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The effect of laser beam cuts on the strength of paper edges

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THE EFFECT OF LASER BEAM CUTS ON THE STRENGTH OF PAPER EDGES

by

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A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in the School of Printing Management and Sciences in the College of Graphic Arts and Photography of the Rochester Institute of Technology

February, 1988

Thesis Advisors: Mr. Chester Daniels
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Certificate of Approval -- Master's Thesis

School of Printing
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CERTIFICATE OF APPROVAL

MASTER'S THESIS

This is to certify that the Master's Thesis of

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With a major in Printing Technology
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satisfactory for the thesis requirement for the
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The Influence of Laser Beam Cuts on the Strength of Paper Edges

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NOMENCLATURE

- Die : any of various tools or devices for molding, stamping, cutting or shaping. [25]

- Free sheet : a sheet of paper made without mechanical pulp.

- Infra-red : designating or of those invisible rays just beyond the red end of the visible spectrum. [25]

- Laser : (an acronym for light amplification by stimulated emission of radiation) a device, containing a crystal, gas, or any suitable substance, in which atoms, when stimulated by focused light waves, amplify and concentrate these waves, then emit them in a narrow, very intense beam. [25]

- μm : symbol standing for micrometer (i.e., a thousands of a millimeter).

- nm : symbol standing for nanometer (i.e., a thousands of a micrometer).

- Rewinder : the device which rewinds the roll of paper obtained at the end of a paper machine and slits it, if necessary, into two or more rolls of determined widths using either circular blades or laser cutting heads.

- Scanning electron microscope : device using a beam of electrons focused electromagnetically on a specimen.

Imaging of surfaces is normally accomplished by utilization of emitted secondary electrons arising from impigement of the primary beam.

For secondary electron imaging, the collected signal is used to synchronously modulate the brightness of various cathode ray tubes (CRT) in such a way that each point of the specimen is represented by a corresponding point on the observation screen.

The specimen, if not electrically conductive, is normally coated by evaporation at low pressure of a 5 to 10 nm layer of metal (e.g., gold) to avoid build up of charge fields and to increase secondary electron emission.

The scanning electron microscope allows a remarkable depth of field (several millimeters), a wide range of magnifications (5 x to 30,000 x), and a resolution in the range of 5 to 10 nm. [23]

- Waist : the narrowest part of a focused laser beam, where the energy per surface unit is the highest.

ABSTRACT

Some paper-mill rewinders use infra-red carbon dioxide laser heads to slit the web into rolls of given widths. However, despite certain advantages, laser slitters are not widely used mainly because of installation and operation costs.

Web breaks on a printing press cause both time and paper waste. Breakings occur because of defects in the edges of the web, such as notches or extra fibers, which offer less resitance to the high tensions existing during the printing of a web at high speed. Since paper edges are much cleaner when cut with an infra-red carbon dioxide laser than when cut with knives, it is possible that laser-cut edges might be stronger and thus reduce the number of breakings of the web.

In order to test this hypothesis, tensile tests were carried out on laser-cut and shear-cut strips for three different types of papers (coated and uncoated stock, and newsprint) and results were compared.

Since direct observation of the effect of a laser beam on cellulose fibers might help in explaining differences in behavior between the two types of paper samples, a scanning electron microscope was used to take photographs of paper areas following treatment with a CO_2 laser functioning in pulsed mode. The photographs revealed a drastic change in the structure of hit cellulose: the latter looks melted. This "melted" portion of a laser-cut paper edge is extremely thin (several micrometers) and seems to slow down the combustion of adjacent fibers.

Regarding tensile tests results, laser-cut strips are always stronger, but the relative difference between the two types of specimens is very low (less than 1% in the case of the uncoated stock and the newsprint, and less than 3% in the case of the coated stock).

An analysis of variance proved this 3% difference to be statistically significant, but it remains too low to constitute an incentive for the web printer to order laser-slit paper rolls only, or for the papermaker to replace blade slitters by laser cutting heads.

CHAPTER I

INTRODUCTION

When it was invented in 1960, the laser was ironically nicknamed "a discovery in search of applications". [1] Today, various kinds of lasers exist and are successfully used in many fields, including medicine, metrology, the printing industry, as well as the cutting of metallic, wooden and plastic materials. Industry became interested in cutting paper with a laser beam in the late sixties and today powerful infra-red carbon dioxide lasers are used in the paper industry for three purposes:

1. **For die-board cutting.**

There are three main advantages to the application of this technique: first, the absence of mechanical contact between the cutting device and the wooden form prevents any deformation of the shape to be cut. Secondly, the laser beam can start a cut anywhere on the form, whereas traditional saws cannot. Last but not least, the possibility of guiding the laser head with a computer reduces not only set-up and processing time, but also the amount of waste cuttings. [2,3,4,5,6,7]

2. **To design postcards, logos, etc.**

The lasers used have very high powers (about 10,000 W) and "vaporize" paper in order to draw pictures. The accuracy reached with these powers is exceptional (i.e. $\pm 10^{-4}$ inch). [8,9]

3. **To replace traditional knife slitters in paper-mill rewinders** (which is the subject of this thesis).

As it will be explained further (see page 15), the purpose of this thesis is to find whether laser cutting improves the strength of paper edges. Tensile tests (carried out on laser-cut and shear-cut strips of given papers) will be accompanied by an observation of laser-affected paper fibers through a scanning electron microscope, because transformations in cellulose structure revealed by such an observation might explain differences in behavior occurring during the tensile tests.

Laser cutters offer several advantages. First and foremost, laser-cut edges are of high quality. Secondly, because laser beams cut more accurately than knives do, they allow a reduction in paper waste during the trimming of paper roll edges. Lastly, the possibility to connect laser slitters to a computer makes the design of a fully automated finishing area possible in paper-mills. [1,10,11,12]

However, laser slitters have not yet met success in the papermaking industry. There are several reasons for this:

1) high costs: a 400 W carbon dioxide laser costs more than $40,000 -- excluding high gas consumption expenses as well.

2) cadence problems (the faster the web goes -- rewinders often reach 2000 m/min --, the more powerful the laser must be, which increases its cost).

3) the need for an efficient removal of combustion products (such as gas or solid particles). [1,10,13]

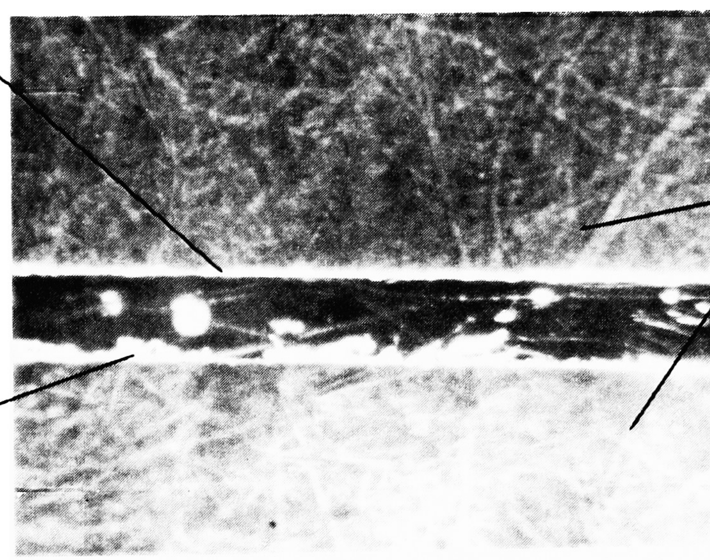
But even if the use of laser slitters in rewinding raises such problems, industry still is interested in this cutting technique, as evidenced by the recent publication of patents [14,15] and the manufacture of new laser cutters (one of which is designed for the specific purpose of cutting corrugated papers) in 1986. [16]

In 1978, following an experiment carried out in a paper-mill to compare the new laser cutting technology to traditional shear slitting, laser use was described as very promising. [13] However, few detailed studies have been carried out in this field since that time and several aspects of laser cutting require additional research. Among these, one question directly concerns the printing industry : does cutting by laser beam influence the strength of paper edges?

Shear-cut paper edges are imperfect and present defects such as extra fibers or notches created when fibers are pulled out during the slitting process. These irregularities may cause the breaking of a paper roll on a web press where paper is submitted to high tensions.

According to photographs taken through optical and electron microscopes, laser-cut edges have a much cleaner and smoother appearance than those cut with traditional devices. [11,13] Photograph A shows the edge of a reel of coated paper (cut with a blade

slitter) and that of a coated paper strip cut with an infra-red laser. The photograph was taken through binocular lenses and the enlargement of 45 x shows the significant improvement in appearance offered by the laser cutting technique, compared with that produced by the conventional slitter.



Photograph A: Comparison between a laser-cut
paper edge and a shear-cut paper edge
Top of the photograph: laser-cut edge
Bottom of the photograph: shear-cut roll edge
Enlargement : 45 x

CHAPTER II

THE LASER PRINCIPLE [17,18,19]

1. History

The word "laser" means "Light Amplification by Stimulated Emission of Radiation". Stimulated emission was described for the first time in 1917 by Einstein and the first application of this phenomenon to ultra-short waves (MASER) was achieved by C. H. Townes in 1954. The first laser was built by T. H. Maiman in the United States in 1960. The "pumping" mechanism, which is still the only way of generating a laser beam, was conceived by A. Kastler and J. Brossel in 1950.

2. Spontaneous absorption and emission

An electromagnetic radiation striking on an atom is either partially or completely absorbed. The absorption and emission processes are described as follows.

The energy carried by a photon is $E = h \times \nu$, where ν is the frequency of the electromagnetic radiation associated with the photon and h is Planck's constant ($h = 6.63 \times 10^{-34}$ J x s).

The absorption of electromagnetic energy by an atom, an ion or a molecule follows the relation $E_2 - E_1 = h \times \nu_{12}$, where E_1 is the initial energy level of the atom and $E_2 > E_1$ is the level it reaches by absorbing a photon of frequency ν_{12} (see figure 1).

The energy acquired by the excited atom may be lost spontaneously by returning to a lower level. This loss of energy may create an electromagnetic radiation emission which follows the same law as in the case of absorption: $E_1 - E_2 = h \times \nu_{21}$, where $E_1 - E_2$ is the energy lost by the atom and ν_{21} the frequency of the emitted radiation (see figure 1).

This spontaneous emission is incoherent : each atom emits independently of the others, at any moment and for a very short time. There is no relation between phases, directions and polarizations of all these emissions and the radiation is omnidirectional. Only its frequency is fixed by the interval of energy levels between which the return takes place.

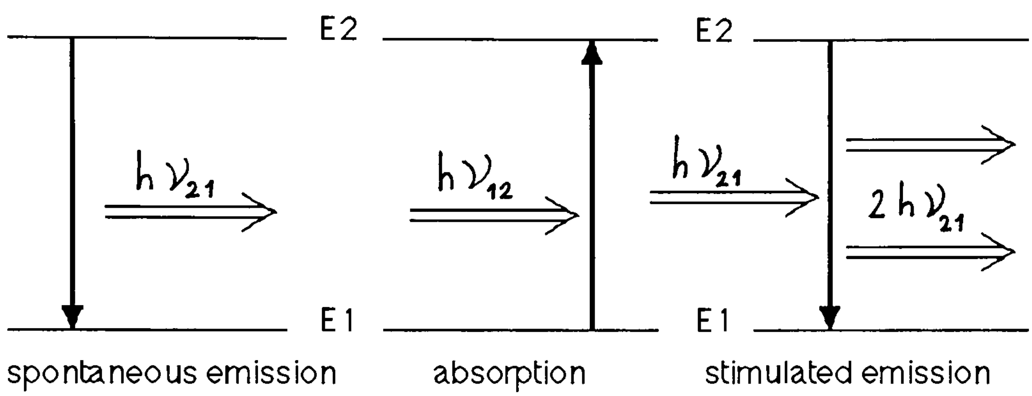


Figure 1: TRANSITIONS BETWEEN TWO ENERGY LEVELS.
Horizontal lines designate the energy levels
Vertical arrows designate the direction of transition.
Double lined arrows refer to incident and resulting photons.

3. Stimulated emission

In stimulated emission, a photon is emitted as an excited atom returns to a lower energy level, but the emission is not spontaneous , rather it is due to an "inducing" photon the frequency of which is the same as that of the emitted photon (see figure 1).

Properties of this stimulated emission are remarkable: the inducing and induced radiations are equal in phase, direction and polarity. Since there is no physical difference between the inducing and induced photons, we can speak of "light amplification", wherein two identical photons are released by the action of one photon. The description, light amplification by stimulated emission of radiation, is the source of the acronym "laser" which is widely used to discuss this phenomenon.

Stimulated emission occurs under very specific conditions: let N atoms be at two possible energy levels (N_1 atoms being at fundamental level E_1 and N_2 atoms being at excited level $E_2 > E_1$). When thermal equilibrium is reached, these atoms are distributed according to Boltzmann's law : $N_2 / N_1 = \exp [(E_1 - E_2) / K \times T]$, where T is the absolute temperature of the environment and K is Boltzmann's constant ($K = 1.38 \times 10^{-23}\ J \times K^{-1}$).

Let us suppose that T = 300 K and that $E_2 - E_1 = h \times \nu$ corresponds to a wavelength of 1 µm ($\nu = 3 \times 10^{14}$ Hz). N_2 / N_1 is about equal to e^{-48}, that is to say 1.4×10^{-21}. Consequently, there are nearly no excited atoms in a body having reached the thermal equilibrium at the ambient temperature. An incident radiation has a probability nearly

equal to zero to create a stimulated emission and has a probability nearly equal to 1 to be absorbed. In order to obtain a light amplification by stimulated emission, we need more excited atoms than atoms at the fundamental state ($N_2 > N_1$). This is achieved, out of thermal equilibrium, by an external excitation called "pumping". Pumping creates a population transposition between the fundamental level E_1 and the excited level E_2. Then the return from the activated state to the ground state will produce a radiation by spontaneous or stimulated emission. More complex laser systems have several energy levels, but the principle of optical pumping remains the same.

4. Resonator cavity

In order to transform the laser amplifier into an oscillator, one uses a "resonator cavity" made with two flat partially reflecting mirrors set perpendicularly to the cavity's axis, on both sides of the amplifying medium.

5. Radiation properties

Stimulated emission properties and the use of a resonator cavity cause identical phase and wavelength output for the photons which the laser emits. The emission is then said to be coherent. This coherence has two aspects: temporal and spatial. Temporal coherence is the monochromaticity of the laser beam: if, at a given time, the phase along a moving front is equal to that of the wave after it has gone over a distance L in a time L/C (C is the velocity of the light, equal to 300,000 km/s), then the radiation is

temporally coherent. Spatial coherence is achieved when each point of the emitted wave reaches the exit mirror in the same phase.

6. Discussion of the specifics of carbon dioxide lasers

Infra-red lasers can work continuously (this is the case of those used in paper-mill rewinders) or in a pulsed mode. Most of them use carbon dioxide or hydrogen fluoride.

Carbon dioxide lasers present several advantages over other types of lasers: they are relatively inexpensive, they are reliable and are remarkable for their efficiency (i.e., the energy provided per unit of consumed energy), which goes from 10% to 20% (values which are to be compared to the efficiencies of other lasers, usually about 0.01%). They emit at the wavelength of 10.6 µm where many materials such as metals, plastics, wood, textiles, and glass offer good absorption. This absorption allows a violent heating of the area hit by the laser beam, which can be compared to a lightning pyrolisis: the material is then deformed, partly burnt, or even sublimized. That is why CO_2 lasers are used not only to cut, but also to engrave, weld, or modify the surface of given materials. In the case of cellulose, cutting by laser beam is possible because cellulose also presents a strong absorption near 10 µm (see figure 2).

In rewinders, laser cutting heads usually use a coaxial gas jet : the first purpose of this gas jet is to remove the cut part of the material and to prevent any particle from depositing on the lens. But if an inert gas (e.g., nitrogen) is employed, it will also prevent any local inflammation of the paper. [1]

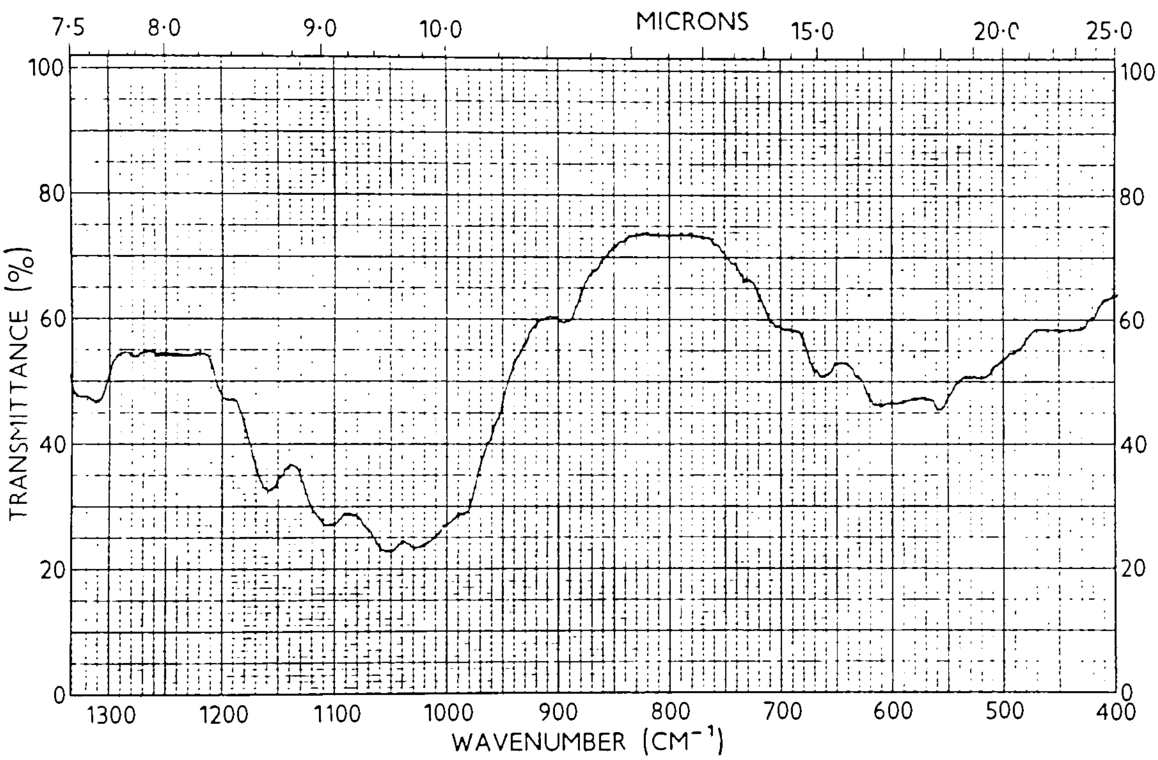


Figure 2: Transmission spectrum of cellulose [11]

Infra-red CO_2 lasers actually use a carbon dioxide/nitrogen/helium mixture, in a "resonant transference of energy system" (see figure 3). Its principle is as follows: when two gases A and B have high energy levels, in which $E_3(A)$ and $E_2(B)$ are very close, they are said to have "resonant levels". If the mixture is submitted to external pumping such as an electric discharge, upper levels $E_3(A)$ and $E_2(B)$ of the two gas are populated. Gas A is always chosen in such a way that $E_3(A)$ is a metastable energy level: atoms or molecules of this gas are then used as a reservoir to store energy. As a matter of fact, when these atoms, stored at level $E_3(A)$, hit those of gas B, most of which are at fundamental level $E_0(B)$, they directly exchange their energies by resonance.

It can be shown that the probability for such resonant transferences is very high: gas A returns to the fundamental level, making gas B reach resonant excited level $E_2(B)$. Thus level $E_1(B)$ remains nearly empty (it quickly loses its excitation): the population transposition is automatically realized.

In the CO_2-N_2 laser, the upper level of nitrogen is a metastable level which maintains the excitation for a long time and only yields it during a resonant collision in which it transfers its energy by making the carbon dioxide reach the upper level $E_2(B)$: then the transition takes place by readjustment of vibration energies.

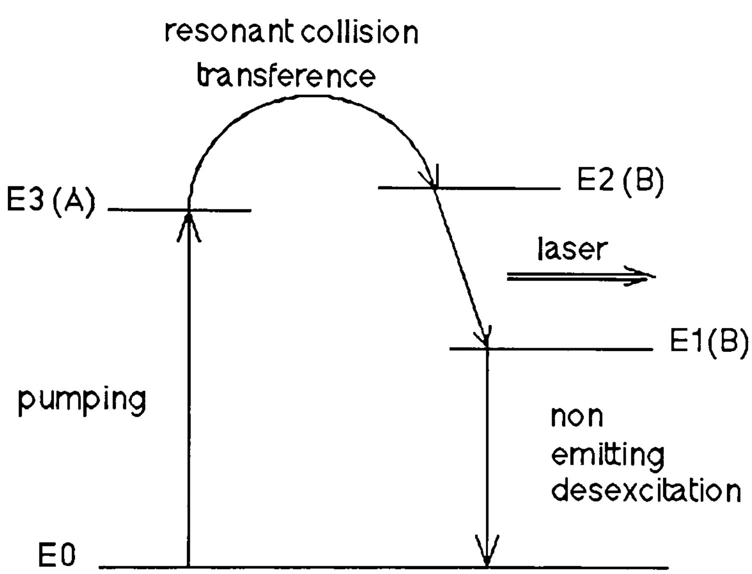


Figure 3: Mechanism of the resonant energy transfer

CHAPTER III

MODEL AND HYPOTHESIS

The "clean" appearance of laser-cut edges was mentioned before. Upon inspection it looks as if paper fibers had "melted". One must be careful with this terminology, because the fusion temperature of cellulose is higher than its decomposition temperature; which means that cellulose should not melt before it is decomposed. [11] However, since the process of the cutting of paper by laser beam is not well known (the structure of a paper sheet, unlike that of materials such as metals, is not homogeneous), it cannot be affirmed that a fusion of cellulose is impossible to obtain. That is why observing different states of cellulose degradation (when hit by a laser beam) could provide useful information relating to the actual effect of laser on cellulose fibers.

There are two possible interpretations of the "melted' aspect of laser-cut paper edges. On the one hand, "melting" corresponds to a step in cellulose degradation and thus could weaken fibers. On the other hand, it may create many additional links between cut fibers (in the thickness of the edge), and so strengthen the edge.

Since laser slitting creates smoother edges than shear slitting, the hypothesis is that a stronger laser-cut edge will result. *

Therefore the working hypothesis of this thesis will be:

"Laser-cut paper edges are stronger than knife-cut paper edges."

In order to verify this hypothesis, experiments were designed to allow a comparison between laser-cut and knife-cut edges.

* *Remark: another advantage of cutting paper with infra-red lasers might be the suppression of paper dust. Trailing edge dust usually consists of fibers, coating debris, or tearouts which impose frequent cleanings on printing presses.* [20]

Assuming that products of laser combustion are efficiently removed (which has been stated to be essential [1,10,13] *), the amount of particles likely to be removed from the edges during the process of printing should be negligible in comparison to the usual trailing edge dust.*

CHAPTER IV

EXPERIMENTAL DESIGN

The experiments carried out consisted of three parts:

1) The cutting and perforating of paper samples with a focused infra-red laser, working respectively in continuous and pulsed modes.

2) The observation of degraded cellulose through a scanning electron microscope.

3) A tensile test comparing paper strips cut with a laser beam and paper strips cut with blades.

Three types of paper were submitted to these experiments. They were chosen because they represent commonly used papers and because their properties are different.

Paper A is an uncoated stock the basis weight of which is 101g/m^2 (i.e., 68 lb.).

Paper B is a coated stock the basis weight of which is 118 g/m^2 (i.e., 80 lb.).

Paper C is a newsprint the basis weight of which is 47 g/m^2 (i.e., 29 lb.).

These three papers were cut into 1in. x 8 in. (2.54 cm x 20.32 cm) strips to meet the TAPPI standard for tensile tests. [21]

In order to observe the actual effect of a pulsed laser beam on cellulose fibers, a fourth type of paper was used (paper D): this is a free sheet which was handmade out of pure cellulose pulp (a mixing of hardwood and softwood pulps). Its thickness was 190 μm (75 x 10^{-4} in.).

1. The cutting and perforating of paper samples

The samples were cut with a carbon dioxide laser (Coherent Everlase 150) made available by the Eastman Kodak Company for this project. This laser, which has a maximum power of 185 W, can work in continuous or pulsed mode.

Remark: both edges of each strip were cut by the laser beam. Assuming that there is a difference in strength between laser-cut and shear-cut paper edges, carrying out a tensile test on a specimen with only one laser-cut edge would not have revealed any particular change in behavior since the breaking would have taken place on the weakest edge, as if the specimen had had two such edges. A comparison could only be made between paper strips with two laser-cut edges and strips with two shear-cut edges.

a) In order to reproduce as closely as possible the conditions under which a paper web is cut by laser beam in a rewinder, a rotating drum, driven by a variable speed motor, was positioned next to the vertical beam path. Paper strips, fixed on the surface of

the drum and projecting beyond the drum, would then have crossed the cutting beam at high speed (see figure 4).

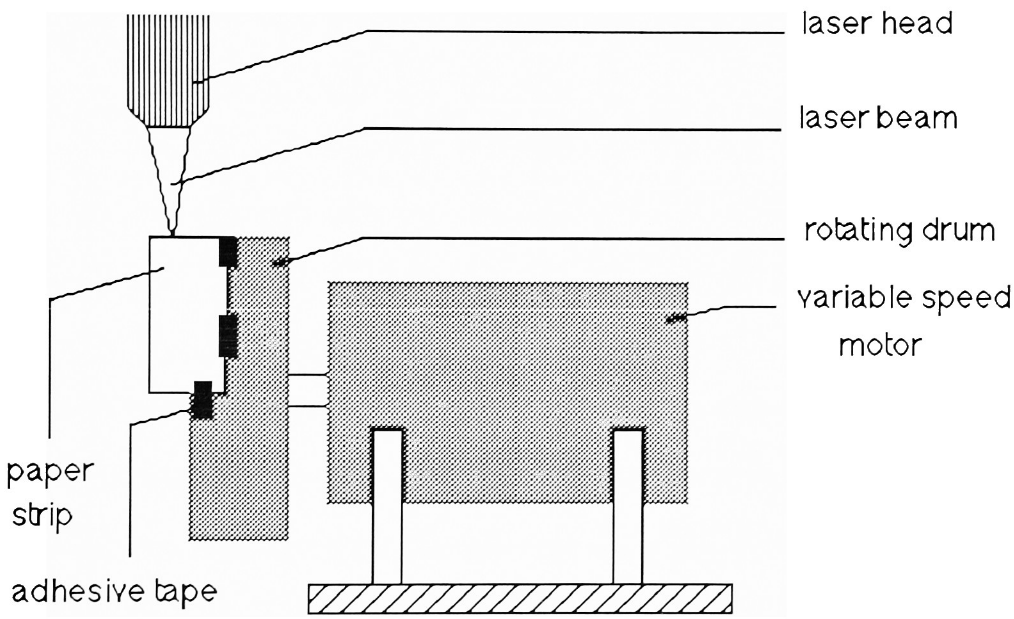


Figure 4: First setting of the cutting experiment

However, because of the instability of the motor and difficulty to place the paper specimens properly on the drum, this arrangement was not satisfactory and it was decided to lay paper samples flat and cut them on an Anorad 16 in. x 16 in. (40.64 cm x 40.64 cm) X-Y table.

It was then possible to cut strips with good accuracy, which was of great importance to meet the TAPPI tensile test standards. [21] But this new way of proceeding resulted in very slow cutting velocity (7.62 m/min or 8.89 m/min, i.e., respectively 300 or 350 in./min), which has been proven to thermally damage the paper edge. (There is a maximum cutting velocity V_M -- over which the cut is incomplete -- and a minimum cutting velocity V_m -- under which the edge is damaged . [1] Here, the speed used is undoubtedly lower than the velocity V_m corresponding to a paper-mill rewinder).

So the choice of cutting paper at low speed is a compromise: accuracy has been preferred but one should know that if edges had been cut faster by the laser beam (all other conditions remaining identical), they would probably have been of better quality (stronger).

Strips were cut in the following dimensions: 1in. x 8 in. (2.54 cm x 20.32 cm). The TAPPI standard for tensile tests recommends the use of 1 inch wide specimens only if their thicknesses are greater than 3 mm (0.12 in.) [21] , but this dimension was used because this width conformed with that of the double bladed hand paper cutter (the latter

enabling us to get 1 inch wide strips with good precision) which was going to be used to obtain non laser-cut samples.

<u>Remark</u>: as laser slitters cut the paper web in the grain direction in a rewinder, paper strips were cut so that their lengths correspond to the grain directions of the papers used. Tensile tests carried out cross-grain would have been meaningless since the transversal tension of a paper web is of little influence on its breaking.

Papers A (uncoated stock) and C (newsprint) were cut with a cutting power of 40 W (in the continuous mode) and a cutting rate of 8.89 m/min (350 in./min).

Paper B (coated stock) was cut with a cutting power of 95 W (in the continuous mode) and a cutting rate of 7.62 m/min (300 in./min).

30 samples were cut for each type of paper, knowing that the number of valid tensile tests would probably be lower (i.e., about 25) since there are always aberrant results during such tests (for instance due to a breaking occurring close to the jaw).

b) The other part of this first experiment consisted of hitting a paper sample with the laser beam functioning in pulsed mode, in order to hit the paper specimens during a time both short enough to minimize paper combustion and reproducible for greater precision in comparisons.

The idea was to create five different impacts on the same paper sample with constant properties of the laser beam (same power and same duration of the impact).

This was made possible by placing the specimen at five different positions between the laser head and the focal point of the lens. This lens, located at the exit of the laser head, focuses the beam so that it becomes as thin as possible (this is the "waist" of the beam), thus providing the maximum energy per unit area (see figure 5).

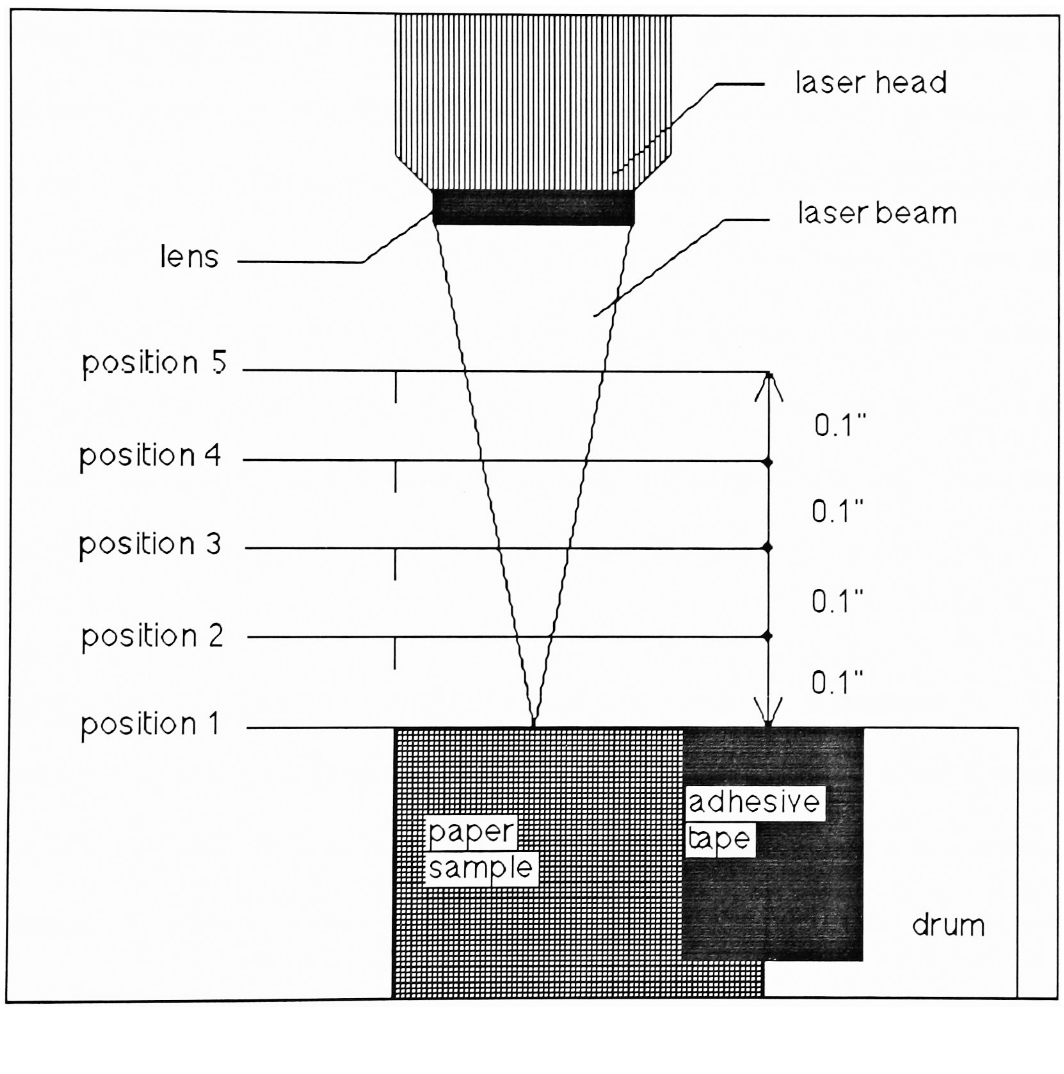


Figure 5: The obtaining of five different impacts on a single piece of paper, using a pulsed laser beam

For a given power of the laser beam, when a piece of paper is placed at the focal point of the lens, the greatest possible impact results. But if the sample is hit between the laser head and the waist of the beam (where the energy per surface unit is lower than at the waist), the paper will be less affected.

In this experiment, a specimen was laid on the rotating drum (in a similar manner to that described in paragraph 1.a., pages 17 and 18), the part to be hit being in the focal plan of the laser head lens. Then another area of the sample was hit 2.54 mm (0.100 in.) above the point of focus (between the latter and the lens) and this was repeated setting the piece of paper at 5.1 mm (0.200 in.), 7.6 mm (0.300 in.), and finally 10.2 mm (0.400 in.) above the focal point, all other conditions remaining identical. The power of the pulsed laser was 35 W. This procedure was carried out with papers A, B, C, and D. Photographs dealing with the laser-affected zones of the latter would then show the actual effect of an infra-red laser beam on cellulose fibers.

2. Observation of hit samples through a scanning electron microscope

Access to the scanning electron microscope in the College of Science (Department of Biology) at R.I.T. was possible, but time of use was limited and only a few photographs could be made. Therefore studies were carried out on papers B, C, and D only, knowing that paper A (uncoated stock) would present results similar to paper D (hand-sheet). For each paper, only the weakest and strongest impacts were considered.

The photographs will allow observation of typical cellulose transformations.

3. Tensile tests

In preliminary tests, carried out by H. S. Ainsworth, no significant change was observed when laser-cut paper samples were submitted to an initial edge tear test or when moisture hysteresis and hygroexpansivity were tested.[13] So such tests were not undertaken and the one which appeared to be best designed to compare the strengths of edges was a tensile test, for it reproduces the tension conditions to which a paper web is submitted on a printing press.

All conditons specified in the TAPPI standard [21] were followed, except that of temperature and humidity. The Instron tensile tester was located in a room which was not air-conditioned. Temperature was recorded during the tests (though it was not the specified one) but humidity could not be measured.

In order to normalize experimental conditions, all tensile tests were carried out during the same afternoon, that is to say in the same conditions of temperature and humidity.

Tensile tests were performed on a previously calibrated Instron 1122 tensile tester.

For each kind of paper (A, B, and C), two types of samples were compared:

- samples with two laser-cut edges, obtained as described in paragraph 4.1. (page 19).
- samples with two knife-cut edges, obtained with a double bladed hand paper cutter.

The three papers A, B, and C were tested in the following conditions:

- temperature: 26 °C (78 °F)
- number of specimens tested: 30 of each kind (laser-cut and shear-cut)
- size of the samples: 25.4 mm x 203 mm (1 in. x 8 in.)
- distance between jaws at the beginning of each test: 180 mm (7.09 in.)
- average value of the "time to break": paper A: 10s
  - paper B: 12s
  - paper C: 7s

These times meet the TAPPI standard specifications (10s ± 5s).[21]

Recall: the tensile tests were only carried out in the grain direction (see the remark in paragraph 1., page 20).

CHAPTER V

RESULTS AND DISCUSSION

1. Observation of papers affected by the laser beam

As stated in chapter IV (page 22), photographs deal with papers B, C, and D only. Since availability of the scanning electron microscope was limited, only the initial and final impact samples were analyzed. The photographs (# 1 to 10) evidence several common trends:

- Whatever the nature of the paper, the affected fibers present the same "melted" appearance.

  The affected area is extremely thin (it does not exceed several micrometers).

  The affected region of the paper seems to protect the fibers located next to it . Intact portions of fibers are adjacent to their "melted" ends (i.e., at a distance of several micrometers only from them).
- The shapes of the contours of the affected areas are slightly elliptical, which must

correspond to that of the laser ray itself. However, the holes created in paper specimens will be referred to as "circles".

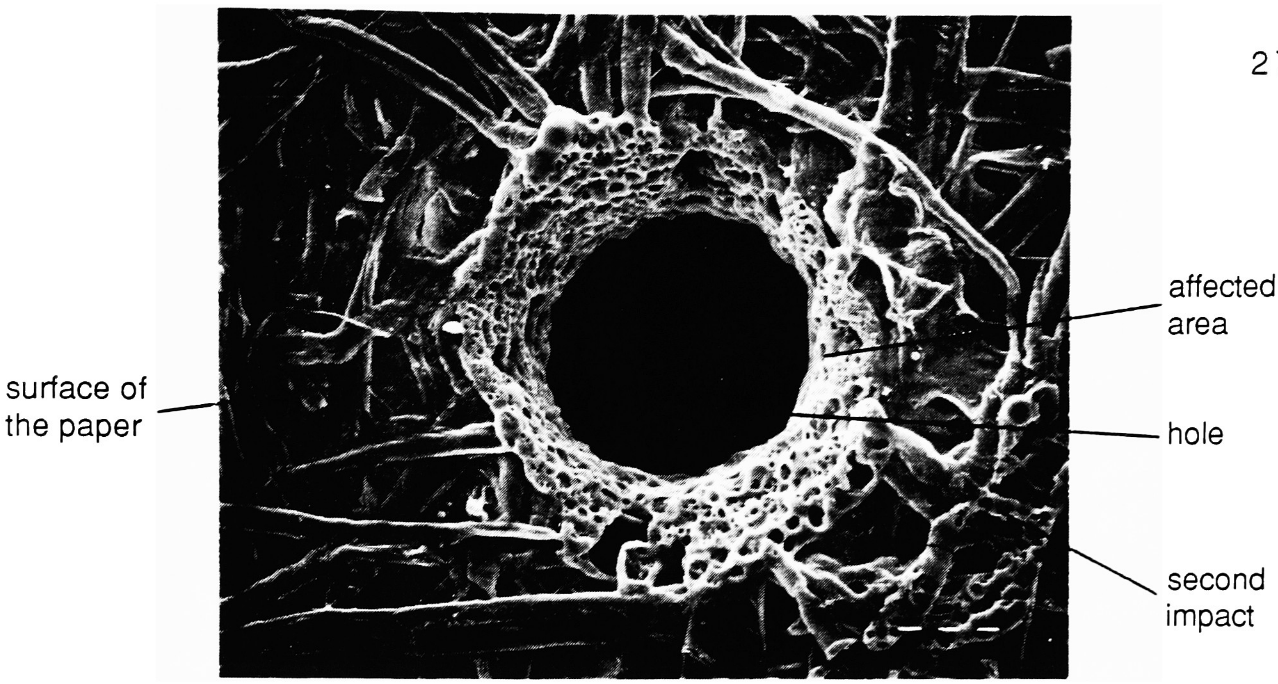
A conservation of the beam contour means that the spreading of the combustion is uniform across the surface of the sheet.

Remark: every photograph contains a coding system (in the lower right corner) made of juxtaposed dashes. This code is a distance scale and works as follows:

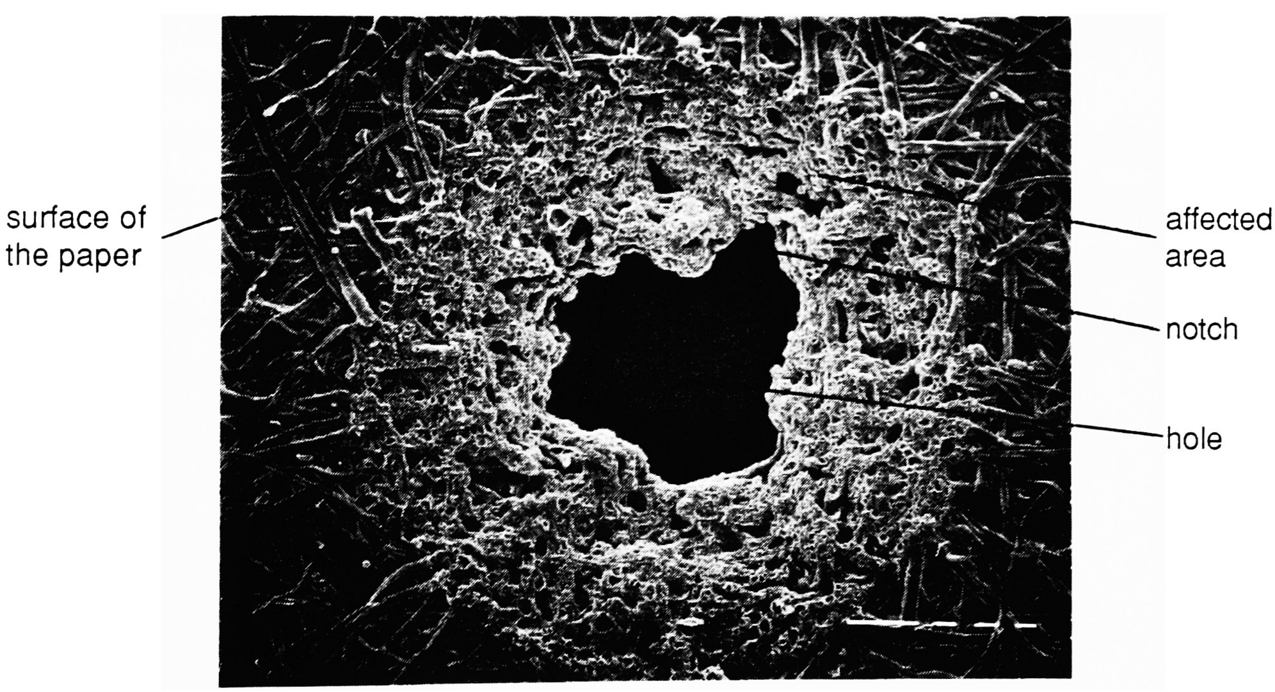
- *when it comprises three dashes, the length of the left dash corresponds to 10 μm.*
- *when it comprises four dashes, the same length corresponds to 100 μm.*

It was hypothesized that this melted aspect might come from sizing or coating components, latex, or even lignin. To make sure that this was not the case, the experiment was also conducted using paper D, a free sheet made of pure cellulose (see page 17). The resulting photographs (#1 to 4) clearly demonstrate that the visible transformation is that of laser-cut fibers, since their appearance is similar to that of hit cellulose in the other types of papers tested.

Photograph #1 displays the laser impact when paper D (hand-sheet) was placed at the focal point of the lens. The edges of the hole are very smooth and the circular shape of the ray has been conserved. The conical shape of the impact will be explained saying that the "melting" of layers of fibers spreads laterally while the beam is subliming the lower layers. Of course, the greater the energy per surface unit, the faster the "digging" of the hole; in such a case, the slope of the conical hole is very steep.



Photograph #1: hand-sheet hit at the waist of the focused pulsed laser beam, seen from above (enlargement 280 x)



Photograph #2 : hand-sheet hit 0.3" above the waist of the focused pulsed laser beam, seen from above (enlargement 88 x)

<u>Remark</u>: a second impact of the beam can be observed near the hole (lower right of the photograph). This impact is lighter and has the shape of an arc. The hole and this arc are concentric, which is typical of a "multimode laser". Here, it is necessary to explain one of the important properties of infra-red carbon dioxide laser beams used to cut thin materials: this property is called "Transverse Excitation Mode" (and noted TEM) and refers to the ability of a laser to oscillate in the resonator cavity along one or more paths parallel to the axis of the cavity. This acronym is followed by subscripts such as $_{00}$, $_{01}$, $_{02}$, etc.

The subscripts $_{00}$ indicate the fundamental mode: lasing occurs only along the axis of the cavity. But as greater amounts of energy are pumped into the laser medium, the laser has a tendency to oscillate in some other mode such as TEM_{01} or TEM_{02} (a pulsed laser precisely requires the pumping of high energy). Then the spatial distribution of the illumination (intensity of light per unit area) is not Gaussian but presents several peaks, as shown on figure 6 (page 29). Each peak is susceptible to creating an impact which will be circle-shaped and have the same center as the central hole.[11,19]

The principal inconvenience of a multimode type oscillation is that, even though the total power of the beam may equal that of a TEM_{00} beam, the power density of the former at its focus point will be lower.[19] Then, not only a second impact appears on the material to be cut, but the slitting is less efficient and a wider cut results. (A similar impact is visible on photograph #8, page 35).

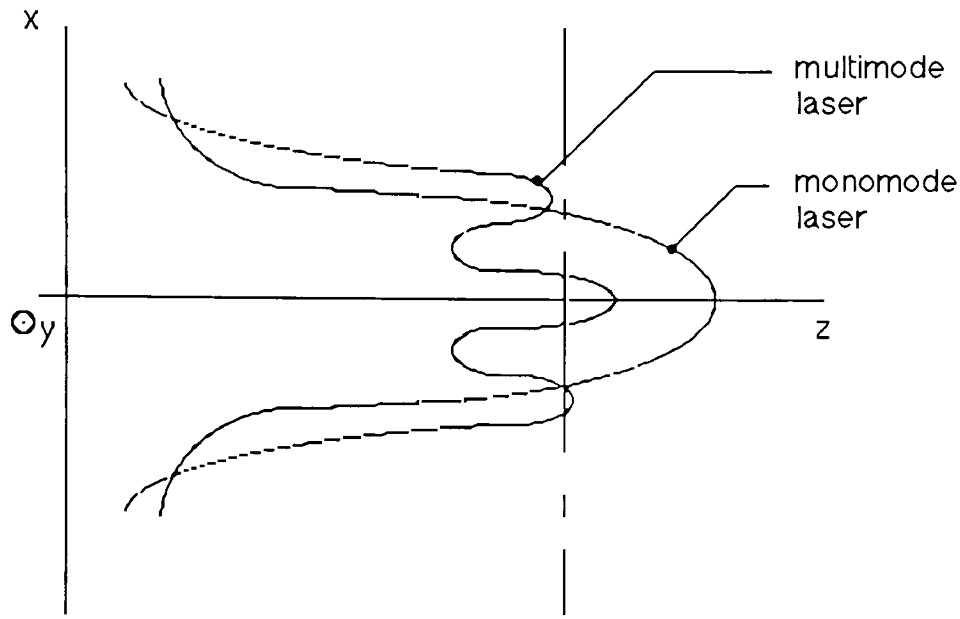


Figure 6: Distributions of the illuminations
of monomode and multimode lasers
The vertical axis is an estimation of the position of the paper specimen
when hit above the waist of the focused pulsed laser beam.

Photograph #2 corresponds to the same hand-sheet hit 0.3 in. above the focal point. The damaged area is much larger (about 975 µm in diameter vs. about 200 µm when the specimen is hit at the point of focus) but its external shape, as stated before, remains circular (though slightly elliptical).

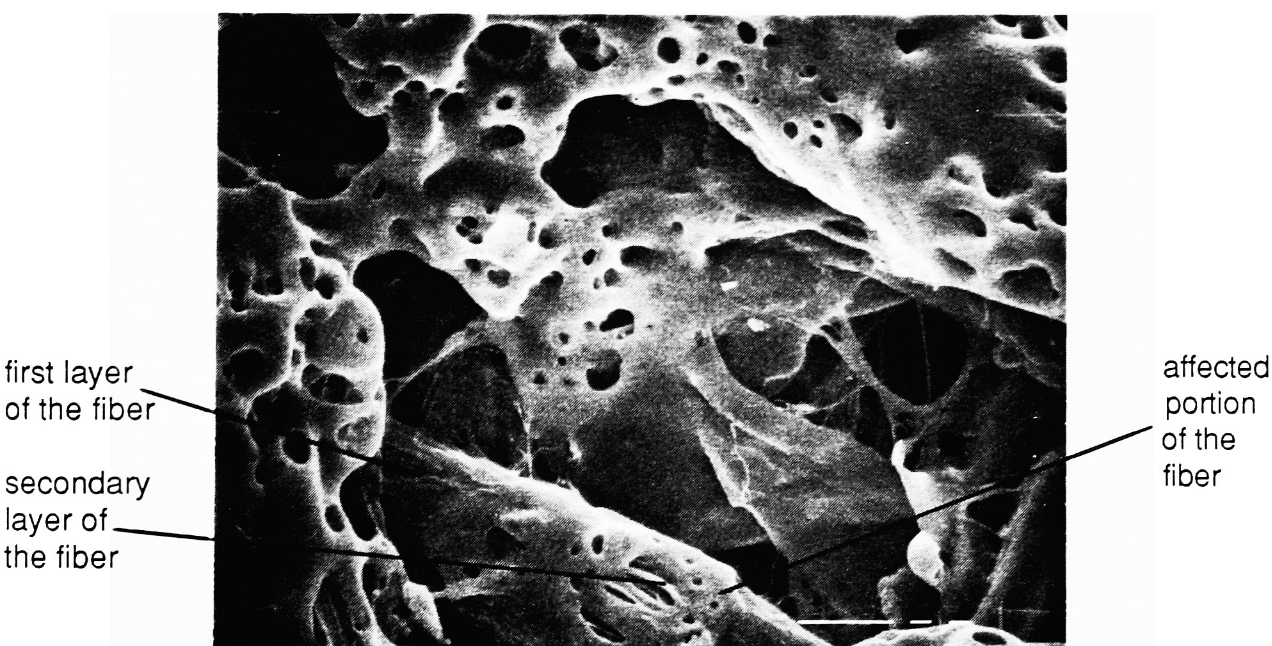
As the energy per surface unit has decreased from its maximum value (at the waist of the beam), the slope of the conic-shaped hole is much smoother than it is in photograph #1. This phenomenon can also be observed on photograph #6 (page33).

Another phenomenon revealed by this photograph is that, though the larger extremity of the conical hole (on the top side of the sheet) has an almost circular shape, this is not the case of the narrower extremity (on the bottom side of the sheet). The irregularity in the contour of the hole seems to correspond to the second impact observed when paper D was hit by a focused beam: here the paper sample intersects the beam before the focal point of the lens (see figure 5, page 21). The resulting impact will not be doubled, but will consist of a hole presenting a large notch, this notch being created by the gradient existing in the spatial distribution of the energy of the beam.

Photographs #3 and 4 have been selected to show how superficial the impact of the laser can be.

On photograph #3 (paper D), many fibrils remain intact despite their being located next to "melted" areas. Moreover, it can be observed that some fibers in a layer situated under a hit portion of the sheet are entirely intact (this view has been taken at the limit of the damaged area obtained by placing the sample 0.3 in. above the focal point).

Photograph # 3 : hand-sheet hit 0.3" above the waist of the focused pulsed laser beam, seen from above (enlargement 890 x)



Photograph # 4: handsheet hit 0.3" above the waist of the focused pulsed laser beam, seen from above (enlargement 1,200 x)

This tends to confirm the hypothesis stating that the melted surface has a lower heat absorption and seems to slow the propagation of the sublimation.

Photograph #4 (hand-sheet) presents the advantage of showing a fiber that has been affected only superficially: the impact was so light that only the primary layer of the fiber has been removed, revealing some of the structure of the secondary layer.

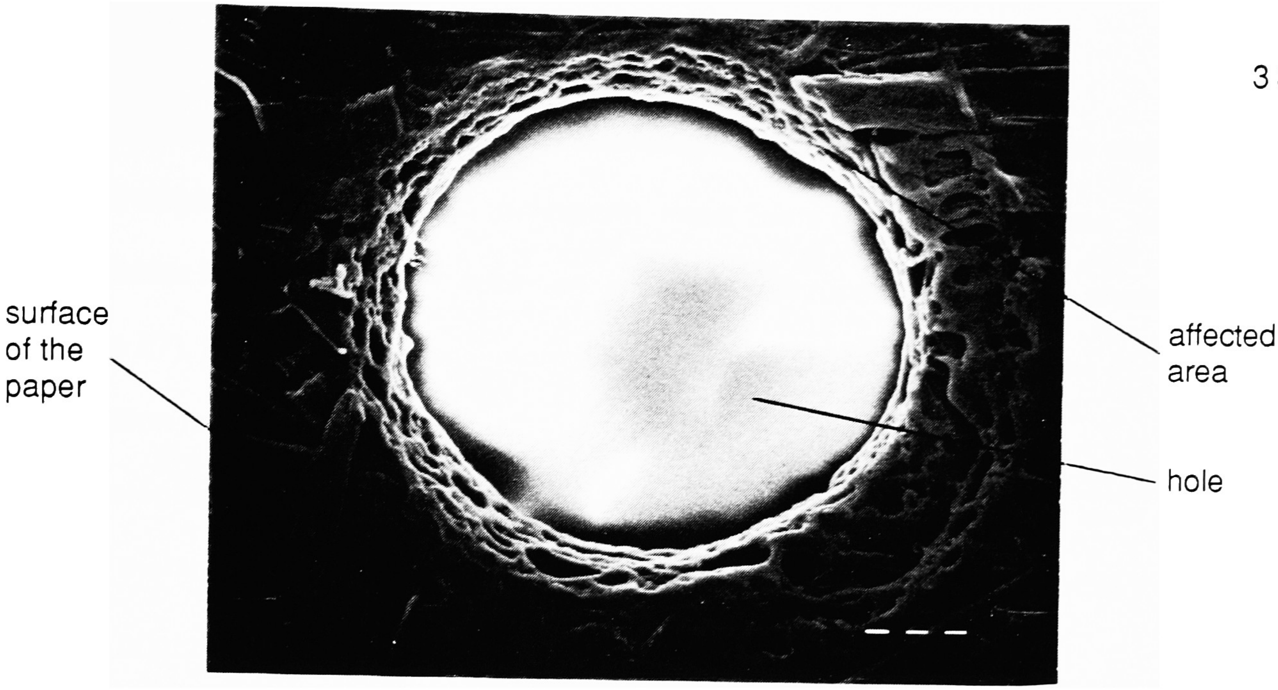
By observation, it seems that when the fiber absorbs the infra-red radiation, its primary layer swells and starts "melting", thus uncovering the secondary layer.

Photographs #5, 6, and 7 concern paper C (newsprint). The bright appearance of photograph #5 is an artifact due to the scanning electron microscope and has nothing to do with the nature of the paper. The image displays the impact of the beam at its point of focus. Two differences with photograph #1 (page 27) immediately appear:

- paper C is thinner than paper D (the hand-sheet).
- the diameter of the hole in paper C is about 50% larger than that in paper D.

The two properties are probably linked: as there is less material to sublime in paper C, the spreading of the degradation is faster and the size of the hole is consequently larger.

In photograph #6, representing the impact of the beam when the newsprint was placed 0.3 in. above the focal point, the diameter of the degraded area is just slightly larger than the one on photograph #2 (page 27, representing the hand-sheet in the same conditions). However, the width of the hole in paper C is here again about 50% greater than that of the hole in paper D (on photograph #2).



Photograph # 5 : newsprint hit at the waist of the focused pulsed laser beam, seen from above (enlargement 280 x)

Photograph # 6 : newsprint hit 0.3" above the waist of the focused pulsed laser beam, seen from above (enlargement 60 x)

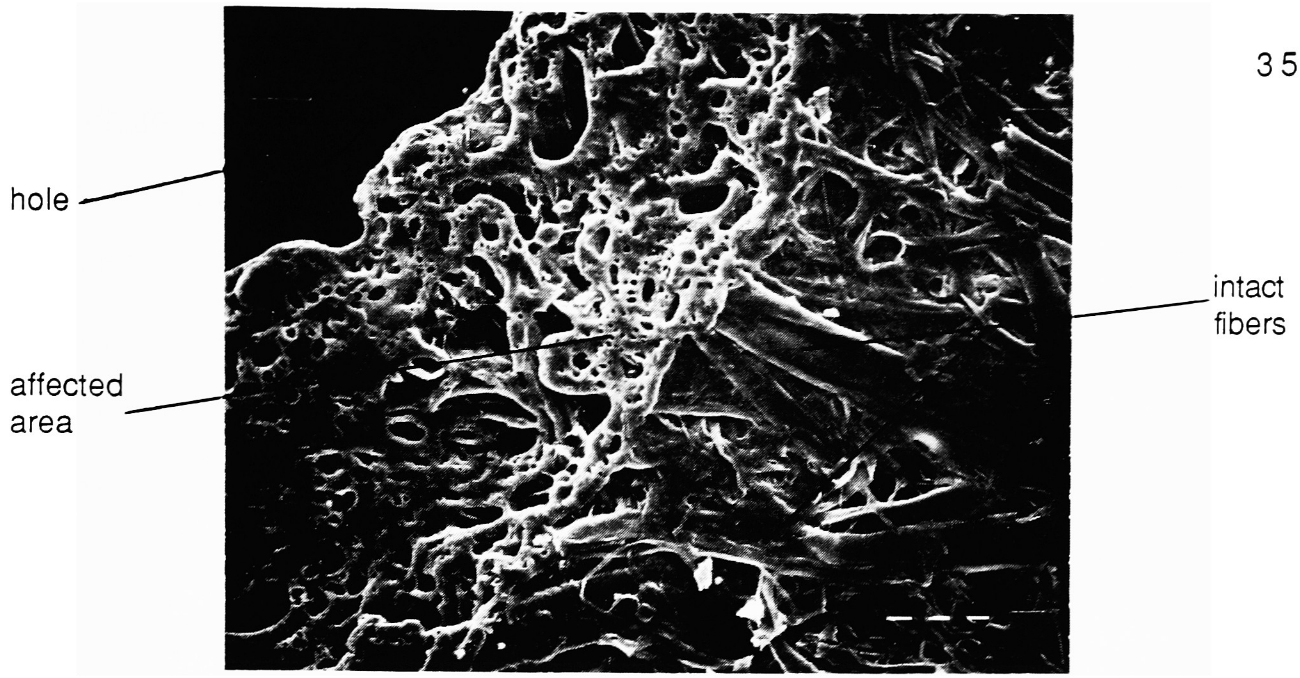
<u>Photograph #7</u> is an enlargement of photograph #6 displaying the three steps of cellulose degradation on paper C (newsprint): intact fibers (right) have become "melted" fibers (center left) or have been sublimed, leaving a hole (upper left). Unlike photograph #2 (page 27), the damaged area is narrow: since paper C is thinner, complete vaporization is shown.

Photographs #8, 9, and 10 deal with paper B (coated stock). Here, the laser not only affected the fibers, but also the coating.

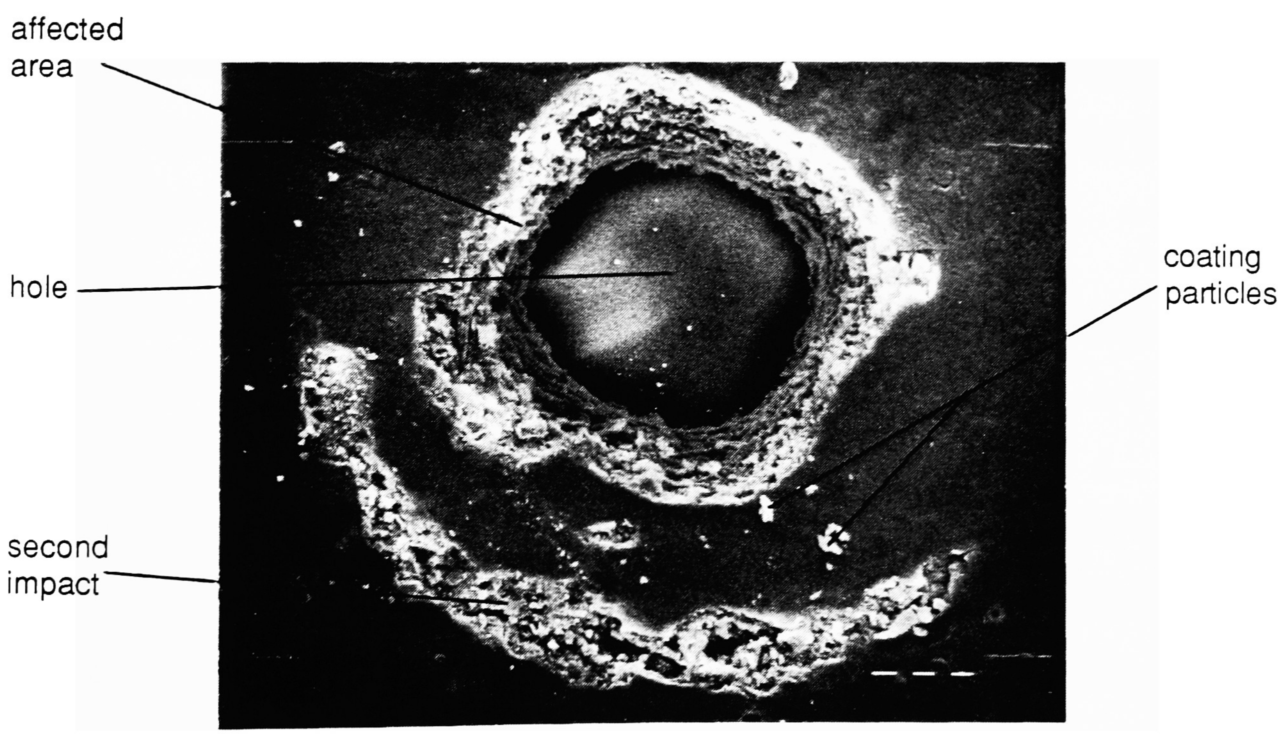
On <u>photograph #8,</u> the impact presents two characteristics:

- unburnable coating particles have been fixed onto the edges of the hole. These particles were located on both sides of the paper before the impact but are distributed all over the "melted" edges afterwards. High turbulences occurring during the laser cutting process probably explain this phenomenon, as well as the numerous holes (created by gas bubbles) present at the "melted" surface.

- the presence of a second impact of the beam, already mentioned in the paragraph describing photograph #1. Seemingly, the second impact is not visible when the paper sample is placed over the focal point, probably because of the distribution of the beam (see remark on pages 28 and 29).



Photograph #7 : newsprint hit 0.3" above the waist of the focused pulsed laser beam, seen from above (enlargement 300 x)



Photograph #8 : coated stock hit at the waist of the focused pulsed laser beam, seen from above (enlargement 280 x)

Photograph #9 represents a portion of the degraded area created by hitting paper B (coated stock) 0.4 in. above the focal point of the lens . The radius of the circular impact is about 550 μm long but that of the central hole is only about 25 μm long.

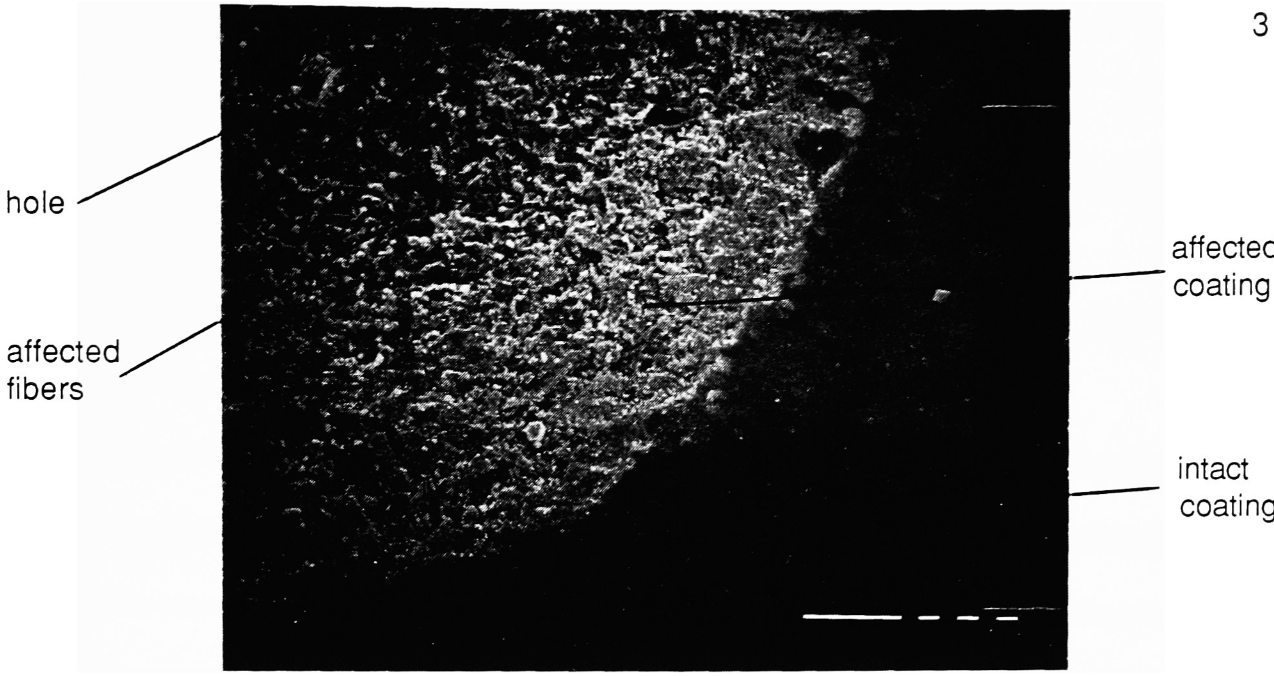
This phenomenon (i.e., a large affected area but a small hole) is similar to the one observed on photograph #2: paper thickness makes the creation of a hole difficult when the energy per surface unit is lower. Moreover, the coating has to be eliminated before the fibers can be reached; and as most of it cannot be burned (because the coating is composed of mineral matter), it slows down the degradation of the sheet.

Then, four zones can be distinguished on this photograph: the intact coated surface, the degraded coating, the degraded fibers (most of the coating has been removed there, but some particles remain fixed to the "melted" fibers), and last the hole (upper left).

Photograph #10 uses a high enlargement to show an area of photograph #9 where both degraded fibers and coated particles are present.

Remark: the composition of the coating is unknown, but some starch was detected using iodine (when exposed to iodine, starch fugitively turns blue).

The coating has been turned into aggregates which have been fixed to the "melted' parts of the fibers during the combustion. There is no change in the appearance of hit fibers, and coating particles seem to attach themselves superficially and not internally. So they should not decrease the strength of a laser-cut edge since the relatively homogeneous "melted" layer is preserved.



Photograph #9 : coated stock hit 0.4" above the waist of the focused pulsed laser beam, seen from above (enlargement 120 x)

Photograph #10 : coated stock hit 0.4" above the waist of the focused pulsed laser beam, seen from above (enlargement 1,200 x)

2. <u>Tensile tests results</u>

- The breaking loads will be expressed in kilograms-force per 15 mm (as specified in the TAPPI standard [21]).
- The temperature recorded during the experiments was 26 °C (78 °F). The high humidity could not be recorded.
- The width of the test specimen (either laser-cut or shear-cut) was 25.4 mm (1 in.).
- The distance between the jaws at the start of the test was 180 mm (7.09 in.).
- The tensile strength was recorded in the machine direction only (see the remark in paragraph 1.a., page 20).
- The average values of the "times to break" were :
  - 10 s for paper A
  - 12 s for paper B
  - 7 s for paper C

- 30 samples of each type of paper and corresponding to each kind of cutting were tested. However, because tensile tests always involve unexpected results (e.g., when there is a defect in the paper strip or when the break occurs next to the jaw), only 26 results were recorded for each test.

On the Instron tensile tester, results are given in the form of elongation test curves plotting the crosshead displacement versus the load to which the specimen is submitted (see figure 7). The strength is recorded by measuring the load at the breaking point.

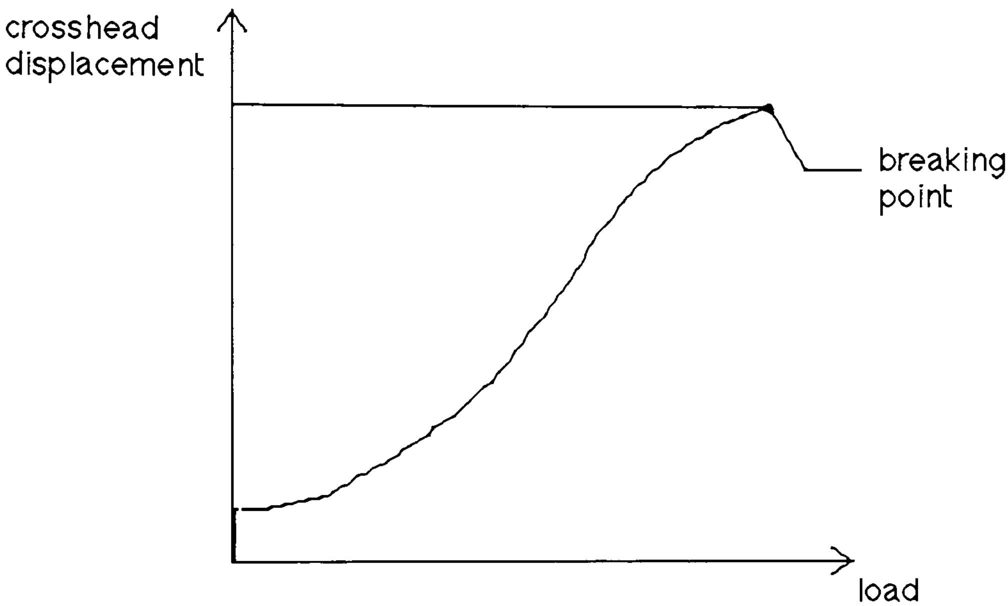


Figure 7: Elongation test curve

LOAD IN KILOGRAMS-FORCE PER 15 MM

	Paper A (uncoated, 101 g/m2)		Paper B (coated, 118 g/m2)		Paper C (newsprint, 47 g/m2)	
	Shear-cut	Laser-cut	Shear-cut	Laser-cut	Shear-cut	Laser-cut
	7.15	7.2	5.55	6.02	3.54	3.37
	7.03	7.38	5.43	5.73	3.96	3.72
	7.09	7.5	5.43	5.55	3.25	3.78
	7.15	7.2	5.49	6.02	3.43	3.84
	7.44	7.56	5.43	5.67	4.25	4.13
	6.97	7.5	5.31	5.91	3.66	3.84
	6.85	7.15	5.55	5.79	3.72	3.96
	7.26	7.5	5.55	5.79	3.9	3.84
	7.38	7.44	5.26	5.55	3.43	3.96
	7.32	7.56	5.67	5.85	3.84	3.96
	7.38	7.44	6.02	5.55	4.13	4.02
	7.38	7.38	5.85	6.02	3.31	4.07
	7.44	7.5	5.55	5.79	3.54	3.37
	7.56	6.85	5.79	5.91	4.43	3.78
	7.68	7.26	5.67	5.73	4.02	3.96
	7.68	7.2	5.67	5.85	3.48	3.9
	7.2	7.91	5.91	5.91	3.96	4.13
	6.97	7.74	5.43	6.14	3.9	3.96
	7.03	7.2	5.91	5.73	3.78	3.78
	6.91	6.73	6.02	5.67	4.07	4.31
	7.38	6.97	5.55	6.08	3.9	3.19
	7.2	6.97	5.79	6.02	3.43	3.96
	7.15	7.32	5.55	5.55	4.25	3.96
	6.56	7.09	5.55	5.43	4.07	3.84
	7.56	6.97	5.67	5.43	4.02	3.72
	7.2	6.97	5.43	5.55	3.96	3.84
Mean	S = 7.23	L = 7.29	S = 5.62	L = 5.78	S = 3.82	L = 3.85
Standard deviation	0.266	0.282	0.205	0.197	0.315	0.244
Relative difference: (L - S) / S	0.83 %		2.85 %		0.79 %	

Table 1: tensile tests results

Two conclusions can be drawn from this table. For papers A, B, and C:

1) laser-cut strips are always stronger than knife-cut strips.

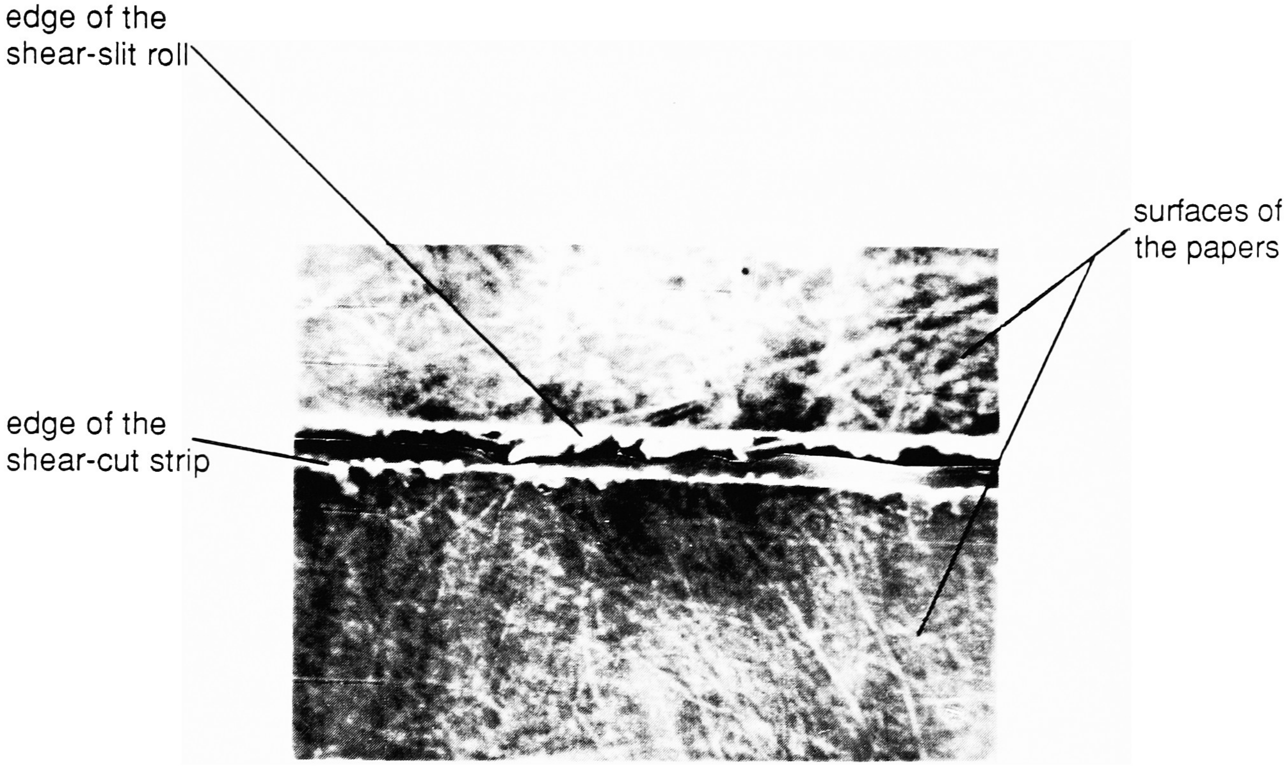
2) the relative difference in load per 15 mm between laser-cut and shear-cut specimens never exceeds 3%.

The standard deviation of each series of measurements remains low: less than 4% of the average load per 15 mm for papers A (uncoated stock) and B (coated stock), and less than 9% of the average load per 15 mm for paper C (newsprint).

Paper C presents a higher standard deviation because of its lower basis weight. Newsprint is subject to defects in formation (lack of uniformity of the sheet) and this creates shifts in tensile tests results. However, in the case of paper C, the results of the laser-cut specimens show a much lower standard deviation than the results of the knife-cut specimens (0.244 vs. 0.315). Here, cutting by laser beam has improved the uniformity of the edges, by "fuseing" some defects of the newsprint.

Remark: the difference in tensile strength properties between laser-cut and shear-cut samples is very low, but it must be realized that the double bladed hand paper cutter creates knife-cut edges that are much more regular than that obtained with circular slitters in rewinders (see photograph B).

It was obviously impossible to cut a 1 inch wide paper web in a rewinder (which would have been necessary to get knife-cut edges similar to that of a paper web); but using a hand paper cutter decreased the number of notches and extra fibers in the edge, that is to say the number of defects in the strip. So there would certainly be a larger shift in tensile strength between rewinder slitter-cut edges and laser-cut ones, but this does not mean that such a difference would be significant.



Photograph B : Comparison between the edge of a knife-slit paper roll and that of a strip cut with a hand paper cutter

Top of the photograph: edge of the roll

Bottom of the photograph: edge of the strip

Enlargement : 45 x

Paper A provides the higher results because of its high basis weight. That of paper B is higher, but paper B is a coated stock and coating does not bring tensile strength to a sheet of paper. Thus the lower values of the loads per 15 mm for paper B.

It is difficult to say whether the 3% difference between laser-cut and knife-cut paper strips results from the nature of paper B, since its composition is unknown. Though this 3% shift is about three times larger than the ones obtained with papers A and C, it remains small and of apparently little significance regarding the practical advantage of slitting paper rolls with laser beams.

That is why it was decided to carry out an analysis of variance (also called ANOVA) to evaluate the test results in a different manner.

This ANOVA will be the object of paragraph 3.

3. Statistical analysis (ANOVA)

a) Principle of an ANOVA. [22]

In the case of this experiment, there is only one variable of classification, which is the means employed to cut paper strips. The results corresponding to each group (laser-cut and shear-cut samples) are displayed in two separate columns of a table.

If there is no advantage in a laser slitter over a knife slitter and if the population mean scores corresponding to the two cutting devices are denoted by μ_1 and μ_2 , then the problem reduces to one of testing the hypothesis

(1) $$H_0 : \mu_1 = \mu_2$$

When the hypothesis H_0 is true, the classification of the data into two columns is meaningless and the entire set of measurements can be treated as a sample of size 52 from a single population. This assumes that the variation in results is due to the paper variations and has nothing to do with cutting differences. Essentially, therefore, if the hypothesis is true, the experiment is assumed to be equivalent to one in which each of the 52 samples was cut in the same manner. If σ denotes the population variance, an estimate of σ^2 can be obtained by means of the familiar sample variance based on all measurements. However, there are several other ways of obtaining valid estimates of σ^2. For instance, the sample variance of the first column of measurements is a valid unbiased estimate of σ^2 although it is not as good as the estimate based on all the measurements. Similarly, the sample variance of the second column is also a valid estimate of σ^2. Furthermore, the mean of the two column estimates of σ^2 is a valid unbiased estimate of σ^2 and nearly as good as the familiar estimate based on combining the two sets of measurements. If s_1^2 and s_2^2 denote the sample variances for the two columns, this last estimate, which will be denoted by V_c, is

(2) $$V_c = (s_1^2 + s_2^2)/2$$

The subscript $_c$ is used here to indicate that the estimate is based on the column variances.

Another quite different type of estimate of σ^2 can be obtained by using the relationship between the variance of a sample mean and that of the population, namely $\sigma_x{}^2 = \sigma^2 / n$. Or, expressed in another form:

(3) $\sigma^2 = n \times \sigma_x{}^2$

Suppose several samples of size n have been taken from some population. If the sample means have been calculated, then the sample variance of those sample means will be a valid estimate of the population variance of the measurements regardless of whether those measurements happen to be simple measurements or other functions of simple measurements. From (3) it follows that if an estimate of $\sigma_x{}^2$ is available, it may be multiplied by n to yield an estimate of σ^2. In this problem there are two such sample means that may be used to construct an estimate of $\sigma_x{}^2$. They are the two column means, which will be denoted by x_1 and x_2. Now the mean of these two column means is equal to the grand mean, x, of all the measurements; hence, an unbiased estimate of $\sigma_x{}^2$ is given by $(x_1 - x)^2 + (x_2 - x)^2$. Since these means are based on samples of size 26 each, it follows that $n = 26$ and, hence, from (3) that the desired estimate of σ^2 is given by:

(4) $V_m = 26 \times [\,(x_1 - x)^2 + (x_2 - x)^2\,]$

The subscript $_m$ is used here to indicate that the estimate is based on the means of the columns.

Since V_c and V_m are both valid unbiased estimates of σ^2 when H_0 is true, it follows that they should be approximately equal in value, and therefore that their ratio should have a value close to 1.

If, however, H_0 is not true and the column means differ considerably, the two estimates V_c and V_m will be seen to differ considerably in value. Because the estimate V_c is based on calculating the variances of each column separately, it will be unaffected by changing the means of the columns, for the variance of a set of measurements is independent of the value of their mean. It is clear from (4), however, that the estimate V_m will be directly affected and will increase in value as the sample means move apart. Thus it appears that the ratio V_m to V_c will exceed 1 when H_0 is not true. This ratio will be used as the desired quantity for testing the hypothesis H_0 . It is denoted by the letter *F*. Hence:

(5) $F = V_m / V_c$

To know if the value of F is too large when compared to the value of F that might be expected in repeated tensile tests using shear-cut strips only, it is necessary to determine the sampling distribution of F.

It is possible to approximate the distribution of F by carrying out repeated sampling

experiments of the type being considered here and constructing a histogram of the resulting values; however, the exact sampling distribution of F can be obtained by mathematical methods, provided that the proper asumptions are made. The assumptions needed here are that the 52 variables are independant normal variables, all having the same mean μ and variance σ^2.

It turns out that the distribution of F depends only on how many data were available for the numerator estimate of σ^2 and how many were available for the denominator estimate. A table lists the 5 percent and the 1 percent right-tail critical values of F corresponding to different values of the parameters v_1 and v_2, which are called the number of degrees of freedom in the numerator and denominator of F (see appendix).[22]

The degrees of freedom here are those that one would naturally associate with the sample variances being used. Since the number of degrees of freedom for the usual estimate of σ^2 is given by $v = n - 1$, or one less than the number of measurements, the number of degrees of freedom for the numerator of F in this problem is given by $v_1 = 1$ because the estimate is based on the two sample means. The number of degrees of freedom for the denominator of F in this problem is $v_2 = 50$ because each variance contributes 25 degrees of freedom and all 2 column variances are employed .

From the table (see appendix) it will be found that the 5 percent critical value of F corresponding to $v_1 = 1$ and $v_2 = 50$ is 4.03.

Remark: in this problem of testing the equality of two column means, the F test is actually equivalent to the t test for the same problem.

b) Application to the tests results

For papers A and C (see tables 2 and 4) the value of F is much lower than the critical value: it appears that the very small difference obtained in tensile strength measurements is proved not to be significant statistically. It can be said that there is no difference in behavior between knife-cut and laser-cut edges.

In the case of paper B (see table 3), the value of F is sensibly higher than the critical value: the small relative difference (about 3%) in tensile strength between the two types of strips is statistically significant. Cutting by laser beam did increase the strength of the paper strips.

Here it has to be recalled that paper B was cut with a much higher power than papers A and C (95 W vs. 40 W). This higher power reduced the time of cutting and, even if the slitting rate used was slower, the energy provided by the beam during the slitting was soon dissipated under the slit and thus did not affect paper B as much as it did for papers A and C. The cutting was more efficient and a stronger edge probably results from this.

One will notice that the ANOVA carried out on all three papers considered together does not reveal any statistically significant difference in behavior between the two types of samples (see table 5). Then it can be concluded that the effect of laser slitting on a broad range of papers is almost unnoticeable.

Source of variation	Sum of squares	Degrees of freedom	Mean square	F
Column means	0 .047	1	0.047	0.62
Experimental error	3.8	50	0.075	
Totals		51		

Table 2 : ANOVA results for paper A

Source of variation	Sum of squares	Degrees of freedom	Mean square	F
Column means	0.33	1	0.33	8.24
Experimental error	2	50	0.04	
Totals		51		

Table 3 : ANOVA results for paper B

Source of variation	Sum of squares	Degrees of freedom	Mean square	F
Column means	0.012	1	0.012	0.15
Experimental error	4	50	0.079	
Totals		51		

Table 4 : ANOVA results for paper C

Source of variation	Sum of squares	Degrees of freedom	Mean square	F
Column means	0.25	1	0.25	0.12
Experimental error	315	154	2.05	
Totals		155		

Table 5 : ANOVA results for papers A, B, and C together

CHAPTER VI

CONCLUSION

Though the hypothesis stating that laser-cut paper edges are stronger than knife-cut paper edges was proved to be true, the measured relative difference remained low .

The average load (in kilograms-force per 15 mm) was, in the case of paper A (uncoated stock), 0.83% greater for laser-cut samples than for knife-cut samples. In the case of paper B (coated stock), it was 2.85% greater for laser-cut samples than for knife-cut samples. In the case of paper C (newsprint), it was only 0.79% greater for laser-cut samples than for knife-cut samples.

Moreover, the ANOVA showed that this shift was not significant in the case of papers A and C. And even if this difference proves to be statistically significant for paper B, it does not mean that the resistance of a laser-cut roll of this paper to tension will have drastically improved over that of a roll of the same paper cut with traditional means.

However it must not be forgotten that the quality of the laser-cut specimens was lower than what can normally be expected (since the slitting rate was especially low) and

that of the shear-cut ones was , to the contrary, higher than normal (due to the use of a double-bladed hand paper cutter).

Regarding the observation of laser-affected samples through a scanning electron microscope, the "melting" of fibers proved to strengthen the edges. Moreover, using a free sheet made of pure cellulose pulp proved that this "melting" was not caused by the presence of lignin or fillers, but was a reaction of fibers when hit by laser beam.

Since laser slitting does not bring much more strength to the edges of a roll of paper, the web press user will not be advised to specify "laser-cut webs" on his orders for paper rolls. Thus the papermaker will not find in this report a decisive argument for equipping his rewinder(s) with laser slitting heads, which remain expensive and imply maintenance as well as gas consumption costs.

CHAPTER VII

RECOMMENDATIONS FOR FURTHER INVESTIGATION

In my opinion, there is no point in testing the strength of laser-cut paper edges again, under the set of conditions described in this study. The results obtained in this thesis do not justify further investigation in this direction.

But changing the conditions of these experiments (e.g., the cutting power of the laser, the cutting velocity, or even the wavelength at which the carbon dioxide laser is used to cut paper) might modify the strength of paper edges more significantly.

Moreover, a deeper study of the chemical effect of an infra-red laser on cellulose might be of value to the paper chemist. In particular, infra-red spectrometry (the multiple internal reflectance type, using the Fourier transform for data analysis), when applied to paper samples having been more or less affected by the beam, would enable one to record any transformation in cellulose structure. The slightest change can be observed thanks to the capability this device has to record and analyze high numbers of readings (each reading corresponding to the reflectance of a sample placed between two plates

-- the latter being transparent to infra-red light --, and which a laser ray crosses several times as it reflects itself on the plates).[11,23]

This study of a non-traditional paper cutting device may also lead to that of another non-traditional cutting device, known as water jet cutting (or WJC). Typically, it employs a fine, high-pressure (about 250 MPa), high velocity (ultrasonic) jet of water. The small nozzle (about 0.2 mm -- 8×10^{-3} in. -- in diameter) produces a very narrow kerf.

This cutting means presents most of the advantages of laser-slitting (possibility of cutting materials in almost any pattern, minimal loss of material, no deformation of the material since no mechanical contact is involved in the cutting process, and easy automation) without some of its inconveniences (WJC will not burn or thermically damage surfaces, will not produce environmental pollution since the water jet drains any dust simultaneously when cutting).[24]

The installation and maintenance costs of laser-cutting and WJC devices are similar, but WJC does not require gas consumption[12] and paper-mills usually do not lack water... However, WJC seems to be more suitable for thick materials such as cardboard or pulp sheets, probably because of the high pressure involved.

All this shows that WJC is certainly worth being studied, for it offers new possibilities and also advantages which are very similar to that of laser-cutting.

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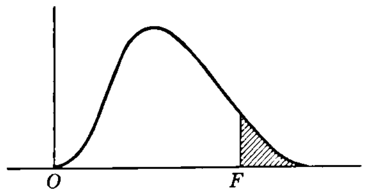
APPENDIX 22

TABLE IX. *F* Distribution

5% (Roman Type) and 1% (Boldface Type) Points for the Distribution of *F*



Degrees of freedom for denominator (ν_2)	Degrees of freedom for numerator (ν_1): 1	2	3	4	5	6	7	8	9	10	11	12	14	16	20	24	30	40	50	75	100	200	500	∞
1	161 **4052**	200 **4999**	216 **5403**	225 **5625**	230 **5764**	234 **5859**	237 **5928**	239 **5981**	241 **6022**	242 **6056**	243 **6082**	244 **6106**	245 **6142**	246 **6169**	248 **6208**	249 **6234**	250 **6258**	251 **6286**	252 **6302**	253 **6323**	253 **6334**	254 **6352**	254 **6361**	254 **6366**
2	18.51 **98.49**	19.00 **99.01**	19.16 **99.17**	19.25 **99.25**	19.30 **99.30**	19.33 **99.33**	19.36 **99.34**	19.37 **99.36**	19.38 **99.38**	19.39 **99.40**	19.40 **99.41**	19.41 **99.42**	19.42 **99.43**	19.43 **99.44**	19.44 **99.45**	19.45 **99.46**	19.46 **99.47**	19.47 **99.48**	19.47 **99.48**	19.48 **99.49**	19.49 **99.49**	19.49 **99.49**	19.50 **99.50**	19.50 **99.50**
3	10.13 **34.12**	9.55 **30.81**	9.28 **29.46**	9.12 **28.71**	9.01 **28.24**	8.94 **27.91**	8.88 **27.67**	8.84 **27.49**	8.81 **27.34**	8.78 **27.23**	8.76 **27.13**	8.74 **27.05**	8.71 **26.92**	8.69 **26.83**	8.66 **26.69**	8.64 **26.60**	8.62 **26.50**	8.60 **26.41**	8.58 **26.30**	8.57 **26.27**	8.56 **26.23**	8.54 **26.18**	8.54 **26.14**	8.53 **26.12**
4	7.71 **21.20**	6.94 **18.00**	6.59 **16.69**	6.39 **15.98**	6.26 **15.52**	6.16 **15.21**	6.09 **14.98**	6.04 **14.80**	6.00 **14.66**	5.96 **14.54**	5.93 **14.45**	5.91 **14.37**	5.87 **14.24**	5.84 **14.15**	5.80 **14.02**	5.77 **13.93**	5.74 **13.83**	5.71 **13.74**	5.70 **13.69**	5.68 **13.61**	5.66 **13.57**	5.65 **13.52**	5.64 **13.48**	5.63 **13.46**
5	6.61 **16.26**	5.79 **13.27**	5.41 **12.06**	5.19 **11.39**	5.05 **10.97**	4.95 **10.67**	4.88 **10.45**	4.82 **10.27**	4.78 **10.15**	4.74 **10.05**	4.70 **9.96**	4.68 **9.89**	4.64 **9.77**	4.60 **9.68**	4.56 **9.55**	4.53 **9.47**	4.50 **9.38**	4.46 **9.29**	4.44 **9.24**	4.42 **9.17**	4.40 **9.13**	4.38 **9.07**	4.37 **9.04**	4.36 **9.02**
6	5.99 **13.74**	5.14 **10.92**	4.76 **9.78**	4.53 **9.15**	4.39 **8.75**	4.28 **8.47**	4.21 **8.26**	4.15 **8.10**	4.10 **7.98**	4.06 **7.87**	4.03 **7.79**	4.00 **7.72**	3.96 **7.60**	3.92 **7.52**	3.87 **7.39**	3.84 **7.31**	3.81 **7.23**	3.77 **7.14**	3.75 **7.09**	3.72 **7.02**	3.71 **6.99**	3.69 **6.94**	3.68 **6.90**	3.67 **6.88**
7	5.59 **12.25**	4.74 **9.55**	4.35 **8.45**	4.12 **7.85**	3.97 **7.46**	3.87 **7.19**	3.79 **7.00**	3.73 **6.84**	3.68 **6.71**	3.63 **6.62**	3.60 **6.54**	3.57 **6.47**	3.52 **6.35**	3.49 **6.27**	3.44 **6.15**	3.41 **6.07**	3.38 **5.98**	3.34 **5.90**	3.32 **5.85**	3.29 **5.78**	3.28 **5.75**	3.25 **5.70**	3.24 **5.67**	3.23 **5.65**
8	5.32 **11.26**	4.46 **8.65**	4.07 **7.59**	3.84 **7.01**	3.69 **6.63**	3.58 **6.37**	3.50 **6.19**	3.44 **6.03**	3.39 **5.91**	3.34 **5.82**	3.31 **5.74**	3.28 **5.67**	3.23 **5.56**	3.20 **5.48**	3.15 **5.36**	3.12 **5.28**	3.08 **5.20**	3.05 **5.11**	3.03 **5.06**	3.00 **5.00**	2.98 **4.96**	2.96 **4.91**	2.94 **4.88**	2.93 **4.86**
9	5.12 **10.56**	4.26 **8.02**	3.86 **6.99**	3.63 **6.42**	3.48 **6.06**	3.37 **5.80**	3.29 **5.62**	3.23 **5.47**	3.18 **5.35**	3.13 **5.26**	3.10 **5.18**	3.07 **5.11**	3.02 **5.00**	2.98 **4.92**	2.93 **4.80**	2.90 **4.73**	2.86 **4.64**	2.82 **4.56**	2.80 **4.51**	2.77 **4.45**	2.76 **4.41**	2.73 **4.36**	2.72 **4.33**	2.71 **4.31**
10	4.96 **10.04**	4.10 **7.56**	3.71 **6.55**	3.48 **5.99**	3.33 **5.64**	3.22 **5.39**	3.14 **5.21**	3.07 **5.06**	3.02 **4.95**	2.97 **4.85**	2.94 **4.78**	2.91 **4.71**	2.86 **4.60**	2.82 **4.52**	2.77 **4.41**	2.74 **4.33**	2.70 **4.25**	2.67 **4.17**	2.64 **4.12**	2.61 **4.05**	2.59 **4.01**	2.56 **3.96**	2.55 **3.93**	2.54 **3.91**
11	4.84 **9.65**	3.98 **7.20**	3.59 **6.22**	3.36 **5.67**	3.20 **5.32**	3.09 **5.07**	3.01 **4.88**	2.95 **4.74**	2.90 **4.63**	2.86 **4.54**	2.82 **4.46**	2.79 **4.40**	2.74 **4.29**	2.70 **4.21**	2.65 **4.10**	2.61 **4.02**	2.57 **3.94**	2.53 **3.86**	2.50 **3.80**	2.47 **3.74**	2.45 **3.70**	2.42 **3.66**	2.41 **3.62**	2.40 **3.60**
12	4.75 **9.33**	3.88 **6.93**	3.49 **5.95**	3.26 **5.41**	3.11 **5.06**	3.00 **4.82**	2.92 **4.65**	2.85 **4.50**	2.80 **4.39**	2.76 **4.30**	2.72 **4.22**	2.69 **4.16**	2.64 **4.05**	2.60 **3.98**	2.54 **3.86**	2.50 **3.78**	2.46 **3.70**	2.42 **3.61**	2.40 **3.56**	2.36 **3.49**	2.35 **3.46**	2.32 **3.41**	2.31 **3.38**	2.30 **3.36**
13	4.67 **9.07**	3.80 **6.70**	3.41 **5.74**	3.18 **5.20**	3.02 **4.86**	2.92 **4.62**	2.84 **4.44**	2.77 **4.30**	2.72 **4.19**	2.67 **4.10**	2.63 **4.02**	2.60 **3.96**	2.55 **3.85**	2.51 **3.78**	2.46 **3.67**	2.42 **3.59**	2.38 **3.51**	2.34 **3.42**	2.32 **3.37**	2.28 **3.30**	2.26 **3.27**	2.24 **3.21**	2.22 **3.18**	2.21 **3.16**
14	4.60 **8.86**	3.74 **6.51**	3.34 **5.56**	3.11 **5.03**	2.96 **4.69**	2.85 **4.46**	2.77 **4.28**	2.70 **4.14**	2.65 **4.03**	2.60 **3.94**	2.56 **3.86**	2.53 **3.80**	2.48 **3.70**	2.44 **3.62**	2.39 **3.51**	2.35 **3.43**	2.31 **3.34**	2.27 **3.26**	2.24 **3.21**	2.21 **3.14**	2.19 **3.11**	2.16 **3.06**	2.14 **3.02**	2.13 **3.00**
15	4.54 **8.68**	3.68 **6.36**	3.29 **5.42**	3.06 **4.89**	2.90 **4.56**	2.79 **4.32**	2.70 **4.14**	2.64 **4.00**	2.59 **3.89**	2.55 **3.80**	2.51 **3.73**	2.48 **3.67**	2.43 **3.56**	2.39 **3.48**	2.33 **3.36**	2.29 **3.29**	2.25 **3.20**	2.21 **3.12**	2.18 **3.07**	2.15 **3.00**	2.12 **2.97**	2.10 **2.92**	2.08 **2.89**	2.07 **2.87**
16	4.49 **8.53**	3.63 **6.23**	3.24 **5.29**	3.01 **4.77**	2.85 **4.44**	2.74 **4.20**	2.66 **4.03**	2.59 **3.89**	2.54 **3.78**	2.49 **3.69**	2.45 **3.61**	2.42 **3.55**	2.37 **3.45**	2.33 **3.37**	2.28 **3.25**	2.24 **3.18**	2.20 **3.10**	2.16 **3.01**	2.13 **2.96**	2.09 **2.89**	2.07 **2.86**	2.04 **2.80**	2.02 **2.77**	2.01 **2.75**
17	4.45 **8.40**	3.59 **6.11**	3.20 **5.18**	2.96 **4.67**	2.81 **4.34**	2.70 **4.10**	2.62 **3.93**	2.55 **3.79**	2.50 **3.68**	2.45 **3.59**	2.41 **3.52**	2.38 **3.45**	2.33 **3.35**	2.29 **3.27**	2.23 **3.16**	2.19 **3.08**	2.15 **3.00**	2.11 **2.92**	2.08 **2.86**	2.04 **2.79**	2.02 **2.76**	1.99 **2.70**	1.97 **2.67**	1.96 **2.65**
18	4.41 **8.28**	3.55 **6.01**	3.16 **5.09**	2.93 **4.58**	2.77 **4.25**	2.66 **4.01**	2.58 **3.85**	2.51 **3.71**	2.46 **3.60**	2.41 **3.51**	2.37 **3.44**	2.34 **3.37**	2.29 **3.27**	2.25 **3.19**	2.19 **3.07**	2.15 **3.00**	2.11 **2.91**	2.07 **2.83**	2.04 **2.78**	2.00 **2.71**	1.98 **2.68**	1.95 **2.62**	1.93 **2.59**	1.92 **2.57**
19	4.38 **8.18**	3.52 **5.93**	3.13 **5.01**	2.90 **4.50**	2.74 **4.17**	2.63 **3.94**	2.55 **3.77**	2.48 **3.63**	2.43 **3.52**	2.38 **3.43**	2.34 **3.36**	2.31 **3.30**	2.26 **3.19**	2.21 **3.12**	2.15 **3.00**	2.11 **2.92**	2.07 **2.84**	2.02 **2.76**	2.00 **2.70**	1.96 **2.63**	1.94 **2.60**	1.91 **2.54**	1.90 **2.51**	1.88 **2.49**
20	4.35 **8.10**	3.49 **5.85**	3.10 **4.94**	2.87 **4.43**	2.71 **4.10**	2.60 **3.87**	2.52 **3.71**	2.45 **3.56**	2.40 **3.45**	2.35 **3.37**	2.31 **3.30**	2.28 **3.23**	2.23 **3.13**	2.18 **3.05**	2.12 **2.94**	2.08 **2.86**	2.04 **2.77**	1.99 **2.69**	1.96 **2.63**	1.92 **2.56**	1.90 **2.53**	1.87 **2.47**	1.85 **2.44**	1.84 **2.42**
21	4.32 **8.02**	3.47 **5.78**	3.07 **4.87**	2.84 **4.37**	2.68 **4.04**	2.57 **3.81**	2.49 **3.65**	2.42 **3.51**	2.37 **3.40**	2.32 **3.31**	2.28 **3.24**	2.25 **3.17**	2.20 **3.07**	2.15 **2.99**	2.09 **2.88**	2.05 **2.80**	2.00 **2.72**	1.96 **2.63**	1.93 **2.58**	1.89 **2.51**	1.87 **2.47**	1.84 **2.42**	1.82 **2.38**	1.81 **2.36**
22	4.30 **7.94**	3.44 **5.72**	3.05 **4.82**	2.82 **4.31**	2.66 **3.99**	2.55 **3.76**	2.47 **3.59**	2.40 **3.45**	2.35 **3.35**	2.30 **3.26**	2.26 **3.18**	2.23 **3.12**	2.18 **3.02**	2.13 **2.94**	2.07 **2.83**	2.03 **2.75**	1.98 **2.67**	1.93 **2.58**	1.91 **2.53**	1.87 **2.46**	1.84 **2.42**	1.81 **2.37**	1.80 **2.33**	1.78 **2.31**
23	4.28 **7.88**	3.42 **5.66**	3.03 **4.76**	2.80 **4.26**	2.64 **3.94**	2.53 **3.71**	2.45 **3.54**	2.38 **3.41**	2.32 **3.30**	2.28 **3.21**	2.24 **3.14**	2.20 **3.07**	2.14 **2.97**	2.10 **2.89**	2.04 **2.78**	2.00 **2.70**	1.96 **2.62**	1.91 **2.53**	1.88 **2.48**	1.84 **2.41**	1.82 **2.37**	1.79 **2.32**	1.77 **2.28**	1.76 **2.26**
24	4.26 **7.82**	3.40 **5.61**	3.01 **4.72**	2.78 **4.22**	2.62 **3.90**	2.51 **3.67**	2.43 **3.50**	2.36 **3.36**	2.30 **3.25**	2.26 **3.17**	2.22 **3.09**	2.18 **3.03**	2.13 **2.93**	2.09 **2.85**	2.02 **2.74**	1.98 **2.66**	1.94 **2.58**	1.89 **2.49**	1.86 **2.44**	1.82 **2.36**	1.80 **2.33**	1.76 **2.27**	1.74 **2.23**	1.73 **2.21**
25	4.24 **7.77**	3.38 **5.57**	2.99 **4.68**	2.76 **4.18**	2.60 **3.86**	2.49 **3.63**	2.41 **3.46**	2.34 **3.32**	2.28 **3.21**	2.24 **3.13**	2.20 **3.05**	2.16 **2.99**	2.11 **2.89**	2.06 **2.81**	2.00 **2.70**	1.96 **2.62**	1.92 **2.54**	1.87 **2.45**	1.84 **2.40**	1.80 **2.32**	1.77 **2.29**	1.74 **2.23**	1.72 **2.19**	1.71 **2.17**

TABLE IX. *(Continued)*

Degrees of freedom for denominator (ν_2)	Degrees of freedom for numerator (ν_1) 1	2	3	4	5	6	7	8	9	10	11	12	14	16	20	24	30	40	50	75	100	200	500	∞
26	4.22 7.72	3.37 5.53	2.89 4.64	2.74 4.14	2.59 3.82	2.47 3.59	2.39 3.42	2.32 3.29	2.27 3.17	2.22 3.09	2.18 3.02	2.15 2.96	2.10 2.86	2.05 2.77	1.99 2.66	1.95 2.58	1.90 2.50	1 85 2.41	1.82 2.36	1.78 2.28	1.76 2.25	1.72 2.19	1.70 2.15	1 69 2.13
27	4.21 7.68	3.35 5.49	2.96 4.60	2.73 4.11	2.57 3.79	2.46 3.56	2.37 3.39	2.30 3.26	2.25 3.14	2.20 3.06	2.16 2.98	2 13 2.93	2.08 2.83	2 03 2.74	1.97 2.63	1 93 2.55	1.88 2.47	1.84 2.38	1.80 2.33	1.76 2.25	1.74 2.21	1 71 2.16	1.68 2.12	1.67 2.10
28	4 20 7.64	3.34 5.45	2.95 4.57	2.71 4.07	2.56 3.76	2.44 3.53	2.36 3.36	2.29 3.23	3.24 3.11	2.19 3.03	2.15 2.95	2.12 2.90	2.06 2.80	2.02 2.71	1.96 2.60	1.91 2.52	1.87 2.44	1.81 2.35	1.78 2.30	1.75 2.22	1 72 2.18	1.69 2.13	1 67 2.09	1 65 2.06
29	4.18 7.60	3.33 5.52	2.93 4.54	2.70 4.04	2.54 3.73	2.43 3.50	2.35 3.33	2.28 3.20	2.22 3.08	2 18 3.00	2.14 2.92	2 10 2.87	2.05 2.77	2.00 2.68	1 94 2.57	1.90 2.49	1.85 2.41	1.80 2.32	1.77 2.27	1.73 2.19	1.71 2.15	1.68 2.10	1.65 2.06	1 64 2.03
30	4.17 7.56	3.32 5.39	2.92 4.51	2.69 4.02	2 53 3.70	2.42 3.47	2 34 3.30	2.27 3.17	2.21 3.06	2 16 2.98	2.12 2.00	2.09 2.84	2.04 2.74	1.99 2.66	1.93 2.55	1.89 2.47	1.84 2.38	1.79 2.29	1.76 2.24	1.72 2.16	1.69 2.13	1 66 2.07	1 64 2.03	1 62 2.01
32	4.15 7.50	3.30 5.34	2.90 4.46	2.67 3.97	2.51 3.66	2.40 3.42	2.32 3.25	2.25 3.12	2.19 3.01	2.14 2.94	2.10 2.86	2 07 2.80	2 02 2.70	1.97 2.62	1.91 2.51	1 86 2.42	1.82 2.34	1.76 2.25	1.74 2.20	1.69 2.12	1 67 2.08	1 64 2.02	1.61 1.98	1 59 1.96
34	4.13 7.44	3.28 5.29	2.88 4.42	2.65 3.93	2.49 3.61	2.38 3.38	2.30 3.21	2.23 3.08	2.17 2.97	2.12 2.89	2.08 2.82	2 05 2.76	2.00 2.66	1.95 2.58	1.89 2.47	1.84 2.38	1 80 2.30	1.74 2.21	1.71 2.15	1.67 2.08	1 64 2.04	1 61 1.98	1.59 1.94	1 57 1.91
36	4.11 7.39	3.26 5.25	2.86 4.38	2.63 3.89	2.48 3.58	2.36 3.35	2.28 3.18	2.21 3.04	2.15 2.94	2.10 2.86	2.06 2.78	2 03 2.72	1.89 2.62	1.93 2.54	1.87 2.43	1.82 2.35	1 78 2.26	1.72 2.17	1.69 2.12	1.65 2.04	1.62 2.00	1.59 1.94	1.56 1.90	1.55 1.87
38	4.10 7.35	3.25 5.21	2.85 4.34	2.62 3.86	2.46 3.54	2.35 3.32	2.26 3.15	2.19 3.02	2.14 2.91	2.09 2.82	2.05 2.75	2.02 2.69	1.96 2.59	1.92 2.51	1.85 2.40	1 80 2.32	1.76 2.22	1.71 2.14	1.67 2.08	1 63 2.00	1.60 1.97	1.57 1.90	1.54 1.86	1.53 1.84
40	4.08 7.31	3.23 5.18	2.84 4.31	2.61 3.83	2.45 3.51	2.34 3.29	2.25 3.12	2.18 2.99	2.12 2.88	2.07 2.80	2.04 2.73	2.00 2.66	1.95 2.56	1.90 2.49	1 84 2.37	1.79 2.29	1.74 2.20	1.69 2.11	1.66 2.05	1 61 1.97	1.59 1.94	1.55 1.88	1.53 1.84	1.51 1.81
42	4.07 7.27	3.22 5.15	2.83 4.29	2.59 3.80	2.44 3.49	2.32 3.26	2.24 3.10	2.17 2.96	2.11 2.86	2.06 2.77	2.02 2.70	1.99 2.64	1.94 2.54	1.89 2.46	1.82 2.35	1 78 2.26	1.73 2.17	1.68 2.08	1.64 2.02	1.60 1.94	1.57 1.91	1.54 1.85	1.51 1.80	1 49 1.78
44	4.06 7.24	3.21 5.12	2.82 4.26	2.58 3.78	2.43 3.46	2.31 3.24	2.23 3.07	2.16 2.94	2.10 2.84	2.05 2.75	2.01 2.68	1.98 2.62	1.92 2.52	1 88 2.44	1 81 2.32	1.76 2.24	1.72 2.15	1.66 2.06	1 63 2.00	1.58 1.92	1.56 1.88	1.52 1.82	1.50 1.78	1.48 1.75
46	4.05 7.21	3.20 5.10	2.81 4.24	2.57 3.76	2.42 3.44	2.30 3.22	2.22 3.05	2.14 2.92	2.09 2.82	2 04 2.73	2.00 2.66	1.97 2.60	1.91 2.50	1.87 2.42	1.80 2.30	1 75 2.22	1.71 2.13	1.65 2.04	1.62 1.98	1.57 1.90	1.54 1.86	1.51 1.80	1 48 1.76	1.46 1.72
48	4.04 7.19	3 19 5.08	2.80 4.22	2.56 3.74	2.41 3.42	2.30 3.20	2.21 3.04	2.14 2.90	2.08 2.80	2.03 2.71	1.99 2.64	1.96 2.58	1.90 2.48	1.86 2.40	1.79 2.28	1.74 2.20	1.70 2.11	1.64 2.02	1.61 1.96	1.56 1.88	1.53 1.84	1.50 1.78	1 47 1.73	1 45 1.70
50	4.03 7.17	3 18 5.06	2.79 4.20	2.56 3.72	2.40 3.41	2.29 3.18	2.20 3.02	2.13 2.88	2 07 2.78	2.02 2.70	1.98 2.62	1.95 2.56	1.90 2.46	1 85 2.39	1.78 2.26	1.74 2.18	1.69 2.10	1.63 2.00	1 60 1.94	1.55 1.86	1.52 1.82	1 48 1.76	1.46 1.71	1 44 1.68
55	4.02 7.12	3.17 5.01	2 78 4.16	2.54 3.68	2.38 3.37	2.27 3.15	2.18 2.98	2.11 2.85	2 05 2.75	2.00 2.66	1.97 2.59	1.93 2.53	1.88 2.43	1.83 2.35	1.76 2.23	1.72 2.15	1 67 2.06	1.61 1.96	1.58 1.90	1 52 1.82	1.50 1.78	1.46 1.71	1.43 1.66	1.41 1.64
60	4.00 7.08	3.15 4.98	2.76 4.13	2.52 3.65	2.37 3.34	2.25 3.12	2.17 2.95	2.10 2.82	2.04 2.72	1.99 2.63	1.95 2.56	1.92 2.50	1 86 2.40	1.81 2.32	1.75 2.20	1.70 2.12	1.65 2.03	1.59 1.93	1 56 1.87	1.50 1.79	1 48 1.74	1.44 1.68	1.41 1.63	1.39 1.60
65	3.99 7.04	3.14 4.95	2.75 4.10	2.51 3.62	2 36 3.31	2.24 3.09	2.15 2.93	2.08 2.79	2.02 2.70	1.98 2.61	1.94 2.54	1.90 2.47	1.85 2.37	1.80 2.30	1.73 2.18	1.68 2.09	1.63 2.00	1.57 1.90	1.54 1.84	1.49 1.76	1.46 1.71	1 42 1.64	1.39 1.60	1.37 1.56
70	3.98 7.01	3.13 4.92	2.74 4.08	2.50 3.60	2.35 3.29	2.32 3.07	2.14 2.91	2.07 2.77	2.01 2.67	1.97 2.59	1.93 2.51	1.89 2.45	1.84 2.35	1.79 2.28	1.72 2.15	1.67 2.07	1.62 1.98	1.56 1.88	1.53 1.82	1.47 1.74	1.45 1.69	1.40 1.63	1.37 1.56	1.35 1.53
80	3.96 6.96	3.11 4.88	2.72 4.04	2.48 3.56	2.33 3.25	2.21 3.04	2.12 2.87	2.05 2.74	1.99 2.64	1.95 2.55	1 91 2.48	1.88 2.41	1.82 2.32	1.77 2.24	1.70 2.11	1.65 2.03	1.60 1.94	1.54 1.84	1 51 1.78	1.45 1.70	1.42 1.65	1.38 1.57	1.35 1.52	1.32 1.49
100	3.94 6.90	3 09 4.82	2.70 3.98	2.46 3.51	2.30 3.20	2 19 2.99	2.10 2.82	2.03 2.69	1.97 2.59	1.92 2.51	1.88 2.43	1.85 2.36	1.79 2.26	1.75 2.19	1.68 2.06	1 63 1.98	1.57 1.89	1.51 1.79	1.48 1.73	1.42 1.64	1.39 1.59	1.34 1.51	1.30 1.46	1 28 1.43
125	3.92 6.84	3.07 4.78	2.68 3.94	2.44 3.47	2.29 3.17	2.17 2.95	2.08 2.79	2.01 2.65	1.95 2.56	1 90 2.47	1 86 2.40	1.83 2.33	1.77 2.23	1.72 2.15	1.65 2.03	1.60 1.94	1.55 1.85	1.49 1.75	1.45 1.68	1.39 1.59	1.36 1.54	1.31 1.46	1.27 1.40	1.25 1.37
150	3.91 6.81	3.06 4.75	2.67 3.91	2.43 3.44	2.27 3.13	2.16 2.92	2.07 2.76	2.00 2.62	1.94 2.53	1.89 2.44	1.85 2.37	1.82 2.30	1.76 2.20	1.71 2.12	1.64 2.00	1.59 1.91	1.54 1.83	1.47 1.72	1.44 1.66	1.37 1.56	1.34 1.51	1.29 1.43	1.25 1.37	1.22 1.33
200	3.89 6.76	3.04 4.71	2.65 3.88	2.41 3.41	2.26 3.11	2.14 2.90	2.05 2.73	1.98 2.60	1.92 2.50	1.87 2.41	1.83 2.34	1.80 2.28	1.74 1.17	1.69 2.09	1.62 1.97	1.57 1.88	1.52 1.79	1.45 1.69	1 42 1.62	1.35 1.53	1.32 1.48	1.26 1.39	1 22 1.33	1.19 1.28
400	3.86 6.70	3.02 4.66	2.62 3.83	2 39 3.36	2.23 3.06	2.12 2.85	2.03 2.69	1.96 2.55	1.90 2.46	1.85 2.37	1.81 2.29	1.78 2.23	1.72 2.12	1.67 2.04	1 60 1.92	1.54 1.84	1 49 1.74	1.42 1.64	1.38 1.57	1.32 1.47	1.28 1.42	1 22 1.32	1.16 1.24	1.13 1.19
1000	3.85 6.66	3.00 4.62	2.61 3.80	2.38 3.34	2.22 3.04	2.10 2.82	2.02 2.66	1.95 2.53	1.89 2.43	1.84 2.34	1 80 2.26	1.76 2.20	1.70 2.09	1.65 2.01	1.58 1.89	1.53 1.81	1.47 1.71	1.41 1.61	1.36 1.54	1.30 1.44	1.26 1.38	1.19 1.28	1.13 1.19	1.08 1.11
	3.84 6.64	2.99 4.60	2.60 3.78	2.37 3.32	2.21 3.02	2.09 2.80	2.01 2.64	1.94 2.51	1.88 2.41	1 83 2.32	1 79 2.24	1 75 2.18	1.69 2.07	1.64 1.99	1.57 1.87	1.52 1.79	1.46 1.69	1.40 1.59	1.35 1.52	1.28 1.41	1.24 1.36	1.17 1.25	1.11 1.15	1.00 1.00