2.4.1) is plotted using equation for \bar{P} as given in Eqn. (1.4.3). From the figure we can see that the single pass gain has an exponential decrease with increase in internal power.

2.5 Output and Input power

The output power $\overline{P_{out}}$ and \bar{P} are related as [2]

$$\bar{P} = \frac{(1 + R_2 e^{gL})(e^{gL} - 1)\overline{P_{out}}}{(1 - R_2)e^{gL}gL} = \overline{P_{out}}Y. \quad (2.5.1)$$

where R_2 is the reflectivity of the second mirror. The input power $\overline{P_{in}}$ and \bar{P} are related as

$$\overline{P_{out}} = \overline{P_{in}} \frac{(1 - R_1)(1 - R_2)e^{gL}}{(1 - (R_1 R_2)^{1/2} e^{gL})^2 + 4(R_1 R_2)^{1/2} e^{gL} \sin^2 \phi} = \overline{P_{in}}X. \quad (2.5.2)$$

where R_1 is the reflectivity of the first mirror. Since \bar{P} is the known, $\overline{P_{out}}$ and $\overline{P_{in}}$ are readily found as

$$\begin{aligned} \overline{P_{out}} &= \frac{\bar{P}}{Y}, \\ \overline{P_{in}} &= \frac{\overline{P_{out}}}{X} = \frac{\bar{P}}{XY}. \end{aligned}$$

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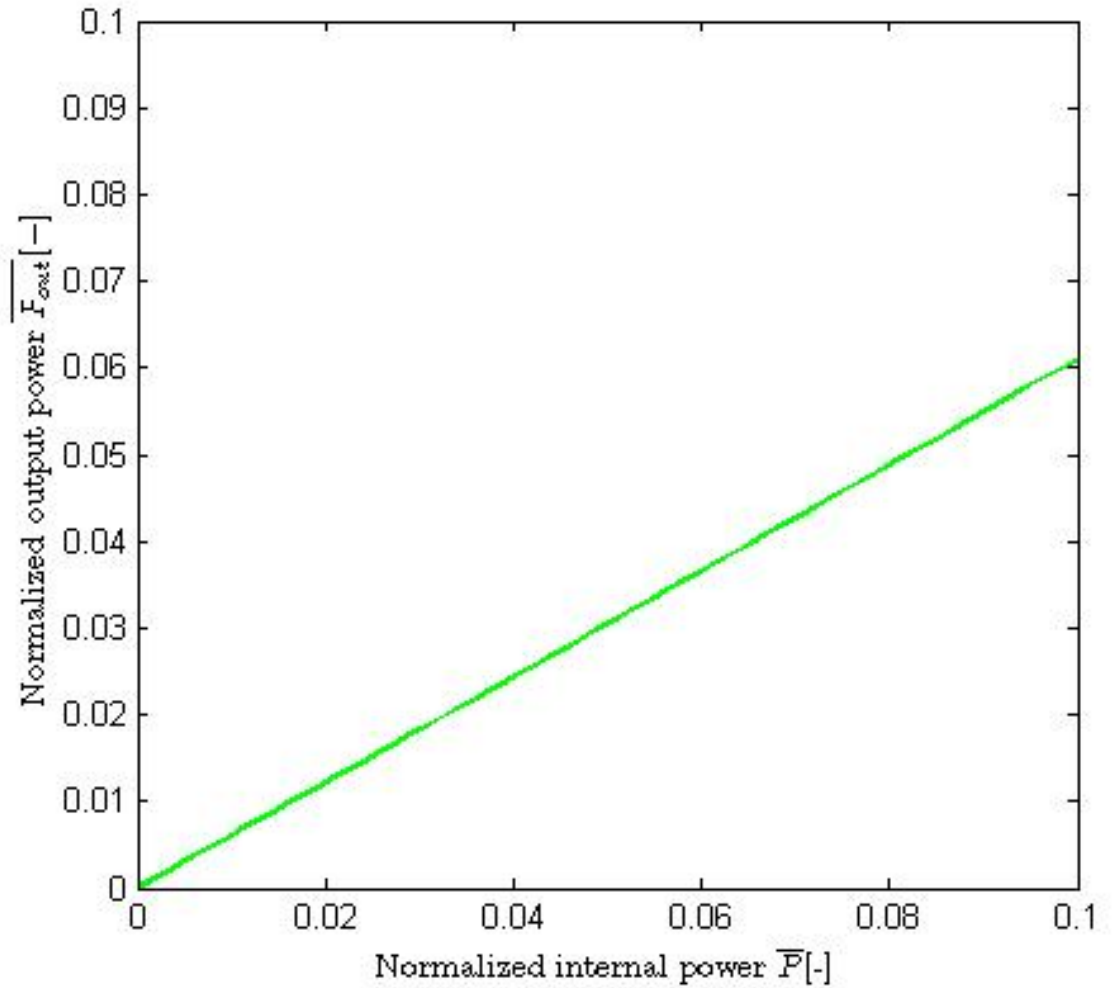


Figure 2.5.1: Normalized output Power versus normalized internal power

Fig (2.5.2) is plotted using equation for \bar{P}_{out} as given in Eqn. (2.5.1) and manipulating that equation to get the normalized average power. From the figure we can see internal power increase increases the output power. The internal power and output power are directly proportional. The SOA increases the input power producing an amplified output.

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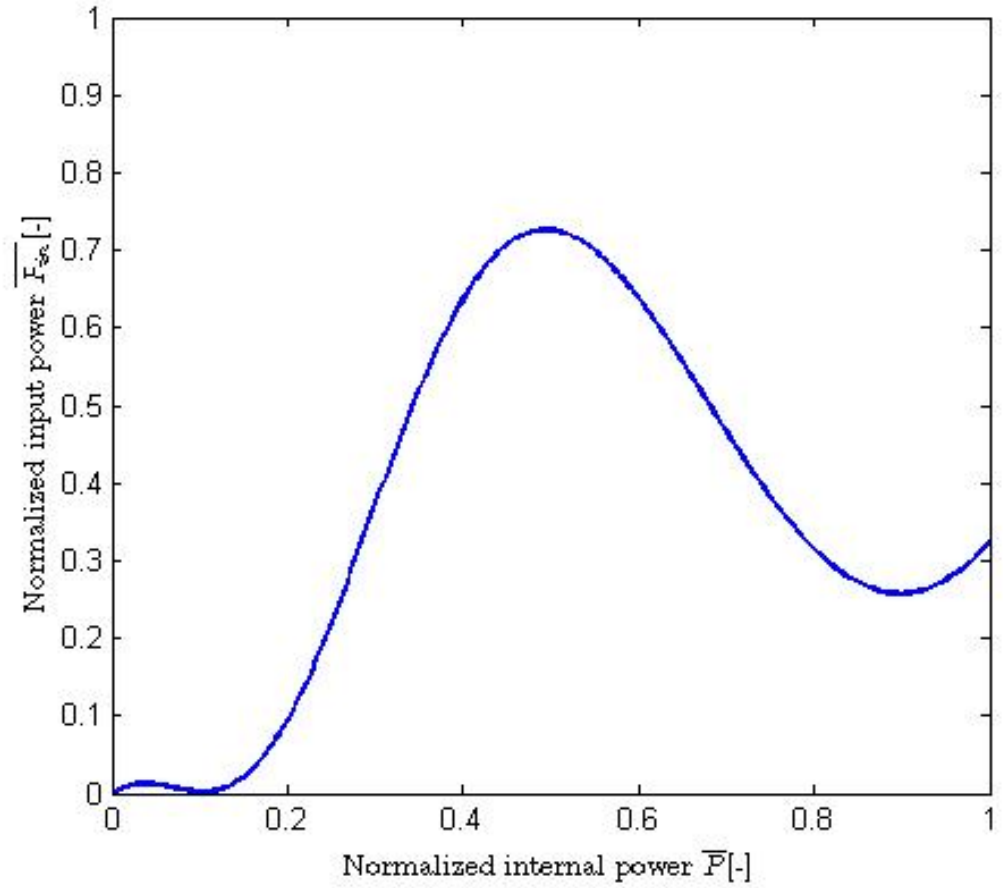


Figure 2.5.2: Normalized input power versus normalized internal power

Fig (2.5.1) is plotted using equation for \bar{P} as given in Eqn. (2.5.1) and Eqn.(2.5.2). From the figure we can see that the normalized internal input power varies non-monotonically with normalized internal power.

2.6 In terms of $\overline{P_{in}}$

Note that we know $\overline{P_{in}}$, we can recreate the previous graphs as a function of $\overline{P_{in}}$:

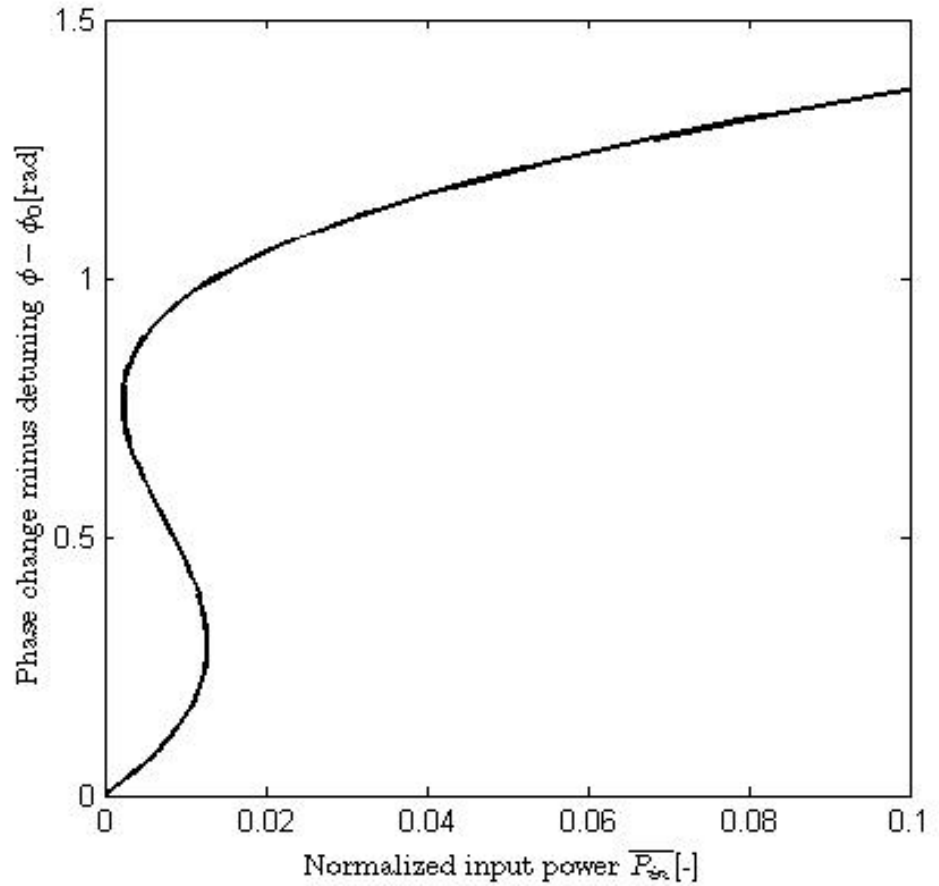


Figure 2.6.1: Phase change minus detuning versus normalized input power

Fig. (2.6.1) use the same parameter values as the previous figures. The graph shows a hysteresis curve with switching to high phase occurring at a value of normalized power of 0.013 and 0.002.

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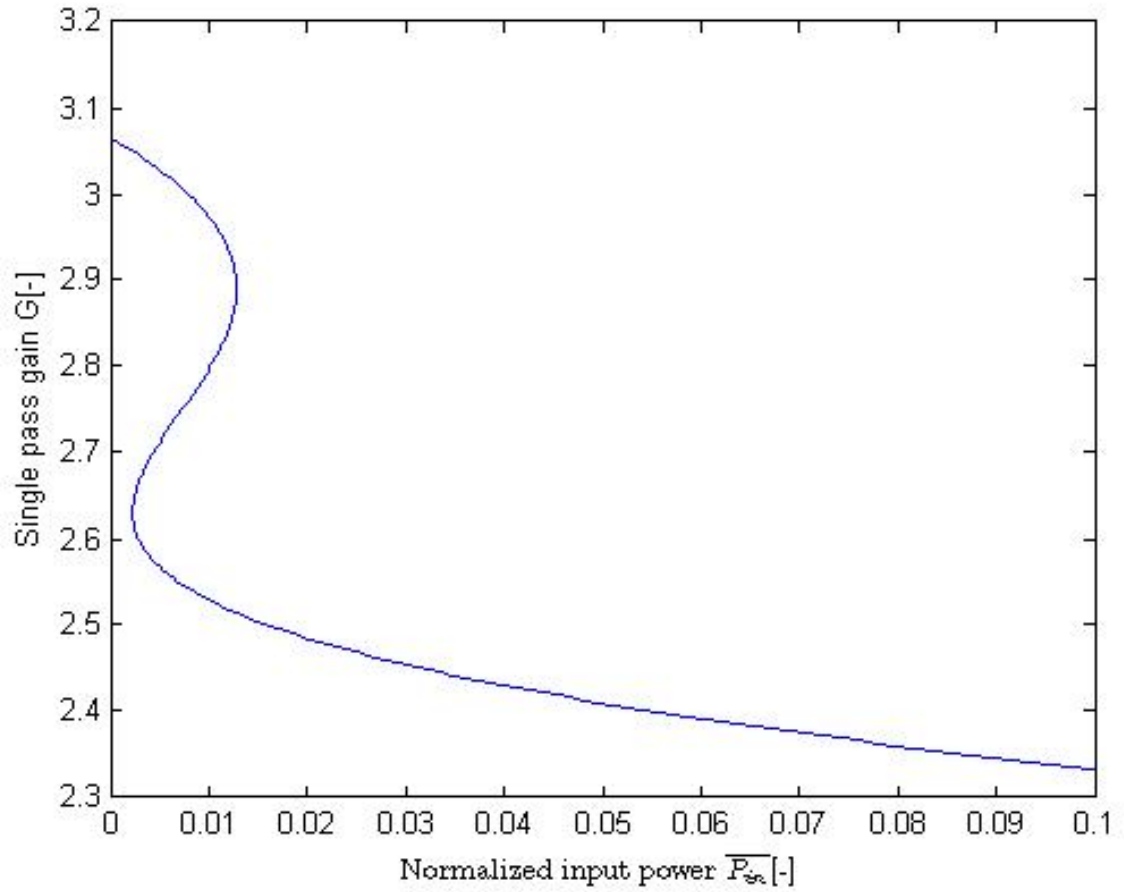


Figure 2.6.2: Single pass gain versus normalized input power

Fig. (2.6.2) uses the same parameter value as the previous figures. The graph shows a hysteresis curve with switching occurring at a value of normalized power of 0.013 and 0.002. The single pass gain is high for a normalized power value of 0.

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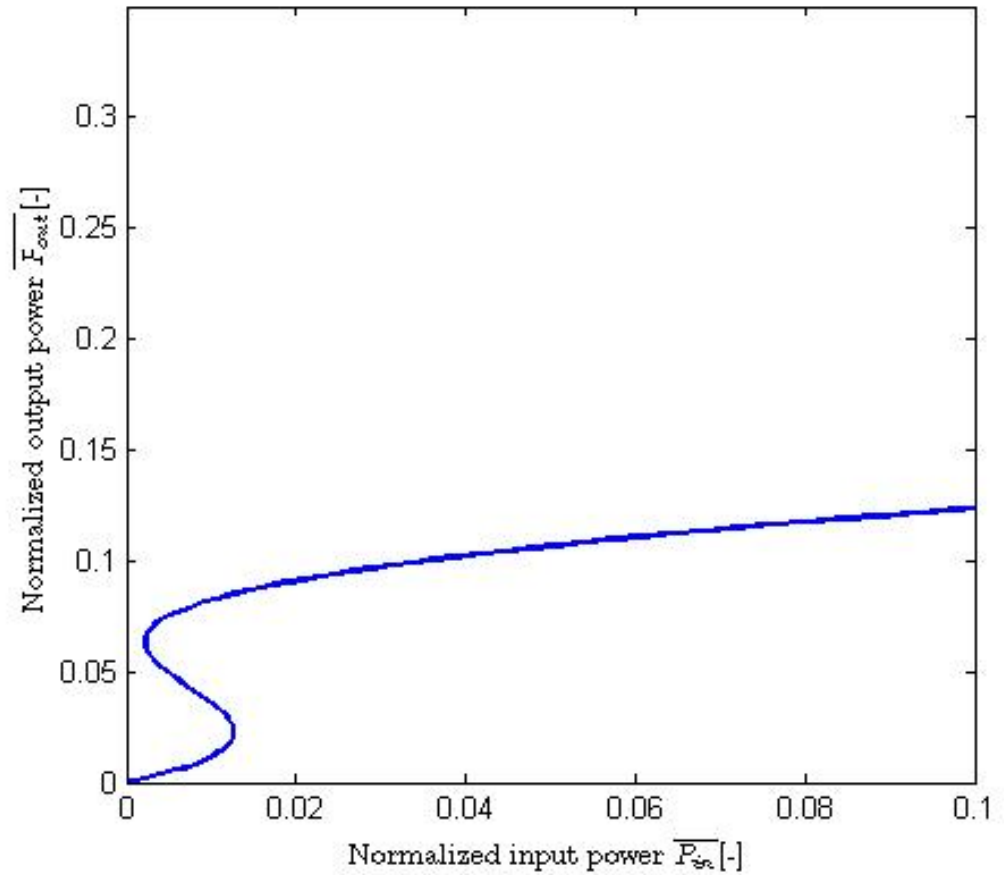


Figure 2.6.3: Normalized output power versus normalized input power

Fig. (2.6.3) uses same parameter values as the previous figures. The graph shows a hysteresis curve with switching occurring at a value of normalized power of 0.013 and 0.002. The output power is initially 0 for 0 input power. As the power increases the output also increases and following a hysteresis curve.

2.7 Output phase

The relation between output phase ψ_{out} and input phase ψ_{in} is given by [2]

$$\psi_{out} = \psi_{in} - \phi - \tan^{-1}\left(\frac{(R_1 R_2)^{1/2} e^{gL} \sin 2\phi}{1 - (R_1 R_2)^{1/2} e^{gL} \cos 2\phi}\right). \quad (2.7.1)$$

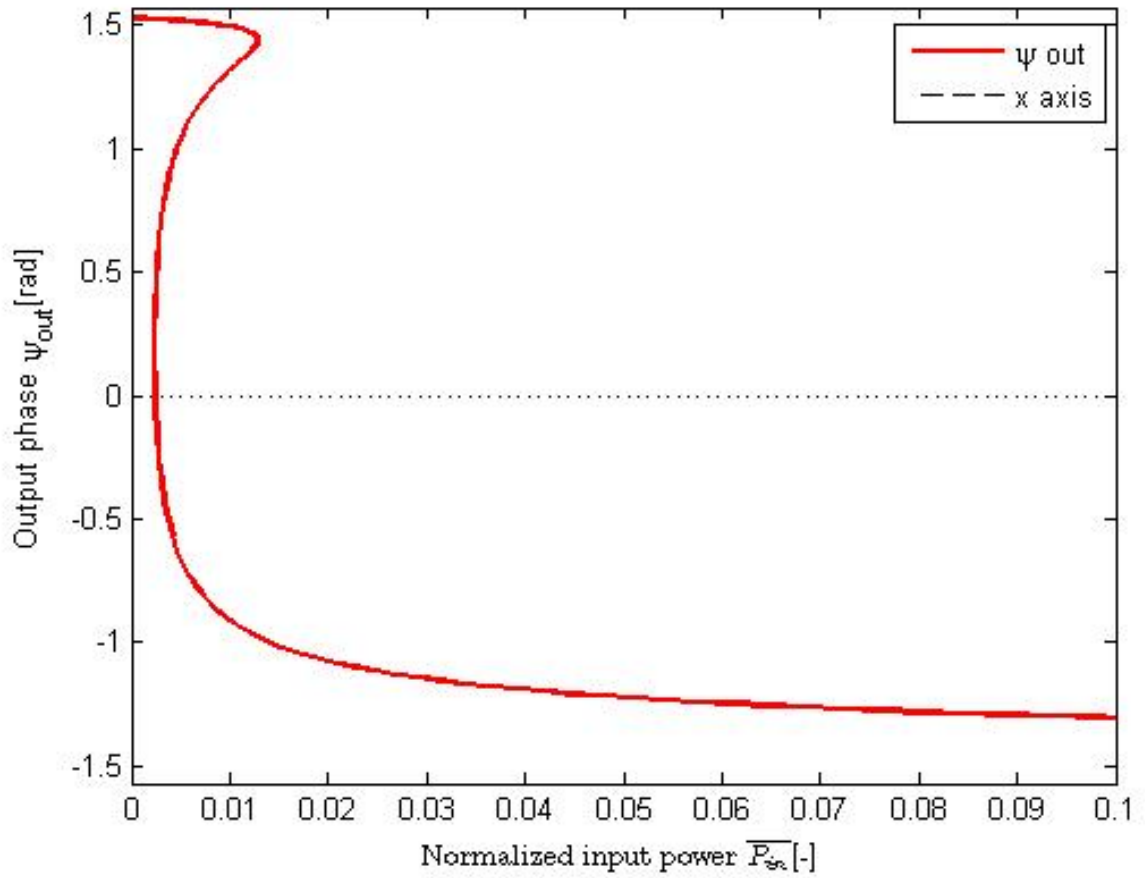


Figure 2.7.1: Output phase versus normalized input power

Fig. (2.7.4) uses same parameter value as the previous figures. The graph shows hysteresis curve with switching to high peaks occurring at a value of normalized

power of 0.013 and 0.002.

2.8 Gain coefficient rate equation

As derived in the previous section, the net gain is given by

$$g^- = g - \alpha_{int}.$$

The rate equation for the net gain is given by

$$\begin{aligned} \frac{dg^-}{dt} &= \frac{dg}{dt} \\ \frac{dg^-}{dt} &= \frac{g_0 - g(1 + \bar{P})}{\tau_c}, \\ g^- &= g - \alpha_{int}, \\ g^- + \alpha_{int} &= g, \\ \tau_c \frac{dg^-}{dt} &= g_0 - (g + \alpha_{int})(1 + \bar{P}), \\ \tau_c \frac{dg^-}{dt} &= g_0 - g(1 + \bar{P}) - \alpha_{int}(1 + \bar{P}). \end{aligned} \tag{2.8.1}$$

2.9 Phase rate equation revisited

The phase rate equation is given by [2]

$$\phi = \phi_0 + \frac{2\pi L}{\lambda_0} (N - N_0) \frac{\partial n}{\partial N} \Gamma,$$

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$$\begin{aligned}
\alpha &= -\frac{4\pi}{\lambda_0 a} \frac{\partial n}{\partial N}, \\
\frac{-2\pi}{\lambda_0} \frac{\partial n}{\partial N} &= \frac{\alpha a}{2}, \\
\phi &= \phi_0 - \frac{\alpha a}{2} (N - N_0) L \Gamma, \\
\phi &= \phi_0 - \frac{\alpha a}{2} (N - N_T - N_0 + N_T) L \Gamma, \\
\phi &= \phi_0 - \frac{\alpha a}{2} (N - N_T) \Gamma + \frac{\alpha a}{2} (N_0 - N_T) L \Gamma, \\
\phi &= \phi_0 - \frac{\alpha g}{2} + \frac{\alpha g_0 L}{2}, \\
\phi &= \phi_0 + \alpha \frac{(g_0 - g)}{2} L. \tag{2.9.1}
\end{aligned}$$

Differentiating the phase equation with respect to time, we get

$$\begin{aligned}
\frac{d\phi}{dt} &= -\frac{\alpha L}{2} \frac{dg}{dt}, \\
\frac{d\phi}{dt} &= -\frac{\alpha L}{2} \left(\frac{g_0}{\tau_c} - \frac{g}{\tau_c} (1 + \bar{P}) \right), \\
\tau_c \frac{d\phi}{dt} &= -\frac{\alpha L}{2} (g_0 - g - g \bar{P}), \\
\tau_c \frac{d\phi}{dt} &= -\frac{g_0 \alpha L}{2} + \frac{g \alpha L}{2} + \frac{g \alpha L}{2} \bar{P}, \\
\tau_c \frac{d\phi}{dt} &= -(\phi - \phi_0) + \frac{g \alpha L}{2} \bar{P}, \\
\phi &= \phi_0 + \frac{\alpha g_0 L}{2} - \frac{\alpha g L}{2}, \\
\frac{\alpha g L}{2} &= (\phi_0 - \phi) + \frac{\alpha g_0 L}{2}, \\
\tau_c \frac{d\phi}{dt} &= -(\phi - \phi_0) + ((\phi_0 - \phi) + \frac{\alpha g_0 L}{2}) \bar{P}, \\
\tau_c \frac{d\phi}{dt} &= (\phi_0 - \phi) + (\phi_0 - \phi) \bar{P} + \frac{\alpha g_0 L}{2} \bar{P},
\end{aligned}$$

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$$\tau_c \frac{d\phi}{dt} = (\phi_0 - \phi)(1 + \bar{P}) + \frac{\alpha g_0 L}{2} \bar{P}. \quad (2.9.2)$$

This expression matches the one derived earlier.

CHAPTER 3

Single Optical Input Signal: Simpler Model

The previous chapter studied optical bistability of a single optical signal injected into an FP-SOA by using the average internal optical power, which was distinguished from both the input power and the output power. In the literature there was a simple model introduced that used output power as an approximation to the internal power [8]. This chapter uses this simpler model to study optical bistability of the FP SOA. We again use the technique of parameterization to determine various quantities like gain, phase, and power. At the end of the chapter, we compare the results of the simple model and the internal power model of the previous chapter.

3.1 Basic equations

The basic equations used in this chapter are given below. These equations are then modified as per the parameterization requirement to derive various solutions [8].

$$P_{out} = \frac{T^2 G}{[(1 - GR)^2 + 4GR \sin^2 \phi]} P_{in}, \quad (3.1.1)$$

$$G = \exp[(g - \alpha_{int})L], \quad (3.1.2)$$

$$g = \frac{g_0}{1 + \bar{P}}, \quad (3.1.3)$$

$$\phi = \phi_0 + \frac{\alpha}{2} g_0 L \frac{\bar{P}}{1 + \bar{P}}, \quad (3.1.4)$$

$$\overline{P_{out}} = \bar{P}. \quad (3.1.5)$$

where $\overline{P_{out}} = \frac{P_{out}}{P_{sat}}$, $\bar{P} = \frac{P}{P_{sat}}$, R is the reflectivity of the mirror, T=1-R is the transmittance, α_{int} is the loss coefficient, ϕ is the single path phase change, ϕ_0 is the initial detuning, and L is the length of the cavity. The gain co-efficient g is derived from the carrier density rate equation. The g and ϕ , G and $\overline{P_{out}}$ equations are familiar from Chapter 2. This set of equations differs from those of chapter 2 by the equation for output power in terms of internal power \bar{P} .

3.2 Output optical power parameterization

In this section, $\overline{P_{out}} = \frac{P_{out}}{P_{sat}}$ is parameterized, which allows equations for gain coefficient g and phase ϕ to be solved. The result obtained is used to calculate the optical

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gain G . Then the solution for normalized input power P_{in} is found. The results obtained are used to graphically plot the normalized output power vs normalized input power.

For the calculations, the same parameters are used as in chapter 2. This would give us better understanding and comparison of the two techniques. The values used are reflectivity of 0.3, confinement factor of 0.5, loss coefficient of 0.5, input phase of 0, phase detuning value of $-\pi/4$ and a single pass unsaturated gain of 3.24 [-]. The same parameters are used throughout the chapter for further calculations.

3.2.1 Gain coefficient and single-pass phase

Since we define the value for $\overline{P_{out}}$ and small signal gain g_0 , we can use that to determine the value of g and ϕ from Eqns.(3.1.3) and (3.1.4).

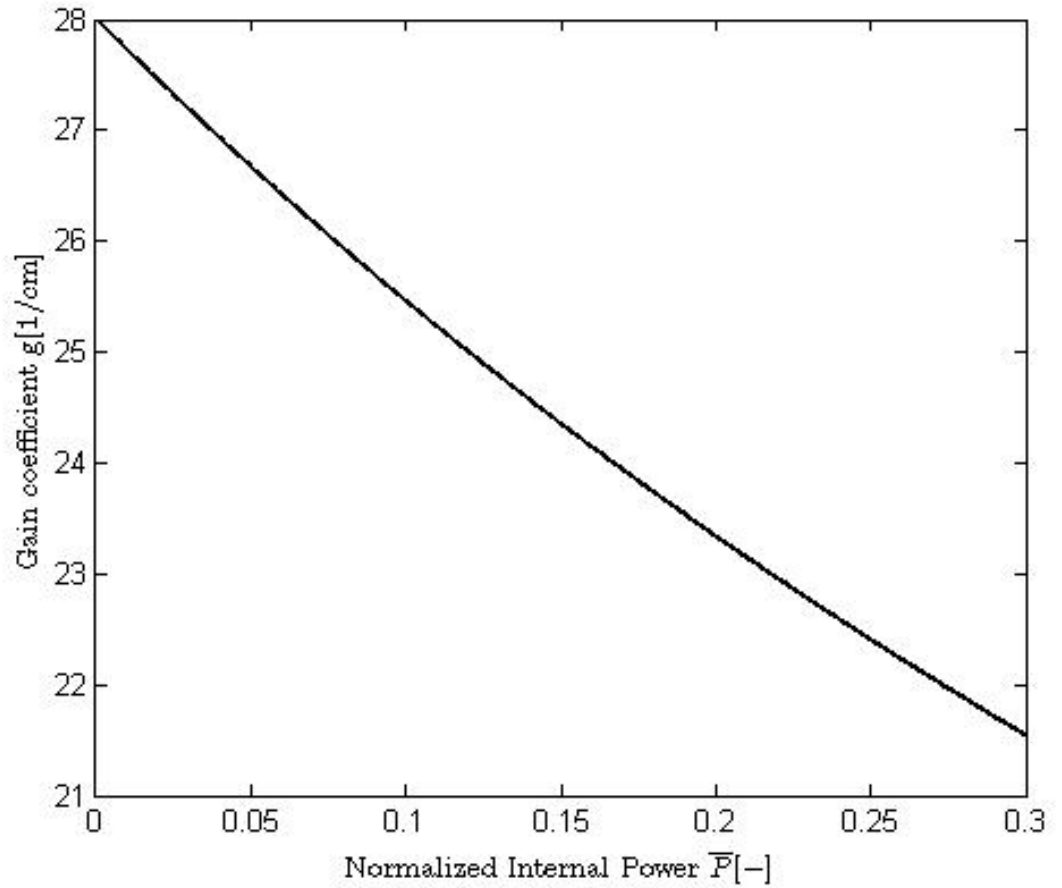


Figure 3.2.1: Gain coefficient vs Normalized internal power

The gain coefficient exponentially decreases with the increase in the internal power. The value is maximum for lower power and decreases as the power increases.

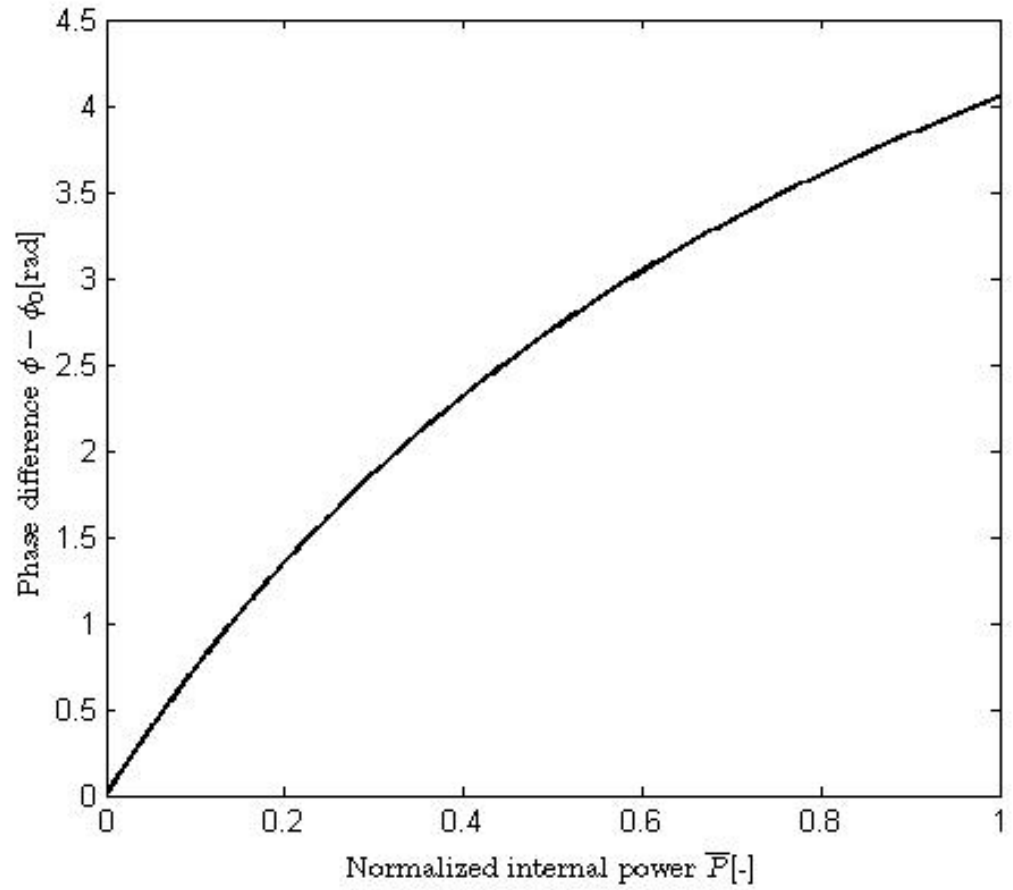


Figure 3.2.2: Phase difference vs Normalized internal power

The phase difference increases exponentially with the increase in the internal power. The phase is lower for small input power and increases as the power increases.

3.2.2 Gain

Since we have calculated the value for g , we can readily find the value for optical gain G . The optical gain varies exponentially along the length of the semiconductor L and the expression is given by Eqn.(3.1.2).

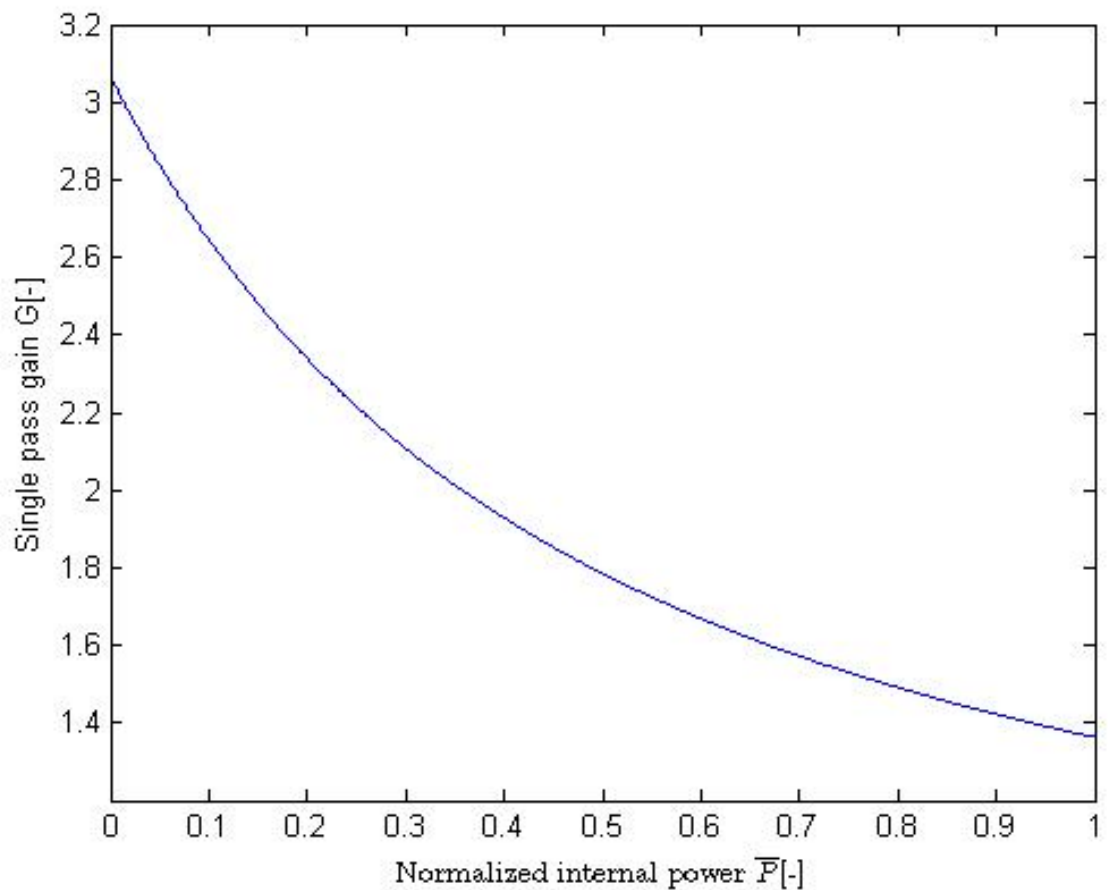


Figure 3.2.3: Single pass gain vs Normalized internal power

The single pass gain follows the same graph as it is an exponential function of the gain coefficient. The single pass gain also exponentially decreases with the

increase in the internal power.

3.2.3 Normalized input and output power

In the simplified model of this chapter, the output power $\overline{P_{out}}$ is simply equal to the internal power \overline{P} . Eqn.(3.1.1) can be re-written in the normalized form by dividing the input and output power terms by P_{sat} terms yielding [2]

$$\frac{P_{out}}{P_{sat}} = \frac{P_{in}}{P_{sat}} \frac{G(1-R)^2}{(1-GR)^2 + 4GR \sin^2 \phi}.$$

The normalized terms can be written in a simplified form as $\overline{P_{in}}$ and $\overline{P_{out}}$ as

$$\overline{P_{out}} = \overline{P_{in}} \frac{G(1-R)^2}{(1-GR)^2 + 4GR \sin^2 \phi}.$$

From this equation, the solution for normalized input power is determined,

$$\overline{P_{in}} = \overline{P_{out}} \frac{(1-GR)^2 + 4GR \sin^2 \phi}{G(1-R)^2}. \quad (3.2.1)$$

Alternatively, η can be defined as a part of P_{in} .

Note that we have calculated the input power in terms of the output power and now we can graph all quantities (previously graphed in terms of the output power in terms of input power).

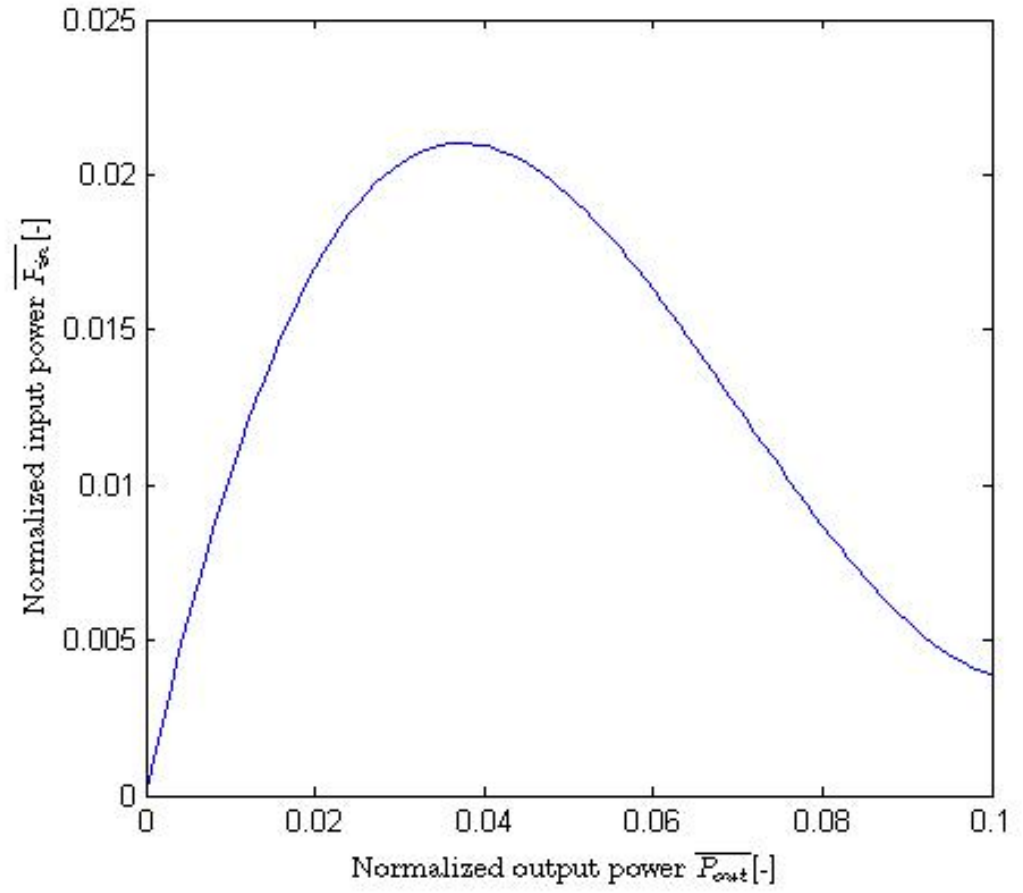


Figure 3.2.4: Normalized input power vs Normalized output power

The input power increases with the increase in the output power.

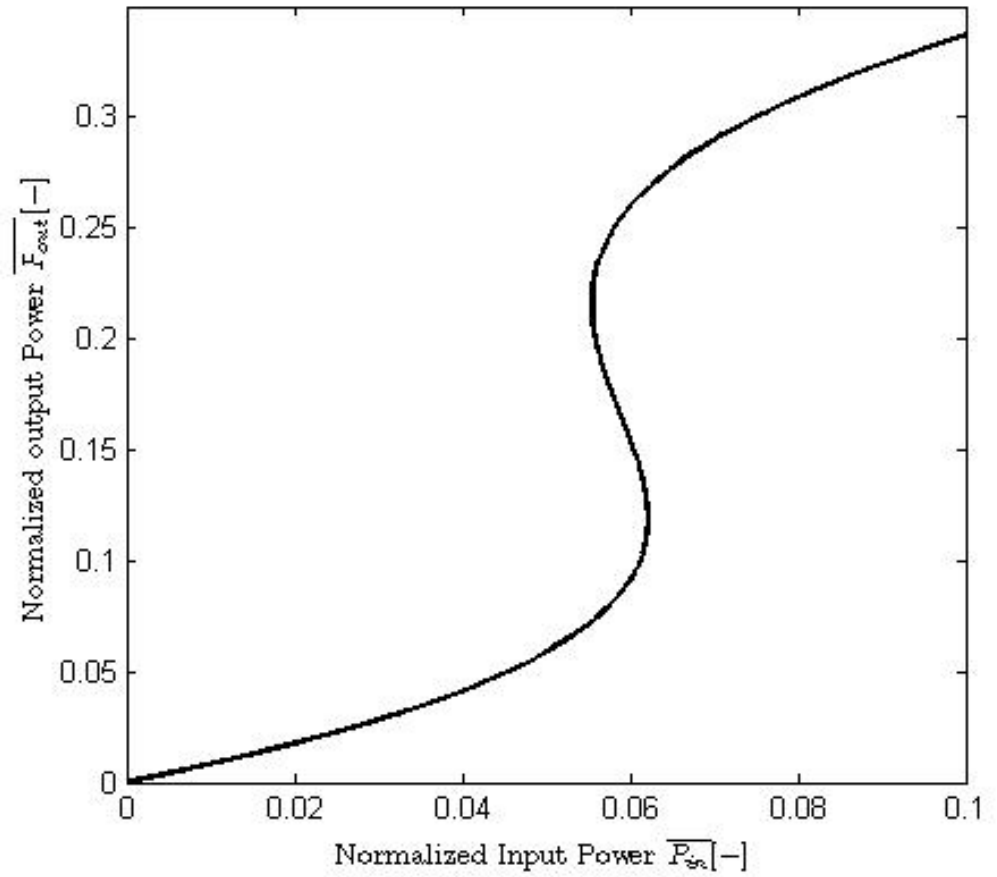


Figure 3.2.5: Normalized output power vs Normalized input power

The output power increases with the increase in the input power.

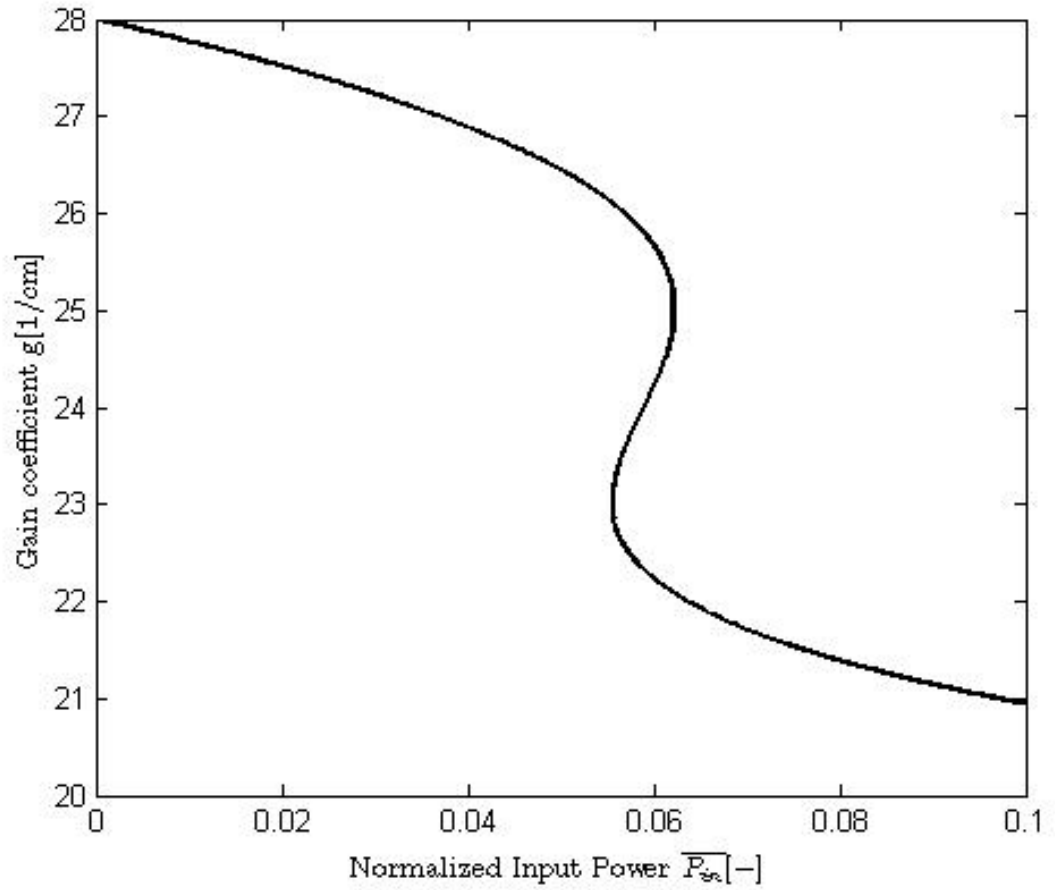


Figure 3.2.6: Gain coefficient vs Normalized input power

The gain coefficient decreases with the increase in the input power. The value is maximum for lower power and decreases as the power increases.

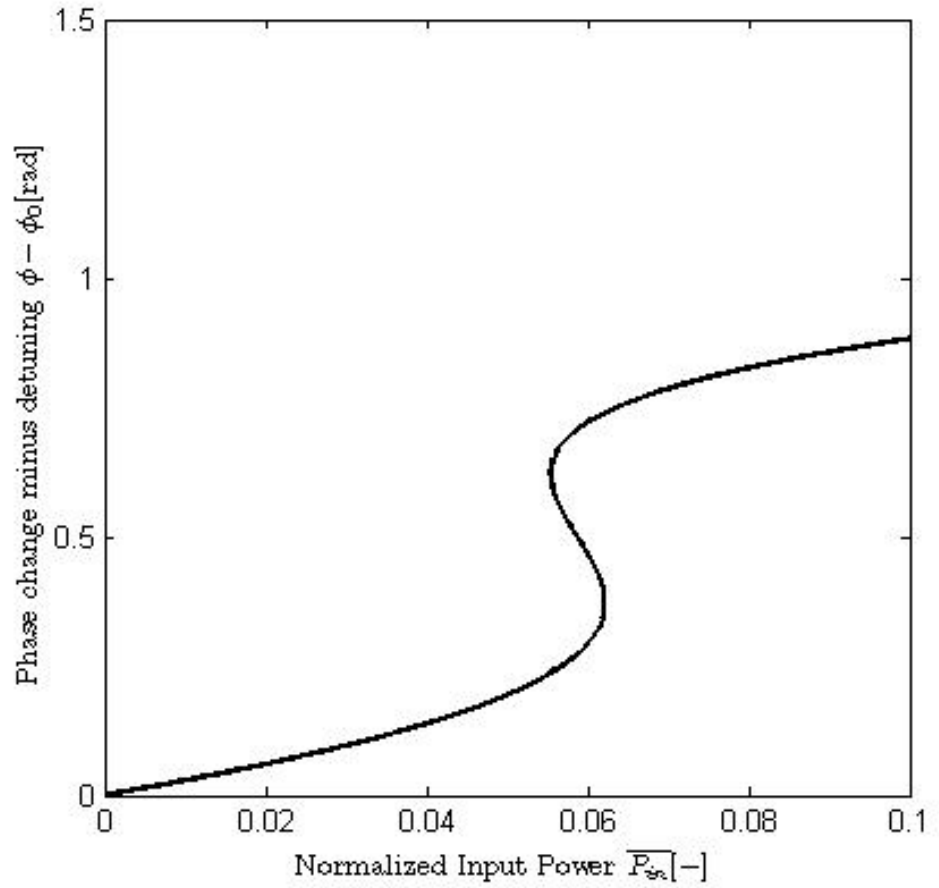


Figure 3.2.7: Phase difference vs Normalized input power

The phase difference increases with the increase in the internal power. The phase is lower for small input power and increases as the power increases.

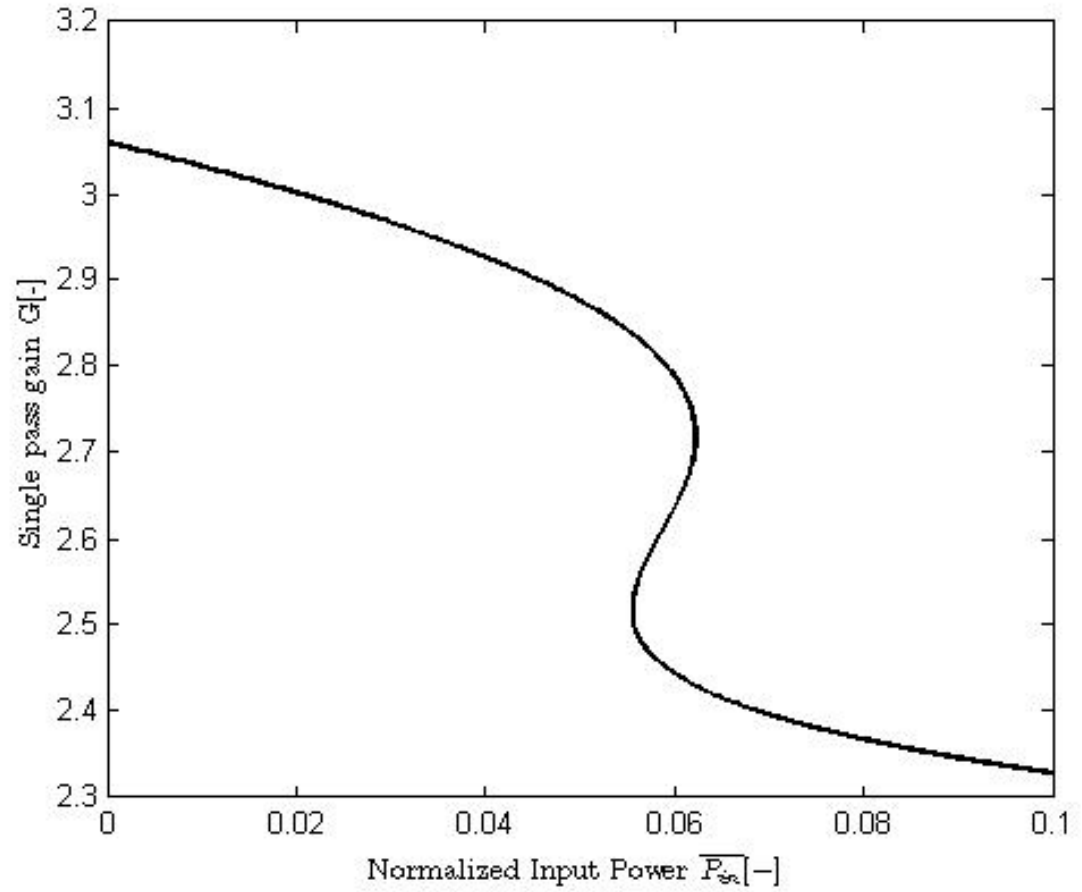


Figure 3.2.8: Single pass gain vs Normalized input power

The single pass gain follows the same graph as it is an exponential function of the gain coefficient. The single pass gain also decreases with the increase in the input power.

3.3 Output phase

The relation between output phase ψ_{out} and input phase ψ_{in} is given by [2]

$$\psi_{out} = \psi_{in} - \phi - \tan^{-1}\left(\frac{(R_1 R_2)^{1/2} G \sin 2\phi}{1 - (R_1 R_2)^{1/2} G \cos 2\phi}\right). \quad (3.3.1)$$

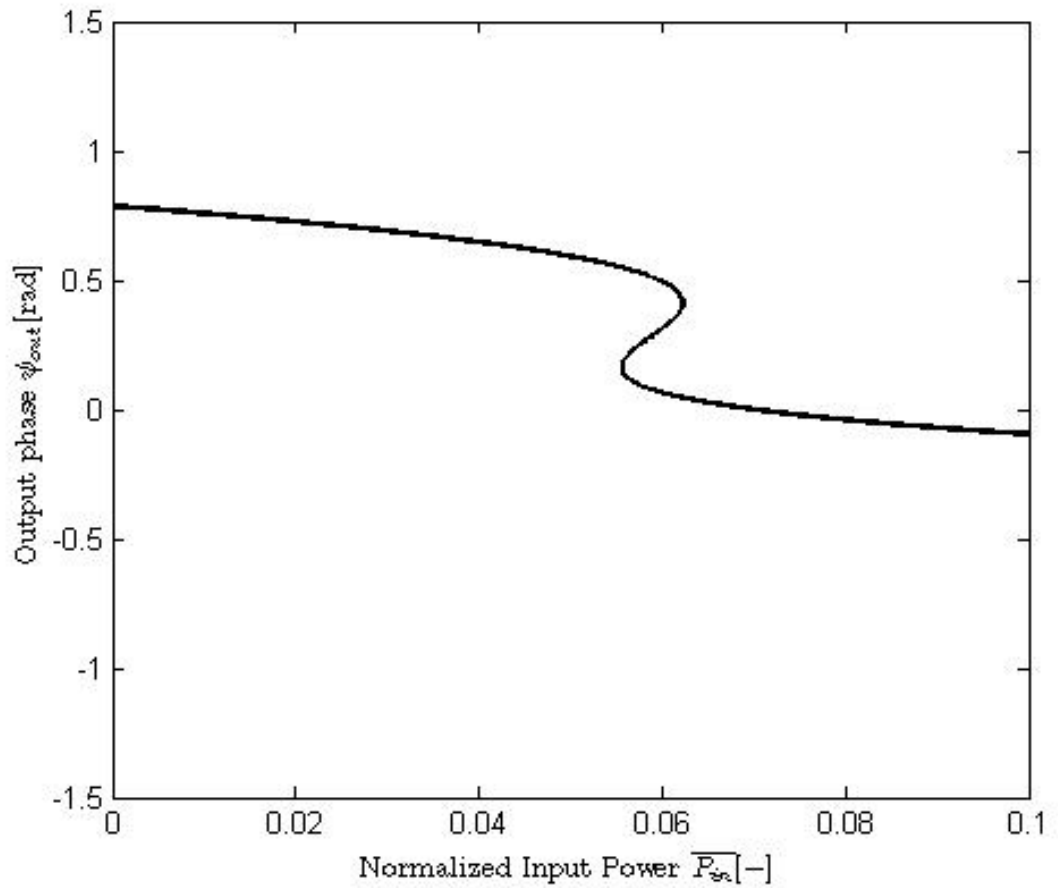


Figure 3.3.1: Output phase versus normalized input power

The output phase decreases with the increase in the input power.

3.4 Comparison of Models

As explained at the beginning of this chapter, this chapter uses a simpler model than the Chapter 2. The core difference in the model is that in the simpler model the output power equals the internal power. The difference is shown in the figures below. In other words, the simpler model (Chapter 3) uses

$$P''_{out} = P,$$

whereas the more involved approach (Chapter 2) uses

$$P'_{out} = P \frac{(1 - R_2)GgL}{(1 + R_2G)(G - 1)} = \frac{P}{X}.$$

Note that

$$\frac{P''_{out}}{P'_{out}} = \frac{P}{\frac{P}{X}} = X,$$

where

$$X = \frac{(1 + R_2G)(G - 1)}{(1 - R_2)GgL}.$$

Here is a plot of X vs normalized internal power \bar{P} . Since $X > 1$, the output power of the simpler approach is greater than that of the more involved approach.

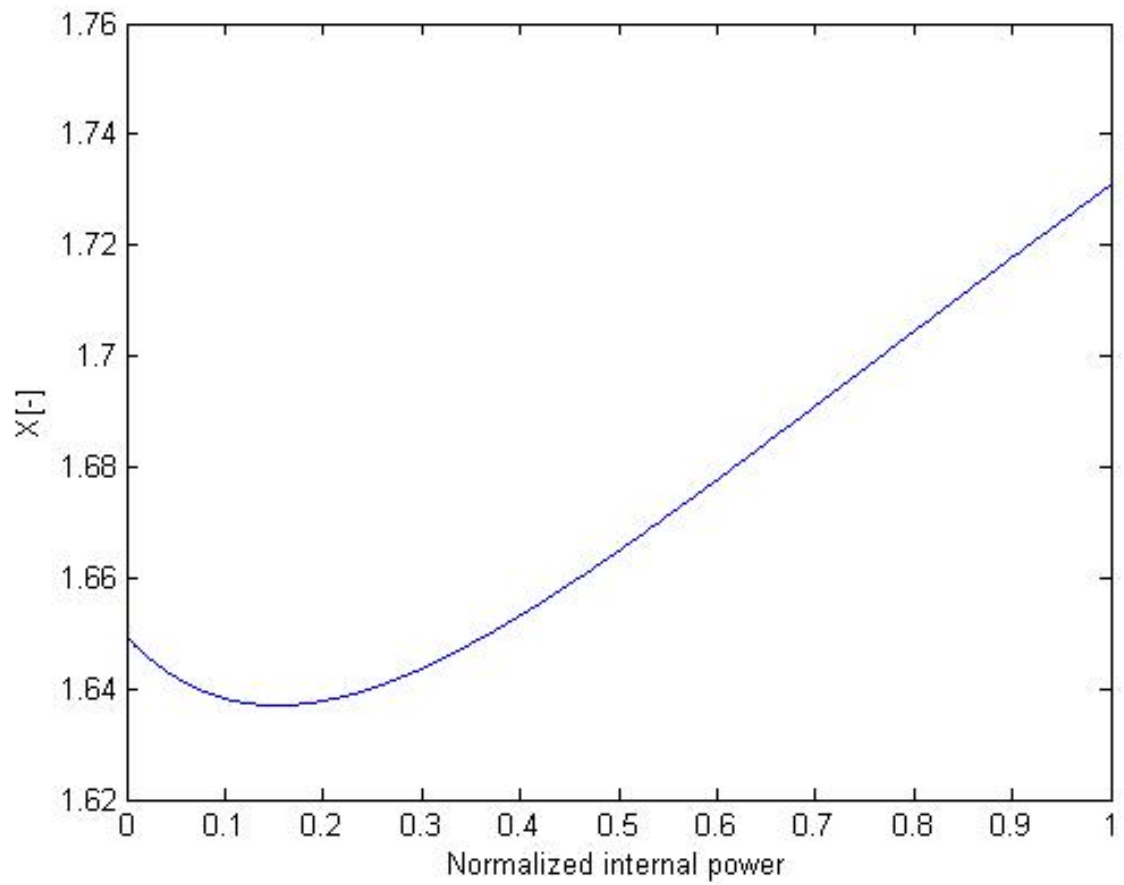


Figure 3.4.1: X vs normalized internal power

The X terms decreases with the increase in the internal power.

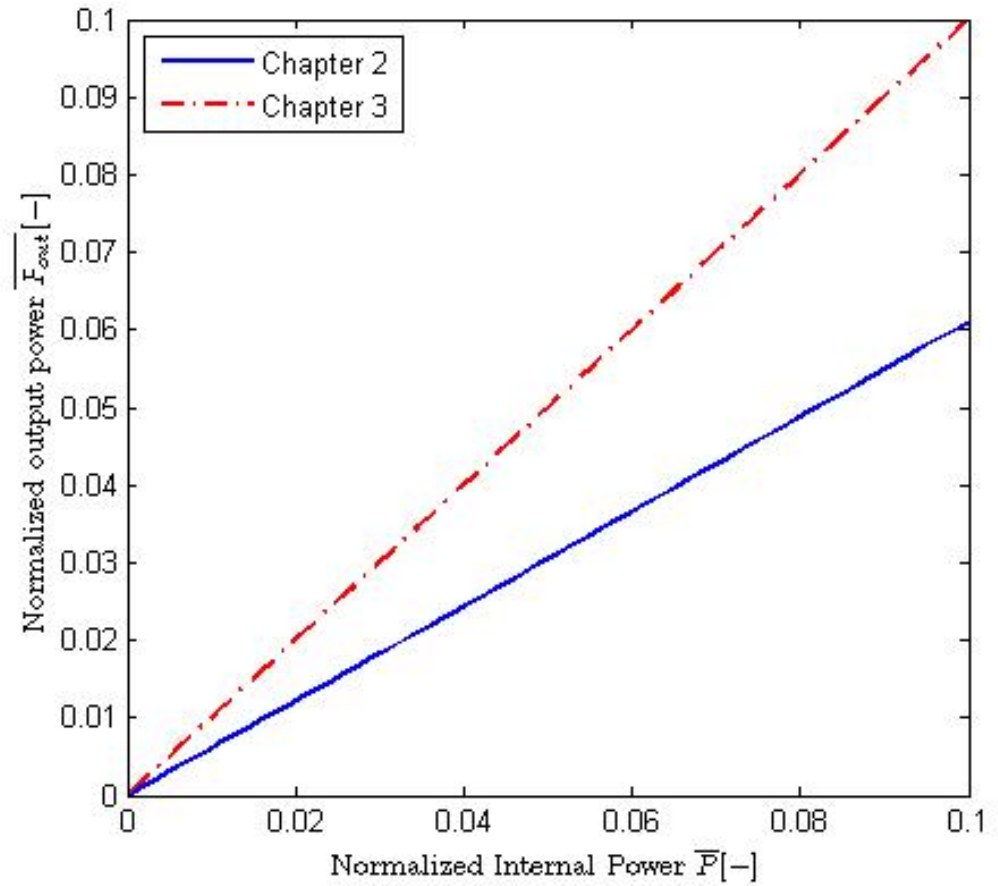


Figure 3.4.2: Normalized output power vs normalized internal power

The figure shows that in both the chapters, the function varies linearly in an increasing fashion. Chapter 2 uses a more complicated approach and for the same internal power, the output power is comparatively more than the approach in Chapter 3.

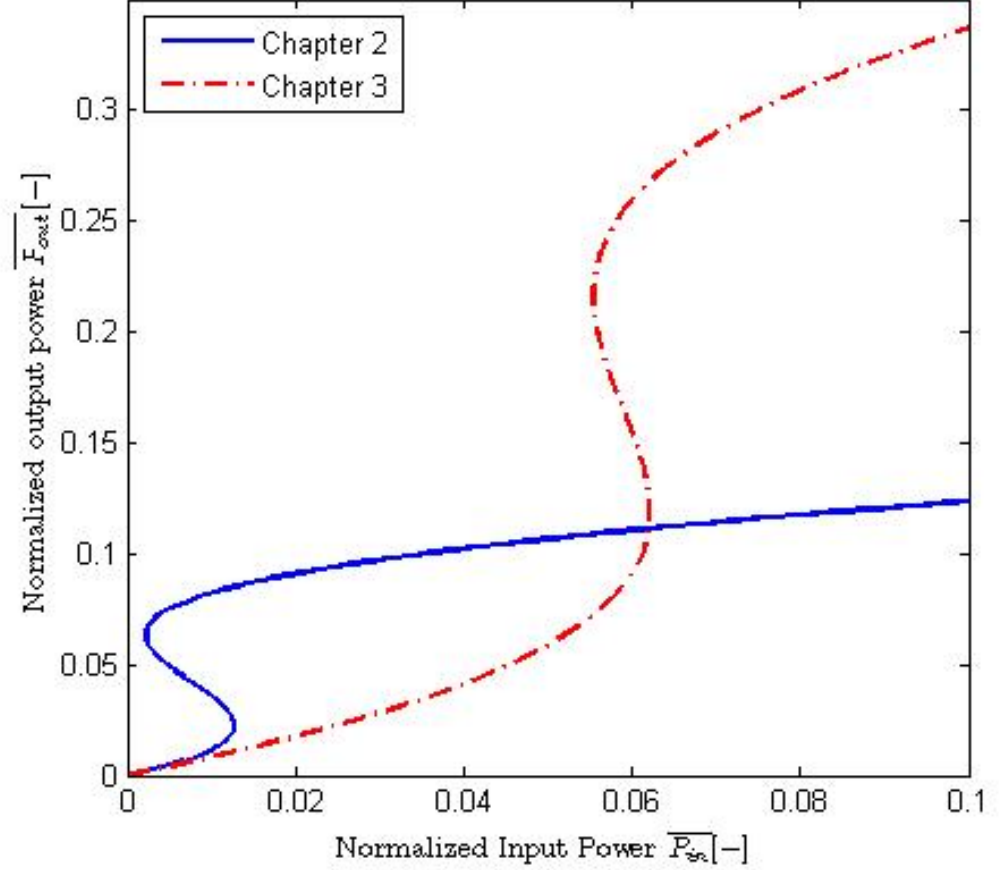


Figure 3.4.3: Normalized output power vs normalized input power

The output power in both the cases varies with hysteresis. The output power is greater in case of the chapter 3 for the same input power range which is demonstrated in the figure at high values of input power. Note that it takes less input power to achieve bistability for the Chapter 2 model than for the Chapter 3 model. This is because a lower input power is related to a lower output power, and the Chapter 3 model gives a lower output power than the Chapter 2 model for the

same amount of input power.

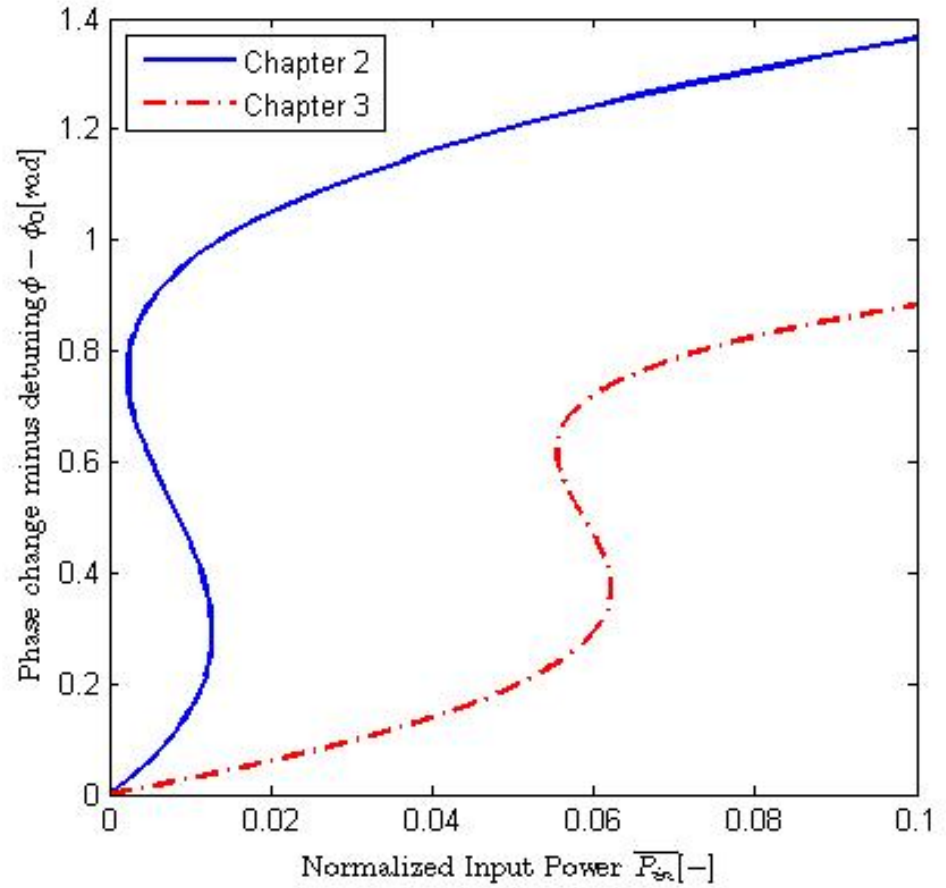


Figure 3.4.4: Phase difference vs normalized input power

The phase difference calculated in both the chapter approach varies with hysteresis and they curve is an increasing fashion for the increase in the input power. The Chapter 2 curve has higher values of the phase difference for the same input power, and the phase jump from lower and higher stable branches is larger.

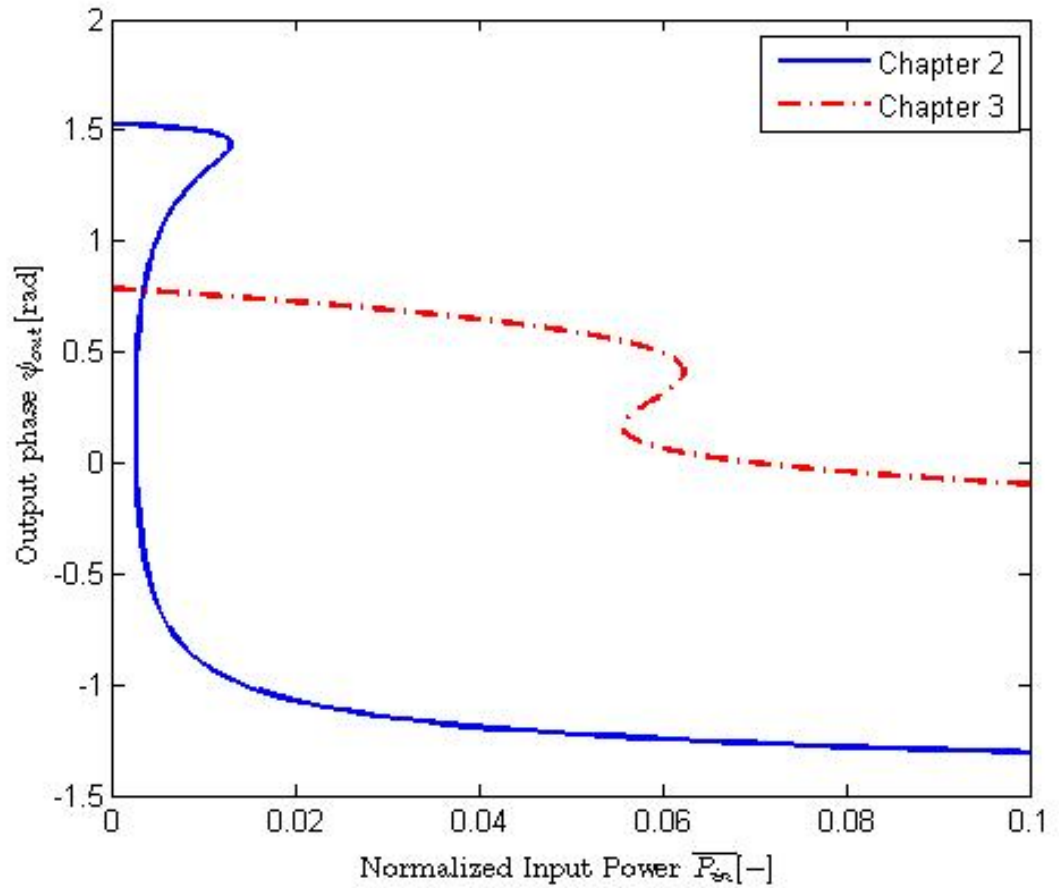


Figure 3.4.5: Phase difference vs normalized input power

The output phase calculated using each approach decreases with the increase in the input power. The Chapter 2 curve has greater range of the output phase than the Chapter 3 approach. The maximum and minimum limit of the output phase is from -1.5 rad to 1.5 rad for the input power varying from 0 to 0.1 whereas the approach in chapter 3 has output phase only varying within -0.5 to 0.5 for the same x axis values.

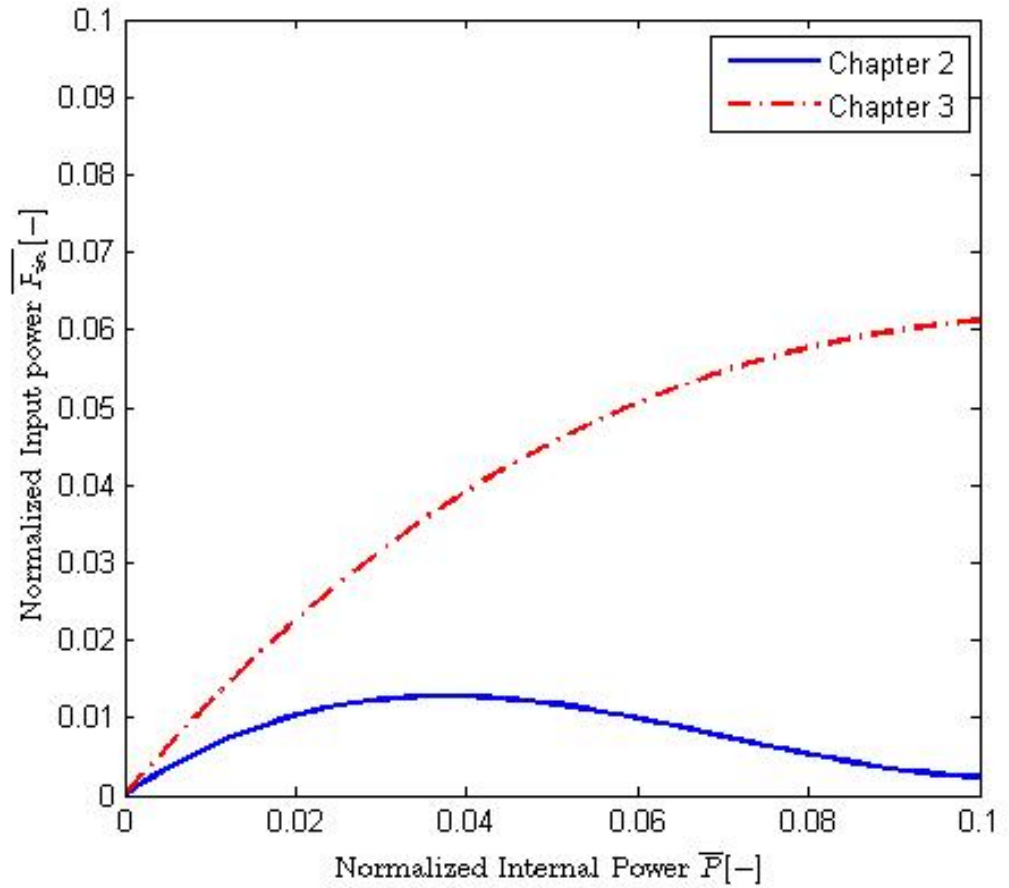


Figure 3.4.6: Normalized input power vs normalized internal power

The input power for the same range of internal power is lesser for the Chapter 2 and more for the simpler model in Chapter 3. This is the reason that all the figures in chapter 3 has a greater power than the chapter 2 models. But the power required to achieve bi-stable action is lessor for the more involved approach.

CHAPTER 4

Dual Optical Input Signals:Baseline Model

Optical bistability, such as studied in the previous two chapters, can be the basis of optical memory devices and optical signal processing devices. The previous two chapters considered a single optical signal, but the behavior when more than one signal is present may open up more applications in optical signal processing. In this chapter we consider two optical signals. We follow the model proposed in reference [8], which is similar in its simplicity as the single signal model proposed by [2]. The schematic of the FP-SOA using two signal is given below.



Figure 4.0.1: Schematic of FP-SOA using 2 signals

The FP-SOA using two input signals schematic is given. The current is applied on the SOA and they produce two amplified output signals.

4.1 Output power parameterization

The Fabry-Perot SOA is designed to have two input powers P_{in1} and P_{in2} with output powers P_{out1} and P_{out2} respectively. The relation between the input and output powers is given by [9]

$$\frac{P_{out1}}{P_s} = \frac{P_{in1}}{P_s} \frac{T^2 G}{(1 - GR)^2 + 4GR \sin^2(\phi_1)}, \quad (4.1.1)$$

$$\frac{P_{out2}}{P_s} = \frac{P_{in2}}{P_s} \frac{T^2 G}{(1 - GR)^2 + 4GR \sin^2(\phi_2)}, \quad (4.1.2)$$

CHAPTER 4: DUAL OPTICAL INPUT SIGNALS:BASELINE MODEL

where P_{in1} is the input power of signal 1, P_{in2} is the input power of signal 2, P_{sat} is saturation output power of the amplifier, $T=1-R$ is transmittivity, R is the reflectivity, ϕ_1 is the single path phase change of signal 1 and ϕ_2 is the single path phase change of signal 2. It is assumed that the saturation power is the same for each signal. These equations for the two-signal case are just two instance of the equation given in Chapter 3. These equations further simplify to

$$\overline{P_{out1}} = \overline{P_{in1}}F_1, \quad (4.1.3)$$

$$\overline{P_{out2}} = \overline{P_{in2}}F_2, \quad (4.1.4)$$

where

$$F_x = \frac{T^2 G}{(1 - GR)^2 + 4GR \sin^2(\phi_x)},$$

$$\overline{P_{outx}} = \frac{P_{outx}}{P_{sat}},$$

and

$$\overline{P_{inx}} = \frac{P_{inx}}{P_{sat}},$$

and x values are 1 and 2.

The gain of the amplifier G is assumed to be the same for each signal and is given as [9]

$$G = \exp[(g - \alpha_{int})L], \quad (4.1.5)$$

where g is the saturated gain coefficient, α_{int} is the loss coefficient and L is cavity length. The saturated gain coefficient can be expressed in terms of small signal

gain coefficient as

$$g = \frac{g_0}{1 + \frac{P_1 + P_2}{P_{SAT}}}. \quad (4.1.6)$$

Here, the gain is saturated by the sum of the internal power of both signals, and is a simple extension of the gain equation used in Chapter 3. Note that it is convenient to use the normalized internal powers $\overline{P}_1 = \frac{P_1}{P_{sat}}$ and $\overline{P}_2 = \frac{P_2}{P_{sat}}$, which yields

$$g = \frac{g_0}{1 + \overline{P}_1 + \overline{P}_2}.$$

The phase change is given by

$$\phi_1 = \phi_{01} + \frac{\alpha}{2} g_0 L \frac{\overline{P}_1 + \overline{P}_2}{1 + \overline{P}_1 + \overline{P}_2}, \quad (4.1.7)$$

$$\phi_2 = \phi_{02} + \frac{\alpha}{2} g_0 L \frac{\overline{P}_1 + \overline{P}_2}{1 + \overline{P}_1 + \overline{P}_2}, \quad (4.1.8)$$

$$\phi_2 = \phi_1 + \phi_{02} - \phi_{01}.$$

Again, the sum of power is used. For the simpler model used in this chapter, the internal power and the output powers are the same

$$P_{out} = P.$$

4.2 Normalized internal power for signal 1 and 2 parameterization

The equations above are solved by using parameterization. In this method $\overline{P}_1 + \overline{P}_2$ is treated as a single quantity and a vector of values is created for further solving.

From a parameterized $\overline{P_1} + \overline{P_2}$, equations for ϕ_1 , ϕ_2 , g and G are immediately solved. From the value of g , G is solved. ϕ_1 and G provides F_1 while ϕ_2 and G provides F_2 . By knowing all these values, the input normalized power value can be determined. The input power $\overline{P_{in}}$ can be calculated using the equation (4.1.3) and (4.1.4). The summed power is

$$\overline{P_{out1}} + \overline{P_{out2}} = \overline{P_{in1}}F_1 + \overline{P_{in2}}F_2,$$

where the left hand side is equal to output parameter. From the known equation for normalized output power, if we assume that P_{in2} is a constant value and a known quantity, then P_{in1} can be calculated as follows:

$$\begin{aligned} \overline{P_{in1}}F_1 &= \overline{P_{out1}} + \overline{P_{out2}} - \overline{P_{in2}}F_2, \\ \overline{P_{in1}} &= \frac{\overline{P_{out1}} + \overline{P_{out2}} - \overline{P_{in2}}F_2}{F_1}. \end{aligned} \quad (4.2.1)$$

By knowing the value of P_{in2} and P_{in1} we can have a plot with normalized input power vs normalized output power.

4.2.1 Gain coefficient

Since we define the value of $\overline{P_1} + \overline{P_2}$ and small signal gain g_0 , we can use them to find the gain coefficient g using (4.1.6). The gain coefficient exponentially decreases with the increase in the total output power.

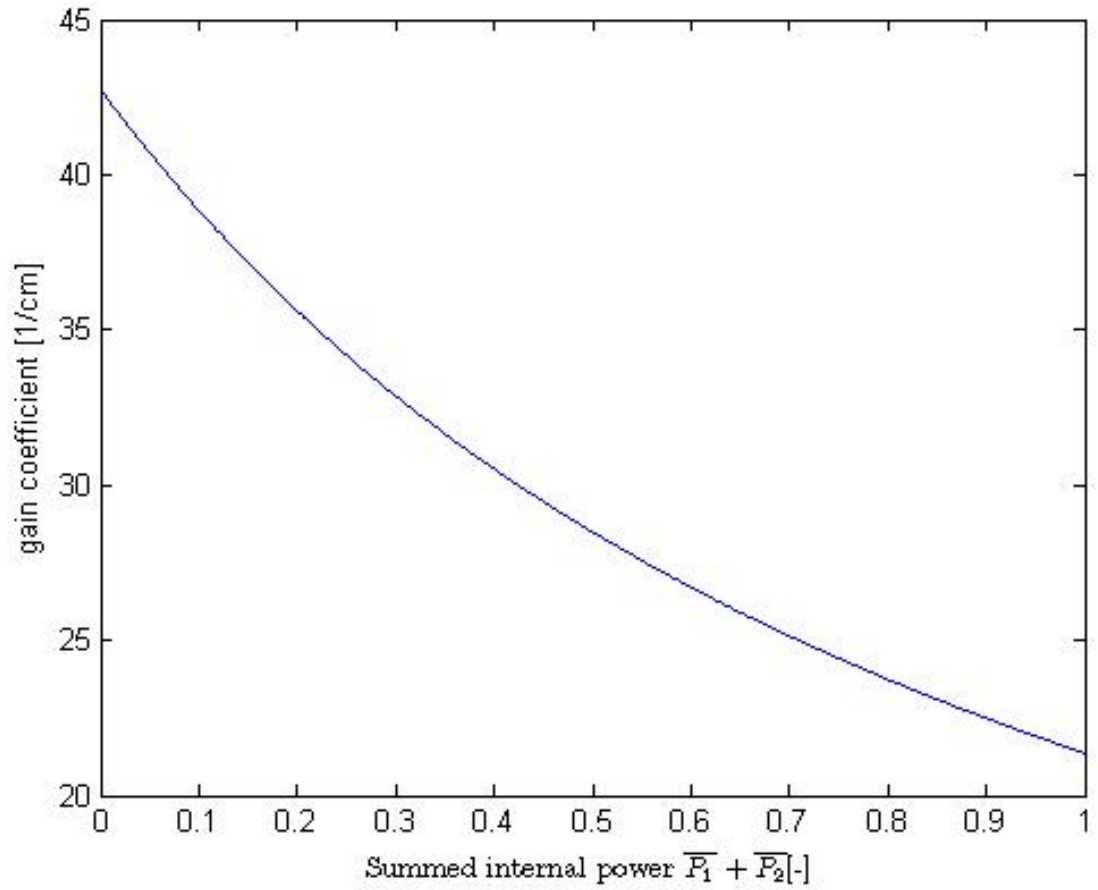


Figure 4.2.1: Gain coefficient vs normalized total internal power

Unless otherwise specified, all figures in this chapter are plotted using $L=0.03$ cm, $R=0.35[-]$, $g_0=42.75$ [1/cm], $\alpha_{int}=10$ [1/cm], $b=4[-]$, $\phi_{01}=2.3$ [rad] and $\phi_{02}=\pi$ [rad], $\Gamma=0.4$ and normalized input power of the signal 2 as 0.01 [9].

4.2.2 Gain

Since we have calculated the value of g , we can find the value for optical gain G . The optical gain varies exponentially along the length of the semiconductor L and the expression is given as

$$G = \exp[(g - \alpha_{int})L]. \quad (4.2.2)$$

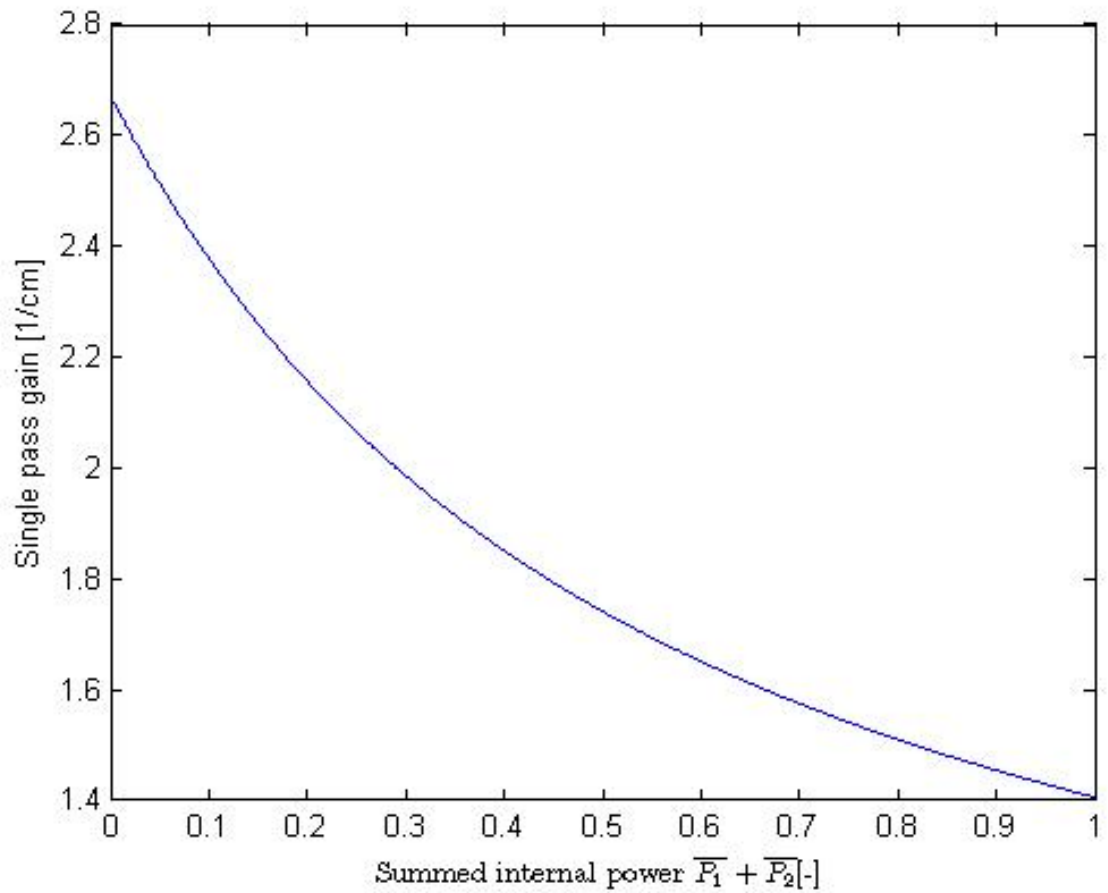


Figure 4.2.2: Single pass gain vs normalized total internal power

The single pass gain is an exponential function of the gain coefficient. Thus

it follows the same pattern and decreases exponentially with increase in the total output power.

4.2.3 Single-pass optical phase

Since we define the value of $\overline{P}_1 + \overline{P}_2$, and small signal gain g_0 , we can use that to find the optical phase ϕ_1 and ϕ_2 using equation (4.1.7) and (4.1.8).

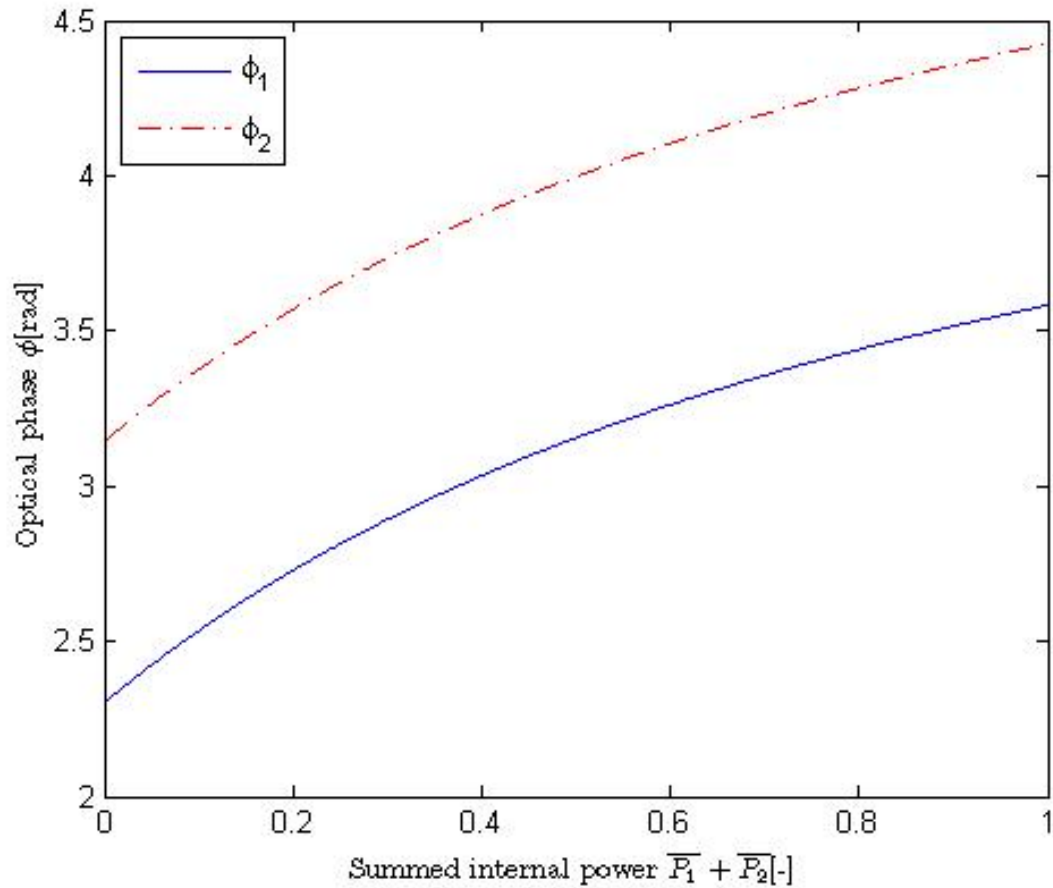


Figure 4.2.3: Optical phase vs normalized total internal power

The phase quantities increase as the internal power increases. They are directly proportional to each other.

4.2.4 Output phase

The output phase for both the signal is given by the equations [2]

$$\psi_{out1} = \psi_{in} - \phi_1 - \tan^{-1}\left[\frac{RG\sin(2\phi_1)}{1 - RG\cos(2\phi_1)}\right] \quad (4.2.3)$$

$$\psi_{out2} = \psi_{in} - \phi_2 - \tan^{-1}\left[\frac{RG\sin(2\phi_2)}{1 - RG\cos(2\phi_2)}\right] \quad (4.2.4)$$

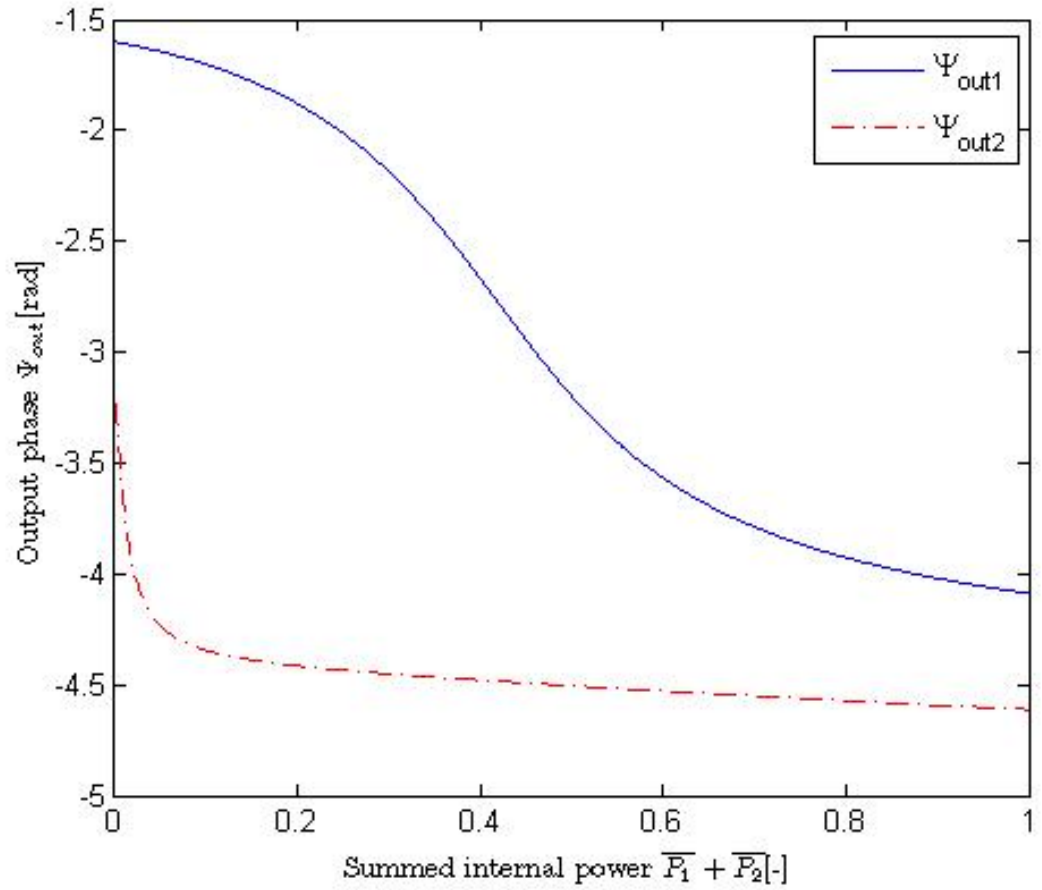


Figure 4.2.4: Output phase vs normalized total internal power

The output phase and power are inversely proportional. Hence the value decreases with increase in the internal power.

4.3 Input power

The sum of the output power is used above to determine g , ϕ , G , F_1 and F_2 . Moreover, $\overline{P_{in1}}$ can be found if we assume a value for $\overline{P_{in2}}$, using Eqn.(4.2.1).

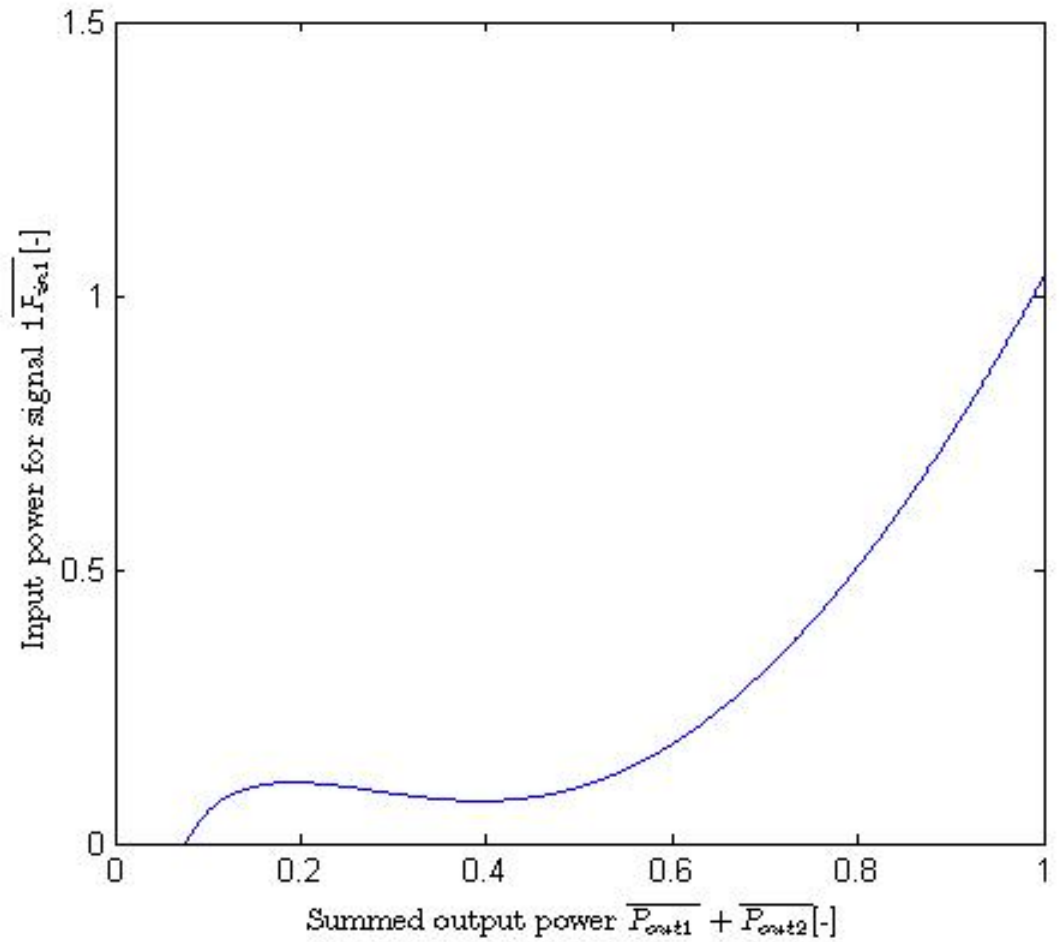


Figure 4.3.1: Normalized input power for signal 1 vs normalized total output power

Note that we know the input power, all other quantities can be plotted in terms

of the input power.

4.3.1 Separated output power

Once the input power for each signal is known, the output power can be found using

$$\overline{P_{out1}} = \overline{P_{in1}}F_1,$$

$$\overline{P_{out2}} = \overline{P_{in2}}F_2.$$

Alternatively, because $\overline{P_{out1}} + \overline{P_{out2}}$ is known, either $\overline{P_{out1}}$ or $\overline{P_{out2}}$ can be found if the other power is found first. The two output powers are plotted in Figure(4.3.2) as a function of input power. Note that the bistable behavior is evident. Both signals have the same input power, that triggers bistability.

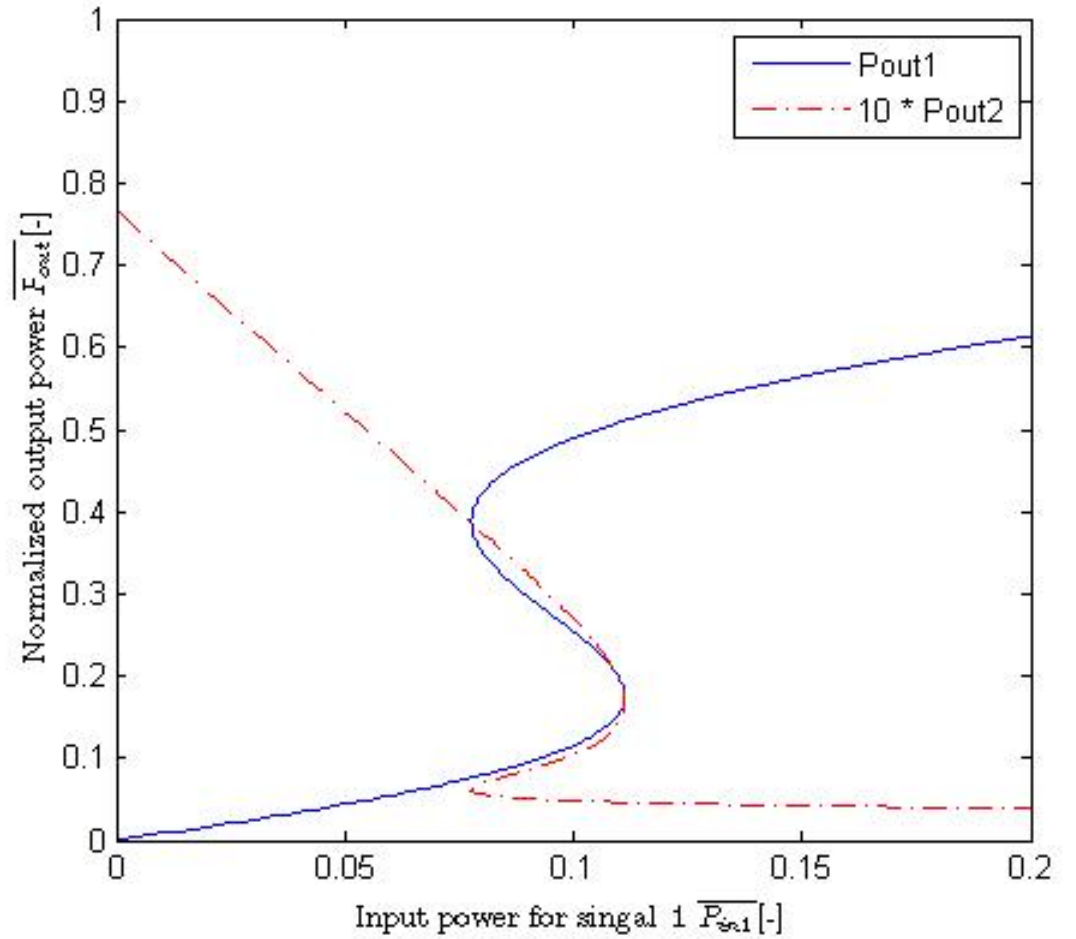


Figure 4.3.2: Normalized output power vs normalized input power of signal 1

4.3.2 Gain and phase

Now that the input power of signal 1 is known, the already calculated gain and phase can be plotted as a function of it. These graphs clearly show hysteric behavior.

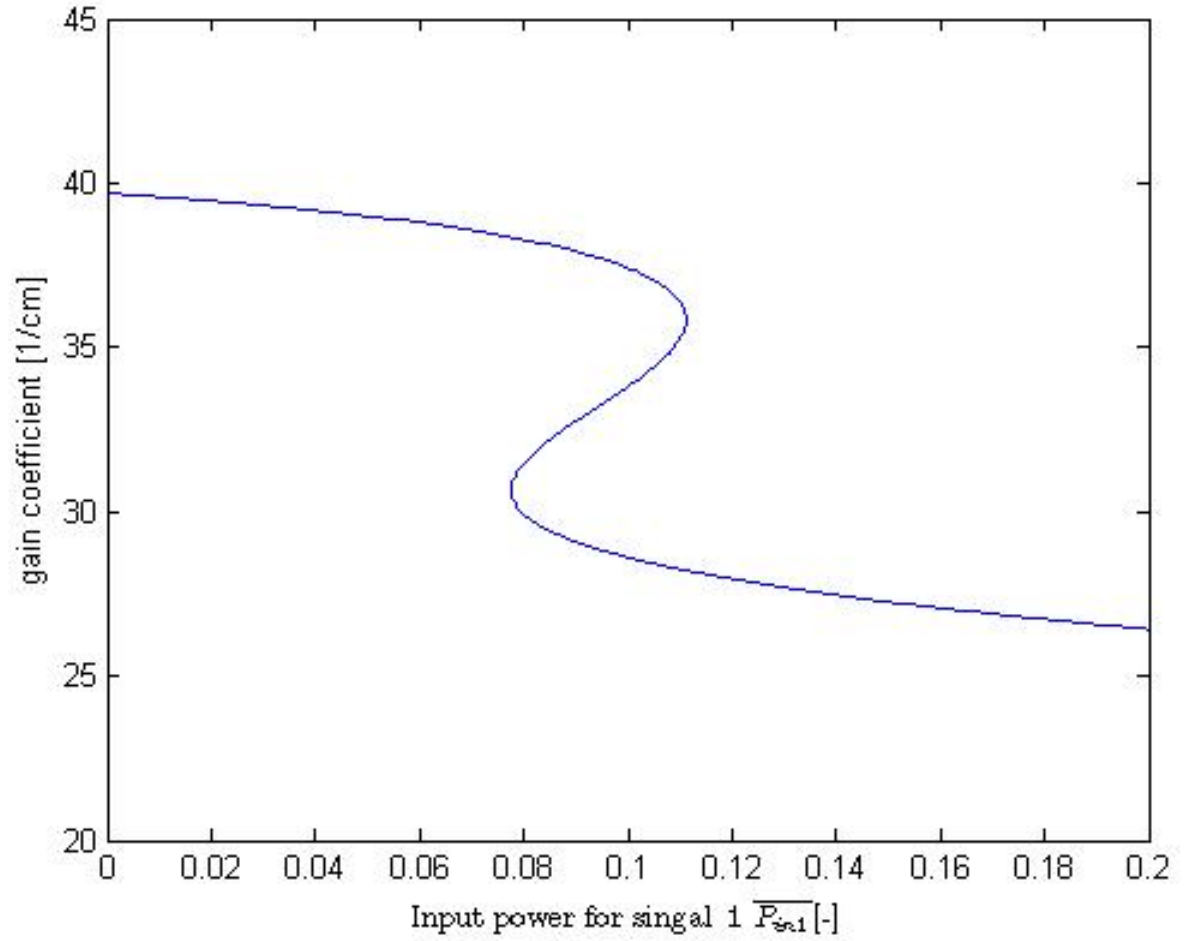


Figure 4.3.3: Gain coefficient vs normalized input power of signal 1

The gain coefficient decreases with the increase in the input power.

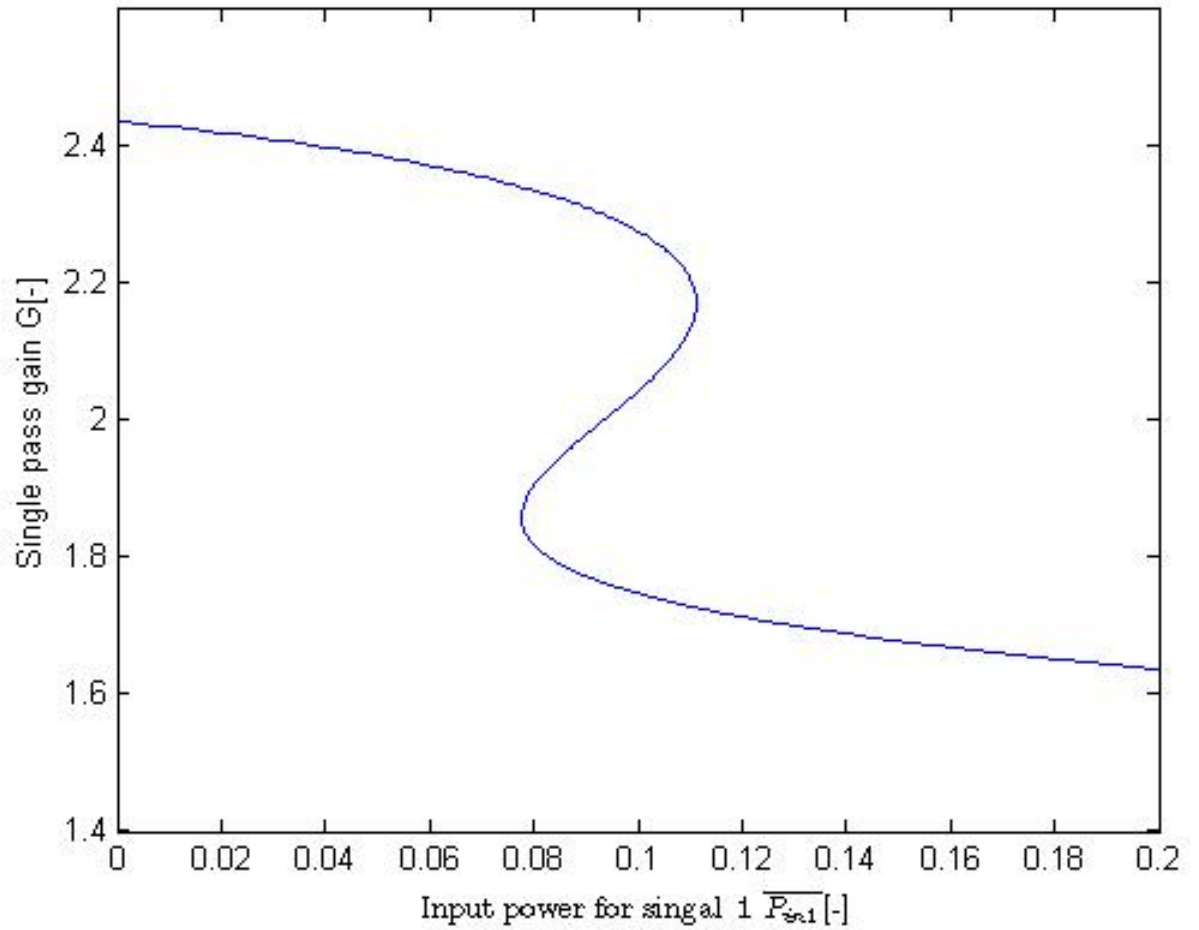


Figure 4.3.4: Gain coefficient vs normalized input power of signal 1

The single pass gain follows a similar pattern to that of the gain coefficient. The single pass gain decreases with the increase in the input power.

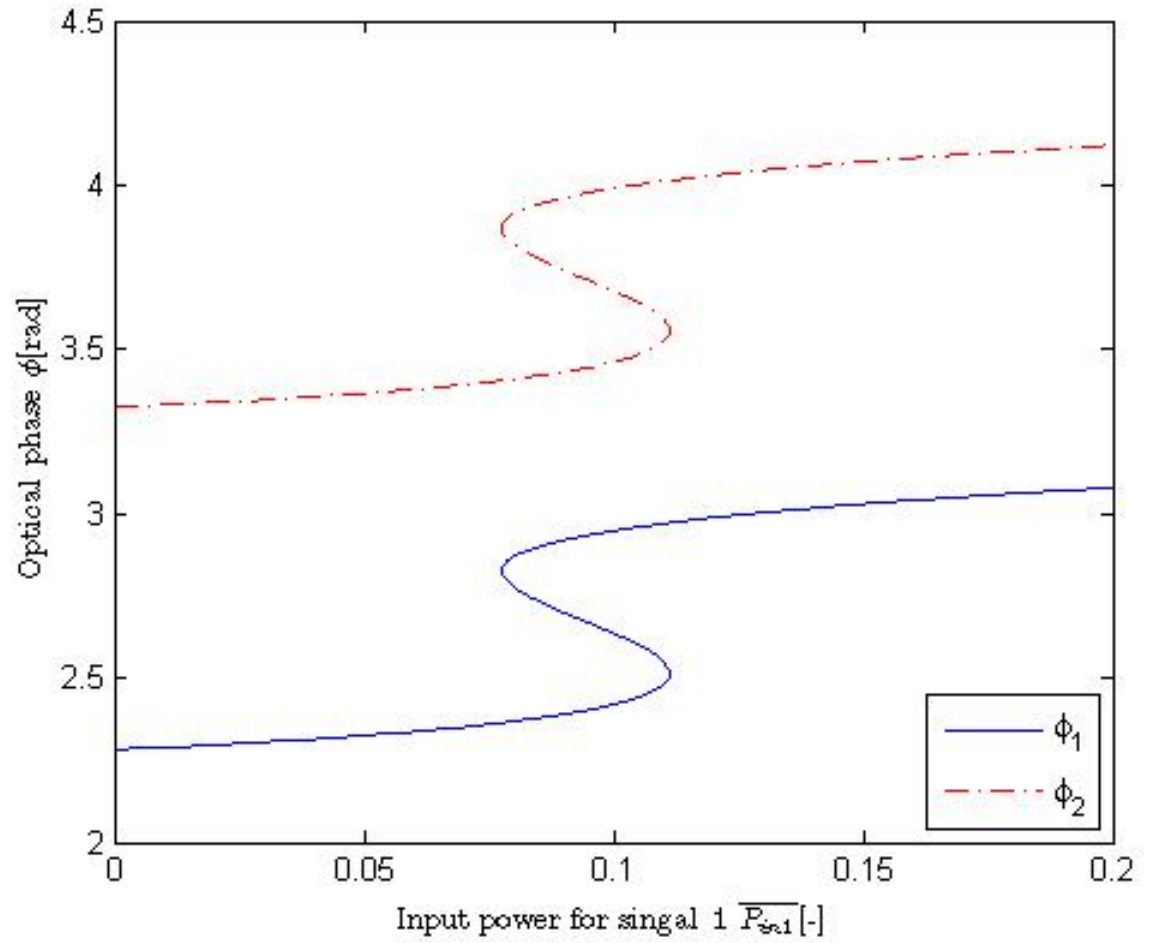


Figure 4.3.5: Optical phase vs normalized input power of signal 1

The optical phase varies with hysteresis with increase in the input power.

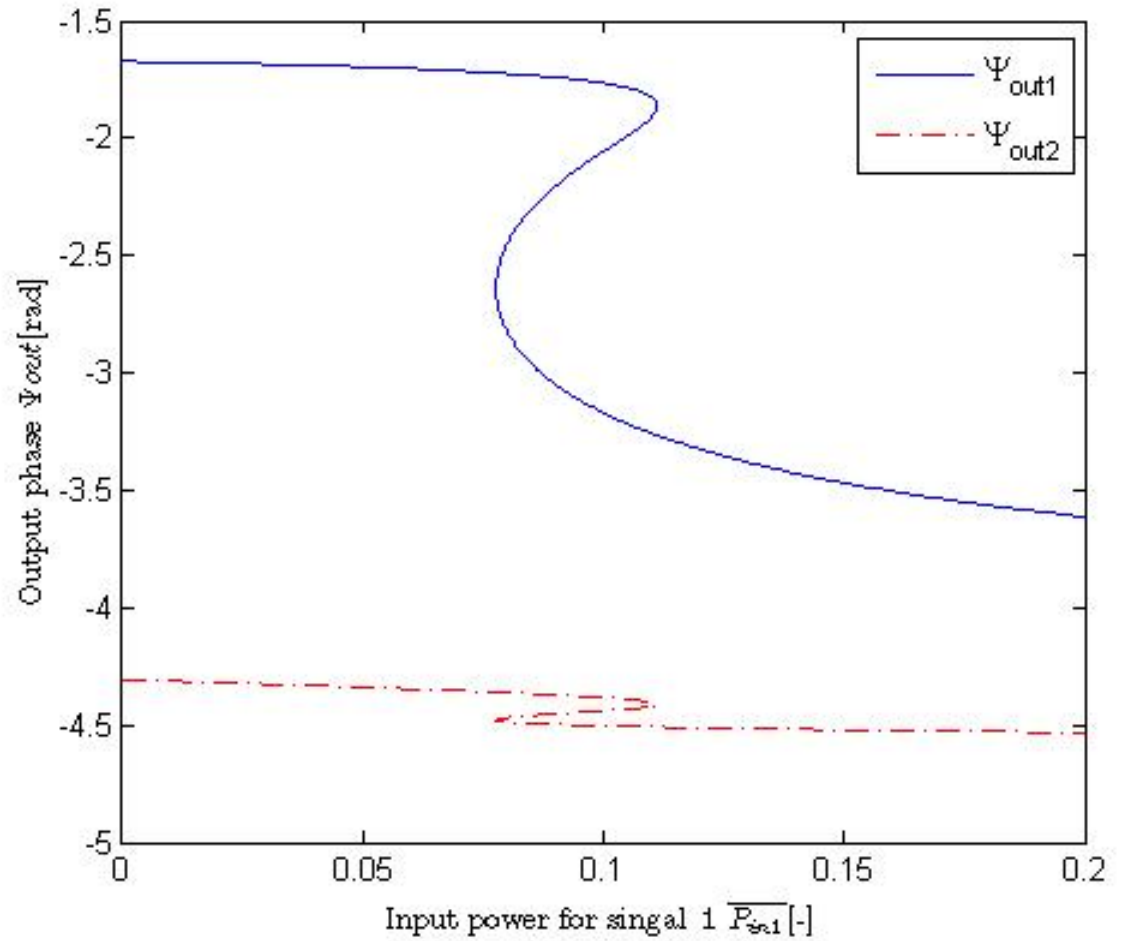


Figure 4.3.6: Output phase vs normalized input power of the signal 1

4.4 Dependence on Initial Detuning

In this section, varying the quantity ϕ_{02} by keeping ϕ_{01} as a constant value as 2.3[rad] and varying the ϕ_{02} as π , $\pi/2$ and 0 in case 1,2, and 3 respectively, the output power is varied and plotted as a function of the input power.

4.4.1 $\phi_{01}=2.3[\text{rad}]$ and $\phi_{02} = \pi$

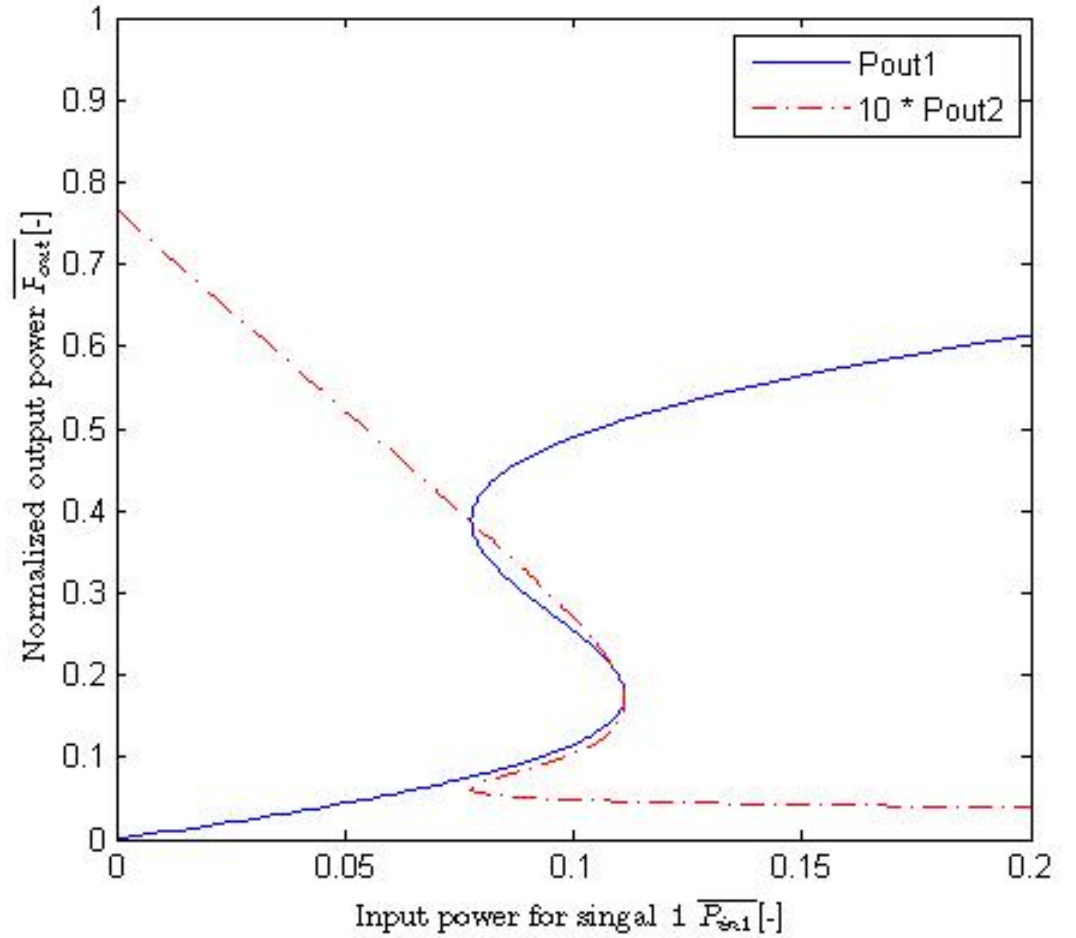


Figure 4.4.1: Output phase vs normalized input power of the signal 1

The output power for signal 1 increases with the increase in input while output power for signal 2 decreases. This corresponds to the difference in phase applied to the signals.

4.4.2 $\phi_{01}=2.3[\text{rad}]$ and $\phi_{02} = \pi/2$

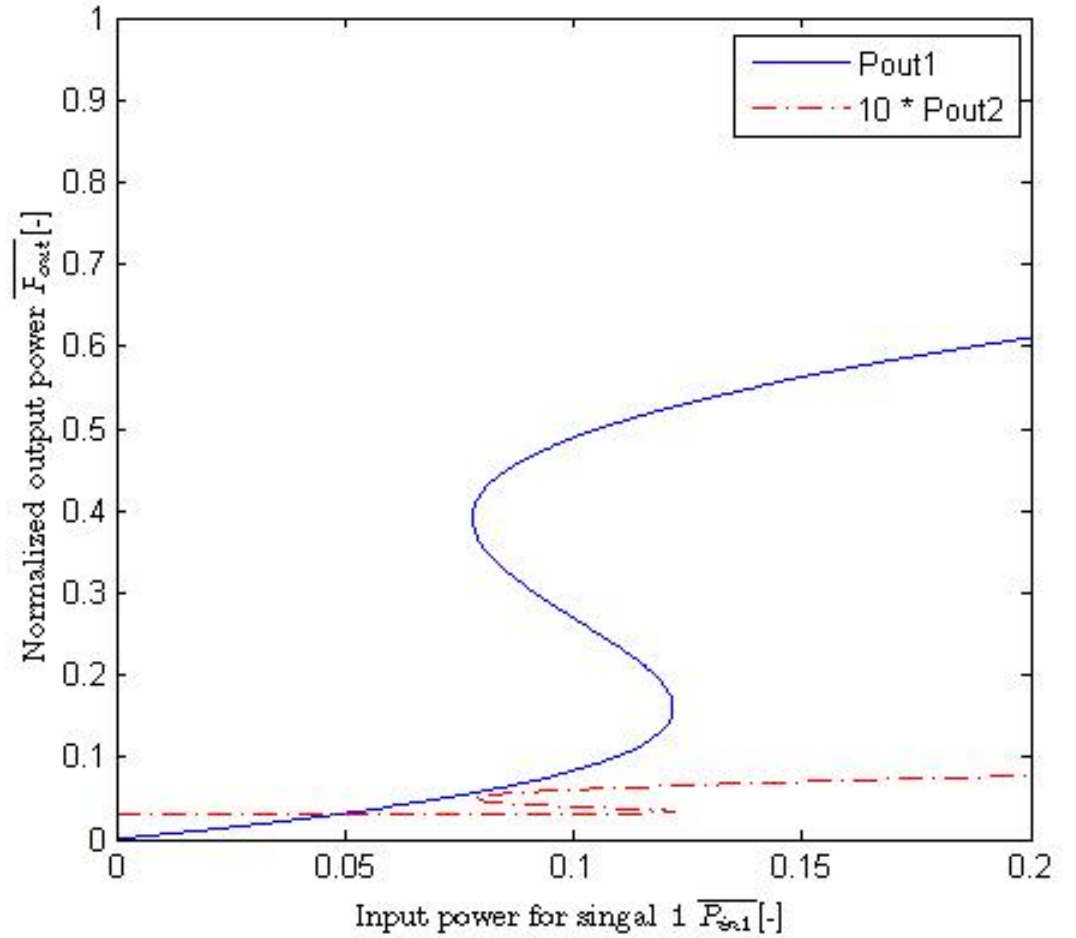


Figure 4.4.2: Output phase vs normalized input power of the signal 1

The output power of signal 1 increases like the previous case since the value of ϕ_{01} is kept as a constant. But since ϕ_{02} is varied, it remains as a constant value in this case with the increase in the input power.

4.4.3 $\phi_{01}=2.3[\text{rad}]$ and $\phi_{02} = 0$

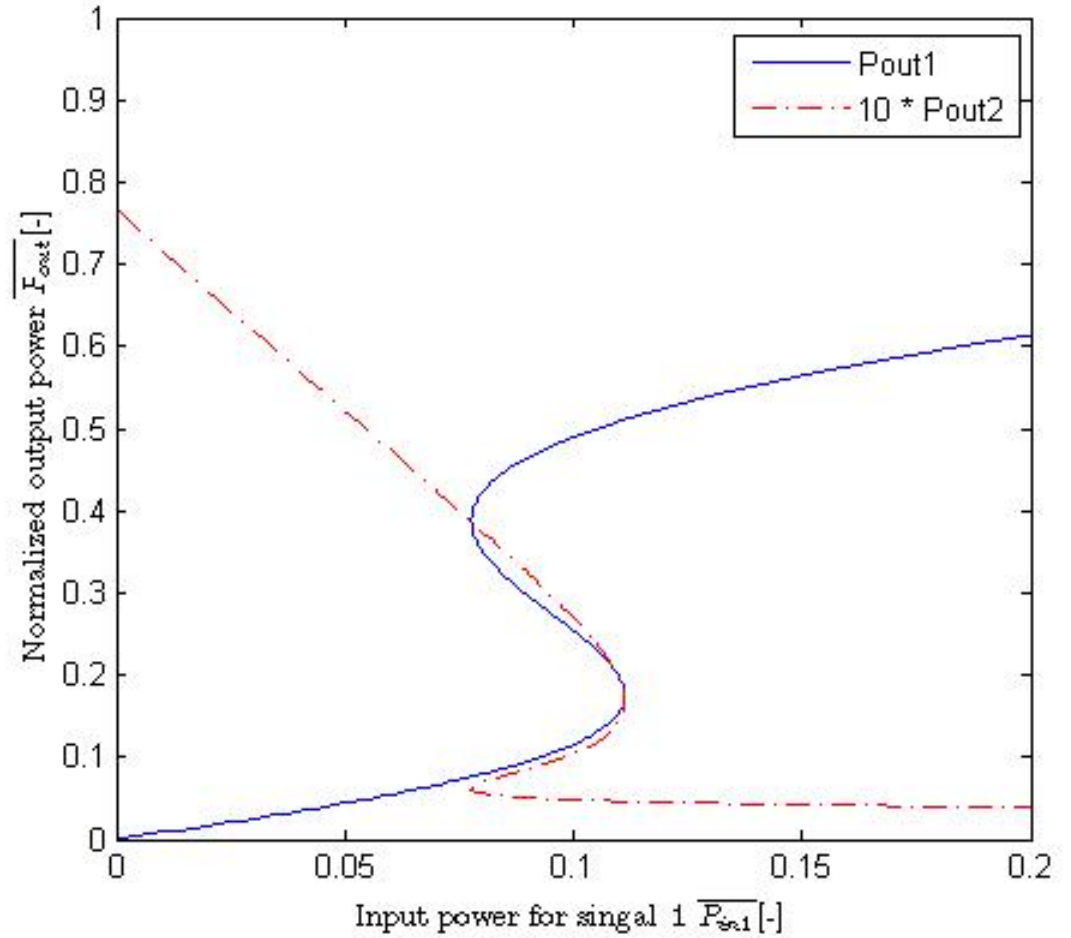


Figure 4.4.3: Output phase vs normalized input power of the signal 1

Since the output power depends on the phase shifts, the output in this case of signal 2 is similar to the first case because the phase shift in current case is 0 and it is π for the initial case. The input power for signal 2 decreases while signal 1 increases with the increase in the input power.

CHAPTER 5

Dual Optical Input Signals: Improved Model

5.1 Introduction

In this chapter, we again consider a Fabry-Perot SOA being injected with two signals. Unlike the previous chapter, however, we will consider a more accurate relation between the output power and the average internal power as the parameterized quantity. Doing so is following the lead of the analysis in Chapter 2 in which the internal power is distinguished from the output power. This work is expected to improve the accuracy of simulations of the two-signal case.

5.2 Gain coefficient

The gain coefficient is common to both signals "1" and "2" and is given by

$$g = \Gamma a(N - N_T),$$

where the steady state value is given by

$$g = \frac{g_0}{1 + \overline{P}_1 + \overline{P}_2},$$

where \overline{P}_1 and \overline{P}_2 is the sum of the internal powers and is the parameterized quantity. As in previous chapter, the bar indicates that the power have been normalized by the saturation power P_{sat} :

$$\overline{P}_1 = \frac{P_1}{P_{sat}},$$

$$\overline{P}_2 = \frac{P_2}{P_{sat}}.$$

A graph of the gain coefficient as a function of total internal power is shown in Fig(x.x.x). Unless otherwise specified , all figures in this chapter are plotted using $L=0.03$ cm, $R=0.035[-]$, $g_0=42.75[1/\text{cm}]$, $\alpha_{int}=10[1/\text{cm}]$, $\phi_{01}=2.3[\text{rad}]$, and $\phi_{02}=\pi[\text{rad}]$, $\Gamma=0.4$, and $\overline{P}_{in2}=0.01$. Note that these are the same nominal parameter values used in Chapter 4 for the simpler model.

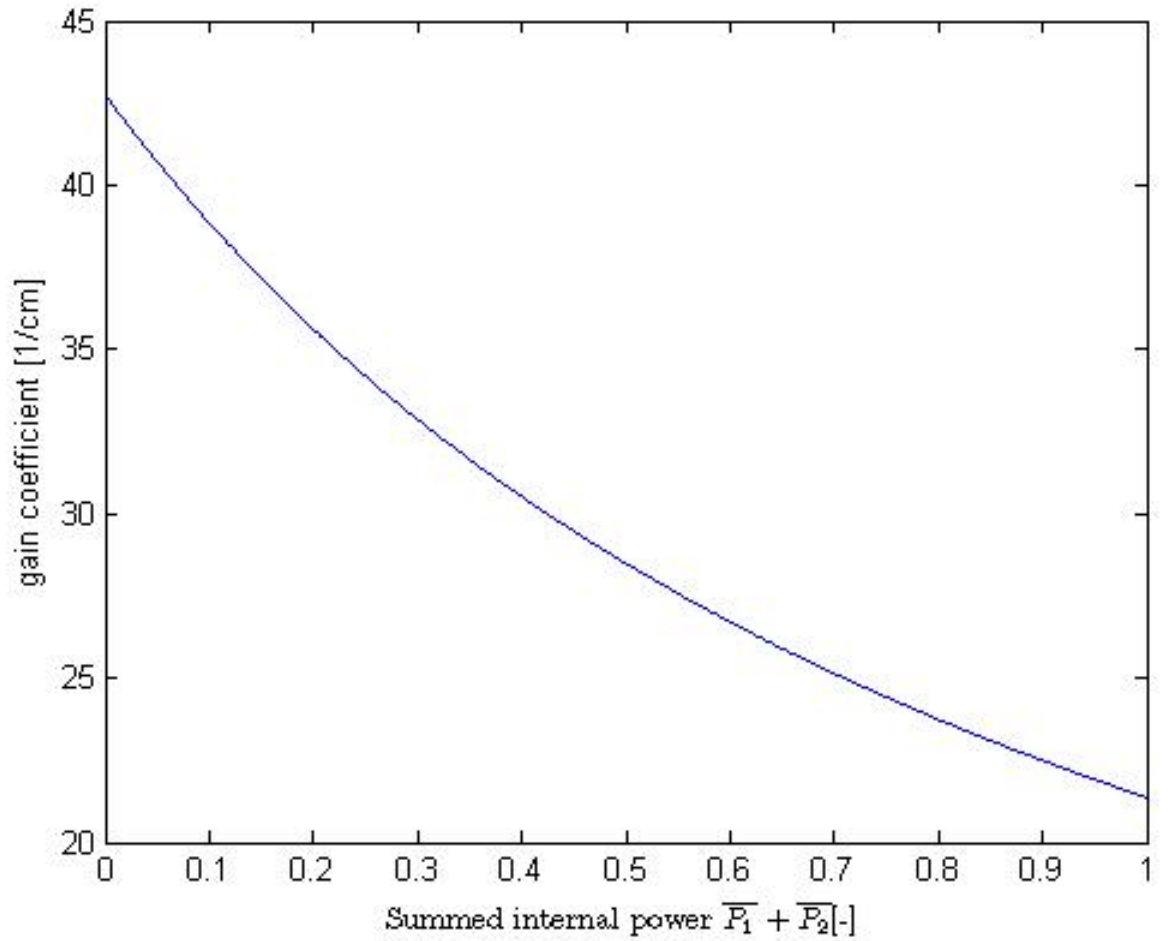


Figure 5.2.1: Gain coefficient vs summed internal power

The gain coefficient is same as that of Chapter 4 and it exponentially decreases with the increase in the summed internal power.

5.3 Gain

The total single pass gain is given by

$$G = \exp[(g - \alpha_{int})L],$$

which is equivalent to [2]

$$G = \Gamma g_0 L + \Gamma(\phi_{01} + \phi_{02} - (\phi_1 + \phi_2))\frac{2}{b} - aL. \quad (5.3.1)$$

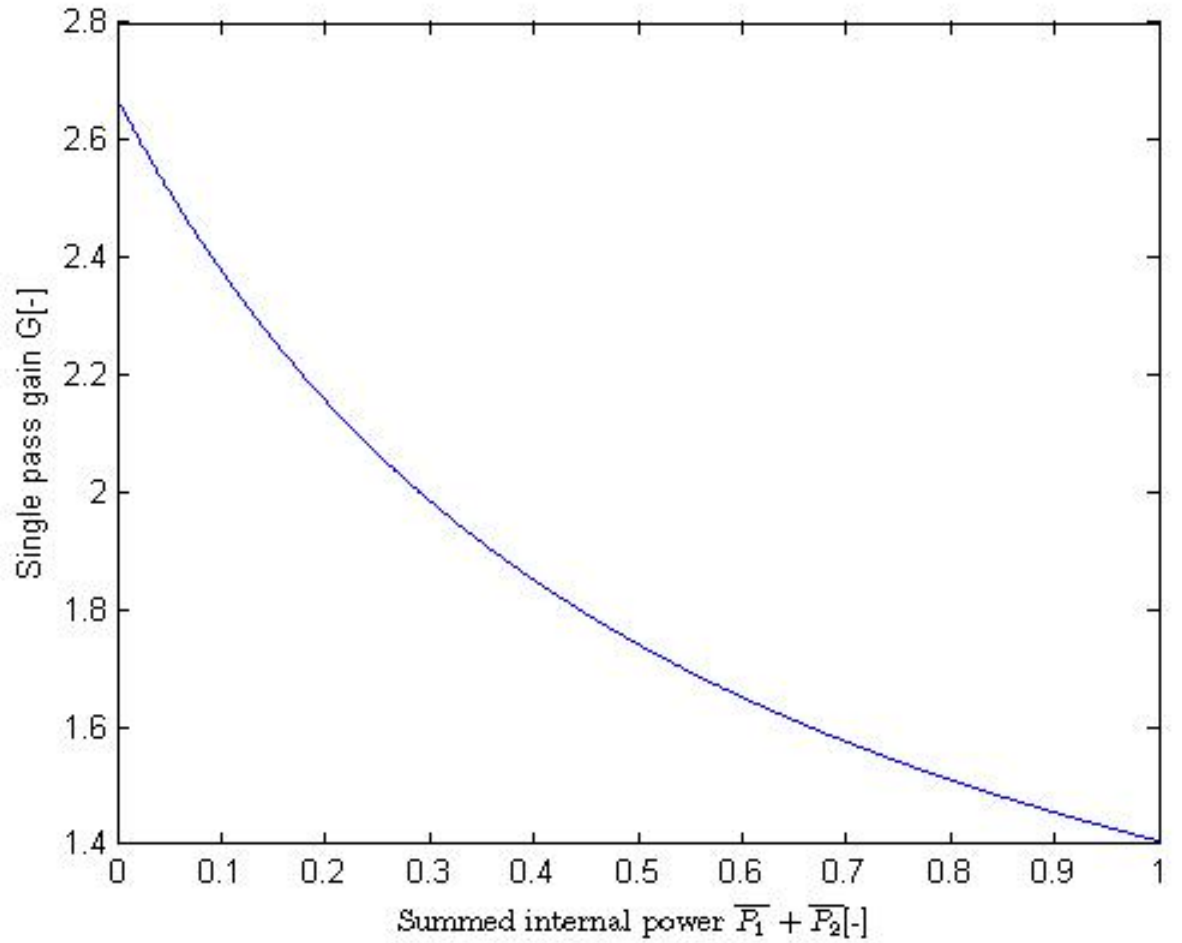


Figure 5.3.1: Single pass gain vs summed internal power

The gain value decreases with an increase in the internal power.

5.4 Single-pass optical phase

We know the average optical internal power traveling across the device. Knowing the power, initial detuning of the signal 1 and 2, we can calculate the optical phase

as

$$\phi_1 = \phi_{01} + \frac{\alpha}{2}g_0L\left(\frac{\overline{P}_1 + \overline{P}_2}{1 + \overline{P}_1 + \overline{P}_2}\right),$$

and

$$\phi_2 = \phi_{02} + \frac{\alpha}{2}g_0L\left(\frac{\overline{P}_1 + \overline{P}_2}{1 + \overline{P}_1 + \overline{P}_2}\right),$$

$$\phi_2 = \phi_1 + \phi_{02} - \phi_{01}.$$

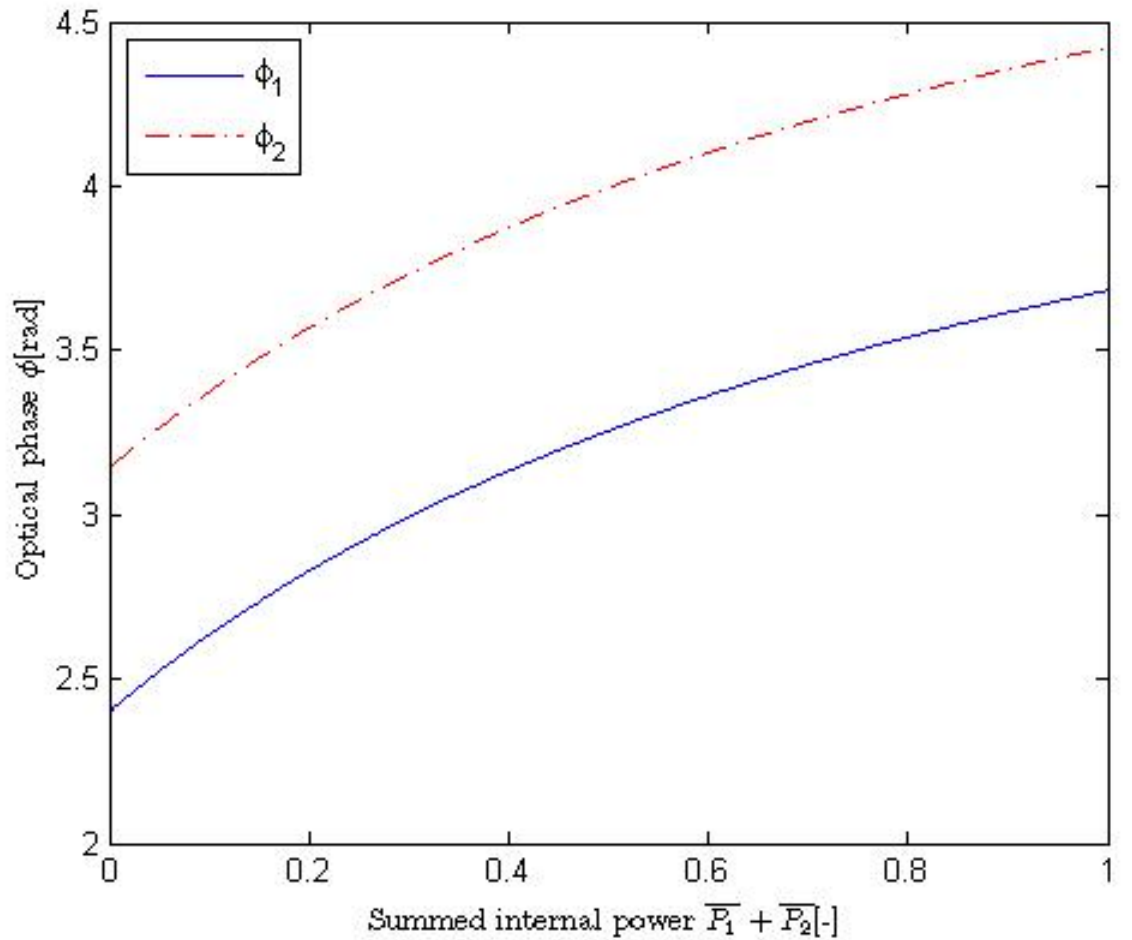


Figure 5.4.1: Optical phase vs normalized internal power

The optical phase exponentially increases with the increase in the internal power.

5.5 Output phase

The relation between output phase ψ_{out1} and input phase ψ_{in} is given by

$$\psi_{out1} = \psi_{in} - \phi_1 - \tan^{-1}\left(\frac{(R_1R_2)^{1/2}e^{GL}\sin 2\phi_1}{1 - (R_1R_2)^{1/2}e^{GL}\cos 2\phi_1}\right).$$

For simplification, let us assume M_1 as

$$M_1 = \left(\frac{(R_1R_2)^{1/2}e^{GL}\sin 2\phi_1}{1 - (R_1R_2)^{1/2}e^{GL}\cos 2\phi_1}\right).$$

Thus the phase of the signal 1 is given by

$$\psi_{out1} = \psi_{in} - \phi_1 - \tan^{-1}(M_1).$$

Likewise,

$$\psi_{out2} = \psi_{in} - \phi_2 - \tan^{-1}\left(\frac{(R_1R_2)^{1/2}e^{GL}\sin 2\phi_2}{1 - (R_1R_2)^{1/2}e^{GL}\cos 2\phi_2}\right).$$

For simplification, let us assume M_B as

$$M_2 = \left(\frac{(R_1R_2)^{1/2}e^{GL}\sin 2\phi_2}{1 - (R_1R_2)^{1/2}e^{GL}\cos 2\phi_2}\right).$$

Thus the phase of signal 2 is given by

$$\psi_{out2} = \psi_{in} - \phi_2 - \tan^{-1}(M_2).$$

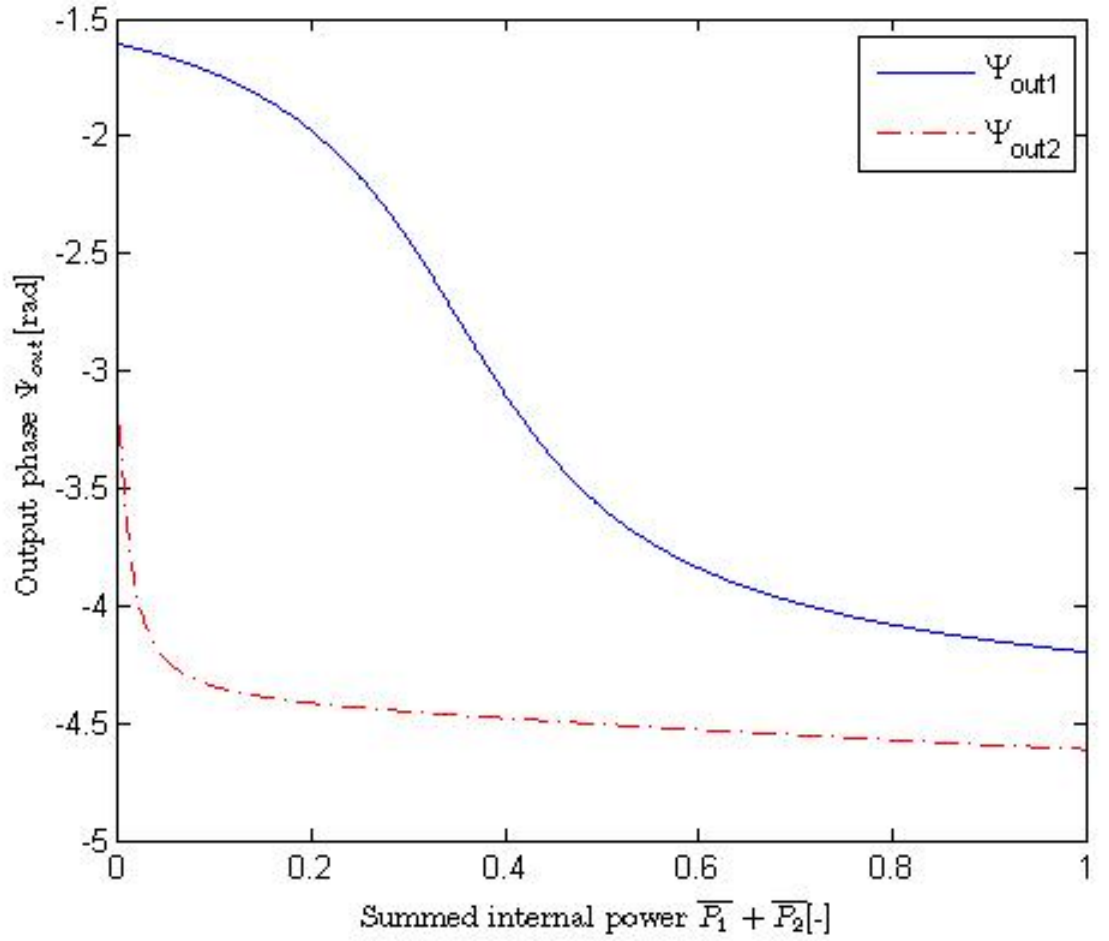


Figure 5.5.1: Output phase vs normalized internal power

The output phase also decreases with the increase in the internal power for both the signals.

5.6 Output Power

In the previous chapter, simpler approach was used in which the internal power and the output powers were assumed to be the same. The relation between the averaged internal and output power in this current chapter is made more realistic:

$$\overline{P}_1 = \overline{P_{out1}}H,$$

where H is given by

$$H = \frac{(1 + R_2G)(G - 1)}{(1 - R_2)GgL},$$

and \overline{P}_1 is the averaged internal power and $\overline{P_{out1}}$ is the output power. Hence the output power is given by

$$\overline{P_{out1}} = \frac{\overline{P}_1}{H}.$$

Similarly we have another normalized averaged internal signal \overline{P}_2 producing the output signal of $\overline{P_{out2}}$. The governing equations are

$$\overline{P}_2 = \overline{P_{out2}}H.$$

Hence the output power of the "2" signal is given by

$$\overline{P_{out2}} = \frac{\overline{P}_2}{H}.$$

The internal powers are known in their combined form. Therefore the output power will be the sum of averaged internal powers:

$$\overline{P}_1 + \overline{P}_2 = \overline{P_{out1}}H + \overline{P_{out2}}H,$$

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Taking H as a common factor ,

$$\overline{P}_1 + \overline{P}_2 = (\overline{P_{out1}} + \overline{P_{out2}})H,$$

The sum of the output power is given by

$$\overline{P_{out1}} + \overline{P_{out2}} = \frac{\overline{P}_1 + \overline{P}_2}{H}. \quad (5.6.1)$$

This expression gives only the sum of output powers, whereas we seek expressions for each output powers independently in terms of $\overline{P}_1 + \overline{P}_2$. Note that Eqn.(5.6.1) reveals the difference between this current model and the simpler model of the previous chapter is simply the inclusion of H. Thus, H is an important quantity and is graphed here:

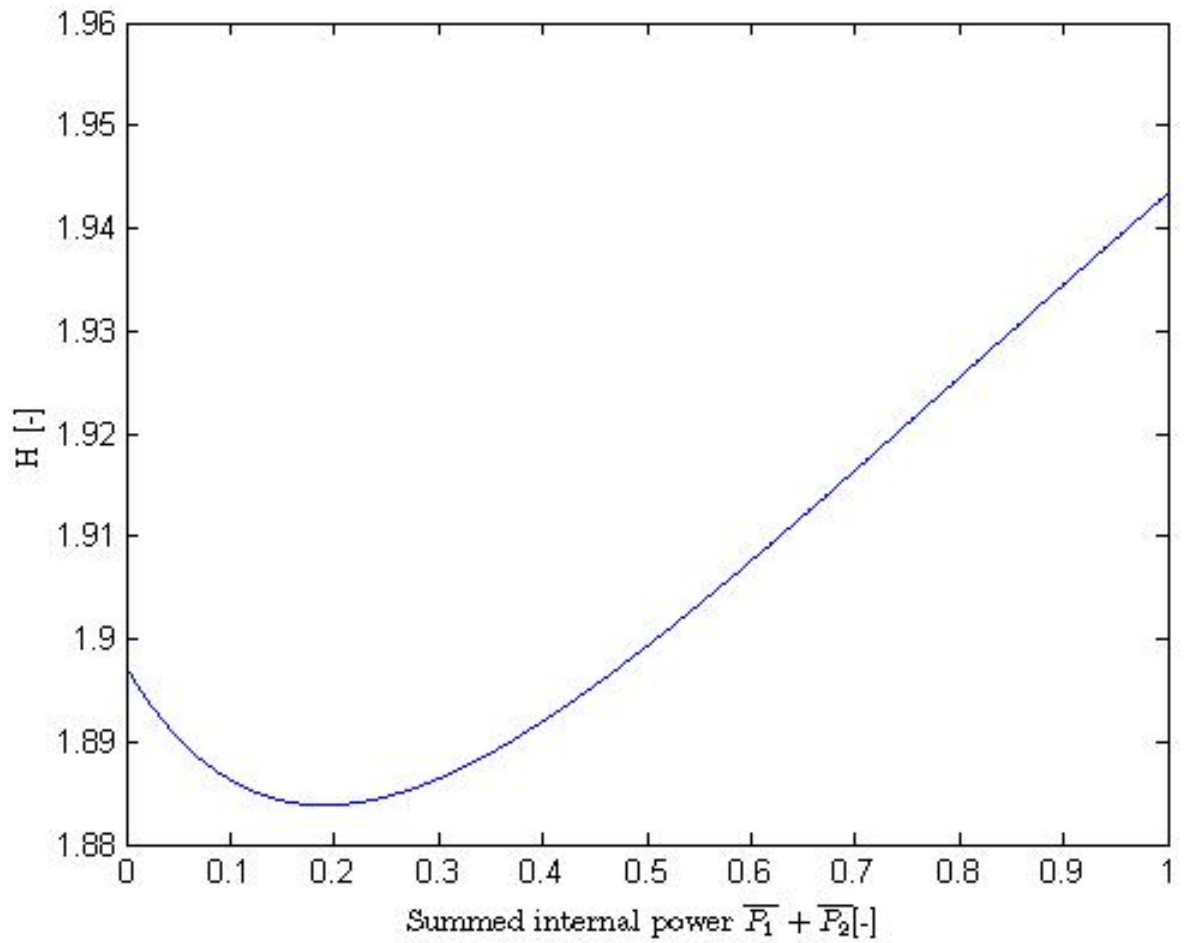


Figure 5.6.1: H vs summed internal power

The H parameter decreases with the increase in the internal power and it is common for both the signals.

5.7 Input power

The normalized output power for signal "1" and the normalized input power are related by the formula

$$\overline{P_{out1}} = \overline{P_{in1}} F_1,$$

where F_1 is given by

$$F_1 = \frac{(1 - R_1)(1 - R_2)G}{(1 - \sqrt{R_1 R_2} G)^2 + 4\sqrt{R_1 R_2} G \sin(\phi_1)}.$$

Hence the input power is given by

$$\overline{P_{in1}} = \frac{\overline{P_{out1}}}{F_1} = \frac{\overline{P_1}}{F_1 H},$$

where we have related the input power to the internal power. Likewise,

$$\overline{P_{out2}} = \overline{P_{in2}} F_2,$$

where F_2 is given by

$$F_2 = \frac{(1 - R_1)(1 - R_2)G}{(1 - \sqrt{R_1 R_2} G)^2 + 4\sqrt{R_1 R_2} G \sin(\phi_2)}.$$

Hence the input power is given by

$$\overline{P_{in2}} = \frac{\overline{P_{out2}}}{F_2} = \frac{\overline{P_2}}{F_2 H}.$$

Working with summed power gives,

$$\overline{P_{out1}} + \overline{P_{out2}} = \overline{P_{in1}} F_1 + \overline{P_{in2}} F_2,$$

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The R.H.S can be equated to

$$\frac{\overline{P}_1 + \overline{P}_2}{H} = \overline{P}_{in1}F_1 + \overline{P}_{in2}F_2.$$

We assume that \overline{P}_{in2} is known to us. Thus \overline{P}_{in1} can be found as:

$$\begin{aligned}\overline{P}_{in1}F_1 &= \frac{\overline{P}_1 + \overline{P}_2}{H} - \overline{P}_{in2}F_2, \\ \overline{P}_{in1} &= \frac{\overline{P}_1 + \overline{P}_2}{HF_1} - \overline{P}_{in2}\frac{F_2}{F_1}.\end{aligned}\tag{5.7.1}$$

This expression gives \overline{P}_{in1} in terms of the $\overline{P}_1 + \overline{P}_2$ internal power sum.

Now that the input power is known, the output power can be calculated using the formula

$$\begin{aligned}\overline{P}_{out1} &= F_1\overline{P}_{in1}, \\ \overline{P}_{out2} &= F_2\overline{P}_{in2}.\end{aligned}$$

The full expression for the output power of signal "1" is

$$\overline{P}_{out1} = \frac{\overline{P}_1 + \overline{P}_2}{H} - \overline{P}_{in2}F_2.\tag{5.7.2}$$

After the input power is found, all the other parameters like gain, phase can be plotted against a function of it.

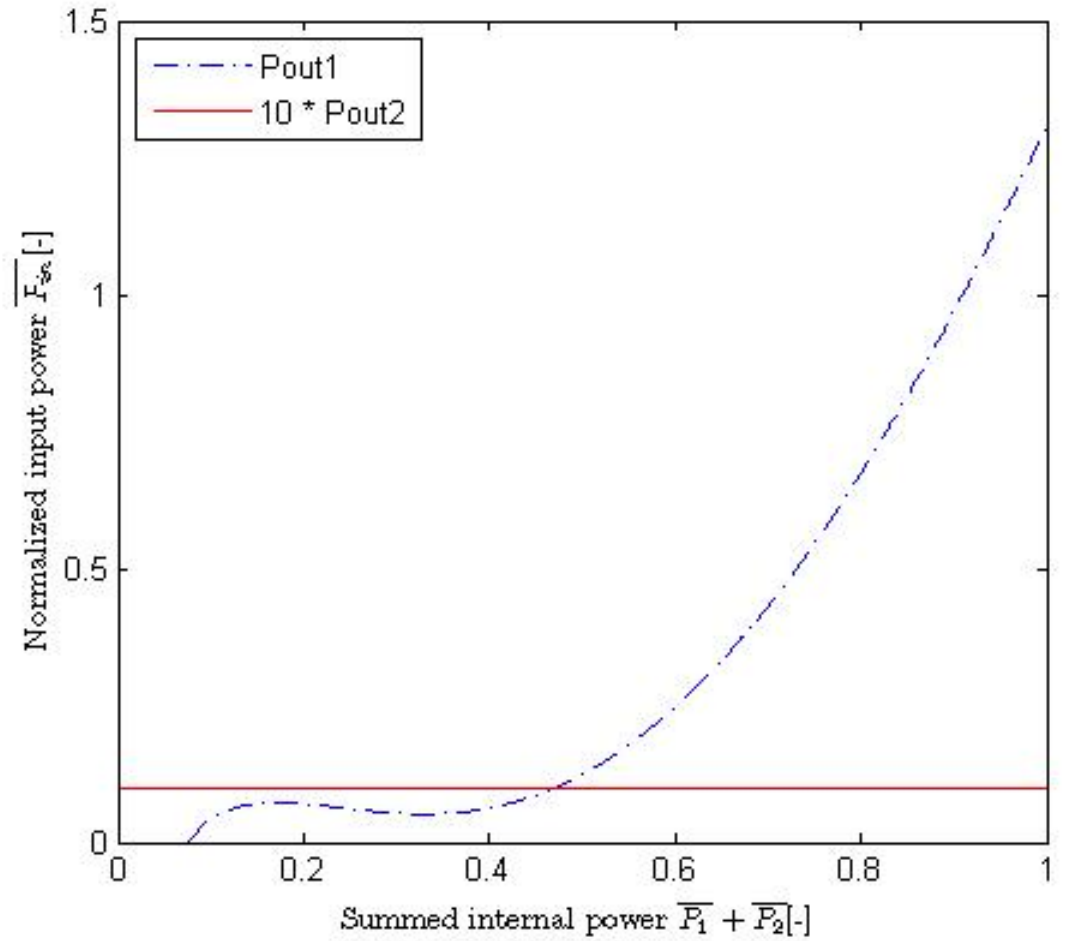


Figure 5.7.1: Normalized input power vs normalized internal power

The input power for signal one tends to increase with the increase in the internal power whereas the input power for signal 2 tend to remain a constant value denoted by a straight line.

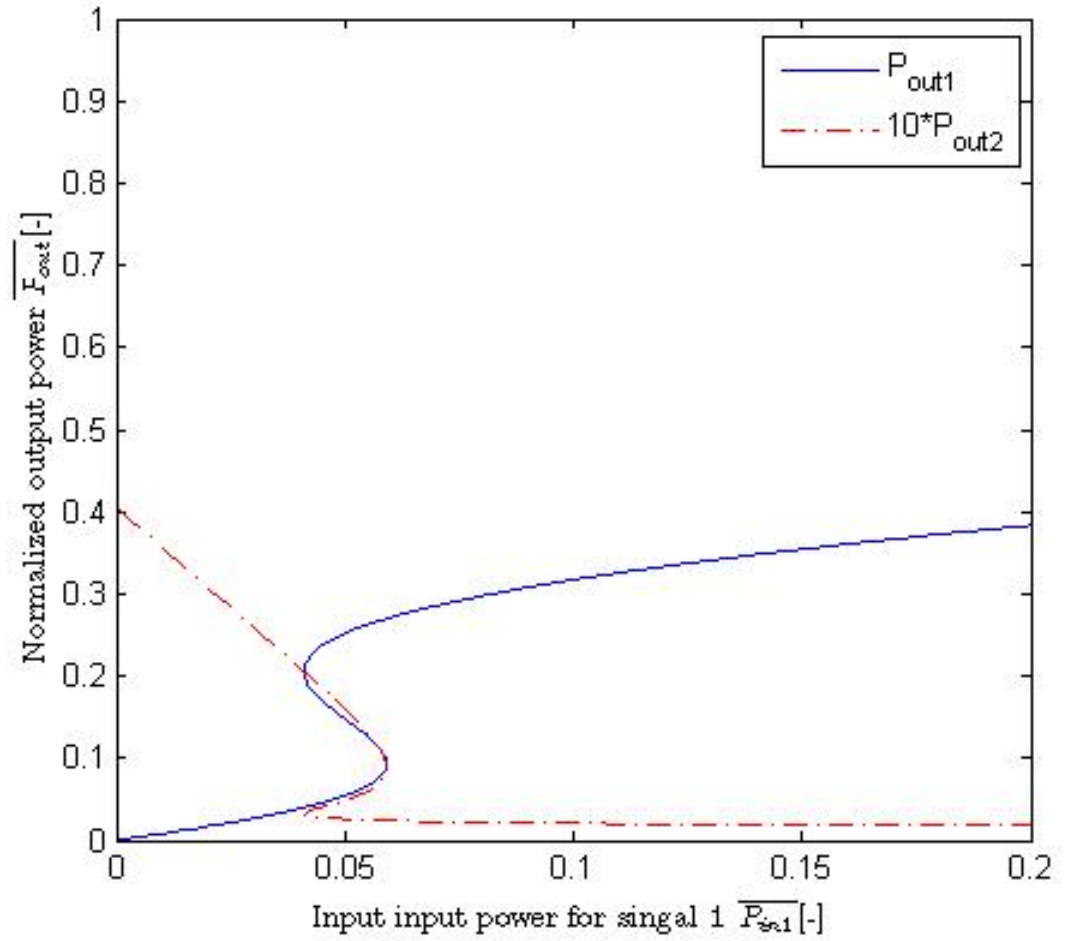


Figure 5.7.2: Normalized output power vs normalized input power

The output power increases with hysteresis with the increase in the input power.

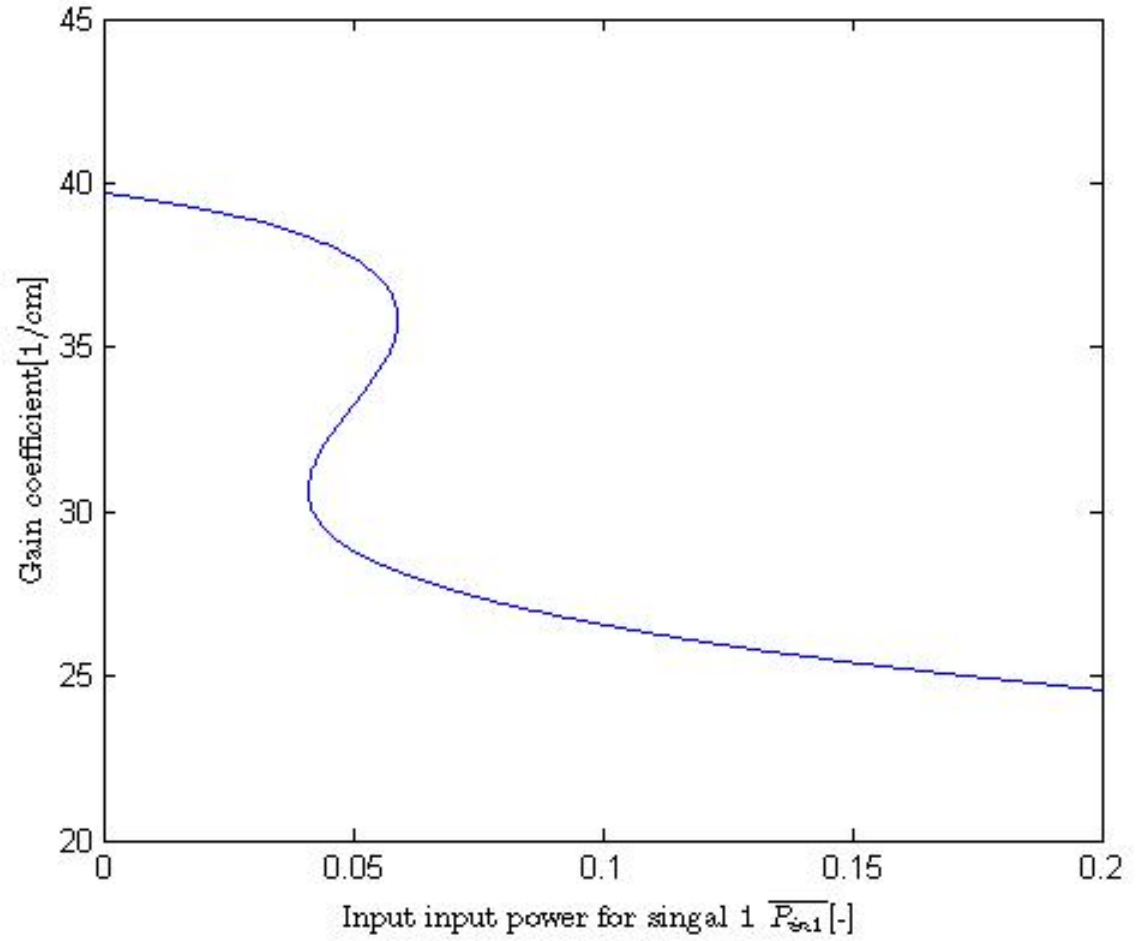


Figure 5.7.3: Gain coefficient vs normalized input power

The gain value decreases hysterically with the increase in the input power.

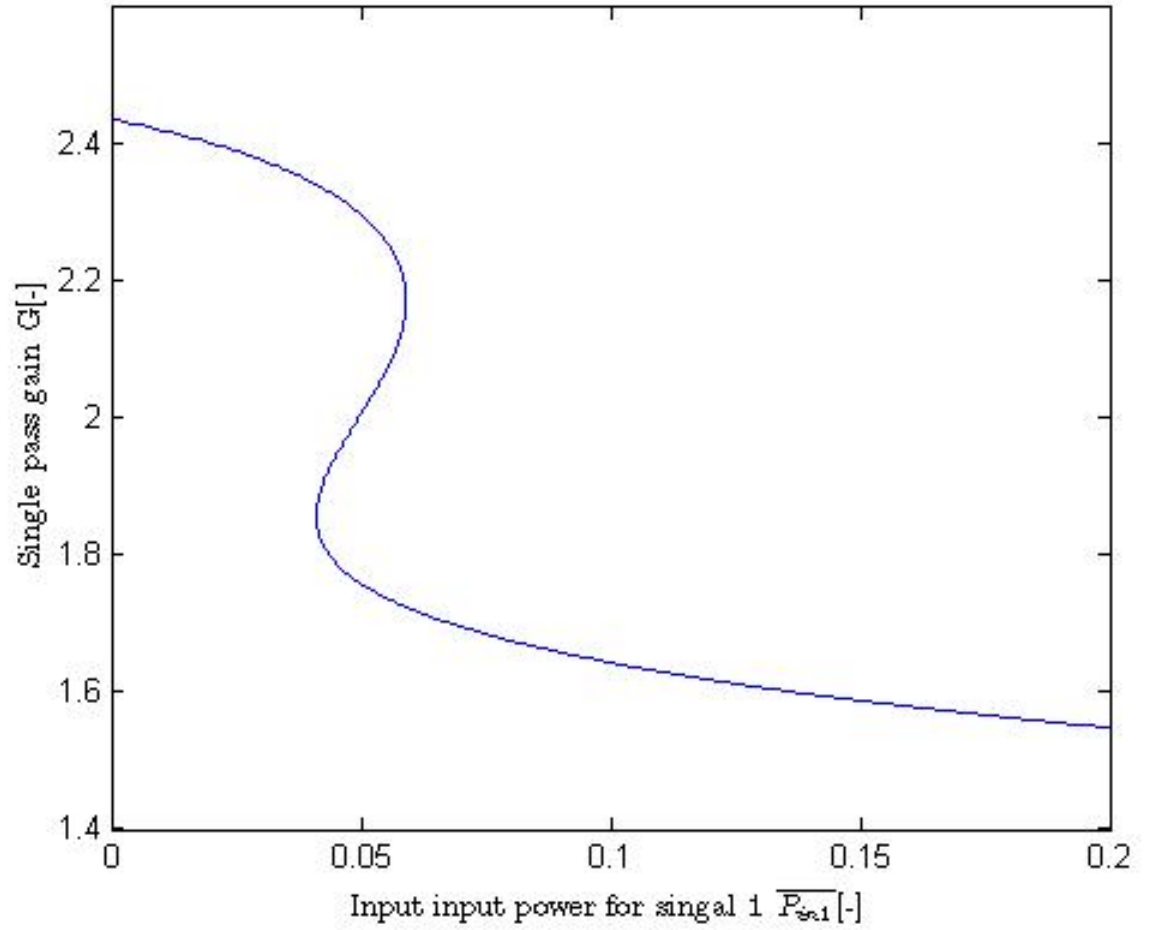


Figure 5.7.4: Single pass gain vs normalized input power

The gain value decreases with the increase in the input power.

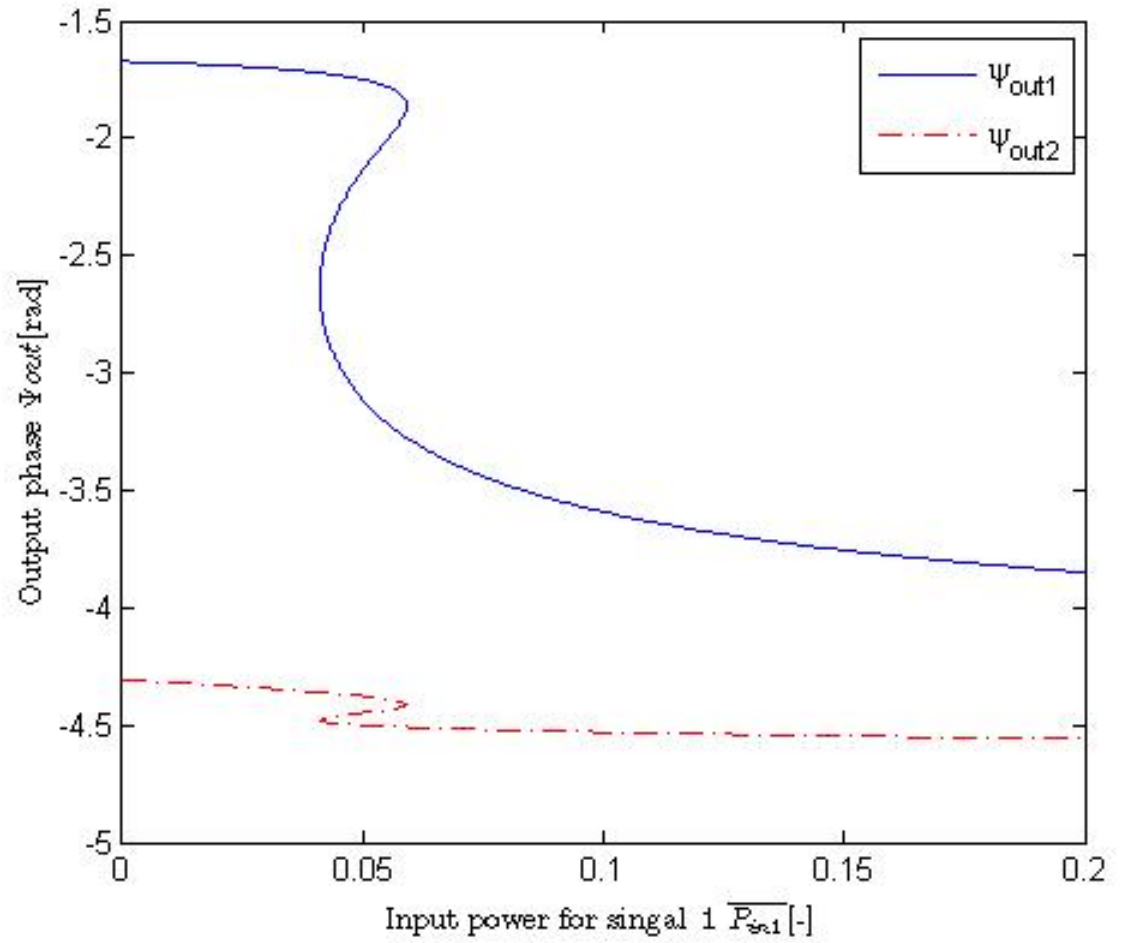


Figure 5.7.5: Output phase vs normalized input power

The output phase decreases hysterically with the increase in the input optical power.

5.8 Comparison of two models

This section compares the dual-optical signal case using the simplified model of Chapter 4 and the more detailed model of this chapter.

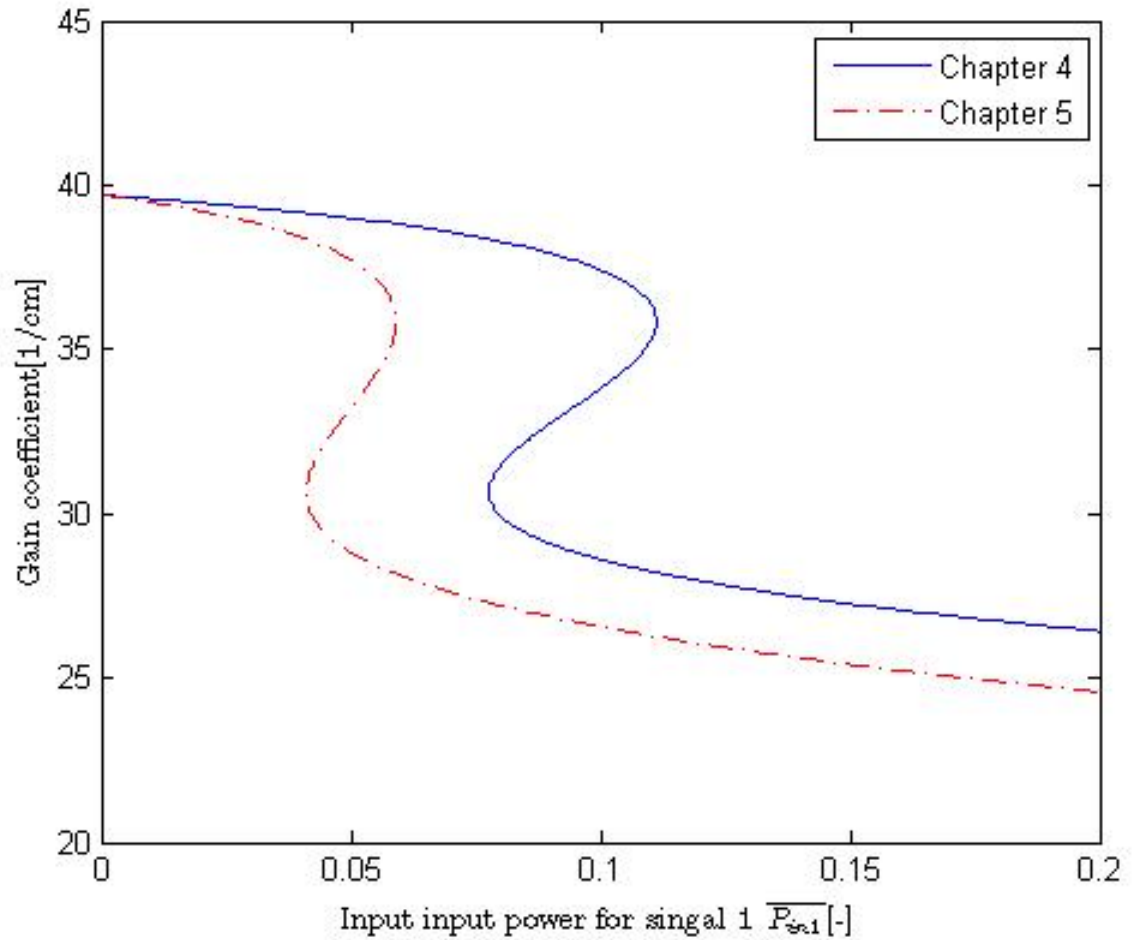


Figure 5.8.1: Gain coefficient vs normalized input power

The gain coefficient decreases with the increase in the input power.

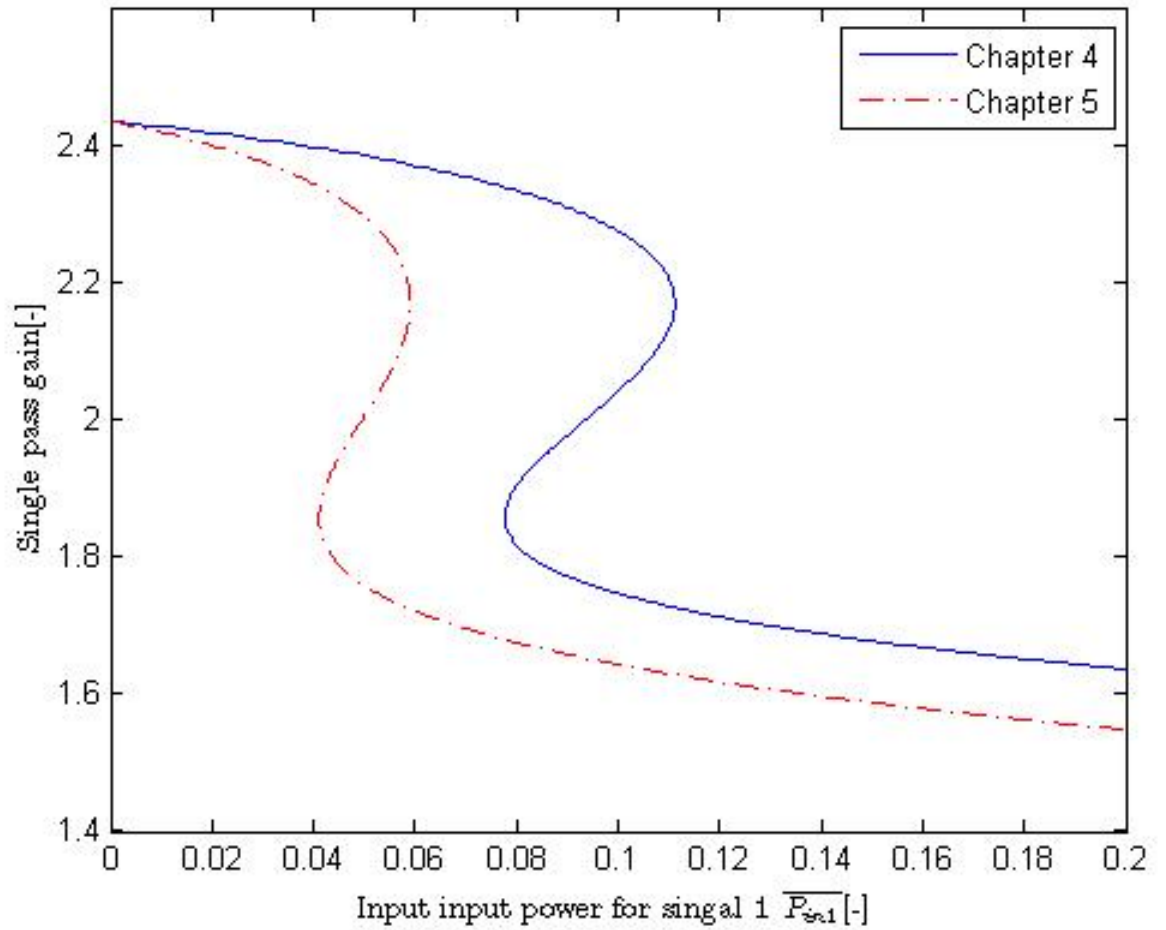


Figure 5.8.2: Single pass gain vs normalized input power

The single pass gain value decreases with the increase in the input power in both the cases. The gain values are found to be higher in the case of the chapter 4 than in case of the chapter 5 which is an extension of the model proposed by [2].

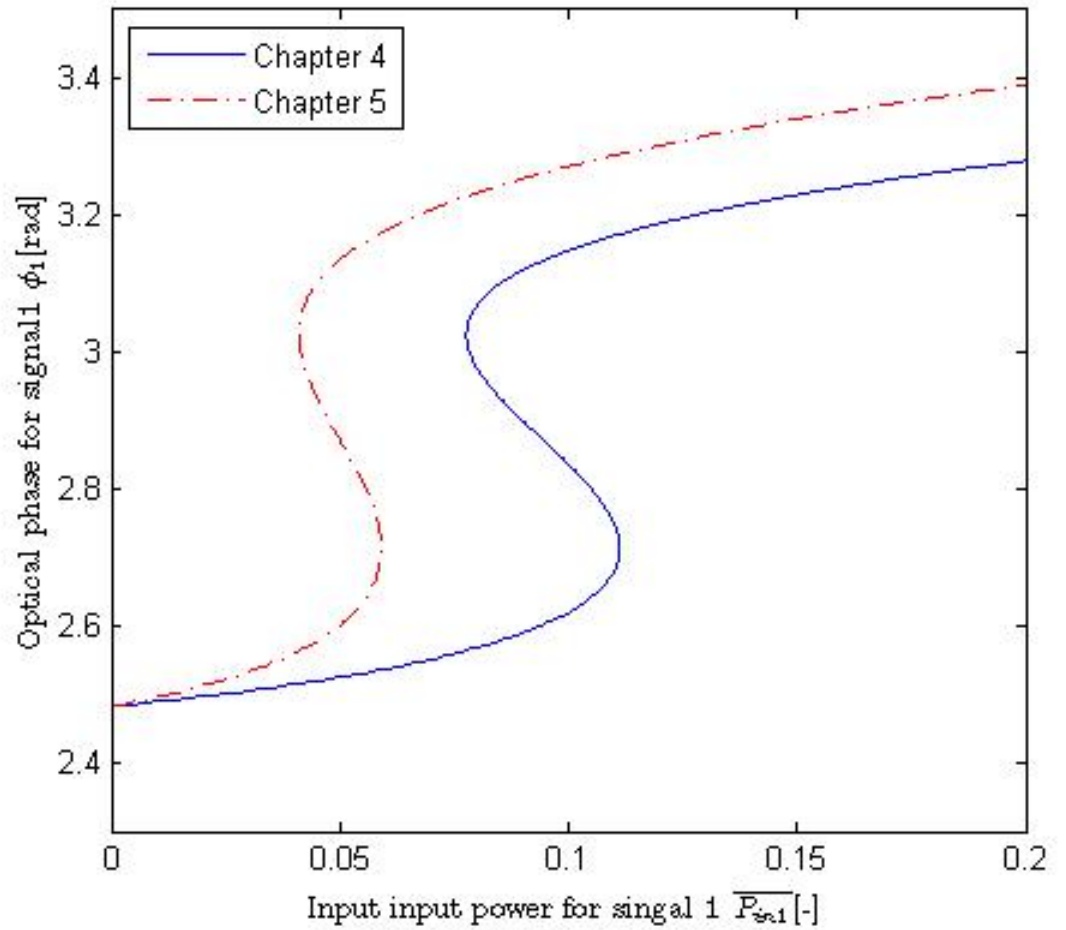


Figure 5.8.3: Optical phase for signal 1 vs normalized input power

The output phase calculated in both the chapters are increasing with respect to the input power. The chapter 5 method has initially lower phase for lower input power and tends to increase than the phase calculated in chapter 4 as the input power increases.

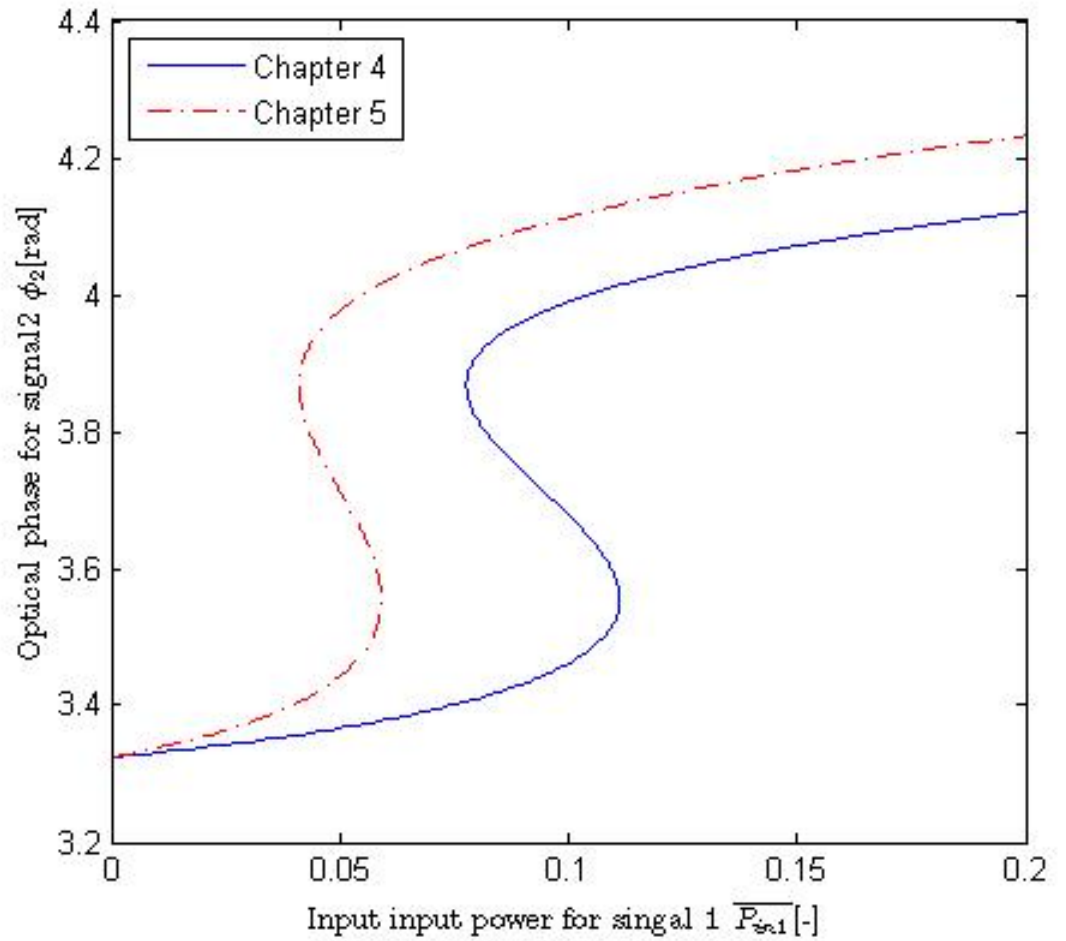


Figure 5.8.4: Optical phase for signal 2 vs normalized input power

The phase 2 in both in chapters are calculated with a initial detuning angle of π which is higher than the initial detuning of signal 1 which is 2.4. Hence the phase is higher range than the phase of signal 1 but the pattern is follows is similar. The phase calculated in chapter 5 starts at a lower value for lower range of input power and it increases as the input power increases.

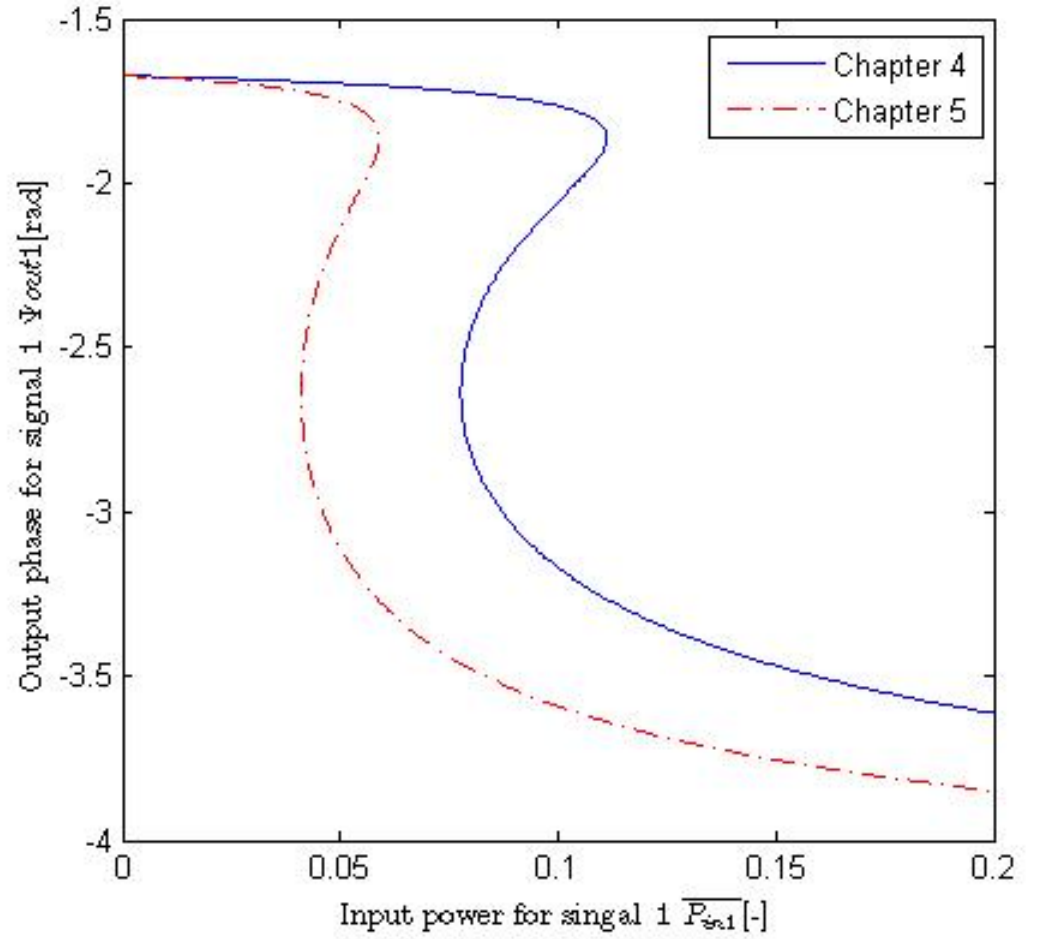


Figure 5.8.5: Output phase for signal 1 vs normalized input power

The output phase decreases with the increase in the input power.

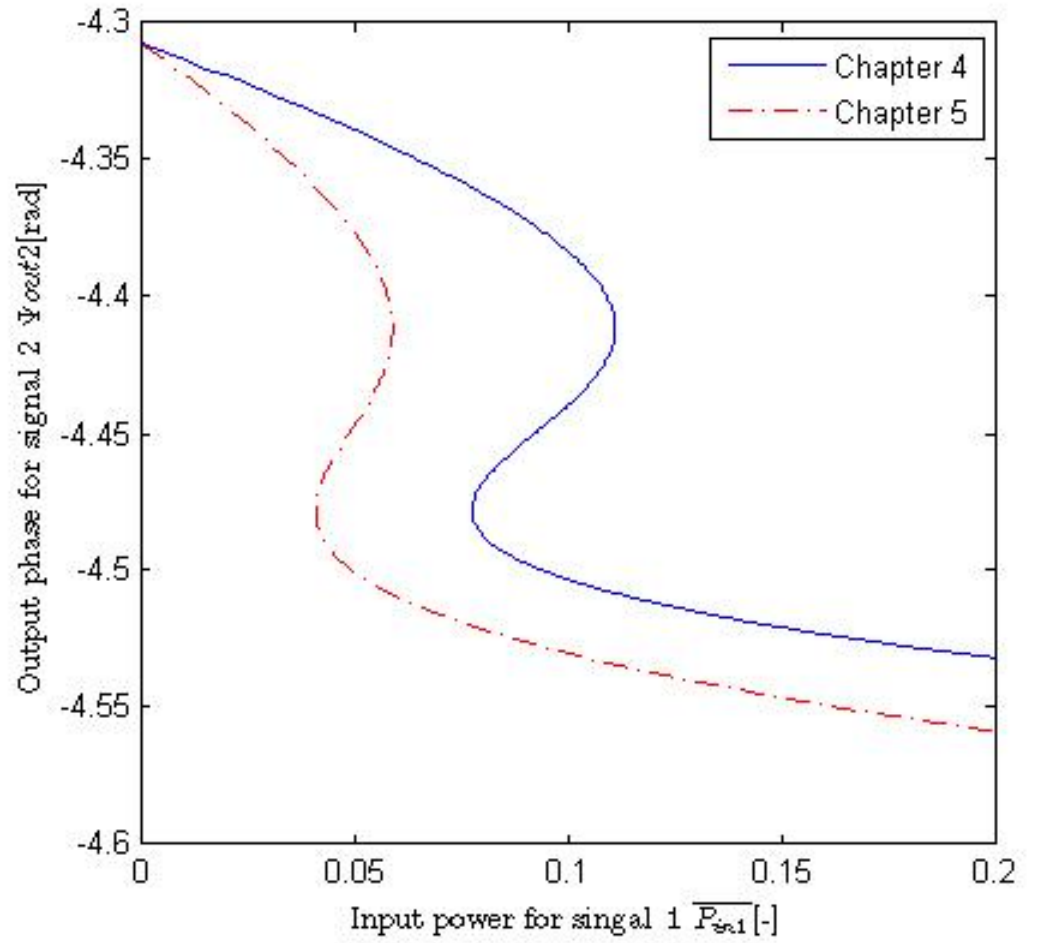


Figure 5.8.6: Output phase for signal 2 vs normalized input power

The output phase decreases with the increase in the input power.

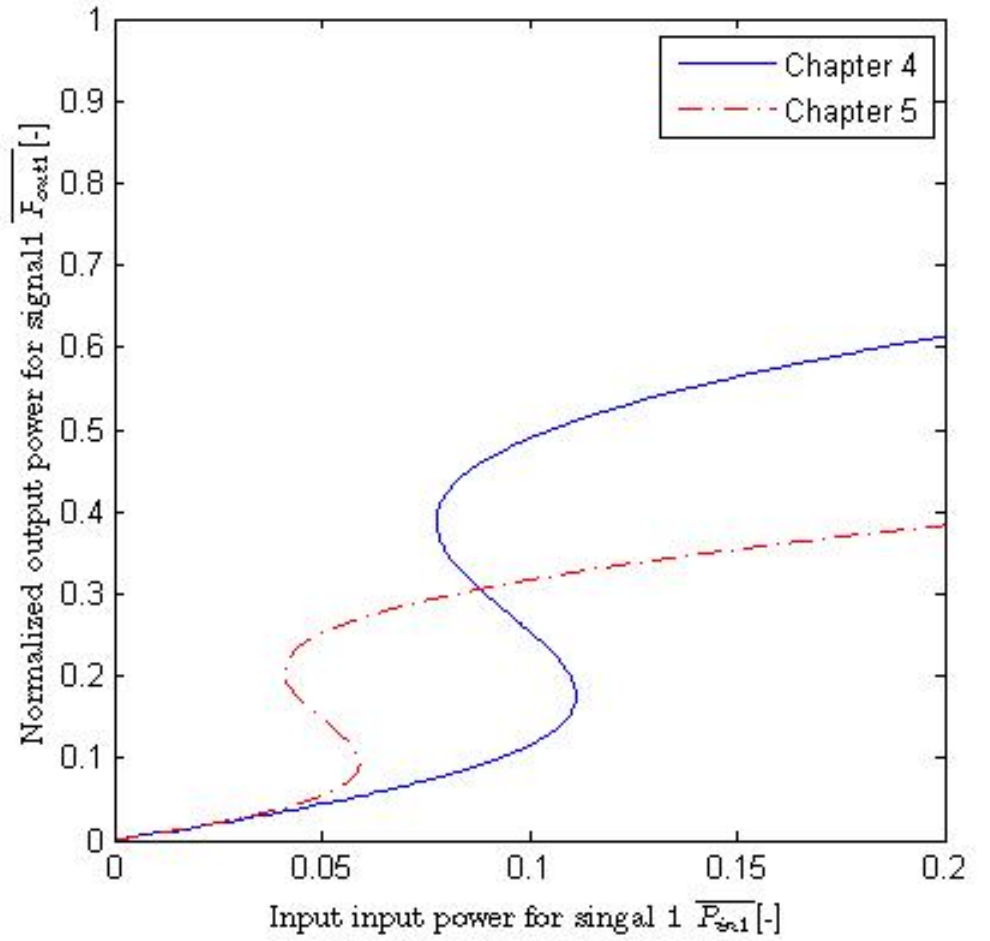


Figure 5.8.7: Normalized output power for signal 1 vs normalized input power

The output power increases with the increase in the input power.

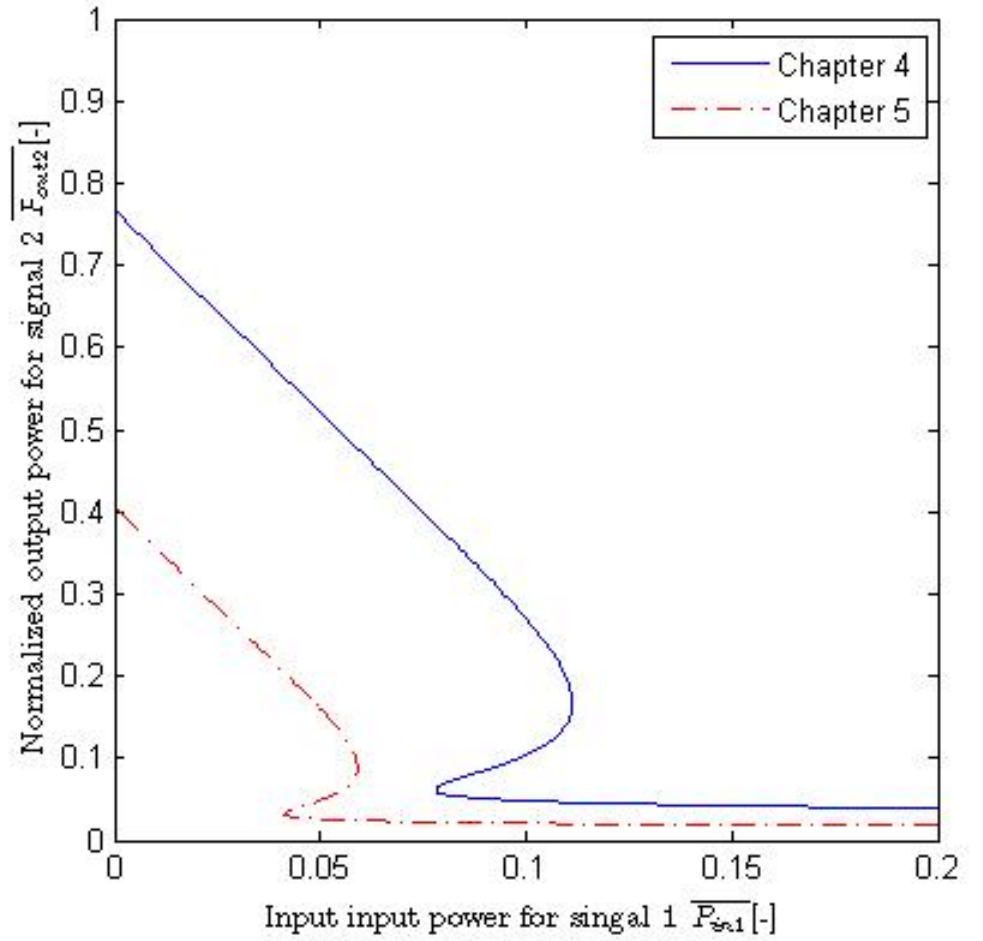


Figure 5.8.8: Normalized output power for signal 2 vs normalized input power

The output power increases with the increase in the input power.

5.9 Strong hysteresis

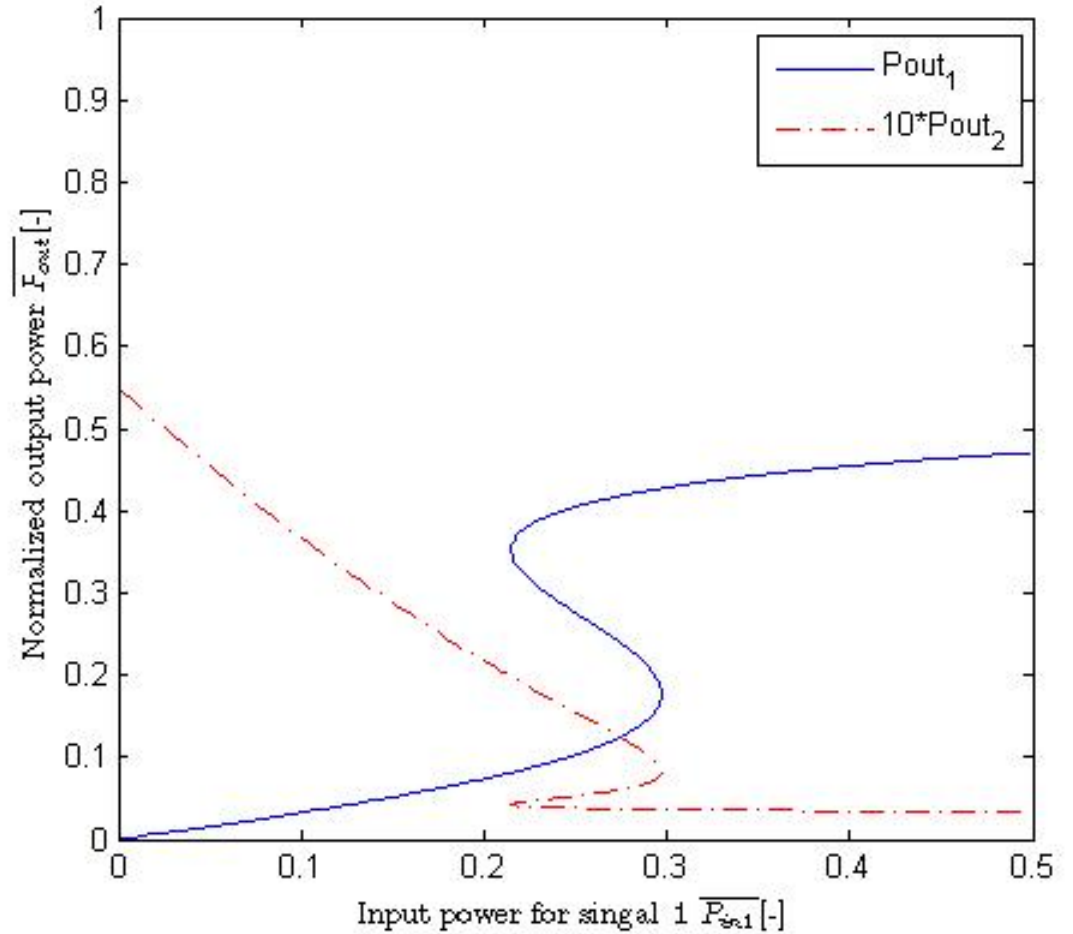


Figure 5.9.1: Normalized output power for signal 1 vs normalized input power
for signal 1

This is an example that proves that by varying ϕ_{01} , initial detuning angle, we can produce a strong hysteresis curve even using the approach in Chapter 5. The nature of the depth of the hysteresis curve depends on the detuning angle. In this

specific case, we use ϕ_{01} as 2 rad. Thus by varying the detuning angle, we can achieve strong hysteresis pattern in the calculation of output power. This is proved in the figure plotted against the normalized input power of signal 1.

CHAPTER 6

Conclusion

This thesis developed an improved model to simulate the performance of two optical signals propagating through a bistable semiconductor optical amplifier. The improved simulations show that the critical switching powers for bistability are lower than previously published. In the process of improving the model, we studied two published models of optical bistability for a single optical signal. These two models informed our work on the dual-signal model. This result and the model developed herein may be useful in future optical signal applications requiring multiple signals as input.

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