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Multispectral imaging and analysis of the Archimedes Palimpsest

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Multispectral Imaging and Analysis of the Archimedes Palimpsest

By

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requirements for the degree of Master of Science in the
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ABSTRACT

The Archimedes Palimpsest is a manuscript that has been preserved for approximately 1,000 years. Among its pages are some of the few known sources of treatises from the Greek mathematician Archimedes. The writing has been overwritten with prayer text, called the Euchologion, and portions of the faded Archimedes text are difficult to read. This research investigates methods to detect the presence of ink in the Archimedes Palimpsest using state-of-the-art image processing techniques applied to data from X-ray fluorescence (XRF) scans.

In an effort to extract more legible text, various methods of imaging have been applied to the Archimedes manuscript. Recent X-ray fluorescence images of the palimpsest suggest the possibility of detecting individual text layers and isolating them from each other. This is encouraging, since many of the pages have also been partially masked by gold-leafed, Byzantine-style artwork, making the Archimedes writing difficult to see with the human eye.

The scans measure the X radiation emitted by atoms on the pages that have been excited by other higher energy X rays incident to the parchment. This caused certain elements within the manuscript, such as the iron in the ink, to fluoresce at energies that are specific to the particular material. A total of 2,000 different energy levels, or bands, were recorded. To evaluate the data contained in this large number of bands, a single data set was created that included all bands, referred to as a "datacube," which shows the transition of each pixel through the spectrum. Special image processing tools, developed for use in the field of remote sensing to process aerial and satellite data, can be used to detect certain

patterns within the datacube. Each tool is then used to segregate the noise from the relevant data in the datacube. The datacube for this thesis research was created from a small portion of one page of the Archimedes Palimpsest, and may inherently be subject to certain noise limitations.

This study focuses on two main objectives:

- Evaluation of X-ray fluorescence data to determine which energy levels contain useful information about the layers of text.
- Creation of a pseudocolored composite RGB image of a portion of enhanced Archimedes text, similar to previous pseudocolored MSI images.

Results from this study show that only a few regions within the datacube contain information relevant to the layers of text. Certain algorithms, such as principal component analysis and minimum noise fraction, showed distinct information about trace elements fluorescing in the ink and parchment. Meaningful data near the spectral line of each trace element was detected after disbanding the datacube into smaller regions. Enough information was obtained as a result to create colorized RGB composite images that enhance the contrast of the Archimedes writing relative to the overwritten text.

It is hoped that this research can improve the method for identifying useful bands of information within datacubes. The research may also have created a repeatable method for detecting useful bands of information in similar datacubes. State-of-the-art multispectral imaging applications were specifically applied to detect, extract, and enhance previously illegible writings that are of interest to scholars and museums in particular.

CHAPTER 1

INTRODUCTION

1.1 The need to preserve ancient writings

Written records have been preserved on cave walls, plant leaves, tree bark, wax tablets, animal skins, papyrus scrolls, and other materials. Only in the mid-1400s was a new method developed for applying ink to parchment with the advent of movable type. Prior to this time, scribes studied the art of calligraphy and illumination in schools and spent many hours duplicating texts by hand. Many of these duplicated manuscripts have survived to the present day, including the Dead Sea Scrolls discovered in 1947 (Israel Museum, 2005). Other collections of manuscripts were stored in ancient libraries and eventually destroyed. Examples of burned and carbonized scroll fragments can been found in the excavated city of Herculaneum in Italy and the Petra Church in Jordan (Booras & Seely, 1999).

Modern discoveries of ancient manuscripts have piqued the interest of many scholars and researchers. Unfortunately, many of the materials are difficult to decipher due to the effects of mold, rot, humidity, and erosion. More insidious is the gradual destruction of these manuscripts under human hands. Great texts have sometimes been harshly erased in order to make the writing material clean enough for newer texts. This was often a method much less costly than manufacturing a fresh sheet of animal skin or papyrus. When new ink was applied to the freshly scraped parchment, the remnants of what was originally conceived on the manuscript became masked, or palimpsested, by overwriting. The term palimpsest originates from the Greek roots "palin" and "psēn," meaning "to rub or scrape"

and is applied when writing material has been "used one or more times after earlier writing has been erased" (Merriam-Webster, 2005b).

1.2 The Archimedes Palimpsest

Sometime in the tenth century, many of Archimedes' mathematical writings and treatises were copied from papyrus scrolls to goat skins. This Greek manuscript was transcribed in two columns on both recto and verso sides of the folio leaves. The leaves were then stitched into quires and sewn together. The bound volume (Figure 1) was called a codex and was the ancient predecessor to the modern book (Noel, et al., 1999). Few preserved codices still exist today. Those that remain include the Leningrad Codex, written in 1010 AD, housed in the National Library of Russia in St. Petersburg (Vasilieva, 2000), and the Aleppo Codex, which dates back to 900 AD, at the Shrine of the Book in Jerusalem (Israel Museum, 2005)—two of the world's oldest complete copies of the Hebrew Bible.

Figure 1. The Archimedes Palimpsest is an example of a bound codex. *Image copyright resides with Christie's Images, 1998.*

It is likely that the treatises of Archimedes were transcribed in the city of Constantinople. Two hundred years later, probably around the time of the Fourth Crusade in 1204, the pages of the manuscript were ripped apart and the iron-gall ink rubbed away. The cleaned pages were then recycled into a new text, this time to make a Christian prayerbook, the Euchologion, considered at the time to be far more important than mere mathematical theories (Noel, et al., 1999).

1.3 Identifying the Archimedes text

This new prayerbook survived the ages in the hands of Greek monks who lived in the monastery of Mar Saba. In 1846, the scholar Constantine von Tischendorf discovered the book in the Metochion in Constantinople. Though he did not recognize Archimedes' faded mathematics underneath the Euchologion, he found the writing interesting enough to tear out a page and keep it in his possession until he died (Noel, 2001, p. 200). The book's impending significance as an anthology from one of the most influential early Greek mathematicians, however, remained unacknowledged.

The source of the mathematical text was at last identified and attributed to Archimedes in 1906 by Johan Ludvig Heiberg, a Danish scholar. He also identified portions of the text containing unique material, including Archimedes' *Method of Mechanical Theorems*. On July 16, 1907, the front page of *The New York Times* lauded the "big literary find in Constantinople" (Noel, et al., 1999). Shortly afterward, however, the manuscript disappeared, but not before Heiberg captured the first photographs of the palimpsest. These photographs, consisting of sixty-five leaves of the manuscript, are presently archived in Copenhagen's Royal Library.

1.4 A need for multispectral imaging

After its disappearance, the Archimedes Palimpsest resurfaced in Paris in the 1970s (Kolata, 2003, p. 1). A French family had acquired the manuscript and was eager to sell it for profit, though it is not known if the family knew of the connection between the writings and Archimedes. The palimpsest had deteriorated over the years and many portions of text were covered with mold and candle wax. Gold-leafed, Byzantine-style illuminations had also been carefully forged over several pages of Euchologion prayers, masking nearly all of the underlying Archimedes text. The condition of the palimpsest at present is compared to the 1906 photographs in Figure 2.

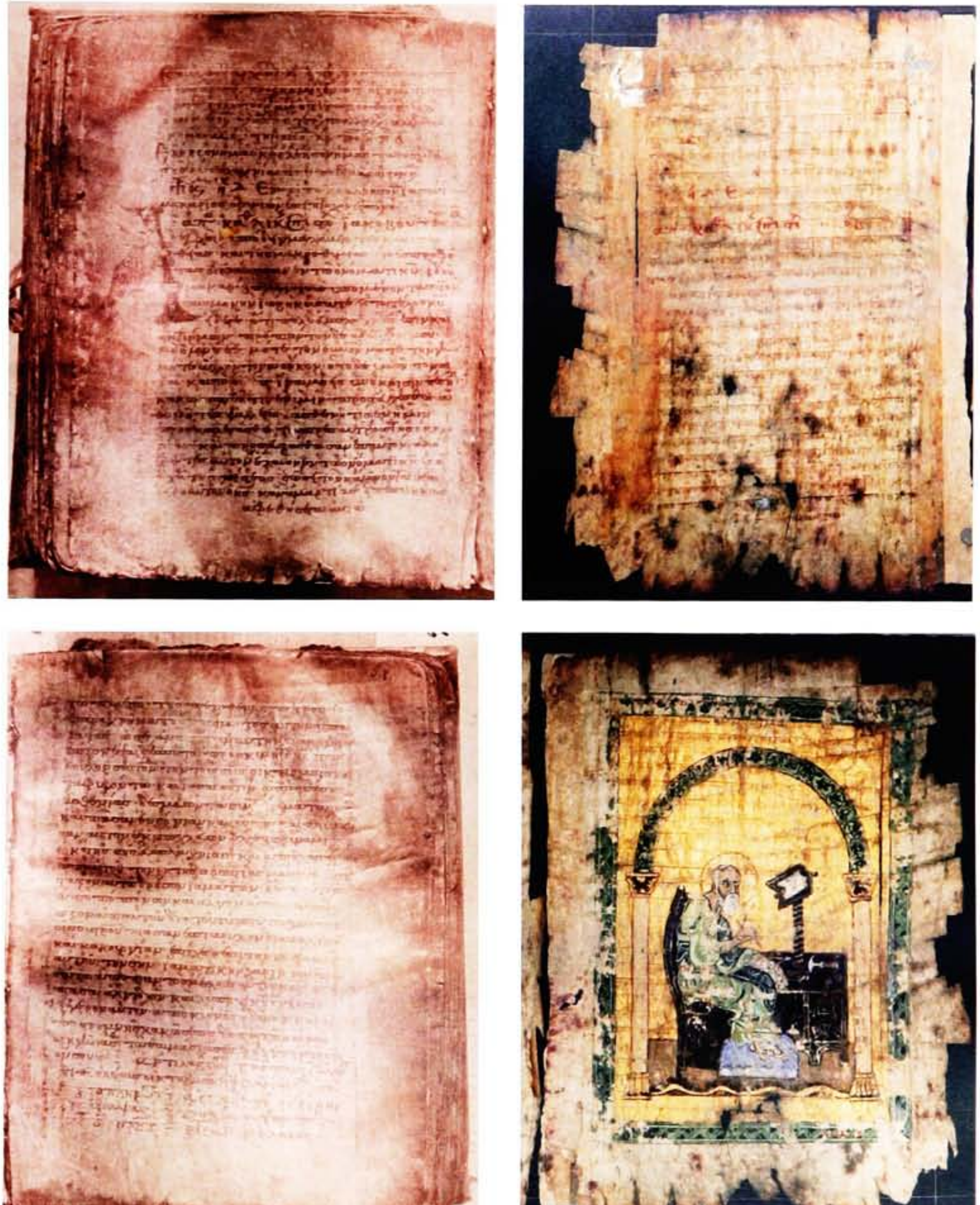


Figure 2. Photographs of the palimpsest taken by Johan Ludvig Heiberg in 1906 (left) compared to the same manuscript pages in their current condition (right). The pages today are worn and moldy. Some pages have been completely masked by forged artwork. *Copyright resides with the owner of the Archimedes Palimpsest.*

Nigel Wilson, from Lincoln College, Oxford, found Tischendorf's stolen leaf at Cambridge University Library in 1983 and recognized that it belonged to the Archimedes Palimpsest (Noel, 2001, p. 200). The manuscript was then auctioned under some controversy in 1998 for $2 million at Christie's in New York to an anonymous owner (Browne, 1998b, p. A27). The Archimedes Palimpsest now resides in the Walters Art Gallery in Baltimore, Maryland. The private owner is interested in having scholars examine the text with advanced imaging techniques to perhaps produce an enhanced facsimile of the manuscript as it may have appeared centuries ago. The manuscript contains parts of seven treatises by Archimedes, and is currently the only existing source for the *Method of Mechanical Theorems* and *Stomachion* treatises and the only source in Greek for *On Floating Bodies* (Walters Art Museum, 2005).

Since 2000, scientists at the Rochester Institute of Technology have been helping to identify and enhance the faded Archimedes text using multispectral analysis (Knox, et al., 2001, p. 206). New data sets acquired in 2005 using X-ray fluorescence remain yet to be examined. Such data, in combination with previous MSI images, could yield improved legibility of the text.

The researcher became interested in applying multispectral imaging (MSI) to ancient manuscripts in 2001 while working for the Institute for the Study and Preservation of Ancient Religious Texts at Brigham Young University. Specific projects included working with digital images of Herculaneum and Petra scrolls. It is hoped that this research on the Archimedes Palimpsest will be beneficial to scholars and provide a method that will be repeatable for future imaging opportunities.

CHAPTER 2

THEORETICAL BACKGROUND

2.1 Introduction

Though infrared photography has been widely applied to ancient studies in the twentieth century, scrolls and other ancient manuscripts have been digitally scrutinized for only a decade. More advanced digital imaging tools, such as multispectral imaging, now involve capturing large arrays of digital information about the manuscripts. Multispectral imaging, developed primarily for satellite earth-imaging and remote sensing, has become an invaluable tool for archaeological imaging studies (Chabries, Booras and Bearman, 2003, p. 360).

Spectral and molecular data can be collected destructively or non-destructively. Several non-destructive imaging methods have been applied to the Archimedes Palimpsest. In 2001, the manuscript was imaged under three different illuminations and the images were captured at different wavelengths. In 2005, the actual energy response of different elements within the manuscript was collected through the use of X-ray fluorescence. Once multispectral data is gathered, special remote sensing algorithms can then be used to organize and filter the data into meaningful conclusions.

2.2 Charge-coupled device (CCD)

Digital cameras and many digital instrument detectors contain an array of sensors that process light into digital signals. One commonly used array of sensors is called a charge-coupled device, or CCD. Such arrays are now available in sizes of up to 6,000 x

4,000 pixels (R. Easton, personal communication, March 14, 2006). Before reaching the sensor, the incident light usually passes through several filters placed over the sensor. Each sensor in the array detects the sample of photons and converts the signal into a digital value. The use of filters enables the CCD to determine whether that digital value corresponds to a specific color, red (R), green (G), or blue (B). The RGB filters are usually arranged in an order commonly known as a Bayer color filter array. One advantage of the Bayer array is that resolution is enhanced by using twice as many green filters than red or blue filters. Once a digital color value has been recorded, then special algorithms are used to interpolate the other two color values based on the values of the surrounding pixels (Palum, 2001, p. 241). After every sensor has sampled the light in terms of RGB values, a camera profile or computer software converts these values into a universal color space. One commonly recognized color space is known as L*a*b*, or CIELAB, from which the color values can be accurately interpolated throughout the rest of the imaging workflow (Sharma, 2004, p. 161).

The latest CCD technology has demonstrated that the range of sensitivity of silicon is between 250-1150 nm, rivaling film (Schott, 1997, pp. 131, 144; see also Chabries, Booras and Bearman, 2003, p. 361). Silicon is naturally adept at detecting longer wavelengths in the near-infrared region than human eyes and film out to approximately 1000 nm. Silicon can also be coated with phosphors that convert ultraviolet light to longer wavelengths within the region of sensitivity, thus allowing the CCD to measure light with shorter wavelengths. "[A] disadvantage of these coatings is the loss in spatial resolution due to light scattering" (Kodak, p. 12). The electromagnetic spectrum is shown as Figure 3.

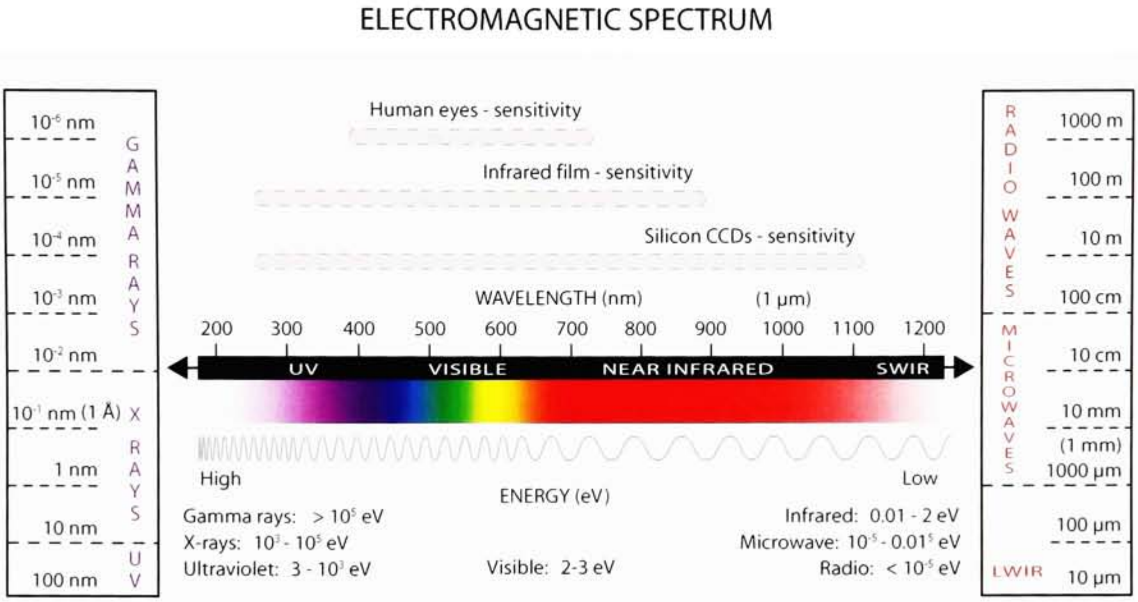


Figure 3. The electromagnetic spectrum. The sensitivities of certain materials are shown. *Illustration by D. Michael Hansen.*

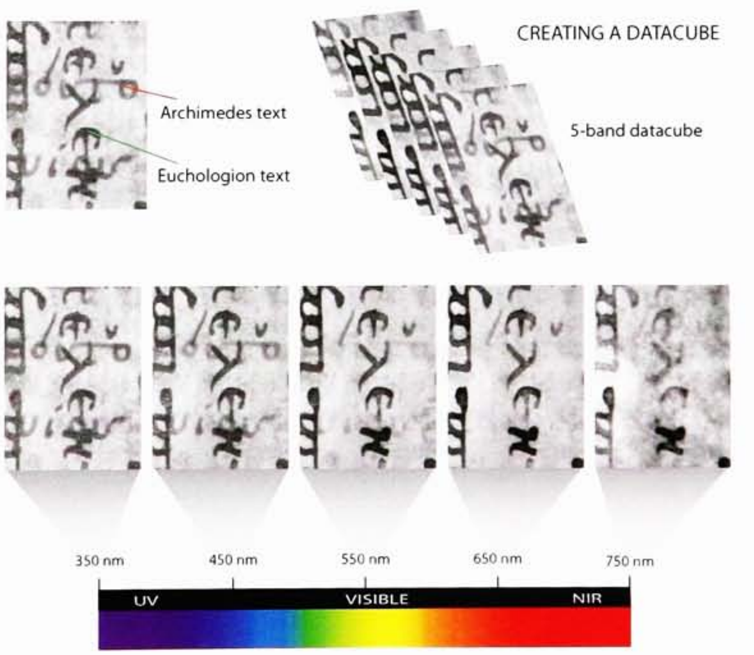


Figure 4. A multispectral datacube contains many bands or channels of spectral data. The Archimedes text is visible in ultraviolet wavelengths, but begins to fade toward the near-infrared region of the spectrum. *Archimedes images copyright resides with the owner of the Archimedes Palimpsest. Illustration by D. Michael Hansen.*

2.3 Multispectral datacube

The purpose of multispectral imaging is to observe the transition of pixels, through reflectance or fluorescence, at different wavelengths. Multispectral data sets contain many channels of spectral information from a given scene, often extending beyond normal human vision. The number of frequencies observed determines the number of information bands in the data set. The capability of the imaging sensor will determine the limitation of the number of bands detected. Each spectral band or channel of data can be rendered in pixels to show a unique image of the same scene at different wavelengths.

Single bandpass and electronic tunable filters are often used in combination with different illuminants to enhance the contrast of the image at different wavelengths. Each band can be examined in isolation or combined as an datacube with other bands, as shown in Figure 4. While sufficient contrast may indeed be available from a single bandpass filter, images from several filters may be stacked into a datacube and processed to detect any existing patterns that may occur over several frequencies (Chabries, Booras & Bearman, 2003, p. 360). These algorithms, such as principal component analysis (PCA), minimum noise transform (MNF), and spectral classification, may be used to isolate useful information from many layers of noise.

2.4 X-ray fluorescence

Wilhelm Conrad Röntgen, a German physicist, first discovered X rays on November 8, 1895 (*Encyclopædia Britannica,* 2006a). Röntgen was conducting experiments with a cathode-ray tube when a piece of barium platinocyanide in the room began to fluoresce. He attempted to cover the barium with paper, wood, and aluminum to block any visible and ultraviolet light that may have caused the fluorescence. When his attempts failed, Röntgen

supposed that some strange invisible radiation was the cause and gave it the nickname X-radiation (*Encyclopædia Britannica,* 2006b).

X-ray fluorescence is a non-destructive method of image analysis that may be used to detect traces of a particular element. Fluorescence from the Archimedes Palimpsest was measured using synchrotron X-ray radiation at the Stanford Linear Accelerator Center (SLAC) in California. A synchrotron X-ray beam is a carefully tuned, highly collimated excitation source. This means that high-energy X rays can be focused on an atom, increasing the energies of electrons within the atom from their ground state (lowest energy) to an excited state (highest energy) (*Encyclopædia Britannica,* 2006a).

When the beam is focused at a specific energy, incident X rays may be absorbed by the inner electron shells of an atom (Figure 5). If an X ray is powerful enough, it can collide with an inner electron, transfer its energy to that electron, and knock it out of the atom's shell, making the atom unstable. As a result, higher-energy electrons from the outer shells

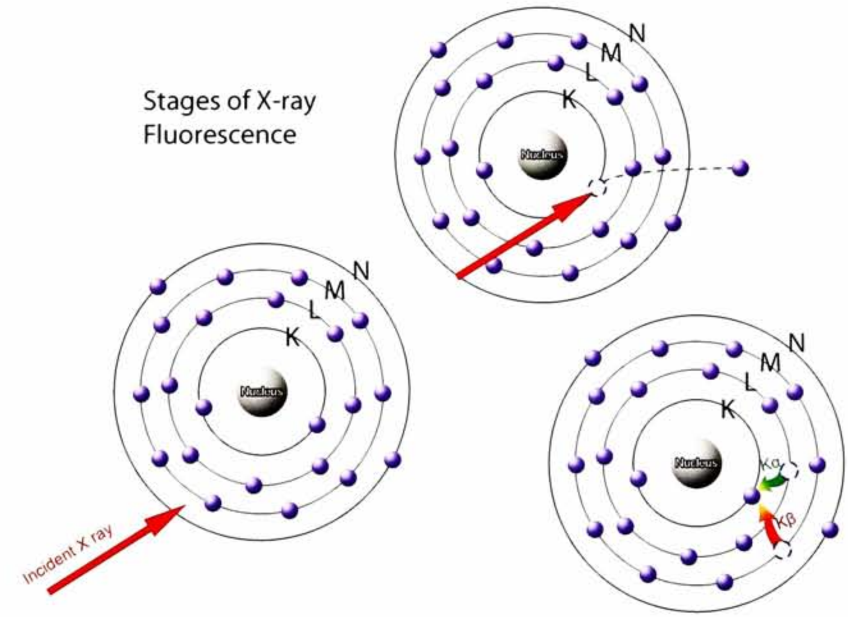


Figure 5. Incident X rays can cause inner electrons to be ejected from the atom. When higher-energy electrons are used to fill in the hole, radiation is emitted. *Illustration by D. Michael Hansen.*

jump down to fill the vacancy. As the electron transitions from an outer valence level to the inner valence level, a loss of energy or radiation produces an emitted X ray photon, such as Kα or Kß radiation. This radiation is a detectable spectral phenomenon called fluorescence. The specific nomenclature for the type of fluorescence that occurs depends on the energy levels of the electron before and after the transition (AmpTek, 2002).

An X-ray detector may then be used to sense the intensities of each fluorescing electron, which can be recorded and plotted as a histogram of energy intensity (keV). The synchrotron X-ray beam then slowly scans across the manuscript in a zig-zag motion, line by line. The spectrum of leaf 163 of the Archimedes Palimpsest is shown as Figure 6.

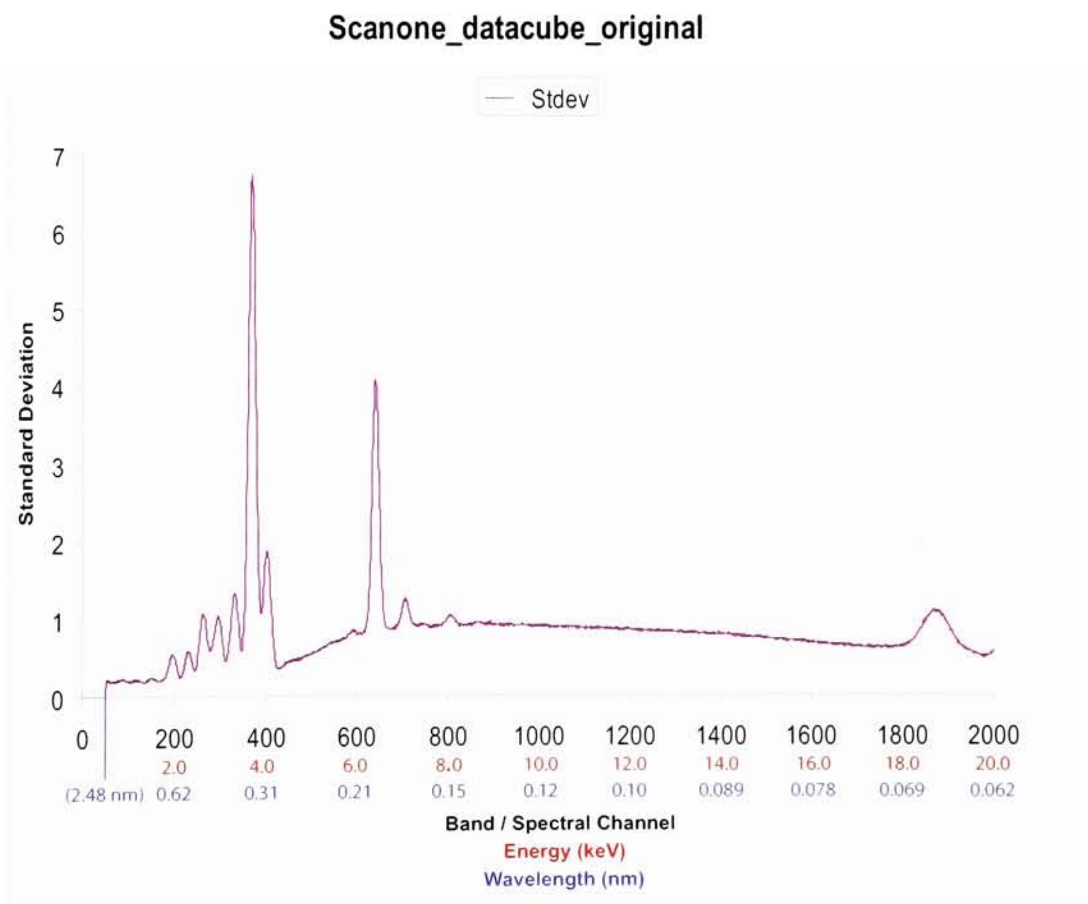


Figure 6. Spectrum of the Scanone_datacube.

Although X-ray fluorescence is technically considered a non-destructive method of imaging, X rays can actually cause damage to the manuscript if the intensity of the beam is concentrated on one spot for more than a few milliseconds (Woods, 2005a). The Canadian Conservation Institute conducted experiments to determine the effects of X-ray emission on parchment. It was concluded that the changes in the thermal stability of the parchment were not significant enough to cause damage to the Archimedes Palimpsest, and that the overall permanence of the manuscript was not compromised (Young, 2005, p. 8).

2.5 ENVI software

Software for processing images and analyzing spectral data in environmental remote sensing was developed by Research Systems Inc. The software is especially helpful for viewing multispectral images and identifying patterns by using advanced algorithms. The name "ENVI" is an acronym for "Environment for Visualizing Images" (RSI, 2004a, p. 24). One advantage of the software is its ability to work with an entire data set or simply a few chosen bands. ENVI software is widely accepted in the remote sensing community as an effective tool for analyzing multispectral data.

2.6 Principal component analysis

Datacubes containing one to ten spectral channels (multispectral), or more than ten spectral channels (hyperspectral), can be processed using principal component analysis (PCA) (Schott, 1997, p. 148). PCA is used to "decorrelate the data and maximize the information content," reducing the number of bands needed for further processing (Schott, 1997, p. 281). Identifying principal components can be especially useful in segregating patterns of noise in the EDAX 2000-band hyperspectral datacube.

Principal components give a visual representation of data based on the statistics of the data. For instance, PCA organizes data into either a covariance or correlation matrix and

calculates a mean value for the data based on this matrix. The axis of the histogram with the largest standard deviation (the greatest amount of variance) from the mean is isolated as the first principal component. The data points are then projected onto an orthogonal axis, and data with the next largest variance become the second principal component. The process of projecting data onto an orthogonal axis continues for every band in the datacube. Each new orthogonal axis creates a new origin for the data, and a new mean can be calculated. In other words, for each principal component, data variance is maximized "by finding a new set of orthogonal axes that have their origin at the data mean" (RSI, 2004b, p. 675).

Rotating the axis orthogonally means that the mean of the first vector becomes the new x-axis of the second vector. Thus, one vector is said to be at right angles with the other. Mathematically, orthogonal rotation involves creating a second rotated matrix, so that "the sum of the products of the corresponding elements [in both matrices] in any two rows or any two columns is equal to one" (Merriam-Webster, 2005a; see also Easton, R., Jr., 2005). The resulting principal components show the largest variance in the first several bands, while the least amount of variance is typical of the last several bands. Eigenvalues are also automatically generated from principal component analysis and should correspond with the visual assessment (Schott, 1997, p. 282; see also Easton, R., Jr., 2005). If a band has a large eigenvalue, chances are that the band contains relevant data. Bands with small eigenvalues are likely to contain mostly noise (RSI, 2004b, p. 680).

2.7 Minimum noise fraction

Another way of reducing the noise and dimensionality of a datacube with many bands is the minimum noise fraction (MNF) transform. Every pixel in each band is compared to adjacent pixels to determine the percentage of a pixel's gray value that is "signal" and that is "noise." Similar to PCA, the statistics are organized into either a covariance or

correlation matrix. Based on these noise statistics, the MNF transform then decorrelates and rescales the noise in the data using principal component analysis. Once the estimated noise has been decorrelated from the original data, PCA is used again to identify principal components in the remaining data. In other words, "the MNF transform…is essentially two cascaded principal component transformations" (RSI, 2004b, p. 682). As a result of PCA, eigenvalues are again calculated for every principal component. Bands with large eigenvalues contain relevant data, while bands with smaller eigenvalues most likely contain only noise.

2.8 Spectral classification

Another method of analyzing multispectral data utilizes spectral classification and mapping of pixel clusters. Spectral classification involves categorizing pixels with similar characteristics within the datacube. Different algorithms can be used, each with a unique method of finding a predetermined estimated number of pixel regions. Both unsupervised and supervised classification algorithms can be used.

Unsupervised classification algorithms, such as K-Means and Iso-Data, analyze corresponding pixels in each band and create regional clusters according to the highest percentages of probability. The K-Means algorithm carefully scrutinizes the image for mean vectors in each of the specified number of classes. The distance of each pixel from its closest mean vector defines its class (Schott, 1997, p. 279). The IsoData classifier similarly identifies evenly distributed mean vectors, but also incorporates a minimum distance algorithm to help classify a particular pixel according to its closest mean (RSI, 2004a, p. 104).

2.9 Spectral classification—supervised

If pixels are already known to belong to a certain class, then these pixels can be labeled as ROIs, or regions of interest (RSI, 2004b, p. 594). Classification algorithms can be guided or supervised with the help of ROIs. The software identifies pixel regions in all vectors with the highest percentages and attributes each pixel to its most probable class. Statistical reports about each class can then be generated and used for post-classification processing. Several algorithms may be used for supervised classification, each based on unique calculation parameters. Common supervised algorithms include maximum likelihood, minimum distance, Mahalanobis distance, and parallelepiped.

Some algorithms produce better results than others, with results varying for different data sets. The maximum likelihood algorithm simply determines the highest probability that a pixel belongs to a particular class (RSI, 2004a, p. 107). The minimum distance classifier works similarly to the unsupervised K-Means classifier, which identifies mean vectors and the shortest distance from each pixel to the closest mean vector (RSI, 2004a, p. 108). The difference is that the mean vectors are calculated from the predetermined ROIs. The Mahalanobis distance algorithm "is a direction sensitive distance classifier...similar to the Maximum Likelihood classification but assumes all class covariances are equal," resulting in a faster algorithm using multispectral sample points to identify cluster centers (RSI, 2004a, p. 108; see also Schott, 1997, p. 274).

The parallelepiped algorithm also identifies the mean for each vector. However, only pixels that fall into a defined threshold around the mean will belong to the class with that mean. The threshold is the shape of a parallelepiped, or a many-sided shape with parallelograms for each side (RSI, 2004a, p. 106). The number of bands in the datacube usually determines the number of sides of the parallelepiped. Including too many bands in

the datacube will likely result in improperly defining the shape of the spectrum, resulting in unsuccessful classification results (Schott, 1997, p. 271).

2.10 EDAX datacube limitation

It is assumed that remote sensing algorithms will be useful for identifying patterns within the Archimedes Palimpsest datacube. However, the effectiveness of each algorithm in processing 2,000 bands of data may be compromised because of its size, with a greater likelihood that many of the bands contain a large amount of noise.

CHAPTER 3

A REVIEW OF LITERATURE IN THE FIELD

3.1 Introduction

Extracting spectral information digitally from ancient manuscripts was first developed just over a decade ago. The purpose of this literature review is to provide some background on the development of multispectral imaging for ancient studies, including reproduction of manuscripts. The early use and preparation of iron-gall ink and parchment is also discussed.

3.2 Developing MSI for ancient manuscripts

Methods of multispectral imaging have been shown to improve text legibility in several collections of ancient manuscripts. Many collections of old writings have been rescued from harsh climates and some form of corrosion. Documents may be rotted or even charred and crumble under the slightest pressure. Legibility of any existing writing is perhaps the greatest concern. Because some documents are extremely difficult to read, scholars often wait until the proper time of day, when the sunlight is just right, to carefully examine these fragile manuscripts. For this reason, scholars have been interested in various multispectral imaging techniques to detect spectral information beyond the limits of the human visual spectrum.

Infrared photography has revealed much of what the eye cannot see within burnt and damaged scrolls. Still, depending on the ink composition, the amount of carbonization, and the condition of the scrolls, the text may still be difficult to read. Since 1993, multispectral

imaging has been successfully used to examine ancient manuscripts by extracting data in the extreme ends of the light spectrum, including ultraviolet and near-infrared regions, using digital sensors more sensitive to light than infrared film (Chabries, Booras and Bearman, 2003, p. 362; see also Bearman and Spiro, 1996). Such data may then be processed and enhanced to classify spectral signatures and improve text legibility.

Near-infrared and ultraviolet photographic imaging of ancient manuscripts produced varied results in the mid-1900s. Older manuscripts seemed to respond better to infrared, while younger manuscripts responded well to ultraviolet wavelengths. Infrared imaging finally improved in the early 1990s with the development of a small-scale electronic imaging system at NASA's Jet Propulsion Laboratory, which included a liquid-crystal tunable bandpass filter ranging from 400 nm to 1100 nm, surpassing even the sensitivity of film. This imaging detector was made of smaller instruments, including a new spectrometer "that did not require motion for image acquisition," and could be mounted on a variety of small imaging devices (Chabries, Booras and Bearman, 2003, p. 362). Powerful remote sensing instruments used in aerial and space exploration could now be used for experiments at a much closer range.

This improved multispectral imaging device was then chosen to test a deteriorating fragment of the Dead Sea Scrolls, called the *Genesis Apocryphon,* at the Getty Conservation Institute in California. Sufficient contrast for the entire *Genesis Apocryphon* was achieved using a bandwidth of 15 nm and centered about a wavelength of 970 nm, increasing the amount of recognizable text by approximately 20 percent. The result demonstrated that text legibility could be improved with the use of an infrared filter and CCD sensor. This worked especially well for carbon-based inks found in the *Genesis Apocryphon* and most fragments of the Dead Sea Scrolls (Chabries, Booras and Bearman, 2003, p. 364).

3.3 Other ancient manuscript projects involving MSI

In 1998, scholars from the Center for the Preservation of Ancient Religious Texts (CPART) at Brigham Young University applied multispectral imaging to archaeology and ancient manuscripts. One of CPART's initial projects involved recovering handwriting from 180 papyri discovered in Petra, a Byzantine church built into the face of a sheer rock wall in Jordan. Around approximately 600 AD, the church was destroyed in a fire that also consumed several scrolls (Chabries and Booras, 2001, p. 195). An attempt at multispectral imaging revealed that the amount of carbonization varied throughout the scrolls. A number of filters would ultimately be needed to capture the best spectral response of each image, ranging from 450 nm to 950 nm wavelength with a 7 nm bandwidth. This time the images were captured at a higher spatial resolution. Until then, near-infrared imaging had never been applied to so many scroll fragments at such a high resolution (Chabries, Booras & Bearman, 2003, p. 367). Scientists from CPART and BYU concluded that imaging using various bandpass filters could indeed provide enough contrast for further textual analysis and enhancement.

Following this project, others also approached Brigham Young University about conducting imaging research in the Italian villa of Herculaneum and on ancient caves in Guatemala. Approximately 1,800 papyrus scrolls had been found while excavating Herculaneum. The papyri had been badly burned and carbonized after the volcanic eruption of nearby Mt. Vesuvius in 79 AD, which covered the entire city and its inhabitants almost instantly with thick layers of volcanic ash. Since their discovery, the scrolls have proven not only difficult to read, but also complicated to unravel, with pieces of outer layers adhering to the inner layers (Chabries, Booras and Bearman, 2003, p. 368).

Scholars from CPART traveled to Naples, Italy, to conduct imaging experiments on the Herculaneum scrolls. Using eight narrow-band filters with 40 nm bandwidth and a spectral range of 400 nm to 950 nm wavelengths, the team was able to image several fragments, including both those that were well-preserved and those that were significantly charred. The results showed that the greatest contrast between the ink and papyrus was achieved in the near-infrared region, at approximately 950 nm (Booras and Chabries, 2001, p. 216). The entire collection of papyrus scrolls, which was housed in 4,400 glass frames, was imaged during a subsequent eleven-month trip to the Biblioteca Nazionali in Naples (Chabries, Booras and Bearman, 2003, p. 368).

When compared with the Petra scrolls, the Herculaneum scrolls showed greater contrast in near-infrared wavelengths, perhaps because the ink was vegetable-based rather than based on carbon or iron-gall. The extent of carbonization may also have affected the portions of the manuscripts with varying contrast (Booras and Seely, 1999, p. 100). According to CPART, "the results of even simple computer manipulation and enhancement suggest that the multispectral images preserve enough data to make further work in computer enhancement possible to vastly improve the reading of the text" (Booras and Chabries, 2001, p. 217).

Depending on the imaging device, the capture of an entire page of a manuscript in a single frame may yield inadequate resolution. For this reason, images may be taken one small section at a time and digitally stitched together. This method was employed in both the Herculaneum and the Petra Church scroll projects (Chabries & Booras, 2001, pp. 197-198). A scanning device can also alleviate this problem.

Many museums, including the Vatican Library in Italy, maintain an interest in preserving access to their collections. In 2002, CPART scholars captured 14,000 images of

ancient Syriac Christian documents housed in the Vatican Library, including multispectral images of a palimpsest (Walch, 2004). The MSI images were captured within a spectral range of 400 nm to 1000 nm wavelengths. Principal component analysis and unique classification algorithms were used to successfully identify the textual layers of the palimpsest and enhance the underwritten text (Wheeler, 2004). Additionally, CPART used multispectral imaging to enhance the Pauline Epistles as part of a project to image biblical codices archived in the Smithsonian's Freer Gallery (CPART, 2005).

3.4 Different methods of MSI used in ancient studies

Numerous drawings found on the walls of Naj Tunich, an ancient Mayan cave in Guatemala, were also subjects for multispectral analysis. It was initially supposed that ten different artists had composed the artwork. In 1998, multispectral images were captured using optical interference filters ranging from 400 to 1000 nm wavelength with 40 nm bandwidth. While the images appeared mostly dark at visible wavelengths, the reflectance at near-infrared wavelengths was sufficient to identify distinct spectral signatures using remote sensing classification algorithms. Classification revealed spectral signatures of three unique pigments, including carbon-based and mineral-based inks. Many of the drawings were even found to originate from the same pigments. It was deduced that the original carbon-based drawings had actually been retouched using mineral-based pigments, and that only a few individual artists were responsible for the artwork. In this case, multispectral analysis both supported and contested previous theories (Ware, Brady and Martin, 2001, pp. 211-214).

X-ray fluorescence has also been a useful multispectral imaging technique for detecting trace elements that may remain in parchment and even stone. The Cornell High Energy Synchrotron Source (CHESS) in New York was used to detect 2,000 year-old stone

inscriptions that had faded over time. The Cornell University News Service fascinatingly described the stone and the use of X-ray fluorescence to enhance the writings:

> The chosen inscriptions—one in Classical Greek and two in Latin—each presented different levels of wear. XRF imaging detected minute amounts of iron, zinc and lead in the inscribed regions, among other elements. Iron chisels were commonly used to inscribe the stones, and the letters were usually painted with pigments containing metal oxides and sulfides. These may account for the iron and lead, but the source of the zinc is a mystery. In the most worn stone, the trace elements measured by XRF clearly revealed the contours of the original letters, even where they were no longer visible to the eye. For modestly worn stones, XRF imaging will help to decipher texts and may provide new information on how the inscriptions were made (Crawford, 2005).

Another non-destructive tool for multispectral analysis is Raman spectroscopy, which supersedes even X-ray fluorescence by extracting molecular information about a particular particle, such as ink pigment. Leaves of several Gutenberg Bibles from libraries throughout Europe were analyzed using Raman spectroscopy to identify and compare the the diversity of the ink pigments. Although the bulk of the text was printed using Johann Gutenberg's system of movable type, many capital letters and ornaments were left to be fashioned by other scribes and artists. It was supposed that the ink palettes used by the artists likely varied according to the artist's location. An analysis of pigments in all the pages, including the King George III version in the British Library, showed that the artist's palette was likely to contain a mix of both common and unique pigments. Such information

about the ink can be useful to curators in preservation efforts, which may involve cleaning, retouching, and determining archival conditions for the manuscripts (Henry, 2005, p. 49).

3.5 Character recognition and facsimile reproduction

High-resolution digital imaging has also been implemented by archaeologists to produce images suitable for archiving and reproduction, giving scholars greater access to limited collections. Such noteworthy facsimile reproductions include that of the Leningrad Codex, photographed by Bruce Zuckerman in 1990 (Freedman, et al., 1998). Digital images of the Dead Sea Scrolls have been compiled on a CD-ROM with accompanying translations, packaged with WordCruncher™, a proprietary optical character recognition (OCR) program developed at Brigham Young University (FARMS, 2000). Character pattern recognition software has also been developed by RIT students for the Archimedes Palimpsest and a 15th Century Florentine codex, providing scholars with tools to identify whole, incomplete, and missing characters based on spectral, statistical, and even contextual analysis (Walvoord, 2004; see also Walvoord, Easton, Knox and Heimbeuger, 2004).

3.6 A tale of two inks

An ink is typically composed of colorants, vehicles, and additives. Colorants include insoluble pigments and soluble dyes. The vehicles are the resins used to bind the colorant to the substrate, while the additives affect the flow behavior and abrasion resistance of the ink. Inks characteristic of early manuscripts are often either carbon- or iron-based, and were often dependent on the choice of substrate, which was usually papyrus or parchment.

Carbon-based inks made from the leftover soot of oil lamps can be traced back to the third century BC. From cave paintings to papyrus, this type of ink was often mixed with water and gum to improve its consistency and permanence. Whether used on papyrus or parchment, however, the ink did not stain the substrate, often smudged, and was prone to

flaking (Fruen, 2002; see also Eusman, 1999). For this reason, many artists sought a recipe for a more permanent, indelible ink.

A dye known to turn papyrus black was discovered by Pliny the Elder, who lived from 23-79 AD (Fruen, 2002). The dye was produced by mixing tannic acid and iron (II) sulfate, also known as ferrous sulfate or vitriol. The most robust tannic acid that caused the ink mixture to have a rich blue-black hue was found in gall nuts. Insects infested the trees and left behind tiny growths on the leaves and branches, which could be crushed and boiled to release the acid (Karnes, 1999).

As the tannin mixes with ferrous sulfate, the oxygen in the air causes a chemical reaction with the solution, darkening the mixture. The concentrated pigment is then diluted with water or vinegar, after which gum arabic is added to disperse the pigment uniformly throughout the solution. The gum arabic also acts as a binding agent to keep the ink from becoming immediately absorbed into the substrate during application (Karnes, 1999).

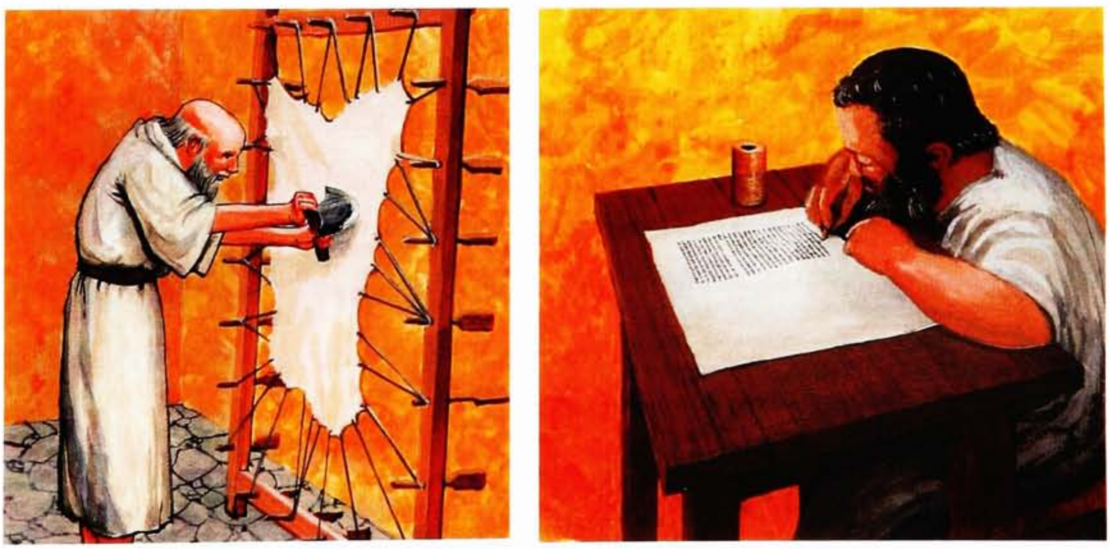
One significant drawback of using iron-gall ink was the nature of the ink's components; if not mixed in correct proportion, they could significantly damage the material. Excessive amounts of tannic acid could eventually eat through the parchment, leaving holes where the written characters used to be. The acid would also sometimes discolor the colorant left on the parchment. Furthermore, large amounts of gum arabic would prevent the ink from absorbing into the manuscript fibers and contribute to flaking (Karnes, 1999). Such effects are commonly found in surviving manuscripts, including the Archimedes Palimpsest.

The use of iron gall ink was widespread from the Middle Ages to the twentieth century. The Library of Congress Conservation Division mentions that iron-gall ink was typical of early "manuscripts, music scores, drawings, letters, maps, and official documents

such as wills, bookkeeping records, logs, real estate transactions, etc." (Eusman, 1999). The Dead Sea Scrolls are known to contain carbon inks, iron-gall inks, and even red ink containing mercury sulfide, or cinnabar (Tov, 2002, p. 199). The two layers of writing found in the Archimedes Palimpsest are both iron-gall, though centuries apart in age (Noel, 2001, p. 199).

3.7 Parchment preparation

Parchment was carefully prepared by removing the skin from the animal and soaking it in a solution made of salt or lime. After several days, the skin was removed, rinsed, and carefully stretched on a wooden frame (Figure 7). A pumice stone was often used to smooth and whiten the skin without puncturing the material (Lyon, 1997, p.40). Next, a tanning solution was applied to the skin to improve its appearance and perhaps the permeability of the new parchment. Finally, the parchment was cut into the desired number of pieces and dimensions, which were usually estimated beforehand by the length of the text (Tov, 2002, p. 189). Then the scribe essentially became the artist, often sitting hunched



Figures 7 & 8. Skin was stretched on a frame and scraped with a stone, after which it could be used for writing. *Illustrations by Michael Lyon. Copyright 1997. Used with permission.*

over the manuscript and fashioning each written character or design by hand with great care (Figure 8).

Only a few of the Dead Sea Scrolls were written on papyrus; most were written on the skins of cows, sheep, goats, gazelles, and even ibexes (Tov, 2002, p. 189). The parchment used for the Archimedes Palimpsest was prepared from the skin of goats (Noel, 2001, p. 199). Animal skin was chosen over papyrus not so much for its appearance, but rather for its structural and mechanical properties. First, the leather is more durable, even when folded into tiny pieces and housed in phylacteries. Parchment was also less porous than papyrus, and therefore less susceptible to water damage. Substrates for writing were often expensive commodities in ancient times, especially papyrus imported from Egypt. Parchment, on the other hand, could be produced locally (Lyon, 1997, p. 40).

This is likely one of the main reasons the Archimedes Palimpsest was considered a good candidate for erasing: not only were mathematics secondary to Christian writings, but the parchment itself was sufficiently strong to withstand harsh rubbing, unlike papyrus. Once a manuscript was written, the only way to alter or remove the text was by "scribal intervention," which included blotting or crossing the word out; adding supplementary elements, such as cancellation dots, to the margins; reshaping the existing character, transforming it into the desired character; and readjusting word spacing (Tov, 2002, p. 201). In the case of the palimpsest, writing could also be removed by harshly rubbing and washing the text away altogether (Noel, 2001, p. 199).

3.8 Imaging the palimpsest

Since 2000, imaging analysis has been used to recover faint underwriting beneath layers of ink, mold, wax, glue, and forged illuminations in the Archimedes Palimpsest. Using Heiberg's 1906 photographs as references, an international team of scientists from

Rochester Institute of Technology, Johns Hopkins University, and Xerox Corporation began examining the delicate manuscript under tungsten, longwave ultraviolet, and xenon strobe illumination (Knox, et al., 2001, p. 207).

The sensor used for imaging the palimpsest was a specially manufactured Kodak 1602E "Blue Plus" CCD, with greater sensitivity at short wavelengths. From images taken using a Bessel filter set with bandwidths of 100 nm centered at wavelengths between 350 nm to 750 nm, the two inks demonstrated different behaviors at each 100 nm interval. The Archimedes text showed a significantly larger reflectance under longwave ultraviolet illumination than tungsten (Figure 9). When the image was viewed as separate monochromatic red (R), green (G), and blue (B) channels, the writing appeared most visible in the red and green channels. A similar, but less dramatic result was achieved under tungsten illumination, in which the Archimedes text showed the greatest reflectance in the blue channel. Overall, the variance of the Archimedes text at every 100 nm interval was greater than the variance of the Euchologion text (Knox, et al., 2001, pp. 206-209). Thus, each ink could be described as having a unique signature. When compared to the spectral response of the parchment, the two normalized signatures showed distinctly different responses, with the Archimedes text having a greater transmittance across the spectrum (Figure 10).

The blue channel from the ultraviolet lighting and the red channel from the tungsten images were selected to form a composite pseudocolor image (Figure 11). The Euchologion text in both images was balanced, or normalized, with respect to the parchment. The tungsten red channel image was used as the red channel for the RGB composite image, and the ultraviolet blue channel was used for both the green and blue composite channels. When the normalized images were combined, the Euchologion text appeared very dark or black, and the Archimedes text appeared to have a reddish tint, shown as Figure 12. This

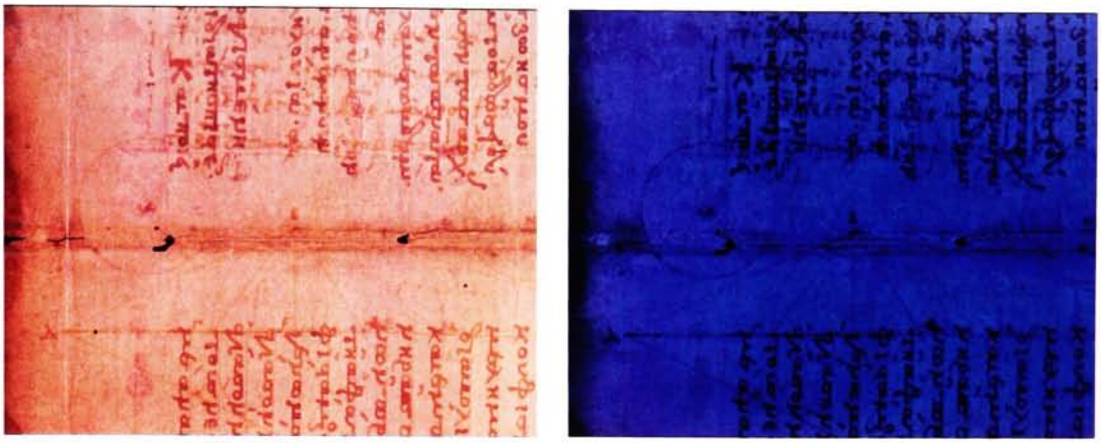


Figure 9. Diagrams in the palimpsest that are barely visible under tungsten illumination (left) are easily seen under ultraviolet illumination (right). *Copyright resides with the owner of the Archimedes Palimpsest.*

SPECTRAL SIGNATURES OF THE INK LAYERS

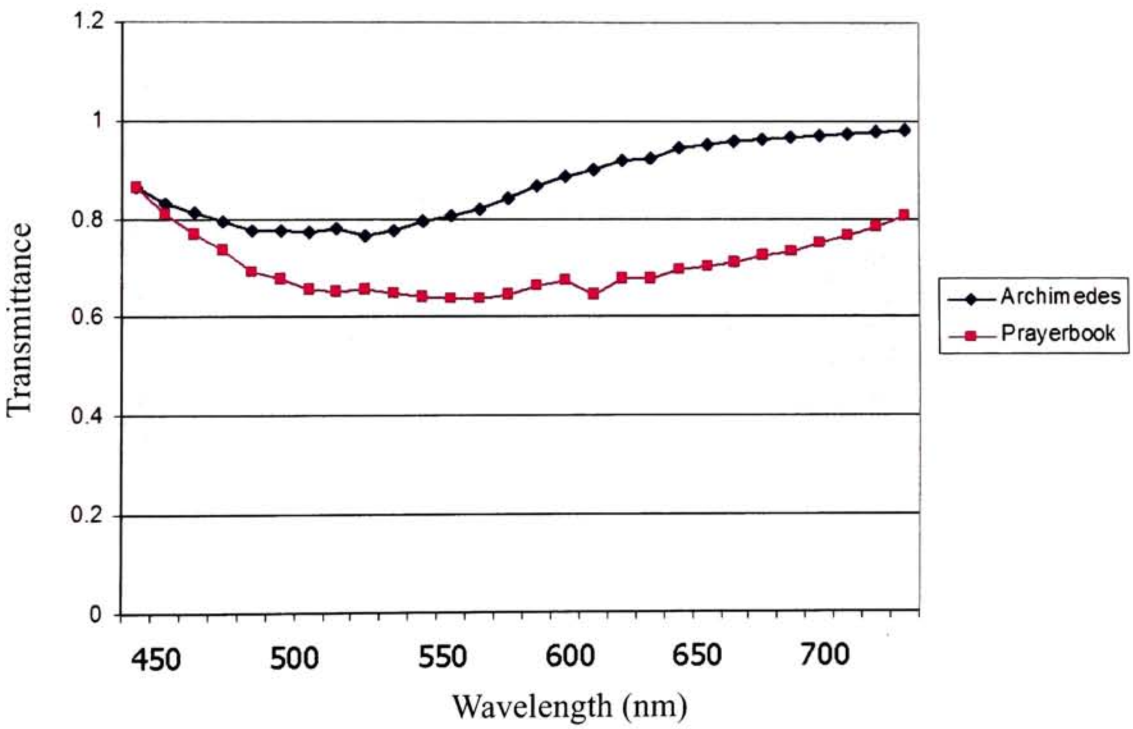


Figure 10. The spectral signatures of both ink layers after dividing out the response of the parchment.

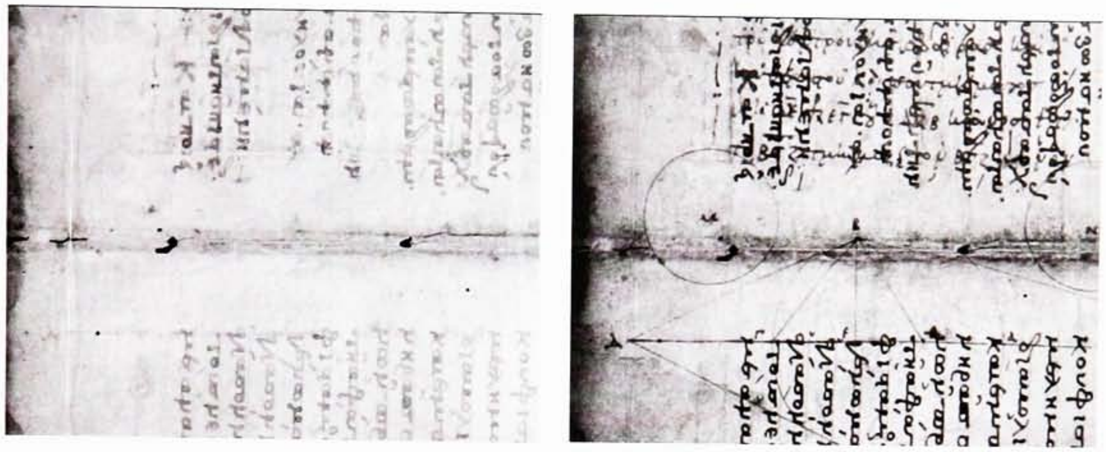


Figure 11. The tungsten monochromatic image placed in the red channel (left) combined with the UV monochromatic image placed in the blue channel (right) will create a pseudocolored image. *Copyright resides with the owner of the Archimedes Palimpsest.*

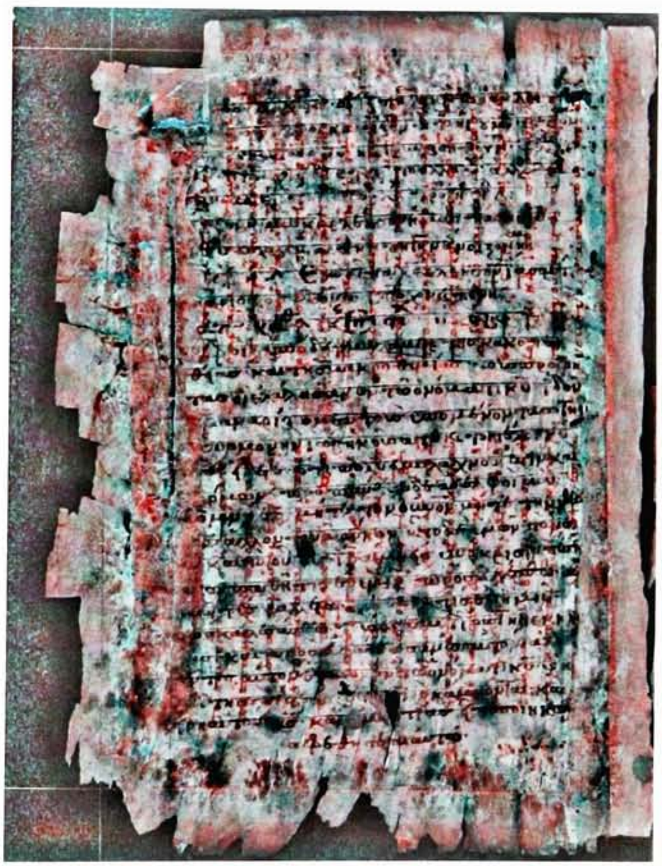


Figure 12. A pseudocolored leaf of the palimpsest. The dark Euchologion text can be seen running horizontally, while the reddish Archimedes text runs vertically underneath. *Copyright resides with the owner of the Archimedes Palimpsest.*

pseudocolor method greatly improved the legibility of the Archimedes writing, allowing the layers of ink and parchment to be distinguished, recovering about 80 percent of the faded text (Device Reads, 2005, p. D10).

The palimpsest was imaged between 350 nm and 750 nm at 100 nm intervals, resulting in five spectral bands for each illuminant. These spectral bands were then processed using a linear mixing model for spectral classification. The resulting spectral signatures showed six individual classes, representing both ink layers, as well as mold and other components within the parchment. The contrast between the text and the parchment could then be enhanced to increase the legibility of the writing (Knox, et al., 2001, pp. 208-209).

3.9 X-ray fluorescence and the Archimedes Palimpsest

In April 2005, the EDAX Company in New Jersey measured X-ray fluorescence from small sections of a few leaves of the palimpsest. The X-ray fluorescence occured as unstable atoms began replacing lower valence electrons with higher-energy electrons, which emitted an X ray characteristic of a particular element, such as iron or potassium (Figure 13) (Woods, 2005b). X-ray iron fluorescence of the Archimedes Palimpsest showed the two layers of ink from both sides of the leaf in the same image.

The encouraging results from the EDAX test prompted another experiment in May 2005 at the Stanford Linear Accelerator Center in California. This time, several pages were scanned by a synchrotron X-ray beam, including the most difficult pages with the gold-leafed illuminations (Figure 14). Because the ink was only 1-2 millionths of an inch in thickness (Davidson, 2005), software designed specifically for imaging the Archimedes manuscript was required to control the beam. Resting the X-ray beam on a particle for more than a few milliseconds could possibly damage the fibers in the parchment (Woods,

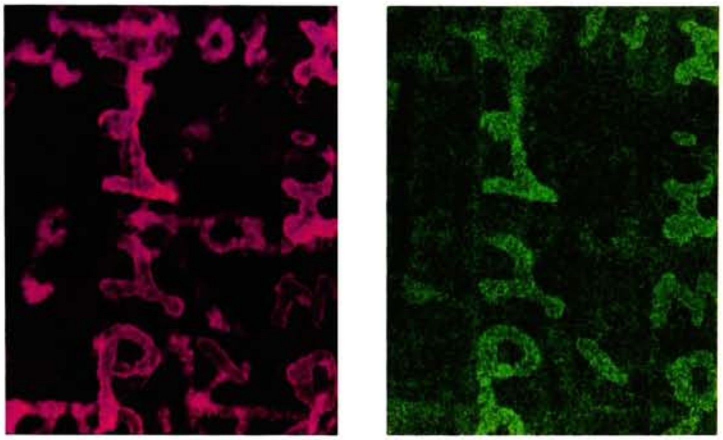


Figure 13. X-ray fluorescence images of the Scanone_datacube showing iron (left) and potassium (right). *Copyright resides with the owner of the Archimedes Palimpsest.*

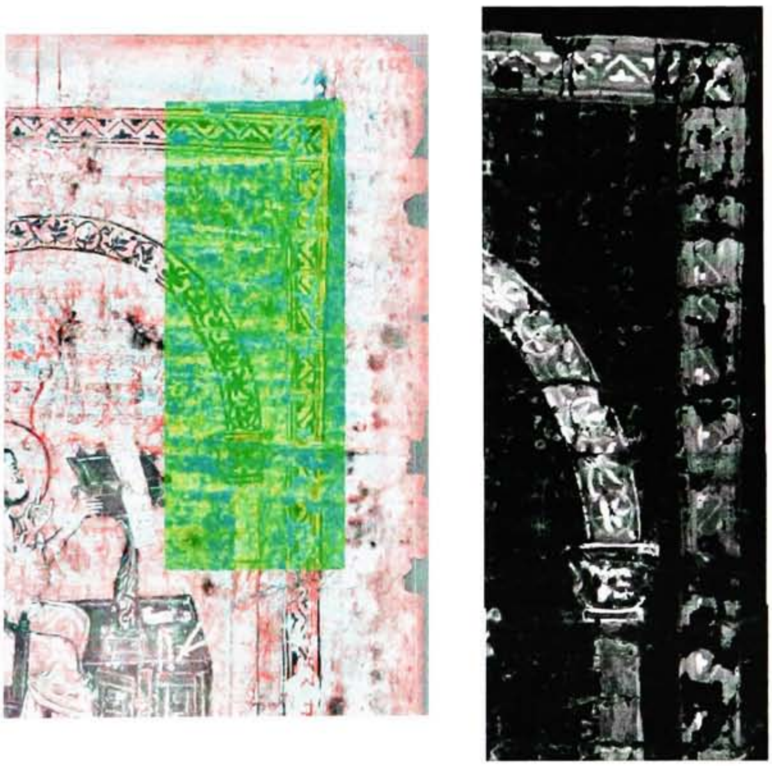


Figure 14. X-ray fluorescence reveals the writing hidden beneath the gold-leafed illumination. *Copyright resides with the owner of the Archimedes Palimpsest.*

2005a). Fluorescence from the iron particles in the ink was successfully detected through the obscuring layer of gold-leaf. The results from the EDAX and SLAC experiments showed that X rays could be used to detect trace elements in the ink and parchment. It is hoped that analysis of these data sets will be useful in distinguishing and classifying textual layers in the palimpsest for future imaging experiments.

Further plans for the Palimpsest Project at the Walters Art Museum to image the entire Archimedes text are already underway, allowing scientists to decipher and transcribe the text for inclusion on an interactive DVD (Woods, 2005). The final results will be published, and an exhibition of the palimpsest is planned at the Walters Art Museum in 2008 (Walters Art Museum, 2005).

3.10 Conclusion

Many different methods of multispectral imaging can be applied to various aspects of ancient studies. Multispectral imaging has proved useful in detecting iron and carbon ink remnants in ancient manuscripts. Iron-gall inks, such as those found in the Archimedes Palimpsest and Petra scrolls, will fluoresce in the ultraviolet region. Inks which contain carbon-based pigments will show greater contrast in the longer, near-infrared wavelengths, as shown in the Herculaneum scrolls.

Imaging analysis of early manuscripts has successfully aided scientists in identifying and deciphering previously illegible writing, and the process is continually developing. Different materials respond with unique spectral characteristics under different conditions. In many cases, ink analysis has either revealed hidden writings or created new understanding of certain writings and their origins. In many cases, additional information has been beneficial to both scholars and conservationists.

CHAPTER 4

HYPOTHESIS

4.1 Hypothesis

Enough useful information about the ink layers in the Archimedes Palimpsest can be extracted from the multispectral X-ray fluorescence data to isolate the Archimedes text layer from the Euchologion text layer.

4.2 Other questions for research

The research should be able to answer the following questions:

- Does the X-ray fluorescence data cube provide enough usable data to distinguish between layers of writing and parchment?
- Is it possible to separate the regions where the characters from the two texts overlap?
- Can layers of the X-ray fluorescence data cube be pseudocolored to match the colorization of previously pseudocolored MSI images with enhanced Archimedes text?
- Will any extracted information be useful to scholars?
- Is the method of analysis repeatable for future imaging experiments?

4.3 Research limitations

The research is structured within the following limits:

- Only data from the X-ray fluorescence EDAX image cube has been analyzed, although the previously collected MSI data sets may also be used for comparison.

- Many algorithms exist to disseminate spectral information from image cubes, including specialized programming code, often written by the researcher. This research is limited to using the analysis tools provided by the ENVI remote sensing software.

CHAPTER 5

METHODOLOGY

5.1 Objectives

This research thesis investigates ink detection in ancient manuscripts, specifically the Archimedes Palimpsest, using state-of-the-art imaging and remote sensing techniques. This research builds upon a decade of developments in applying multispectral imaging techniques to detecting and reading illegible writing on ancient manuscripts.

The research focuses on three main objectives:

- Evaluation of X-ray fluorescence data to determine which of the 2,000 bands contain useful information about the layers of text.
- Creation of a pseudocolored composite RGB image of a portion of enhanced Archimedes text, similar to previous pseudocolored MSI images.
- Review of secondary research in the areas of multispectral imaging, ancient manuscripts, and ink detection, including a discussion of early inks and parchment.

5.2 XRF data evaluation

X-ray fluorescence datacubes were collected by EDAX in 2005. ENVI software and UNIX workstations were used to process the "Scanone_datacube" using remote sensing analysis. Different algorithms were applied with varying results. One objective

is to identify and limit the number of bands needed for processing and for future imaging experiments.

5.3 RGB composite image

The ENVI software allows a single spectral band of information to be isolated, enhanced, and saved as a separate image file. ENVI also provides a method for combining grayscale images in RGB channels to make a three-band pseudocolor image. Different combinations can be used to match colors that are similar to previously pseudocolored MSI images of the palimpsest. Adobe Photoshop also was used to create composite RGB images. The objective is two-fold: to enhance the contrast of the Archimedes text from the Euchologion text, and to emulate the colorization of previous pseudocolored MSI images.

5.4 Literature review preparation

The objective of the literature review is to examine the development and application of multispectral imaging to ancient studies. The review is based on secondary research. A glossary and extensive bibliography accompany the thesis as part of the literature review. Because the development of MSI for ancient manuscripts began less than two decades ago, much of the research will be derived from studies from 1993 to the present.

CHAPTER 6

RESULTS

6.1 Scanone_datacube

The datacube selected for analysis, referred to as "Scanone_datacube," was the first of several datacubes created by Keith Knox using X-ray fluorescence data from the EDAX company in New Jersey. Several small sections of the palimpsest were analyzed using X-ray fluorescence, especially in areas where Euchologion text and Archimedes text overlapped and where gold-leafed illuminations covered both texts. The objective was to detect X-ray fluorescence at 2,000 frequencies to find which fluorescence bands provided the most useful information.

The spectrum of the Scanone_datacube is shown in Figure 15. Two peaks of intense fluorescence are clearly shown in bands 374 and 645. These correspond to specific levels of energy, known as atomic spectral lines. Band 374 corresponds to an energy of 3.7 keV, which is an atomic emission line for calcium (Ca). Band 645 corresponds to an energy of 6.41 keV, which is an atomic emission line for iron (Fe). The images of these bands, shown in Figure 16, would suggest that the parchment accounts for much of the calcium fluorescence, and that the fluorescing layers of ink contain traces of iron. The four layers of ink from the Euchologion and Archimedes text on both sides of the page are visible in band 347 (Figure 17).

Noise is also present in the datacube and is introduced with each instrument in the imaging process. Because the instrument was detecting 2,000 different energy levels, it is likely that information in many bands is not useful for segmenting the texts. Subsequent

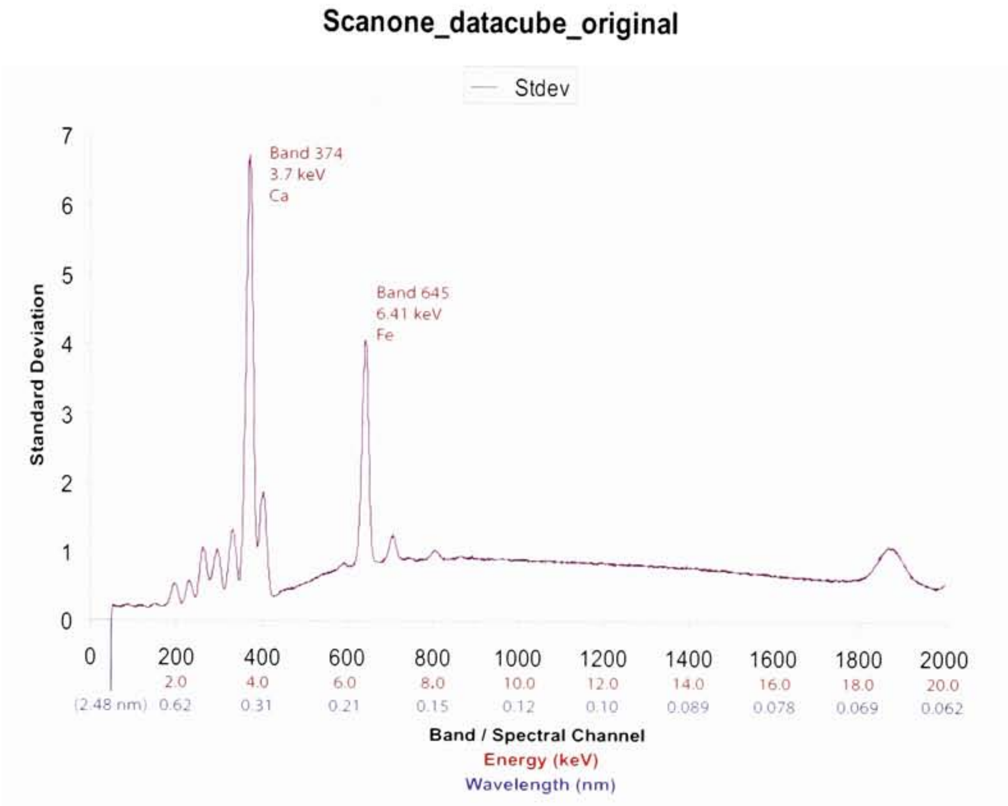


Figure 15. Bands in the datacube roughly correspond with energy levels of the same number (within a few bands). Band 374 is fluorescing at 3.7 keV. Band 645 is fluorescing at 6.41 keV. The energy levels are assumed to be equally spaced for all 2000 bands.

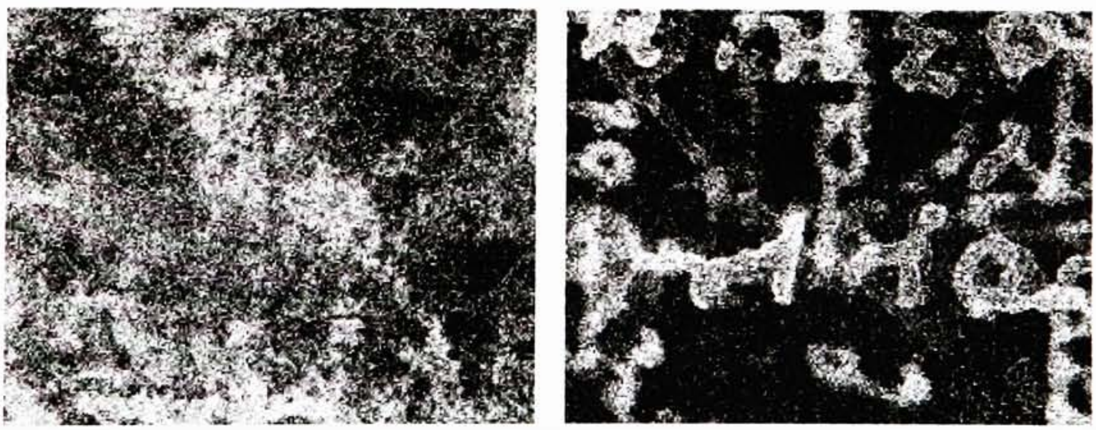


Figure 16. The two largest regions of intensity in the Scanone_datacube have peaks at band 374 (left) and band 645 (right). Band 374 has an energy corresponding with calcium (Ca), while band 645 has an energy corresponding with iron (Fe). *Copyright resides with the owner of the Archimedes Palimpsest.*

Figure 17. Band 374 from the Scanone_datacube shows writing from the front of 163 verso (left) and behind, 163 recto (right). The image on the right has been flipped to show the characters in context. *Images copyright owner of the Archimedes Palimpsest.*

image processing, such as spectral classification, principal component analysis (PCA), and minimum noise fraction (MNF), can be used to segregate the noise from the relevant data and reduce the dimensionality of the data set.

6.2 Classification

The first step was to use ENVI to determine if it is possible to group pixels into distinct classes to identify significant patterns. Based on knowledge of the palimpsest, the original data was assumed to consist of five classes: parchment and four layers of text. Unsupervised classification algorithms were used to estimate the number of class means and create pixel clusters based on these values. Five separate classes were identified by the K-Means classifier. The IsoData classifier, a similar technique that also uses a

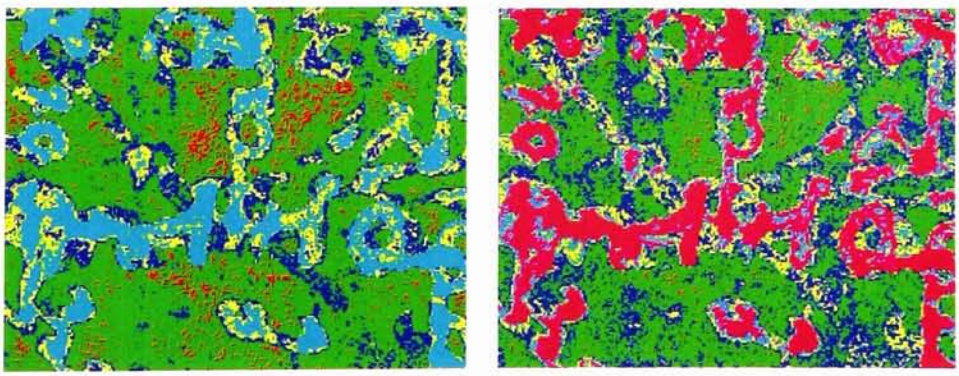


Figure 18. Five classes were found using the K-Means algorithm (left), while the IsoData algorithm (right) found six. *Copyright resides with the owner of the Archimedes Palimpsest.*

minimum distance algorithm, found six classes (Figure 18). When compared to previous pseudocolored images of the palimpsest, both classifiers mistakenly classified Archimedes text with Euchologion text. The dark blue regions in the IsoData results show most of the Archimedes text from the other side of the leaf, though neither of the algorithms successfully isolated one text layer from another.

The next method involved supervised classification. A few pixels were identified as belonging to specific classes and that could be labeled as regions of interest, or ROIs (Figure 19). The ROIs were used to identify clusters and calculate means based on those clusters. In this case, the maximum likelihood and Mahalanobis distance classifiers struggled to locate much of the bottom two text layers, and there appeared to be no distinction between the Euchologion and Archimedes text.

The parallelepiped algorithm was perhaps the least successful of all, identifying only three classes mixed with Euchologion and Archimedes text. Only the minimum distance algorithm found five classes, including all four layers of writing; however, the classes were still mixed with pixels from both Euchologion and Archimedes writings (Figure 20).

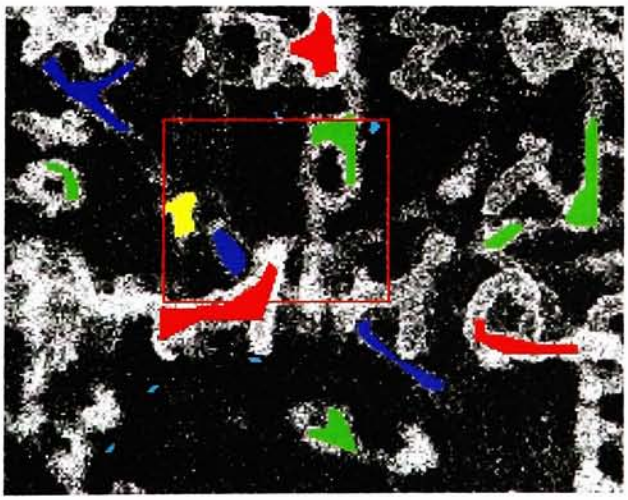


Figure 19. Pixels that belong to a specific class can be selected as regions of interest (ROIs) using ENVI. *Copyright resides with the owner of the Archimedes Palimpsest.*

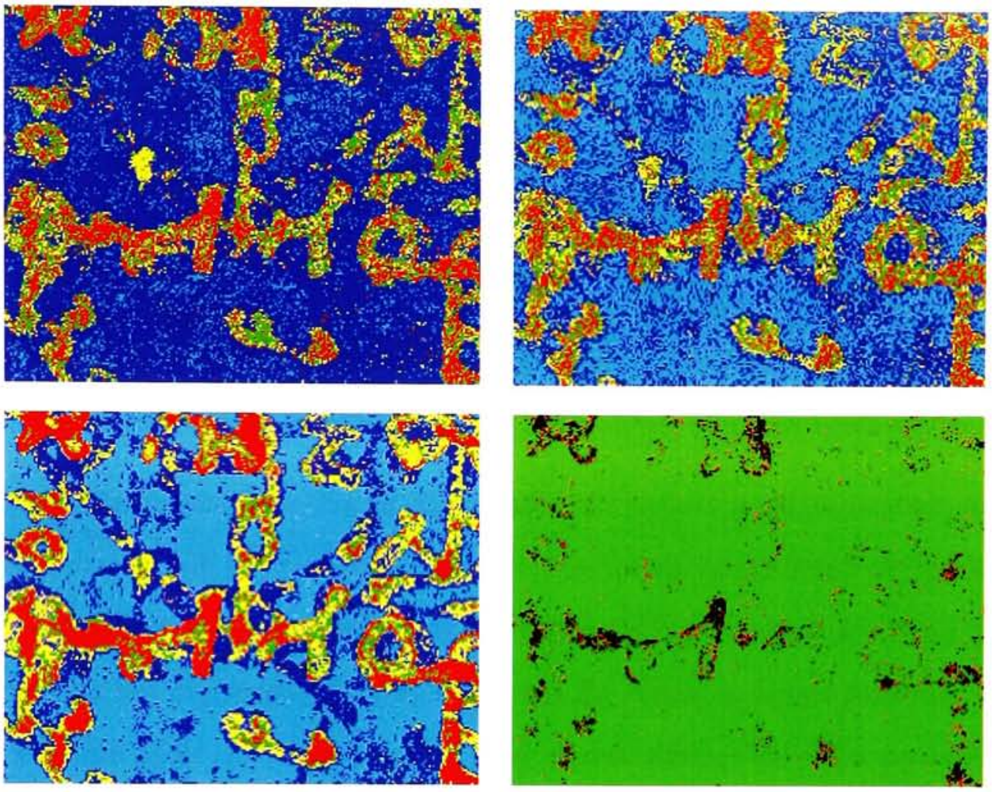


Figure 20. Results from supervised classification algorithms: maximum likelihood (top left), Mahalanobis distance (top right), minimum distance (bottom left), and parallelepiped (bottom right). *Copyright resides with the owner of the Archimedes Palimpsest.*

6.3 Scanone_datacube analysis—PCA

Principal component analysis was then applied to the EDAX Scanone_datacube. The first calculated principal component contained groups of pixels with the largest variance from the vector mean, which seemed to include fluorescence from the parchment (Figure 21). The second principal component, based on a new mean, showed all four layers of text. The image was similar to that obtained from band 374 alone, but was less noisy. Beginning with the third principal component, the remaining 1,998 bands seemed to contain only noise, and only very faint patterns were discernible. A typical multispectral data set usually contains at least three useful principal components, the lack of which suggests that this data set contains many bands of unnecessary data. Eigenvalues were also calculated for each principal component, which verified that only two main principal components

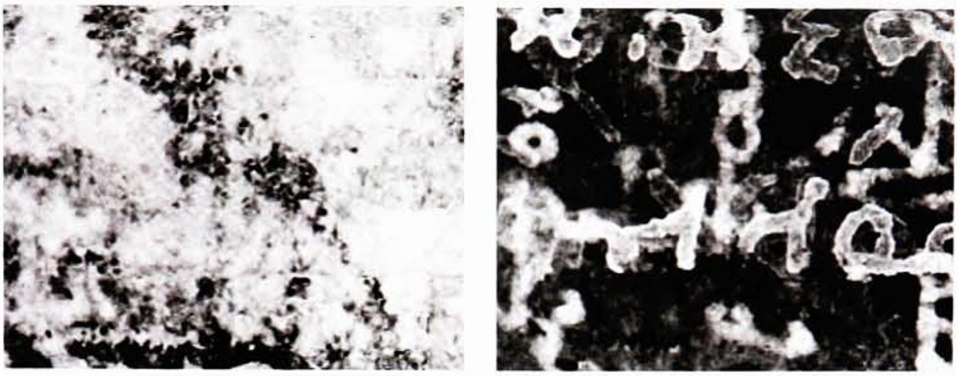


Figure 21. Results of PCA. *Copyright resides with the owner of the Archimedes Palimpsest.*

held most of the useful data (Figure 22). Bands with large eigenvalues contain relevant data, while bands with low eigenvalues are likely to contain mostly noise (RSI, 2004b, p. 680). The first two principal components had extremely high eigenvalues, while the rest had relatively low eigenvalues. It was likely that the remaining bands contained mostly noise, although other algorithms could be used to parse through the noise to identify less obvious bands containing relevant information.

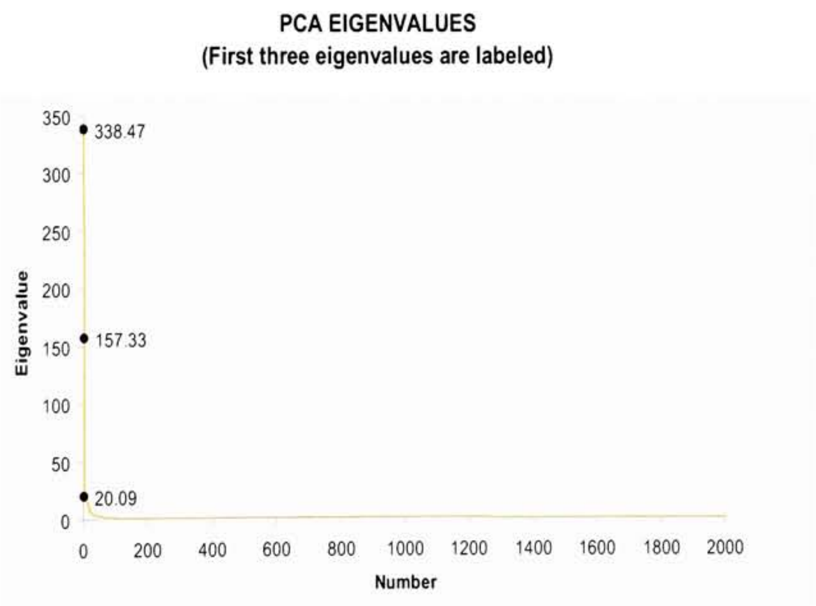


Figure 22. Eigenvalues as a result of PCA. Only the first three are labeled. Eigenvalues confirm that only a few bands actually contain relevant information. Most of the bands contain some noise.

6.4 Scanone_datacube analysis—MNF

The minimum noise fraction algorithm (MNF) uses "two cascaded principal component transformations," and provides another way of reducing the noise and dimensionality of a particular data set (RSI, 2004b, p. 682). MNF was very effective when tested on a few hundred bands of the Scanone_datacube. However, when trying to process all 2,000 bands, the MNF transform exceeded the limited number of iterations allowed by the ENVI software. The software also was not capable of estimating noise statistics from the data. After several unsuccessful attempts to run the MNF algorithm, it was decided to disband the datacube into smaller subsets.

6.5 Creating subsets

The 2,000-band datacube was segmented into a limited number of subsets, each centered around a specific atomic emission energy level (Figure 23). Using the standard deviation data of the original Scanone_datacube, twelve regions were identified and

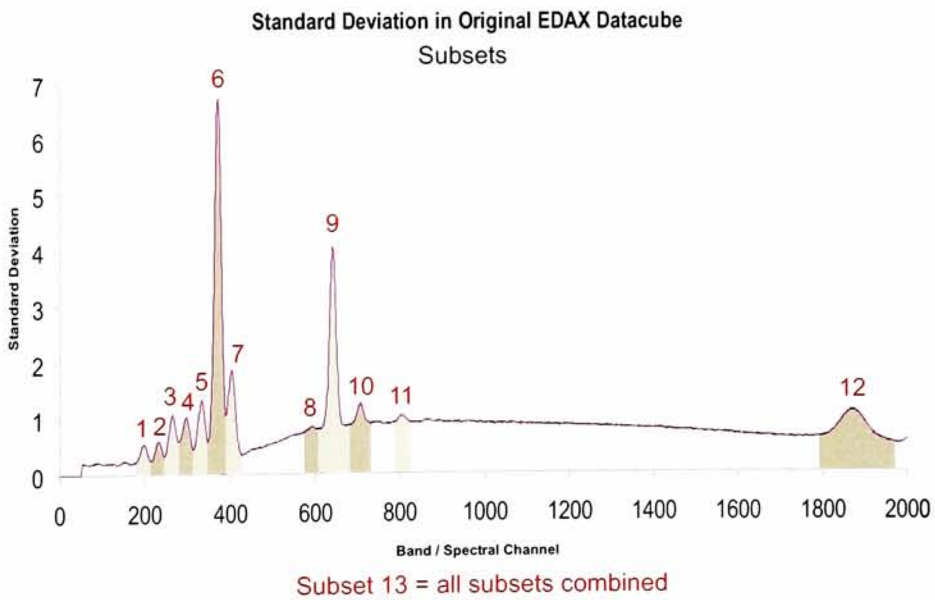


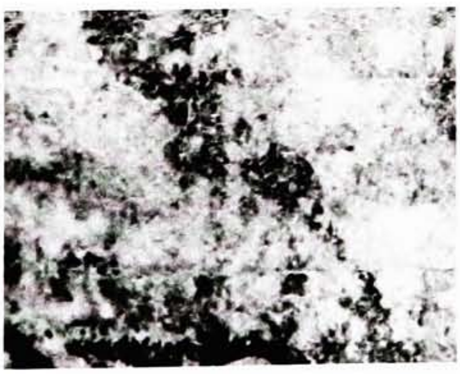
Figure 23. Twelve regions centered around significant atomic emission lines were identified and disbanded into smaller datacubes, called subsets. A thirteenth subset was created by combining all twelve regions. (Also see Appendix B.)

disbanded into individual subsets. Each subset contained anywhere from 31 to 128 total bands (also see Appendix B). A thirteenth subset was created by combining all twelve subsets together, which totaled 582 bands.

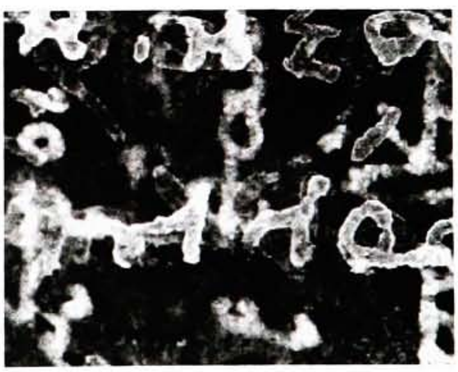
6.6 Subset analysis—PCA

PCA was performed on each of the thirteen subsets individually (Figures 24 and 25). In each case, the first principal component appeared to contain useful image information, while the remaining components consisted only of noise. The most significant image is perhaps PCA Band 1 from Subset 2 (Figure 25), which shows Euchologion text completely isolated from the rest of the writings. It is interesting to note that the bottom layer of Euchologion text from the other side of the leaf is not visible in the image. PCA Band 1 from Subset 9 (Figure 25) shows all four layers of writing, while the Archimedes text is only slightly visible in PCA Band 1 from Subset 5. The images of the parchment in Subsets 6 and 7 are exact opposites of each other.

The remaining subsets seem to show mostly noise and shadows, making it difficult to determine how much information contained in the shadows is relevant to distinguishing between individual layers of writing. Subset 13 (Figure 24), which contains bands from all



Subset 13 (combined sets):
PCA Band 1



Subset 13 (combined sets):
PCA Band 2

Figure 24. Subset 13 showed similar PCA results to that of the original Scanone_datacube.
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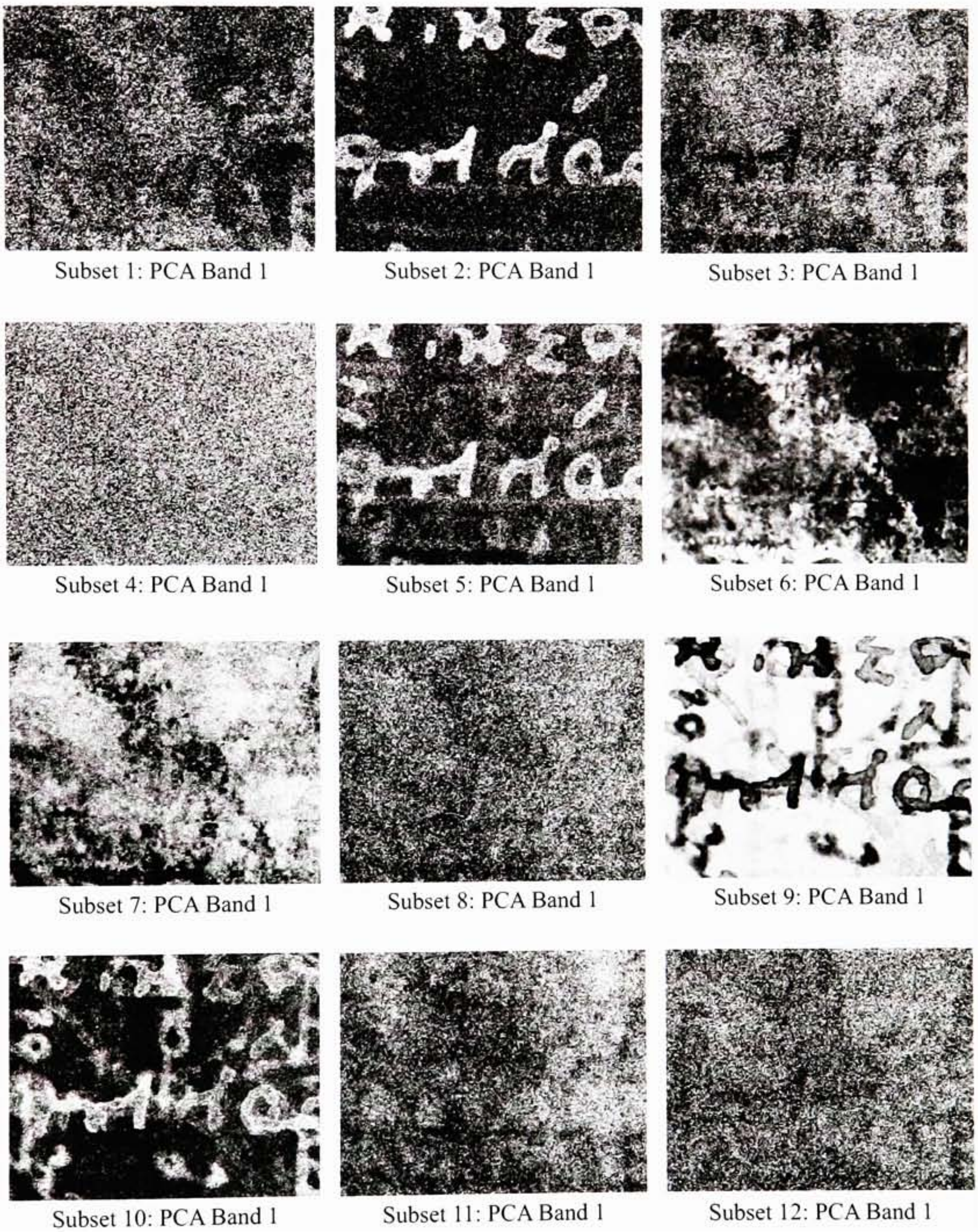


Figure 25. The primary component of each subset after PCA. The first layer of Euchologion text appears to be isolated in Subset 2. The Archimedes text is faintly visible in Subsets 5 and 10. All four layers of text are clearly visible in Subset 9. Patterns in the parchment are especially visible in Subsets 6, and 7. The other subsets appear to contain mostly noise. *Copyright resides with the owner of the Archimedes Palimpsest.*

subsets combined, showed similar PCA results to that of the original Scanone_datacube with less noise saturation. This seems to suggest that bands from the original Scanone_ datacube that are not included in any subset are likely irrelevant and do not provide any new information.

6.7 Subset analysis—MNF

Principal component analysis seemed to be effective in identifying the useful information contained in each subset. Therefore, minimum noise fraction (MNF) was perormed only on the combined subset—Subset 13—to determine if certain patterns of text and noise were correlated. The results in MNF Band 1 show a significant distinction between the text layers and the parchment (Figure 26). MNF Band 2 shows a mixture of parchment and text. Perhaps the most significant image created is MNF Band 3, which distinguished between the first layer of Euchologion text and the other text layers. Certain

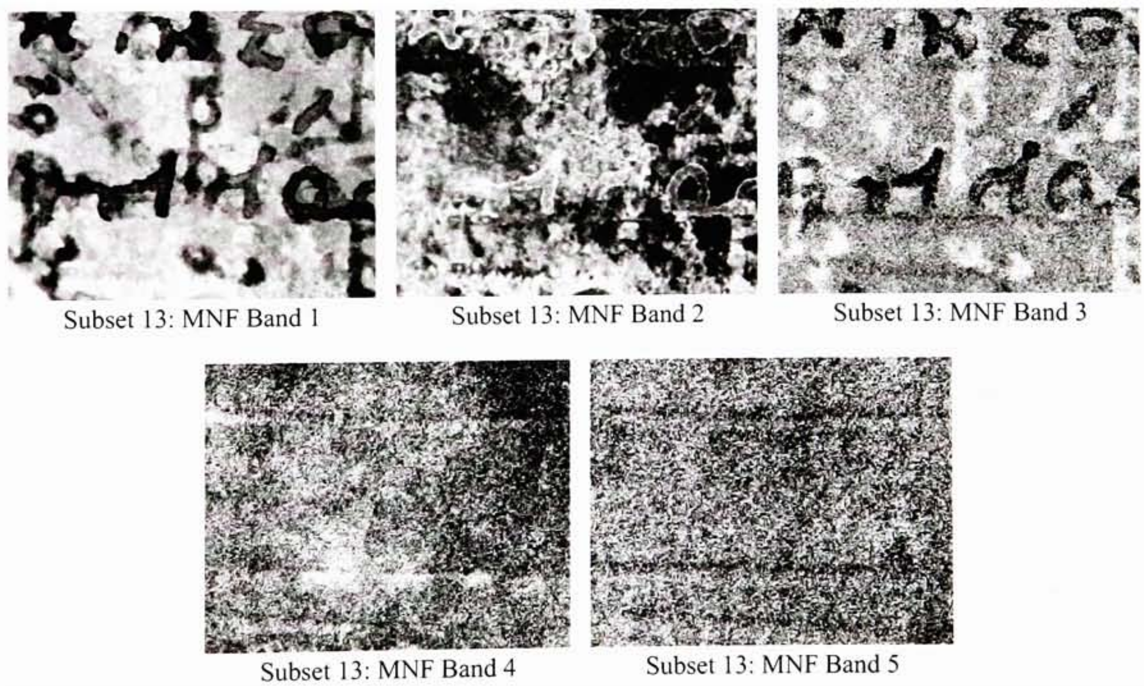
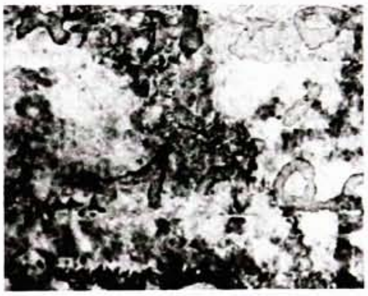


Figure 26. Minimum noise fraction (MNF) distinguished between the Euchologion text and Archimedes text in Subset 13 Band 3. Ruling lines are visible in Bands 4 and 5. *Copyright resides with the owner of the Archimedes Palimpsest.*

characters of Archimedes text were actually visible through the Euchologion layer. The remaining MNF bands seemed saturated with noise, though an interesting pattern was discovered in Bands 4 and 5. The ruling marks scored into the parchment by the scribe were clearly evident, although the characters themselves were not visible.

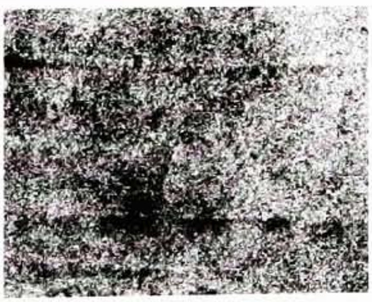
For the purpose of comparison, MNF was also applied to the set of principal components from Subset 13 (Figure 27). Because MNF already incorporates two principal component transformations, a third PCA transformation reversed the brightness values of each image, making the new MNF bands the exact counterparts of the previous MNF bands. It is significant to note that MNF Band 3 still distinguished the first layer of Euchologion text from the other layers of text.

Subset 13 (after PCA):
MNF Band 1

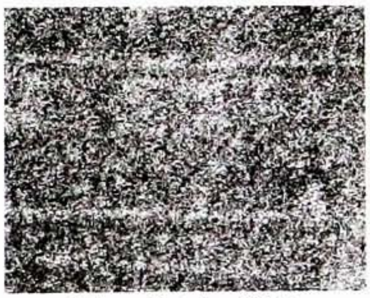


Subset 13 (after PCA):
MNF Band 2

Subset 13 (after PCA):
MNF Band 3



Subset 13 (after PCA):
MNF Band 4



Subset 13 (after PCA):
MNF Band 5

Figure 27. These unique images appear as exact opposites of the MNF images in Figure 26. The MNF algorithm was applied to Subset 13 after first performing PCA. *Copyright resides with the owner of the Archimedes Palimpsest.*

6.8 Pseudocoloring

Five images selected from PCA and MNF results were combined to form pseudocolored RGB images. ENVI allows up to three grayscale images to be combined in RGB channels to create a colorized composite image. The images could be assigned to any channel, but always in the order R+G+B. Adobe Photoshop provided a similar alternative with the advantage of working in different layers and blending modes. Each image could be displayed in a specific layer, with those on top having priority over those underneath (Figure 28). Under the Blending Modes option, each layer could be applied to either the red, green, or blue channel, or a combination of channels. With each layer having a unique RGB combination, the layers themselves could then be rearranged to create distinctive color composites. The objective was two-fold: to enhance the contrast between the Archimedes and Euchologion text, and to emulate the colorization of previous pseudocolored MSI

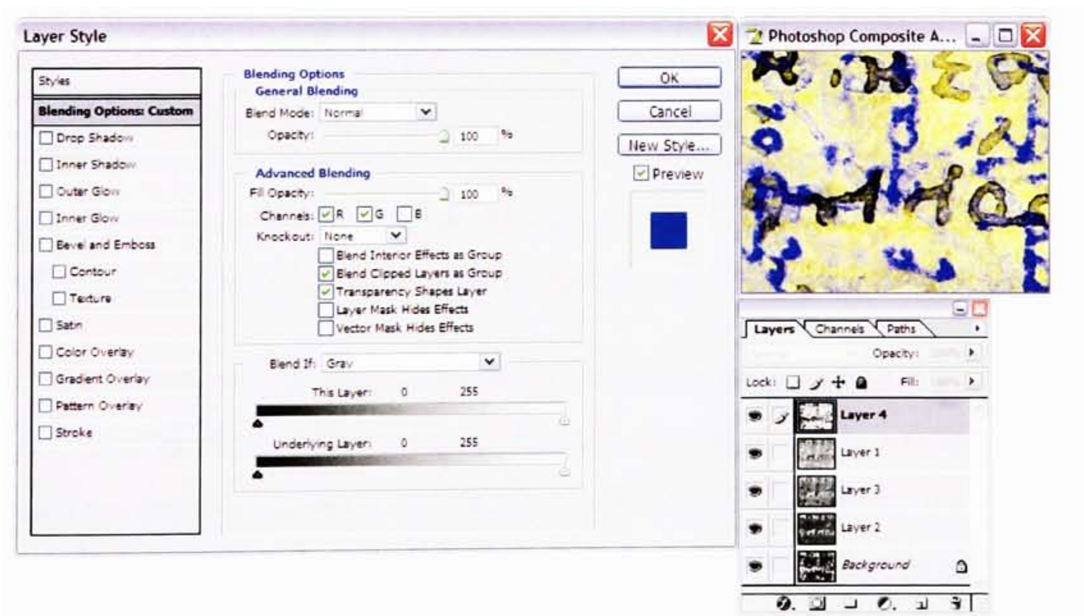


Figure 28. Adobe Photoshop was used to combine selected grayscale images into composite RGB images with enhanced Archimedes text. Image layers could be given a number of different RGB combinations. *Archimedes Palimpsest image copyright resides with the owner of the Archimedes Palimpsest.*

Figure 29. Pseudocolored XRF images with enhanced Archimedes text created in ENVI (bottom-left) and Adobe Photoshop (bottom-right) were compared with reference images of the palimpsest, 163 recto (top-left, flipped) and 163 verso (top-right). *Copyright resides with the owner of the Archimedes Palimpsest. Reference images were created by Keith Knox. Pseudocolored XRF images created by D. Michael Hansen.*

images of the palimpsest with enhanced Archimedes text. The composite RGB images are shown in reference to the previous pseudocolored images in Figure 29.

ENVI also can calculate an interesting three-dimensional surface view of various bands created by PCA, shown in Figure 30. Bands from the datacube can be visualized in much the same way at elevation is viewed from MSI satellite images. The 3D image can be moved, rotated, and zoomed (RSI, 2004b, p. 993). Because the "terrain" is based on grayscale values of a single image, this option simply allows for visual assessment of where each text layer is positioned relative to the parchment.

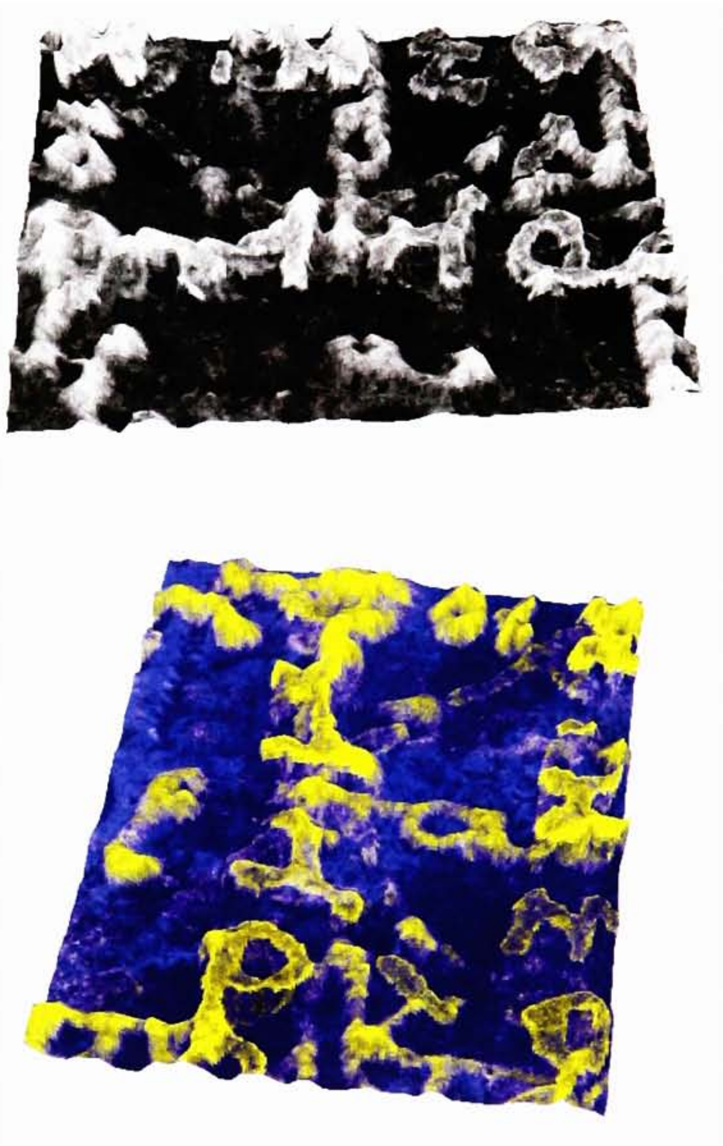


Figure 30. Grayscale and colorized 3D renderings of Band 2 from PCA. *Copyright resides with the owner of the Archimedes Palimpsest.*

CHAPTER 7

SUMMARY AND CONCLUSIONS

7.1 Summary

Methods for digitally imaging ancient manuscripts using multispectral techniques have been developed only within the last two decades. Various methods of multispectral analysis have proven beneficial to scholars in extracting writings that have become illegible over time. X-ray fluorescence of the Archimedes Palimpsest has suggested the possibility of detecting and isolating individual text layers. Future imaging sessions of the palimpsest may be improved by limiting the number of necessary bands to be scanned, and by creating a repeatable method for detecting useful bands in the datacube.

7.2 Conclusions

High-energy X-ray emission caused all four layers of ink and the parchment to fluoresce at detectable wavelengths. Principal component analysis (PCA) was effective in attaining a general idea of how much useful information was contained in the datacube. Smaller datacubes containing a few selected bands were created from the original XRF Scanone_datacube. PCA and minimum noise fraction (MNF) effectively decorrelated the noise within these subsets and created a distinction between the top layer of Euchologion text and Archimedes text. Ruling lines made by the scribe were discovered in the noisy bands using MNF.

Selected images showing the Euchologion text isolated from the other text layers were combined to create composite RGB images that enhanced the contrast between the

text layers. The composite images were also created using a colorization similar to that of previous pseudocolored MSI images. However, other text layers could not be completely isolated in this study and therefore could not be used in creating an actual facsimile reproduction of the palimpsest.

7.3 Recommendations for further investigation

It is hoped that this information will be useful in future X-ray fluorescence imaging experiments, especially as other datacubes become available for analysis. One recommendation is that supervised classification may be improved by selecting smaller regions of interest, creating more accurate clusters. For noise segregation, PCA and MNF are repeatable processes and may provide similar results for other data sets.

Noise in the data set may be reduced by performing inverse PCA and MNF transforms. After segregating the noisy bands from the relevant bands, inverse rotations revert only the relevant bands back into the original datacube. After an inverse PCA or MNF algorithm has been completed, performing the PCA and MNF algorithms again should generate different results. Further transformations may be successful in isolating text layers.

Other possible studies include a spectral comparison of different iron-gall inks or even the manufacturing of a palimpsest. Various multispectral imaging techniques can be used for enhancing other objects, such as paintings and stone inscriptions. It may also be interesting to study which museums have applied multispectral imaging to their collections, and how MSI is becoming incorporated into various imaging workflows.

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LIST OF REFERENCES

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APPENDIX A

GLOSSARY OF TERMS

algorithms—written procedures involving computation and decision-making. Mathematical algorithms are often created as computer programs that perform these tasks.

atomic spectral emission line—a known value of emission, characteristic of a particular element. Standard values for emission lines can be obtained from the National Institute of Standards and Technology (NIST).

band—a specific frequency or energy level; also called a spectral band or channel. All of the bands containing spectral data may be stacked together in one data file, called a datacube. Each band can be rendered in pixels to show how the manuscript appears at different frequencies.

bandpass filter—a crystal filter that allows only one wavelength of energy to pass through to the camera or sensor.

Bayer color filter array—a pattern of red, green, and blue filter patches commonly used in many current imaging systems. One unique feature is that there are twice as many green patches as red or blue patches, which is shown to preserve image sharpness (see Palum, 2001). Invented by Bryce Bayer at the Eastman Kodak Company in 1976.

Charge-coupled device (CCD)—an array of sensors that convert photon samples into digital values. The number of sensors in the array define the dimensions of the CCD: 6,000 x 4,000 pixels = 24 megapixels.

classes—specific groups to which data belong.

classification—using decision rules to assign data to specific classes. Usually accomplished by estimating the mean value of a particular class and calculating the probability that a piece of data either belongs or does not belong to that class.

codex—bound parchment leaves, like a bound book. The parchment folios were folded into quires, or signatures, and stitched on one side. A piece of wood or leather was usually used for the front and back cover pieces.

datacube—a set of data containing many channels, or bands, of spectral data. The histogram of a datacube shows the transition of a pixel over several frequencies (also see Appendix B).

eigenvalue—values associated with eigenvectors that represent the relevance of an eigenvector to the overall system. Within the datacube, eigenvalues are calculated to show how much of the data is estimated to be either relevant or noisy. Noise can then be disbanded from the datacube, leaving the relevant information remaining.

Euchologion—a Christian prayerbook kept in the possession of monks at Mar Saba and later housed at the Methochion in Constantinople until the twentieth century. Other Greek writings could be seen faintly beneath this prayerbook text, including mathematical treatises belonging to Archimedes.

ENVI—acronym for "Environment for Visualizing Images;" remote sensing software developed by Research Systems Inc. The software is widely accepted in the remote sensing community as an effective tool for multispectral data analysis.

fluorescence—a phenomenon that occurs when electrons in excited states return to lower orbitals. Also see X-ray fluorescence.

hyperspectral—having more than ten spectral channels. Hyperspectral datacubes may contain several hundred bands of spectral data. The EDAX XRF datacube of the Archimedes Palimpsest is unique in that it contains 2,000 bands.

illuminant—a source of light or luminous flux. The ink in the Archimedes Palimpsest reflected differently under tungsten, ultraviolet, and xenon illuminants.

illumination—a hand-painted picture, border, decoration, or character in a manuscript. Illuminations were often made from colored inks and gold-leaf. Many ancient manuscripts contain illuminations that are extremely vivid and ornate.

infrared—low-energy wavelengths outside the visible spectrum. The near-infrared region contains wavelengths between 0.7 and 1.1 μm.

interpolate—using a function to calculate or estimate unknown values of a data set. Interpolation occurs when one is trying to match a desired mathematical curve with only a few measured values.

iron-gall ink—an ink commonly used from the Middle Ages to the twentieth century. The dye was produced from a mixture of tannic acid and ferrous sulfate. The most acidic tannin could be extracted from tiny gall nuts found on tree branches and leaves.

IsoData algorithm—an unsupervised classification algorithm that identifies evenly distributed mean vectors and classifies pixels according to the distance from its closest mean vector using a minimum distance technique.

K-Means algorithm—an unsupervised classification algorithm that identifies mean vectors and classifies each pixel according to the distance from its closest mean vector.

Mahalanobis distance—a supervised classification algorithm that uses multispectral sample points to identify cluster centers, similar to the maximum likelihood classifier. The Mahalanobis distance classifier then classifies pixels according to their direction and distance from the cluster centers.

matrix—a set of numbers or elements organized in rows and columns. Matrices are used in algebra and linear mathematics, and can be added, multiplied, and inverted to solve problems. Covariance and correlation matrices are used in principal component analysis and minimum noise fraction to calculate eigenvalues and eigenvectors.

minimum noise fraction (MNF)—a method of estimating and decorrelating the noise in a data set; also called minimum noise fraction transform. MNF uses two principal component rotations to segregate noise from the relevant data and calculate eigenvalues.

monochromatic—all photons having the same energy level.

multispectral—containing more than one spectral channel or band. The channels can be stacked together into a multispectral datacube, which is commonly used for remote sensing analysis.

orthogonal—creating a second matrix so that when compared to the original, "the sum of the products of the corresponding elements [in both matrices] in any two rows or any two columns is equal to one."

palimpsest—a manuscript that has been erased and overwritten with other writing. Several treatises of Archimedes can be seen faintly beneath Euchologion text in the Archimedes Palimpsest.

parallelepiped—a many-sided shape with parallelograms for each side. The parallelepiped classifier is used for supervised classification. Pixels that fall into a predetermined threshold around the mean will belong to that particular class. The threshold is in the shape of a parallelepiped. The number of bands in the datacube usually determines the number of sides of the parallelepiped.

photon—particles of light, or electromagnetic energy, that have wave-like characteristics and can be detected with a measuring instrument, such as a spectrometer.

principal component analysis (PCA)—a method of identifying useful bands within a datacube by segregating patterns of noise from relevant information; also called principal component transform. PCA organizes data into matrices and identifies vectors with the largest variance from the mean. Eigenvalues are then calculated, which can be used to reduce the number of bands needed for further processing.

pseudocolor—creating a false-color, composite image using several spectral bands. Combining different bands in various combinations can enhance the contrast of a particular color or image area, such as the ink in the Archimedes Palimpsest.

Raman spectroscopy—a method of molecular analysis that measures photon absorption from scattered radiation. An object is irradiated by a laser at a certain frequency, causing molecules to vibrate or rotate at a specific frequency, which can then be detected using a monochromator.

RGB values—intensity values given to the primary components of light, which consist of red, green, and blue wavelengths. Filters disperse the light into RGB separations. Then sensors detect the light and give a value to the intensity of the signal. Each value is stored in one of the R, G, or B channels of an image.

remote sensing—detecting radiation from a particular object. Remote sensing is particularly helpful in analyzing high-energy and low-energy frequencies in the non-visible regions of the spectrum. Multispectral imaging is a common tool for remote sensing analysis.

spectrometer—an instrument that measures dispersed wavelengths of light across the spectrum.

treatise—an examination of a particular theory that discusses facts and conclusions. Several treatises contained in the Archimedes Palimpsest are the only existing sources of Archimedes mathematical theories.

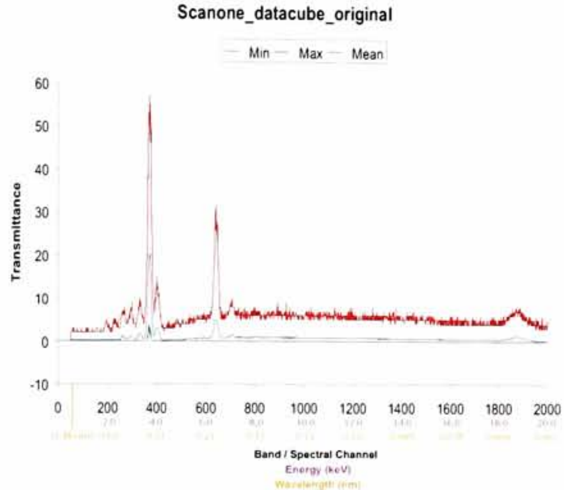
ultraviolet—high-energy wavelengths outside the visible spectrum. The ultraviolet region contains wavelengths between 0.01 and 0.4 μm.

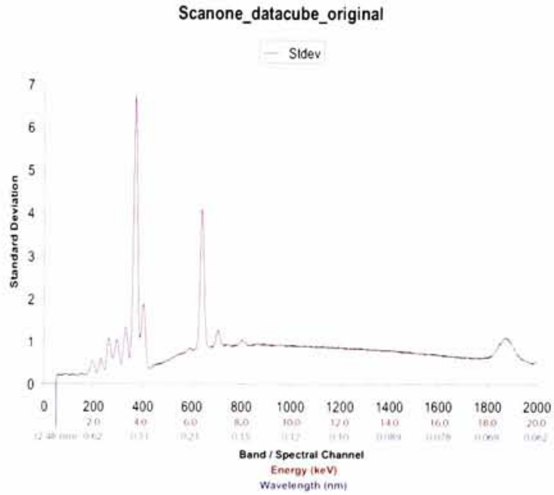
vector—a system of scalars or coordinates that has direction, dimensionality, and amplitude. Vectors give direction to measured quantities, such as an arrow or line segment drawn in a particular direction. Vectors are especially useful in matrix algebra; for instance, vectors are used in spectral classification to attribute a particular data point to an estimated mean value.

X-ray fluorescence—a non-destructive method of image analysis that may be used to detect traces of a particular element. Powerful X rays can knock inner electrons out of an atom. When higher-energy electrons are used to fill in vacancies in the lower valence levels, the difference in energy is emitted as fluorescing radiation. The type of fluorescence that occurs is characteristic of a particular element, such as iron.

APPENDIX B

DATACUBE HISTOGRAM & SUBSETS

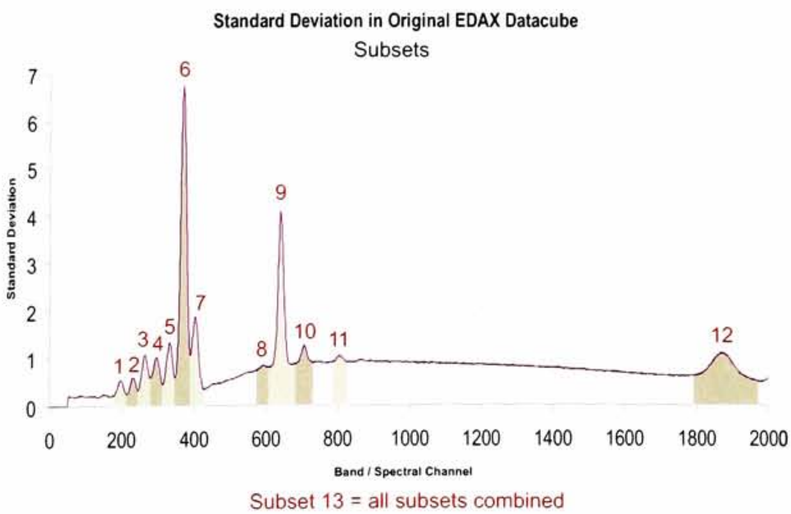




Scanone_datacube subsets

Subset	Range	Max	Stdev
1	Band 173	2	0.205305
	Band 196	5	0.541846
	Band 212	2	0.244547
2	Band 213	2	0.247242
	Band 231	5	0.585627
	Band 244	3	0.324023
3	Band 245	3	0.332522
	Band 269	8	0.894118
	Band 277	4	0.588791
4	Band 278	4	0.595003
	Band 299	9	0.981006
	Band 313	3	0.447231
5	Band 314	4	0.465518
	Band 330	10	1.298584
	Band 347	4	0.584499
6	Band 348	4	0.604013
	Band 369	57	6.693197
	Band 388	6	1.066931
7	Band 389	7	1.062406
	Band 400	15	1.365406
	Band 427	3	0.339736
8	Band 582	6	0.795572
	Band 593	6	0.847076
	Band 612	6	0.804761
9	Band 613	5	0.789097
	Band 640	32	4.075556
	Band 673	6	0.856143
10	Band 674	5	0.849087
	Band 706	10	1.263296
	Band 728	5	0.880923
11	Band 786	6	0.896856
	Band 803	7	1.039386
	Band 837	6	0.90128
12	Band 1817	4	0.68133
	Band 1872	8	1.071643
	Band 1944	4	0.56137

The first histogram shows the maximum, minimum, and mean fluorescence intensities. The standard deviation histogram was calculated using the maximum values and mean values. Thirteen smaller datacubes were disbanded from the original "Scanone_ datacube," and were centered around a particular spectral atomic emission line. Each subset contained anywhere from 31 to 128 total bands, except for subset 13, which contained 582 bands.



APPENDIX C

COMPOSITE RGB IMAGES CREATED USING ENVI SOFTWARE

COMPOSITE RGB IMAGES CREATED USING ADOBE PHOTOSHOP