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n-Values and paper opacity

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MASTER'S THESIS

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n-VALUES AND PAPER OPACITY

by
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ABSTRACT

The n-value is a factor which allows the Murray-Davies equation to be correctly solved for physical dot area or tint density. This equation becomes known as the Yule-Nielsen equation when the n-value is used.

Many variables affect the value of n; paper characteristics, screen ruling, even the equation variables themselves. The level of solid ink density may have a very significant effect on the n-value. This fact, although suspected, has not been investigated until now.

A study into this variance was undertaken in Appendix II with a method for adjusting the value of n to a standard solid ink density given in the body of the paper.

If screen ruling is held constant, the only other significant factor affecting the n-value is the light-scattering properties of the substrate. This light-scattering may be inferred through TAPPI opacity measurements. This opacity figure may then be adjusted to a sheet of "standard" caliper to discount the influence of various thicknesses.

If the measurements can be made accurate enough, a mathematical relationship between the adjusted n-value and TAPPI opacity may be discovered through regression analysis.

The attempt made in this paper may be considered unsuccessful because of the low reliability of the opacity readings and other factors. More accurate measurements may establish a better correlation.

CHAPTER I

INTRODUCTION

THE HALFTONE PROCESS

Most printing processes are incapable of reproducing original images as continuous tone. Instead, original continuous tone images must be printed as discrete areas of solid ink covering blank paper. These discrete areas of ink on paper are usually printed as a pattern of small dots which are not resolvable at normal viewing distance and which create the illusion of continuous tone. By varying the ratio between the areas of solid ink and the areas of blank paper, levels of gray are created to reproduce the levels of gray in the original.

The number of times which a dot pattern repeats itself over a given linear distance is known as the screen ruling and usually varies from 65 to 200 lines (dots) per inch. However, very high quality halftone printing may sometimes be done with screen rulings of up to 300 lines per inch. The choice of screen ruling is usually dependent upon the printing process, paper characteristics, and the overall level of quality desired by the customer.

One of the largest influences affecting a halftone print is the paper. Since printing inks are fluid, they tend to spread when printed on paper. This is especially

true of absorbant papers. This phenomenon is known as "physical dot gain" and its occurance is associated with ink, paper, press, and even human factors.[1]

"Optical Dot Gain" (ODG) is a term that has been coined to describe the darker-than-expected tones produced by a halftone tint when printed on paper. It is generally stated that optical dot gain is a function of the light-scattering properties of paper which tend to diffuse the halftone pattern and reflect proportionally less light to the observer.[2]

STATEMENT OF THE PROBLEM

Because of these variables of optical and physical dot gain, the actual area of a press sheet halftone dot is a difficult thing to measure. To discover the actual size of a halftone dot, it is necessary to factor out the optical dot gain. This is done by using the Yule-Nielsen equation (discussed later as Equation 9) and the variables of measured solid ink density, measured tint density, and an n-value which allows the equation to be correctly solved.

This n-value is not a directly measurable quantity but rather must be found empirically or by a cumbersome method of inference. Lists of recommended n-values for paper types have been published. Since paper characteristics may vary widely within such general classes as coated, uncoated, etc., these values should be used with caution. For

example, uncoated paper has been quoted as having n-values which range from 1.41[3] to 2.70[4]. This would result in an error in calculating percent dot size of 11.5% for a solid ink density of 1.3.

Coated	1.65
Uncoated	2.70
Dupont Gevaproof	1.40
3M Color Key	4.00
3M Transfer Key	1.90

Table 1.
Quoted n-values.[5]

LPI	Coated	Uncoated
65	1.30	2.00
150	1.80	--
300	3.00	--

Table 2.
Quoted n-values.[6]

The main problem with using an n-value is that it cannot be easily and confidently determined for a given paper and screen ruling. Since n has long been stated to be an optical factor, a simple optical test of paper might be available to infer its value with reasonable accuracy. That is the premise for this paper.

FOOTNOTES TO CHAPTER I

[1]Southworth, Miles. "Dot Gain: Causes and Cures." Quality Control Scanner. Vol. 2 No. 9, pp. 1-4.

[2]Ibid. p. 1.

[3]Clark, Timothy W. "A Preliminary Investigation into the Effect of Select Paper Characteristics on Dot Gain in Web Offset Lithography." Master Thesis, School of Printing, Rochester Institute of Technology, June 1978. p. 16.

[4]Southworth. p. 4.

[5]Southworth. p. 4.

[6]Yule, J. A. C. and Nielsen, W. J. "The Penetration of Light into Paper and its Effect on Halftone Reproduction." TAGA Procedings, Vol. 3 (1951), p. 72.

CHAPTER II

THEORETICAL DISCUSSION AND LITERATURE REVIEW

HALFTONE THEORY

In investigating the behavior of a halftone print on paper, it is useful to first think in terms of a simplified illustration. Imagine that a halftone dot in a unit area looks as is does in Figure 1.

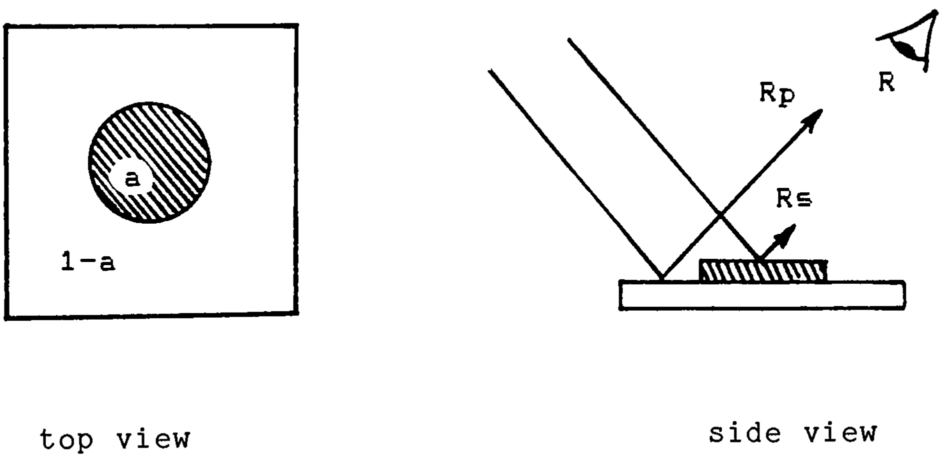


Figure 1.
Unit Area of a Halftone Tint

The symbols for Figure 1 are defined as follows:

R = Total reflectance of the tint
R_p = Reflectance of the paper; 0 to 1
R_s = Reflectance of the solid ink; 0 to 1
a = Relative area of the solid ink; 0 to 1
$1-a$ = Relative area of the blank paper; 0 to 1

An equation may be drawn to explain the relationship

between the total reflectance of light from the tint, given the relative reflectances of the paper and solid ink areas.

$$R = a(R_s) + (1\text{-}a)R_p \qquad (1)$$

For an ideal system where ink is a perfect absorber of light ($R_s = 0$) and paper is perfect reflector of light ($R_p = 1$), Equation 1 simplifies to Equation 2.

$$R = 1\text{-}a \qquad (2)$$

However, reflectance values are rarely used as units of measurement in the graphic arts. Instead, the modulation of light by a reflective surface is expressed in terms of density which is a logarithmic transformation of reflectance. The symbols for Figure 1 must by redefined.

D = Total density of the tint
D_p = Density of the paper; 0 to infinity
D_s = Density of the solid ink; 0 to infinity
a = Relative area of the solid ink; 0 to 1
$1\text{-}a$ = Relative area of the blank paper; 0 to 1

Following the rule that $D = -\log R$ and that $R = 10^{-D}$, Equation 1 may be rewitten in terms of density.

$$D = -\log[a10^{-Ds} + (1\text{-}a)10^{-Dp}] \qquad (3)$$

For an ideal system where ink and paper are perfect (D_s = infinity, $D_p = 0$, $10^{-Ds} = 0$ and $10^{-Dp} = 1$), Equation 3

simplifies fo Equation 4.

$$D = -\log(1-a) \tag{4}$$

However, it is not a perfect world and a different equation was necessary to take into account the inefficiencies of the printing process.

THE MURRAY-DAVIES EQUATION

The first significant study of halftone prints was conducted by Alexander Murray in 1936.[1] Murray felt that the density of a halftone tint was a function of "black dot area" and "the reflecting power of the ink." To test this relationship, he printed various levels of tints and then measured and calculated the dot areas with a microscopic reticule. The "reflecting power of the ink" was taken as solid ink density.

In collaboration with E.R. Davies, Murray wrote an equation to illustrate the relationship between dot area, solid ink denstiy, and tint density. This equation came to be known as the Murray-Davies equation.

$$D = \log \frac{1}{1-a(1-r)} \tag{5}$$

where:
D = Denstiy of the tint
a = Dot area
r = Reflecting power of the ink.

The selection of r as a variable is analogous to our use of R_s above. Since $R_s = 10^{-Ds}$ and $\log 1/x = -\log x$, then Equation 5 may be rewritten as Equation 6 which is the most common form of the Murray-Davies equation used to solve for the densities of tints. The more descriptive term, D_t, has been substituted for D.

$$D_t = -\log[1-a(1-10^{-Ds})] \quad (6)$$

To solve for dot area, the Murray-Davies equation takes the form of Equation 7. This dot area value is also known as Optical Dot Area (ODA).[2]

$$a = \frac{1-10^{-Dt}}{1-10^{-Ds}} \quad (7)$$

In deriving Equation 6 from Equation 1 which was written in reflectance terms, we find that the reflectance of paper (R_p) was taken as perfect.

$$R = a(R_s) + (1-a)R_p \qquad \text{for } R_p = 1 \quad (1)$$

$$R = a(R_s) + (1-a)$$

$$D_t = -\log\,[a(R_s) + (1-a)] \qquad \text{since } D = -\log R$$

$$D_t = -\log\,[a10^{-Ds} + (1-a)] \qquad \text{since } R_s = 10^{-Ds}$$

$$D_t = -\log\,[(1-a) + a10^{-Ds}]$$

$$D_t = -\log\,[(1-a(1-10^{-Ds})] \quad (6)$$

By zeroing his densitometer to blank paper, Murray felt that he could disregard the reflection of paper in his calculations. He concluded that the equation worked as expected.

By his own admission, Murray's methods of measurement and calculation of dot areas were somewhat inexact. To obtain the best measurements of dot area possible, he sought the best quality papers to proof his tints. Because of the inaccuracies in measurement and because he was using highly efficient paper, Murray did not introduce a disproportionate amount of error into his calculations by taking paper as a perfect reflector ($R_p = 1$). If Murray and Davies felt that paper did play any role at all in solving their equations, they did not feel that it was important enough to consider. This idea that paper was an insignificant factor in measuring dot areas persisted into the early 1950s.[3]

However, when the Murray-Davies equation was used to solve for tint densities and dot areas, it was found to give inconsistent results. Since ink is not a perfect absorber of light, it was expected that a 50% tint would absorb less than 50% of the light reaching it. Yet this was not observed to be the case. A 50% tint was found to absorb more than 50% of the light.

John Yule noted this darkening effect in a 1943 research paper and assumed that the penetration of light into the paper might possibly have been at fault.[4] In 1950,

Williams confirmed that light scattering within the paper can interfere with densitometric analysis.[5]

THE YULE-NIELSEN EQUATION

In 1951, Yule and Nielsen published the results of a study which indicated that the reflection of light from a halftone print was not a simple matter.[6] They postulated that multiple internal reflections and other factors influenced the correct solution to the Murray-Davies equation. They attempted to write an equation that would take into account some of the optical variables but soon realized that such an equation would be very involved and would require more investigation.

Instead, they settled on what has come to be known as an n-value which modified the Murray-Davies equation so that it could be solved with reasonable accuracy. This n-value was applied to construct the Yule-Nielsen equation which follows.

$$D_t = -n \log[1-a(1-10^{-Dt/n})] \qquad (8)$$

To solve for dot area, the Yule-Nielsen equation takes the form of Equation 9. This dot area value is also known as Physical Dot Area (PDA).[7] For the purpose of what follows, Physical Dot Area (PDA) should not be confused with Actual Dot Area (ADA) since these quantities may be different under different conditions.

Actual Dot Area shall be defined as the area of a halftone dot as it truely exists on a press sheet. This area may be discovered by direct measurement with a planimeter.

Physical Dot Area shall be defined as the area of a halftone dot as computed by the Yule-Nielsen equation using a given n-value to factor the equation to obtain Actual Dot Area.

It should be noted that the quantites of Actual Dot Area and Pysical Dot Area are not equal when the solid ink density is changed and the n-value is held constant (see Appendix II).

$$a = \frac{1-10^{-Dt/n}}{1-10^{-Ds/n}} \tag{9}$$

It has already been stated that the n-value has the disadvantage of not being easily determined. Its value must be inferred by comparing a known value for actual dot area to a value of that same area measured on diffusing materials, i.e. paper. By comparing the results, a value for n may be derived by iteration.

In use, the n-value has become sort of a "catch-all" for all of the variables which prevented the Murray-Davies equation from being accurately solved. Yule and Nielsen were quick to point out that this n-value had no theoretical

basis and represented an empirical derivation. Yet, they found that it did fit the observed facts quite well.

FACTORS AFFECTING THE VALUE OF n

A study published in 1979 by Milt Pearson commented on the variables which affect the value of n.[8]

> "of the factors contributing to a correct value of n for a given condition, dot area level is the least significant contributor. Screen frequency is the most significant. This implies that the value of n changes faster with the changes in frequency than with any other parameter. However, in practice the frequency is usually fixed which leaves variations in the substrate as the biggest factor affecting n."

LIGHT-SCATTERING: These "variations in the substrate" have been traditionally catagorized in terms of light-scattering properties. In a 1953 article, Clapper and Yule discovered that multiple internal reflections (light-scattering within paper) do indeed play a major part in the darkening of halftone prints.[9] They also identified the non-specular portion of first-surface reflections as playing a smaller role. It was believed necessary to incorporate this quantity because of the pick-up geometries common in densitometers of the time. On the light scattering properties of papers, they commented:

> "with a fine screen and translucent paper, multiple internal reflection and scattering could result in a density as much as three times as great as that predicted from dot area by the original simple formula (of Murray-Davies). This

> is more than enough to account for the greatest discrepencies between dot size and calculate density which have been observed."

To illustrate the role which paper plays in the darkening of halftone prints, Clapper and Yule included a simple model in the form of a drawing which depicted all of the possible interactions of light with the materials of ink and paper.

Accompanying this drawing was a rigorous mathematical analysis of the relationships involved taking into account the number of times light would cycle through the paper and what portion of it would ultimately be reflected or absorbed. The equation which was written was a significant step forward in describing and understanding the mechanics of light reflection, scattering, and absorption by the paper and ink.

Yet, Clapper and Yule's equation was of no more practical use than the Yule-Nielsen equation. Instead of an n-value as an all encompassing variable which could not be directly measured, this new equation contained six variables, five of which could not be directly measured. In the final analysis, the Yule-Nielsen equation continues to be quoted and used primarily because of its simplicity and simply because it has been shown to work.

Clapper and Yule sought to keep their illustration simple for mathematical modeling. They analyzed internal light scattering in terms of cycles which were determined by

the number of times a ray of light would strike an air/solid interface and be reflected.

Of course, paper is made up of an enormous quantity of fibers and fillers which create an enormous quantity of air/fiber, air/filler, air/ink, fiber/filler, fiber/ink, and filler/ink interfaces which in turn influence the amount of light scattering within a sheet. Every time light strikes a fiber, filler particle, or an ink film, a portion is reflected, a portion transmitted, and a portion absorbed. Transmitted light will also be refracted at the interfaces which further contributes to the diffusion of light within the paper. Clapper and Yule's equation sought to quantitize all of these variables.

The effect that this light-scattering has on densitometer measurements has already been indicated. The mechanism of this effect should be discussed.

Before any readings are taken, a densitometer must be calibrated to some known standard. This usually consists of one or more calibration plaques which are supplied with the instrument. The plaques are read by the instrument and the readings adjusted until they agree with the stated densities on the plaques. Having accomplished this, the densitometer is ready to read a paper sample.

Figures 2 and 3 represent stylized diagrams of light-scattering within paper. Figure 2a illustrates the light from the instrument striking, entering, and being scattered

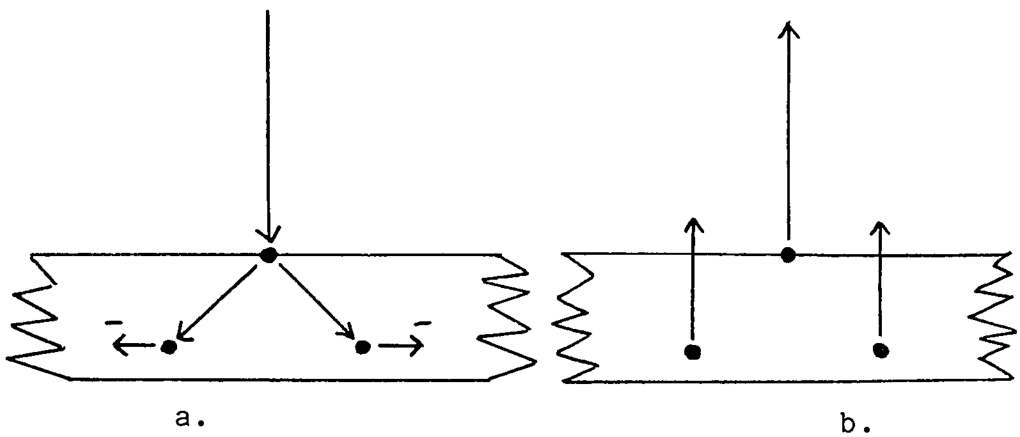


Figure 2.
Light-Scattering and Light-Absorption in Blank Paper.

within the paper. The bullets (•) are imaginary reflection points which represent all of the air/solid interfaces which redirect the light. The smaller arrows with minus signs represent the quantity of incident light absorbed by the paper and lost to the instrument. It is this quantity that the instrument will convert to a density reading.

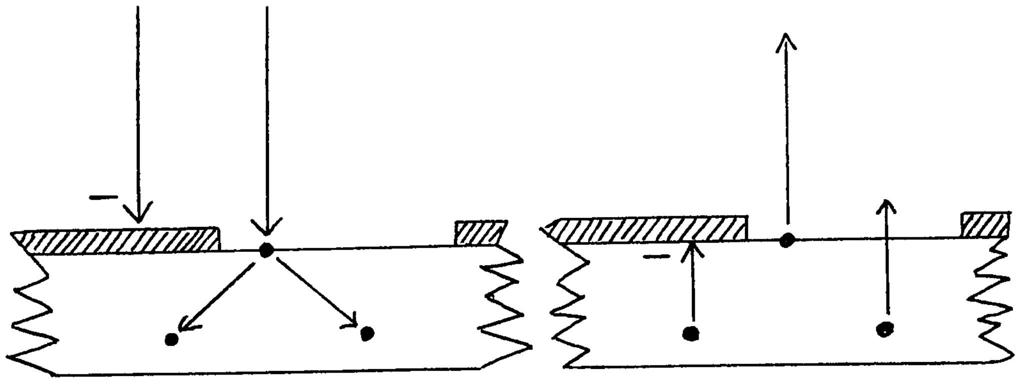
Figure 2b illustrates the light being returned to the instrument. The large central arrow represents the main quantity of light which has been returned by the paper surface. The two smaller arrows (adjacent to the larger arrow) represent the smaller quantity of light which has re-emerged from the interior of the paper after having been scattered. All three arrows together represent the total amount of light received by the instrument.

Since the Murray-Davies and Yule-Nielsen equations call for paper to be taken as a perfect reflector, these quantities of absorbed light must be removed from the

reading. This is done by "nulling" the densitometer to paper by adjusting the instrument to read zero while reading the paper surface. Thus, the quantities of light absorbed by the paper are ignored.

The halftone pattern introduces a special problem in the measurement of tints in terms of dot area. Figure 3a represents the light from the instrument striking and being scattered within the paper. It also shows a portion of the light being absorbed by the halftone pattern. Note that the quantity of light absorbed by the paper has been removed.

Figure 3b illustrates the light being returned to the instrument from the paper surface and interior. However, note that a quantity of light which has been scattered under the ink layer has been absorbed by the halftone underside of that layer. This makes the tint appear darker to the instrument.

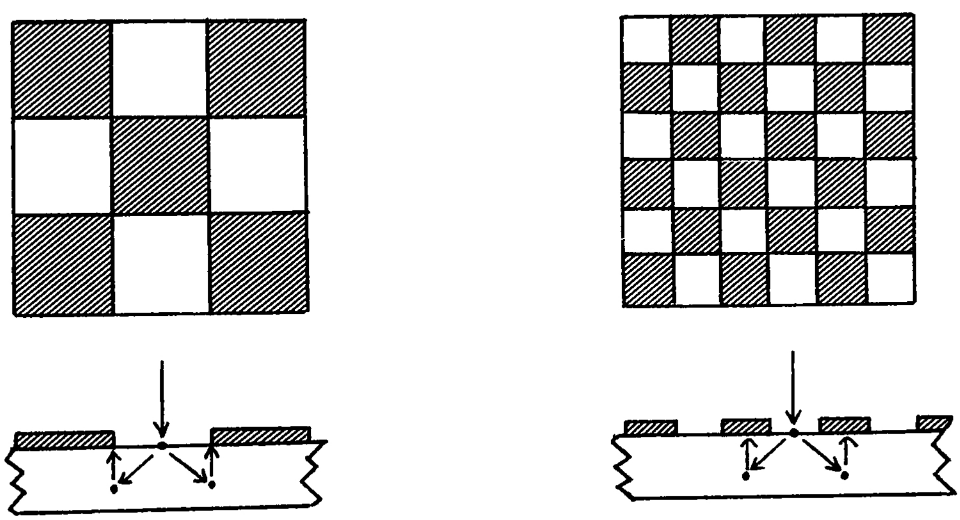


a. b.

Figure 3.
Light-Scattering and Light-Absorption
in a Halftone Print on Paper.

Screen ruling is also a factor affecting the value of n. For two identical papers, a 50% tint printed with a fine screen ruling will appear darker than the same tint level printed with a coarse screen ruling. This is because of the shorter distance light must travel within the paper to be absorbed by a dot instead of retransmitted to the instrument (see Figure 4).

Since finer screen rulings cause densitometers to measure halftone tints as more dense than they otherwise would be, the n-values must be higher to properly factor the solid ink density to obtain a correct solution for dot area.



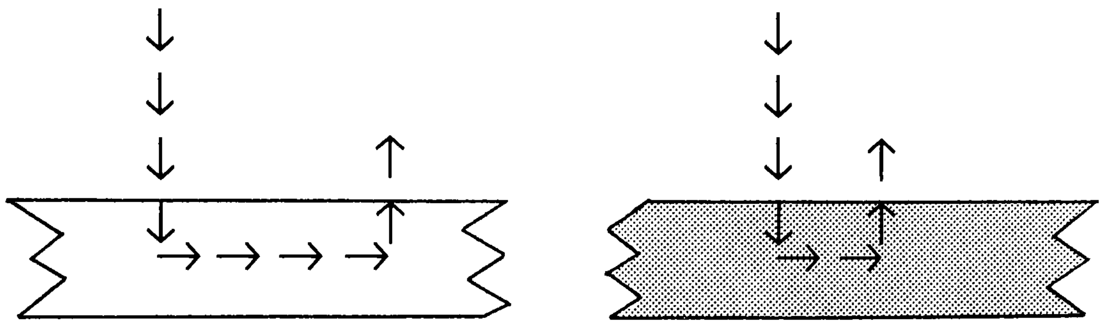
a. 100 Lines per Inch

b. 200 Lines per Inch

Figure 4.
Light Retransmittance and Halftone Dot Interference

LIGHT-ABSORPTION: Just as important as the light-scattering properties of a sheet are its light-absorbing properties. The distance which light can travel within a

sheet before being retransmitted is limited by the amount of light which the sheet will absorb. Sheets with low bright-



High Brightness Sheet

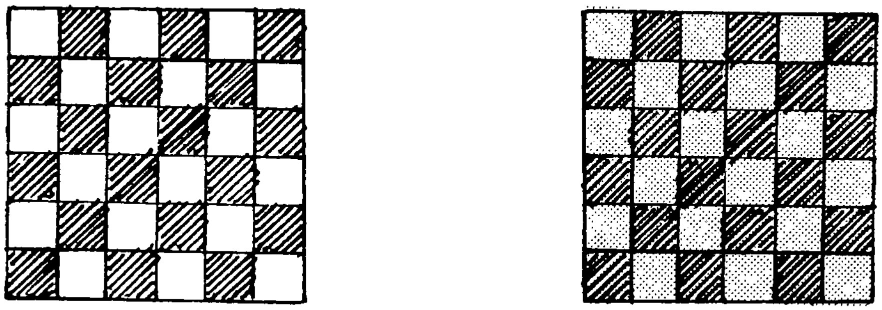
Low Brightness Sheet

Figure 5.
Limitation of Light-Scattering
and Retransmittance by Absorption of Light

ness will absorb more light than brighter sheets. Paper may also absorb selective wavelengths of light giving them a colored appearance.

By nulling a densitometer to a press sheet, the density of the paper is taken as zero. This is done to eliminate the effect of the paper when measuring solids and tints. As we have seen, this is not entirely true because of light-scattering within the paper. Yet, even though the paper has now been taken as zero density, the distance that light will travel within the paper has not been taken into consideration. As scattering is inhibited by absorptions, we might expect the n-values for darker papers to decline. As scattering increases throughout brighter sheets, n-values should increase. How far light travels within a sheet is

determined as much by the scattering within that sheet as by how much light will be absorbed. Figure 5 illustrates this effect for a light colored paper and a dark colored paper.



a. 200 LPI (Light Sheet)
Higher n-Value

b. 200 LPI (Dark Sheet)
Lower n-Value

Figure 6.
The Hypothetical Change in n-Value
as a Function of Sheet Brightness.

Figure 6 shows the hypothetical effect that a darker paper may have on n-values. Figure 6a should have a higher n-value than Figure 6b. This is due to the decreased amount of light scattering due to the increase in light absorption in the lower brightness sheet. This effect may be offset to some degree by the reduction in solid ink density when the densitometer is nulled on darker sheets.

EQUATION VARIABLES: There are certain assumptions associated with the Murray-Davies and Yule-Nielsen equations. We assume that it operates properly for all dot area levels (tonal values) due to experience. Yet no mathematical model has ever been devised to test this assumption.

Also, if we accept the fact that optical dot gain is caused by light-scattering within paper (all other things being equal), then we should assume that the optical dot gain may be factored out by finding a unique value for n. In fact this assumption is not true because the n-value is not unique with respect to solid ink density.

Dot Area Level: It has already been mentioned that dot area levels were not very significant as contibuting factors to the variance of n (see page 12). Theoretically, the relationship between the n-value and dot area should be mathematically describable and should provide information as to how the Yule-Nielsen equation behaves throughout the entire range of tonal values, not just selected dot areas.

A mathematical model was developed to test this relationship in Appendix I and it was found that the Yule-Nielsen equation does indeed behave remarkably well throughout the range of dot areas. Of course, this agrees with experimental results and is not really a surprise. The only substantial variance noted was in the extreme shadows, and only with relatively high n-values.

Tint Density: Both tint density and solid ink density are inextricably linked. As solid ink density increases, so does the density of the tint. This relationship has been well established by the Murray-Davies equation and need not be challenged. So, in investigating the variance of n, we need only look at solid ink density.

Solid Ink Density: Changes in solid ink density do indeed have an effect on n-value. This effect can be significant and is not the fault of any ink/paper mechanism but rather in the Yule-Nielsen equation itself.

If the actual dot area (ADA) is known, then n-value may be calculated by iteration using Equation 9, obtaining the n-value when physical dot area (PDA) is found to equal actual dot area (ADA). The optical dot area (ODA) may directly calculated using Equation 7. The optical dot gain (ODG) may be obtained by subtracting ADA from ODA.

If we were to change the solid ink density and recalculate the n-value using Equation 6 directly and Equation 9 by iteration, we find that he n-value is different. In some cases, this difference can be substantial (see Appendix II).

To find how these different n-values may affect optical dot area, we can use the latter n-value with the new solid ink density to find ODA using Equations 8 and 7. The new optical dot gain may then be calculated and compared to the original optical dot gain number. Again, these differences may be substantial given certain conditions (see Appendix II).

By just how much solid ink density influences the value of n may be quickly illustrated using an example. Table 3 on the following page contains the data for such an example. Column 1 contains a hypothetical set of numbers which is the

solution to some imaginary function.

It is believed that each number is related to the corresponding n-value in Column 2. These n-values were calculated from information provided in Columns 3, 4, and 5.

1	2	3	4	5	6	7	8	9
Units	Measured n-value	D_s	D_t	ADA	ODG	D_s	D_t	Adjusted n-value
.07	1.643	.62	.306	.60	6.5%	1.3	.433	1.276
.08	1.234	1.89	.456	.59	6.8	1.3	.426	1.294
.20	1.578	1.15	.406	.55	10.4	1.3	.422	1.510
.25	1.894	.95	.398	.56	11.6	1.3	.446	1.600
.31	1.547	1.70	.554	.61	10.6	1.3	.521	1.708
.39	2.176	1.06	.534	.64	13.5	1.3	.579	1.852
.42	1.694	1.65	.565	.60	14.5	1.3	.534	1.906
.53	1.755	1.85	.517	.54	16.5	1.3	.482	2.104
.65	1.891	1.73	.662	.63	16.7	1.3	.615	2.320
.77	2.195	1.50	.690	.65	15.6	1.3	.659	2.536
.81	2.174	1.55	.602	.58	16.7	1.3	.571	2.608
.90	2.002	1.95	.619	.57	15.6	1.3	.568	2.770

Units: arbitrary units to be compared to n-values. (1)
Measured n-value: The n-value as calculated from the sample. (2)
D_s: Solid ink density for measured sample (3) and adjusted sample. (7)
D_t: Tint denstiy for measured sample (4) and adjusted sample. (8)
ADA: Actual dot area of the measured sample. (5)
ODG: Optical dot gain of the sample (from Murray-Davies). (6)
Adjusted n-value: The n-value calculated from D_s 1.3. (9)

Table 3.
Measured n-Values Adjusted to a "Standard" Solid Ink Density

To discover the relationship between the numbers in Column 1 and 2, it was decided to perform a regression, the results of which are given in Figure 7. In using a second degree polynomial, note that the correlation is relatively loose and not really good.

However, by adjusting the value of n to a "standard"

solid ink density of 1.3, the correlation can be improved. Columns 5 and 6 were kept as constants in calculating the new tint densities and n-values.

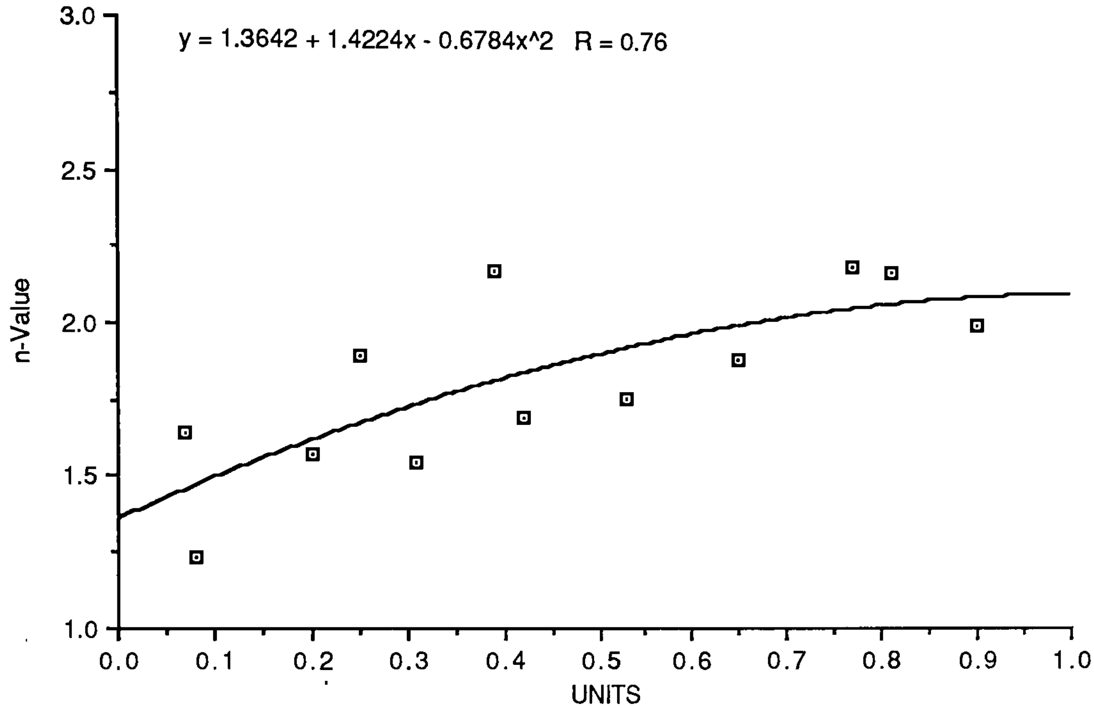


Figure 7.

A Regression/Correlation Plot of
Units and n-Values.

By regressing Column 1 against the adjusted n-values in Column 9, we find that the correlation has been improved to such a point that the relationship may be described as linear and perfect (see Figure 8).

It may be concluded that n-values are not only dependant on the light-scattering characteristics of paper and on screen ruling, but also very dependant on solid ink density. This is significant when trying to characterize

substrates with high optical dot gain (such as off-press proofs) and when using inks of widely differing target densities (such as process inks). It has been discovered that lower solid ink densities exhibit higher n-values. This helps to explain the very high n-values which have been

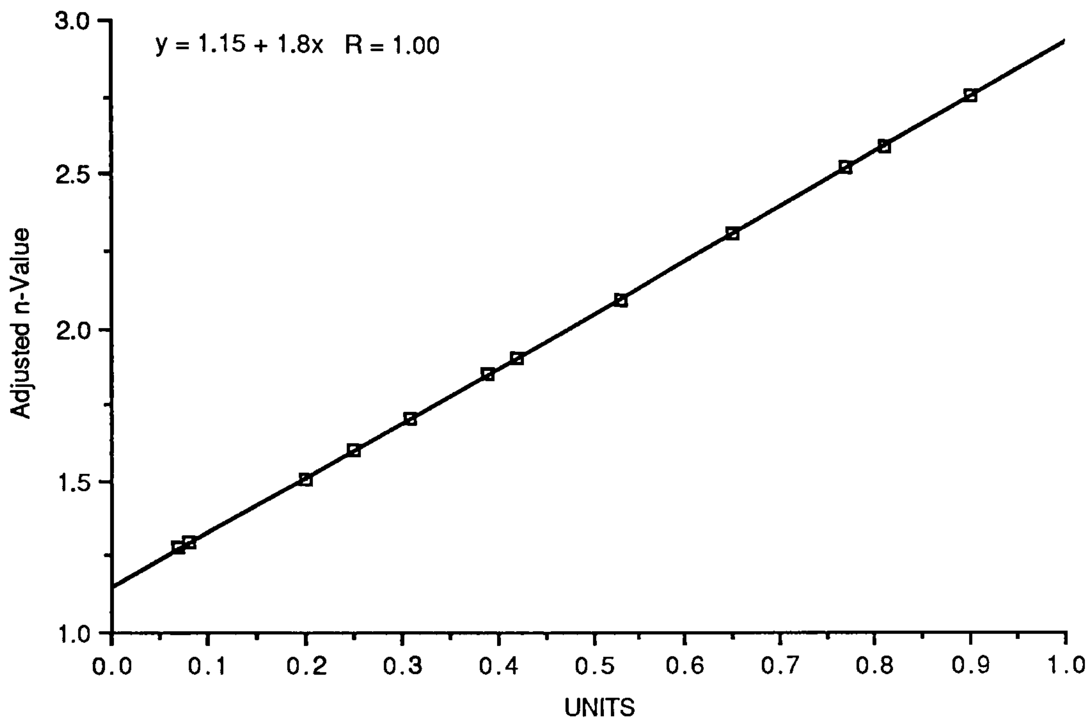


Figure 8.

A Regression/Correlation Plot of
Units and Adjusted n-Values

PAPER TESTS AS POSSIBLE n-VALUE PREDICTORS

Standardized tests have been developed to evaluate the properties of paper. While the use of these tests are geared primarily towards manufacturing quality control and printability evaluation, their use as predictors of print quality has also been acheived.

PAPER SURFACE EFFICIENCY: One such example of a print quality predictor is Paper Surface Efficiency (PSE) which was first described in a 1962 paper by Frank Preucil.[10] He devised an equation whereby he defined Paper Surface Efficiency as a function of paper absorptivity and paper gloss (Equation 10).

$$PSE = \frac{(100 - A) + PG}{2} \tag{10}$$

where: PSE = Paper Surface Efficiency
A = Absorptivity
PG = Paper Gloss

A value for absorptivity is derived from a K&N test as shown in Equation 11.

$$A = 4/3\ (100 - K\&N\%) \tag{11}$$

where: A = Absorptivity
K&N% = Percent of light absorbed by K&N test spot

Paper Surface Efficiency has proven to be a good predictor of the variation in ink color when printed on various substrates. Very good correlations have been found between the changes in PSE values and the changes in hue and grayness for process color inks and their overprints. Data has also been shown to be useful for mask factor prediction

and to evaluate process ink efficiency.

The two PSE variables of paper gloss and absorptivity are curious companions. Paper gloss is an optical property of paper which may be measured directly. It is an indicator as to the degree of surface reflecting power of the paper.

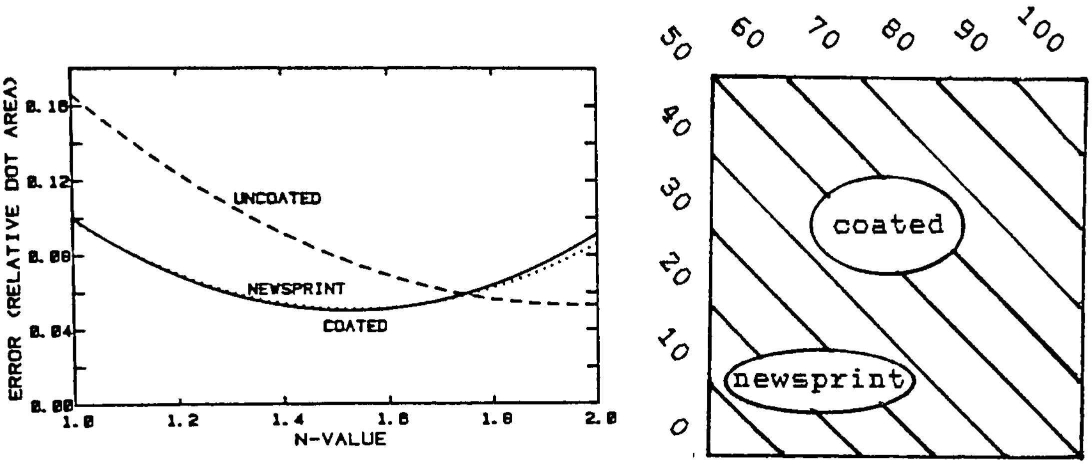
Unlike paper gloss, absorptivity is not an optical property of paper but a derivation of ink absorption, a physical property of paper which may be loosely correlated to internal structure. For example, if paper fibers are closely packed or if fillers are added, the ink absorption and light scattering within the sheet would be expected to be low. Conversely, if paper fibers were loosely packed with no fillers added, ink absorption and light-scattering within the sheet would be high.

By analogy, there should be a relationship between n-value and PSE. In practice this has been found not to be so.

The lack of correlation between n-value and Paper Surface Efficiency may be inferred in the Milt Pearson study[11] and a chart constructed by Zenon Elyjiw from experimental data.[12] This chart grouped various paper types with PSE.

In the Pearson study, it was found that the n-values for newsprint and coated paper were almost identical (about 1.5) for a given set of general conditions (Figure 9a). Since newsprint has been shown to have a PSE of about 20 and

a coated paper a PSE of about 60 (Figure 9b), it can readily be seen that PSE would be a poor predictor of n-values.



a. n-Values (newsprint & coated) (best value = low curve point)

b. PSE Values (newsprint and coated)

Figure 9.

It is assumed that newsprint does not fit into the scheme of things because of its dual properties of high ink absorbancy and relatively high opacity. This relatively high opacity is due to the use of groundwood and unbleached fibers which increase light absorption. The high opacity of coated papers is of course due to filler content.

POROSITY: Paper porosity has often been stated as the reason for light scattering within paper. All other conditions being equal, this is certainly true. The more porous a paper, the more air/fiber interfaces. This will cause more light-scattering within the sheet. However, as with ink absorbancy, porosity is not an optical

characteristic of paper but a physical one. Porosity refers to the ability of a paper to resist or enable the transmittance of gases, not light. It does not take into account the optical properties of light reflectance and light absorbance and would therefore be inappropriate to use it as a predictor of n-values.

SPREAD-FUNCTION: In a 1967 paper, Yule, Howe, and Altman investigated the effects of the sideways scattering of light within paper which they called the spread-function.[13]

Using an intense light source and a condensing lens, the image of a very sharp knife-edge plate was projected onto a sheet of paper. The image was diffused by the internal structure of the paper. The trace of the diffused edge was measured with a scanning microdensitometer.

The profile of the trace (taken as the spread-function) could be expressed in terms of standard deviation, since it approximates a gaussian distribution. Wide spread functions indicate a high degree of light-scattering. Narrow spread functions indicate a low degree of light scattering.
By plotting the width of the spread-function against the difference in dot size for the same dots printed on paper (which inluded the effects of both physical and optical dot gain), they found a correlation.

However, further investigation into this topic by Paul E. Lewis at Rochester Institute of Technology found no

correlation between n-values and the spread function of paper.[14]

OPACITY: Using the spread function as a simple predictor of n-values is precluded by the fact that it is a delicate procedure and somewhat difficult to measure. Yule et.al., suggested an alternative to the spread-function approach.[15] Citing a paper by Jorgensen,[16] they proceeded to suggest that light-spreading in uncoated papers could be estimated by "TAPPI opacity, the reflectance, and paper thickness using the Kubelka-Munk formulas" (see Equations 13 thru 19). It was beleived that this method would not be applicable to coated papers since they are not homogeneous. This shoud be questioned.

It has been stated as a general assumption that light-scattering in a pulp-filler mixture is proportional to filler content.[17] This would tend to exclude coated papers from the benefit of Kubelka-Munk analysis. However, because papers are manufactured with varying levels of filler loading, it is difficult to say at what point the scattering properties are significantly changed by the filler content. It may even turn out that filler content is not very significant when using opacity as a method to predict the value of n. Whether or not this is true remains unknown.

Opacity has long been used to infer light scattering within paper. Kubelka and Munk first published their theory on light scattering in 1931.[18] They used

differential equations to describe the behavior of diffusing materials in terms of light scattering and light absorption.

It was Steele in 1935 who extended the work of Kubelka and Munk to specifically include paper.[19] Although calculus was used to solve the original differential equations, Steele rewrote them in terms of hyperbolic functions. These he found to be easier to work with and enabled him to provide a graphical solution to the various paper relationships concerning opacity.

In 1938, Judd investigated the Kubelka and Munk theory as it applied to paper to discover the magnitude of departure from theory.[20] He concluded that "except for deviations of less than one percent, the Kubelka and Munk theory applies to paper."

In 1948, Kubelka published equations for the explicit solutions of the hyperbolic functions first outlined by Steele.[21] These solutions enabled the construction of diagrams for an improved graphical solution and a direct mathematical one. It is from these equations that an adjusted version of opacity will be constructed and used to infer the light scattering properties of paper based on a sheet of "standard" thickness.

Opacity is measured by a test instrument known as an opacity meter or opacimeter. Basically, an opacity meter consists of a light source, a photocell to measure the amount of light reflected from the sheet, a black cavity to

(as nearly as possible) absorb all light transmitted by the sheet, a white tile of known reflectance, and a form of readout.

The value of opacity (sometimes called the contrast ratio) may be calculated by first measuring a specimen of paper backed by a black body (black felt lined cavity) of .005 reflectance or less. This quantity is termed R_0. A second measurement is then taken of the same specimen backed with a white body (white tile) having an absolute reflectance of .89. This quantity is termed $R_{0.89}$. The mechanics and optics of the instrument may be arranged to read these values as the ratio of reflected light to incident light and it is this value that we regard as contrast ratio opacity, commonly termed $C_{0.89}$ and hereafter called simply opacity.

It is useful to have the separate quantities of R_0 and $R_{0.89}$ instead of the single quantity of $C_{0.89}$ so that they may be used as input variables into the Kubelka-Munk equations. It is necessary to use these equations to apply a known value of opacity to sheets of differing thicknesses or basis weights because of the non-linear relationship of opacity to basis weight. Figure 10 shows this relationship and demonstrates a close correlation between the hyperbolic solutions and actual experimental data.[22]

It should be noted that the veritcal axis plots basis weight. Actually either basis weight or caliper may be used

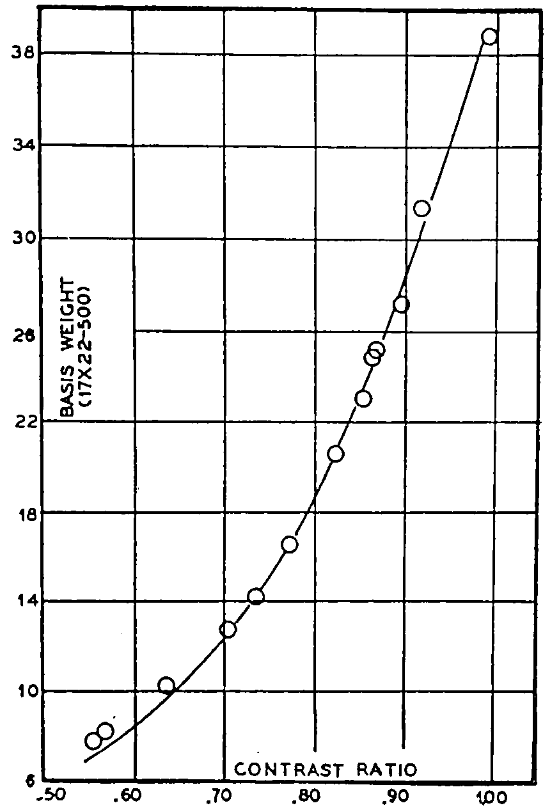


Figure 10.
Contrast Ratio (TAPPI Opacity) versus Basis Weight
Calculated Curves and Experimental Values

interchangably without affecting the outcome of the calculations. Van den Akker has pointed out that this substitution is perfectly justified.[23]

If it were not for the non-linearity of the functional relationship of opacity to sheet caliper, the calculation of an opacity value which has been adjusted to a sheet of standard caliper would be relatively simple. Many sheets of printing papers are around .004" in thickness. If we were to use .004" as the "adjusted" caliper for a sheet of paper, the equation to convert the value of opacity from a known sheet caliper, to a value of opacity for a standard sheet

sheet caliper of .004", would look like Equation 12. This new value of opacity may be termed "adjusted" opacity, $C_{0.89(.004)}$.

$$C_{0.89(.004)} = \left(\frac{R_{0.89}}{R_0}\right) \times \left(\frac{.004''}{X}\right) \tag{12}$$

However, because a linear solution would not be correct, a series of equations becomes necessary to incorporate the hyperbolic solutions into Kubelka and Munk's original theory. Therefore, the following sequence of equations were used to provide a new value of opacity which can also be termed adjusted opacity. Note: a and b are intermediate solutions to the equations.

$$a = 1/2 \left[0.89 + \left(\frac{R_0 + R_{0.89} - 0.89}{R_0 \, (R_{0.89})}\right)\right] \tag{13}$$

$$b = (a^2 - 1)^{1/2} \tag{14}$$

$$SX = 1/b \left[\text{Ar ctgh} \left(\frac{1 - a(R_0)}{b \, (R_0)}\right)\right] \tag{15}$$

$$SX_{.004} = SX \left(\frac{.004''}{X}\right) \tag{16}$$

$$R_{0(.004)} = \frac{1}{(a + b \text{ ctgh } bSX_{.004})} \tag{17}$$

$$R_{0.89(.004)} = \frac{1 - 0.89(a - b \text{ ctgh } bSX_{.004})}{a - 0.89 + b \text{ ctgh } bSX_{.004}} \quad (18)$$

$$C_{0.89(.004)} = R_{0(.004)}/R_{0.89(.004)} \quad (19)$$

The above equations were translated into a computer program which was used to find standard opacity and other equation components (see Table 7 for results, Figure 45 for the program in MBASIC).

FOOTNOTES TO CHAPTER II

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[18]Kubelka and Munk. Z. Tech. Physik, 12 (1931), p. 539.

[19]Steele, F. A. "The Optical Characteristics of Paper. I. The Mathematical Relationships Between Basis Wieght, Reflectance, Contrast Ratio, and Other Optical Properties." Paper Trade Journal, (March 21, 1935), pp. 299-304.

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[22]Steele, (1935), p. 302.

[23]Van den Akker, J. A., "Scattering and Absorption of Light in Paper and Other Diffusing Media." TAPPI 32, No. 11 (1949), pp. 498-501.

CHAPTER III

HYPOTHESIS

It is hypothesized that there is a positve correlation between adjusted TAPPI opacity and adjusted n-value for various paper types and a given screen ruling.

CHAPTER IV

METHODOLOGY

The RIT Symmetrical Scales were selected as test objects and printed onto 37 different substrates of average caliper. These substrates included newsprint, uncoated, coated, and proofing stock. Three versons of the scales were used: 65, 100, and 150 lines per inch.

For each scale, a 50% parallel line tint patch was selected for measurement with a gravure scope equipped with a video monitor.

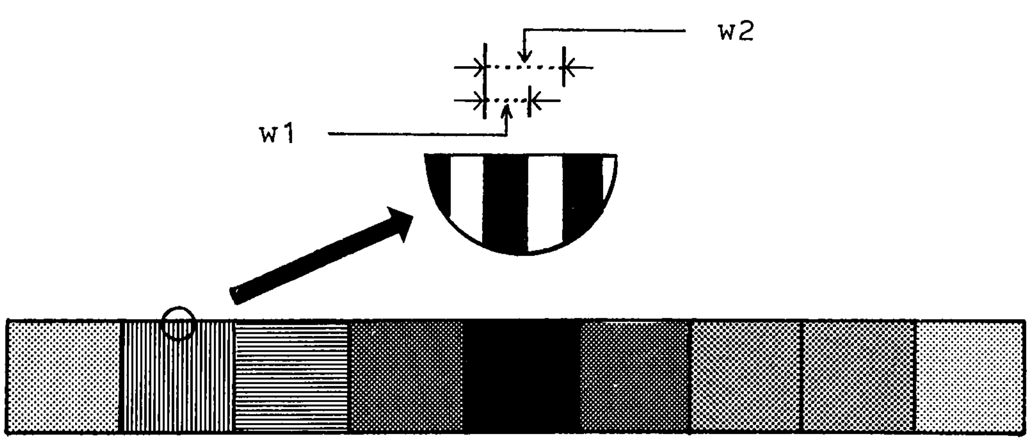


Figure 11.
The RIT Symmetrical Scale (50% Line Tint Enlarged)

Two dimensions (w1 and w2) were measured as illustrated by Figure 11. All measurements were made to the nearest micron. Five pairs of measurements were taken to obtain a

reasonable sampling of the tint.

The actual area of the tint was found by dividing each w1 by its corresponding w2 for each of the five pairs of data. These results were then added and averaged to obtain an average tint value in terms of percent of ink coverage.

The density of the ink solid and tint were each taken five times to obtain a good sample. These were then averaged to obtain an average density value for both the solid and the tint.

The value of n was then calculated for each of the three tints and 37 substrates using the computer program in Appendix I. Optical Dot Area (ODA), Optical Dot Gain (ODG), and Physical Dot Area (PDA) were also calculated. The value of n was also adjusted to a standard solid ink density of 1.3 using the method outlined in Appendix II.

Each test sample was measured with an opacity meter to find the reflectances of the sample backed by a black cavity of near zero reflectance and a white tile with an absolute reflectance of .89.

These values were used as input values for the Kubelka and Munk equations to adjust the reflectances to a standard sheet of .004" caliper. A value of adjusted TAPPI opacity could then be obtained for each of the test samples. The scattering coefficients and absorption coefficients were also obtained.

The adjusted TAPPI opacities of each sheet were then

plotted and regressed against the n-values for each screen ruling to search for a correlation between them. A least squares regression program was used to analyze the correlation.

CHAPTER V

RESULTS

The results of the study are published in the following pages. The data have been arranged into four tables. Tables 4, 5, and 6 contain the original readings and measurements for deriving the dependant variable of adjusted n-value. Table 7 contains the readings and measurements for deriving the independant variable of adjusted TAPPI opacity.

The code in each table refers to the type of substrate, printing method, and sample number within the substrate class. The first letter(s) refers to one of four substrate types (P = off-press proofing stock, N = newsprint, C = coated, and UC = uncoated). The last letter refers to the printing method (W = web offset, S = sheetfed, and D = duplicator). The decimal number refers to the number of the sample with the substrate class.

The remaining nomenclature is as follows:

ADA	=	Actual Dot Area
ODA	=	Optical Dot Area
ODG	=	Optical Dot Gain
D_s	=	Solid Ink Density
D_t	=	Tint Density
n	=	n-Value
Adj. n	=	n-Value Adjusted to 1.3 Solid Ink Density
R_0	=	Reflectance of a sheet backed by a black cavity of zero reflectance

Caliper	=	Thickness of the sheet in inches
$SX_{.004}$	=	Kubelka-Munk scattering coefficient adjusted to a sheet of .004 caliper
$C_{.89(.004)}$	=	Adjusted TAPPI Opacity as computed by the Kubelka-Munk equations and standardized to a sheet of .004" caliper

The graphs of the data appear in Figures 12 through 23. The graphs concerned with the hypothesis are in Figures 13, 15, and 17. Additional graphs have been included in an attempt to discover improvements in correlation using other data sets. Each graph contains a line which is defined by the regression equations in Tables 8 and 9.

The statistical data is published in Tables 8 and 9 for all 37 samples. Tables 10 and 11 contain the statistical analysis for the 12 samples of uncoated paper.

65

#	CODE	ADA	D_t	D_s	ODA	ODG	n	Adj. n
1.	P.1	.489	.369	1.532	.590	.101	1.433	1.493
2.	P.2	.491	.342	1.472	.564	.073	1.297	1.324
3.	NW.1	.683	.459	.813	.771	.081	1.863	1.456
4.	NW.2	.662	.368	.678	.745	.083	2.003	1.270
5.	CS.1	.562	.376	1.348	.606	.044	1.172	1.763
6.	CW.2	.592	.505	1.709	.701	.109	1.443	1.558
7.	CW.3	.600	.393	1.240	.632	.032	1.125	1.123
8.	CS.4	.574	.348	1.179	.590	.016	1.063	1.060
9.	CW.5	.622	.399	1.057	.659	.037	1.168	1.143
10.	CW.6	.666	.469	1.233	.701	.035	1.147	1.141
11.	CD.7	.585	.362	1.189	.605	.020	1.076	1.072
12.	CD.8	.572	.350	.996	.615	.043	1.211	1.172
13.	CD.9	.611	.393	1.173	.638	.027	1.110	1.103
14.	CD.10	.580	.363	1.208	.604	.024	1.093	1.089
15.	CD.11	.544	.343	1.077	.596	.052	1.245	1.211
16.	CD.12	.532	.334	1.129	.580	.048	1.213	1.192
17.	CD.13	.567	.353	1.209	.593	.026	1.102	1.098
18.	CD.14	.585	.371	1.222	.611	.026	1.102	1.098
19.	CD.15	.574	.379	1.137	.628	.054	1.245	1.221
20.	CD.16	.594	.419	1.317	.650	.056	1.230	1.223
21.	CW.17	.588	.382	1.154	.629	.041	1.176	1.162
22.	CD.18	.577	.392	1.148	.640	.063	1.294	1.266
23.	CD.19	.575	.370	1.159	.616	.041	1.175	1.161
24.	CS.20	.568	.378	1.397	.605	.037	1.139	1.145
25.	CW.21	.619	.456	1.453	.674	.055	1.210	1.228
26.	UCW.2	.637	.425	.977	.689	.061	1.341	1.261
27.	UCW.3	.636	.402	.909	.689	.053	1.304	1.219
28.	UCS.4	.575	.382	1.028	.646	.071	1.378	1.305
29.	UCW.5	.663	.440	.903	.728	.065	1.424	1.291
30.	UCD.6	.590	.336	.879	.621	.031	1.159	1.117
31.	UCW.7	.633	.398	.898	.687	.054	1.317	1.225
32.	UCW.8	.637	.417	.956	.694	.057	1.320	1.241
33.	UCW.9	.660	.428	.874	.723	.063	1.425	1.282
34.	UCW.10	.588	.384	.960	.659	.071	1.415	1.310
35.	UCW.11	.603	.390	1.023	.655	.052	1.258	1.211
36.	UCW.12	.574	.369	1.077	.625	.051	1.238	1.205
37.	UCD.13	.566	.367	.928	.647	.081	1.511	1.363

Table 4.
Data from samples (n-Values for 65 Lines per Inch)

100

#	CODE	ADA′	D_t	D_s	ODA	ODG	n	Adj. n
1.	P.1	.482	.386	1.533	.607	.125	1.579	1.672
2.	P.2	.479	.362	1.490	.584	.105	1.471	1.528
3.	NW.1	.677	.479	.824	.786	.109	2.329	1.631
4.	NW.2	.662	.368	.678	.723	.061	1.570	1.270
5.	CS.1	.606	.420	1.403	.645	.039	1.147	1.154
6.	CW.2	.659	.572	1.701	.747	.088	1.351	1.154
7.	CW.3	.614	.400	1.219	.641	.027	1.105	1.100
8.	CS.4	.618	.398	1.173	.643	.025	1.102	1.095
9.	CW.5	.619	.419	1.077	.676	.057	1.277	1.236
10.	CW.6	.661	.469	1.230	.702	.041	1.172	1.165
11.	CD.7	.614	.412	1.161	.658	.044	1.192	1.176
12.	CD.8	.588	.375	.932	.655	.067	1.393	1.287
13.	CD.9	.662	.462	1.178	.701	.039	1.172	1.159
14.	CD.10	.612	.405	1.185	.649	.037	1.153	1.143
15.	CD.11	.574	.369	1.060	.627	.053	1.255	1.216
16.	CD.12	.561	.365	1.125	.615	.054	1.244	1.219
17.	CD.13	.596	.377	1.145	.625	.029	1.120	1.110
18.	CD.14	.615	.406	1.212	.647	.032	1.129	1.123
19.	CD.15	.616	.428	1.127	.677	.061	1.294	1.260
20.	CD.16	.622	.483	1.292	.707	.085	1.403	1.401
21.	CW.17	.608	.424	1.121	.674	.066	1.325	1.286
22.	CD.18	.609	.444	1.113	.694	.085	1.458	1.393
23.	CD.19	.621	.415	1.165	.661	.040	1.169	1.156
24.	CS.20	.604	.408	1.365	.637	.033	1.121	1.125
25.	CW.21	.651	.509	1.456	.715	.064	1.260	1.284
26.	UCW.2	.663	.483	.976	.750	.087	1.610	1.436
27.	UCW.3	.674	.446	.917	.730	.056	1.348	1.247
28.	UCS.4	.612	.433	1.001	.701	.089	1.559	1.421
29.	UCW.5	.670	.458	.913	.742	.072	1.500	1.339
30.	UCD.6	.611	.372	.808	.681	.D70	1.512	1.309
31.	UCW.7	.652	.446	.900	.734	.082	1.603	1.395
32.	UCW.8	.669	.476	.959	.748	.079	1.536	1.381
33.	UCW.9	.663	.475	.891	.763	.100	1.900	1.534
34.	UCW.10	.617	.423	.953	.700	.083	1.542	1.388
35.	UCW.11	.644	.449	1.064	.705	.061	1.318	1.265
36.	UCW.12	.620	.431	1.091	.685	.065	1.327	1.280
37.	UCD.13	.623	.415	.912	.701	.078	1.522	1.357

Table 5.
Data from samples (n-Values for 100 Lines per Inch)

150

#	CODE	ADA	D_t	D_s	ODA	ODG	n	Adj. n
1.	P.1	.485	.427	1.523	.645	.160	1.851	2.022
2.	P.2	.486	.401	1.492	.623	.137	1.677	1.778
3.	NW.1	.710	.549	.828	.843	.133	4.414	2.039
4.	NW.2	.667	.424	.671	.792	.125	5.040	1.790
5.	CS.1	.598	.468	1.352	.690	.092	1.424	1.439
6.	CW.2	.705	.701	1.746	.816	.111	1.515	1.700
7.	CW.3	.590	.450	1.232	.685	.095	1.482	1.458
8.	CS.4	.644	.458	1.179	.698	.054	1.246	1.227
9.	CW.5	.651	.489	1.093	.735	.084	1.486	1.405
10.	CW.6	.723	.547	1.232	.761	.038	1.171	1.164
11.	CD.7	.676	.497	1.210	.726	.050	1.229	1.216
12.	CD.8	.647	.462	1.017	.725	.078	1.466	1.361
13.	CD.9	.729	.592	1.152	.801	.072	1.426	1.374
14.	CD.10	.683	.524	1.253	.742	.059	1.274	1.266
15.	CD.11	.616	.476	1.202	.703	.087	1.442	1.410
16.	CD.12	.611	.465	1.225	.699	.088	1.438	1.414
17.	CD.13	.645	.484	1.231	.714	.069	1.323	1.308
18.	CD.14	.690	.502	1.224	.729	.039	1.168	1.161
19.	CD.15	.675	.544	1.160	.767	.092	1.548	1.482
20.	CD.16	.695	.615	1.291	.798	.103	1.607	1.602
21.	CW.17	.641	.465	1.138	.709	.068	1.340	1.301
22.	CD.18	.676	.549	1.118	.777	.101	1.665	1.553
23.	CD.19	.674	.516	1.173	.745	.071	1.368	1.334
24.	CS.20	.613	.478	1.405	.695	.082	1.352	1.375
25.	CW.21	.689	.545	1.524	.737	.048	1.184	1.207
26.	UCW.2	.713	.577	1.001	.817	.104	1.943	1.640
27.	UCW.3	.702	.518	.932	.789	.087	1.715	1.466
28.	UCS.4	.622	.486	.974	.753	.131	2.233	1.784
29.	UCW.5	.701	.514	.907	.792	.091	1.812	1.498
30.	UCD.6	.707	.460	.842	.763	.056	1.409	1.257
31.	UCW.7	.708	.518	.904	.796	.088	1.784	1.481
32.	UCW.8	.701	.545	.970	.801	.100	1.873	1.578
33.	UCW.9	.703	.523	.884	.805	.102	2.104	1.606
34.	UCW.10	.709	.543	.959	.807	.093	1.793	1.524
35.	UCW.11	.664	.541	1.010	.789	.125	2.174	1.785
36.	UCW.12	.654	.491	1.106	.735	.081	1.456	1.385
37.	UCD.13	.681	.491	.894	.776	.095	1.852	1.512

Table 6.
Data from samples (n-Values for 150 Lines per Inch)

#	CODE	R_0	$R_{.89}$	$C_{.89}$	Caliper	$SX_{.004}$	$C_{.89(.004)}$
1.	P.1	.878	.911	.964	.00932	3.79	.866
2.	P.2	.770	.785	.981	.00958	2.83	.854
3.	NW.1	.646	.702	.920	.00284	4.37	.970
4.	NW.2	.606	.679	.892	.00307	3.30	.945
5.	CS.1	.807	.846	.954	.00349	6.93	.967
6.	CW.2	.765	.817	.936	.00270	6.98	.977
7.	CW.3	.694	.757	.917	.00256	5.44	.976
8.	CS.4	.829	.865	.958	.00371	7.33	.965
9.	CW.5	.771	.840	.918	.00283	6.22	.961
10.	CW.6	.768	.808	.950	.00362	5.70	.962
11.	CD.7	.815	.843	.967	.00482	5.73	.948
12.	CD.8	.769	.820	.938	.00374	5.15	.946
13.	CD.9	.749	.820	.913	.00241	6.69	.974
14.	CD.10	.819	.859	.953	.00360	7.01	.963
15.	CD.11	.845	.860	.983	.00827	4.56	.912
16.	CD.12	.824	.840	.981	.00796	4.21	.907
17.	CD.13	.826	.849	.973	.00434	7.06	.967
18.	CD.14	.788	.863	.913	.00307	5.94	.947
19.	CD.15	.696	.724	.961	.00317	5.59	.983
20.	CD.16	.780	.823	.948	.00377	5.60	.955
21.	CW.17	.747	.823	.908	.00252	6.22	.968
22.	CD.18	.770	.821	.938	.00366	5.28	.949
23.	CD.19	.697	.721	.967	.00356	5.20	.978
24.	CS.20	.823	.858	.959	.00408	6.55	.957
25.	CW.21	.725	.819	.885	.00239	5.67	.962
26.	UCW.2	.749	.810	.925	.00297	5.70	.963
27.	UCW.3	.741	.850	.872	.00269	5.05	.936
28.	UCS.4	.740	.816	.907	.00419	3.65	.899
29.	UCW.5	.767	.835	.919	.00384	4.54	.925
30.	UCD.6	.697	.730	.955	.00421	4.04	.948
31.	UCW.7	.740	.842	.879	.00262	5.29	.945
32.	UCW.8	.745	.812	.917	.00293	5.53	.959
33.	UCW.9	.756	.810	.933	.00275	6.56	.975
34.	UCW.10	.730	.810	.901	.00384	3.79	.909
35.	UCW.11	.750	.832	.901	.00357	4.33	.920
36.	UCS.12	.741	.809	.916	.00261	6.10	.970
37.	UCD.13	.757	.845	.869	.00273	5.66	.950

Table 7.
Data from samples (TAPPI Opacity and Scattering Coefficient)

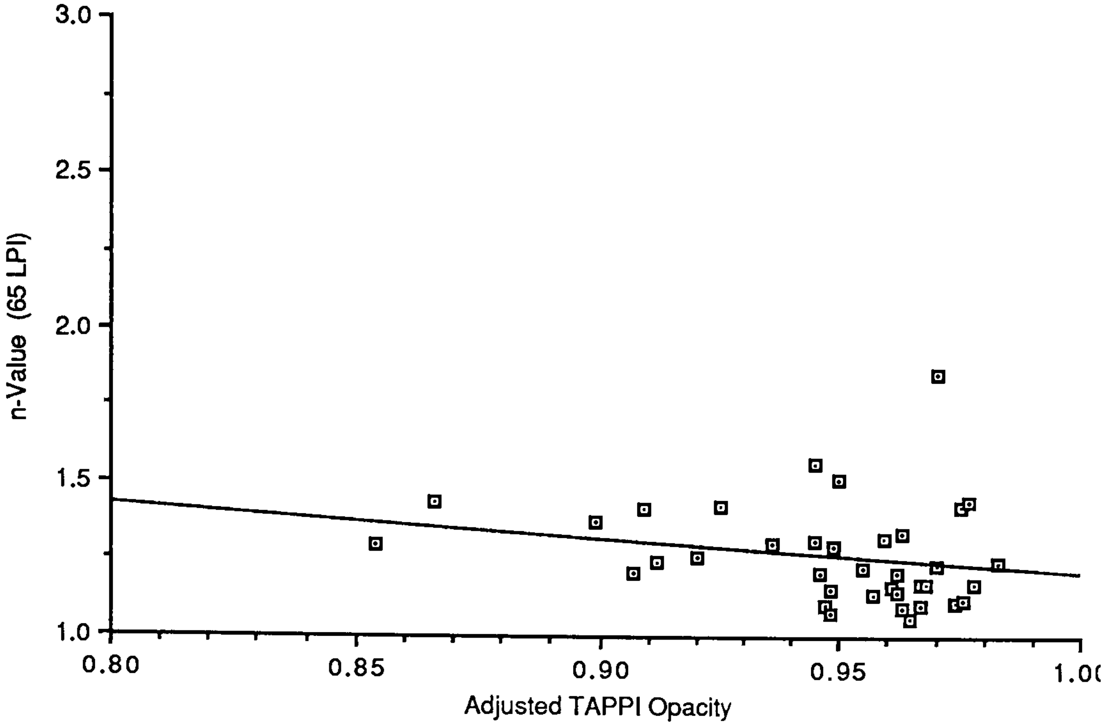


Figure 12.
Plot of n-Values & Adjusted TAPPI Opacity (65 LPI)

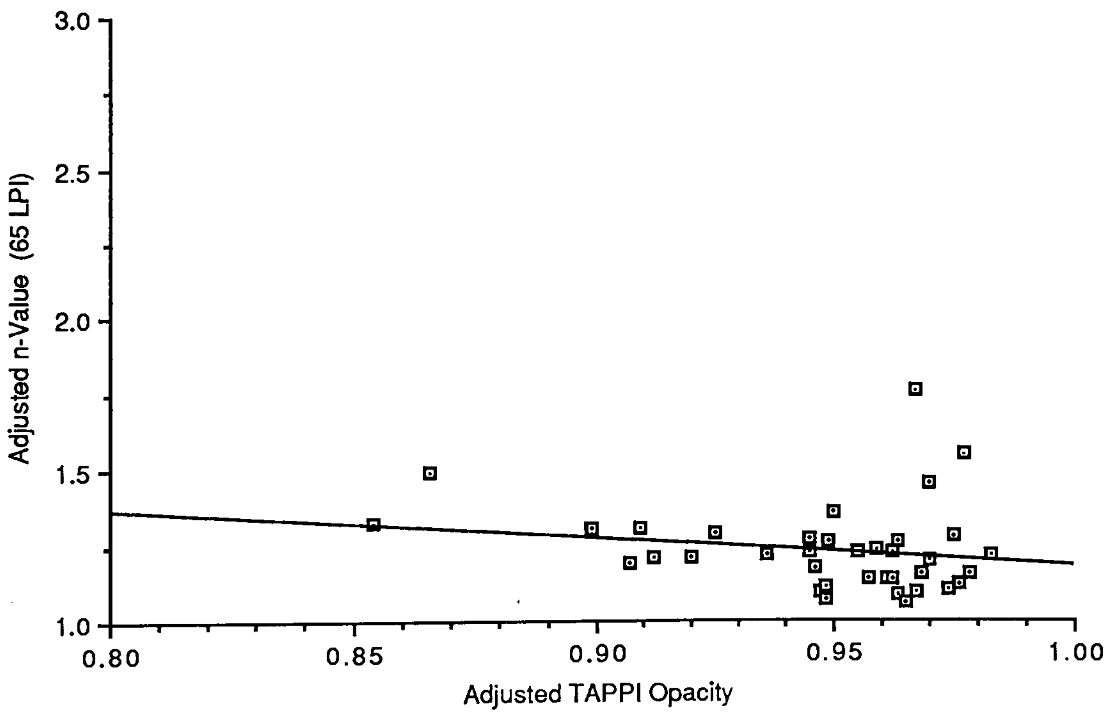


Figure 13.
Plot of Adjusted n-Values & Adjusted TAPPI Opacity (65 LPI)

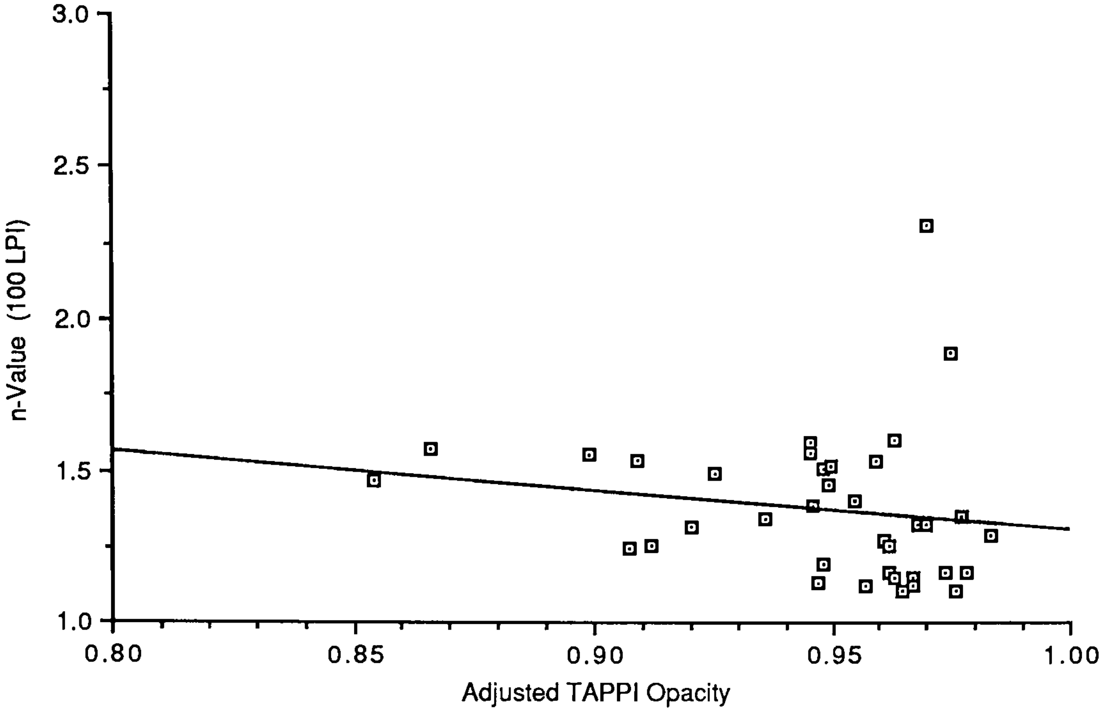


Figure 14.
Plot of n-Values & Adjusted TAPPI Opacity (100 LPI)

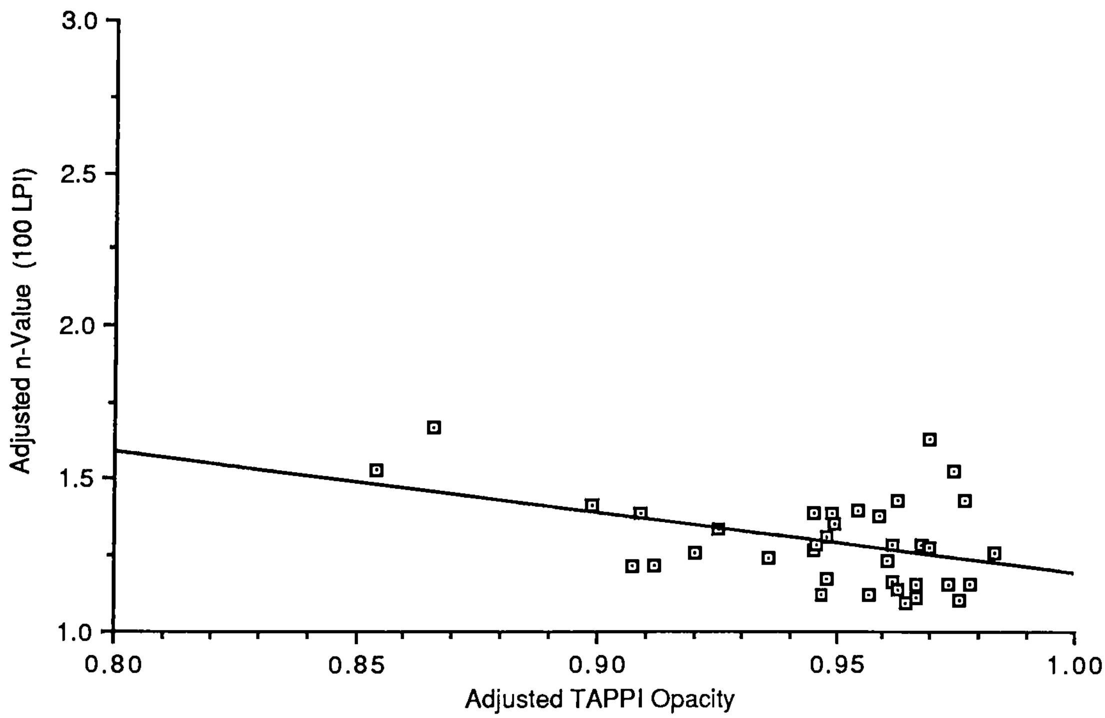


Figure 15.
Plot of Adjusted n-Values & Adjusted TAPPI Opacity (100 LPI)

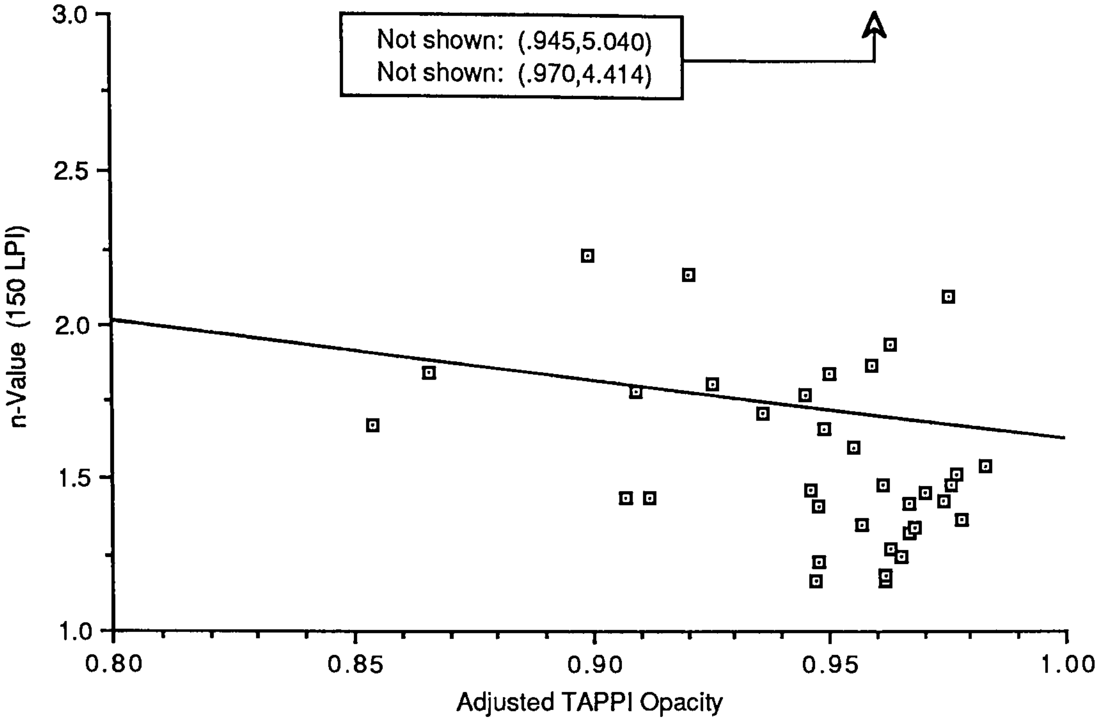


Figure 16.
Plot of n-Values & Adjusted TAPPI Opacity (150 LPI)

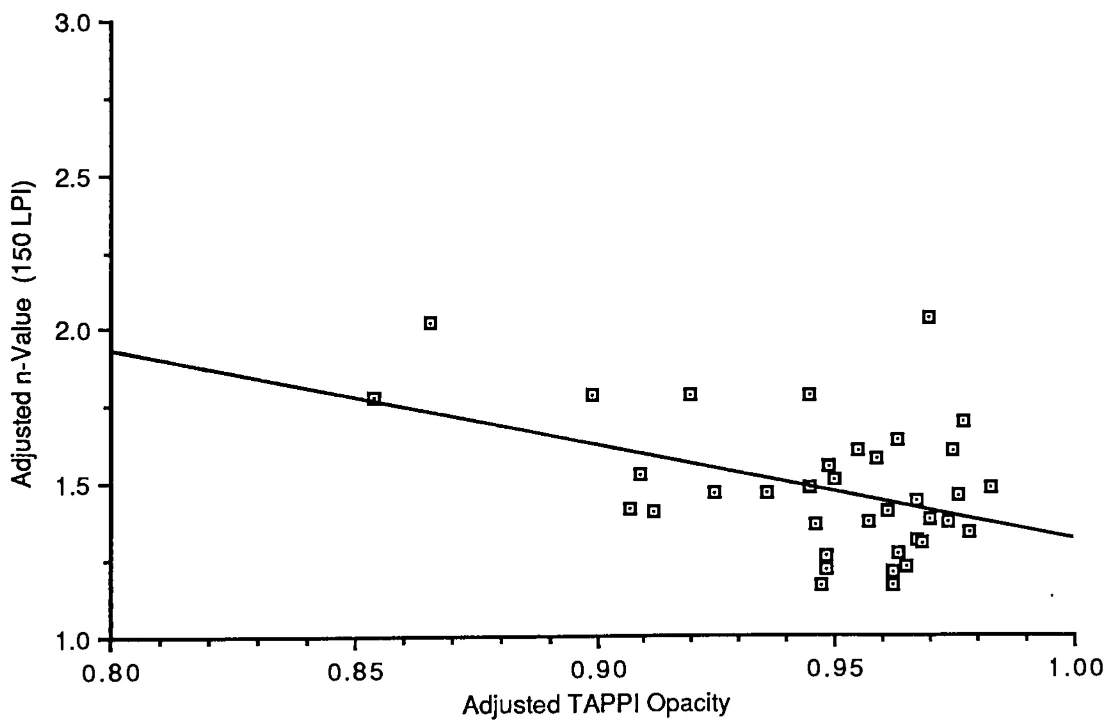


Figure 17.
Plot of Adjusted n-Values & Adjusted TAPPI Opacity (150 LPI)

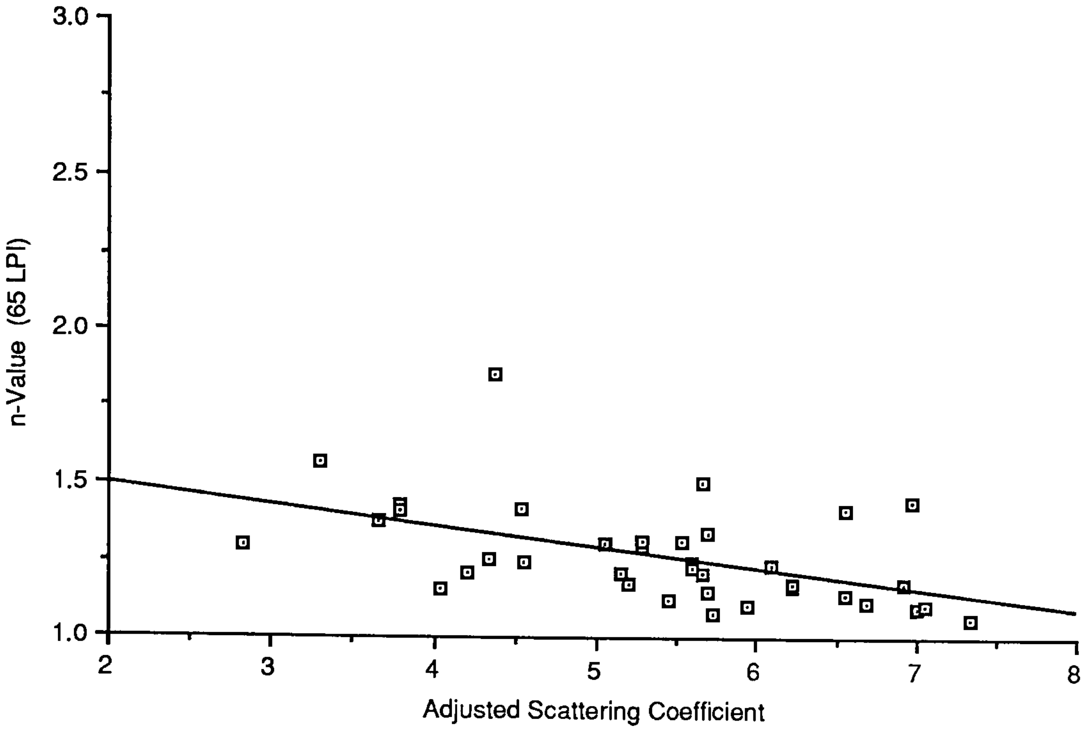


Figure 18.
Plot of n-Values & Adj. Scattering Coefficient (65 LPI)

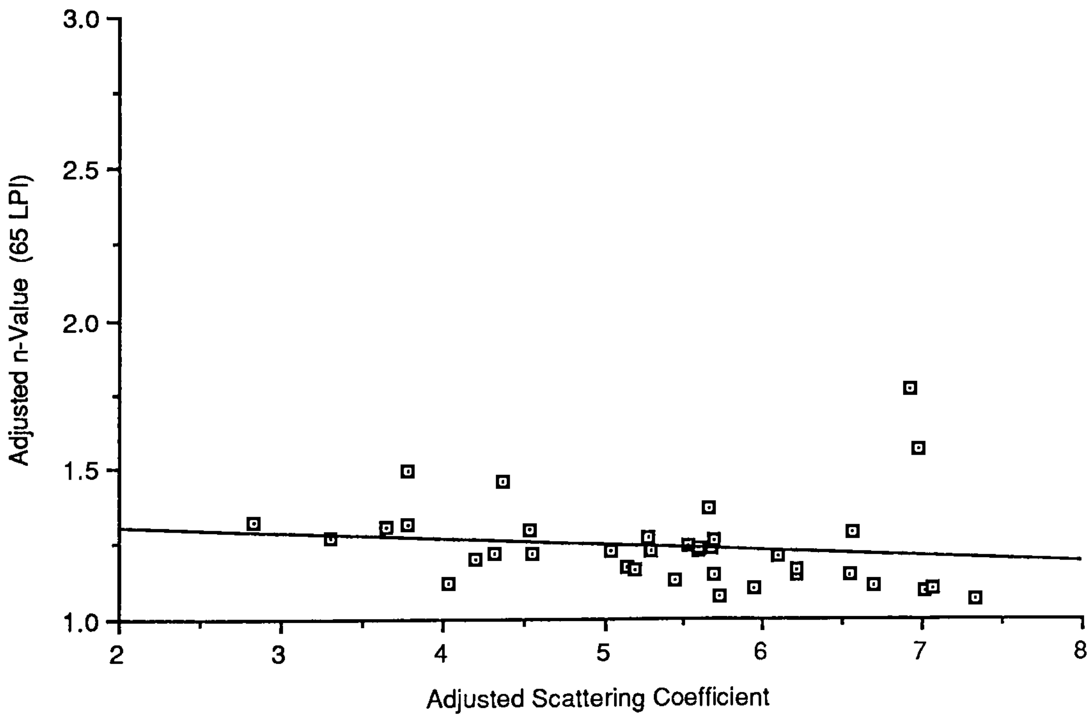


Figure 19.
Plot of Adj. n-Values & Adj. Scattering Coefficient (65 LPI)

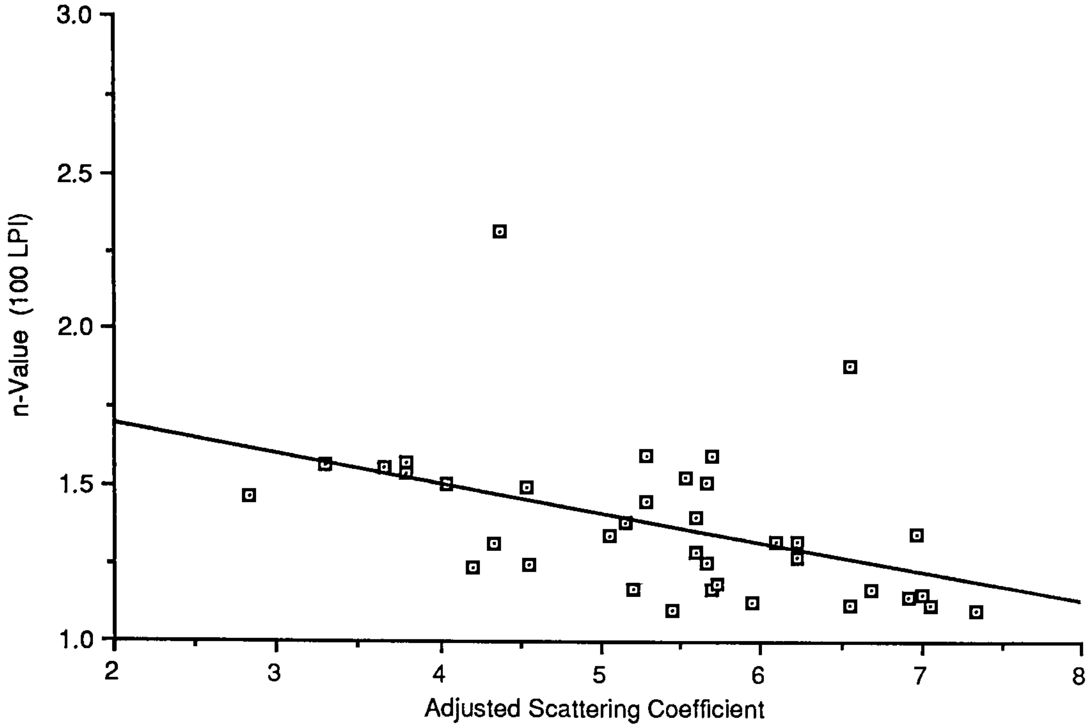


Figure 20.
Plot of n-Values & Adj. Scattering Coefficient (100 LPI)

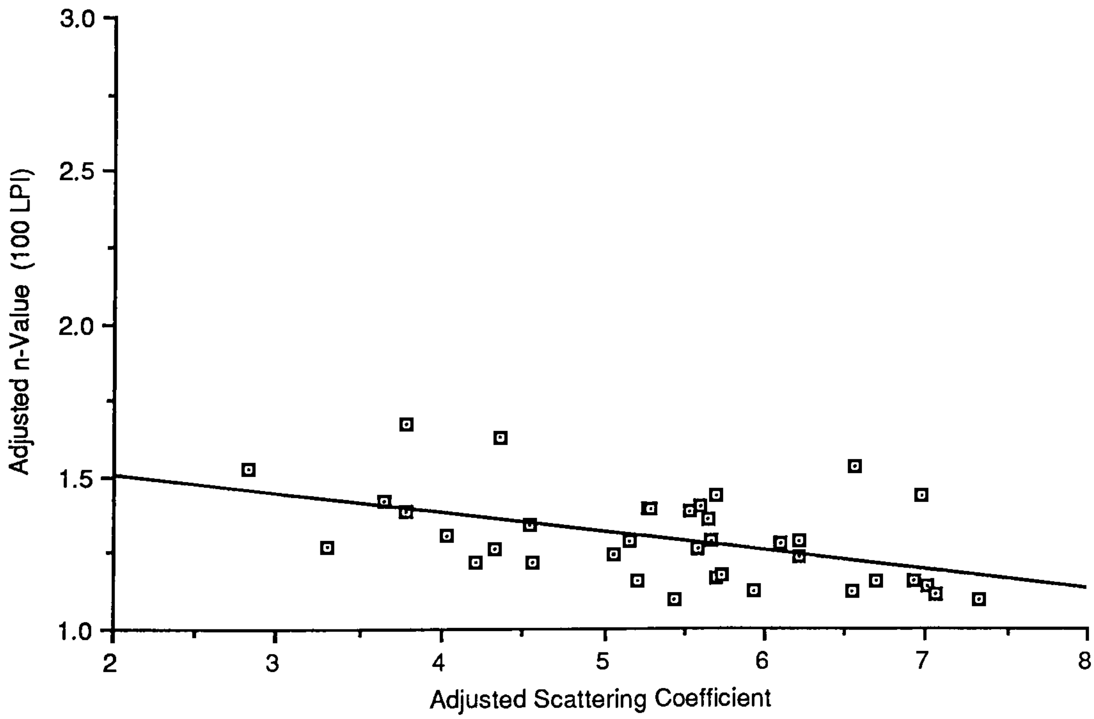


Figure 21.
Plot of Adj. n-Values & Adj. Scattering Coefficient (100 LPI)

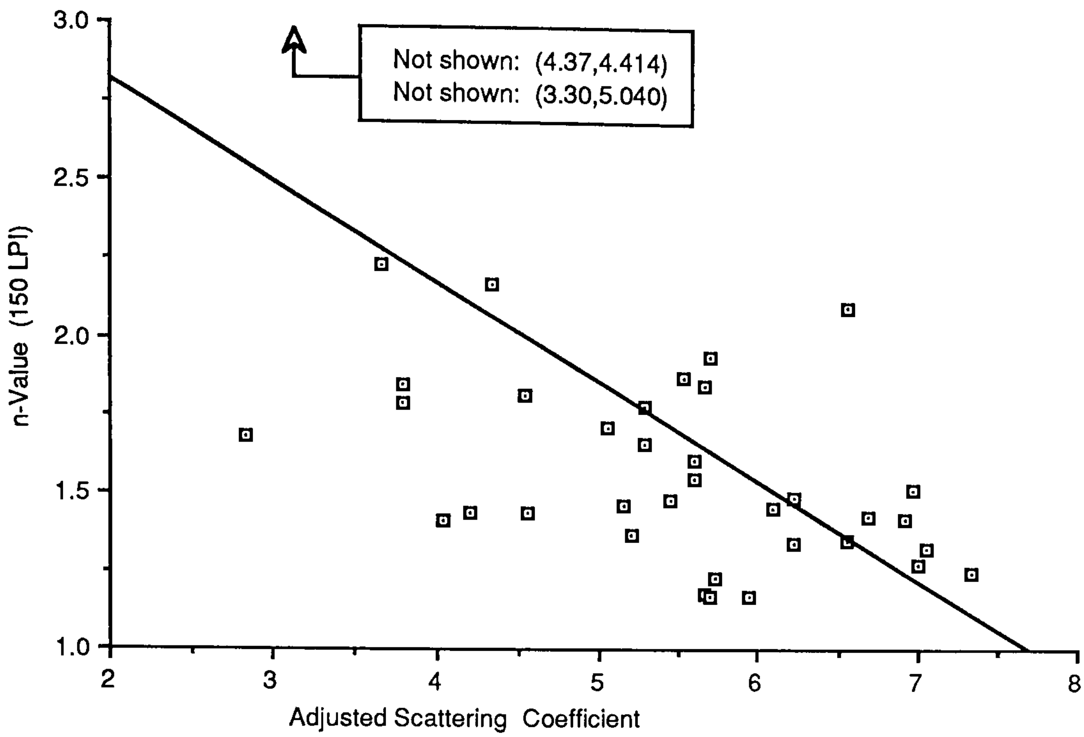


Figure 22.
Plot of n-Values & Adj. Scattering Coefficient (150 LPI)

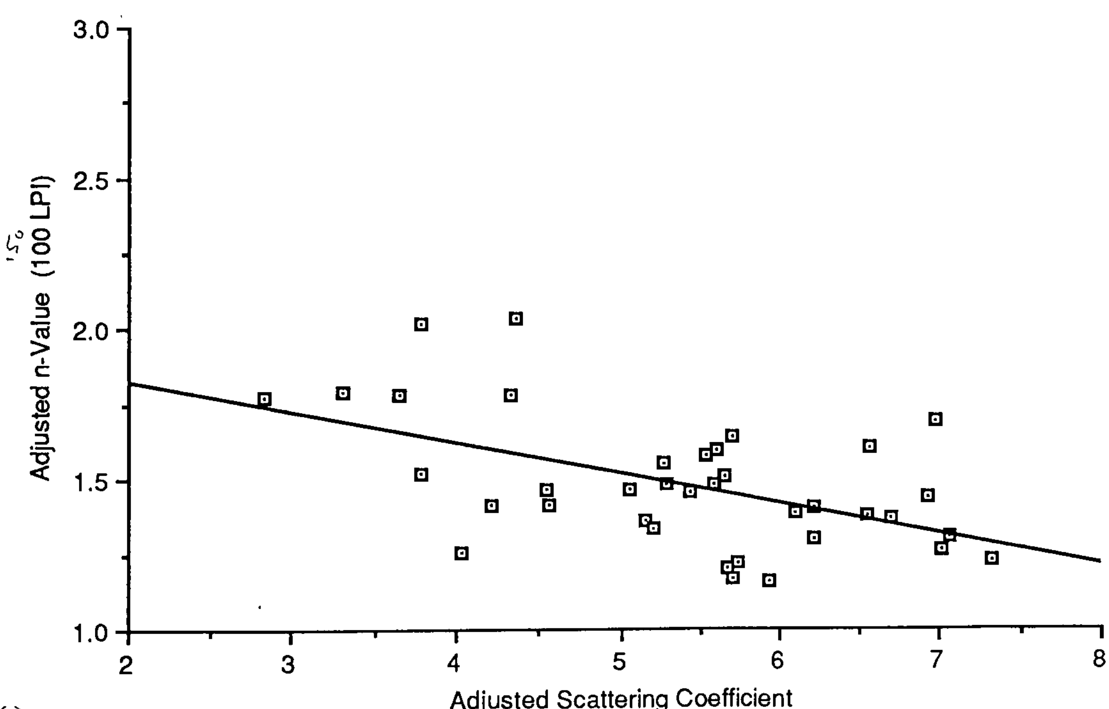


Figure 23.
Plot of Adj. n-Values & Adj. Scattering Coefficient (150 LPI)

C.89(.004)

Dependant Variable	Regression Equation	Standard Error of the Coefficient	Standard Error of the Y Estimate	r^2
n-Value (65 LPI)	y = -1.061x + 2.275	.898	.620	.038
* Adjusted n-Value (65 LPI)	y -0.909x + 2.100	.792	.143	.036
n-Value (100 LPI)	y = -1.291x + 2.603	1.368	.248	.024
* Adjusted n-Value (100 LPI)	y -1.996x + 3.188	.752	.136	.167
n-Value (150 LPI)	y = -1.903x + 3.540	4.350	.789	.005
* Adjusted n-Value (150 LPI)	y - -3.100x + 4.421	1.120	.203	.180

Table 8.
Statistical Data for Adjusted TAPPI Opacity Regressions
(all samples)

SX.004

Dependant Variable	Regression Equation	Standard Error of the Coefficient	Standard Error of the Y Estimate	r^2
n-Value (65 LPI)	y -0.068x + 1.635	.021	.146	.225
Adjusted n-Value (65 LPI)	y = -0.020x + 1.344	.024	.144	.024
n-Value (100 LPI)	y = -0.095x + 1.892	.033	.225	.194
Adjusted n-Value (100 LPI)	y = -0.061x + 1.627	.019	.131	.228
n-Value (150 LPI)	y = -0.320x + 3.461	.101	.698	.221
Adjusted n-Value (150 LPI)	y = -0.103x + 2.039	.027	.189	.286

Table 9.
Statistical Data for Adjusted
Scattering Coefficient Regressions
(all samples)

$C_{.89(.004)}$

Dependant Variable	Regression Equation	Standard Error of the Coefficient	Standard Error of the Y Estimate	r^2
n-Value (65 LPI)	y = -.568x + 1.876	1.248	.100	.020
Adjusted n-Value (65 LPI)	y = -.632x + 1.848	.811	.065	.057
n-Value (100 LPI)	y - 2.005x - .364	1.942	.156	.096
Adjusted n-Value (100 LPI)	y .746x + .660	1.040	.084	.049
n-Value (150 LPI)	y = -3.626x + 5.260	3.092	.248	.121
Adjusted n-Value (150 LPI)	y -2.431x + 3.832	1.816	.146	.152

Table 10.
Statistical Data for Adjusted TAPPI Opacity Regressions
(uncoated samples)

$SX_{.004}$

Dependant Variable	Regression Equation	Standard Error of the Coefficient	Standard Error of the Y Estimate	r^2
n-Value (65 LPI)	y = .017x + 1.257	.032	.100	.026
Ajusted n-Value (65 LPI)	y .006x + 1.223	.021	.067	.007
n-Value (100 LPI)	y = .055x + 1.246	.049	.155	.111
Adjusted n-Value (100 LPI)	y = .026x + 1.232	.026	.082	.091
n-Value (150 LPI)	y = -.021x + 1.232	.084	.264	.006
Adjusted n-Value (150 LPI)	y = -.020x + 1.646	.050	.157	.016

Table 11.
Statistical Data for Adjusted
Scattering Coefficient Regressions
(uncoated samples)

CHAPTER VI

CONCLUSIONS

The hypothesis restated: It is hypothesized that there is a positve correlation between adjusted TAPPI opacity and adjusted n-value for various paper types and a given screen ruling.

The hypothesis cannot be supported by the results. In no case did a significant correlation appear in the data which would aid in the accurate prediction of the n-value given TAPPI opacity.

The data in Chapter V (Tables 3 through 6) were analyzed by regression and correlation, a summary of which appears at the end of that chapter. The graphs of both the data and the regression equation have been given in Figures 12 through 23.

The hypothesis was tested by the results obtained in the rows marked by asterisks in Table 8. As may be seen, the correlations are very low.

Other regressions were performed with other combinations of data to try and improve the correlation. The orignal n-values and the Kubelka-Munk scattering coefficient were also looked at in combination with adjusted n-value and adjusted opacity. While some slight improvements in correlation were noted in using the adjusted

n-value with the scattering coefficient, there was a deterioration in the slope of the regression equation which would tend to offset any gains made in correlation.

Regressions were also performed on uncoated papers (Tables 10 and 11). These correlations and slopes were even lower than those for all paper types. No graphs were constructed because of the poor correlation.

In an attempt to discover the error (if any) that prevented the anticipated correlation, let us first look at the method of finding the actual dot area on paper. As mentioned in the methodology, a gravure scope was used to take five very accurate readings per tint patch. While some subjectivity was necessary to interpolate readings on rough surfaces, the readings were consistent enough within the tint patch to discount this as a significant source of error.

There were some other problems with data collection which could very well have had an adverse effect upon the correlations. While taking opacity readings, it was noticed that the readout on the opacity meter fluctuated over time. It was assumed that this fluctuation was due to variations in voltage within the building caused by the periodic demand of heavy equipment. A voltage stabilizer was obtained and the readings performed in the quieter hours but this fluctuation was never entirely eliminated. Perhaps it was a problem with the instrument or stabilizer as both of them

were of old design. The opacity meter was of the analog-readout type which made it difficult to carry readings out to three decimal places.

While it was hoped to obtain accurate opacity readings to three decimal places (which newer meters are capable of), this was never really practically possible with the older one. Indeed, the accuracy of some two-decimal place readings may be considered questionable.

Another factor which could have affected the correlation was the finding that the n-values for the paper samples tested were lower than the published n-values for the same paper types. This would result in correlation equations with a lower slope making predictions for the n-value more difficult and subject to additional error.

The reason for this discrepency probably lies in the way n-values have been traditionally determined. Rather than measure the ink coverage on a page directly to obtain actual dot area, tints were first made on non-diffusing transparent materials which were then laminated onto the paper in question. The n-value could then be determined by measuring the tint density and solid ink density on paper.

The problem with using this method is that it is not really ink on paper and so it is not truely representative of real conditions. The materials which were laminated to paper were often very smooth and shiny which aggravated the diffusion of light under the halftone pattern. This would

lead to the conclusion that the n-values for those paper/ink combinations were higher than they really were.

The correlations that are exhibited in the results were usually negative and not positve as hypothesized. This cannot be readily explained although, given the correlations, we can not be sure that is truely the case. Perhaps more could have been said about this had there been a greater quantity of paper samples in the .8 to .9 opacity range. Perhaps the correlation is actually negative. If so, then the assumptions on pages 17 through 19 would have to be reconsidered.

When the conditions of increased optical dot gain and lower solid ink density exist (as in uncoated paper), the n-value is very sensitive to slight changes in the other equation variables. This may be inferred from Figure 39. This would tend to magnify any errors which exist in the measurements given these conditions.

APPENDIX I

A MATHEMATICAL MODEL FOR TESTING THE SOLUTION ACCURACY OF THE YULE-NIELSEN EQUATION WITH RESPECT TO CHANGES IN DOT AREA LEVEL

It has been stated that the change in dot area level for a given screen ruling is not a significant factor in the change of the value of n as applied to the Yule-Nielsen equation. While Pearson has conducted a study (discussed on page 11) to investigate this and other effects, his selection of dot areas to study was limited to three levels (35, 46, and 70%). Although we should accept this conclusion that the Yule-Nielsen equation behaves as expected for this extended middletone range, the behavior of the equation throughout the tonal range and into the extreme highlight and shadow regions has yet to be modeled. The purpose of this study is to investigate the solution accuracy of the Yule-Nielsen equation with respect to the dot area levels of 5 to 95 % using a geometric model of dot growth. The error attributable to n will be calculated with respect to the middletone (50%) region. A computer program based on this geometric model will be used to find the error for a selection of dot area levels.

One need only know the solid ink density and and n-value to solve a mathematical model investigating the effects on dot area level. To construct such a model,

consider that a halftone unit area may be divided into two discrete areas by inscribing a circle within a square unit area (Figure 24a). For the purposes of what follows, assume that the area inside of the circle equals the area outside of it. Thus, a 50% dot may be illustrated with respect to the highlight areas (Figure 24b) or the shadow areas (Figure 24c).

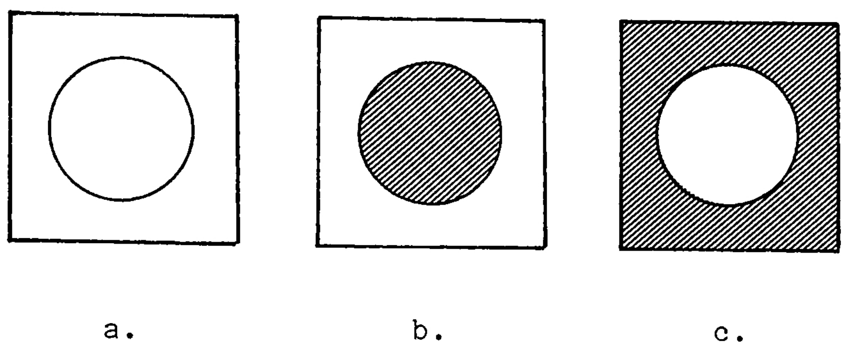


Figure 24.
Simplified 50% Dot Area Models

The transition in the 50% area from the type of dot exhibited in Figure 24b to the type of dot exhibited in Figure 24c is necessary for constructing a simplified mathematical model. The 50% dot is used only as a point of reference from which to measure the error of n into both the highlights and shadows.

The model is therefore not strictly rigorous due to the fact that dots do not go through an immediate transition at the 50% stage as illustrated by Figures 24b and 24c. Rather, as round dots grow, their perimeters would theoretically touch at the 78.5% level. However, due to the

nature of photographic materials and the spreading of ink on paper, dots will connect and "bridge-over" long before this with a more realistic transition point occuring much closer to the middletones.

Regardless of exactly where this transition takes place, the equations which follow operate with the transition at the 50% level. In point of fact, it matters little where in the middletone region the transition does take place as preliminary studies by the author have shown that the results remain essentially the same.

As for dot shapes, a round dot was chosen as the basis for this study for both the sake of simplicity and due to the fact that a round dot was most representative of the various dot shapes for their full range. Other dot shapes, such as square and elliptical, are only perceivable in the middletone regions. Toward the highlight and shadow areas, they graduallly become rounded. Investigations have indicated that dot shapes are not a significant factor affecting the value of n.

Consider a modification of Figure 24a in Figure 25. Figure 25 is representation of a 50% dot of radius r which is defined by the solid circular line within the unit square. The broken lines on either side of this line define two annuli of equal area. If the 50% dot were to be increased or decreased by an area equivalent to one of the annuli, then the new radius of the dot would be defined by

$r + r_1$ in the case of the dot growing larger and by $r - r_2$ in the case of the dot becoming smaller.

The values of r_1 and r_2 may be discovered if the areas of the larger and smaller dots are known. The areas may be calculated from equations derived from the Yule-Nielsen equation. Thus, if both solid ink density and an n-value are provided, both r_1 and r_2 may be found.

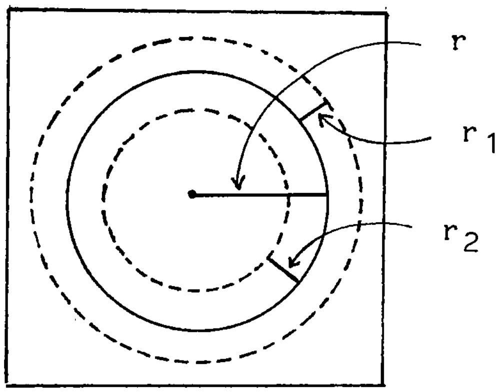
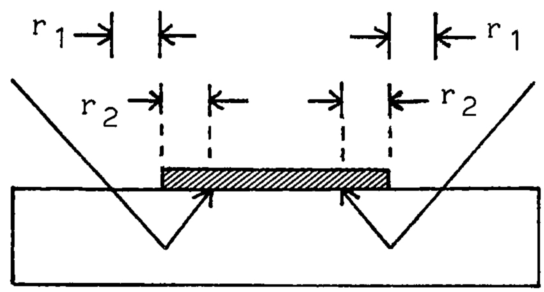


Figure 25.
Increase and Decrease of Dot by Constant Area

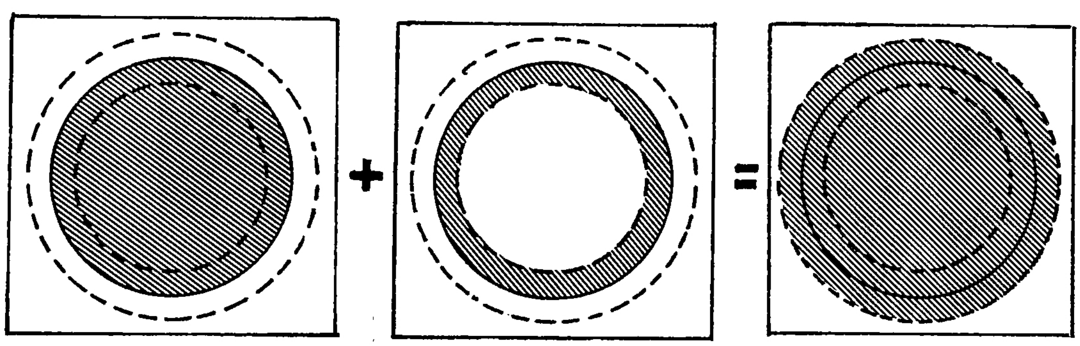
$$r_1 = \left(\frac{1 - 10^{-n} \log[1-.5(1-10^{-Ds/n})]}{(1 - 10^{-Ds})(\pi)}\right)^{1/2} - \left(\frac{.50}{\pi}\right)^{1/2} \qquad (20)$$

$$r_2 = \left(\frac{.50}{\pi}\right)^{1/2} - \left(\frac{1 - 10^{-n} \log[1-.5(1-10^{-Ds/n})]}{(1 - 10^{-Ds})(\pi)}\right)^{1/2} \qquad (21)$$

The value of r_1 and r_2 are both useful in quantifying optical dot gain, the mechansim of which has been briefly discussed in the body of this paper. By applying that mechanism to the highlight-to-midtone type of dot in Figure 24b, a new, detailed version of that dot may be constructed in Figure 26.



a.



b. c. d.

Figure 26.
Optical Dot Gain (highlight to midtone model)

A side and top view of the dot is given in Figures 26a and 26b respctively. By reading a tint of such dots with a

densitometer, some quantity of light will be absorbed by the top of the halftone dot (Figure 26b) and some quantity of light which enters the paper will be absorbed by the underside of the halftone dot (Figure 26c). Thus, a quantity of light is absorbed and lost to the measuring instrument which is equivalent to Figures 26b plus 26 causing the dot to appear to grow in area to that of Figure 26d.

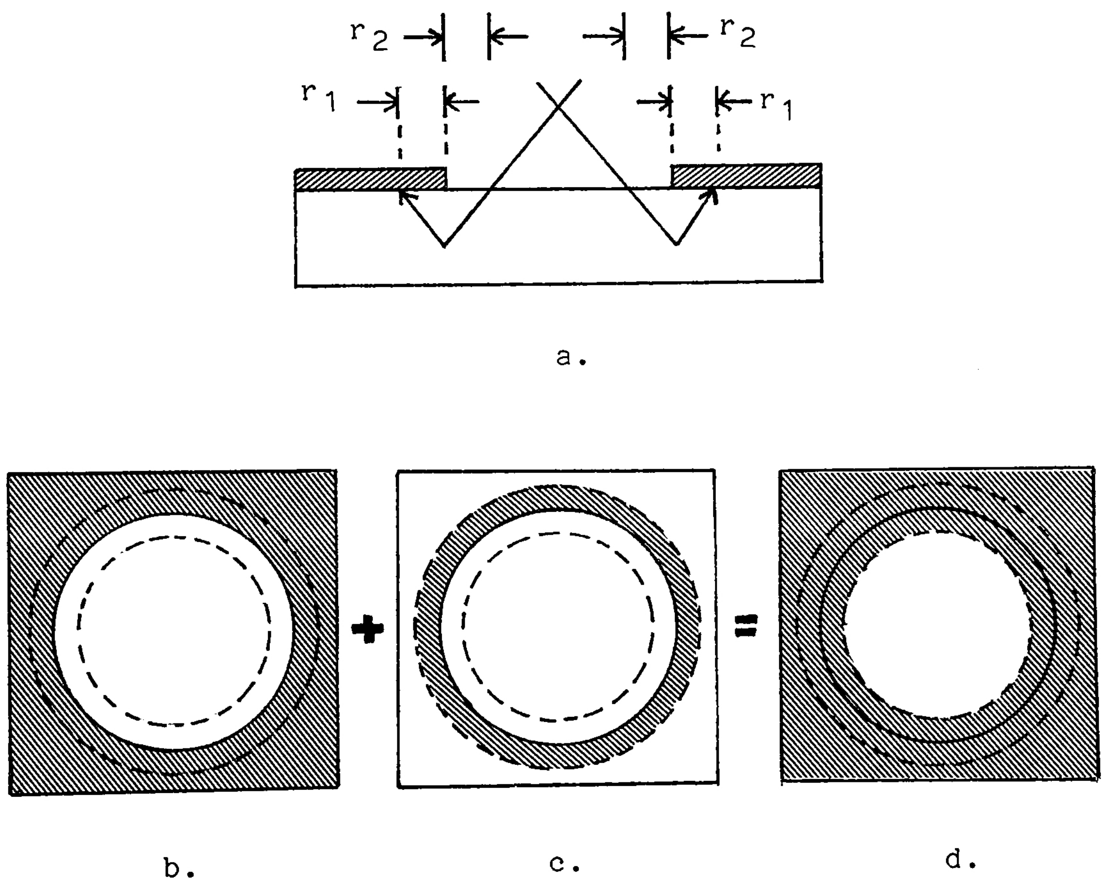


Figure 27.
Optical Dot Gain (midtone to shadow model)

A similar effect happens to the midtone-to-shadow type of dot in Figure 27. How each of the former equations is

used is dependant on which model is being used (Figure 26 or Figure 27). For Figure 26, r_2 represents the amount of additional light absorbed and r_1 represents the corresponding and apparent (optical) growth in incremantal dot size. For Figure 27, r_1 represents the amount of additional light absorbed and r_2 represents the corresponding and apparent (optical) growth in incremental dot size.

Since this model of light absorption by the under side of the halftone dot should hold true for all dot area levels (except for perhaps the extremes), the distance defined by r_2 will remain constant for for all dot areas less than 50%. Likewise, the distance defined by by r_1 will remain constant for dot areas greater than 50%. By finding the values for both r_1 and r_2, it is possible to calculate the optical dot areas for any level from 0 to 100%.

The optical dot areas may then be factored by the chosen n-value to again obtain the actual dot area, again using the Yule-Nielsen equation. This n-factored actual dot area value may then be compared to the original actual dot area value to detect any error that could be attibutable to n. This may be done for any level using equations 22, 6, and 9 for the highlight-to-midtone model of Figure 26 and equations 23, 6, and 9 for the midtone-to-shadow model of Figure 27.

$$ODA = ADA + \{ADA - [\pi(\sqrt{ADA/\pi} - r_2)^2]\} \tag{22}$$

$$ODA = 1 - (1-ADA) - \pi(\sqrt{(1-ADA)/\pi} + r_1)^2 - (1-ADA) \tag{23}$$

$$D_t = -\log\,[1-ODA(1-10^{-Ds})] \tag{6}$$

$$PDA = \frac{1 - 10^{-Dt/n}}{1 - 10^{-Ds/n}} \tag{9}$$

where ADA = Actual dot area
ODA = Optical dot area
PDA = n-factored dot area (to be compared to A).

NOTE: Equations have been left unsimplified for illustrative purposes.

By subtracting the actual dot area ADA from from the n-factored dot area PDA, the error in dot area attributable to n may be found.

A computer program was written to test for this error (see Appendix III) and the output from this program is given as Tables 12 through 20. The output was then graphed and appear as Figures 28 through 36.

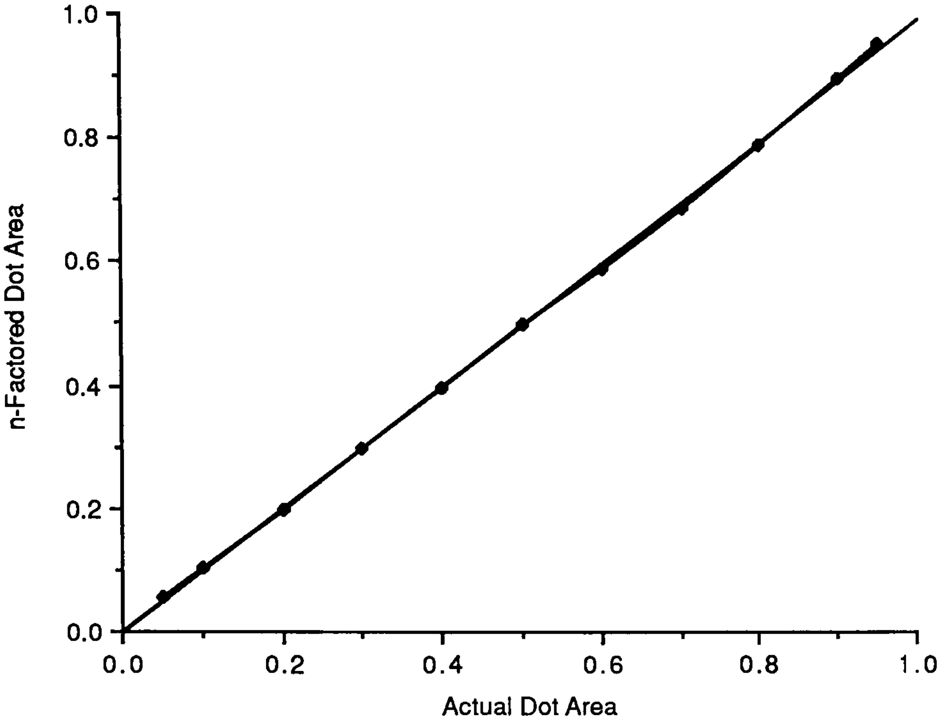


Figure 28.
Plot of Data in Table 12.

DENSITY OF SOLIO: .8
n-VALUE: 1.5

ACTUAL DOT AREA	n-FACTOREO DOT AREA	ERROR IN DOT AREA ATTRIBUTABLE TO n
.05	.0565	.0065
.1	.1052	.0052
.2	.2007	.0007
.3	.2972	-.0028
.4	.3966	-.0034
.5	.5	0
.6	.594	-.006
.7	.6927	-.0073
.8	.7968	-.0032
.9	.9067	.0067
.95	.9627	.0127

Table 12.
Output From Computer Program in Figure 46 (D_S=.8, n=1.5).

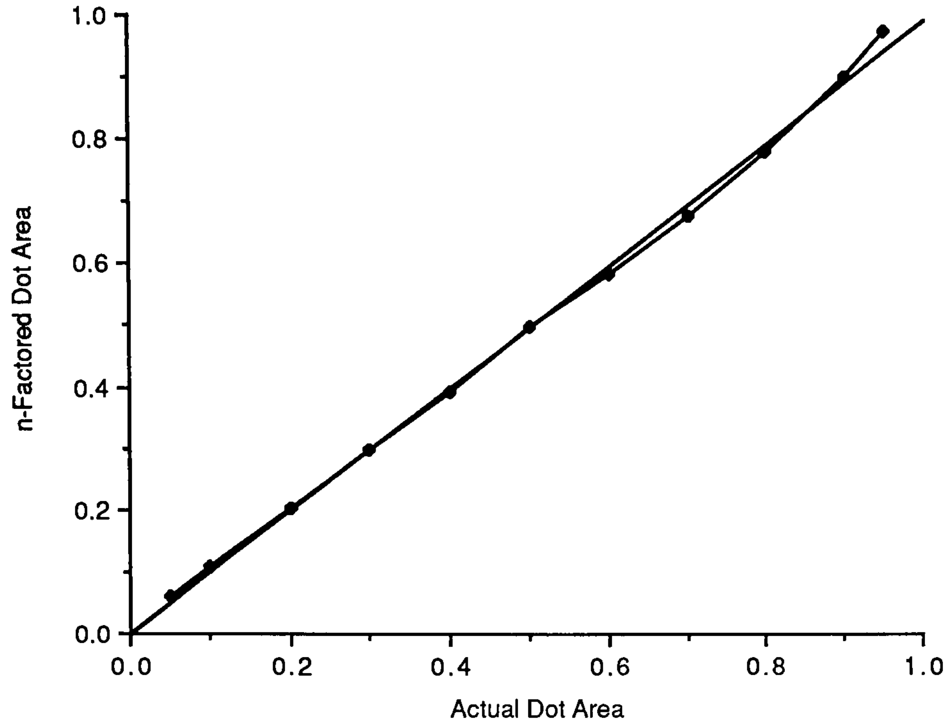


Figure 29.
Plot of Data in Table 13.

DENSITY OF SOLID: .8
n-VALUE: 2.5

ACTUAL DOT AREA	n-FACTORED DOT AREA	ERROR IN DOT AREA ATTRIBUTABLE TO n
.05	.0557	.0057
.1	.1027	.0027
.2	.1943	-.0057
.3	.289	-.011
.4	.39	-.01
.5	.5	0
.6	.593	-.007
.7	.6947	-.0053
.8	.807	.007
.9	.9303	.0303
.95	.9926	.0426

Table 13.
Output From Computer Program in Figure 46 (D_s=.8, n=2.5).

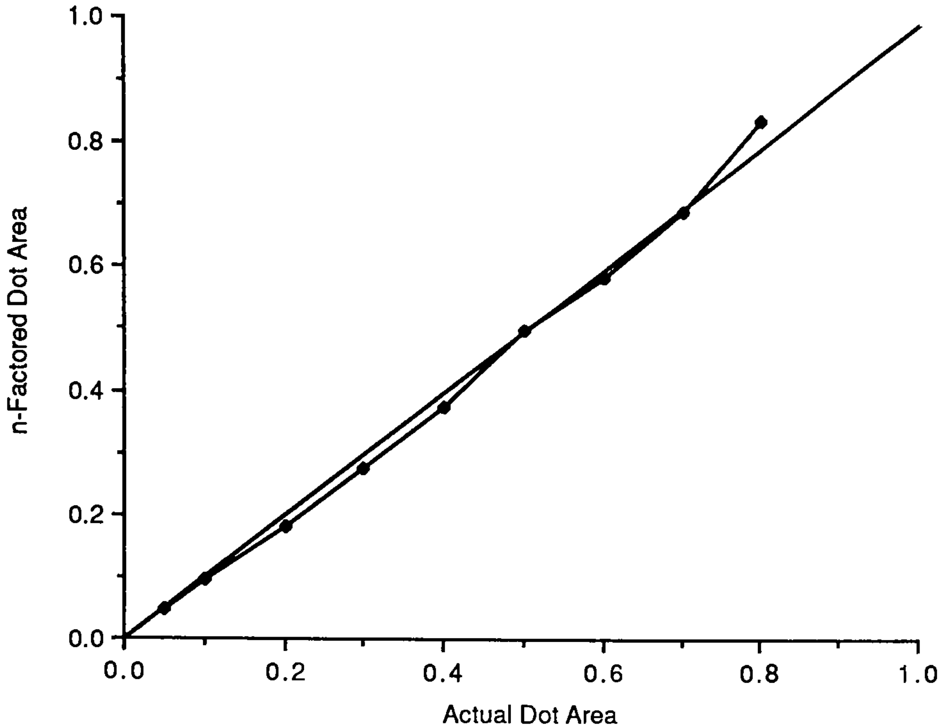


Figure 30.
Plot of Data in Table 14.

DENSITY OF SOLID: .8
n-VALUE: 3.5

ACTUAL DOT AREA	n-FACTORED DOT AREA	ERROR IN DOT AREA ATTRIBUTABLE TO n
.05	.0539	.0039
.1	.0998	-.0002
.2	.1897	-.0103
.3	.2837	-.0163
.4	.3859	-.0141
.5	.5	0
.6	.5938	-.0062
.7	.6984	-.0016
.8	.8163	.0163
.9	.9479	.0479

CALCULATED DOT AREA IS > 100%. CANNOT COMPUTE THE NEXT FUNCTION.

Table 14.
Output From Computer Program in Figure 46 (D_s=.8, n=3.5).

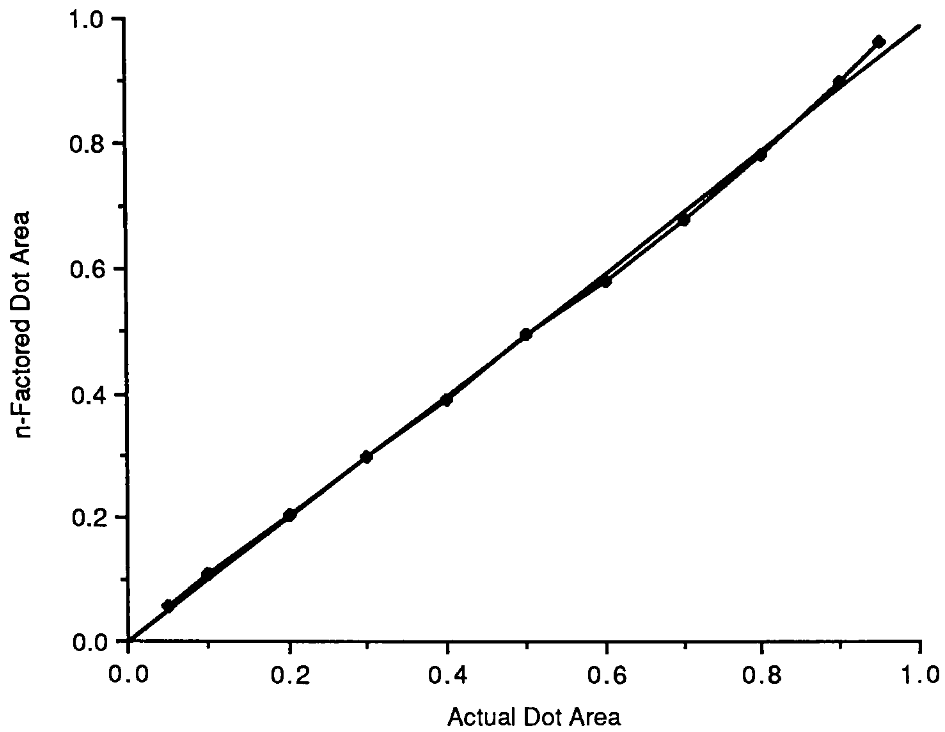


Figure 31.
Plot of Data in Table 15.

DENSITY OF SOLID: 1.3
n-VALUE: 1.5

ACTUAL DOT AREA	n-FACTORED DOT AREA	ERROR IN DOT AREA ATTRIBUTABLE TO n
.05	.0581	.0081
.1	.1069	.0069
.2	.2014	.0014
.3	.2968	-.0032
.4	.3957	-.0043
.5	.5	0
.6	.5904	-9.600001E-03
.7	.6872	-.0128
.8	.7924	-.0076
.9	.9093	.0093
.95	.9724	.0224

Table 15.
Output From Computer Program in Figure 46 (D_s=1.3, n=1.5).

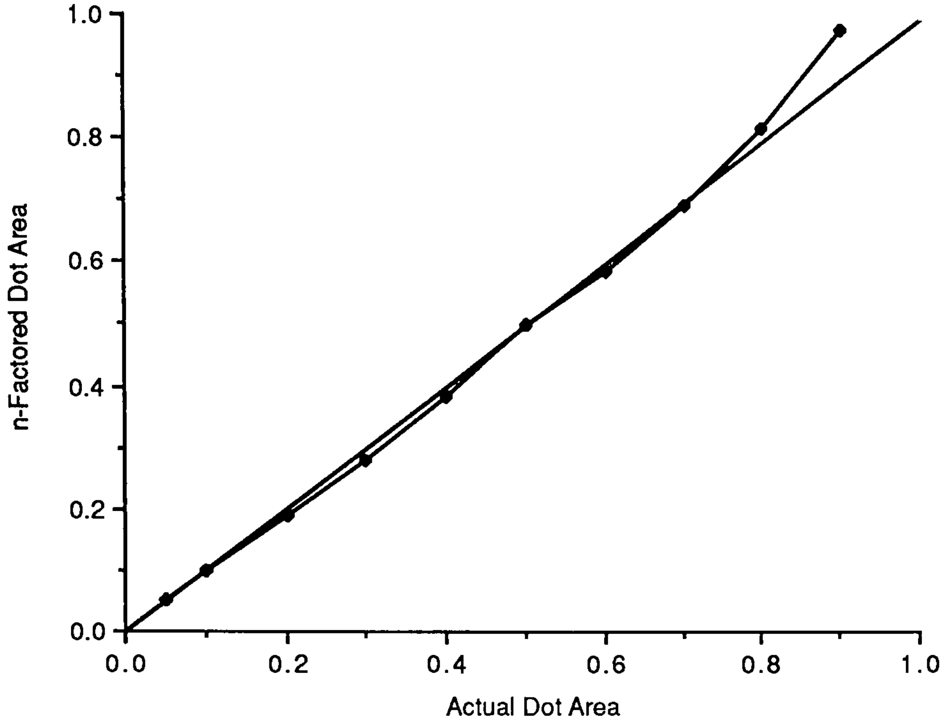


Figure 32.
Plot of Data in Table 16.

OENSITY OF SOLIO: 1.3
n-VALUE: 2.5

ACTUAL DOT AREA	n-FACTOREO OOT AREA	ERROR IN DOT AREA ATTRI8UTA8LE TO n
.05	.0532	.0032
.1	.0995	-.0005
.2	.1888	-.0112
.3	.2819	-.0181
.4	.3838	-.0162
.5	.5	0
.6	.5908	-.0092
.7	.6955	-.0045
.8	.8216	.0216
.9	.9837	.0837

CALCULATED OOT AREA IS > 100%. CANNOT COMPUTE THE NEXT FUNCTION.

Table 16.
Output From Computer Program in Figure 46 (D_S=1.3, n=2.5).

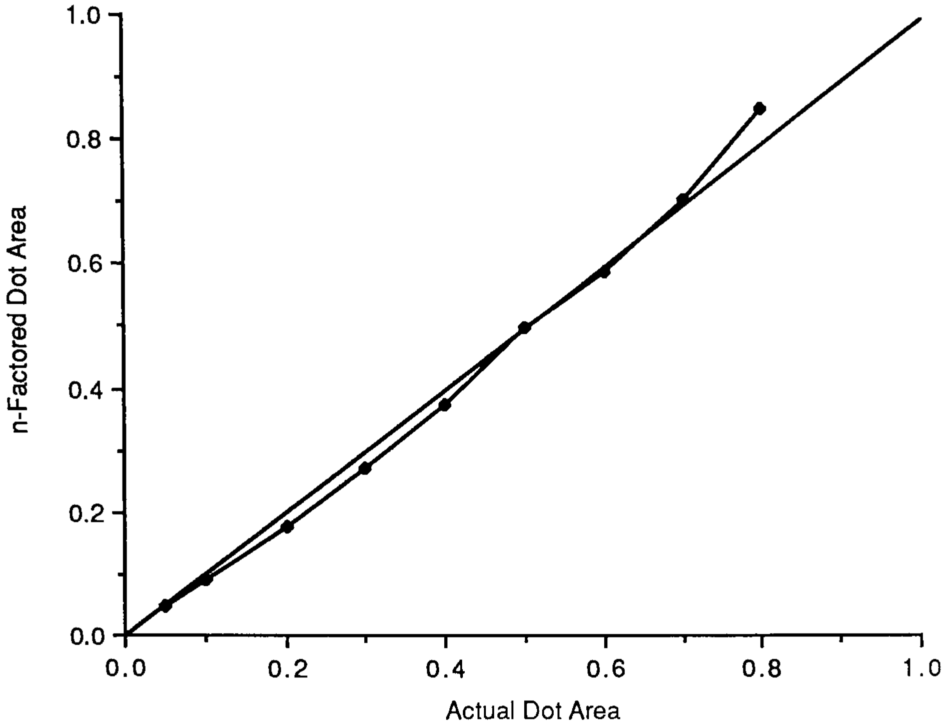


Figure 33.
Plot of Data in Table 17.

OENSITY OF SOLIO: 1.3
n-VALUE: 3.5

ACTUAL OOT AREA	n-FACTORED OOT AREA	ERROR IN DOT AREA ATTRIBUTABLE TO n
.05	.0483	-.0017
.1	.0928	-.0072
.2	.1792	-.0208
.3	.2712	-.0288
.4	.3753	-.0247
.5	.5	0
.6	.5945	-.0055
.7	.7086	.0086
.8	.8556	.0556

CALCULATEO DOT AREA IS > 100%. CANNOT COMPUTE THE NEXT FUNCTION.

Table 17.
Output From Computer Program in Figure 46 (D_s=1.3, n=3.5).

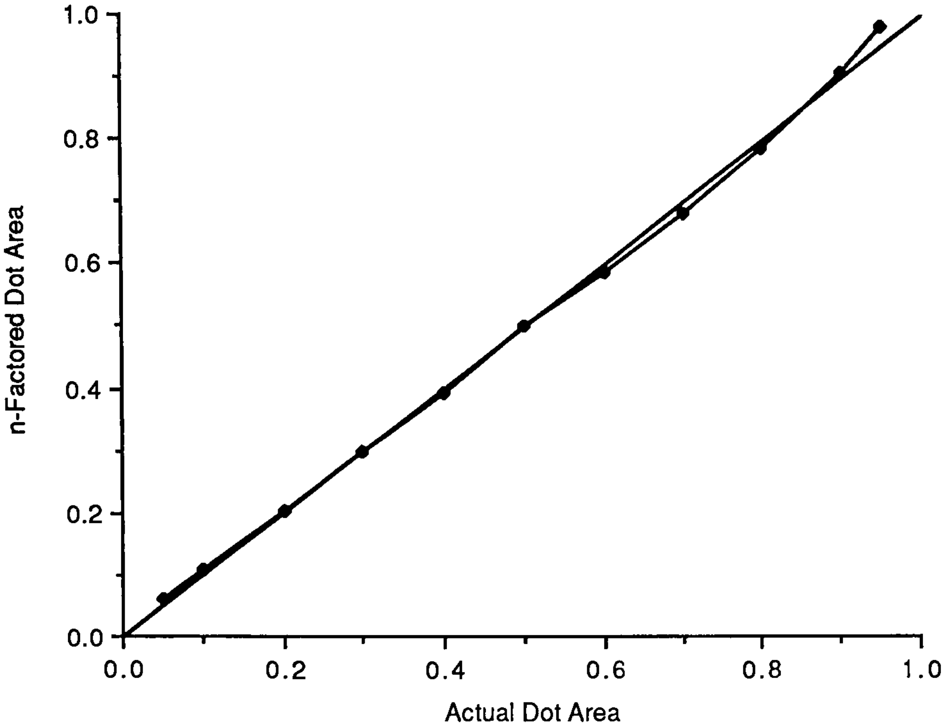


Figure 34.
Plot of Data in Table 18.

OENSITY OF SOLIO: 1.8
n-VALUE: 1.5

ACTUAL OOT AREA	n-FACTOREO OOT AREA	ERROR IN OOT AREA ATTRI8UTABLE TO n
.05	.0588	.0088
.1	.1078	.0078
.2	.2019	.0019
.3	.2968	-.0032
.4	.3954	-.0046
.5	.5	0
.6	.5878	-.0122
.7	.6827	-.0173
.8	.7877	-.0123
.9	.9097	.0097
.95	.9823	.0323

Table 18.
Output From Computer Program in Figure 46 (D_S=1.8, n=1.5).

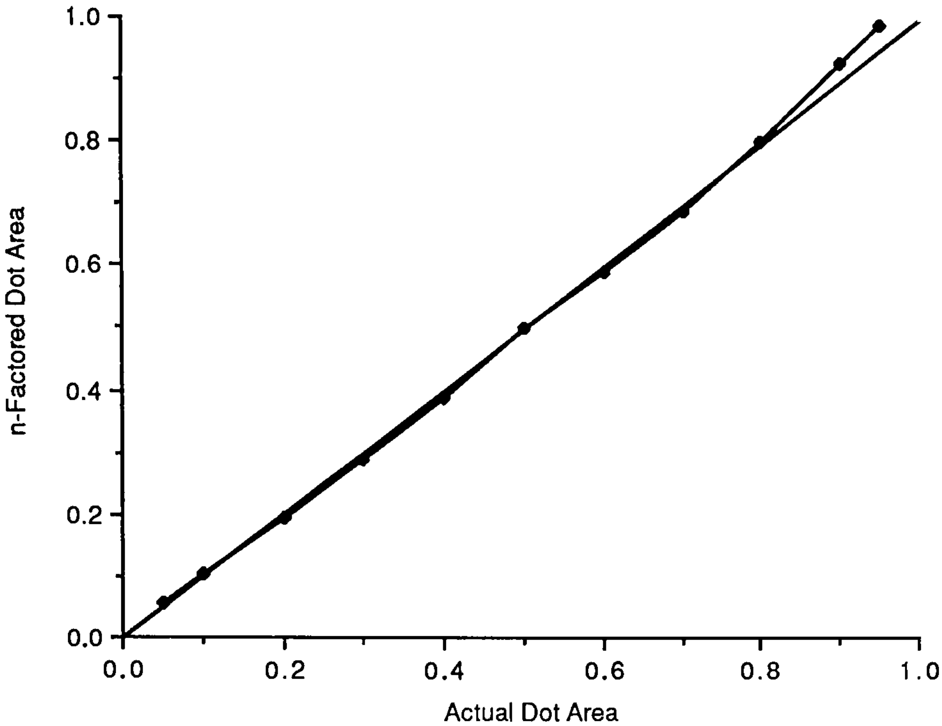


Figure 35.
Plot of Data in Table 19.

OENSITY OF SOLIO: 1.8
n-VALUE: 2.5

ACTUAL DOT AREA	n-FACTORED DOT AREA	ERROR IN DOT AREA ATTRI8UTA8LE TO n
.05	.0497	-.0003
.1	.0957	-.0043
.2	.1837	-.0163
.3	.2759	-.0241
.4	.3786	-.0214
.5	.5	0
.6	.5899	-.0101
.7	.6989	-.0011
.8	.8454	.0454

CALCULATED OOT AREA IS > 100%. CANNOT COMPUTE THE NEXT FUNCTION.

Table 19.
Output From Computer Program in Figure 46 (D_S=1.8, n=2.5).

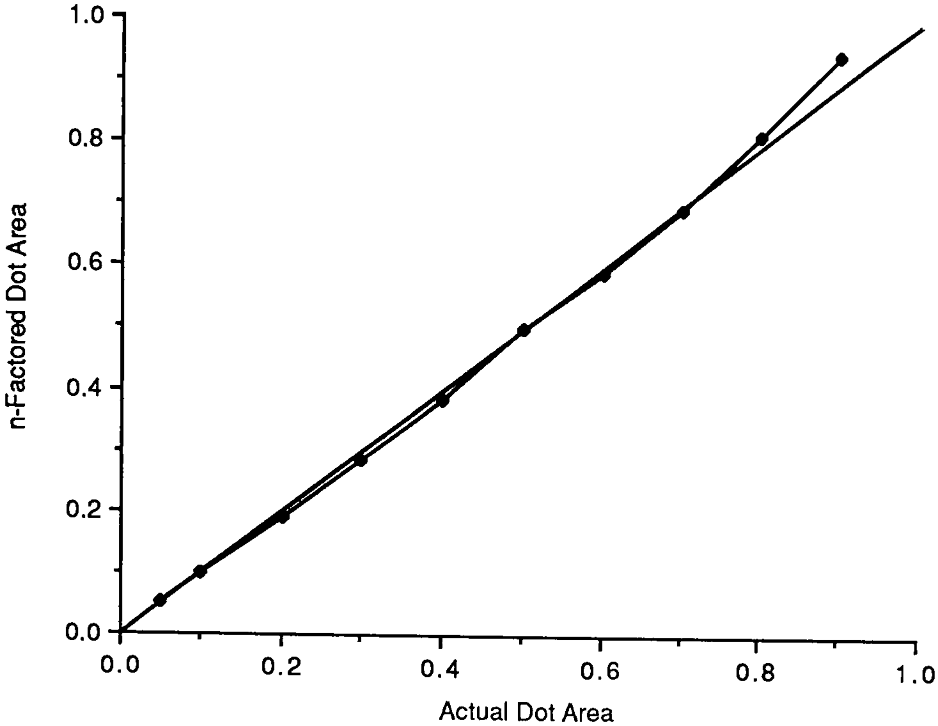


Figure 36.
Plot of Data in Table 20.

DENSITY OF SOLID: 1.8
n-VALUE: 3.5

ACTUAL DOT AREA	n-FACTORED DOT AREA	ERROR IN DOT AREA ATTRIBUTABLE TO n
.05	.0418	-.0082
.1	.0851	-.0149
.2	.169	-.031
.3	.2595	-.0405
.4	.3651	-.0349
.5	.5	0
.6	.598	-.002
.7	.7291	.0291
.8	.9568	.1568

CALCULATED DOT AREA IS > 100%. CANNOT COMPUTE THE NEXT FUNCTION.

Table 20.
Output From Computer Program in Figure 46 (D_s=1.8, n=3.5).

APPENDIX II.

MATHEMATICAL MODELS FOR TESTING THE SOLUTION ACCURACY OF THE YULE-NIELSEN EQUATION WITH RESPECT TO n-VALUE AND DOT AREA GIVEN CHANGES IN SOLID INK DENSITY

PART I. THE VARIANCE OF n-VALUE

The stated purpose of the n-value in to factor optical dot gain out of the Murray-Davies equation. Optical dot gain has long been stated to be primarily a function of the light-scattering charactetistics of the substrate and of the screen ruling. Because of this assumption, it has been assumed that the n-value is unique for these conditions and that only one n-value may be paired with any given quantity of optical dot gain. This is not true. There may be many values for n for any one optical dot gain quantity.

It is the level of solid ink density that also has a significant effect on the value of n. Lower solid ink densities will effect a higher value of n. Conversely, higher solid ink densities will effect a lower value of n.

These effects are aggravated by the existing level of optical dot gain. The greater the level of optical dot gain in a given situation, the greater the n-value will be affected by any changes in solid ink density. The opposite is true of lower levels of optical dot gain.

This effect may be investigated with the solution

procedure outlined in Figure 37. Given the three variables of actual dot area (ADA), optical dot gain (ODG), and a range of solid ink densities (D_s and D_{s2}), the effect on the n-value may be observed. To make these computations easier, the equations were included in a computer program which appears in Figure 47 in Appendix III.

	REQUIRED	SOLUTION	METHOD
1.	ADA	(input variable)	given
2.	ODG	(input variable)	given
3.	D_s - .8 to 1.8	(input variables)	given
4.	ODA	ODA ADA ODG	direct
5.	D_t	$D_t = -\log[1-ODA(1-10^{-Ds})]$	direct
6.	n	$PDA = \frac{1 - 10^{-Dt/n}}{1 - 10^{-Ds/n}}$	by iteration when PDA = ADA
7.	Repeat steps 3 through 6 to completion.		

Figure 37.
Solution Procedure for the Discovery of n-Value Variance.

The change in n-value was investigated in terms of three levels of optical dot gain over a selection of solid ink densities which ranged from .8 to 1.8. The results of this investigation are published in Tables 21, 22, and 23 and graphed in Figure 39.

Each level of optical dot gain in this graph will have a unique n-value curve which is derived from changes in

solid ink density. Once this curve is known, it is easy to adjust the value of n to a "standard" solid ink density. This will standardize the n-value to a given set of conditions which should render it more useful in describing the light-scattering properties of substrates. This value may be termed "adjusted n-value."

PART II.
THE VARIANCE OF DOT AREA

If we accept the commonly held notion that the n-value is unique and constant for paper type and screen ruling, then we would not expect any changes in the amount of optical dot gain for any changes in solid ink density. As we have seen, this is not true for the n-value and it will not be true for optical dot gain if n is held as a constant.

This effect was investigated by using the solution procedure outlined in Figure 38 to test the effect of solid ink density changes on optical dot gain using the n-value as a constant. This was done for the 50% level of dot area. Three levels of optical dot gain were chosen (5%, 10%, and 15%) along with three levels of solid ink density (1.8, 1.3, and .8). The error in dot area, found by comparing the original optical dot area figure with an n-factored optical dot area figure, was noted for different levels of solid ink density. A computer program was written to find these relationships. The program appears in Appendix III.

The results for 5% optical dot gain appear in Tables 24

through 26; for 10% optical dot gain in Tables 27 though 29; and for 15% optical dot gain in Tables 30 through 32. These results were graphed in Figure 40 for 5% optical dot gain, Figure 41 for 10% optical dot gain, and Figure 42 for 15% optical dot gain.

	REQUIRED	SOLUTION	METHOD
1.	ADA	(input variable)	given
2.	ODG	(input variable)	given
3.	D_s	(input variable)	given
4.	ODA	ODA = ADA - ODG	direct
5.	D_t	$D_t = -\log[1-ODA(1-10^{-Ds})]$	direct
6.	n	PDA $\frac{1 - 10^{-Dt/n}}{1 - 10^{-Ds/n}}$	by iteration when PDA = ADA
7.	D_{s2} = .8 to 1.8	(input variables)	given
8.	D_{t2}	$D_{t2} = -n \log[1-ADA(1-10^{-Ds2/n})]$	direct
9.	DDA_2	$ODA_2 = \frac{1 - 10^{-Dt2}}{1 - 10^{-Ds2}}$	direct
10.	DAE	DAE ODA_2 - ODA	direct
11.	Repeat steps 7 though 10 to completion.		

Figure 29.
Solution Procedure for the Discovery of Dot Area Variance

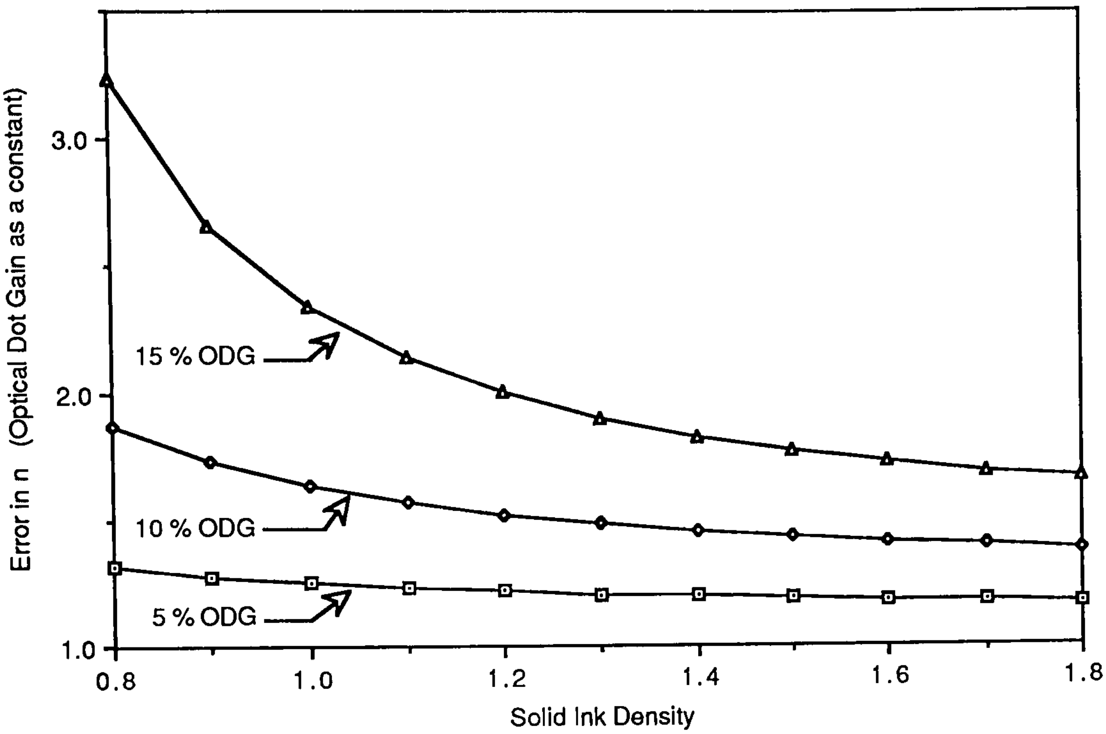


Figure 39.
Plots of n-value vs. D_s (Three Levels of Optical Dot Gain)
From Data in Tables 21, 22, and 23.

FOR 5 % OPTICAL DOT GAIN AT 50 %

SID	n
1.8	1.173
1.7	1.1772
1.6	1.1823
1.5	1.1886
1.4	1.1963
1.3	1.2057
1.2	1.2176
1.1	1.2327
1	1.2522
.9	1.2779
.8	1.3129

Table 21.
Output From Computer Program in Figure 47 (5% ODG).

FOR 9.999999 % OPTICAL DOT GAIN AT 50 %

SID	n
1.8	1.3861
1.7	1.3986
1.6	1.4139
1.5	1.4327
1.4	1.4561
1.3	1.4856
1.2	1.5234
1.1	1.5729
1	1.6397
.9	1.7332
.8	1.8716

Table 22.
Output from Computer Program in Figure 47 (10% ODG).

FOR 15 % OPTICAL OOT GAIN AT 50 %

SID	n
1.8	1.6636
1.7	1.6925
1.6	1.7283
1.5	1.7732
1.4	1.8304
1.3	1.9049
1.2	2.0047
1.1	2.1433
1	2.3461
.9	2.6666
.8	3.2393

Table 23.
Output From Computer Program in Figure 47 (15% ODG).

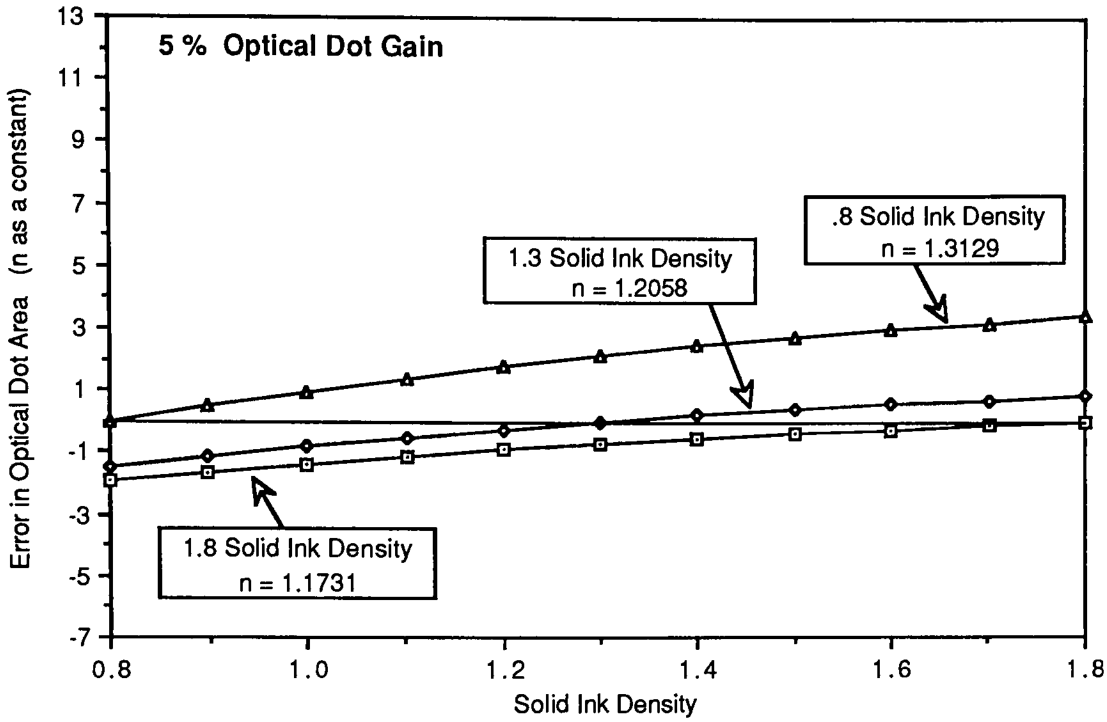


Figure 40.
Plots of Dot Area Error vs. D_s (Three Levels of D_s, 5% ODG) From Data in Tables 24, 25, and 26.

FOR 5 % OPTICAL OOT GAIN AT 50 % AND .8 SOLIO INK DENSITY.
n = 1.312919

SOLID INK DENSITY	TINT DENSITY	OPTICAL DOT AREA	ERROR IN DOT AREA ATTRIBUTABLE TO n HELD CONSTANT AGAINST SID
1.8	.3714	58.41 %	3.41 %
1.7	.367	58.2 %	3.2 %
1.6	.3617	57.98 %	2.98 %
1.5	.3555	57.72 %	2.72 %
1.4	.3482	57.44 %	2.44 %
1.3	.3396	57.12 %	2.12 %
1.2	.3296	56.76 %	1.76 %
1.1	.3178	56.38 %	1.38 %
1	.3041	55.95 %	.95 %
.9	.2882	55.49 %	.49 %
.8	.2698	55 %	0 %

Table 24.
Output From Computer Program in Figure 48 (D_s=.8, 5% ODG).

FOR 5 % OPTICAL OOT GAIN AT 50 % AND 1.3 SOLIO INK OENSITY.
n = 1.205799

SOLIO INK DENSITY	TINT DENSITY	OPTICAL OOT AREA	ERROR IN DOT AREA ATTRI8UTA8LE TO n HELD CONSTANT AGAINST SID
1.8	.3464	55.84 %	.84 %
1.7	.3429	55.71 %	.71 %
1.6	.3388	55.56 %	.56 %
1.5	.3339	55.4 %	.4 %
1.4	.328	55.21 %	.21 %
1.3	.3209	55 %	0 %
1.2	.3125	54.76 %	-.24 %
1.1	.3025	54.5 %	-.5 %
1	.2906	54.21 %	-.79 %
.9	.2766	53.89 %	-1.11 %
.8	.2601	53.54 %	-1.46 %

Table 25.
Output from Computer Program in Figure 48 (D_s=1.3, 5% ODG).

FOR 5 % OPTICAL OOT GAIN AT 50 % AND 1.8 SOLID INK OENSITY.
n = 1.173051

SOLIO INK DENSITY	TINT OENSITY	OPTICAL OOT AREA	ERROR IN OOT AREA ATTRI8UTA8LE TO n HELO CONSTANT AGAINST SID
1.8	.3384	55 %	0 %
1.7	.3353	54.89 %	-.11 %
1.6	.3315	54.76 %	-.24 %
1.5	.3269	54.62 %	-.38 %
1.4	.3214	54.46 %	-.54 %
1.3	.3148	54.29 %	-.71 %
1.2	.3069	54.09 %	-.91 %
1.1	.2974	53.86 %	-1.14 %
1	.2861	53.62 %	-1.38 %
.9	.2727	53.35 %	-1.65 %
.8	.2568	53.05 %	-1.95 %

Table 26.
Output From Computer Program in Figure 48 (D_s=.8, 5% ODG).

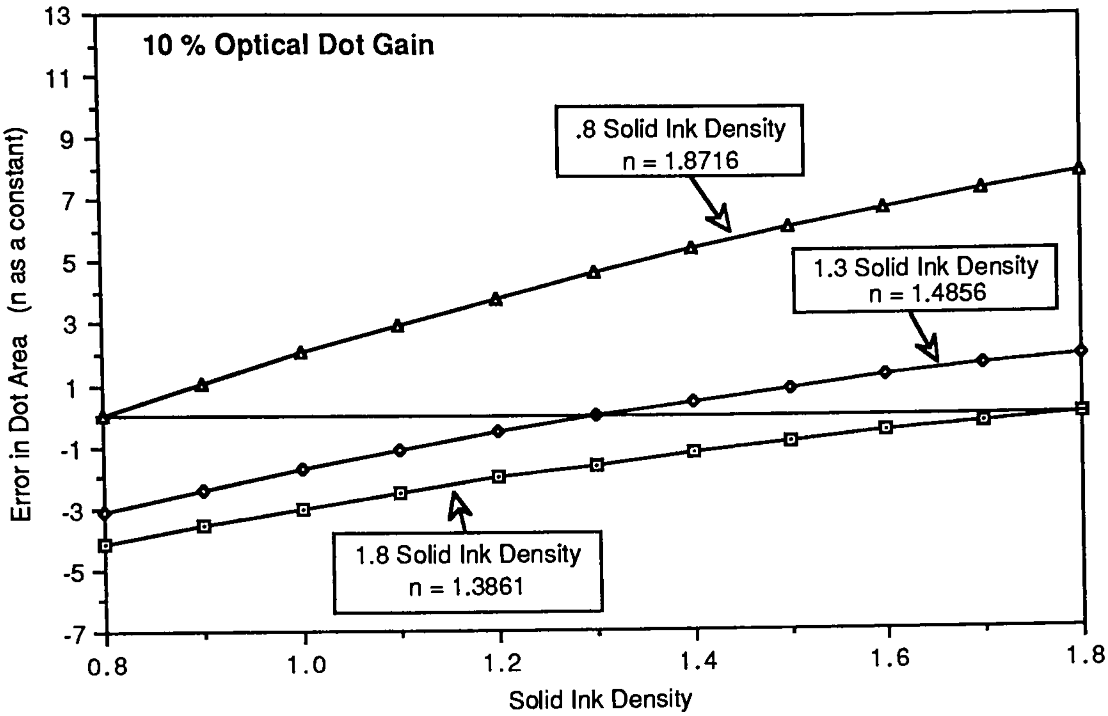


Figure 41.
Plots of Dot Area Error vs. D_s (Three Levels of D_s, 10% ODG)
From Data in Tables 27, 28, 29.

FOR 10 % OPTICAL OOT GAIN AT 50 % AND 1.8 SOLIO INK DENSITY.
n = 1.386143

SOLID INK OENSITY	TINT OENSITY	OPTICAL OOT AREA	ERROR IN DOT AREA ATTRIBUTABLE TO n HELD CONSTANT AGAINST SIO
1.8	.3877	60 %	0 %
1.7	.3825	59.74 %	-.26 %
1.6	.3764	59.46 %	-.54 %
1.5	.3693	59.15 %	-.85 %
1.4	.3611	58.8 %	-1.2 %
1.3	.3515	58.41 %	-1.59 %
1.2	.3403	57.99 %	-2.01 %
1.1	.3274	57.52 %	-2.48 %
1	.3125	57.01 %	-2.99 %
.9	.2954	56.46 %	-3.54 %
.8	.2758	55.B7 %	-4.13 %

Table 27.
Output From Computer Program in Figure 48 (D_s=.8, 10% ODG).

FOR 10 % OPTICAL DDT GAIN AT 50 % AND 1.3 SOLID INK DENSITY.
n - 1.485615

SOLID INK DENSITY	TINT DENSITY	OPTICAL DOT AREA	ERROR IN DOT AREA ATTRIBUTABLE TO n HELD CONSTANT AGAINST SID
1.8	.4087	61.96 %	1.96 %
1.7	.4025	61.64 %	1.64 %
1.6	.3953	61.29 %	1.29 %
1.5	.387	60.9 %	.9 %
1.4	.3774	60.47 %	.47 %
1.3	.3664	60 %	0 %
1.2	.3538	59.47 %	-.53 %
1.1	.3394	58.91 %	-1.09 %
1	.323	58.29 %	-1.71 %
.9	.3043	57.63 %	-2.37 %
.8	.2832	56.92 %	-3.08 %

Table 28.
Output from Computer Program in Figure 48 (D_s=1.3, 10% ODG).

FOR 10 % OPTICAL DOT GAIN AT 50 % AND .8 SOLID INK DENSITY.
n 1.871627

SOLID INK DENSITY	TINT DENSITY	OPTICAL DOT AREA	ERROR IN DOT AREA ATTRIBUTABLE TO n HELD CONSTANT AGAINST SID
1.8	.4791	67.89 %	7.89 %
1.7	.4687	67.36 %	7.36 %
1.6	.4571	66.77 %	6.77 %
1.5	.4442	66.13 %	6.13 %
1.4	.4298	65.43 %	5.43 %
1.3	.4138	64.67 %	4.67 %
1.2	.3961	63.86 %	3.86 %
1.1	.3765	62.98 %	2.98 %
1	.355	62.05 %	2.05 %
.9	.3313	61.05 %	1.05 %
.8	.3053	59.99 %	-.01 %

Table 29.
Output From Computer Program in Figure 48 (D_s=1.8, 10% ODG).

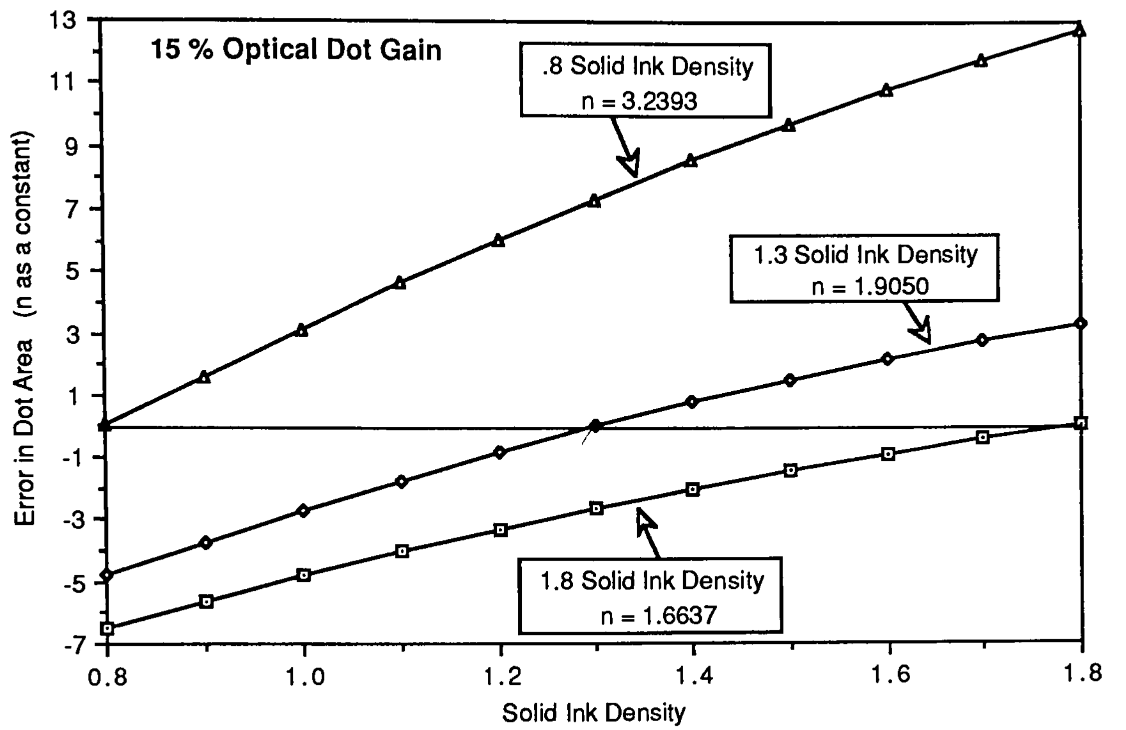


Figure 42.
Plots of Dot Area Error vs. D_s (Three Levels of D_s, 15% ODG) From Data in Tables 30, 31, 32.

FOR 15 % OPTICAL OOT GAIN AT 50 % ANO .8 SOLIO INK OENSITY.
n 3.239332

SOLIO INK OENSITY	TINT OENSITY	OPTICAL OOT AREA	ERROR IN OOT AREA ATTRI8UTA8LE TO n HELO CONSTANT AGAINST SIO
1.8	.6298	77.78 %	12.78 %
1.7	.6074	76.84 %	11.84 %
1.6	.5838	75.83 %	10.83 %
1.5	.5588	74.75 %	9.75 %
1.4	.5325	73.59 %	8.59 %
1.3	.5048001	72.35 %	7.35 %
1.2	.4757	71.04 %	6.04 %
1.1	.4451	69.65 %	4.65 %
1	.4129	68.17 %	3.17 %
.9	.3792	66.62 %	1.62 %
.8	.3438	64.99 %	-.01 %

Table 30.
Output From Computer Program in Figure 48 (D_s=.8, 15% ODG).

FOR 15 % OPTICAL DOT GAIN AT 50 % AND 1.3 SOLID INK DENSITY.
n = 1.904996

SOLID INK DENSITY	TINT DENSITY	OPTICAL DOT AREA	ERROR IN DOT AREA ATTRIBUTABLE TO n HELD CONSTANT AGAINST SID
1.8	.4844	68.31 %	3.31 %
1.7	.4737	67.75 %	2.75 %
1.6	.4617	67.15 %	2.15 %
1.5	.4484	66.49 %	1.49 %
1.4	.4336	65.77 %	.77 %
1.3	.4172	65 %	0 %
1.2	.3991	64.16 %	-.84 %
1.1	.3792	63.26 %	-1.74 %
1	.3573	62.3 %	-2.7 %
.9	.3332	61.28 %	-3.72 %
.8	.3068	60.2 %	-4.8 %

Table 31.
Output from Computer Program in Figure 48 (D_s=1.3, 15% ODG).

FOR 15 % OPTICAL DOT GAIN AT 50 % AND 1.8 SOLID INK DENSITY.
n = 1.663664

SOLID INK DENSITY	TINT DENSITY	OPTICAL DOT AREA	ERROR IN DOT AREA ATTRIBUTABLE TO n HELD CONSTANT AGAINST SID
1.8	.4433	65 %	0 %
1.7	.4351	64.57 %	-.43 %
1.6	.4259	64.1 %	-.9 %
1.5	.4154	63.59 %	-1.41 %
1.4	.4035	63.02 %	-1.98 %
1.3	.3902	62.4 %	-2.6 %
1.2	.3751	61.73 %	-3.27 %
1.1	.3582	61.01 %	-3.99 %
1	.3392	60.23 %	-4.77 %
.9	.318	59.4 %	-5.6 %
.8	.2945	58.51 %	-6.49 %

Table 32.
Output From Computer Program in Figure 48 (D_s=1.8, 15% ODG).

APPENDIX III

MBASIC COMPUTER PROGRAMS

```
10 PRINT "ENTER ACTUAL TINT AREA (as a decimal)."
20 INPUT A
30 PRINT "ENTER OENSITY OF TINT ON PAPER."
40 INPUT 01
50 PRINT "ENTER OENSITY OF SOLIO ON PAPER."
60 INPUT 02
70 PRINT "   "
80 PRINT "   ","   ","   "," SIO   1.3"," AOJUSTEO n"
90 LPRINT "   ","   ","   "," SIO   1.3"," AOJUSTEO n"
100 PRINT "   ","   ","   ","OOG CONSTANT","SIO CONSTANT","OOG ERROR IN"
110 LPRINT "   ","   ","   ","OOG CONSTANT","SIO CONSTANT","OOG ERROR IN"
120 PRINT "   ","   ","MEASURED","  AOJUSTEO","  AOJUSTED","  MEASUREO"
130 LPRINT "   ","   ","MEASURED","  AOJUSTEO","  AOJUSTEO","  MEASUREO"
140 PRINT "   ","   "," SAMPLE","   SAMPLE","   SAMPLE","   SAMPLE"
150 LPRINT "   ","   "," SAMPLE","   SAMPLE","   SAMPLE","   SAMPLE"
160 PRINT "   ","   ","-----------------------------------------------------"
170 LPRINT "   ","   ","-----------------------------------------------------"
175 LET F   0
180 LET A1   (1-10^(-01))/(1-10^(-02))
190 LET 6   A1   A
200 LET N   1
210 LET I   1
220 GOTO 260
230 IF I < .000001 THEN 305
240 LET N   N   I
250 LET I   I/10
260 LET N   N + I
270 LET A2   (1-10^(-D1/N))/(1-10^(-02/N))
280 IF A >= A2 THEN 230
290 IF N > 6 THEN 560
300 GOTO 260
305 IF F   1 THEN 400
310 LET 04   D2
320 LET 03   D1
330 LET A3   A1
340 LET G3   G
350 LET N3   N
360 LET D2   1.3
370 LET F   1
380 LET D1   -LOG(1-A1*(1-10^(-D2)))/LOG(10)
390 GOTO 190
```

Figure 43.
Solution for n-value, adjusted n-value, and ODG error.

```
400 REM use adjusted n (N) with original density (D4)
410 LET D12 = -N*LOG(1-A*(1-10^(-D4/N)))/LOG(10)
420 LET A12 = (1-10^(-D12))/(1-10^(-D4))
425 LET M = 10000
430 PRINT "SOLID INK DENSITY:",D4,D2,D4
440 LPRINT "SOLID INK DENSITY:",D4,D2,D4
450 PRINT "TINT DENSITY:      ",D3,INT(D1*M)/M,INT(D12*M)/M
460 LPRINT "TINT DENSITY:      ",D3,INT(D1*M)/M,INT(D12*M)/M
470 PRINT "OPTICAL DOT AREA:",INT(A3*M)/M,INT(A1*M)/M,INT(A12*M)/M
480 LPRINT "OPTICAL DOT AREA:",INT(A3*M)/M,INT(A1*M)/M,INT(A12*M)/M
490 PRINT "ACTUAL DOT AREA:",A,A,A
500 LPRINT "ACTUAL DOT AREA:",A,A,A
510 PRINT "OPTICAL DOT GAIN:",INT(G3*M)/M,INT(G*M)/M,INT((A12-A)*M)/M,INT((A12-A1)*M)/M
520 LPRINT "OPTICAL DOT GAIN:",INT(G3*M)/M,INT(G*M)/M,INT((A12-A)*M)/M,INT((A12-A1)*M)/M
530 PRINT "n-VALUE:          ",INT(N3*M)/M,INT(N*M)/M,INT(N*M)/M
540 LPRINT "n-VALUE:          ",INT(N3*M)/M,INT(N*M)/M,INT(N*M)/M
550 GOTO 580
560 PRINT "ONE VALUE OF n IS > 6.  CHECK YOUR DATA."
570 LPRINT "ONE VALUE OF n IS > 6.  CHECK YOUR DATA."
580 PRINT "   "
590 LPRINT "   "
600 LPRINT "   "
610 LPRINT "   "
620 END
```

Figure 43 (continued).
Solution for n-value, adjusted n-value, and ODG error.

	MEASURED SAMPLE	SID = 1.3 ODG CONSTANT ADJUSTED SAMPLE	ADJUSTED n SID CONSTANT ADJUSTED SAMPLE	ODG ERROR IN MEASURED SAMPLE
SOLID INK DENSITY:	.671	1.3	.671	
TINT DENSITY:	.424	.6065	.3787	
OPTICAL DOT AREA:	.7922	.7922	.7396	
ACTUAL DOT AREA:	.6670001	.6670001	.6670001	
OPTICAL DOT GAIN:	.1252	.1252	.0726	-.0527
n-VALUE:	5.0402	1.7899	1.7899	

Figure 44.
Sample of Output for Program in Figure 43.

```
80 PRINT "Enter Sample Code."
90 INPUT N
100 IF N = 0 THEN 420
110 PRINT "Enter reflectance with black backing (as a decimal)."
120 INPUT R0
130 PRINT "Enter reflectance with white backing (as a decimal)."
140 INPUT R89
150 PRINT "Enter paper caliper (to four decimal places)."
160 INPUT X
170 LET C89   R0/R89
175 LPRINT "Sample Code:";N
180 LPRINT "Paper Caliper:";X
190 LPRINT "Reflectance with Black Backing:";R0
200 LPRINT "Reflectance with White Backing:";R89
210 LPRINT "Opacity of Sample:";C89
220 PRINT "Opacity of Sample:";C89
230 LET A   (((R0-R89+.89)/(R0*.89))+R89)/2
240 LET B   SQR(A^2-1)
250 LET H   (1-(A*R0))/(B*R0)
260 LET S   (LOG((H+1)/(H-1))/2)/B
290 LET S   S*(.004/X)
300 LET K   S*(A-1)
310 LET H1   EXP(-B*S)/(EXP(B*S)-EXP(-B*S))*2+1
320 LET R0   1/(A+B*H1)
330 LET R89   (1-.89*(A-B*H1))/(A-.89+B*H1)
340 LET C89 = R0/R89
350 PRINT "Standard Opacity of Sample (adjusted to .004):";C89
360 LPRINT "Standard Opacity of Sample (adjusted to .004):";C89
370 PRINT "Coefficient of Scatter (adjusted to .004):";S
380 LPRINT "Coefficient of Scatter (adjusted to .004):";S
390 PRINT "Coefficient of Absorption (adjusted to .004):";K
400 LPRINT "Coefficient of Absorption (adjusted to .004):";K
410 GOTO 80
420 END
```

Figure 45.
Solution for Adjusted TAPPI Opacity and Adjusted Scattering Coefficient Using the Kubelka-Munk Equations

```
10  REM THE OPERATION OF N-VALUES ON OOT AREA LEVEL
20  REM A COMPUTER MOOEL
30  PRINT "ENTER OENSITY OF SOLID."
40  INPUT 02
50  PRINT "ENTER n-VALUE."
60  INPUT N
70  PRINT "    "
80  LPRINT "OENSITY OF SOLIO:";02
90  LPRINT "n-VALUE:";N
100 LPRINT "    "
110 PRINT " ACTUAL","n-FACTOREO","ERROR IN OOT AREA"
120 LPRINT " ACTUAL","n-FACTOREO","ERROR IN OOT AREA"
130 PRINT "OOT AREA"," OOT AREA","ATTRIBUTABLE TO n"
140 LPRINT "OOT AREA"," OOT AREA","ATTRIBUTABLE TO n"
150 PRINT "---------------------------------------------"
160 LPRINT "---------------------------------------------"
170 LET F - 0
180 LET P   3.1415927#
190 REM CALCULATE LARGE (R1) ANO SMALL (R2) CHANGES IN OOT RAOIUS
200 LET 01   -N*LOG(1-.5*(1-10^(-02/N)))/LOG(10)
210 LET B = (1-10^(-01))/(1-10^(-02))
220 LET R1   (B/P)^(1/2)-(.5/P)^(1/2)
230 LET B - 1-B
240 LET R2 - (.5/P)^(1/2)-(B/P)^(1/2)
250 GOTO 270
260 LET F = 1
270 REAO A
280 IF F - 1 THEN 360
290 REM HIGHLIGHT TO MIOOLETONE MOOEL
300 LET B   A+(A-(P*(((A/P)^(1/2)-R2)^2)))
310 IF B < -.0001 THEN 480
320 LET 01 - -LOG(1-B*(1-10^(-02)))/LOG(10)
330 LET A1 = (1-10^(-01/N))/(1-10^(-02/N))
340 GOTO 400
350 REM MIOOLETONE TO SHAOOW MOOEL
360 LET B   A+((P*((((1-A)/P)^(1/2)+R1)^2))-(1-A))
370 IF B >= 1 THEN 450
380 LET 01   -LOG(1-B*(1-10^(-02)))/LOG(10)
390 LET A1   (1-10^(-01/N))/(1-10^(-02/N))
400 LET A1   A1 + .00005
410 PRINT A,INT(A1*10000)/10000,INT((A1-A)*10000)/10000
420 LPRINT A,INT(A1*10000)/10000,INT((A1-A)*10000)/10000
422 IF A - .95 THEN 502
430 IF A   .5 THEN 260
440 GOTO 270
450 PRINT "CALCULATEO OOT AREA IS > 100%.  CANNOT COMPUTE THE NEXT FUNCTION."
460 LPRINT "CALCULATEO OOT AREA IS > 100%.  CANNOT COMPUTE THE NEXT FUNCTION."
470 GOTO 502
480 PRINT "CALCULATEO OOT AREA IS < 0%.  CANNOT COMPUTE THE NEXT FUNCTION."
490 LPRINT "CALCULATEO OOT AREA IS < 0%.  CANNOT COMPUTE THE NEXT FUNCTION."
495 GOTO 502
500 OATA .05,.1,.2,.3,.4,.5,.6,.7,.8,.9,.95
502 LPRINT "    "
504 LPRINT "    "
506 LPRINT "    "
510 ENO
```

Figure 46.
Solution for Error in Dot Area Level (n constant)

```
20 PRINT "ENTER ACTUAL TINT AREA (as a decimal)."
30 INPUT A
40 PRINT "ENTER OPTICAL DOT GAIN (as a decimal)."
50 INPUT G
60 PRINT "FOR";G*100;"% OPTICAL DOT GAIN AT";A*100;"%"
70 LPRINT "FOR";G*100;"% OPTICAL OOT GAIN AT";A*100;"%"
71 PRINT "   "
72 PRINT "   "
73 PRINT "   "
74 LPRINT "   "
80 PRINT " SID","   n"
90 LPRINT " SID","   n"
100 PRINT "---------------------"
110 LPRINT "---------------------"
120 READ 02
130 LET A1   A + G
140 LET D1   -LOG(1-A1*(1-10^(-D2)))/LOG(10)
150 LET N   1
160 LET I   1
170 GOTO 210
180 IF I < .000001 THEN 260
190 LET N   N   I
200 LET I   I/10
210 LET N = N + I
220 LET A2   (1-10^(-D1/N))/(1-10^(-02/N))
230 IF A >= A2 THEN 180
240 IF N > 6 THEN 300
250 GOTO 210
260 PRINT 02,INT(N*10000)/10000
270 LPRINT D2,INT(N*10000)/10000
280 IF D2   .8 THEN 330
290 GOTO 120
300 PRINT "THE VALUE OF n IS GREATER THAN 6.  CHECK YOUR DATA."
310 LPRINT "THE VALUE OF n IS GREATER THAN 6.  CHECK YOUR DATA."
320 DATA 1.8,1.7,1.6,1.5,1.4,1.3,1.2,1.1,1.0,.9,.8
330 LPRINT "   "
340 LPRINT "   "
350 LPRINT "   "
360 ENO
```

Figure 47.
Solution for n-value Given D_S and ODG as Variables.

```
5   REM THE ERROR OF N WITH RESPECT TO CHANGES IN SOLIO INK DENSITY
10  PRINT "ENTER ACTUAL TINT AREA (as a decimal)."
20  INPUT A
30  PRINT "ENTER DENSITY OF SOLID."
40  INPUT D2
50  PRINT "ENTER OPTICAL DOT GAIN (as a decimal)."
60  INPUT G
70  PRINT "    "
80  PRINT "    "
90  PRINT "    "
100 LET A1   A + G
110 LET D1 = -LOG(1-A1*(1-10^(-D2)))/LOG(10)
200 LET N = 1
210 LET I = 1
220 GOTO 260
230 IF 1 < .000001 THEN 310
240 LET N = N   I
250 LET I = I/10
260 LET N   N + I
270 LET A2   (1-10^(-D1/N))/(1-10^(-D2/N))
280 IF A >= A2 THEN 230
290 IF N > 6 THEN 470
300 GOTO 260
310 PRINT "FOR";G*100;"% OPTICAL DOT GAIN AT";A*100;"% ANO";02;"SOLIO INK OENSITY."
315 PRINT "n =";N
320 LPRINT "FOR";G*100;"% OPTICAL OOT GAIN AT";A*100;"% AND";02;"SOLIO INK OENSITY."
325 LPRINT "n =";N
330 PRINT "    "
340 LPRINT "    "
350 PRINT "SOLID INK","  TINT"," OPTICAL","ERROR IN DOT AREA ATTRI8UTA8LE"
360 PRINT " DENSITY","DENSITY","OOT  AREA","TO n HELD CONSTANT AGAINST SIO"
370 LPRINT "SOLIO INK","  TINT"," OPTICAL","ERROR IN OOT AREA ATTRI8UTABLE"
380 LPRINT " OENSITY","DENSITY","OOT  AREA","TO n HELO CONSTANT AGAINST SIO"
390 PRINT "---------------------------------------------------------------------"
400 LPRINT "---------------------------------------------------------------------"
410 READ D22
420 LET D12 = -N*LOG(1-A*(1-10^(-D22/N)))/LOG(10)
430 LET A12 = (1-10^(-D12))/(1-10^(-D22))
440 PRINT "  ";022,INT(012*10000)/10000,INT(A12*10000)/100;"%","        "INT((A12-A1)*10000)/100;"%"
450 LPRINT "  ";D22,INT(D12*10000)/10000,INT(A12*10000)/100;"%","        "INT((A12-A1)*10000)/100;"%"
452 IF D22   .8 THEN 492
460 GOTO 410
470 PRINT "THE VALUE OF n IS GREATER THAN 6.  CHECK YOUR DATA."
480 LPRINT "THE VALUE OF n IS GREATER THAN 6.  CHECK YOUR DATA."
490 DATA 1.8,1.7,1.6,1.5,1.4,1.3,1.2,1.1,1.0,.9,.8
492 LPRINT "    "
494 LPRINT "    "
496 LPRINT "    "
500 END
```

Figure 48.
Solution for Error in Dot Area for Changes in D_s.
(Optical Dot Gain and n-value Held Constant)

GLOSSARY OF SYMBOLS

a	1. (opaciity)- An interim solution to the Kubelka-Munk equations. 2. (halftone)- The area of the ink solid on blank paper.
ADA	Actual Dot Area- The actual area of a halftone dot as it truely exists on the press sheet.
Adj. n	Adjusted n-Value- The n-value normalized to a standard solid ink density (1.3) to eliminate the variance in the Yule-Nielsen equation caused by changes in solid ink density.
b	An interim solution to the Kubelka-Munk equations.
$C_{0.89}$	TAPPI Opacity of a paper specimen. Sometimes called the contrast ratio, it is a measure of by how much paper blocks the transmission of light.
$C_{0.89(.004)}$	Adjusted TAPPI Opacity- What the TAPPI Opacity would be if the paper specimen were .004" thick.
DAE	Dot Area Error- The difference in dot area found by comparing an actual dot area with a computed dot area.
D	Density of the tint.
Dp	Density of the paper.
D_p	Density of the paper.
Ds	Solid Ink Density
D_s	Solid Ink Density
Dt	Density of the tint.
D_t	Density of the tint.
ODA	Optical Dot Area- As computed by the Murray-Davies equation given the solid ink density and tint density.

LIST OF SYMBOLS
(continued)

ODG	Optical Dot Gain- The apparent growth in optical dot area, usually ascribed to light-scattering within paper.
n	n-Value- A variable which is used to divide the tint density and solid ink density to find the physical dot area.
PDA	Physical Dot Area- The dot area as computed by the Yule-Nielsen equation given solid ink density, tint density, and an n-value which is used to factor the variables to seek the actual dot area.
PDG	Physical Dot Gain- The growth of a dot because of ink spreading on paper, over/underexposure of halftone films or plates, or some other physical means.
r_1	The incremental increase in dot radius.
r_2	The incremental decrease in dot radius.
R	(halftone) The total reflectance of a halftone tint.
Rp	(halftone) Reflectance of the paper.
R_p	(halftone) Reflectance of the paper.
Rs	(halftone) Reflectance of the solid ink.
R_s	(halftone) Reflectance of the solid ink.
R_0	(opacity) Reflectance of a paper specimen backed by a black cavity of zero reflectance.
$R_{0.89}$	(opacity) Reflectance of a paper specimen backed by a white tile of .89 reflectance.
$R_{0(.004)}$	(opacity) Reflectance of a paper specimen backed by a black cavity of zero reflectance as if that specimen were .004" thick.

LIST OF SYMBOLS
(continued)

$R_{0.89(.004)}$	(opacity) Reflectance of a paper specimen backed by a white tile of .89 reflectance as if that specimen were .004" thick.
S	The scattering coefficient in the Kubelka-Munk equations.
SID	Solid Ink Density
SX	The scattering coefficient in the Kubelka-Munk equations which is dependant on the thickness of the sheet.
$SX_{(.004)}$	The scattering coefficient in the Kubelka-Munk equations which is dependant on a standardized sheet of .004" thickness.
X	The thickness of a sheet of paper (in mils).

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