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Unbalanced Workload Allocation in Large Assembly Lines.

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by

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Abstract

In modern production systems that perform under high cost environments, even small improvements in line efficiency represents large savings over the lifetime of an assembly line. In the beginning of modern production systems, it was thought that a ‘perfectly balanced’ line was the most efficient way to design the line. However in practice, the ideal perfectly balanced line seldom occurs, because some degree of imbalance is inevitable.

Recent studies have found that unbalanced lines with a bowl shape workload configuration can yield performance in throughput as good as, or even better than those of a perfectly balanced line. This thesis studied the “bowl phenomenon” in large unpaced assembly lines under stochastic processing times. The control variables analyzed in this study were line length, buffer capacity, task time variability, and percentage of imbalance. A full factorial experiment was designed in order to characterize the main and interaction effects, and computational simulation was used to replicate the behavior of the unbalanced assembly lines. The results of the experiment suggest that unbalancing a large assembly line in a bowl shape workload configuration could provide statistical significant improvements in throughput. Moreover, the results also suggest that the Work in Process (WIP) and the Cycle Time (CT) increase linearly as the Throughput (TR) of the line increases. Even though, the rate at which the TR increases is greater than the rate at which the WIP and CT increases, line designers and production managers need to make an important managerial decision on how much they are willing to increase the WIP and CT of their lines in order to improve the throughput when implementing a bowl shape workload configuration. Furthermore, the results suggested that as the buffer capacity and the number of workstations in the line decreases, and the coefficient of variation of the workstations

increases the benefits the bowl phenomenon and the percentage of imbalance of the “best bowl configuration” increases.

In this research, the relationship between the production rate of large assembly lines with a bowl shape workload configuration and its line length, buffer capacity, task time variability, and percentage of imbalance has been studied for the first time. The results would provide valuable guidelines for line designers and managers that want to improve their assembly lines.

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1. Introduction

Assembly lines are key components of modern production systems. The first real example of an assembly line is attributed to Henry Ford, with the assembly line of the Ford Model T in 1913. In the beginning of modern production systems, it was thought that a ‘perfectly balanced’ line (equal workload along all workstations) was the most efficient way to design the line. This stimulated a great amount of research in heuristic and near optimal algorithms for the Assembly Line Balancing Problem (ALBP) that aimed for the perfectly balanced workload allocation.

The design and planning of production systems is a vital task for line designers and production managers. In modern production systems that perform under high cost environments, even small improvements in line efficiency represents large savings over the lifetime of an assembly line. In practice, the ideal perfectly balanced line seldom occurs, because some degree of imbalance is inevitable. A perfectly balanced line might not be possible due to some technological and/or organizational constraints, task variability or due to the performance rate of workers.

Recent studies have found that unbalanced lines, in which workstations have different workloads, can yield performance as good as, or even better than those of a perfectly balanced line. The “bowl configuration” of workload, in which greater workload is allocated towards the ends of the lines and decreasingly less in a symmetric pattern toward the center, have been shown to improve the production rate of assembly lines. Since the discovery of the “bowl phenomenon” numerous studies have been done to understand its benefits. Even though assembly lines consist of hundreds or even of thousands of tasks, and a large number of workstations, many research efforts done on the bowl phenomenon have not experimented with larger assembly lines due to computational limitations.

This thesis aims to study the “bowl phenomenon” in large unpaced assembly lines under stochastic processing times. This will improve the understanding of the relationship between the production rate of assembly lines with a bowl shape workload configuration and its line length, buffer capacity, and task time variability. Furthermore, it will provide valuable guidelines for line designers and managers to improve their production systems. Section 2 provides a background; section 3 is a detailed review of existing literature of balanced and unbalanced assembly lines, and outlines the research questions. In section 4 the methodology used in this study is presented. Furthermore, in section 5 the results of the experiments are provided, and in section 6 the conclusions draw from the analysis of the results and futures works are presented.

2. Background

2.1. Simple Assembly Line Balancing Problem

The problem of planning for the allocation of work elements into assembly lines has been the subject of interest for a long time. According to Baybars (1986) the first analytical statement of the Assembly Line Balancing Problem (ALBP) in a mathematical form was published by Salveson (1955).

Assembly lines can be defined as a finite set of workstations arranged along material handling equipment. Workpieces are successively passed down the assembly line and moved from one workstation to the next. To produce any product on an assembly line it is required to divide the total amount of work into a finite set of elementary tasks. Performing a particular operation requires a task time, and certain equipment and/or skilled workers. The total workload necessary to assemble a workpiece is calculated by the sum of all the task times. Due to some technological and/or organizational conditions, the precedence constraints between tasks need to be taken into consideration at the moment of assigning the elementary tasks to the workstations on the line.

These constraints can be visualized in a precedence graph. A precedence graph contains a node for each task, a node weights for the task times, an arcs for the direct precedence constraints (see Table 1), and a paths for the indirect precedence constraints. For example, in Figure 1 task# 6 and #7 needs to be completed after starting task #8. This precedence graph contains a set of 10 tasks, with task times ranging between 1 and 5 time units.

Table 1: Task Information

Task	Task Time	Precedence
1	3	-
2	1	1
3	2	1
4	3	2
5	1	3,4
6	2	5
7	4	3,5
8	3	6,7
9	2	8
10	5	9

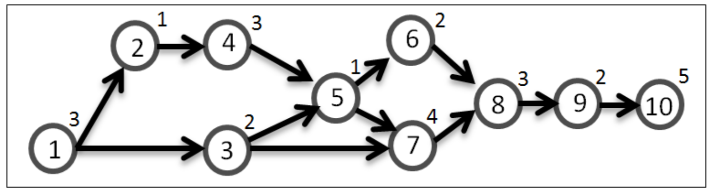


Figure 1.Precedence Graph

The fundamental objective of ALBP is the assignment of tasks to an ordered sequence of workstations, such that the precedence relations and other constraints are not violated and some measure of effectiveness is optimized. Most of the research done in assembly lines has focused on solving the Simple Assembly Line Balancing Problem (SALBP) (Scholl and Becker, 2006). This type of the ALBP is based on the following assumption (Baybars, 1986):

- *The line is designed for a unique model of a single product*
- *Deterministic task times*
- *A task cannot be split among two or more stations*
- *There are no assignments restrictions beside the precedence constraints.*
- *All tasks must be processed*
- *All workstations are equally equipped*
- *The line has a serial layout with N one-sided workstations.*

All these assumptions reduce the complexity of the problem. However, the balancing of real assembly lines requires the consideration of additional technical and/or organizational constraints, which increases the complexity of the problem. There exist four different versions of the SALBP, depending on the objective to optimize, as shown in Table 2.

Table 2: Versions of SALBP
(Scholl and Becker, 2006)

	Cycle Time	
	Given	**Minimize**
Number of workstations		
Given	SALBP-F	SALBP-2
Minimize	SALBP-1	SALBP-E

SALBP-F is a feasibility problem that is objective is to establish whether or not a feasible line balance exists for a given combination of workstations and cycle time. SALBP-1 aims to minimize the number of workstations given a fixed cycle time. SALBP-2 aims to minimize the cycle time given a set of workstations. SALBP-E is the most general version of the problem; it aims to maximize the line efficiency by simultaneously minimizing cycle time and the number of workstations.

2.2. Generalized Assembly Line Balancing Problem

Balancing of real assembly lines requires considering additional technical and/or organizational constraints, in contrast with the SALBP. Any ALBP that does not follow all the assumption of the SALBP are considered Generalized Assembly Line Balancing Problems (GALBP) (Baybars, 1986). One of the main assumptions of the SALBP is that all processing times are known with certainty (deterministic task times). In assembly lines with highly automated workstations, where tasks time variance is sufficiently small, the tasks time might be considered to be deterministic. However, when human operations are involved, the variance of the processing times increases. This is generally attributed to the variability of humans with respect to work rates, skill, and motivation levels (Becker and Scholl, 2006).

The stochastic version of the GALBP introduces the concept of task time variability. When processing times are considered to be stochastic many other issues arise, in comparison with

SALBP. Under stochastic processing times, workstations could finish their work in different periods. To accommodate for this variability buffer spaces can be allocated between workstations. Furthermore, the launch rate of the workstations could be controlled to either be paced or unpaced.

In paced assembly lines systems a common cycle time limits the processing times of all the workstations. This is achieved either by continuous or intermittent conveyor belts, which force the operators to finish the tasks before the workpiece reaches the end of the workstation. If continuous material handling equipment is used to pace the line, the workstations length needs to be defined taking in consideration the workload configuration of the line.

In contrast, unpaced lines are not limited to a given time span to transfer the workpieces. Therefore, production rates are no longer given by a fixed cycle time. Some authors classify unpaced lines in asynchronous and synchronous unpaced lines. In asynchronous unpaced lines the workpieces are always moved whenever the required operations are completed, and if the following workstation is not blocked by another workpiece. After transferring the workpiece the workstation continues to work, unless the preceding workstation is unable to deliver a new workpiece. When this happens the workstation waiting for the new workpiece is considered to be starved. In order to minimize the “blocking” and “starving” of workstations buffers capacity can be implemented. In synchronous unpaced lines all workstations wait until the slowest one finishes its work before the workpieces are transferred. In contrast to the asynchronous unpaced lines, buffers between workstations are not required. Under deterministic processing time a synchronous unpaced line works as an intermittent paced line, with the cycle time determined by the slowest workstations. However, synchronous unpaced lines can transfer the workpieces if the tasks were performed unexpectedly fast, so they do not need to wait for a fixed time span.

2.3. Definitions

In this section the general concepts used in this study are defined using the nomenclatures of Table 3.

Table 3: Nomenclatures

N	Number of workstations in the line
B	Buffer capacity
B_n	Buffer capacity of workstation *n*
Op_n	Operator in the workstation *n*
RM	Raw material
FG	Finished goods
DI	Degree of imbalance
MAD	Mean absolute deviation of workload
t_n	Processing time of workstation *n*
T	Line total operating time
w_n	Workload of workstation *n*
TR	Throughput

c	Cycle time
E	Line efficiency
WR	Workload range
Wmax	Maximum workload
Wmin	Minimum workload
V	Workload Variance
SI	Smoothness index
FI	Flow index
TT	Takt time
IT	Idle Time
ABL	Average Buffer Levels
D	Task Time Distribution

Workstation and Workload:

Assembly lines can be defined as a finite set of workstations arranged along a material handling equipment. The workstation workload is the sum of all the tasks times allocated to that workstation (See Figure 2).

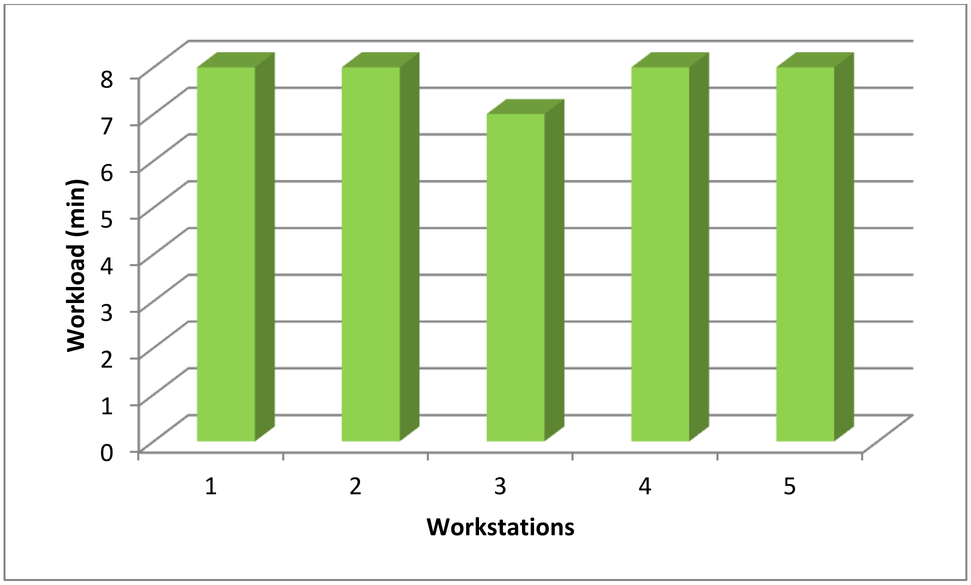


Figure 2.Workload distribution

Buffers:

Buffers are physical locations used to temporarily store work in process (WIP) in the assembly line. For an assembly line with a set of N workstations, there will be a total of N - 1 buffers, represented as: B_1, B_2, B_3, …, B_{n-1}. The buffer after workstation 1 is referred to as B_1, the buffer after the 2nd workstation as B_2, and so on until the last buffer. The last buffer is referred to as B_{n-1}. For example, Figure 3 shows a line with 3 workstations, 3 operators, and two buffer spaces (B_1 and B_2).

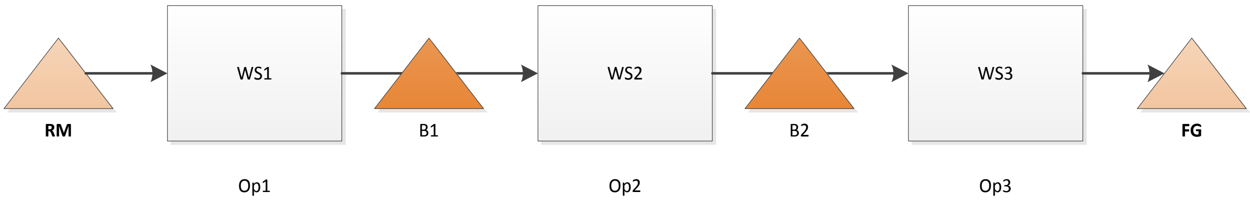


Figure 3. Example of traditional assembly line with buffer spaces.

Degree of Imbalance:

The degree of imbalance (DI) of a line configuration is the percentage of workload imbalance. It is measured by the mean absolute deviation of workload (MAD). (*N* is number of workstations in the line, *T* is the line total operation time, and w_n is the workload of workstation *n*).

$$MAD = \frac{\sum_{n=1}^{N} \left| w_n - {}^{T}/_{N} \right|}{N}$$

Percentage of Imbalance (x)

The Percentage of Imbalance is the percentage difference in mean processing time between the inner workstations of a two level bowl configuration and its balanced counterparts. (See example in Figures 4-5) $x = \frac{{}^{T}/_{N} - w_{n-1}}{{}^{T}/_{N}}$

Workstations	*Mean processing time*
1	1.04
2	0.99
3	0.99
4	0.99
5	0.99
6	0.99
7	0.99
8	0.99
9	0.99
10	1.04
x	0.01

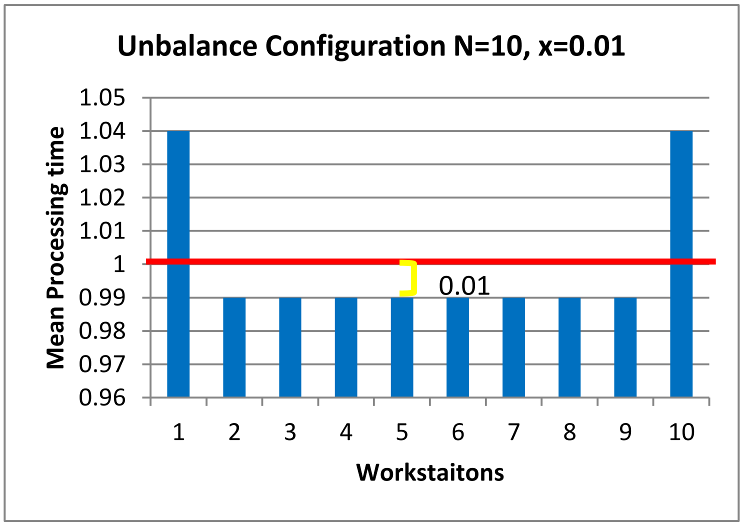


Figure 4. Example one of Percentage of Imbalance

Workstations	Mean Processing time
1	1.065
2	0.99
3	0.99
4	0.99
5	0.99
6	0.99
7	0.99
8	0.99
9	0.99
10	0.99
11	0.99
12	0.99
13	0.99
14	0.99
15	1.065
x	0.01

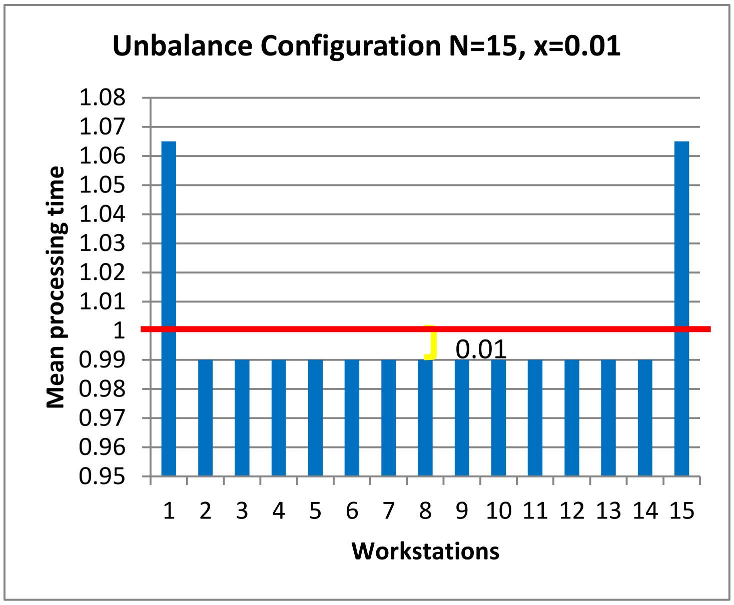


Figure 5. Example two of Percentage of Imbalance

Throughput:

Throughput (TR) is defined as the average output of a production process (machine, workstation, line, plant) per unit time. Therefore, in an assembly line throughput is the average quantity of nondefective parts produced in the line per unit time. The importance of using throughput as a measurement of a production system is that it is the most frequently used measure by engineers and managers when designing and operating a line.

3. Literature Review

3.1. Assembly Line Balancing

The fundamental ALBP seeks to assign tasks to an ordered sequence of workstations, such that the precedence relations are not violated and some measure of effectiveness is optimized. The measures of effectiveness used in ALBP can be divided into two categories: economic and technical measures (Ghosh & Gagnon, 1989).

The use of economic measures could be encouraged by the impact in the profitability and operational cost that the design and planning of assembly lines might have on an organization. Many authors have implemented cost-oriented models to solve ALBP. Chakravarty (1985) and Silverman (1986) implemented heuristics that focused on minimizing the total cost of the line, while Askin (1997) implemented a heuristic that focused on minimizing the operational and equipment cost of the line. Rosenberg (1992) made the assumption that the operation of a workstation causes a wage rate that was directly related to the maximum wage rate of all tasks that are assigned to that workstation. The objective was to minimize the aggregate wage rate over all workstations, which was equivalent to minimizing the number of workstations in the case that all tasks have the same wage rates. Amen (2000) extends Rosenberg's heuristic by adding a cost of capital, which could be explained as the initial investment cost for the workstations.

Since ALBPs have a long to mid-term planning horizon, the criteria used to measure the effectiveness of the lines need to be carefully selected considering the strategic goals of the organization. From an economic point of view profitability and cost measures are preferable. However, measuring and predicting the cost of running a line over months or years, and the

profits achieved by selling the products assembled is moderately complicated and error prone. These might be the reasons why technological measures are more popular in the ALBP (Becker & Scholl, 2006)

The technological measures commonly used are related to the throughput and/or operational efficiency of the assembly lines. Many authors have used the line efficiency as an indicator for operational efficiency (Scholl et al., 2006). McMullen (1997), Macaskill, (1972) and Gokcen et al. (1999) used line efficiency in their heuristic for solving the mixed-model assembly line balancing problem. Line efficiency (E), consists in maximizing the line utilization which is measured as the productive fraction of the line total operating time (T) over the cycle time (*c)* and the number of workstations (*N)*. The maximization of the line efficiency is a measure that directly addresses the minimization of workstations and the idle time of the line.

$$E = {}^{T}/_{(N.c)}$$

The primary objective of the line designer should be to minimize the number of workstations, and as a secondary objective to distribute the amount of workload as evenly as possible among the workstations (Talbot, 1991). Although, Sparling (1998), Miltenburg (1998), Sabuncuoglu (2000), and Kara (2007) had the minimization of workstation as the primary objective, the minimization of workload differences among the workstations was a secondary objective.

There are many criteria which can be used to optimize the distribution of workload among the workstations on a line. The workload range (WR) measures the extreme values of the workloads without regards to their distribution among the workstations. The workload range is the

difference between the maximum (Wmax) and the minimum workload (Wmin) of the workstations on an assembly line.

$$WR = Wmax - Wmin$$

The workload variance (V) penalizes deviations from the mean workload quadratically (*N* is the total number of workstations, *T* is the line total operating time, and w_n is the workload of workstation *n*).

$$V = \frac{\sum_{j=1}^{N}(w_j - {}^{T}/_{N})^2}{N}$$

Moreover, the mean absolute deviation of workload (MAD*)* penalizes deviations from the mean workload linearly.

$$MAD = \frac{\sum_{n=1}^{N}|w_n - {}^{T}/_{N}|)}{N}$$

As Talbot (1991) explained, workload variance and mean absolute deviation of workload are very similar, and from a practical perspective there may be no reason to choose one criteria over the other. However, an important consideration is to select a criterion that allows constructing tractable linear measurements to compare different line balance designs. This linear tractable property is the reason why Talbot (1991) used the MAD as the criterion to measure the workload distribution in his assembly line balancing algorithm.

Baybars (1986) suggested that ALBP could be improved by adding a secondary objective which consists of smoothing the overall workstations workloads. The smoothness index (abbreviated SI or SX in some cases), describe the relative workload smoothness of a given assembly line. The

smoothness index is the root square of all the square differences between the cycle time (c) and the workstations workload (w_n).

$$SI = \sqrt{\sum_{n=1}^{N}(c - w_n)^2}$$

The primary objective of the assembly line designers usually is the minimization of workstations or the maximization of line efficiency, while workload balance is a secondary objective. The main reason for this is that as the number of workstations in a line increases the overall cost also increases. Moreover, to achieve the maximum potential of an assembly line its efficiency should be 100%. However, in the case that 100% efficiency is not possible (due to some technological and/or organizational constraints), it was thought that the flow of the line, the output rate, the lead time, and work in progress (WIP) were optimized by reducing the workload differences among workstations.

Kathiresan et al. (2012) presented a method that assigned work elements to workstations with a criterion to meet takt time and achieved workload balance among workstations. They introduce a new line efficiency criterion called flow index (*FI*), which penalizes deviation of workstation workload (w_n) from the takt time of the line (TT). By minimizing the flow index the workload smoothness among workstations with respect to takt time is reduced. They defined the flow index as the root mean square of deviation of workstations workload and takt time.

$$FI = \sqrt{\frac{\sum_{n=1}^{N}(TT - w_n/_{TT})^2}{N}}$$

The value of the flow index varies from 0 to 1. A flow index of 'zero' indicates a perfectly balanced line, and a value of 'one' indicates the greatest possible difference between workstations workload and the takt time of the line (extreme condition) (Figure 6). Therefore, smaller values of flow index results in smooth workload distribution among the workstations with respect to takt time.

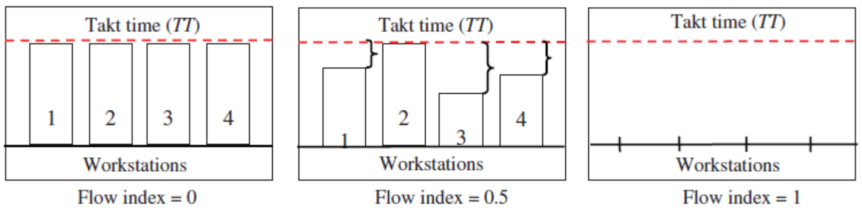


Figure 6. Mechanics of the Flow Index
(Kathiresan, Jayasudhan, Prasad, and Mohanram, 2012)

3.2. Unbalanced Assembly Lines

Boysen et al. (2008) showed that the ALBP has been an important field of research since its first analytical statement was published in 1955. All this research has built a significant body of literature, which covers a lot of different aspects of a production system. However, they were able to recognized only 15 articles which explicitly deal with ALBP of real world. In contrast to the 312 different research publications treated in the latest literatures review of ALBP analyzed in this survey(Scholl & Becker, 2006; Becker & Scholl, 2006; Boysen, Fliedner, & Scholl, 2006). Those 15 articles represent less than 5% of the body of literature studied, which as the authors highlighted is an indication of the noteworthy gap that exist between the current status of research and the requirements of real world problems. Templemeir (2003) indicated that assembly lines are composed of workstations with different mean processing times. Therefore, a

great number of ALBP algorithms, which made the assumption of equal processing time over all the workstations, are not appropriate for real world systems.

In real assembly lines task variability is present due to human labor, production mix and/or machines breakdowns. In these lines different issues arise that are not considered in many ALBP algorithms that make the assumption of deterministic task times. As stated by Ghosh and Gagnon(1989) when task variability is present new problems arise, such as the workstations time exceeding the cycle time, the production of unfinished parts, the pacing effects on worker's processing times, the size and location of inventory buffers, launch rates, and allocation of the workload along the line. So, under stochastic environments the line designers need to answer questions regarding what cycle time to choose, how much and where buffers inventory should be allocated or if planned imbalance should be considered into the system.

Planned imbalance means that workload of all workstations of the assembly line are intentionally designed to be unbalanced and not necessarily equal to each other. According to Carnall and Wild (1976) in real production systems a perfectly balance line may be impossible because:

1 .In most cases equal allocation of workload to workstations may be prevented by precedence and/or technological constraints

2. The variability of the processing times at individual workstations may differ as a result of differences in the nature of the tasks.

3. Individual workers may have different mean work performance rates.

Previous studies in unbalanced lines have concluded that, under real conditions, perfectly balanced lines rarely perform better than their unbalanced equivalents. McNamara (2011)

showed statistically significant improvement on throughput of nearly 3%, on idle time of 32%, and on average buffer levels of 90% by deliberately unbalancing the workload, buffer capacity and variability of the workstations in the line. In modern production systems, that perform under high cost environments, these improvements still represent a large saving over the lifetime of an assembly line (Das et al. 2010). Thus, as stated by Hillier and So (1996) line designers should concentrate more efforts on unbalancing the line in an optimal or near optimal configuration, given that perfectly balanced lines are difficult to achieve and are 'riskier' targets**.**

3.3. The Bowl Phenomenon

The experimental results of Hillier and Boling (1966) were the first to highlight the benefits of unbalancing the mean processing time in a bowl shape configuration , thus discovering the existence of the 'bowl phenomenon'. In this work a queuing model for lines length of up to 4 workstations (N=4) with exponential task time distributions was implemented to study the ALBP in unpaced asynchronous lines with variable processing times. It was shown that the output rate can be improved, compare to a balanced line, by deliberately unbalancing the line by allocating higher and equal workload to the first and last workstations and lower workload to the middle workstations.

Subsequent research done by Rao (1976), Carnall and Wild (1976), and De la Wyche et al. (1977) also demonstrated the benefits and existence of the bowl phenomenon. Rao (1976) experimented with 3 workstation assembly lines with different combinations of task time distributions (exponential, uniform and deterministic). It was shown that in a three station system the improvements in the throughput can be as large as 6.79%.In this study, it was demonstrated that optimum unbalance could be accomplished by allocating some workload from the interior

workstations to the exterior ones (first and last workstation); preferably when the coefficient of variation (CV) of the workstations is less than 0.5. Alternately, when the coefficient of variation (CV) of the workstations is greater than 0.5 (CV>0.5) allocating the workload of the more variable workstations to the less variable ones would provide a better configuration.

Carnall and Wild (1976) experimented with 4 and 10 workstations assembly lines, buffer capacity of 1, 2 and 3 units , under a positively skewed Weibull task time distribution and Coefficient of Variation of 0.1,0.21 and 0.5. In this study, by implementing a bowl shape configuration it was possible to produce improvements in throughput up to 4% over the balanced lines. These results confirmed the existence of the 'bowl phenomenon' and extended it to the case of changing workstations variance rather than workload. It was also discovered that increasing buffer capacity or reducing the CV of the workstations reduces the improvement of unbalancing the line in a bowl shape configuration.

Hillier and Boling (1979) established general guidelines for the bowl configurations. These guidelines were:

-The optimal bowl allocation should be symmetric.

-The optimal bowl allocation should be relatively flat in the middle and very steep towards the end of the line.

-The degree of imbalance should decrease with the inter station buffer storage capacity.

-The degree of imbalance should increase with the length of the line.

-The degree of imbalance should increase with the CV of the processing time.

Based on these general guidelines the line design may consist of a two-level bowl or a multi-level bowl configuration. A two-level bowls configuration consists of equal workload at the first and last workstation of the line, with equal but smaller workload at all workstation in between. A multi-level bowl configuration typically consists of greater workload at the first and Nth workstation (Level 1), equal but smaller workload for the 2nd and (N-1)th workstations (Level 2), and successively smaller paired workload at the remaining workstations (Level 3). For example, Figure 7 shows a two-level bowl configuration, in which workstations 1 and 6 are the Level 1, and the remaining workstations (2-5) are the Level 2. Figure 8 shows a multi-level bowl configuration, in which workstations 1 and 6 are the Level 1, workstations 2 and 5 are Level 2, and workstations 3 and 4 are Level 3.

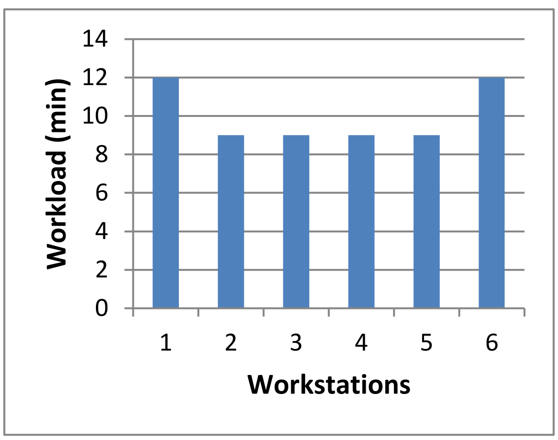


Figure 7. Two-level bowl configuration

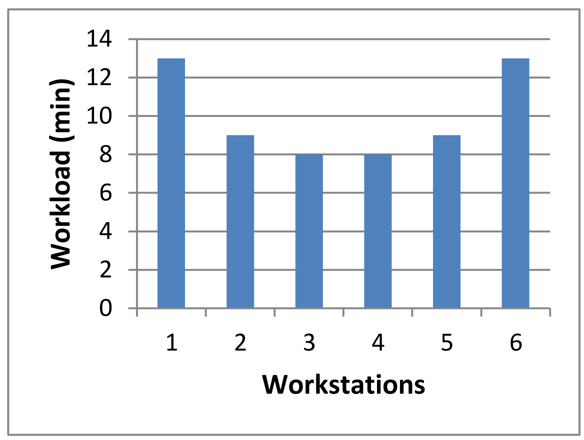


Figure 8. Multi-level bowl configuration

Hillier and Boiling (1979) provided the first extrapolation model for near optimal bowl configurations, and demonstrated that as the number of workstations in the line increase from 2 to 6, under 0 buffer capacity and CV of 1, the degree of imbalance in the optimal bowl configuration tends to stay the same. Moreover, the improvements in output rate become greater as the line length increases. Also, when task time distributions are highly variable (Erlang and Exponential distribution with CV>1) the degree of imbalance in the optimal bowl configuration

substantially decreases as the buffer capacity increases, supporting previous studies of Hatcher (1969), Quarles (1967), Sheskin (1976), Smith and Brumbaugh (1977) and Carnall and Wild (1976) about the effect of inventory buffers in assembly line output rate.

El-Rayah (1979) presented the first study to use simulation to confirm the existence of the bowl phenomenon. In this study, different unbalance configurations were simulated in assembly lines with up to 12 workstations under Normal, Lognormal and Exponential task time distributions and CVs of 0.3 and 1. It was demonstrated that the bowl configuration was the only one to consistently improve the output rate, compared to the balanced line and the other unbalanced configurations tested (ascending, descending, and “low-high-low-high”). But, more important these results demonstrated that unbalancing a line in the wrong configuration might produce negative outcomes.

An interesting experiment that used simulation and analytical models to study the unbalanced stochastic assembly lines was presented by Smunt and Perkins (1985). It suggested that balanced lines are as good as or better than unbalanced lines, when processing times are modeled under more realistic values of task time variance. In this study, assembly lines with 3 and 4 workstations under Normal task time distribution and CV of 0.2, 0.5 and 1 were simulated. It was concluded that unbalancing the lines should only be considered when task time distribution has great variance. Furthermore, that more extensive experiment research with various normal task time distribution and different workstations lengths should be conducted.

The conclusions of Smunt and Perking (1985) regarding the bowl phenomenon motivated that Karwan and Philipoom (1989) published an article that highlighted the flaws of the previous study. It was highlighted that Smunt and Perking (1985) t-test was not powerful enough due to

the small sample size. It was also stated that Smunt and Perking (1985) didn't use Hillier and Boling (1979) optimal bowl configurations. Finally, that Dudley's (1963) and Knott and Sury's (1987) research clearly indicated that the frequency distribution of task times for experienced workers on unpaced lines is positively skewed. Dudley mentioned that times for trainees or when various interventions are designed to pace workers a Normal distribution is perhaps appropriate.

Muth and Alkaff (1987) demonstrated a method that analyzed serial production lines and computed the output rate of assembly lines with unbalanced workstations. In this work, the authors highlighted that an important characteristic of the Hillier and Boling (1979) study was the use of fixed CV. Therefore, in the Hiller and Boling (1979) study it was not possible to select the variance independently of the mean processing times of the tasks. This raised the question of what really caused the bowl phenomenon, the change in service time variance or the changes in mean processing time of workstations. Using an innovative method based on random distributions, Muth and Alkaff (1987) demonstrated that carefully selected bowl configurations do indeed provide some improvements over the balanced lines, when the sum of the total mean processing times and variance are conserved.

So (1989) simulated 3, 4 and 8 workstation assembly lines with buffer capacity of 1,3 and 5 under Exponential task time distribution with CV=1, and Normal task time distribution with CV= 0.2, 0.46 and 0.62. Based on the general guidelines of Hillier and Boling (1979), 4 different bowl configurations were simulated. It was concluded that very small improvements (0.3% in average) in the efficiency of an assembly line with finite buffer could be achieved if the line is unbalanced properly. Therefore, in contrast with Smunt and Perking (1989) results, the authors concluded that improvements in line efficiency could be achieved even when task time variability is small.

Hillier and So (1993) improved the extrapolation procedure of Hillier and Boling (1979) by implementing a new related measure of imbalance. In this study, assembly lines with 3 up to 9 workstations, buffer capacity of 0 up to 5 under Exponential and Erlang task time distributions with CV of 0.25 up to 0.707, were simulated. The authors stated that this study was limited to experiments with small assembly lines (N<9) due to computational requirements. However, it was indicated that many real assembly line systems have a larger number of workstations, hence the importance of extrapolating the "optimal bowl configuration" for larger assembly lines. The study confirmed that the percentage of improvement increases as the number of workstations in the line increases. For example, an assembly line with zero buffer capacity, Exponential task time distribution and 7 workstations shows an improvement of 1.48%, while a line under the same conditions but with 9 workstations shows an improvement of 1.59%.

Pike and Martin (1994) provided an extensive simulation and are the only ones to have studied the bowl phenomenon in assembly line with up to 30 workstations. In this study, assembly lines with 3-12, 15, 20, 25 and 30 workstations, buffer capacity 0-4 units under Normal and positively skewed task time distributions with CV of 0.2, 0.25 and 0.30 were simulated. Different bowl configurations were systematically tested with 0.001increments in the mean processing time of the workstations until it performed more efficiently than the balanced line, according to paired t-test at a 99.95% confidence level. The configuration with the statistically smaller mean output interval was selected as the "optimal bowl configuration". In this study, it was also shown that the maximum degree of imbalance that would still yield a mean output interval that perform statistically no worse than the balanced line. The authors were able to demonstrate that the bowl phenomenon also exists for large assembly lines, with small CV (CV=0.2) values, and relatively large buffer capacity (B=3).

In this study, the effect of the line length over the bowl phenomenon was demonstrated. The percentage values of improvement in mean output interval resulting from the use of optimal bowl configuration, in assembly lines with Normal task time distribution and CV= 0.25, are shown in Table 4 (N is the number of workstations in the line, B is the buffer capacity) and in Figure 9.

Table 4: Percentage decrease in mean output interval (Pike & Martin 1994)

	N													
B	3	4	5	6	7	8	9	10	11	12	15	20	25	30
0	0.15	0.29	0.396	0.456	0.468	0.513	0.526	0.48	0.53	0.513	0.495	0.469	0.375	0.367
1	0.066	0.112	0.149	0.121	0.167	0.166	0.157	0.156	0.147	0.119	0.138	0.128	0.119	0.119
2	X	0.058	0.077	0.096	0.067	0.095	0.114	0.067	0.067	0.086	0.085	0.05	0.085	X
3		X	X	X	X	0.097	0.087	X	0.068	0.106	X	X	X	
4						X	X		X	X				

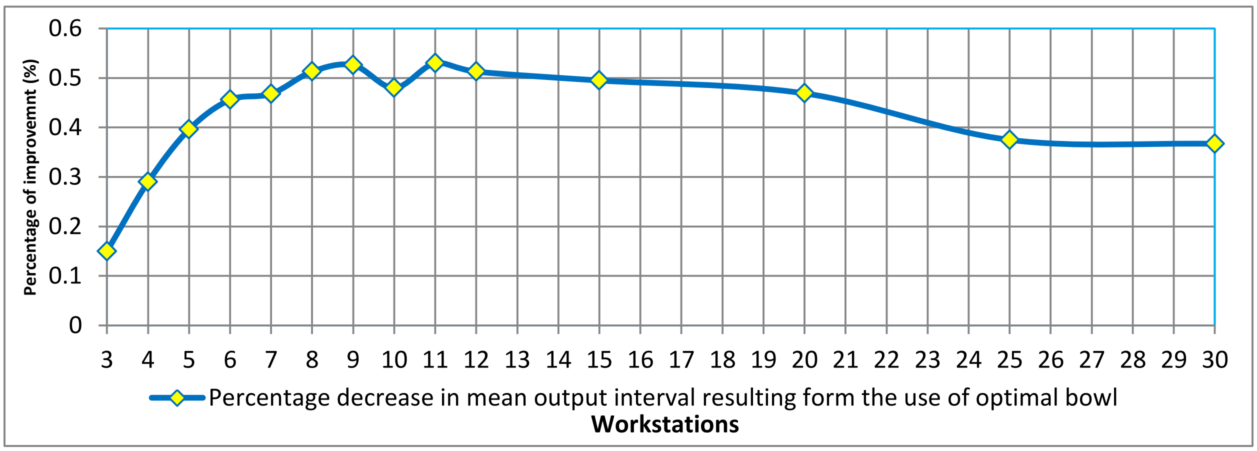


Figure 9. Percentage decrease in mean output interval (0 buffer capacity, CV of 0.25 and normal task time distribution) (Pike & Martin 1994)

In Figure 9 it can be observed that the percentages of improvement of the bowl phenomenon rapidly increases as the number of the workstations reaches to 12. This supports the conclusion of Hillier and Boling (1979), that the improvement in production rate becomes greater as the line length increases. However, for lines larger than 12 workstations it showed a tendency to reduce the percentage of improvements.

Many studies have proven the existence and potential benefits of the bowl phenomenon. However, the benefits that could be achieved by deliberately unbalancing a line with a bowl configuration depend on correctly identifying the line parameters in order to estimate the best bowl shape configuration. Regardless of the proven improvements that the bowl phenomenon provides, perfectly balanced lines remain the norm of the industry (Hillier & So, 1996). A frequently stated reason for this is the uncertainty about the robustness of the bowl phenomenon (Smunt & Perkins, 1989). Hillier and So (1996) studied the robustness of the bowl phenomenon by experimenting with the effects that a poor estimation in the amount of imbalance of the bowl configurations would have over the throughput of the line. In this study, experimental results demonstrated that the bowl phenomenon is relatively robust, because even larger error (50%) in the degree of imbalance of the "optimal bowl configuration" would still provide most of the potential improvement in output rate. Furthermore, the output rate still exceeds the one of a balanced line in most cases even when the workload configuration deviates from the optimal bowl by 10%. It was concluded that unbalanced lines provided a more robust target than the perfectly balanced lines, which are 'riskier' targets.

Hillier et al. (2006) studied both workload and buffer bowl configurations. In this study a cost oriented model, which takes into account the revenue per unit of output and the cost per unit of buffer space, was implemented. Assembly lines with 4 and 5 workstations, under Exponential and Erlang task time distributions with relatively large variance were simulated. The results showed that both of the buffer and workload bowl configuration were very similar. It was concluded that this same pattern would hold for larger lines. The improvement achieved by just optimizing the workload allocation in a bowl configuration and balanced buffer allocation was on average 0.3208%.

Shaaban and McNamara (2009) did an extensive simulation and statistical analysis to study unbalanced workload allocation in non-automated production lines. In this study, assembly lines with 5 and 8 workstations, buffer capacity of 1, 2 and 6 under Weibull task time distribution with CV of 0.274 were simulated. Four different patterns of imbalance (ascending, descending, bowl and inverted bowl) with 2%, 5%, 12% and 18% degree of imbalance were tested. The result showed that improvements in Idle Time (IT) and Average Buffer Levels (ABL) can be achieved by unbalancing the workload of the workstations. The best configuration that resulted in an average IT reduction of 3.46% was the bowl configuration. The monotone decreasing pattern shows improvement of 87.56% in ABL.

McNamara (2011) continued researching the effect of multiple sources of imbalance in unpaced assembly lines. In this study, assembly lines with 5, 8 and 10 workstations (10 workstation for the configuration with best results), buffer capacity of 4,8,14,24 and 42 (B=8 and 24 with N=5; B=14 and 42 with N=8, and B=4 to the best configuration), degree of imbalance of 2%,5%,12% and 18%(18% for the best configuration) and four unbalance patterns (ascending, decreasing, bowl and inverted bowl pattern) were simulated. The combination that demonstrated the greatest improvements in throughput was the combination of an inverted bowl of mean processing time, bowl configuration for the CV and a descending buffer capacity configuration. The best combination that reduced the idle time was the inverted bowl of mean processing time, a bowl configuration of CV and a decreasing configuration of buffer. Regarding the average buffer level, the combination of descending mean processing time, a bowl configuration of CV and an ascending buffer capacity provided the best results. Concluding that it was possible to predict the best patterns of imbalance in terms of mean processing times, CV and buffer capacity based on the results obtained from the experiments of two sources imbalance. This study demonstrated

that increasing the line length and buffer capacity reduces the improvements of unbalancing the line in terms of idle time. As well as Shaaban and McNamara (2009) the authors concluded that line designer needs to decide between reducing IT or ABL since none of the resulting patterns reduced both of them at the same time.

Shaaban and Hudson (2012) studied multiple scenarios when assembly lines operate in a non-steady state condition. In this study, assembly lines with 5 and 8 workstations, buffer capacity of 2 and 6, under Weibull task time distribution with CV of 0.08 up to 0.5 were simulated. The experimental result showed that for only one source of imbalance the pattern of bowl configuration of mean processing time, bowl configuration of CV and unequal buffer capacity offered the best improvements in idle time. Regarding the average buffer levels the descending patter of mean processing time, the bowl configuration of CV, and concentrating the buffer capacity at the end of the line achieved better results. When two sources of imbalance were simulated the best patters for idle time was the combination of an inverted bowl of mean processing time and the "unequal patter" for the CV. In term of average buffer levels, the descending order of mean processing time with a bowl configuration of CV resulted in the best solution. More important, it was concluded that the best unbalanced configuration under non steady state conditions, in term of idle time and average buffer level, were not so different from the results of previous studies done in steady-state conditions.

The robustness and efficiency of unbalancing the workload in assembly lines with a bowl shape configuration have been studied to a great extent. More recent works had started to investigate the benefits of unbalancing not just the workload, but also the interaction of unbalancing inventory buffer levels and the CV. Furthermore, many different scenarios have been simulated under a wide variety of buffer capacity (0 up to 42), task time distributions (Exponential, Erlang,

Normal, Weibull and Uniform) and coefficients of variation (0.1 up to 3). One reason for these wide variety of scenarios simulated might be because the optimal bowl configuration is very dependent upon the line length, the buffer capacity and the coefficient of variation (Smunt & Perkins, 1985). It has been proven that the line length has a significant impact on the benefits of the bowl phenomenon. Hillier and Boling (1979) stated that the percentage of improvement of unbalancing the line in a bowl configuration, compared to a perfectly balance line, increases as the number of workstations in the line increases.

3.4. Literature Gap

The benefits of the bowl phenomenon have only been studied in assembly lines with up to 30 workstations. Pike and Martin (1994) suggested that it is possible that the bowl phenomenon also exists for assembly lines with more than 30 workstations. Although extrapolation guidelines that calculate the near-optimal bowl configuration for assembly lines exist, they are limited to configuration for lines with up to 9 workstations, buffer capacity of 0 up to 5, and CV from 0.25 to 0.707 (Hillier and So, 1993).

Even though assembly lines consist of thousands of tasks, (Klindworth, Otto, & Scholl, 2012) and a large number of workstations, many research efforts done on the bowl phenomenon have not experimented with larger assembly lines due to computational limitations (Hillier & So, 1993). Hillier and Boling (1979) and Hillier and So (1993) simulated assembly lines with up to 6 and 9 workstation respectively. In these studies, it was demonstrated that the percentage of improvement of unbalancing the line in a bowl configuration increase as the number of workstations in the line increases. However, Pike and Martin (1994) demonstrated that in

assembly lines with more than 12 workstations the improvements of the bowl phenomenon tends to gradually decrease as the number of workstation increases.

Very limited work exists in the body of literature of the bowl phenomenon that demonstrates the impact of the line length over the bowl phenomenon in large assembly lines. Hence, a literature gap was identified in the area of the bowl phenomenon in large unpaced assembly lines under stochastic processing times.

3.5 Research Questions

In real production systems a perfectly balance line may be impossible because in most cases equal allocation of workload to workstations may be prevented by precedence and/or technological constraints, or the variability of the processing times at individual workstations may differ as a result of differences in the nature of the tasks. The objective of this thesis is to analyze the benefits of unbalancing the workload of large assembly lines in a bowl shape configuration and the effects that the buffer capacity, the line length, and coefficient of variation of the workstation have on the bowl phenomenon. This will improve the understanding of the relationship between the production rate of large assembly lines with bowl shape workload configurations and its line parameters. Furthermore, it will provide valuable guidelines for line designers and managers to improve their production systems and take advantage of inherent variability of their lines. The specific research questions that this thesis aims to address are:

- What is the impact of unbalancing (DI) large assembly lines in a bowl shape configuration on throughput?
- What are the effects of line length (N) in the bowl phenomenon in large assembly lines?

- What are the effects of the buffer (B) capacity in the bowl phenomenon in large assembly lines?
- What are the effects of the task variability (CV) in the bowl phenomenon in large assembly lines?
- What are the impacts of a single and multiple bowl configurations on the throughput of large assembly lines?

4. Proposed Methodology

The method of investigation most frequently used to study the benefits of planned imbalance, in complex dynamic production systems such as unpaced assembly lines, is computer simulation (see Shaaban & McNamara, 2009; McNamara, 2011; Shaaban & Hudson, 2012). Computer simulation allows gaining valuable understanding of the performance and operation characteristics of the production systems simulated. This information improves the decision making process, with regard to the selection of one condition over another. Moreover, computer simulation helps production managers and researches to understand how production systems vary over time, enabling them to understand how certain conditions impact the systems in any given moment (Kelton, Sadowski, & Swets, 2010). Even though queuing theory can be used to study production systems, computer simulation is often preferred when studying complex systems, as shown in the latest papers done on the bowl phenomenon and unbalanced assembly lines (see section 3.3).

In view of the advantages discussed above, it was decided that computer simulation was the most appropriate method to test the bowl phenomenon in large unpaced assembly lines under stochastic processing times. The experimental design aspects of this study will be discussed in the following sections.

4.1. Control and Response Variables

To improve the understanding of the relationship between the production rate of large assembly lines with a bowl shape workload configuration and its line length, buffer capacity and task time variability, a full factorial design was conducted. For the assembly lines studied in this investigation (See section 4.2 and 4.3) the independent variables were:

- Line Length (Number of workstation in the line, N)
- Buffer Capacity (B)
- Coefficient of Variation (CV)
- Percentage of Imbalance (x)

The response variables were:

- Throughput (TR)
- Work-In-Process (WIP)
- Cycle Time (CT)

4.2. Line Design for One Bowl Configurations

To better understand the impact of the line length over the bowl phenomenon in large assembly lines, the scope of this work was to simulate assembly line with 30,50 and 70 workstations. Previous studies (Carnall & Wild, 1976; Hillier & So, 1993; Pike & Martin, 1994) demonstrated that as the buffer capacity increases the benefits of the bowl phenomenon decreases. Therefore, lines with buffer capacity of 0, 1 and 2 units were simulated. Moreover, based on previous works on variation of human performance in assembly lines (Mason, Baines, Kay, & Ladbrook, 2005),

the scope of this research was to experiment with a Gamma task time distribution, and coefficients of variation of 0.2, 0.8 and 1.4.

Following the same methodology implemented by Pike and Martin (1994), simulations were completed for each possible combination of workstations (N), buffer capacity (B) and coefficient of variation. The base model was a perfectly balanced line with workstations means processing times of 1 hour. Hillier and Boling (1979) suggested that a two-level or nearly two level configuration would be the best bowl configuration. Moreover, Pike and Martin (1994) results showed that there is no statistical difference in the improvements of a two-level bowl configuration and a multi-level bowl configuration.   Therefore the scope of this research was to experiment with two-level bowl configurations.

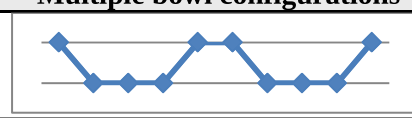
The two-level bowl configurations were tested systematically in 0.001 decrements of mean processing time until no statistical improvement on throughput was achieved, in comparison with the balanced line, according to paired t-test at a 95 % confidence level. The unbalanced configurations exhibited the appropriate conservation of variance and total processing time. The two-level bowl configurations were tested for all the possible combination of the independent variables to determine if an unbalanced allocation of workload exists that statistically outperforms the balanced line. The bowl configuration with biggest statistically significant improvement on throughput was selected as the "best one bowl configuration" for that condition of line length, buffer capacity and coefficient of variation.

4.3. Line Design for Multiple Bowl Configurations

To address the research question of whether a multiple bowl configuration could provide improvements in throughput, in comparison to a single bowl configuration and/or a perfectly balanced line; multiple-bowl configurations were tested.

The multiple-bowl configurations were tested in assembly lines with 30 and 70 workstations. The multiple-bowl configurations followed the same guidelines for the one bowl configuration presented by Hillier and Boling (1979) (See example in Table 5, H= high workload, L= low workload). Furthermore, the new unbalanced configurations exhibited the appropriate conservation of variance and total processing time.

Table 5: Example of a multi-bowl configurations for assembly lines with N=10

Workstations (N)	Multiple bowl configurations	Multiple bowl configurations
10	HLLLHHLLLH	

Simulations were performed for each possible combination of the independent variables (see Table 6). In total 8 (2*2*2) different assembly lines were simulated to test the multiple-bowl configurations. Then, each of the multiple-bowl configurations were systematically tested in .001 decrements of mean processing time until no statistical improvement on throughput was achieved, in comparison with the balanced line, according to paired t-test at a 95% confidence level. The base model was a perfectly balanced line with workstation mean processing time of 1 hour. The multi-bowl configuration with biggest statistically significant improvement on throughput was selected as the “best multi-bowl configuration” for that condition of line length, buffer capacity and task time variability.

Table 6: Independent variable values for multi bowl configurations test

Number of Workstations (N)
30
70
Buffer Capacity (B)
0
2
Coefficient of Variation (CV)
0.2
1.4

4.4. Preliminary Simulation Model

The unbalanced configurations were tested using the simulation package of Arena Version 14.5. A preliminary model was designed to simulate the behavior of an unpaced assembly line with stochastic processing times and buffer capacity between workstation. Based on previous works (Shaaban & Hudson, 2012), the following assumptions for the model were made:

- There are no machine breakdowns
- Defective items are not produced.
- Single product
- No changeover
- The time to move work units in and out of the buffers is negligible.
- Infinite supply of raw material for the first workstations.

- Infinite space for finish good after the last workstation.

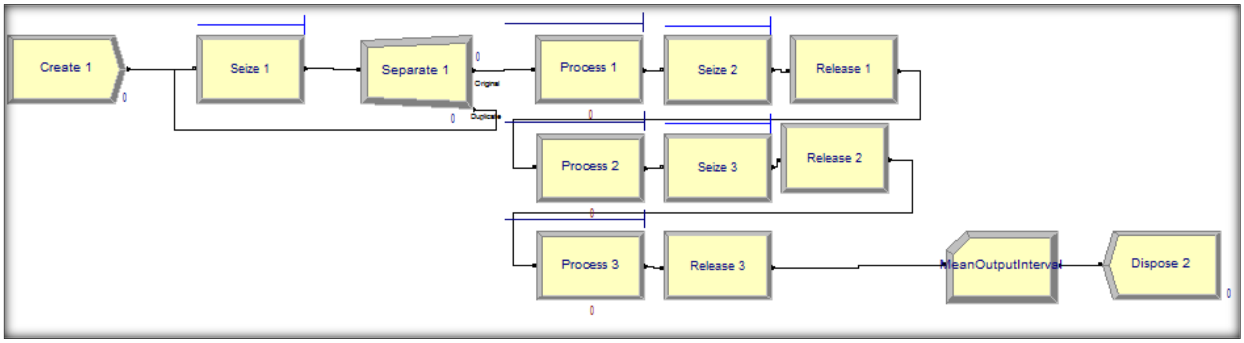


Figure 10. Preliminary Simulation Model (3 workstation line)

Figure 10 demonstrated the preliminary model used in Arena to simulate the unpaced assembly line. The module "Separate 1" allows the simulation of an infinite inventory of raw material for the first workstation. The module "Seize 1" allows the control of the raw material flow. The module "Process 1" simulates the workstation 1. The module "Seize 2" allows the simulation of a buffer between workstation 1 (module Process 1) and workstation 2 (module Process 2), which release an entity (workpiece) when workstation 2 finished processing. The capacity of the resource being seized represents the desired buffer capacity between workstations.

4.4.2. Simulation Run Parameters

In order to ensure that what is being observed is as close to normal operating behavior as possible, a steady-state simulation model was analyzed. A steady-state simulation model has no natural termination time. In such models, the designer is interested in long term dynamics and statistics. An example of a steady-state system is an assembly line that operates 24 hours a day, or an assembly line that always has some WIP at the end of a work day.

The simulation model used in this investigation initiates in an empty and idle state. This means that the model starts with zero WIP and with all the workstation in idle. In terminating systems,

this might be the way things actually start out. However, in a steady-state simulation, initial conditions are not supposed to matter, and the run is supposed to go on forever.

The simulation initial "atypical" history is called transient-state, as opposed to the simulation "typical" history that evolves later, which is called the steady-state. The transient-state regiment is characterized by statistics that vary as a function of time, while steady-state regimen prevails when statistics stabilize and do not carry over time. In between these two regimes, there is typically a transition period when the systems approaches the steady-state regimen, a period characterized by small and generally decreasing variability of the statistics over time. For all practical purposes, the systems may be considered to be approximately in steady-state during that transition period (Kelton, Sadowski, & Swets, 2010). In steady-state simulation, only long term statistics are of interest, but initial systems conditions tend to bias the long term statistics. Therefore, the statistics were collected after a warm-up period, when the biasing effect of the initial conditions decayed to insignificant.

To calculate the necessary warm-up period for the simulation model the method described by Law (2000) was implemented. A pilot test (with a three workstations balanced line, Normal distributed mean processing time of 1 hours, coefficient of variation of 0.25, and buffer capacity of 1unit) was run for 50,000 hours to analyze the behavior of the WIP.

The data of the WIP over time was analyzed in Minitab v.16 and autocorrelation values calculated. After a period of 2,300 hours autocorrelation values between 0.20 and -0.20 were achieved (See Figure 11), which as suggested by McNamara (2011) should ensure steady-state conditions.

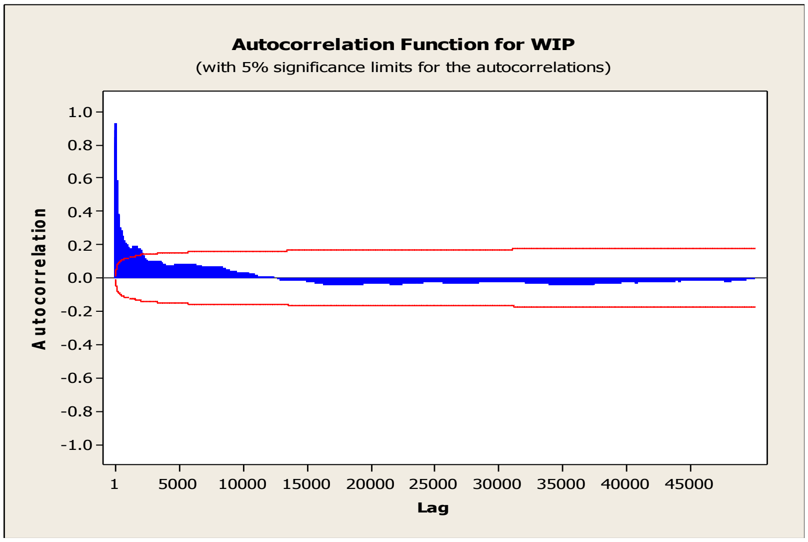


Figure 11. Autocorrelation Function for WIP

A warm-up period 3,000 hours was selected to ensure steady-state conditions. This warm-up period means that all statistical accumulators of the model were cleared after a period of 3,000 hours, and the final reports reflected only the data collected after that period of time.

After calculating the warm-up period a second pilot test was done to calculate the necessary number of simulation replications to achieve a desired half width of the 99% confidence interval (CI). In this pilot test a 30 workstations balanced line with Normal distributed mean processing time of 1 hour, coefficient of variation of 0.25, and buffer capacity of 1 was simulated. A relatively large run length (Run Length> 6x Warm-up Period) of 20,000 hours was used as suggested by McNamara (2011), and simulation replications of 10, 20, 30 and 40 were used to study the behavior of the throughput under these simulation parameters. The output data for each number of replications was analyzed and the half widths of the 99% CI were calculated. Tables 7-10 show the calculation of half with of the 99% CI achieved under these simulation parameters. The half with of the 100(1-α) % CI is calculated with the two tailed T statistic multiplied by the product of the standard deviation of the sample over the root square of the total number of replications.

$$half\ width\ (h) = t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$$

Table 7: 10 simulation replications data

Replication #s 10	
Average	***15486.9***
Standard Deviation	***11.779926***
Tα/2,v	3.25
*α=0.01	
h= 12.1	

Table 8: 30 simulation replications data

Replication #s 30	
Average	***15488.23***
Standard Deviation	***11.146929***
Tα/2,v	2.76
*α=0.01	
h= 5.6	

Table 9: 20 simulation replications data

Replication #s 20	
Average	***15490.8***
Standard Deviation	***10.739009***
Tα/2,v	2.86
*α=0.01	
h= 6.87	

Table 10: 40 simulation replications data

Replication #s 40	
Average	***15488.5***
Standard Deviation	***10.677078***
Tα/2,v	2.7
*α=0.01	
h= 4.5	

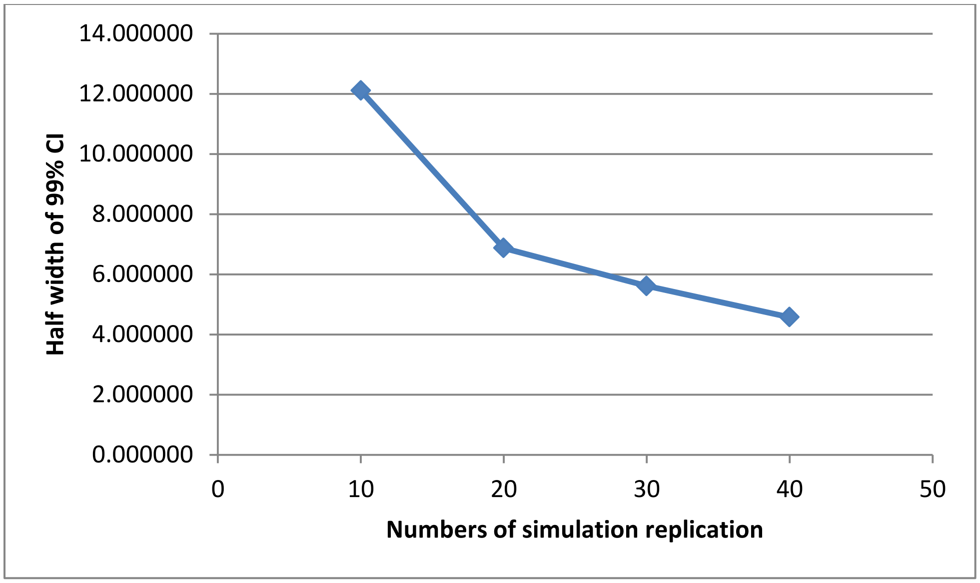


Figure 12. Half width of the 99% CI under different number of simulation replications

As Tables 7-10 and Figure 12 show the half width of the 99% CI reduces as the number of simulation replication increases. However the biggest reduction is seen between 10 and 20 simulation replications. Moreover, the simulation time increases rapidly as the number of simulation replications increases, due to the extra computational time needed. Therefore, it was advantageous to select a 20 simulation replications, which provided a robust output without incurring in unnecessary computational time.

Under the simulation parameter of warm-up period = 3,000 hours, run length = 20,000 hours, and 20 simulation replications a 6.87 units for the half width of the 99% confidence interval of the throughput was achieved. This value of half width represents 0.04% of the average value of the throughput (See Table 9). The data analyzed suggest that a preliminary model with a warm-up period of 3,000 hours, a run length of 20,000 hours, and 20 simulation replications provided a robust output data without incurring in unnecessary computational time. Therefore, the previously mentioned simulation run parameters were selected for the simulation model used in

this study (warm-up period of 3,000 hours, run length of 20,000 hours, and 20 simulation replications)

To ensure that the differences in the output of the models are due to the changes in the control variables and not due to the random seed numbers, the same seed stream was used in each model. The use of the same seed stream generated some correlation between the outputs of the models. Therefore, the results were analyzed using a paired T-test instead of a two-sample T-test, as suggested by Kelton et al (2010).

4.4.3. Model Validation

The acceptable method to validate simulation model and run parameters is to compare the output results to those of an existing model. The results of Hillier and So (1993) and Pike and Martin (1994) were used to compare the preliminary simulation model and run parameters that were used in this study. These works studied the effect of unbalancing the workload in unpaced assembly lines under different processing time distribution and buffer capacities. Furthermore, these studies were the only to provide the values of mean processing time for a two-level bowl configurations.

Pike and Martin (1994) studied the improvement in mean output interval of assembly line with 3-12, 15,20, 25 and 30 workstations, buffer capacity 0-4 units under Normal and positively skewed task time distributions with CV of 0.2, 0.25 and 0.30. The preliminary model was tested with the unbalanced configurations provided for the assembly lines with 3-12, 15,20, 25 and 30 workstations, buffer capacity 0-4 units, under Normal task time distributions with CV of 0.25. Figures 13-16 show the values of the percentage improvements in mean output interval presented in Pike and Martin (1994) study [ ] and results of the preliminary model [ ]. The

"x" axis represents the number of workstation simulated, and the "y" axis represents the percentage of improvement in mean output interval, as calculates by Pike and Martin (1994).

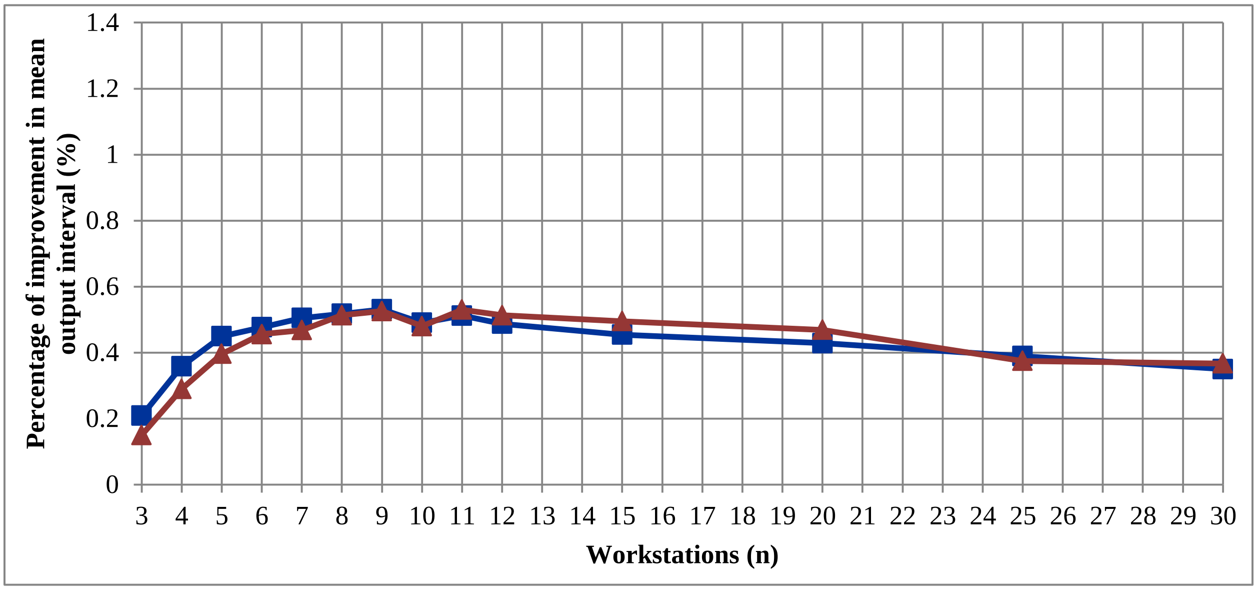


Figure 13: Normal task time distribution, CV= 0.25 and B= 0.

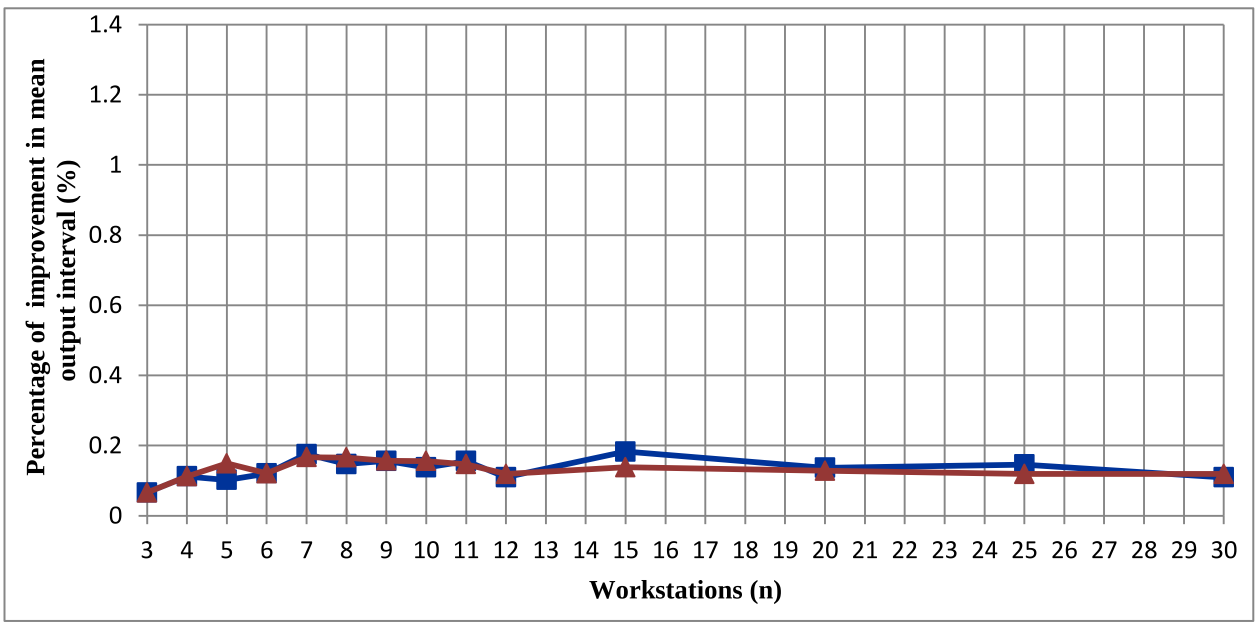


Figure 14: Normal task time distribution, CV= 0.25 and B= 1.

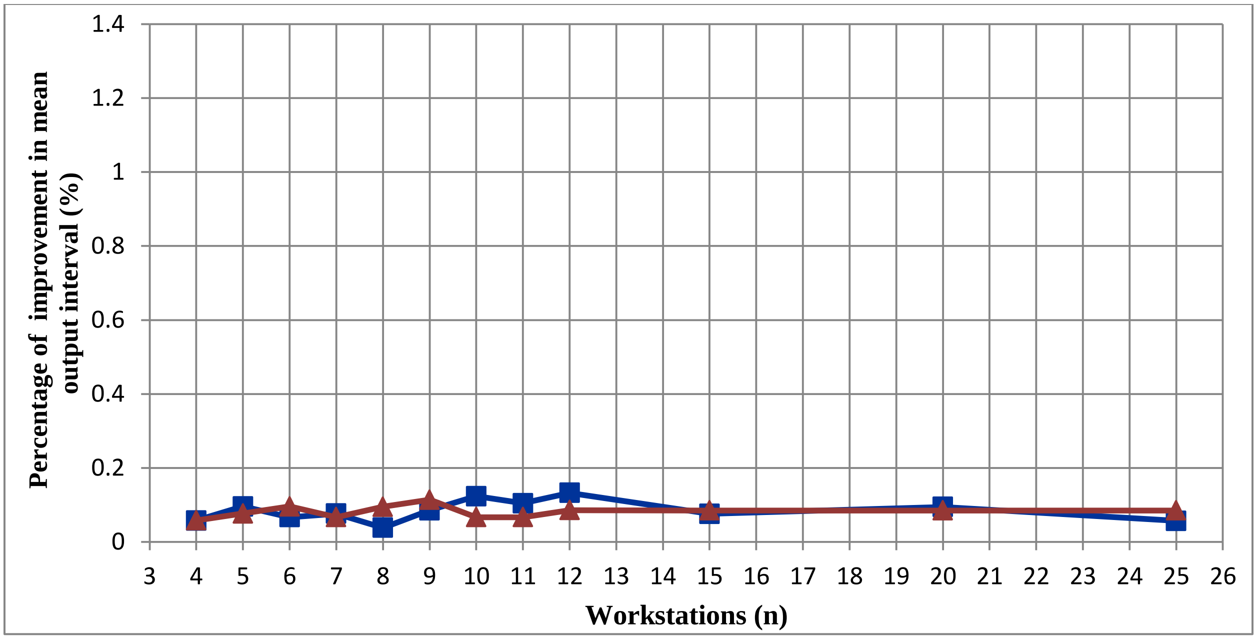


Figure 15: Normal task time distribution, CV= 0.25 and B= 2

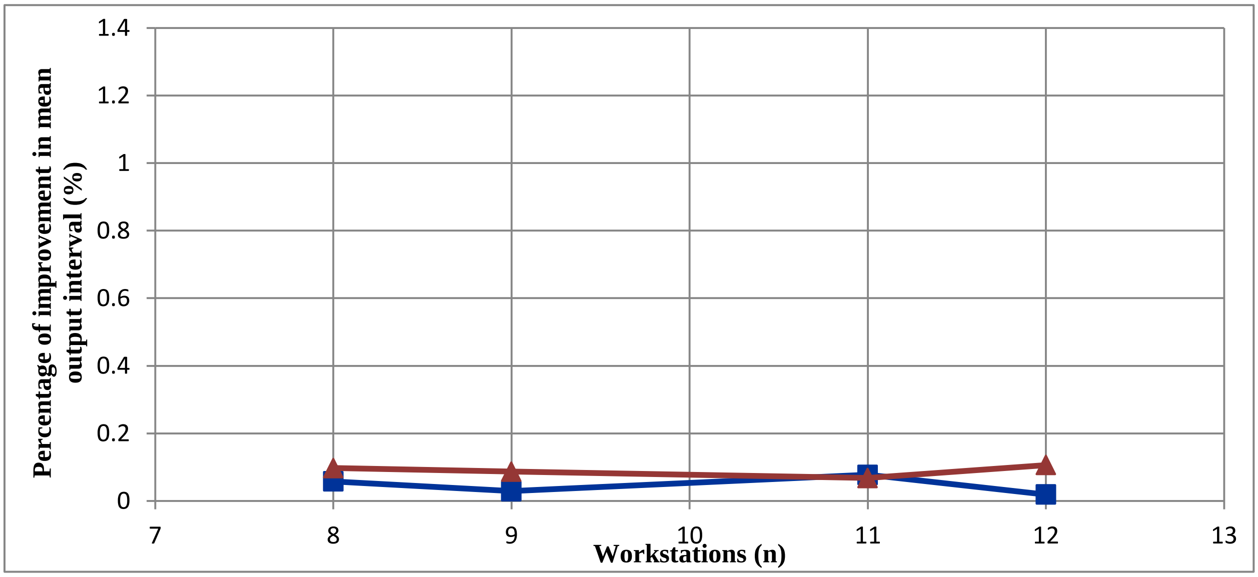


Figure 16: Normal task time distribution, CV= 0.25 and B= 3.

Based on the values of the bowl configurations provided by Hillier and So (1993) the preliminary model was tested in assembly lines with N= 3-9, B=0-5, 10 and 15, under Exponential task time distribution with CV=1, and Erlang task time distribution with k=2-16 (k is the shape parameter of the Erlang distribution, $CV = {}^{1}/_{\sqrt{k}}$). Tables 11-15 show the different

assembly lines scenarios simulated. Furthermore, Figures 17-21 show the percentage of improvement in throughput presented by Hillier and So(1993) [ ] study and the improvements of the preliminary model [ ].The “x” axis represents the scenarios simulated, and the “y” axis represents the percentage change in throughput (%TR) as calculates by Hillier and So (1993).

Table 11: Scenarios with Exponential task time distribution, CV=1, N=7-9 and B=0

Exponential task time distribution					
Scenario #	***N***	***B***	***CV***	***%TR of Hillier and So (1993)***	***%TR of preliminary model***
1	7	0	1	101.48	101.4314
2	8	0	1	101.53	101.6369
3	9	0	1	101.59	101.6809

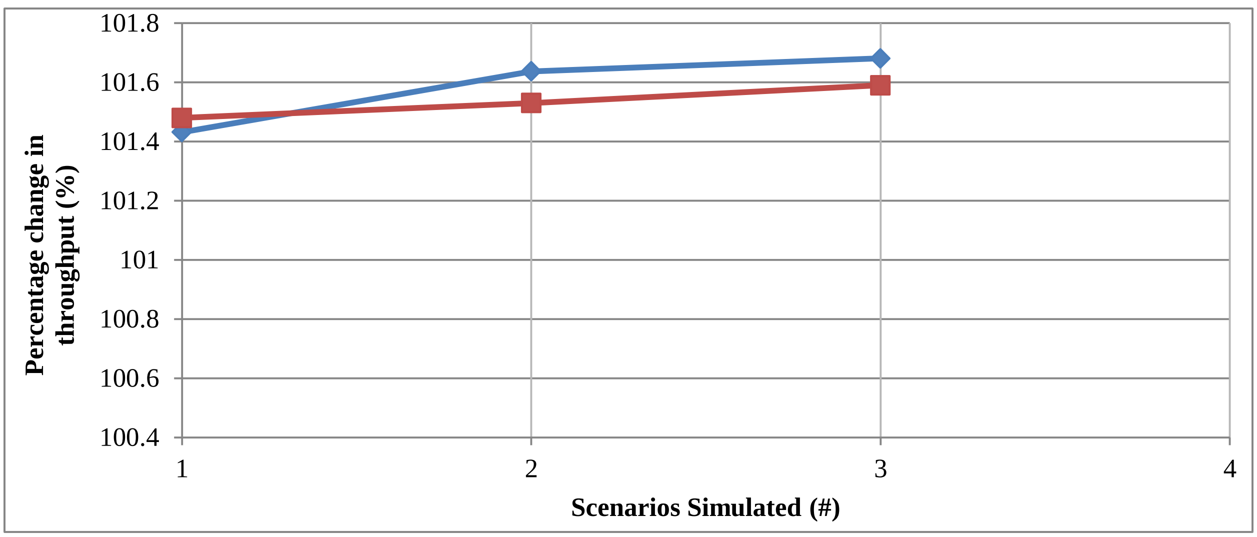


Figure 17. Scenarios with Exponential task time distribution, CV=1, N=7-9 and B=0

Table 12: Scenarios with Exponential task time distribution, CV=1, N=3, and B=5, 10,15

Exponential task time distribution					
Scenario #	*N*	*B*	*CV*	*%TR of Hillier and So (1993)*	*%TR of preliminary model*
1	3	5	1	100.22	100.4
2	3	10	1	100.12	100.09
3	3	15	1	100.08	100.17

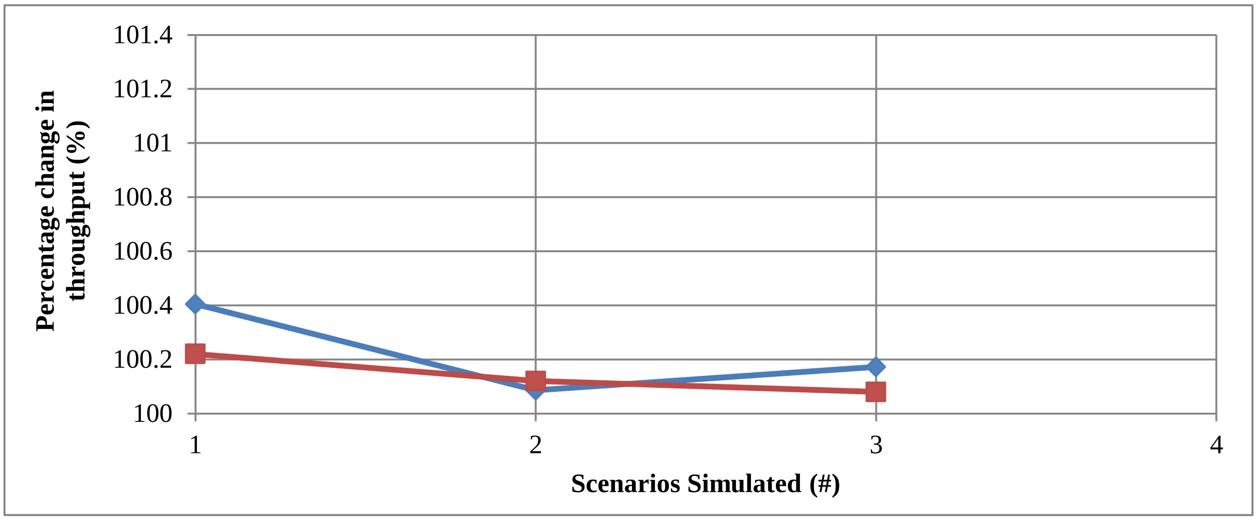


Figure 18. Scenarios with Exponential task time distribution, CV=1, N=3, and B=5, 10 and 15

Table 13: Scenarios with Exponential task time distribution, CV=1, N=4-7, and B=0-5

Exponential task time distribution					
Scenario #	*N*	*B*	*CV*	*%TR of Hillier and So (1993)*	*%TR of preliminary model*
1	4	5	1	100.36	100.38
2	5	3	1	100.63	100.44
3	5	4	1	100.52	100.54
4	5	5	1	100.44	100.4
5	6	2	1	100.82	100.8
6	7	0	1	101.48	101.43
7	7	1	1	101.14	101.26

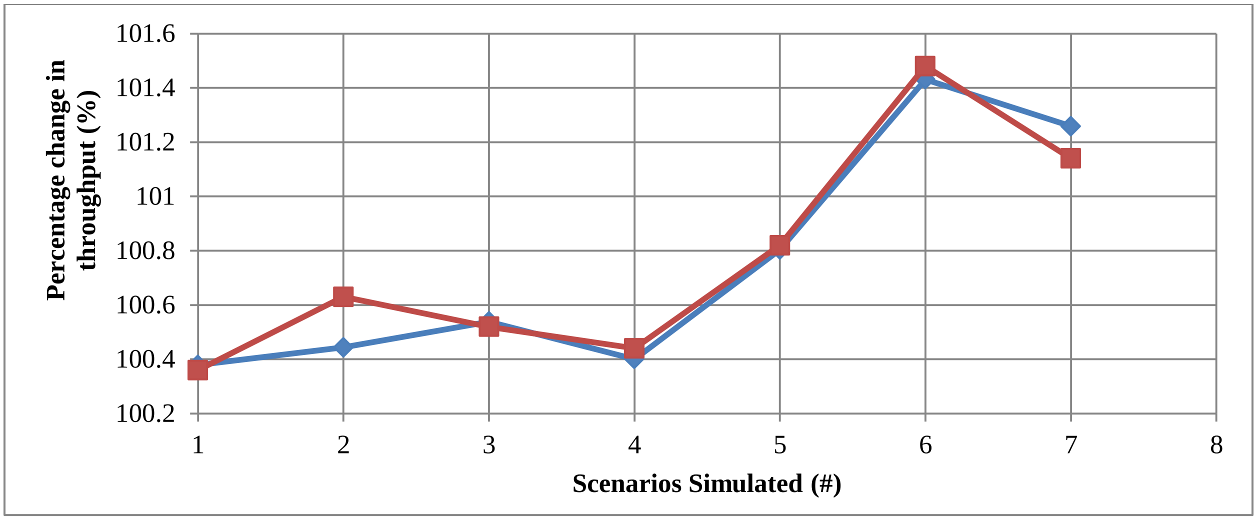


Figure 19. Scenarios with Exponential task time distribution, CV=1, N=4-7, and B=0-5

Table 14: Scenarios with Erlang task time distribution, k=2-16, N=3, and B=0-5

Erlang task time distribution					
Scenario #	***N***	***B***	***k***	***%TR of Hillier and So (1993)***	***%TR of preliminary model***
1	3	5	2	100.11	100.12
2	3	5	3	100.08	100.04
3	3	3	4	100.1	100.08
4	3	4	4	100.08	100.1
5	3	2	5	100.11	100.13
6	3	3	5	100.08	99.97
7	3	1	6	100.16	100.08
8	3	2	6	100.09	100
9	3	0	8	100.32	100.36
10	3	0	9	100.3	100.34
11	3	0	10	100.29	100.37
12	3	0	11	100.29	100.32
13	3	0	12	100.27	100.26
14	3	0	13	100.26	100.28
15	3	0	14	100.25	100.3
16	3	0	15	100.24	100.32
17	3	0	16	100.25	100.34

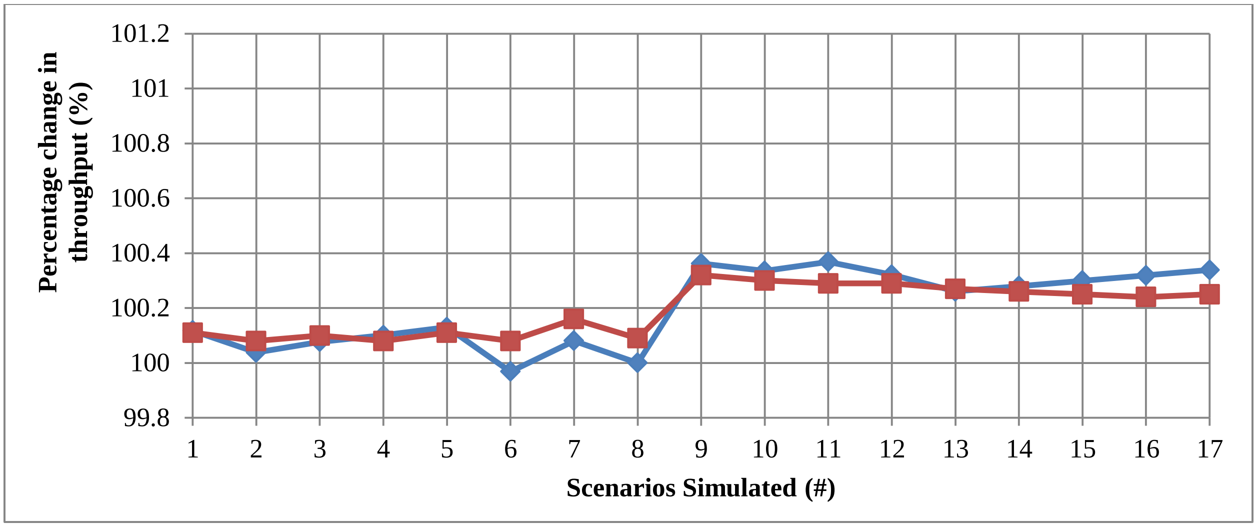


Figure 20. Scenarios with Erlang task time distribution, k=2-16, N=3, and B=0-5

Table 15: Scenarios with Erlang task time distribution, k=2-4, N=4-6, and B=0-4

Erlang task time distribution					
Scenario #	*N*	*B*	*k*	*%TR of Hillier and So (1993)*	*%TR of preliminary model*
1	6	0	2	101.17	101.15
2	5	0	3	100.93	100.88
3	4	0	4	100.68	100.73
4	5	0	4	100.84	100.86
5	5	1	2	100.67	100.73
6	4	1	3	100.42	100.54
7	4	1	4	100.34	100.26
8	4	2	3	100.28	100.23
9	4	3	2	100.29	100.2
10	4	4	2	100.23	100.01

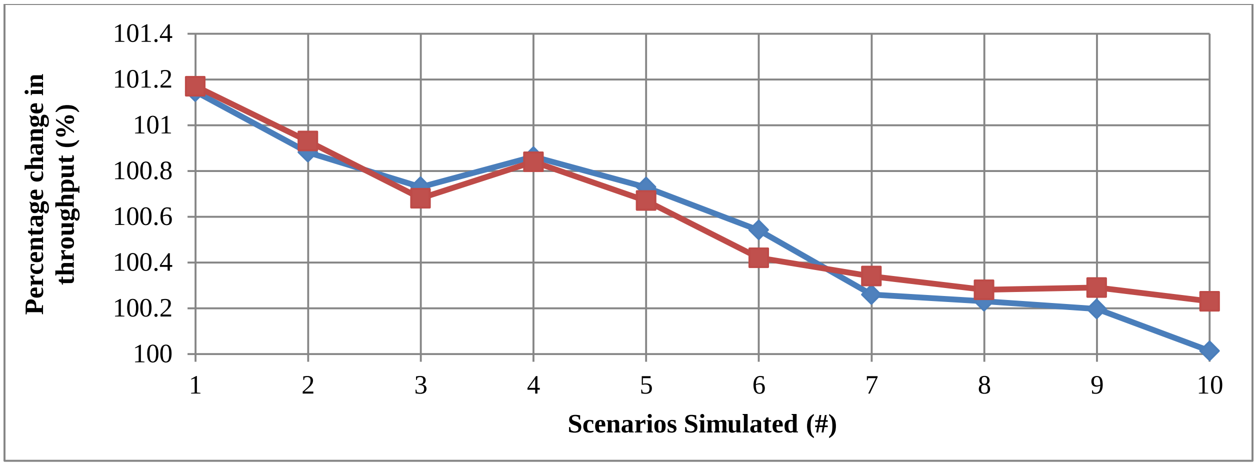


Figure 21. Scenarios with Erlang task time distribution, k=2-4, N=4-6, and B=0-4

In total 84 configurations were simulated with the values provided by Hillier and So (1993) and Pike and Martin (1994). The tendency shown in Figures 13-21 demonstrated that the preliminary model was able to simulate the effects that line lengths, buffer capacity, task time distribution, and CV values have over the out rate of unpaced production lines. Furthermore, the error between the results of the studies simulated and the preliminary model were calculated. Then, via a T-test the error means were analyzed in Minitab v.16. The results show that with a high level of confidence (P value of 0.874) the null hypothesis (H0: μ=0; the mean of the error is equal to zero) should not be rejected (See Table 16 and Figure 22). These results suggest that the preliminary model is able to simulate the behavior of unpaced assembly lines under stochastic processing times in steady-state conditions.

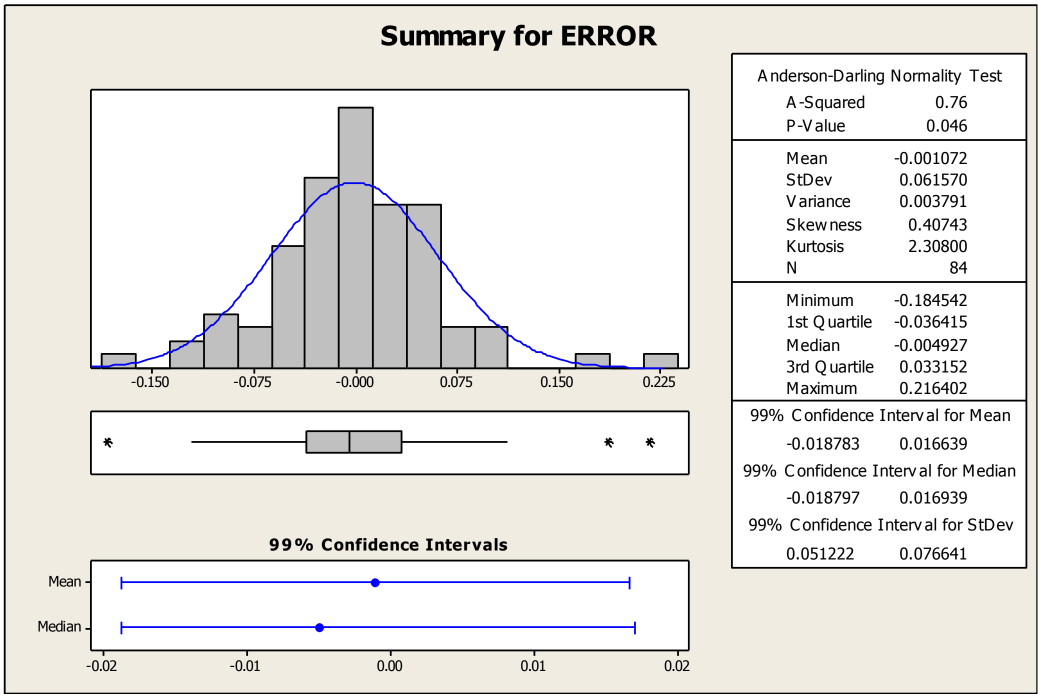


Figure 22. Minitab Summary of Error

Table 16.Minitab One-Sample T-test

One-Sample T: ERROR

Test of mu = 0 vs. not = 0

Variable	***N***	***Mean***	***StDev***	***SE Mean***	***99% CI***	***T***	***P***
ERROR	84	-0.00107	0.06157	0.00672	(-0.01878, 0.01664)	-0.16	0.874

4.5. Data Analysis

The performance measure analyzed was the throughput of the lines. For each one bowl and multi-bowl configuration simulated, the results on throughput were analyzed in Minitab v.16. The average throughput of the unbalanced configurations tested must be statistically greater, according to a paired t-test at a 95% confidence level, than the throughput of the balanced counterpart. The one bowl and multi-bowl configuration with the largest throughput was designated as the “best configuration” for each case, similar to Pike and Martin (1994)

methodology (See section 4.2 and 4.3). An analysis of variance (ANOVA) was conducted to study the main and interaction effects of the independent variables (see section 4.1) on the throughput of the lines.

Additionally, a regression analysis was conducted, similar to Shaaban and McNamara (2009) methodology. The regression analysis provided a better understanding of the relationship between the response and independent variables. Finally, the Work-In-Process (WIP) and Cycle time (CT) of the lines were analyzed to validate that the simulation model behaved accordantly with Little's Law and to better understand the impact of the workload distribution with a bowl shape configuration in large assembly lines.

5. Results and Discussion

Section 4 described the methodology and experimental setup used to study the effects of unbalance workload allocation in large assembly lines. The results of the experiment are discussed in this section.

5.1. One Bowl configuration

The assembly lines were simulated in Arena to determine the effects of line length (N), buffer capacity (B) and coefficient of variation (CV) on the throughput, work in process, and cycle time of unbalanced assembly lines with a bowl shape workload configuration. Section 4.2 presented the different levels of the control variables for the design of experiment used. Table 17 shows the assembly lines simulated, the percentage of improvement achieved by the "best one bowl configuration" (%Improvement in TR), and the Range of Percentage of Imbalance ("x") that produced statistically significant improvement in throughput, compared to the balance line. Furthermore, the Throughput (TR), the Percentage of Imbalance ("x"), the Percentage Increase in Work In Process (%Increase in WIP) , and Percentage Increase in Cycle Time (%Increase in CT) of the "best one bowl configuration" are also shown.

Statistically significant improvements in throughput were achieved in 22 of the 27 assembly lines simulated. The results show improvements in throughput up to 1.7%. The biggest improvements in throughput were achieved in lines with high coefficients of variation (CV=1.4) and low buffer capacity (B=0). Furthermore, it can be observed that as the buffer capacity increases and the coefficient of variation decreases, the throughput of the lines improved. However, the percentage of improvement in throughput achieved by unbalancing the lines with a bowl shape workload configuration decreases (See Table 17, Lines 7, 8, 9).

Furthermore, as the graphs in Table 19-24, the Correlation Analysis presented in Table 18, and the Figures 23-25 show, these improvements in throughput are correlated to an increase of the Work in Process (WIP) and Cycle Time (CT) of the lines. The results show an increase in WIP up to 1.59% and an increase in the Cycle Time up to 0.418%. The biggest percentage increase of WIP were seen in lines with high coefficients of variation (CV=1.4) and low buffer capacity (B=0).While the biggest percentage increase in CT were seen in lines with high coefficients of variation (CV=1.4) and high buffer capacity (B=2).

As shown in Figures 23-25 the WIP and the CT of the lines have a linear relationship with the percentage of improvement in throughput achieved by unbalancing the line in a bowl shape workload configuration. The rate at which the throughput increases is greater than the rate at which the CT of the line increases. In line with low buffer capacity (B=0) the rate at which the throughput increases is greater than the rate at which the WIP of the lines increases. However, as the buffer capacity of the line increases the WIP and CT increases at a faster rate, compared to the lines with low buffer capacity. This can be appreciated by the change in the slope between the scatterplots presented in Figure 23 (B=0) and 25 (B=2).

Table 17. Assembly lines simulated with one bowl configuration, and percentage of improvement achieved.

Line	CV	B *(units)*	N *(workstation)*	X *(%)*	%Improvement in TR *(%)*	TR *(units)*	%Increase in WIP *(%)*	%Increase in CT *(%)*	Range of "x" *(%)*
1	0.2	0	30	0.006	0.348389714	13481.4	0.195439	0.19487	[0.001-0.01]
2	0.2	0	50	0.004	0.243386748	13407.2	0.146429	0.135006	[0.001-0.006]
3	0.2	0	70	0.003	0.171944525	13371.75	0.067242	0.09295	[0.001-0.004]
4	0.2	1	30	0	0	15261.75	0	0	0
5	0.2	1	50	0.001	0.054866529	15871.4	0.045487	0.02348	[0.001]
6	0.2	1	70	0.001	0.046707363	15858.9	0.09765	0.0230189	[0.001]
7	0.2	2	30	0	0	15279.1	0	0	0
8	0.2	2	50	0	0	16330.6	0	0	0
9	0.2	2	70	0	0	16324.55	0	0	0
10	0.8	0	30	0.02	1.147581765	7764.95	0.78898	0.19718379	[0.002-0.037]
11	0.8	0	50	0.014	0.88416479	7626.4	0.77854	0.1451385	[0.003-0.023]
12	0.8	0	70	0.009	0.647376206	7555.15	0.703889	0.103185	[0.002-0.016]
13	0.8	1	30	0.003	0.258048787	10257.35	0.406849	0.06357	[0.003-0.007]
14	0.8	1	50	0.01	0.548009669	10221.8	0.643686	0.144428	[0.002-0.013]
15	0.8	1	70	0.006	0.508730009	10163.65	0.254781	0.141057	[0.001-0.009
16	0.8	2	30	0	0	11472.75	0	0	0
17	0.8	2	50	0.006	0.442940288	11658.25	0.414573	0.18201	[0.002-0.009]
18	0.8	2	70	0.004	0.248132433	11617.2	0.188083	0.0849327	[0.004-0.005]
19	1.4	0	30	0.028	1.705250151	5239.7	1.56329	0.1010825	[0.005-0.06]
20	1.4	0	50	0.023	1.206932647	5059	1.593429	0.0496995	[0.004-0.038]
21	1.4	0	70	0.016	0.85983533	4975.75	1.223698	0.036292	[0.005-0.026]
22	1.4	1	30	0.022	0.752571217	7056.05	0.682433	0.160504	[0.007-0.029]
23	1.4	1	50	0.014	0.845279488	6912.85	0.890556	0.201319	[0.007-0.25]
24	1.4	1	70	0.012	0.874257579	6845.5	0.774843	0.196357	[0.005-0.018]
25	1.4	2	30	0.014	0.323852549	8234.15	0.239083	0.11184	[0.006-0.014]
26	1.4	2	50	0.014	0.932636758	8153.3	0.846386	0.320657	[0.004-0.021]
27	1.4	2	70	0.01	0.954862301	8098.55	0.730867	0.418329	[0.002-0.015]

Tables 19-21 show the 95% CI of the throughput for assembly lines simulated with a one bowl shape workload configuration. Tables 22-24 show the 95%CI of the WIP of the assembly line simulated with a one bowl shape workload configuration. In the 22 lines, where the bowl shape workload configuration provided statistical significant improvement in throughput, it can be observed that as the buffer capacity and the number of workstation decreases, and the coefficient of variation increases, the percentage of imbalance (x) of the “best one bowl configuration” increases.   Moreover, the percentage of imbalance (x) of the worse bowl configuration also increases. This can be seem in the column *“Range of “x””* in Table 17 , that shows the range of Percentage of Imbalance that produced statistically significant improvement in throughput, compared to their balanced counterpart. This suggested that as the buffer capacity and number of workstations decreases and the coefficient of variation increases there are more opportunities to improve the throughput of the line by unbalancing the workload in a bowl shape configuration.

Table 18.Correlation Analysis TR vs WIP

Correlations: TR, WIP

```
Pearson correlation of TR and WIP = 0.633
P-Value = 0.000
```

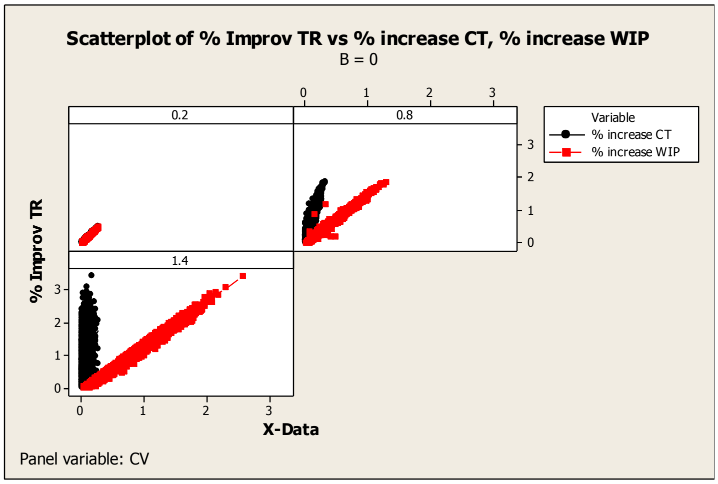


Figure 23.Scatter plot Diagram %TR vs, % CT, and 5 WIP (B=0)

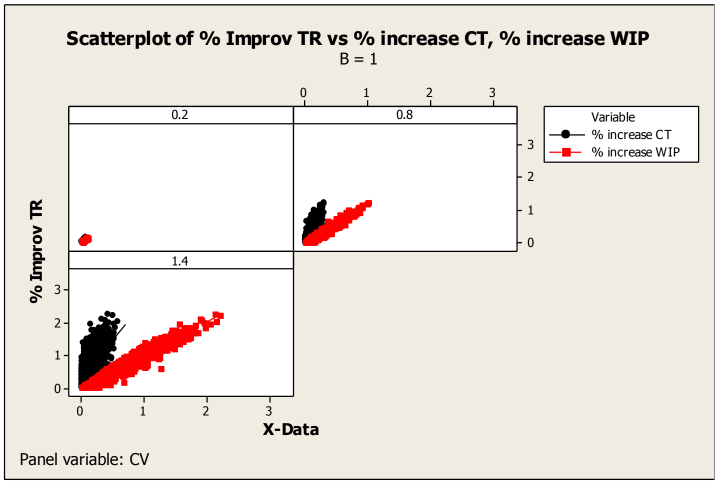


Figure 24.Scatter plot Diagram %TR vs, % CT, and 5 WIP (B=1)

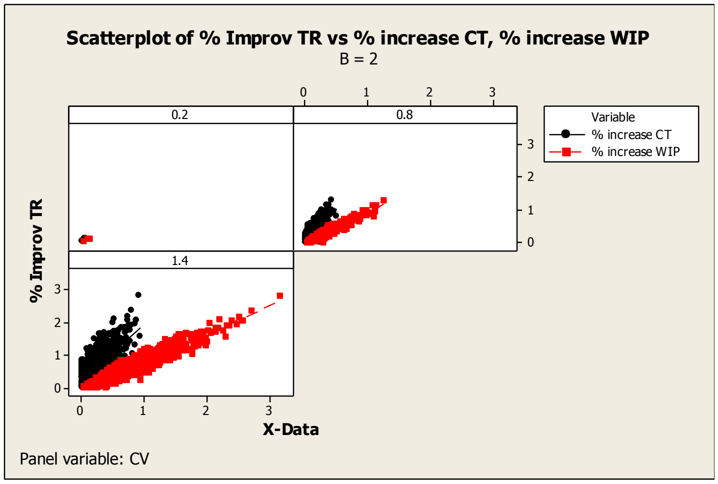


Figure 25.Scatter plot Diagram %TR vs, % CT, and 5 WIP (B=2)

Table 19. 95% CI of the throughput for the unbalanced assembly lines simulated with a one bowl shape configuration and CV of 0.2

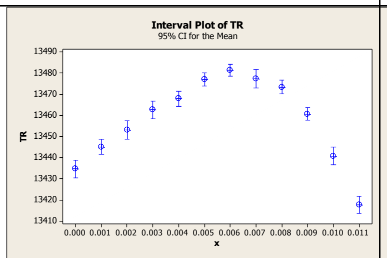
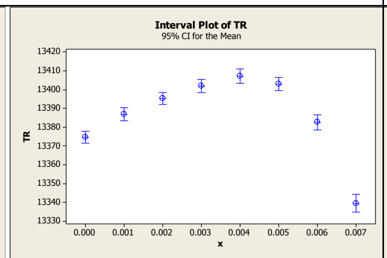
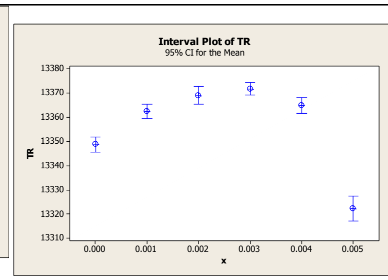
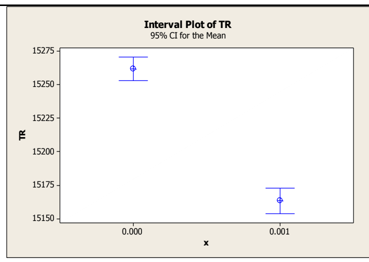
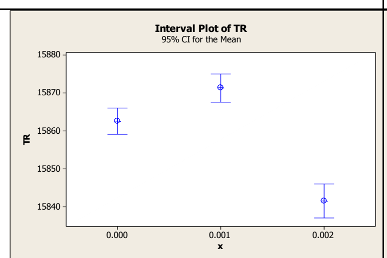
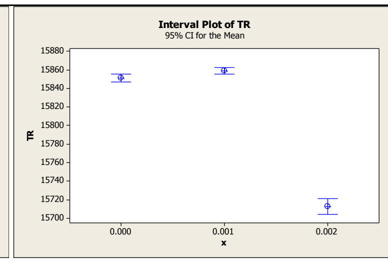
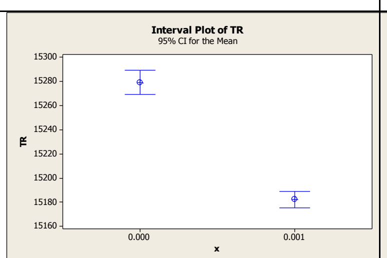
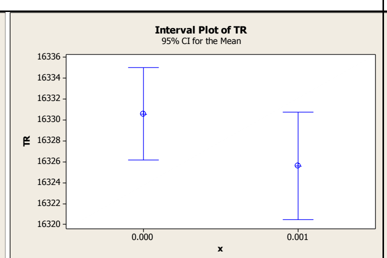
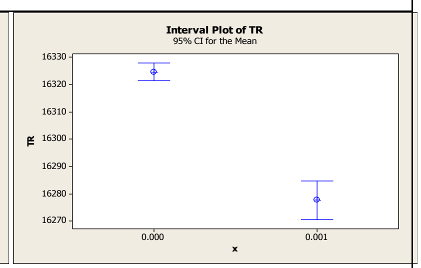
B/N	30	50	70
0			
1			
2			

Table 20. 95% CI of the throughput for the unbalanced assembly lines simulated with a one bowl shape configuration and CV of 0.8

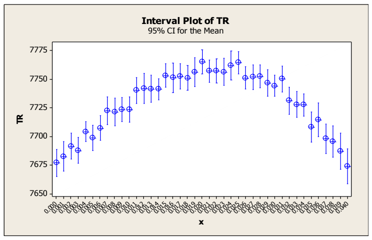
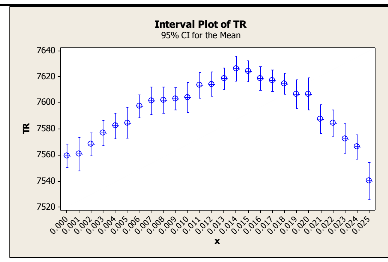
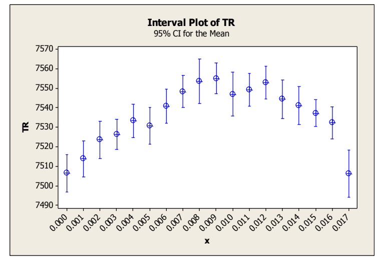
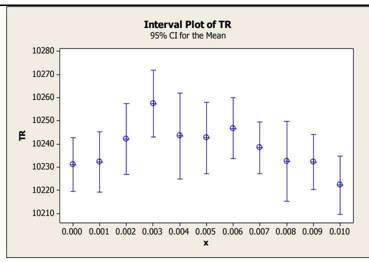
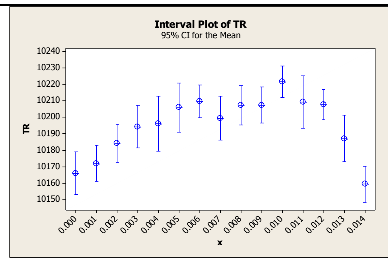
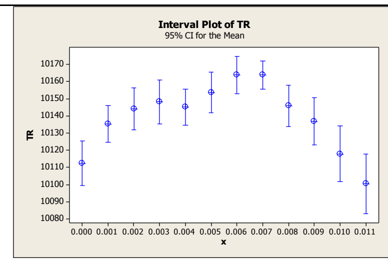
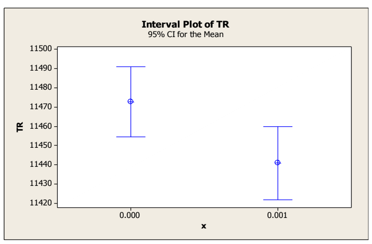
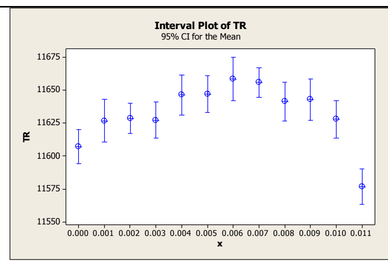
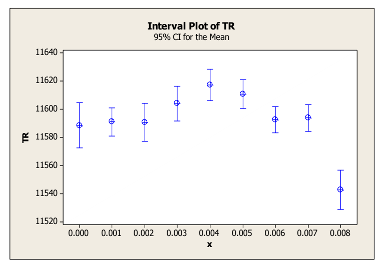
B/N	30	50	70
0			
1			
2			

Table 21. 95% CI of the throughput for the unbalanced assembly lines simulated with a one bowl shape configuration and CV of 1.4

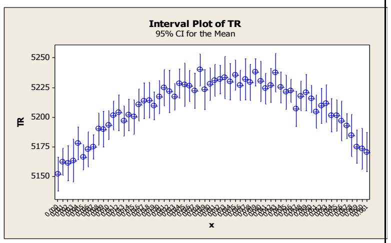
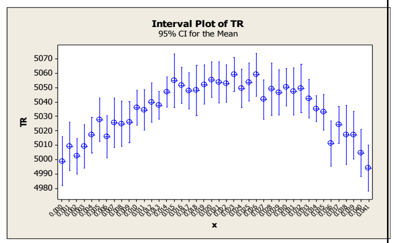
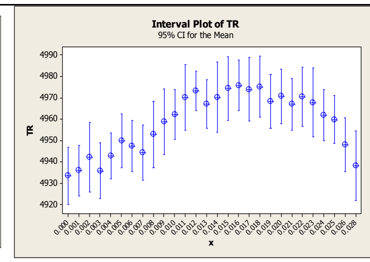
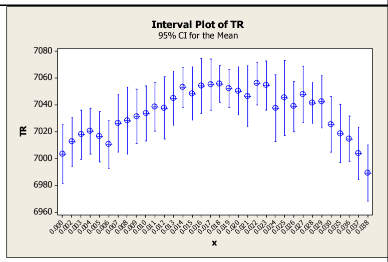
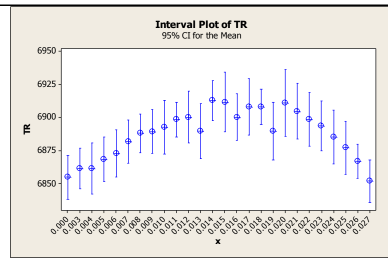
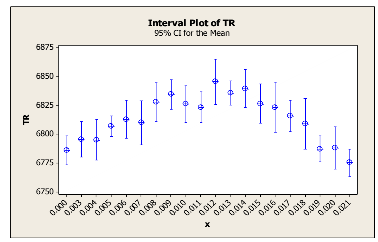
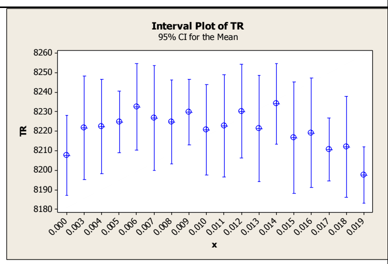
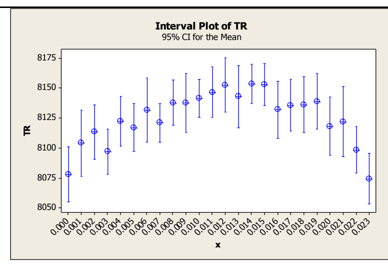
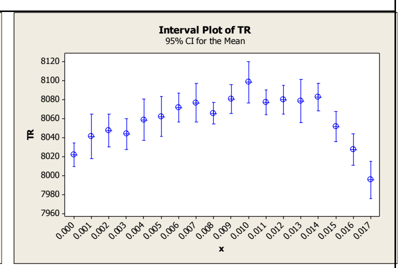
B/N	30	50	70
0			
1			
2			

Table 22. 95% CI of the WIP for the unbalanced assembly lines simulated with a one bowl shape configuration and CV of 0.2

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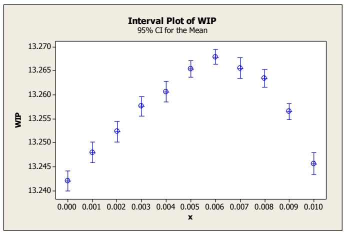
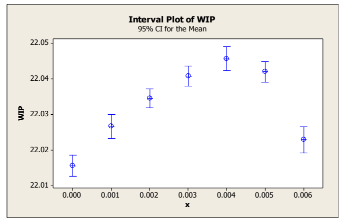
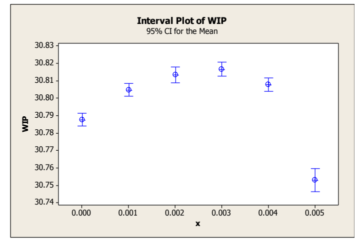
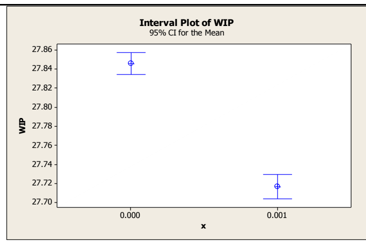
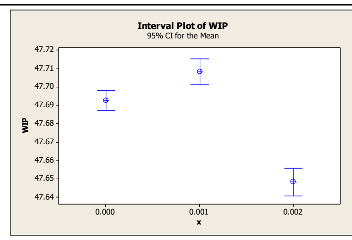
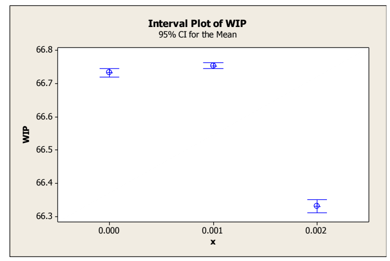
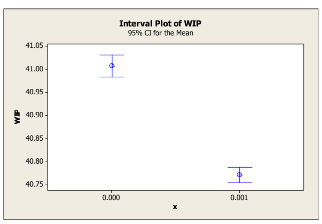
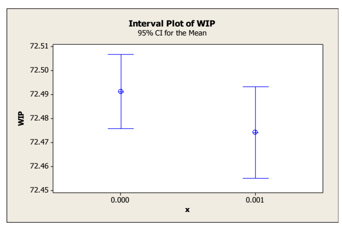
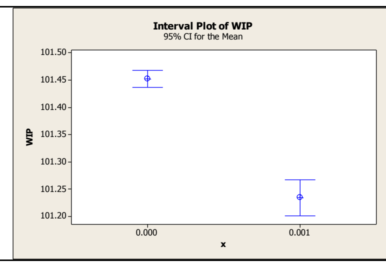
0			
1			
2			

Table 23. 95% CI of WIP for the unbalanced assembly lines simulated with a one bowl shape configuration and CV of 0.8

<table>
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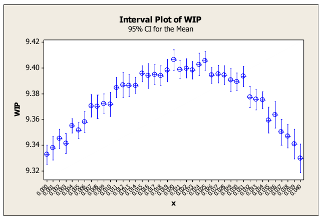
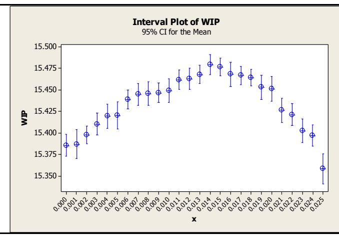
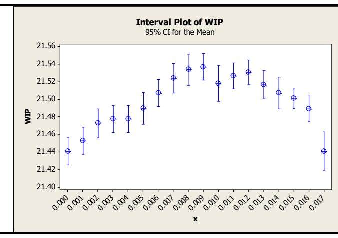
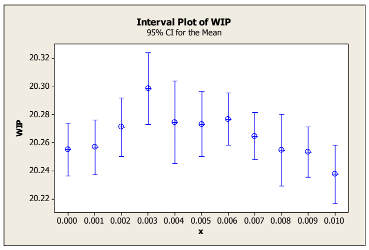
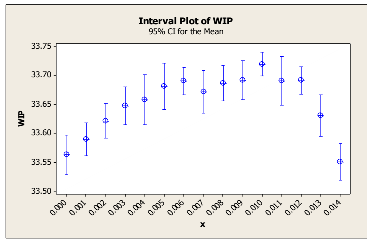
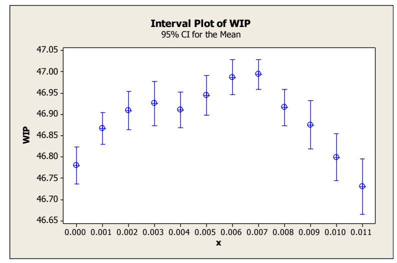
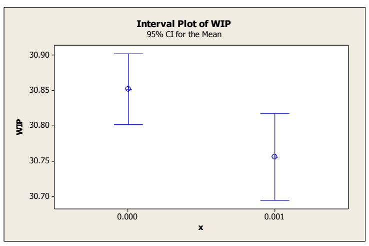
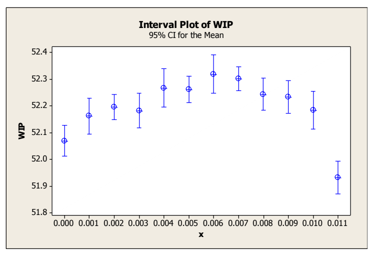
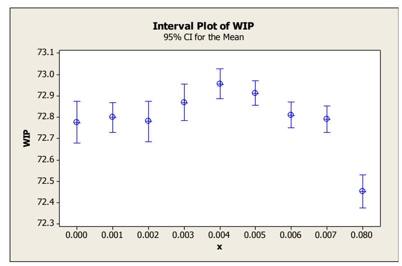
<tr><td>0</td><td></td><td></td><td></td></tr>
<tr><td>1</td><td></td><td></td><td></td></tr>
<tr><td>2</td><td></td><td></td><td></td></tr>
</table>

Table 24. 95% CI of the WIP for the unbalanced assembly lines simulated with a one bowl shape configuration and CV of 1.4

<table>
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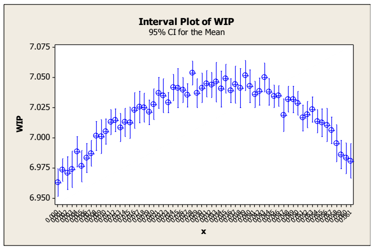
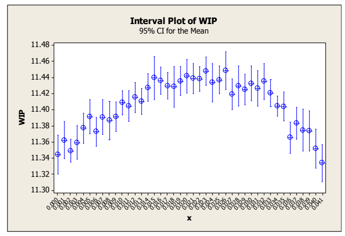
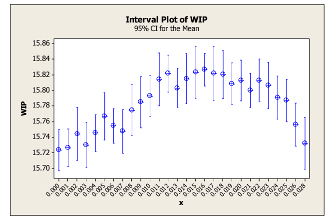
N</th><th>30</th><th>50</th><th>70</th></tr>
<tr><td>0</td><td>
</td><td>
</td><td>
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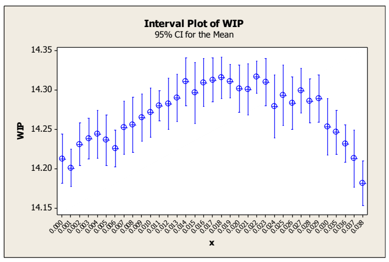
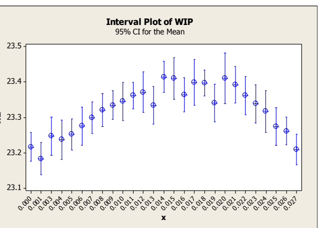
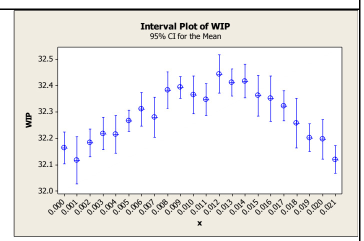
<tr><td>1</td><td>
</td><td>
</td><td>
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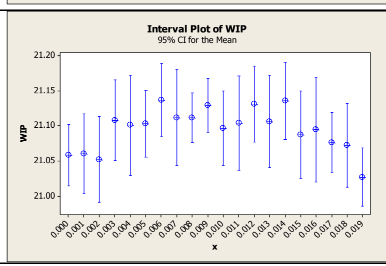
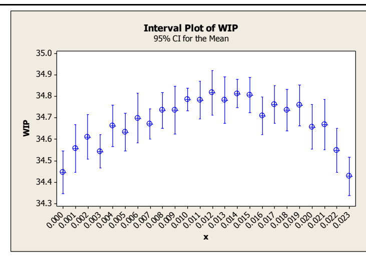
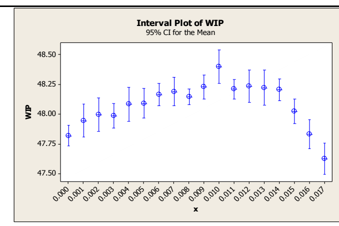
<tr><td>2</td><td>
</td><td>
</td><td>
</td></tr>
</table>

Table 25 shows the lines simulated, the Percentage of Imbalance (“x”), and a rage of ±50% the Percentage of Imbalance of the “best one bowl configuration”. Additionally, the percentage of the Potential Improvements in Throughput (% of Potential TR Improvement) achieved with theses ±50% Percentage of Imbalance of the “best one bowl configuration” are shown. This Percentage of the Potential Improvement in Throughput is the ratio between the Improvements in throughput achieved with the ±50% the Percentage of Imbalance and the Percentage of Imbalance of the “best one bowl configuration”. The results shows that up to 88% of the improvement in throughput of the “best one bowl configuration” can be achieved even if the amount of imbalance is either 50% more or less than the “best one bowl configuration”.

Table 25. Percentage of Potential Improvement (Robustness)

Line	CV	B *(units)*	N *(workstation)*	X *(%)*	±50% X *(%)*	% of Potential TR Improvement -50% X	+50% X
1	0.2	0	30	0.006	[0.003-0.009]	60%	56%
2	0.2	0	50	0.004	[0.002-0.006]	63%	24%
3	0.2	0	70	0.003	[0.002-0.004]	88%	70%
4	0.2	1	30	0	[0-0]	0%	0%
5	0.2	1	50	0.001	[0-0.002]	0%	0%
6	0.2	1	70	0.001	[0-0.002]	0%	0%
7	0.2	2	30	0	[0-0]	0%	0%
8	0.2	2	50	0	[0-0]	0%	0%
9	0.2	2	70	0	[0-0]	0%	0%
10	0.8	0	30	0.02	[0.01-0.03]	52%	45%
11	0.8	0	50	0.014	[0.07-0.021]	73 %	42%
12	0.8	0	70	0.009	[0.004-0.014]	77%	71%
13	0.8	1	30	0.003	[0.002-0.004]	42%	44%
14	0.8	1	50	0.01	[0.005-0.015]	44%	0%
15	0.8	1	70	0.006	[0.003-0.009]	70%	48%
16	0.8	2	30	0	[0-0]	0%	0%
17	0.8	2	50	0.006	[0.003-0.009]	39%	70%
18	0.8	2	70	0.004	[0.002-0.006]	0%	0%
19	1.4	0	30	0.028	[0.014-0.042]	57%	85%
20	1.4	0	50	0.023	[0.012-0.035]	68%	57%
21	1.4	0	70	0.016	[0.008-0.024]	55%	67%
22	1.4	1	30	0.022	[0.011-0.033]	49%	67%
23	1.4	1	50	0.014	[0.007-0.021]	47 %	86%
24	1.4	1	70	0.012	[0.006-0.018]	45%	39%
25	1.4	2	30	0.014	[0.007-0.021]	0%	0%
26	1.4	2	50	0.014	[0.007-0.021]	57%	59%
27	1.4	2	70	0.01	[0.005-0.015]	52%	39%

Table 26 shows the analysis of variance conducted and the P-values for the main effects, multiple-way interactions effects, and blocking effects using a full factorial analysis. Because the values of the percentage of imbalance (*x*) were not balanced (equal) among the different experimental configurations, it was used as a blocking variable in the analysis. To determine the significance of the factors the P-values of the ANOVA test were compared to a standard Alpha (α) of 0.05. Table 26 shows that the main effects, the multiple-way interaction effects, and the blocking effect are significant. However, as the F-statistic and the Adjusted Mean Square (Adj MS) shows that the main effect of CV and B are the most significant ones. The large sample size (8,800 data points) drives the significance of some the variables. Therefore, some of the variables that shown as statistically significant (base on their P-values), might not be noteworthy for practical uses. Thus, to better understand the effect of the control variables over the response variable the Adjusted Mean Square is a better statistic to analyze.

Table 26. Analysis of Variance for throughput (One bowl configuration)

Analysis of Variance for TR, using Adjusted SS for Tests

Source	DF	Seq SS	Adj SS	Adj MS	F	P
x	61	18824299545	1571191	25757	22.05	0.000
CV	2	28361004387	25185500533	12592750266	10782496.24	0.000
B	2	12635496255	5626868254	2813434127	2408992.65	0.000
N	2	11753556	8085904	4042952	3461.76	0.000
CV*B	4	337985248	252907747	63226937	54137.83	0.000
CV*N	4	24533440	51919991	12979998	11114.08	0.000
B*N	4	10813259	28163787	7040947	6028.78	0.000
CV*B*N	8	26744131	26744131	3343016	2862.45	0.000
Error	8712	10174642	10174642	1168		
Total	8799	60242804464				

S = 34.1744   R-Sq = 99.98%   R-Sq(adj) = 99.98%

Figures 26-30 shows the Main Effects Plot and the Interaction Plots for the throughput of the assembly lines simulated with a bowl shape workload configuration. As the F-statistic in the ANOVA test in Table 26 suggested, the main effects of CV and B are the most significant ones. Furthermore, the Interaction Plots in Figures 27-30 show that the two-way interaction of CV*B, CV*N, and CV*x are the most significant interactions.

The main effect plot of throughput (TR) vs. percentage of imbalance (x) shows that on average as the percentage of imbalance (x) increases the throughput of the line decreases. However, the Interaction Plots show that in lines with coefficient of variation of 0.8 and 1.4, or in lines with buffer capacity of 0, the slope of the effects throughput vs percentage of imbalance (x) is less pronounced. Suggesting that a bowl shape workload configuration will provide some improvements in lines with high coefficient of variation, and low buffer capacity, similarly to the conclusion drawn from Tables 19-21.

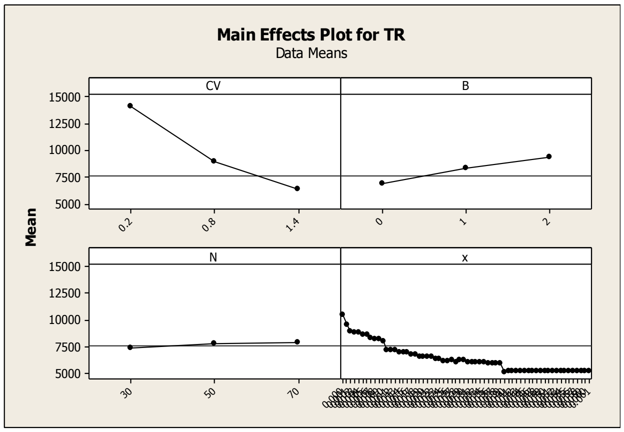


Figure 26.Main Effects Plot for throughput (One bowl configuration)

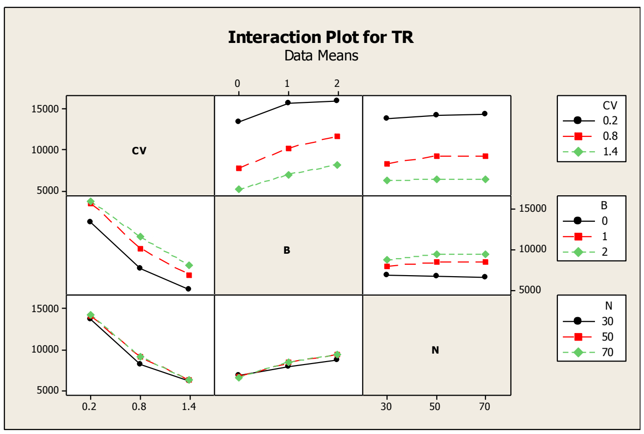


Figure 27. Interaction Plots for throughput (One bowl configuration)

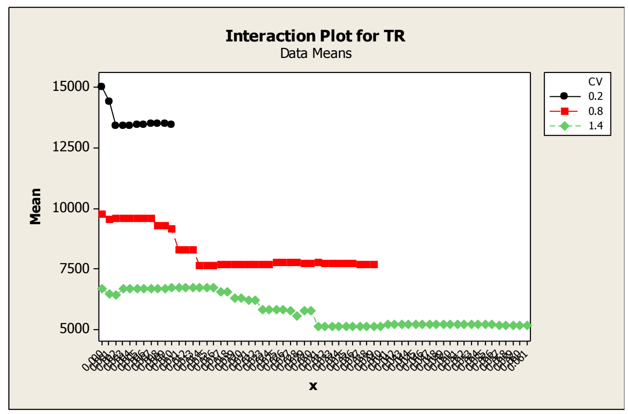


Figure 28. Interaction Plots for throughput, CV vs. x. (One bowl configuration)

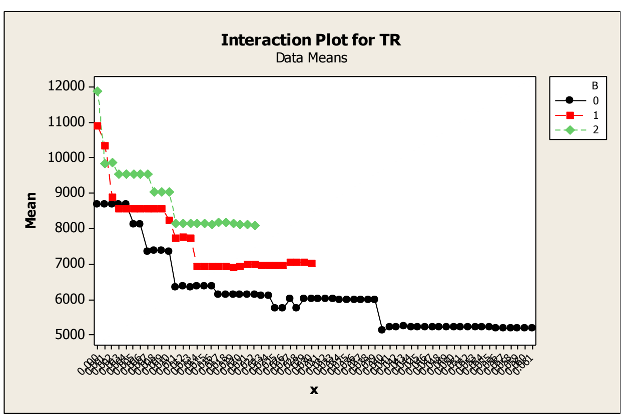


Figure 29.Interaction Plots for throughput, B vs. x. (One bowl configuration)

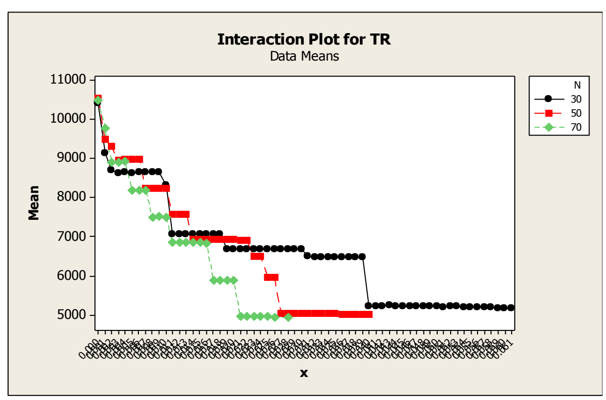


Figure 30.Interaction Plots for throughput, N vs. x. (One bowl configuration)

To validate the assumptions of the analysis of variance presented in Table 26 a residual analysis was conducted. Figure 31 show the residuals normality plot, residuals histogram, residuals vs. fitted values plot, and residuals versus order plot. The normality plot and the histograms suggest that the residual are normally distributed with mean of zero. However, the residual vs. fitted values plot does not support the constant varices assumption. This can be explained by the fact that one of our control variables (Coefficient of Variation) directly impacts the variance of the output data of the simulation model used. As in a normal production system, the variability of each of the workstation in the line directly affects the variability of the flow of the line itself. Therefore, the output data for the scenarios simulated with high CV will have greater variability than those scenarios simulated with low CV. This high, medium and low variability can be seem in the Residual vs Fitted value Plot. Extreme departure from the constant variance assumption might influence the confidence intervals of the analysis of variance.

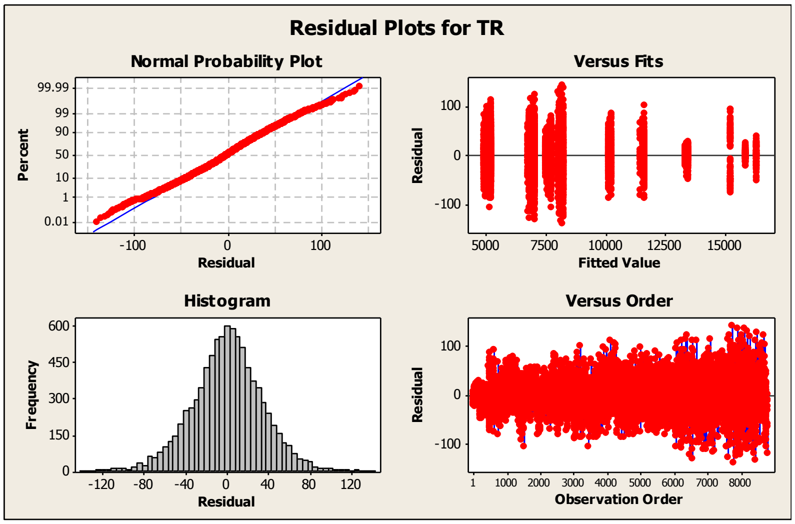


Figure 31. Residual Plots of the ANOVA for throughput (One bowl configuration)

In order to have a better understanding of the relationship between the control variables and the improvements that a bowl shape workload configuration can provide in large assembly lines, a regression analysis was conducted. In Table 19-21 it can be seen that the relationship between the response variable and the percentage of imbalance (x) is not linear, and that a second order polynomial model would fit the data better. To help in the model building and variable selection process a Stepwise Regression analysis was conducted in Minitab using the control variables showed in Section 4.2, their interactions, and a quadratic term of the control variable "x" (See Table 27).

Table 27. Stepwise Regression

Stepwise Regression: %Improvement in TR versus CV, B, ...
Alpha-to-Enter: 0.15 **Alpha-to-Remove**: 0.15
Response is TR Improv % on 15 predictors, with N = 440

Step	1	2	3	4	5	6
Constant	0.24123	0.04417	0.06286	0.07786	0.08279	0.05700
x	20.4	53.2	60.2	33.7	34.7	36.8
T-Value	17.17	17.80	20.23	8.40	9.64	12.56
P-Value	0.000	0.000	0.000	0.000	0.000	0.000
x^2		-205	-249	-301	-377	-446
T-Value		-11.69	-14.15	-17.57	-22.03	-30.48
P-Value		0.000	0.000	0.000	0.000	0.000
B*x			-10.8	-15.1	-16.0	-62.3
T-Value			-7.38	-10.64	-12.50	-19.17
P-Value			0.000	0.000	0.000	0.000
CV*x				25.9	47.3	64.7
T-Value				9.07	14.26	22.08
P-Value				0.000	0.000	0.000
CV*N*x					-0.394	-0.692
T-Value					-10.19	-18.64
P-Value					0.000	0.000
B*N*x						0.974
T-Value						15.03
P-Value						0.000
S	0.322	0.281	0.265	0.243	0.219	0.178
R-Sq	40.23	54.47	59.53	65.96	72.54	81.95
R-Sq(adj)	40.09	54.27	59.25	65.65	72.22	**81.70**
Mallows Cp	1134.4	762.1	631.4	464.3	293.5	48.2

Step	7	8	9	10
Constant	-0.029122	0.008046	0.015058	0.099337
x	41.3	40.4	42.2	39.5
T-Value	13.12	12.82	13.30	12.37
P-Value	0.000	0.000	0.000	0.000
x^2	-444	-460	-482	-491
T-Value	-30.70	-28.68	-27.77	-28.53
P-Value	0.000	0.000	0.000	0.000
B*x	-59.8	-57.9	-61.4	-69.0
T-Value	-18.27	-17.24	-17.50	-17.54
P-Value	0.000	0.000	0.000	0.000
CV*x	63.8	69.5	72.0	78.2
T-Value	21.98	18.16	18.59	19.03
P-Value	0.000	0.000	0.000	0.000
CV*N*x	-0.775	-0.842	-0.840	-0.909
T-Value	-17.96	-16.12	-16.23	-16.92
P-Value	0.000	0.000	0.000	0.000
B*N*x	0.914	0.874	0.848	0.995
T-Value	13.84	12.87	12.52	13.09
P-Value	0.000	0.000	0.000	0.000
CV*N	0.00188	0.00342	0.00352	0.00498
T-Value	3.62	4.02	4.18	5.50
P-Value	0.000	0.000	0.000	0.000
CV		-0.121	-0.184	-0.326
T-Value		-2.27	-3.26	-4.96
P-Value		0.023	0.001	0.000
CV*B			0.055	0.145
T-Value			3.12	5.11
P-Value			0.002	0.000
B*N				-0.00222
T-Value				-4.00
P-Value				0.000
S	0.175	0.174	0.173	0.170
R-Sq	82.49	82.69	83.08	83.69
R-Sq(adj)	82.20	82.37	82.72	**83.31**
Mallows Cp	36.2	32.7	24.7	10.7

The results of the Stepwise Regression Analysis suggested that a regression model with 6 regressor variables would provide an adequate model. Furthermore, the addition of more regressor variables would not provide a significant improvement in the descriptive power of the model. Since the R^2adj of the best 6 variable model was 81.70%, and the R^2adj of the best 10 variable model was 83.31%. Therefore, a regression analysis was conducted using the regressor variables of the best 6 variable model from the Stepwise Regression. (See Table 28)

Table 28. Regression analysis for percentage of improvement in throughput (%Improvement in TR).

The regression equation is

%Improvement in TR = 0.0570 + 36.8 x - 446 x^2 - 62.3 B*x + 64.7 CV*x - 0.692 CV*N*x + 0.974 B*N*x

Predictor	Coef	SE Coef	T	P
Constant	0.05700	0.01674	3.41	0.001
x	36.765	2.928	12.56	0.000
x^2	-446.25	14.64	-30.48	0.000
B*x	-62.255	3.247	-19.17	0.000
CV*x	64.692	2.930	22.08	0.000
CV*N*x	-0.69203	0.03712	-18.64	0.000
B*N*x	0.97367	0.06478	15.03	0.000

S = 0.177703   R-Sq = 82.0%   R-Sq(adj) = 81.7%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	6	62.095	10.349	327.73	0.000
Residual Error	433	13.673	0.032		
Total	439	75.769			

Based in the behavior of normal production systems and the relationship of the regressors with the response variable a no intercept model might seem more applicable. Due to the fact that in the case that the Percentage of imbalance (x) of the bowl configuration is 0 there should not be any improvement compare to the balance line because the line itself is balanced. However, as the T-statistic of β_0 suggest, this is the least significant Beta in the model. Moreover, the value is so insignificant that might not have any practical purpose (0.057%= 0.00057).

Finally, to validate the assumptions of the regression analysis presented in Table 28 a residual analysis was conducted. The normality plot and the histograms in Figure 32 suggest that the assumption that the residual are normally distributed with mean of zero is not heavily violated. Furthermore, the residuals vs. fitted values plot suggest that the variance of the residuals is constant. Even though, there can be seem some outliners in the data points these values relate to the scenarios were extreme values of Percentage of Imbalance were used and in which there were no statistical significant improvement in TR compare to the balance line (last percentage of imbalance simulated under each configuration of CV, B, and N).

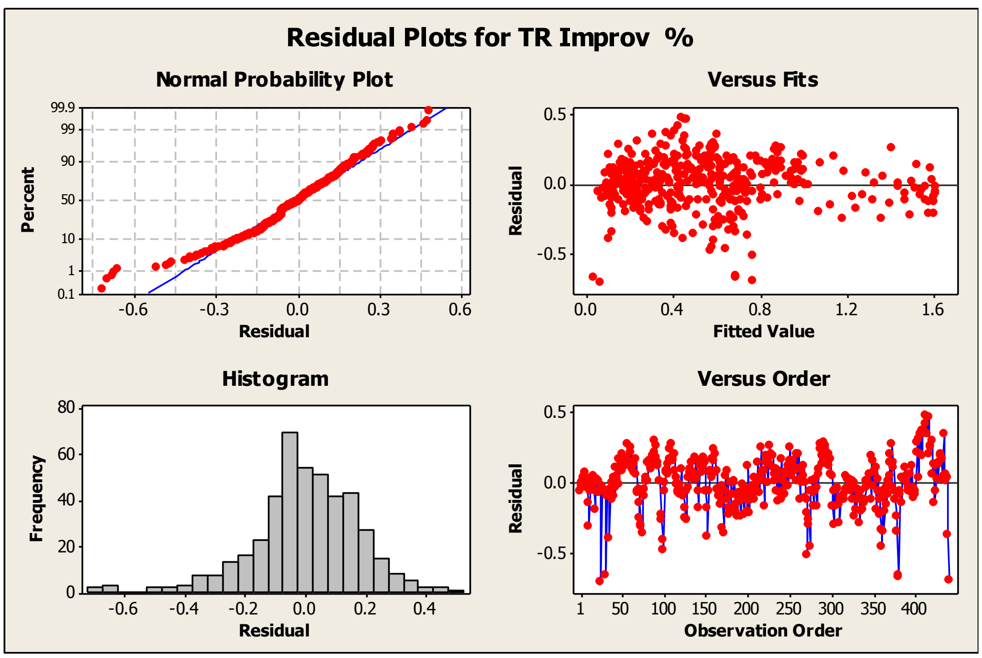


Figure 32. Residual Plots of regression analysis.

5.2. Multiple Bowl configuration

The assembly lines were simulated in Arena to determine the effects of line length (N), buffer capacity (B) and coefficient of variation (CV) on the throughput of an unbalance assembly line with a two bowl shape workload configuration. Section 4.3 presented the different levels of the control variables for the design of experiment used. Table 29 shows the assembly lines simulated, and the improvement achieved by the "best two bowl configuration" as well as the range of Percentage of Imbalance ("x") that produced statistically significant improvement in throughput, compared to the balance counterpart. Furthermore, the Percentage of Imbalance (x), the Percentage Increase in Work In Process (%Increase in WIP) , and Percentage Increase in Cycle Time (%Increase in CT) of the "best two bowl configuration" are also shown.. Statistically significant improvements in throughput are shown in 2 of the 8 assembly lines simulated. The results show improvements up to 0.37%. The improvement were only attained in lines with high coefficient of variation (CV=1.4). The results show an increase in WIP up to 0.30% and an increase in Cycle Time increases of up to .045% in comparison with the perfectly balance lines.

Table 29. Assembly lines simulated with two bowl configuration, and percentage of improvement achieved

Line	CV	B *(units)*	N *(workstation)*	X *(%)*	%Improvement in TR *(%)*	%Increase in WIP *(%)*	%Increase in CT *(%)*	Range of "x"*(%)*
1	0.2	0	30	0	0	0	0	0
2	0.2	0	70	0	0	0	0	0
3	0.2	2	30	0	0	0	0	0
4	0.2	2	70	0	0	0	0	0
5	1.4	0	30	0.024	0.373238703	0.257347	0.01209	[0.009-0.024]
6	1.4	0	70	0	0	0	0	0
7	1.4	2	30	0	0	0	0	0
8	1.4	2	70	0.004	0.292321117	0.307733	0.04578	[0.004-0.007]

Table 30 show the 95% CI of the throughput for assembly lines simulated with a two bowl shape workload configuration under the different line configurations. Table 31 show the 95%CI of the WIP of the assembly line simulated with a bowl shape workload configuration under different line configuration. The improvements achieved with the two bowl shape workload configuration are in most cases insignificant at an Alpha value of 0.05. Only in 2 out of the 8 assembly lines simulated, the improvements were significant at an alpha of 0.05. However, in just one assembly line the improvement was significant at an alpha of 0.01 (See Table 32).

Table 30. 95% CI of the throughput for the unbalanced assembly lines simulated with a two bowl shape configuration

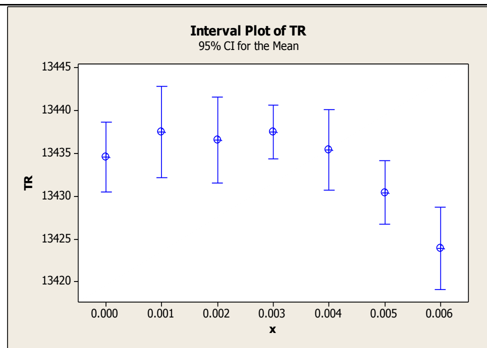
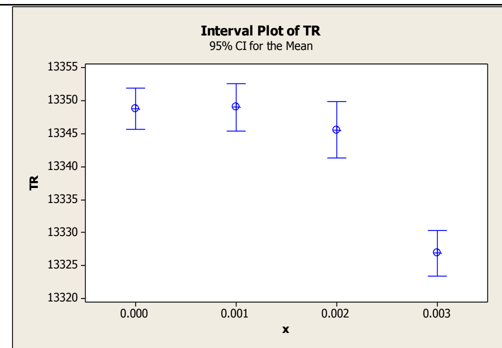
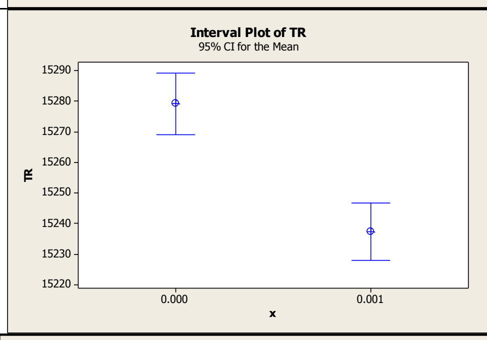
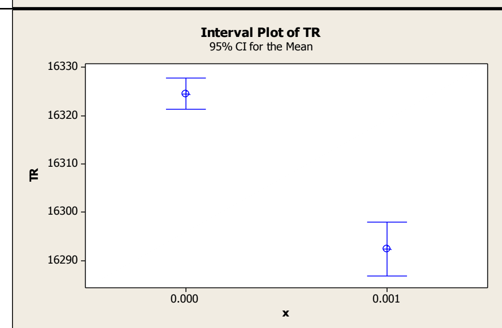
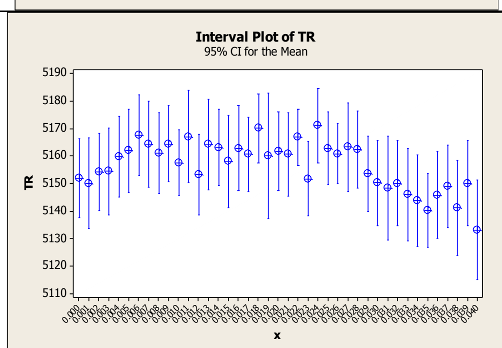
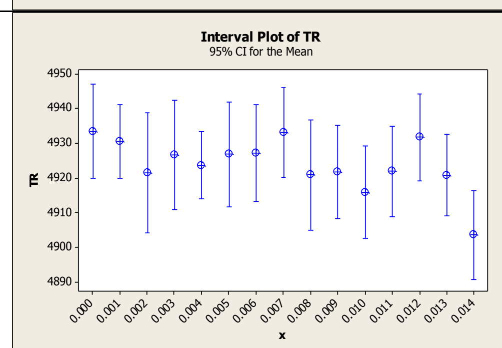
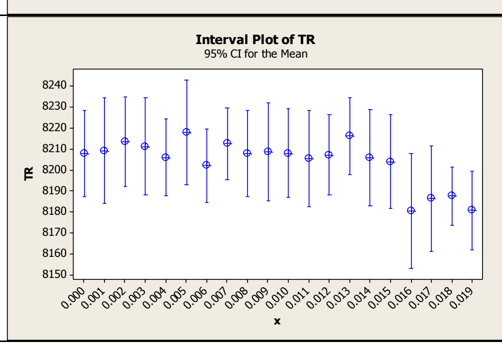
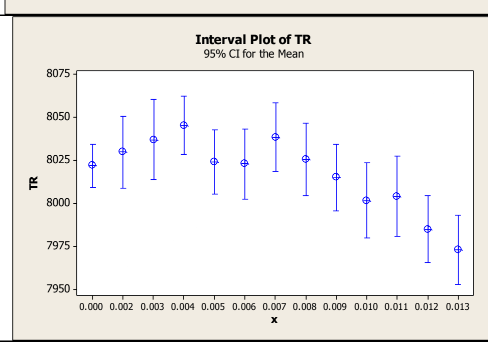
CV	B/N	30	70
0.2	0		
	2		
1.4	0		
	2		

Table 31. 95% CI of the WIP for the unbalanced assembly lines simulated with a two bowl shape configuration

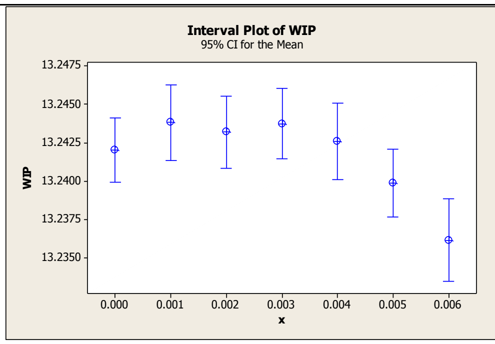
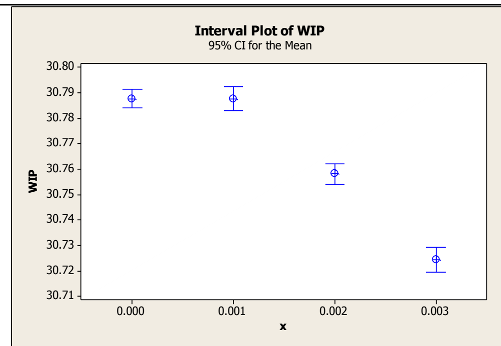
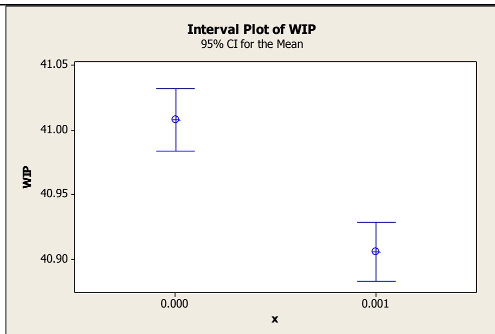
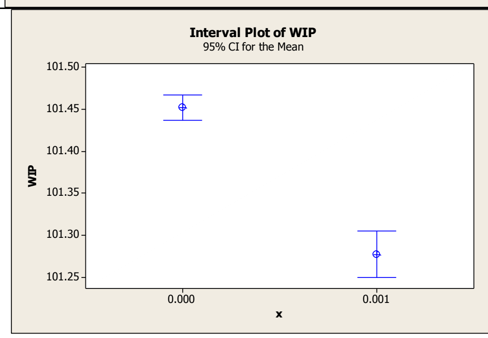
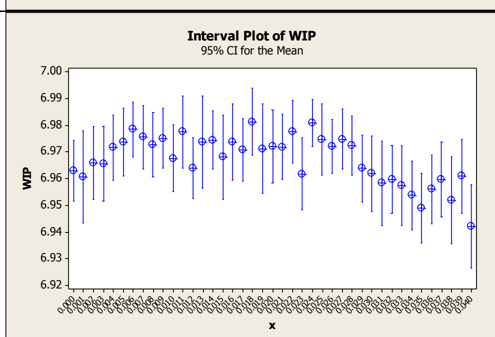
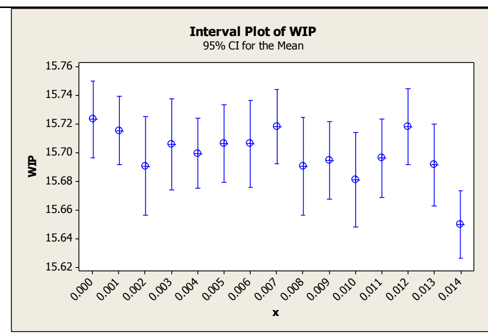
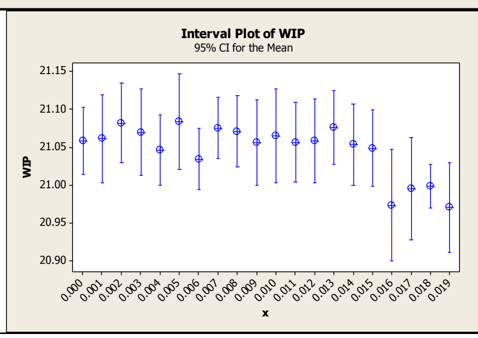
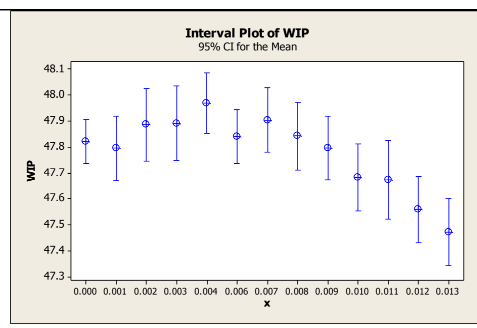
<table>
<tr><th>CV</th><th>B/N</th><th>30</th><th>70</th></tr>
<tr><td>0.2</td><td>0</td><td>
</td><td>
</td></tr>
<tr><td></td><td>2</td><td>
</td><td>
</td></tr>
<tr><td>1.4</td><td>0</td><td>
</td><td>
</td></tr>
<tr><td></td><td>2</td><td>
</td><td>
</td></tr>
</table>

Table 32. Paired T-test for the "best two bowl configuration" (CV=14, B=2, and N=70)

```
Paired T for 0.004 - 0

              N     Mean  StDev  SE Mean
0.004        20  8045.45  36.00     8.05
0            20  8022.00  26.61     5.95
Difference   20    23.45  42.43     9.49


95% CI for mean difference: (3.59, 43.31)
T-Test of mean difference = 0 (vs not = 0): T-Value = 2.47  P-Value = 0.023
```

Table 33 shows the analysis of variance conducted and the P-values of the main effects, multiple-way interactions effects, and blocking effect using a full factorial analysis. Because the values of the percentages of imbalance (x) were not balanced (equal) among the different experimental configurations, it was used as a blocking variable in the analysis. To determine the significance of the factors, the p-values were compared to an alpha (α) of 0.05.

Table 33 shows that the main effects and the multiple-way interaction effects are significant. However, the blocking factor was not significant. The Main Effects Plot in Figure 33 and the F-statistic and Adjusted Mean Square (Adj MS) in Table 33 shows that the main effects of CV and B are the most significant ones. Additionally, some of the variables that shown as statistically significant (base on their P-values), might not be noteworthy for practical uses. Because the large sample size (1,320 data points) drives the significance of some the variables. Therefore, to better understand the effect of the control variables over the response variable the Adjusted Mean Square is a better statistic to analyze.

Table 33. Analysis of Variance for throughput (Two bowl configuration)

Analysis of Variance for TR, using Adjusted SS for Tests

Source	DF	Seq SS	Adj SS	Adj MS	F	P
x	29	5248343723	26367	909	0.78	0.797
CV	1	6836311459	6620524648	6620524648	5650422.79	0.000
B	1	1958389790	961574148	961574148	820675.21	0.000
N	1	741	2226281	2226281	1900.07	0.000
CV*B	1	13971328	13228173	13228173	11289.86	0.000
CV*N	1	12003239	14618768	14618768	12476.69	0.000
B*N	1	4884929	9931873	9931873	8476.56	0.000
CV*B*N	1	9599991	9599991	9599991	8193.31	0.000
Error	1283	1503274	1503274	1172		
Total	1319	14085008474				

S = 34.2299   R-Sq = 99.99%   R-Sq(adj) = 99.99%

The main effect plot of throughput (TR) vs. percentages of imbalance (x) shows that in average as the percentages of imbalance (x) increases the throughput of the line decreases. However, the Interaction Plots show that under high coefficient of variation (CV= 1.4) the slope of the relationship throughput vs. percentages of imbalance (x) is less pronounced. Suggesting that a two bowl shape configuration will provide some improvements in throughput in lines with high coefficient of variation, similar to what the Table 30 suggested.

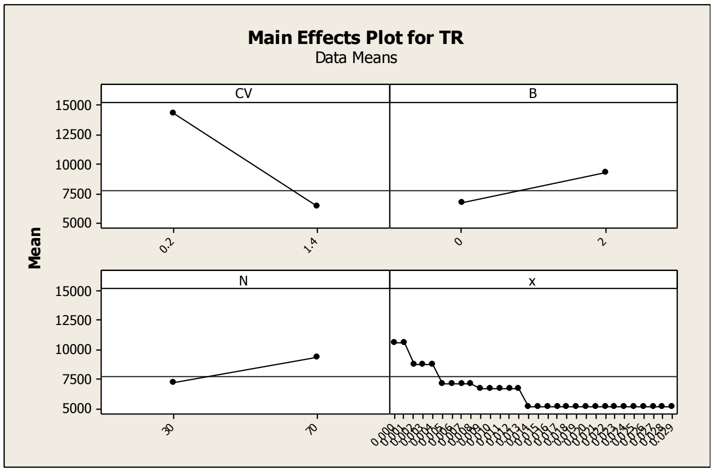


Figure 33. Main Effects Plot for throughput (Two bowl configuration)

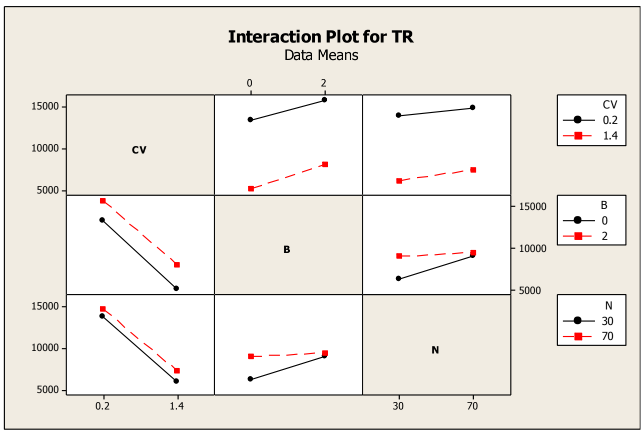


Figure 34. Interaction Plots for throughput (Two bowl configuration)

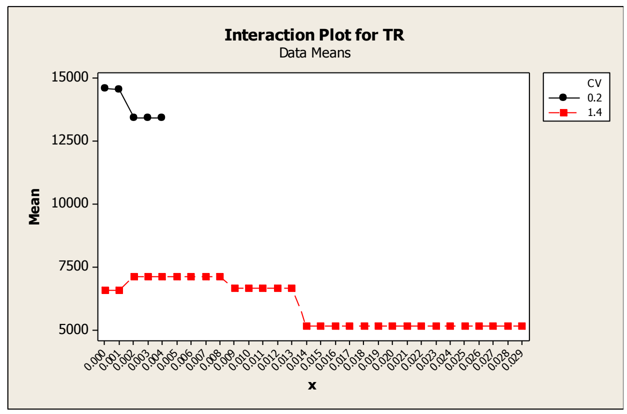


Figure 35. Interaction Plots for throughput, CV vs. x. (Two bowl configuration)

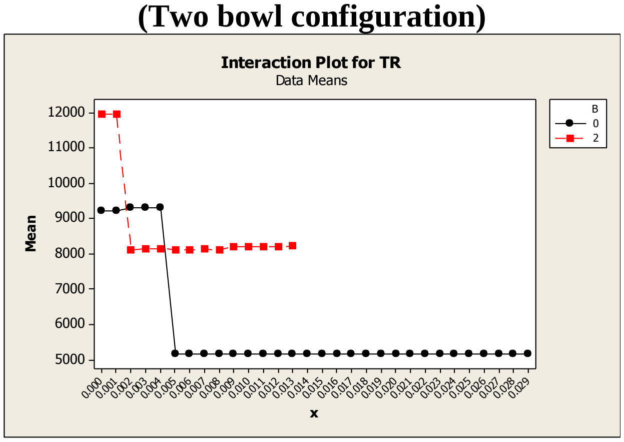


Figure 36. Interaction Plots for throughput, B vs. x. (Two bowl configuration)

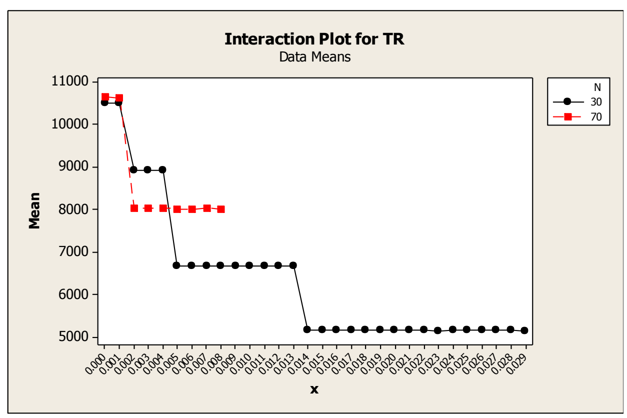


Figure 37. Interaction Plots for throughput, N vs. x. (Two bowl configuration

To validate the assumptions of the analysis of variance presented in Table 33 a residual analysis was conducted. Figure 38 show the residuals normality plot, residuals histogram, residuals vs. fitted values plot, and residuals vs. order plot. The normality plot and the histograms suggest that the assumption that the residual are normally distributed with mean of zero, is not heavily violated. However, the residual vs. fitted values plot suggest that the variance of the residual is not necessarily constant. This can be explained by the fact that one of the control variables (Coefficient of Variation) directly impacts the variance of the output data of the simulation model used. As in a normal production system, the variability of each of the workstations in the line directly affects the variability of the flow of the line itself. Therefore, the output data for the scenarios simulated with high CV will have greater variability than those scenarios simulated with low CV. This high and low variability can be seem in the Residual vs Fitted value Plot. Extreme departure from the constant variance assumption might influence the confidence intervals of the analysis of variance.

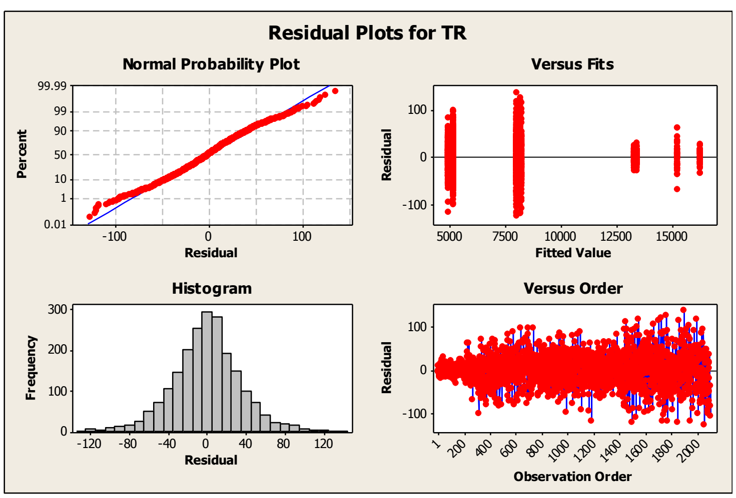


Figure 38. Residual Plots of the ANOVA for throughput (Two bowl configuration)

6. Summary and Conclusions

6.1. Summary

The experiment and analysis carried out in this study provide a better understanding of the relationship between the production rate of large assembly lines with a bowl shape workload configuration and its line length, buffer capacity and task time variability. The results lead us to the following conclusions about effects of the variables analyzed in this study and the bowl phenomenon:

- From the independent variable used in the study the most significant variable that had an impact on the production rate of a line was the coefficient of variation.
- Unbalancing a line in a bowl shape workload configuration could provide statistical significant improvements in throughput. The results of the experiment showed improvements up to 1.7% in comparison to the perfectly balance line.
- Unbalancing a line in a bowl shape workload configuration could increase the work in process of the line. The results of the experiment showed an increase of WIP up to 1.59% in comparison to the perfectly balance line.
- Unbalancing a line in a bowl shape workload configuration could increase the cycle time of the line. It showed increases up to 0.42% in the experiments conducted.
- Unbalancing the workload in a bowl shape configuration could improve the production rate even in lines with coefficient of variation as low as 0.2.
- The results of the experiment showed that as the buffer capacity of workstations in the line decreases from 2 to 0 the benefits the bowl phenomenon increases.

- As the buffer capacity of workstations in the line decreases from 2 to 0 the degree of imbalance of the “best one bowl configuration” increases.
- As the number of workstations in the line decreases from 70 to 30 the benefits the bowl phenomenon increases.
- As the number of workstations in the line decreases from 70 to 30 the degree of imbalance of the “best one bowl configuration” increase.
- The results suggest that as the coefficient of variation of workstations in the line increases from 0.2 to 1.4 the benefits the bowl phenomenon also increases.
- As the coefficient of variation of workstations in the line increases from 0.2 to 1.4 the degree of imbalance of the “best one bowl configuration” also increases.
- Unbalancing the line with a bowl shape workload configuration will produce more improvements than unbalancing it with a two bowl shape configuration.
- The experiments with the two bowl shape workload configuration only showed improvement in throughput in lines with coefficient of variation of 1.4.

6.2. Conclusions

In modern production systems that perform under high cost environments, even small improvements in line efficiency represents large savings over the lifetime of the line. In the beginning of modern production systems, it was thought that a ‘perfectly balanced’ line was the most efficient way to design the line. As shown in section 3.1 this stimulated a great amount of research in heuristic and near optimal algorithms for the Assembly Line Balancing Problem (ALBP) that aimed for the perfectly balanced line. Is interesting to see how line designers and production managers can improve their assembly lines at a low or no cost at all, by just re-distributing the workload of the line. The results of this study support the conclusion of previous

works (see Section 3.3), that have shown that unbalanced lines with a bowl shape workload configuration can yield performance as good as, or even better than their balanced counterpart.

As shown in section 5 as well as in previous studies (see section 3.1) line designers and production managers need to make an important managerial decision on how much they are willing to increase the WIP and CT of their lines in order to improve the throughput when implementing a bowl shape workload configuration. As seen in section 5, the WIP and CT increase linearly as the throughput of the line increases. Whether a production manager consider an increase in throughput as more beneficial than a reduced WIP and CT will depend on the costs of inventory, lost production, and other factors.

In one hand, in production systems which produce highly demandable and fast-moving goods it is probable that the main goal will be to improve throughput. For example, industries that manufacture fast moving goods such as computers, TVs, and appliances are in constant and increasing demand .In these systems, line designers and production managers will need to distribute the workload in a configuration that improves the throughput of their lines. On the other hand, the main priority of certain production systems is to keep WIP and CT low. For example, in the automotive industries, where just-in-time management requires lean buffering; or in industries where some fast moving consumer goods are highly perishable and cannot be stored for long periods of time (Shaaban and McNamar 2009).

Whether the main goal is to increase throughput production managers need to consider that the "optimal" bowl shape workload configuration can be a target that might not be possible to achieve under real conditions, because of technological and/or organizational constraints. Moreover, the percentage of improvement in throughout achieved in some of the lines presented

in this study might discourage many production managers to unbalance their assembly lines with a bowl shape workload configuration. However, they need to consider that the perfectly balanced line is also a target that might not be achieved in practice due to technological and/or organizational constraints, task variability or due to the performance rate of workers (see Carnall and Wild 1976).

Additionally, as seen in Table 17 there are bowl configurations with imbalance percentages above and under the "optimal" bowl configuration that performs statistically better than the balanced workload configuration. Moreover, Table 25 supports the finding of Hillier & So (1996), suggesting that in some cases line designers and production managers can achieved around 88% of the potential improvement in throughput that the "best one bowl configuration" can produced even if the amount of imbalance is either 50% more or less than the "optimal" configuration. Therefore, line designers and production managers should unbalance their assembly lines in a bowl shape workload configuration even though the "optimal" configuration might not be feasible.

In conclusion, considering the large costs involved in operating typical modern production systems achieving even 1.7% improvement in throughput is quite significant. Nevertheless, as shown in section 5 these improvements in throughput are correlated to an increase of the WIP and CT of the line. The WIP and the CT of the lines have a linear relationship with the improvement in throughput achieved by unbalancing the line in a bowl shape workload configuration, following Little's Law.

Therefore, line designers and production managers should concentrate their efforts on ensuring an “optimal” or at least an acceptable bowl shape workload configuration if the main goal is to improve the throughput of their lines and not to reduce WIP and/or CT. The results shown in this study will contribute to the body of literature of unbalanced lines in providing new understandings into how to design such large assembly lines with the objective of improving line efficiency and how the buffer capacity ,the coefficient of variation, and the number of workstations affects the bowl phenomenon.

6.3. Future work

This work only studied the effects of the bowl phenomenon in lines with one source of imbalance (the mean processing time/workload of the workstation), while having a balanced or equal coefficient of variation and buffer capacity throughout the workstations in the line. However, as some studies have shown (See Shaaban and Hudson 2012, and McNamara 2010) the benefits of the bowl phenomenon could provide improvement in efficiency greater than 1.7% in lines with multiple sources of imbalance. Meaning that not just the mean processing time of the workstation would be unbalance, but the coefficient of variation and the buffer capacity of the workstations would be unbalance as well.

Bibliography

Amen, M. (2000). An exact method for cost-oriented assembly line balancing. *International Journal of Production Economics* , 64:187–195.

Askin, R. G. (1997). A parallel station heuristic for the mixed-model production line balancing problem. . *International Journal of Production Research*, 35(11):3095–3105.

Baybars, I. (1986). A survey of exact algorithms for the simple assembly line balancing problem. *Management Science*, 909.

Becker, C., & Scholl, A. (2006). A survey on problems and methods in generalized assembly line balancing. *European Journal of Operation Research*, 694-715.

Boysen, N., Fliedner, M., & Scholl, A. (2006). A classification of assembly line balancing problems. *European Journal of Operation Research*, 674-693.

Boysen, N., Fliedner, M., & Scholl, A. (2008). Assembly line balancing: Which model to use when? *International Journal of Production Economics*, 509-528.

Carnall, C., & Wild, R. (1976). The location of variable work stations and the performance of production flow lines. *International Journal of Production Research*, 703-710.

Chakravarty, A. K. (1985). Balancing mixed model lines with in-process inventories. *Management Science*, 31:1161–1174.

Das, B., Garcia, A., MacDonald, C., & Ghoshal, K. (2010). A computer simulation approach to evaluating bowl versus inverted bowl assembly line arrangement with variable operation times. *International Journal of Advance Manufacturing Technology*, 15-24.

De la Wyche, P., & Wild, R. (1977). The design of imbalanced series queue flow lines. *Operational Research Quarterly*, 695-702.

Dudley, N. (1963). Work-time distributions. *International Journal of Production Research*.

El-Rayah, J. (1979). The efficiency of balanced and unbalanced production line. *International Journal of Production Research*, 61-75.

Ghosh, S., & Gagnon, R. (1989). A comprehensive literature review and analysis of the design, balancing and scheduling of assembly systems. *International Journal of Production Research*, 637-670.

Gokcen, H., & Baykoc, O.(1999). A new line remedial policy for the paced lines with stochastic task times. *International Journal of Production Economics* , 191–197.

Hatcher, J. (1969). The effect of internal storage on the production rate of a series of stages having exponential service times. *AIIE Transactions*, 150-156.

Hillier, F., & Boling, R. (1966). The effect of some design factors on the efficiency of production lines with variable operation times. *Journal of Industrial Engineering*, *17*(12).

Hillier, F., & Boling, R. (1979). On the optimal allocation of work in symmetrically unbalanced production lin systems with variable operation times. *Management Science*, 721-729.

Hillier, F., & So, K. (1993). Some data for applying the bowl phenomenon to large production line systems. *International Journal of Production Research*, 811-822.

Hillier, F., & So, K. (1996). On the robustness of the bowl phenomenon. *European Journal of Operational Research*, 496-515.

Hillier, M., & Hillier, F. (2006). Simultaneous optimization of work and buffer space in unpaced production lines wirh random processing times. *IIE Transactions*, 53-65.

Kara, Y. O. (2007). Balancing and sequencing mixed-model JIT U-lines with multiple objectives. . *Applied Mathematics and Computation*, 184(2):566–588.

Karwan , K., & Philipoom, P. (1989). A note on "Stochastic unpaced line design: review and further experiemental results". *Journal of Operation Management*, 48-54.

Kathiresan, G., Jayasudhan, T., Prasad, P., & Mohanram, P. .. (2012). Flow index based line balancing: a tool to improve the leanness of assemblyline design. *International Journal of Production Research* , 50(12): 3345-3358.

Kelton, D., Sadowski, R., & Swets, N. (2010). *Simulation with Arena.* New York: McGraw-Hill.

Klindworth, H., Otto, C., & Scholl, A. (2012). On a learning precedence graph concept for the automotive industry. *European Journal of Operational Research*, 259–269.

Knott, K., & Sury, R. (1987). A study of work-time distribution on unpaced tasks. *IIE Transactions*, 50-55.

Law, A. (2000). *Simulation Modeling and Analysis.* Boston, MA: McGraw-Hil.

Macaskill, J. L. (1972). Production line balances for mixed-model lines. *Management Science*, 19(4):423-432.

McMullen, P. R. (1997). A heuristic for solving mixed-model line balancing problems with stochastic task durations and parallel stations. *International Journal of Production Economics,*, 51:177–190.

McNamara, T. (2011). Unpaced production lines with three simultaneous imbalance sources. *Industrial Management & Data Systems*, 1356-1380.

Miltenburg, J. (1998). Balancing U-lines in a multiple U-line facility. *European Journal of Operational Research*, 109:1–23 .

Muth, E., & Alkaff, A. (1987). The bowl phenomenonrevisited. *International Journal of Production Research*, 161-173.

Pike, P., & Martin, G. (1994). The bowl phenomenon in unpaced lines. *International Journal of Production Research*, 483-499.

Quarles, J. (1967). An analysis of the effect fo stage allocation on sreies queue production systems. *Unpublished M.S. Thesis, University of Tennesee*.

Rao, N. (1976). A generalization of 'bowl phenomenon' in service production systems. *International Journal of Production Research*, 437-443.

Rosenberg, O. (1992). A comparison of heuristic algorithms for cost-oriented assembly line balancing. *Operations Research* , 36:477–495.

Sabuncuoglu, I. E. (2000). Assembly line balancing using genetic algorithms. *Journal of Intelligent Manufacturing*, 11:295–310.

Salveson, M. (1955). The Assembly Line Balancing Problem. *The Journal of Industrial Engineering*, 18-25.

Scholl, A., & Becker, C. (2006). State-of-the-art exact and heuristics solution procedures for simple assembly line balancing. *European Jornal of Operational Research*, 666-693.

Shaaban, S., & Hudson, S. (2012). Transient behaviour of unbalanced lines. *Flexible Services adn Manufacturing Journal*, 1936-6582.

Shaaban, S., & McNamara, T. (2009). Improving the efficiency of unpaced production lines by unbalancing service time means. *International Journal of Operational Research*, 346-361.

Sheskin, T. (1976). Allocation of interstage storage along an automatic production line. *AIIE Transactions*, 146-152.

Silverman, F. N. (1986). A cost-based methodology for stochastic line balancing with interminnent line stoppages. *Management Science*, 32: 450–462.

Smith , L., & Brumbaugh, P. (1977). Allocating inter-station inventory capacity in unpaced production lines with heteroschedastic processing time. *International Journal of Production Research*, 163-172.

Smunt, T., & Perkins, W. (1985). Stochastic unpaced line design: review and further experimental results. *Journal of Operations Management*, 351-373.

So, K. (1989). On the efficiencyof unbalancing prodcution lines. *International Journal of Production Research*, 717-729.

Sparling, D. (1998). Balancing Just-in time production units: the N u-line balancing Problem. *Information Systems and Operational Research*, 36(4)215-237.

Talbot, R. R. (1991). Improving the equality of workload assigments in assembly lines. *International journal of production research* , 29(3):619-633.

Templemeir , H. (2003). Practical considerations in the optimization of flow production systems. *International Journal of Production Research*, 149–170.