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Rumor propagation on random and small world networks

Deana B. Olles

RUMOR PROPAGATION ON RANDOM AND SMALL WORLD NETWORKS

By

Deana B. Olles

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Applicant's Name: Deana B. Olles
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Applicant's Signature: Deana B. Olles **Date:** 11/7/06

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Advisor's Name: Dr. Bernard Brooks **Signature:** Bernard Brooks

Committee Member 1: Dr. Nicholas DiFonzo **Signature:** Nicholas DiFonzo

Committee Member 2: Dr. H. Shahmohammad **Signature:** Hossein Shahmohamad

Committee Member 3: ——— **Signature:**

Graduate Programs Director: Hossein Shahmohamad **Date:** Dec 5, 0
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For my parents Francis and Bonnie Connell
and
husband Mark Olles

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Abstract

In this work; three specific dynamical systems models, the Basic, Maki-Thompson, and Daley-Kendall, are used to model rumor transmission on social networks. Rumor flow is a measure of the time it takes for the rumor to completely pass through a specified network. Comparisons between random social networks and a small world social networks yield the faster transmission of a rumor over a small world network.

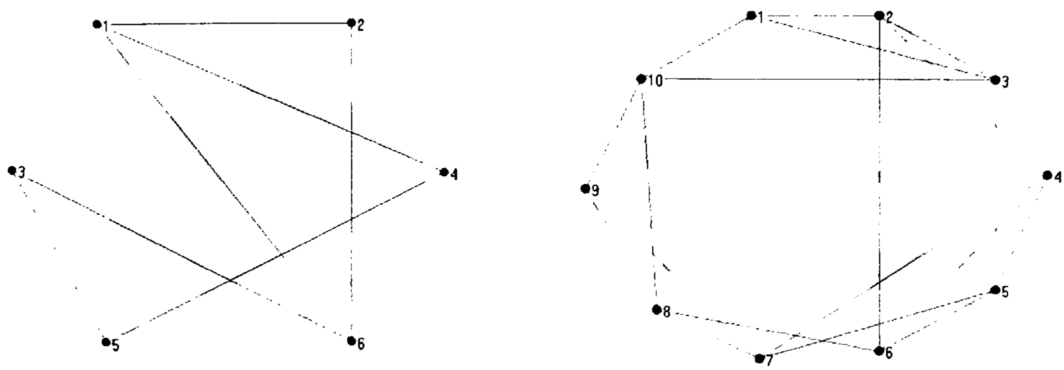
Using unique adjacency matrices that define our random networks, observations of some characteristics of the random networks will be made that are specific to this type of graph. Differences in the constructs of the two networks will be illustrated by comparing these properties to those of the small world networks (created by a certain rewiring scheme of a k-regular network). Interesting comparisons are to be made about the networks' defining characteristics include average clustering coefficients, centrality measures, and average path lengths. The flow of a rumor through each type of network reveals the characteristics of the network. A rumor will clearly flow through a small world network faster than in a random network, mainly due to higher density, increased clustering, and better defined centrality.

1 Introduction

Mathematical study of rumor propagation has played both theoretical and applied roles. Graph theory, dynamical systems and, to a smaller extent, stochastic processes have played prominent roles in many models of rumor propagation. Studying the different patterns of rumor transmission throughout social networks via the changes in dynamical systems, network generation, and even probability models, is significant in predicting how the societal spread of rumors will occur.

Many mathematical models have been designed to illustrate the spread of infectious diseases throughout certain populations. As rumors are spread throughout social networks, in some ways their transfer resembles the spread of an infection passed by word of mouth. Dynamical systems and some well-known stochastic models have been implemented in studying the flow of rumors over many different types of networks of differing conditions. The conditions to consider are clustering, path lengths, and centrality, which ought to affect the transmission of the rumor.

The relationship between the general random network and the small world network is important to observing the differentiation between the two networks' rumor propagation. This paper will show that the characteristics of random and small world networks play crucial roles in the rumor flow. Some properties to consider when making such a comparison are the differing connectivities, path lengths, and clustering coefficients of the two networks' types. Figure 1.1 displays a small random network along side a small world network. The basis for the creation of each type of network will be discussed in Section 2, along with the differing properties. We will continually refer back to these two networks when discussing their properties in Section 2.



Random Network　　　Small World Network

Figure 1.1

Moreover, the difference in the sizes of the two networks is not of question, as we are simply using these as examples. The strong comparisons between random and small world networks is left for Sections 3 and 4, where the results of our codes will display a much more thorough examination of the two. The propagation of a rumor over a small world network is expected to be much quicker than that over a random network. This paper will use graph theory and mathematical modeling to prove this argument.

1.1 What is rumor?

Before considering the application of mathematical modeling to rumor flow over a social network, let us first address the question 'what is rumor'? The reader must also have some insight into the nature of rumors and the sorts of rumors that might spread throughout sociological constructs such as networks.

1.1.1 A definition of rumor

The definition of rumor is made by stating many of its properties. The properties that make a rumor so different from other forms of information are what makes up the fundamental definition of a rumor. This section will briefly discuss the ideas of belief, truth value, and appeal of rumors. We wish to study the context in which rumors begin, contents of rumors' statements and the functions that rumors serve [8]. In defining a rumor we will derive all of these properties.

A rumor is defined as "unverified and instrumentally relevant information" of uncertain origin usually spread by word of mouth, or hearsay [8]. Rumors have often been confused with the idea of gossip or urban legend. However, the definition simply states that a rumor is unverified information passed from person to person, usually between those of some strength in relationship. Rumors will commonly have an unknown origin and can change over time. Rumor as hearsay is not far from reality, as a rumor can change meaning as it passes from one listener to the next. However, rumor mutation will not be addressed in this paper.

It is very important to understand that a rumor is a proposition or proposal for belief in a statement or topic that may not have an analogous verification [30]. The truth value of rumors is important in understanding how they flow, as it may be related to the believability or appeal of the listener. Such statements are considered unverified accounts of events, objects, individuals or concerns and usually circulate throughout the general population of a society [15]. Those rumors with more believability tend to spread quicker and over a larger population.

Rumors are begun for many reasons, sometimes with an underlying agenda at hand. Since rumors are unverified, the truth is sometimes in question. Truthful rumors are more than often begun using the media or academia. False rumors are more than likely started by those individuals with a reason for lying, whether it is to hurt another individual, fit in with a social group, or better their situation in a career. While there are many reasons for starting rumors, there are also many reasons to pass them. Rumors that continue to be spread throughout a network usually have some interesting characteristic about them. Also, the embellishment of a rumor can make it more interesting and thus, allow for its survival, however true it might be [30].

The problem with rumors is that there is no set method for stopping their passage through a network. There are many theories, including introducing stiflers into the social network, starting an opposing rumor with more validity, and even severing the ties between individuals in the social network [28]. This paper should assist future study in how to stop the flow of rumors by examining the characteristics of their flow.

1.1.2 How rumors are spread

In social networks, rumors are mainly spread through media, internet, relationships, or through leadership opportunities [15]. The examination of two individuals in a network and their connection yields the following properties. One of the individuals plays the role of the spreader, or the individual who passes the rumor on by telling an acquaintance. The role of the listener is simply that of the individual who hears the rumor from the spreader and makes their decision about whether or not they believe it. Rumors and the dynamics of their flow are typically misunderstood, some rumors expire after a given amount of time, yet others persist. The details behind each rumor and its origination are the foundations that determine which rumors will fail and which will last. How valid does the rumor sound to listener? Who does the rumor target and who hears it? Many more essential elements take part in how a rumor is either spread or stopped.

Through the constructs of any social network, we can make assumptions about how rumors will flow from unit to unit, and even observe such patterns using collected data or computer programming in simulations. There are many types of networks we can consider, each defining the relationships between individuals or groups in different forms. Some of these social constructs are random, small world, complete, regular, and family or village. Two of the most important networks are the random and small world networks. Random networks are created in a simple manner. Beginning with a set of vertices or nodes, edges are randomly placed between two nodes using an assigned probability. Small world networks can be designed in many ways, but this paper will only consider that of the rewiring procedure of edges on a k-regular graph in the form of a ring. Figure 1.1 showed a randomly designed graph to the left, while on the right, a ring with ten nodes was rewired to create a small world network. These two types of networks are considered over the others as they have more of the characteristics particular to a mass society. The observation of rumor flow over random and small world networks should help us in predicting the characteristics of rumor flow over large societies or civilizations.

1.1.3 Why rumors are spread

Rumors are information passed with low evidentiary basis, but high perceived importance by participants [8]. Individuals within networks spread rumors, in order to fit into or benefit their position in a social group. The media spreads rumors that possess at least some degree of validity regularly. However given the media's track record, there always seems to be room for a fair amount of skepticism The career oriented rumor may be spread for the possible benefit of an employee or client. Some of the numerous reasons for spreading a rumor correspond to attempts to join or impress another individual or group of individuals. Many types of rumors are present in today's society, and each of these has its own motivation for being spread. The different factors in the varieties of rumors eventually lead to diversity in receptivity of the rumor and the role that this rumor will play in satisfying the listener's needs [6]. In other words, the rumor should only appeal to some of its audience, and the events following the passing of the rumor may or may not affect the listeners' state.

One factor that determines if a rumor is spread is believability [15]. It is somewhat trivial but when an individual believes a rumor, they will actively seek to pass it on. The listener may not always believe the rumor immediately and may in fact need to hear it more than once for assurance. This may also lead to their confidence in passing the rumor on. These variations in belief and spread are interesting and important to understand while studying the speed of rumor transmission. Another case of rumor spread is that of doubt and the need for certainty of the rumor's truth value [15]. A listener who may be uncertain of the rumor's accuracy may pass it on in search for clarification from another individual. Again, this process has an affect on the transmission of the rumor and also has a position in the possible modification of the rumor.

The influence rumors have on society is significant to their spread as well. When a rumor influences the decision making process for an individual, it increases the chance of that individual spreading the rumor. Rumors circulated in 1991 regarding the possible sterilization of black men by the Ku Klux Klan through Tropical Fantasy Soda Pop [9]. Since the rumor was so influential to a specific group of individuals, the rumor spread vigorously throughout that social network and caused a major decrease in sales. Children are an extremely persuadable group, and because of this, rumors about candy and soda were easily spread throughout that group. The Pop Rocks candy was believed to explode in the stomachs of children who ate the candy with soda pop, which caused a huge decline in sales of the candy [8]. The influence that rumors have on people and their decision making process certainly has a role in spread of rumors. When individuals find information important to them, whether it is true or false, they tend to spread the information to those they relate to.

1.2 Why study rumor transmission?

Many forms of rumors are present in societies that may pose threats to the well being or survival of that community. Whether passed through governing bodies, media or masses, rumors true or false can have an impact on society's tranquility. Rumors about the possibility of terrorist attacks, disease, or even natural disasters can be easily passed through the media to the majority of the population and trigger responses from the masses. Studying the transmission of rumors and the effects will help in the future study of more complex societies.

In the recent past, we have witnessed media reports about terrorist attacks against a host of nations. The diversity in produced presentations of these broadcasts had triggered widespread debate about the possible outcomes and even the accuracy of the description of how such events began [9]. The SARS epidemic caused extensive panic throughout the world, and traveling to certain destinations became a new found concern. The events following the hurricane Katrina and the Tsunami in the Pacific caused anxiety in those affected communities, as well as the rest of the world, as money and supplies were necessary in assisting these people in their recovery. Also, exaggerated Katrina rumors obstructed rescue worker participation. These are just some examples of the harmful effects of rumor spread.

These events all possess the seeds for rumor growth, as different societies imagined different events and anticipated different outcomes. Whether or not the events would affect many people, rumors still spread between and among them about the situation at hand and its importance. In studying such rumors, we can hopefully determine how people will respond to other future events like those mentioned above. Classifying rumors has become a wide spread pursuit in the hands of psychologists and sociologists. All categories of rumors maintain their exclusive properties for how and why that rumor is spread.

Times of turmoil, war or economic difficulty tend to be associated with "wish-fulfillment", "fear" and "aggression" rumors [17].The desire to fulfill a dream or fantasy or the belief that an attack or disaster is imminent are some general rumors that might spread throughout an affected population. "Spontaneous" rumors are spread in times of crisis or mistrust, and are usually untrue or exaggerated [15]. "Premeditated" rumors are spread with a goal in mind, usually with "Machiavellian purposes" in order to better one's position [15]. These types of rumors are commonly found in the form of advertising or propaganda. The "self-fulfilling" rumor is one that "serves to alter perceptions and behaviors in such a way that it increases the probability that the rumored event with indeed come to pass" [27]. The most obvious example of this type of rumor was the fictitious depletion of funds the banks held during the stock market crash in 1929, which advertently led to the rush for withdraws causing this proposed event to actually take place, contributing to the Great Depression.

Studying rumors and their transmission can not only lead to our understanding of their nature, but also to the possible management and prevention of those rumors that may be harmful in nature. In the generation phase of the rumor, or its origination phase, we can attempt to reduce the hesitation or doubt of the listener [7]. Also, as in illnesses or spread of disease, it is important to combat the rumor during its earliest stages, which will reduce the number of times an individual will hear the rumor, therefore, reducing the chance that individual will believe the rumor [7]. Although rumors have different organizational structures, it is important to study not only their nature in generation and transmission, but also the possible consequences of rumor.

1 Social networks and rumors

Social network data consists of binary social relations. That is, it records the presence or absence of relationships among pairs of persons. There are many kinds of social networks, including random networks, complete networks, regular, bipartite, hypercubes and small world. In the application of acquaintances, there are also directed and undirected graphs. This paper will only consider the undirected graphs, in which a two way communication exists.

2.1 Random Networks

In the study of graph theory, a random graph (network) is defined as one that is generated using a modeled random method involving network size and probability theory. A random network is created beginning with a network size; namely, the number of nodes, or individuals, present in the network. Edges, or connections, are then added in a random manner among the individuals using some probability model. Different probability distributions will in turn create different social constructs.

Random networks are products of a stochastic arrangement of connections between vertices in a graph. A random graph is a graph in which properties such as the number of graph vertices, graph edges, and connections between them are determined in some random way [28]. A random graph is obtained by starting with a set of *n* vertices and adding edges between them at random. Different random graph models produce different probability distributions on graphs. The theory of random graphs lies at the intersection between graph theory and probability theory, and studies the properties of typical random graphs.

The most commonly studied model, and that studied in this paper, called *G(n,p)*, includes each possible edge independently with probability *p*, in a graph of size n (n vertices). A closely related model, *G(n,M)* assigns equal probability to all graphs with exactly *M* edges. Both models can be viewed as snapshots at a particular time of the random graph process, which is a stochastic process that starts with *n* vertices and no edges and at each time unit adds one new edge chosen uniformly from the set of missing edges.

The theory of random graphs studies typical properties of random graphs, those that hold with high probability for graphs drawn from a particular distribution. For example, we might ask for a given value of *n* and *p* what the probability is that *G(n,p)* is *connected*, meaning that it has a path between any two vertices. In studying such questions, random graph theorists often concentrate on the limit behavior of random graphs, or the values that various probabilities converge to as *n* grows very large.

The algorithm for generating a random network is somewhat simple. We begin with a network size *n*, namely, the number of nodes in the network. The possible connections between nodes are determined using probabilities. Assigning a "probability of connectivity" p, to the entire network will allow for edges to be placed randomly throughout. Figure 2.1 demonstrates the process of how to conduct this generation through flow chart form.

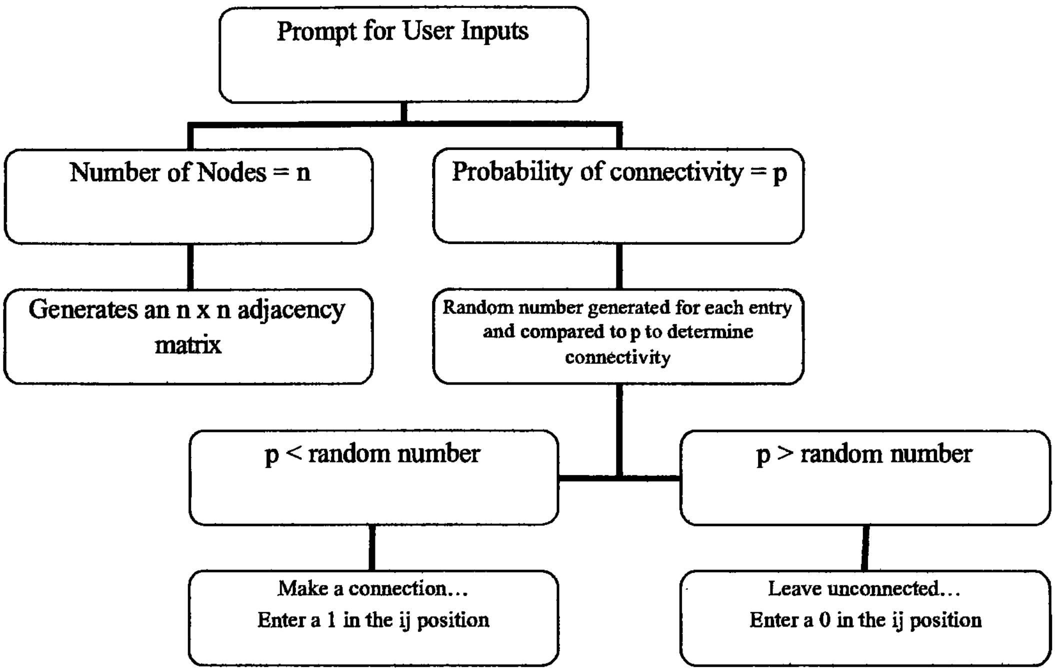


Figure 2.1

A random number $0 \leq q \leq 1$ is generated in each possible edge and then compared to the probability assigned. If $q > p$ an edge is placed between the two nodes and if $q < p$ the edge placement is denied. This process is continued throughout the entire network, until all possible edges are positioned. Figure 2.2 is an example of a network generated using this method.

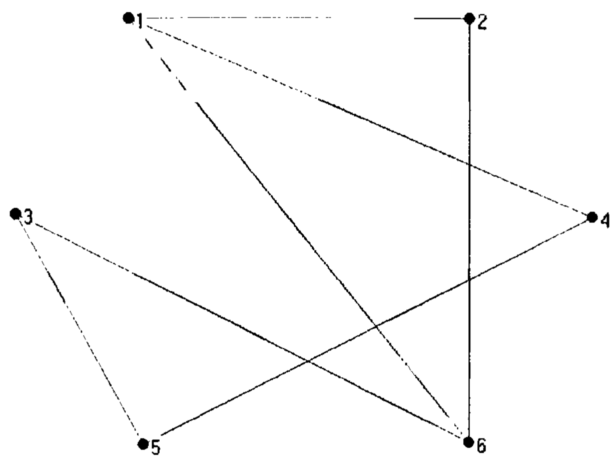


Figure 2.2

In order to generate the actual network, an adjacency matrix must first be established. From the adjacency matrix, we may design the network. An adjacency matrix for a graph G with vertex set V and edge set E, is an n x n matrix A such that

$$A(i,j)=\begin{cases}1 & \text{if} \quad (i,j)\in E \\ 0 & \quad \text{otherwise}\end{cases} \tag{1}$$

where i and j are nodes of G respectively [10]. Begin initially with a matrix of size n x n, where n is the number of nodes in the network, and all entries are 0. The number of rows and columns are equally n in size. It is important to note that adjacency matrices are always symmetric. For each connection made between the i^{th} and j^{th} node, a 1 is placed in the ij^{th} and ji^{th} positions of the matrix. Whenever the connection fails, a 0 remains in the both positions. Note that the diagonal entries must remain 0 as we are dealing with a simple network where no loops exist. This means that no individual is connected to themselves. The adjacency matrix for the random graph is shown below in Figure 2.3.

$$A = \begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \end{pmatrix}$$

Figure 2.3

Observing the first row and its identical first column, it is clear that the 1's represents node 1's connection to the nodes 2, 4, and 6. The rest of the matrix reads the same, with the second row representing node 2's connections and so on.

2.2 Small World Networks

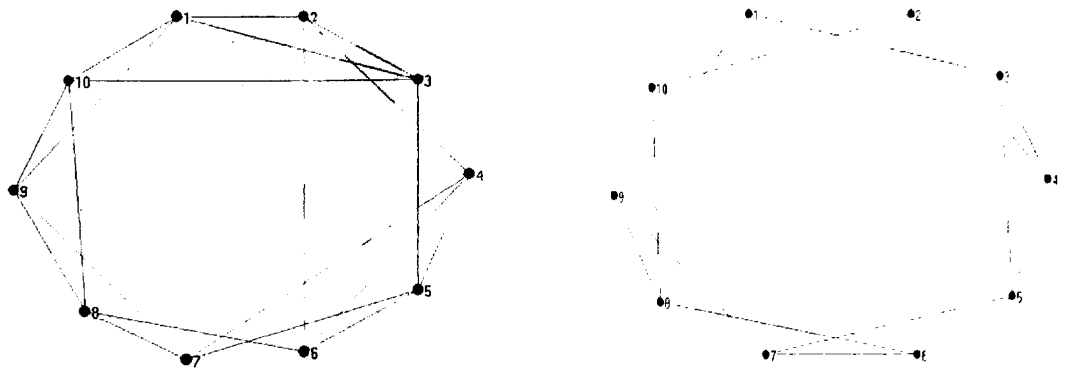
A small world network is one created from any other form of social network. The connections in an original network are systematically removed and placed elsewhere in the network in order to create a network such that some clustering exists between nodes [32]. This simply means that most nodes are also neighbors of one another, but every node can be reached from every other by a small number of hops or steps. A small world network adds some shortcuts, where nodes represent people and edges connect people that know each other, captures the small world phenomenon of strangers being linked by a few mutual acquaintances. The idea of degrees of separation is the theory that anyone on earth can be connected to any other person on the planet through a chain of acquaintances with very few intermediaries. The concept is based on the notion that the number of acquaintances grows exponentially with the number of links in the chain, and so only a small number of links is required for the set of acquaintances to become the whole human population [32]. Small world networks act as a method of (displaying or implementing) this concept.

In mathematics and physics, a small world network is a class of random graphs where most nodes are also neighbors of one another, but every node can be reached from every other by a small number of steps. A small world network, where nodes represent people and edges connect people that know each other, captures the small world phenomenon of strangers being linked by a mutual acquaintance. Taking a connected graph or network with a high graph diameter and adding a very small number of edges randomly, the diameter tends to drop drastically [32]. This is known as the small world phenomenon, sometimes referred to as "six degrees of separation". This reference is made since, in the social network of the world, any person turns out to be linked to any other person by roughly six connections.

By virtue of the above definition, small world networks will inevitably have high representation of cliques, and subgraphs that are a few edges shy of being cliques, i.e. small-world networks will have sub-networks that are characterized by the presence of connections between almost any two nodes within them. This follows from the requirement of a high clustering coefficient. Secondly, most pairs of nodes will be connected by at least one short path, which follows from the requirement that the shortest average path length be small. Additionally, there are several properties that are commonly associated with small world networks even though they are not required for that classification. Typically there is an overabundance of nodes in the network with a high number of connections. These hubs serve as the common connections mediating the short path lengths between other edges.

This property is often analyzed by considering the fraction of nodes in the network that have a particular number of connections going into them (the degree distribution of the network) [32]. Networks with a greater than expected number of hubs will have a greater fraction of nodes with high degree. Consequently the degree distribution will be enriched at high degree values. This type of network is by no means the only kind of small world network. Graphs of very different topology can still qualify as small-world networks as long as they satisfy the two definitional requirements above.

Generating small world networks can be done using a different number of models. The most stable model, it seems, is to cut edges from an original network, such as a k-regular ring, and rewire these edges elsewhere in the network in a systematic fashion. Figure 2.4 was generated using a 4-regular ring with 10 nodes, and rewiring its edges to allow the degree to remain the same. The algorithm is quite simple then, as shown in the flow chart of Figure 2.5.



Small World Network 4-Regular Ring

Figure 2.4

In Figure 2.4, a 10 node ring with each node having degree 4, was rewired to create a small world network. The cut-edges were between nodes 2 and 10, 3 and 4, and 6 and 7. The added-edges were between 3 and 10, 2 and 6, and 4 and 7.

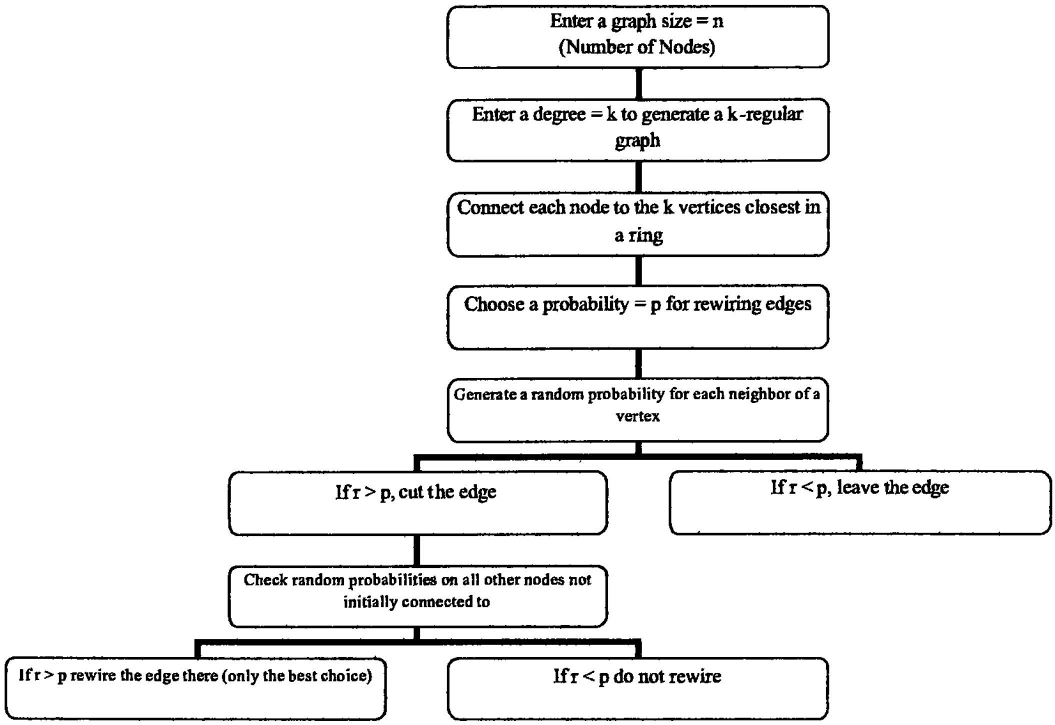


Figure 2.5

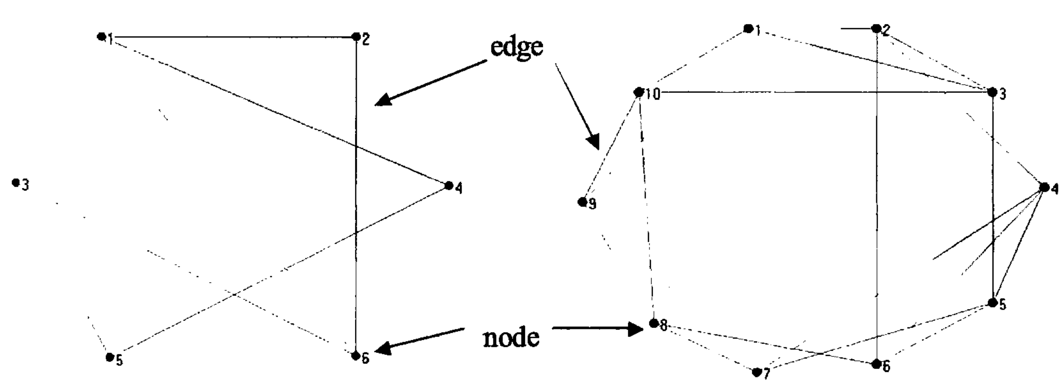
As in the random network, we must first begin with an adjacency matrix. The corresponding matrix to the network in Figure 2.4 is shown below in Figure 2.6.

$$B=\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

Figure 2.6

2.3 Basic graph theory

Mathematically, social networks can be represented as graphs or adjacency matrices. A graph is defined as a set of nodes and a set of lines, or edges, that connect the nodes. The nodes, in a social aspect, represent people or even groups of people, while the edges represent the connection between people or groups of people in that network. In Figure 2.7, the edge between nodes 1 and 2 represents a relationship between node 1 and node 2, in both networks. It is important to keep in mind that the length of the edges does not represent anything in the case of social networking. This is because all the edge represents is a relationship between node 1 and node 2. Similarly, the orientation of the drawing means nothing. For example, node 1 could have been placed in the middle of the graph; this would not mean anything different than if it had been placed in the bottom of the graph.



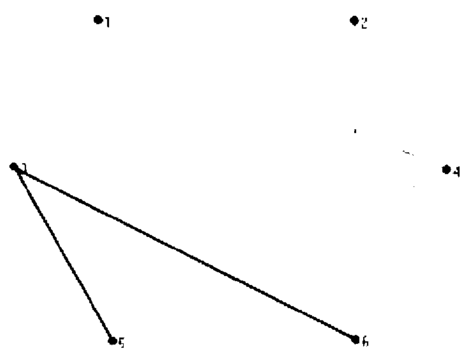
Random Network

Small World Network

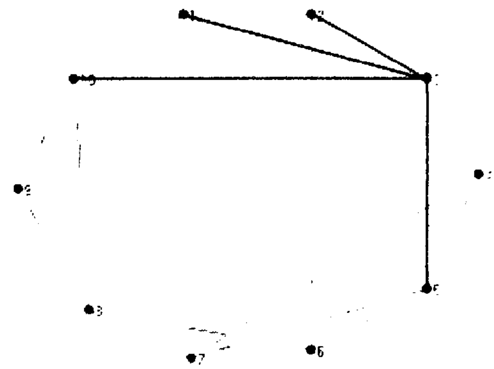
Figure 2.7

The property garnering attention in this paper is "Who is related, or connected, to whom?" The use of the term ' related ' may be somewhat misleading, since we are not talking about relationships, specifically. We are simply concerned with whether or not two people know each other in some way, and not necessarily the strength of their relationship. There are many other terms used in place of "node" and "edge". For example, a node is referred to as a vertex in graph theory. However, it is important to keep some uniformity throughout this paper, so we will continue to refer to these elements as nodes and edges, and become more specific in applications with their defined representation. Figure 2.7 represents drawings of two graphs of different size, one representing a random network and the other a small world network. The larger of the two is the small world as its size, corresponding to the number of nodes, is 10.

Two nodes connected by an edge are said to be "adjacent" and are the endpoints of that edge. An edge that originates or terminates at a given node is said to be "incident" to that node. The degree of a node is defined by the number of edges incident to that node, or even the number of other nodes to which it is connected. If a point has degree 0 it is called an isolate, and has no connections whatsoever within the social network. In the Figure 2.8, we say that 3 is connected to 5 and 6 in the random network, so node 3 has degree 2. In the small world network, node 3 is connected to 1, 2, 5 and 10, and therefore; node 3 has degree 4.



Random Network



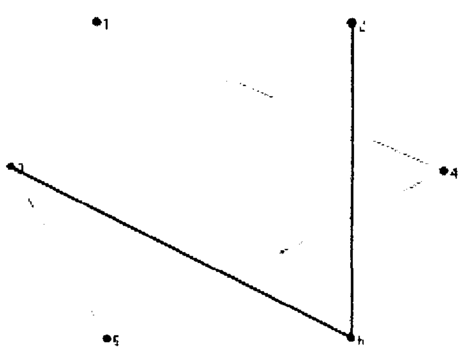
Small World Network

Figure 2.8

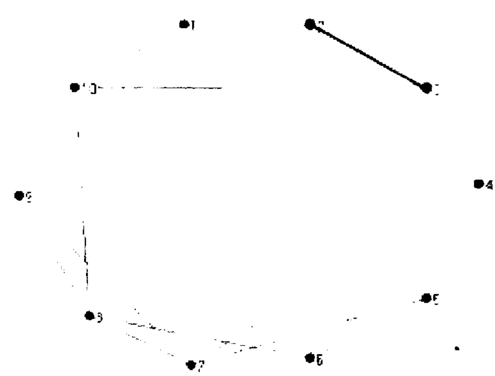
In discussing the specific cases of random and small world networks, we are comparing a network that has a random distribution of connections between nodes to a network manipulated to have fewer connections needed among groups of nodes, in order for people to have common acquaintances. The process of adding edges at a rate proportional to that in creating an original random network will give us one form of small world network. Another way to create a small world network is to cut edges from the random network and systematically place them elsewhere in the network. The process to be presented throughout the remainder of this paper is to rewire the edges of a k-regular ring to yield a small world network. For a simple example, we will specifically use a 4-regular ring of 10 nodes, as seen above after the rewiring process has been conducted.

2.3.1. Paths

A path is an alternating sequence of nodes and edges which does not visit any node more than once. Two (or more) paths are disjoint if they don't share any nodes, also paths are edge-disjoint if they don't share any edges. The length of a path is defined by the number of edges in that path. Small world networks constitute networks that have a smaller average path length than comparable random networks. The significance in this idea is that, the shorter the average path length, the quicker the rumor flow throughout the network. This idea will be utilized in describing the centrality measures discussed later in the section. Observe the differing path lengths between the small world networks in Figure 2.9.



Random Network



Small World Network

Figure 2.9

Here, the path length from node 2 to node 3 is 2 in the random network, as 2 edges were required to get between the 2 nodes. In the small world network, the path length from node 2 to node 3 is only 1. The different conditions set on creating these two networks will explain why this may be a frequent occurrence.

The shortest path between two nodes is not always unique. That is, there may be several paths between the same two points that have the smallest number of edges. The graph-theoretic distance between two nodes is defined as the length of the shortest path between them. Let $W_n(i,j)$ be the length of the shortest path from nodes v_i to v_j [10]. The pseudo code found in Figure 2.10 represents the process for finding the shortest path length between any two nodes.

```
Initialize W_0
for k = 1 to n do
        for i = 1 to n do
                for j = 1 to n do
                        W_k(i,j) ← min((W_{k-1}(i,k) + W_{k-1}(k,j)), W_{k-1}(i,j))
Output W_n
```

Figure 2.10

If a rumor is flowing through a network, the time that it takes to get from one node to another is partly a function of the graph-theoretic distance between them. Nodes that are not far, on average, from all other nodes, tend to receive what's flowing through the network sooner than other nodes.

2.3.2. Clustering Coefficient

The clustering coefficient is a measure of the relationship between a node's neighbors, or the connections to that node. This value indicates the ratio of existing links connecting a node's neighbors to each other to the maximum possible number of such links. Another way of describing this property is as a proportion of actual links to possible links. In a small world network, we expect to have a rather high clustering coefficient, as the connections between groups of nodes will increase, over that of a random network.

The formulas most commonly used to calculate the clustering coefficients of individual nodes are the proportion of edges formula and the proportion of triangles formula. The proportion of edges is given by:

$$C_i = \frac{2\left|\{e_{jk}\}\right|}{k_i(k_i-1)}, \quad \text{for all } v_i, v_k \in N_i, e_{jk} \in E \tag{2}$$

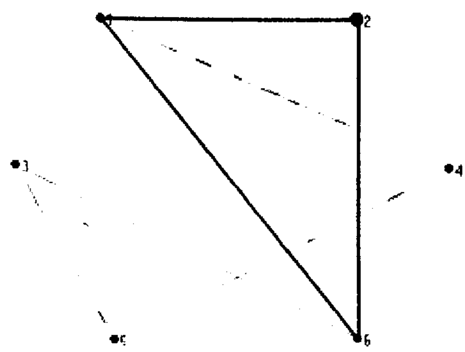
which describes the clustering coefficient as the ratio of the size of the actual edge set to the number of possible edges existing between the i^{th} node and all of its neighbors. This ratio can be derived from the definition of each of these terms, where the size of the edge set is given by $\left|\{e_{jk}\}\right|$ and the number of possible edges is classified as $k_i(k_i-1)/2$. When setting up the actual proportion, we have $\left|\{e_{jk}\}\right|/\left(k_i(k_i-1)/2\right)$ which simplifies to $2\left|\{e_{jk}\}\right|/k_i(k_i-1)$.

The process clearer to the naked eye is that of counting triangles; however, it is somewhat more complicated than it sounds. The formula for the clustering coefficient of any node *i*, is given by:

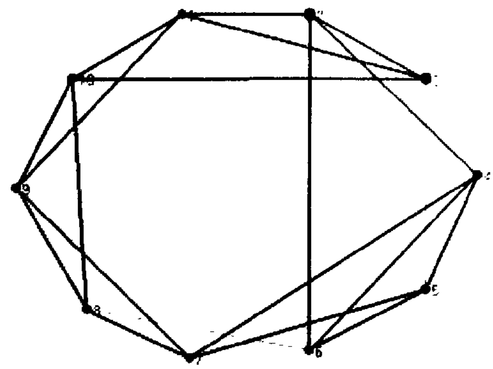
$$C_i = \frac{\lambda_G(i)}{\tau_G(i)}, \text{for all } i \in N_i \qquad (3)$$

where $\lambda_G(i)$ represents the number of subgraphs of G, containing node *i*, that have three nodes and three edges (triangles), and $\tau_G(i)$ represents the number of subgraphs of G, containing node *i*, that have three nodes but only two edges.

The clustering coefficient for each node in the random and small world graphs below, G and H respectively, can be calculated using the method of counting triangles. All triangle subgraphs of each entire graph have been indicated by coloring those edges blue. Figure 2.11 shows the different clustering patterns of the random and small world networks introduced earlier. From these displays, we can easily calculate the clustering coefficients for individual nodes as well as the graphs' average clustering coefficients.



Random Network (G)



Small World Network (H)

Figure 2.11

The difference in the number of triangles in the random vs. small world networks is already evident in Figure 2.11. The clustering coefficients for the individual nodes in G are as follows;

$C_1 = \frac{1}{3}$ since we have one triangle subgraph containing node 1, $\{1,2,6\}$ and 3 subgraphs containing node 1 with 3 nodes and at least 2 edges each, $\{1,2,4\}$, $\{1,2,6\}$ and $\{1,4,6\}$

$C_2 = 1$ since we have only one possible triangle subgraph containing node 2, which has been fullfilled; $\{1,2,6\}$

$C_3 = 0$ since node 3 has no triangle subgraphs associated with it

$C_4 = 0$ since node 4 has no triangle sungraphs associated with it

$C_5 = 0$ since node 5 has no triangle subgraphs associated with it

$C_6 = \frac{1}{3}$ where we have one triangle subgraph containing node 6, $\{1,2,6\}$ and 3 subgraphs containing 6 with at least 2 edges

Calculating the clustering coefficient over the entire network is as easy as averaging the individual clustering coefficients over the number of nodes. The formulation for this process is to sum up all of the n clustering coefficients and then divide by the total number of coefficients, n. Here, we are simply calculating the mean clustering coefficient. So, we have

$$\overline{C} = \frac{1}{n}\sum_{i=1}^{n} C_i \qquad (4)$$

Considering our random graph and its individual nodes' clustering coefficients, the average clustering coefficient over the entire graph is given by:

$\overline{C} = \frac{1}{6}(C_1 + C_2 + C_3 + C_4 + C_5 + C_6)$ as there are 6 nodes in the network. This yields $\overline{C} = \frac{1}{6}\left(\frac{1}{3} + 1 + 0 + 0 + 0 + \frac{1}{3}\right) = \frac{1}{6}\left(\frac{5}{3}\right) = .2777$ as our clustering coefficient for G.

The same process can be followed on H to give the following individual clustering coefficients:

$$C_1=\frac{1}{2}\quad C_2=\frac{1}{3}\quad C_3=\frac{1}{3}\quad C_4=\frac{1}{2}\quad C_5=\frac{1}{3}$$

$$C_6=\frac{1}{3}\quad C_7=\frac{1}{3}\quad C_8=\frac{1}{3}\quad C_9=\frac{1}{2}\quad C_{10}=\frac{1}{2}$$

Thus, our average clustering coefficient over H can be found by averaging these 10 individual coefficients to give us:

$$\overline{C}=\frac{1}{10}\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{3}+\frac{1}{2}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{2}+\frac{1}{2}\right)=\frac{1}{10}\left(\frac{24}{6}\right)=.4000$$

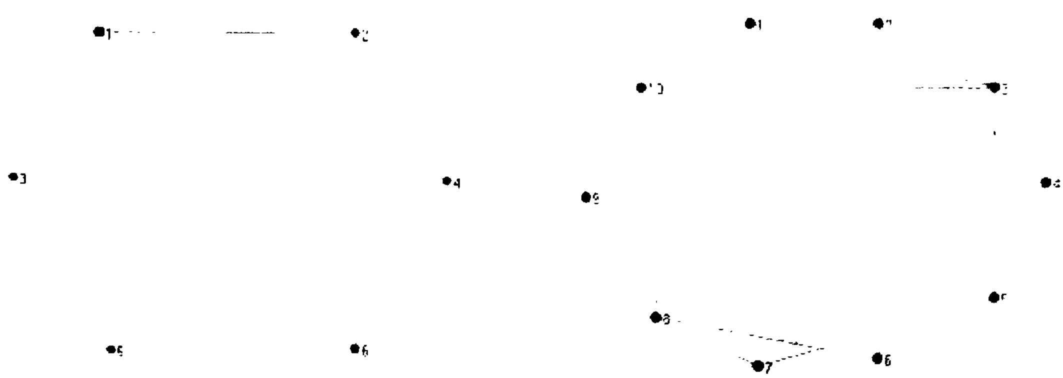
Comparing the two clustering coefficients, we see that the small world network has a higher coefficient than the random, which shows that the rumor would not have to be passed through so many people to get from one individual to the next. We may return to the notion of "six degrees of freedom" to further evaluate the meaning of the previous statement. The small world network assumes a structure that allows for the individuals of the network to few others "between" them and any other node in the network. Such a structure maintains the ideal societal bond between individuals where communication flows very rapidly. However, a random network, without this property, does not seem to model such a network.

2.3.3. Measures of Centrality

Centrality is the real valued function of the vertex set in an undirected, connected graph that defines the individual nodes as a linear combination of each other, according to their relationships, represented by the edge set [16]. The mathematical notation for such a function is given by $f(G,v)$, or a function of the graph G and each individual vertex or node v. Centrality finds its origins in the study and location of the strongest or most popular individuals in a social network. The concept of centrality concerns itself with the individual who will most influence the majority of the group. If a rumor is seeded within this individual, or if the central individual eventually hears the rumor and believes it, it should be assumed that the rumor will spread rapidly. This is due to the fact that the majority of this social network looks to the central individual for guidance on believing the rumor. The measures of centrality are numerous; however, we will consider three such measures: degree, closeness and betweenness.

2.3.3a. Degree Measure

The comparison of the degrees of each node within a network is quite simple. The node with the largest number of connections may be considered the central node. This merely indicates that this person or individual has the most friends, or maybe just more relationships; however strong they may be. We denote the degree of each node as the degree of the i^{th} node, where i represents any node from 1 to n, as d_i. In Figure 2.12, we have both a random network and a small world network which we have dealt with previously in the paper. In looking at the random network, it is easy to see that nodes 1 and 6 might be considered central nodes, since their degrees are 3, which is the highest possible degree in that network. However, in the small world network, the degree of all ten nodes is 4, which does not allow us to draw any conclusions about the centrality of the small world network, through the property of degree. Since all nodes assume the same degree, there is no locatable central node.



Random Network Small World Network

Figure 2.12

The degree of an individual node in comparison with the rest of the network is the key in determining whether that node is of central value, in other words, whether that node has the most influence or dominance over the remaining nodes. Clearly, the random network possesses two such individuals, say nodes 1 and 6. These two are of degree 3, surpassing the degrees of the remainder of the nodes. But in the small world network, we can make no comparison, as all of the nodes have equal degree of 4. This suggests that there is no central node, so no one individual is more powerful or influential over the others.

The possibility to find the average degree over an entire network is also interesting, so that a comparison might be made regarding the uniformity of degree distributions. If each node's degree is considered separately, we can locate a central node. However, calculating the mean degree over the whole network can sometimes illustrate a consistency, or lack thereof, in the distribution of the degrees. Consider the notation for an individual i's, degree, d_i. Then the average degree over a network of size n can be found using

$$\bar{d} = \frac{\sum_{i=1}^{n} d_i}{n} \tag{5}$$

where the sum of all degrees in the network is divided by the total number of individuals in the network. Let us now return to Figure 2.12 and calculate the average degree of the random network as well as the small world network.

The random network has the following degrees associated with its individuals.

$$d_1 = 3 \qquad d_2 = 2 \qquad d_3 = 2$$
$$d_4 = 2 \qquad d_5 = 2 \qquad d_6 = 3$$

The average of these degrees is found simply by summing them and dividing by 6, as there are 6 individuals, as shown below.

$$\sum_{i=1}^{6} d_i = d_1 + d_2 + d_3 + d_4 + d_5 + d_6 = 3+2+2+2+2+3 = 14$$

$$\bar{d} = \frac{\sum_{i=1}^{6} d_i}{6} = \frac{14}{6} = 2.33$$

The small world network calculations follow in the same manner, although in this case, the average will be taken over 10 nodes rather than 6.

$$\begin{array}{ll} d_1 = 4 & d_6 = 4 \\ d_2 = 4 & d_7 = 4 \\ d_3 = 4 & d_8 = 4 \\ d_4 = 4 & d_9 = 4 \\ d_5 = 4 & d_{10} = 4 \end{array}$$

The average degree should be obvious to us as all degrees in this network are equal to 4.

$$\sum_{i=1}^{10} d_i = d_1 + d_2 + d_3 + d_4 + d_5 + d_6 + d_7 + d_8 + d_9 + d_{10} = 4+4+4+4+4+4+4+4+4+4 = 40$$

$$\bar{d} = \frac{\sum_{i=1}^{10} d_i}{10} = \frac{40}{10} = 4$$

The difference in the average degrees is considerable when the size of the networks is taken into account. The random network has only 6 nodes while the small world network has only 10. These networks are considerably small when weighted against real societal networks. So, even with a difference of 1.67 in the average degrees, we can consider this to be a substantial disparity. More importantly, the small world network has an evident consistency in its degree distribution, while the random network does not. The small world network yields less error in determining the average degree than in the random network's case.

Although, we have not considered networks of the same size, it is important to understand that the structure of the network plays a vital role in determining centrality as a result of degree. Later, in Section 4, such essentials will be demonstrated using the comparison of multiple networks.

2.3.3b. Closeness Measure

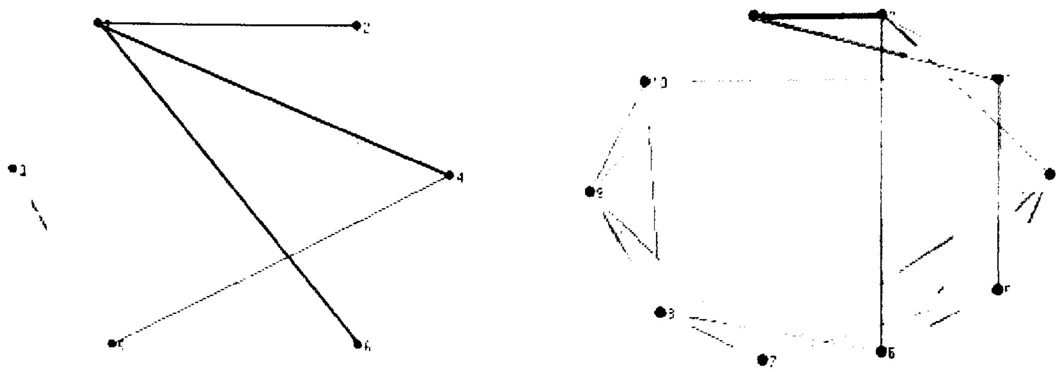
The individual who is closest to most of the others in the network, meaning that they don't need to go through too many others to get to another specific individual, could also be considered the central node. Two ways to observe this idea are through closeness and betweenness. The definition of closeness is as follows:

$$C_c(v) = \frac{1}{\sum_{t \in V} d_G(v,t)} \qquad (6)$$

where V denotes the vertex set and $\sum_{t \in V} d_G(v,t)$ is the sum of the distances between node v and all others in the network [33]. The distance between any two nodes is simply the minimum number of steps it takes to get between them. As a proportion, $0 < C_c(v) \leq 1$. Larger networks have smaller closeness measures as the denominator is the sum of a larger vertex set. But as connectivity increases, the minimal path lengths between nodes decreases and thus, the denominator decreases. By basic calculus, we know that

$$\lim_{\sum d_G \to 1} C_c(v) = \lim_{\sum d_G \to 1} \frac{1}{\sum_{t \in V} d_G(v,t)} = 1$$

Since the minimum value for $\sum d_G$ will always be 1, we take the limit as $\sum d_G$ approaches 1. The limit in this case is in fact 1, which is the maximum possible value for a closeness measure. So, as the denominator decreases, the closeness measure increases. However, complete networks having the highest connectivity, do not actually have closeness measures of 1. This calculation depends solely on their size. We should consider both networks in Figure 2.13 to determine which node is the central node, if there is an obvious choice using this method.



Random Network　　　　　Small World Network

Figure 2.13

The random network has only six nodes, so the calculations of closeness will be somewhat quicker than in the small world network with ten nodes. For the random network we have:

$$C_1 = \frac{1}{1+2+1+2+1} = \frac{1}{7} = .1429$$

$$C_2 = \frac{1}{1+2+2+3+1} = \frac{1}{9} = .1111$$

$$C_3 = \frac{1}{2+2+2+1+1} = \frac{1}{8} = .125$$

$$C_4 = \frac{1}{1+2+2+1+2} = \frac{1}{8} = .125$$

$$C_5 = \frac{1}{2+3+1+1+2} = \frac{1}{9} = .1111$$

$$C_6 = \frac{1}{1+1+1+2+2} = \frac{1}{7} = .1429$$

Since both nodes 1 and 6 have the highest closeness measure, 1 and 6 could be considered central nodes in the random network, just as before with the degree measure. Again, it is quite simple to calculate the average closeness over the entire network. The mean of the closeness measures is found by summing them over the total number of measures as shown below.

$$\overline{C_c(v)} = \frac{\left(\frac{1}{7}+\frac{1}{9}+\frac{1}{8}+\frac{1}{8}+\frac{1}{9}+\frac{1}{7}\right)}{6} = \frac{\left(\frac{191}{252}\right)}{6} = \frac{191}{1512} = .1263$$

Now considering the small world network, the same procedure yields the closeness measurements of:

$$C_1 = \frac{1}{14} \quad C_2 = \frac{1}{14} \quad C_3 = \frac{1}{14} \quad C_4 = \frac{1}{14} \quad C_5 = \frac{1}{14}$$
$$C_6 = \frac{1}{14} \quad C_7 = \frac{1}{14} \quad C_8 = \frac{1}{14} \quad C_9 = \frac{1}{14} \quad C_{10} = \frac{1}{14}$$

where the average closeness measure is clear to be $\overline{C_c(v)} = \frac{1}{14} = .0714$ since all nodes have the same closeness measure. Since all of the closeness measures are equal in the small world network, we cannot distinguish which node should be considered central, if any. The results here are similar to those obtained from observing the degree measure of this same network. Also, notice that the small world network has a smaller closeness measure than the random network. This is due to the fact that the small world network is larger, with 10 nodes rather than 6.

2.3.3c. Betweenness Measure

Lastly, we will consider the centrality measure of betweenness, which considers the comparisons in the distances between any two nodes. By definition, we have that

$$C_B(v) = \sum_{s \neq t \neq v \in V} \frac{\sigma_{st}(v)}{\sigma_{st}} \tag{7}$$

which is the sum of the proportions of the shortest path between nodes s and t through the node v to the shortest path between nodes s and t, with no restriction [33]? This calculation is a little more involved. The shortest path between two nodes passing through a third will be greater than or equal to the shortest path between the two nodes without having the restriction.

$$\sigma_{st} \leq \sigma_{st}(v) \Rightarrow \frac{\sigma_{st}(v)}{\sigma_{st}} > 1$$

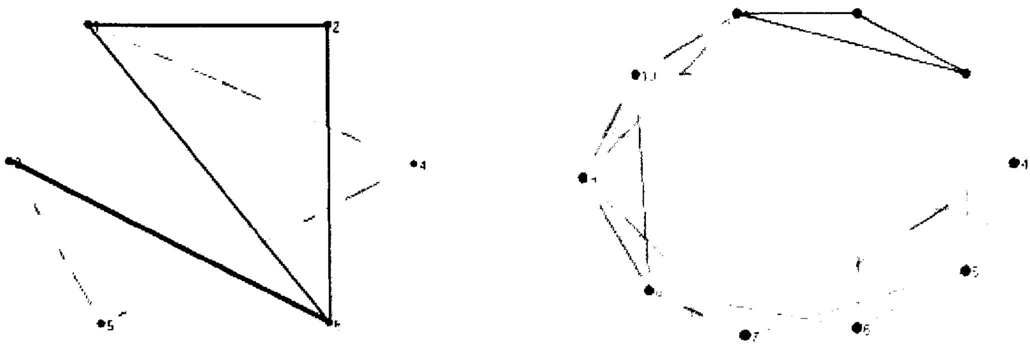
Since only simple graphs are under consideration, the numerator will never be zero. Then,

$$1 < \sum_{s \neq t \neq v \in V} \frac{\sigma_{st}(v)}{\sigma_{st}} \leq |V|$$

where the larger the network, the larger the betweenness measure. The length of the restricted paths will decrease towards 2, and the length of the path lengths in the denominator will decrease towards 1.

$$\lim_{\sigma_{st} \to 1} \left(\lim_{\sigma_{st}(v) \to 2} \left[\frac{\sigma_{st}(v)}{\sigma_{st}} \right] \right) = \frac{2}{1} = 2$$

Figure 2.14 illustrates the betweenness measurement for node 1 in one case on either graph.



Random Network Small World Network

Figure 2.14

In the random network, the betweenness measures are as follows:

$$C_B(1)=\left(\frac{3}{2}+\frac{2}{2}+\frac{3}{3}+\frac{2}{1}\right)+\left(\frac{3}{2}+\frac{4}{1}+\frac{4}{1}\right)+\left(\frac{4}{1}+\frac{2}{2}\right)+\left(\frac{3}{2}\right)=21.5$$

$$C_B(2)=\left(\frac{3}{2}+\frac{5}{1}+\frac{4}{2}+\frac{2}{1}\right)+\left(\frac{4}{2}+\frac{4}{1}+\frac{5}{1}\right)+\left(\frac{5}{1}+\frac{3}{2}\right)+\left(\frac{4}{2}\right)=30$$

$$C_B(3)=\left(\frac{5}{1}+\frac{4}{1}+\frac{3}{2}+\frac{4}{1}\right)+\left(\frac{4}{2}+\frac{3}{3}+\frac{5}{1}\right)+\left(\frac{4}{1}+\frac{3}{2}\right)+\left(\frac{2}{2}\right)=29$$

$$C_B(4)=\left(\frac{5}{1}+\frac{3}{2}+\frac{2}{2}+\frac{4}{1}\right)+\left(\frac{4}{2}+\frac{3}{3}+\frac{5}{1}\right)+\left(\frac{4}{1}+\frac{4}{1}\right)+\left(\frac{3}{2}\right)=29$$

$$C_B(5)=\left(\frac{5}{1}+\frac{3}{2}+\frac{4}{1}+\frac{4}{1}\right)+\left(\frac{4}{2}+\frac{5}{2}+\frac{3}{1}\right)+\left(\frac{2}{2}+\frac{4}{1}\right)+\left(\frac{3}{2}\right)=28.5$$

$$C_B(6)=\left(\frac{2}{1}+\frac{2}{2}+\frac{4}{1}+\frac{3}{2}\right)+\left(\frac{2}{2}+\frac{3}{2}+\frac{3}{3}\right)+\left(\frac{3}{2}+\frac{4}{1}\right)+\left(\frac{4}{1}\right)=21.5$$

Notice that the nodes with the highest degree have the smallest betweenness measure. This indicates that it takes fewer nodes between any two to pass the rumor on. Another important calculation to consider is the average of these betweenness measures, which can easily be found by summing the measures over the total number of measures as shown below.

$$\begin{aligned}\overline{C_B(v)} &= \frac{21.5+30+29+29+28.5+21.5}{6} \\ &= \frac{159.5}{6} \\ &= \boxed{26.6}\end{aligned}$$

As shown below for the betweenness measures of the small world network, all of the outcomes are extremely large, suggesting that the property of "six degrees of separation" does in fact hold for this type of network. The notion of larger networks having larger betweenness measures also holds here.

$$\begin{aligned}C_B(1) &= 85.5 \\ C_B(2) &= 83.5 \\ C_B(3) &= 85 \\ C_B(4) &= 90.5 \\ C_B(5) &= 84.5 \\ C_B(6) &= 81.5 \\ C_B(7) &= 81.5 \\ C_B(8) &= 81.5 \\ C_B(9) &= 81.5 \\ C_B(10) &= 85.5\end{aligned}$$

The average of the above betweenness measures is found simply by adding them and dividing by ten. Below is the solution for this example.

$$\overline{C_B(v)} = \frac{85.5+83.5+85+90.5+84.5+81.5+81.5+81.5+81.5+85.5}{10}$$

$$= \frac{840.5}{10}$$

$$= \boxed{84.05}$$

Comparing the results of the average betweenness measure on the random and small world networks, we can see that the small world network has a much larger betweenness measure. This is due to the larger size and better structure of the network. Recall that the proportion will always be greater than or equal to one, due to the fact that the numerator will always be greater than or equal to the denominator. While the shortest path between any two nodes is extremely small, the denominator will be small, resulting in a larger betweenness measure. But when any two nodes have a larger shortest path length, the denominator is closer to the numerator in size. Therefore, the ratio will be smaller. The larger betweenness measures occur in the cases of more connected networks, or even networks of a higher structure. The comparison to make between the small world and random network results should be obvious.

Centrality measures tend to target the same nodes as central nodes, but locate them in different manners. Some measures are more stable as more mathematical work is done to prove such nodes are to be considered central. Moreover, the average of the centrality measures over whole networks will help us in later determination of the effects such properties have on the flow of rumors over these networks.

2.4 Other Types of Networks

This section aims at describing two other types of networks which are significant to the study of rumor. The bipartite and complete networks are two networks with very different properties. Future study will examine the bipartite network and its rumor propagation, while the complete network is considered in the cases of random and small world networks with connectivity of 1.0.

2.4.1 Bipartite

A bipartite graph is a graph such that the vertex set has been divided into two distinct sets U and W. Any vertex in U can be connected to any vertex in W and visa versa. However; the vertices in either set cannot be connected to any other within their shared set [11]. Thus, the graph of G has edges such that one endpoint is in U and the other is in W. Below, in Figure 2.15, is an example of a bipartite graph.

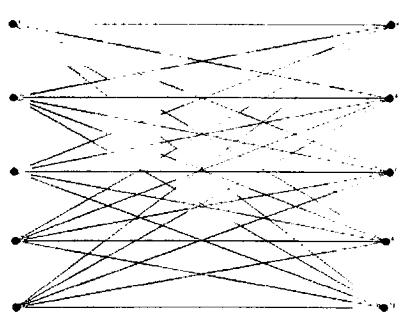


Figure 2.15

2.4.2 Complete

A complete graph is a simple graph in which each pair of graph vertices is connected by an edge. The complete graph with n vertices is denoted K_n and has $n(n-1)/2$ (the triangular numbers) undirected edges [11]. In older literature, complete graphs are sometimes called universal graphs. Below, in Figure 2.16, is an example of a complete graph. Notice that the graph is completely connected.

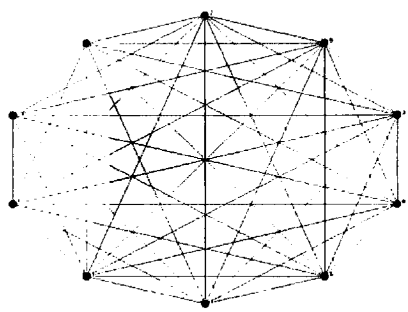


Figure 2.16

3 Stochastic Models of Rumor Flow

The stochastic theory of rumors began with the Daley-Kendall Model for transmission, including individuals representing ignorants, spreaders, and stiflers [13]. The Maki-Thompson model is a variation of the Daley-Kendall Model. The two models "formalize" the social transactions between individuals of a social network. Not only will we examine these two models by applying them to random and small world networks in the attempt to see a differing pattern, but we will also examine two proposed models that may also show significant differences between the two types of networks. These individuals will be further described in the following subsections, as there are more significant details to examine.

3.1 Proposed Basic Stochastic Model

The basic method for simulating the flow of a rumor over any network comes in the form of probability. Once a network has been generated and fixed, the constructs of that network will determine and manage the flow of the rumor throughout. The idea here is to seed the rumor in an individual node and observe the possible spread of the rumor through all other nodes.

Initially, we seed the rumor in only one node, symbolizing the rumor starter. The next step in the process is to then permit the possible spread of the rumor to this node's neighbors. A transmission rate or probability of transmission is assigned to the entire network, stating that the nodes have a certain probability of passing on the rumor. This rate we should denote as t, where $0 \le t \le 1$. In order to make a decision about whether or not the rumor is actually passed on, one must compare the transmission rate to a random probability generated for each individual node. In the case where the random number is smaller than the transmission rate, the rumor is passed to that node and they in turn begin to attempt to pass it to their neighbors. If the random probability is more than the proposed transmission rate, that spreading of the rumor fails and another random number is generated for this node. This process is continued until the node has become infected, or has heard the rumor.

This model may seem somewhat naïve or unsophisticated, considering the fact that most rumors are not forced onto individuals to believe and in turn, pass on. Also, once someone has told another individual the rumor, the chances they will tell them again are small. Figure 3.1 demonstrates the process of this simple model through a flow chart.

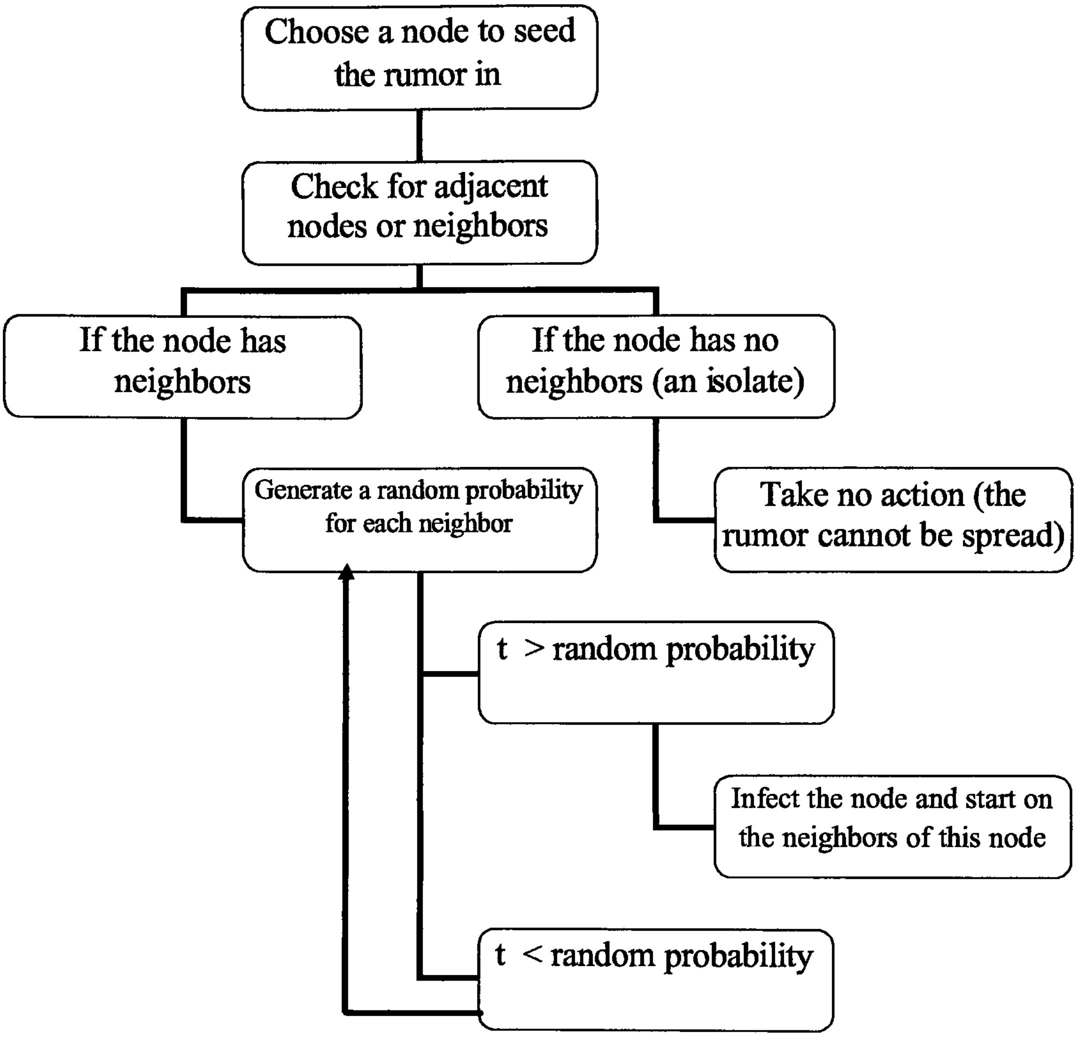
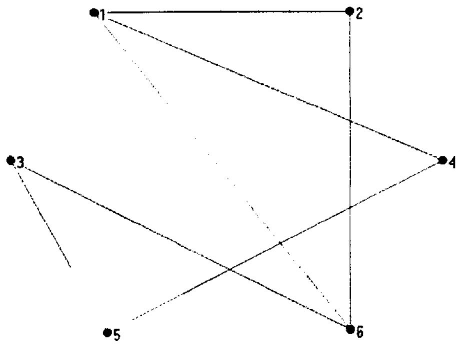


Figure 3.1

If we consider the case of the random network in Figure 3.2, where we seed the rumor in node 1, we can observe the flow of the rumor throughout the network in the form of binary outputs of a computer program as well as node colorings on a graph.



Random Network

Figure 3.2

Let's consider the transmission rate of $t = 0.10$, where there is a 10% chance of transmitting the rumor between two nodes in a time step. Recall that in this model, each node has an equal chance of becoming infected by the rumor. The following binary outputs in Figure 3.3 describe the, nonunique, rumor flow over this random network.

Nodes

1 2 3 4 5 6

	1	[1 0 0 0 0 0]
	2	[1 0 0 1 0 0]
Trials	***3***	[1 1 0 1 0 0]
	4	[1 1 0 1 0 1]
	5	[1 1 0 1 1 1]
	6	[1 1 1 1 1 1]

The above set of binary sequences displays the spread of the rumor over the 6 nodes in the random network. In the initial state, or trial 1, node 1 has been infected with the rumor. Depending on the nodes adjacent to node 1, a spread of the rumor to the remainder of the nodes is evident by the 1's replacing 0's. Every infected node has a 1 in its place while those still susceptible to the rumor have 0's. The last state, or trial 6, shows the rumor in its final stage, where the entire network has become infected.

Figure 3.3

Here, the 1's indicate an infected node while the 0's indicate a still susceptible node. Notice how the 1 in the first position represents the node 1, where we've seeded the rumor. Node 1 then attempts to pass the rumor onto its neighbors, 2, 4 and 6. However; as the transmission rate is somewhat small, only node 4 becomes infected in the first try. Once each node is infected, they continue to pass the rumor on to their neighbors until the entire network has become infected. The entire network needed six time trials to become entirely infected, while the network size was 6 and the probability of transmission was 0.10. If we were to increase the transmission rate, the network would become infected at a quicker pace, and a slower rate if the transmission rate is lower. The depiction of these types of outcomes is a little easier to understand, as in Figure 3.4 below. Once each node has become infected, the color of the node will be changed to red in order to indicate the infection.

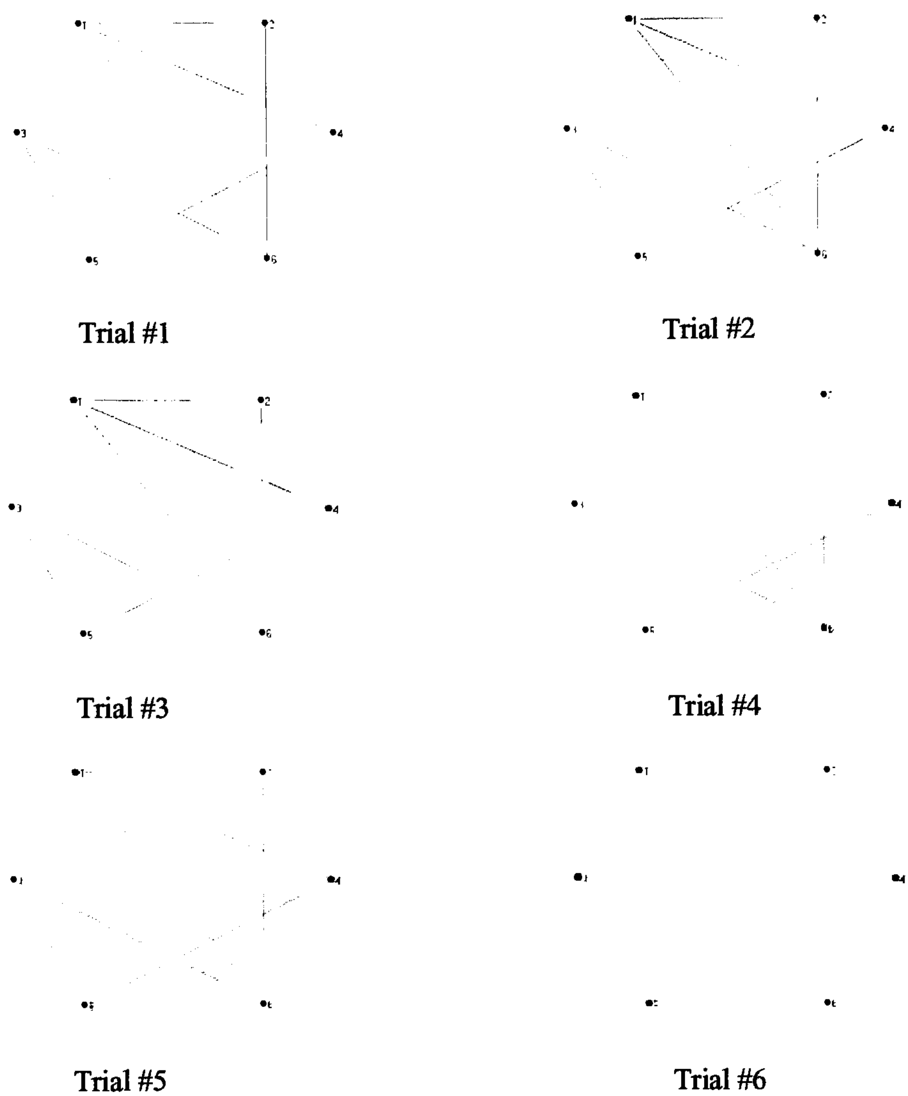
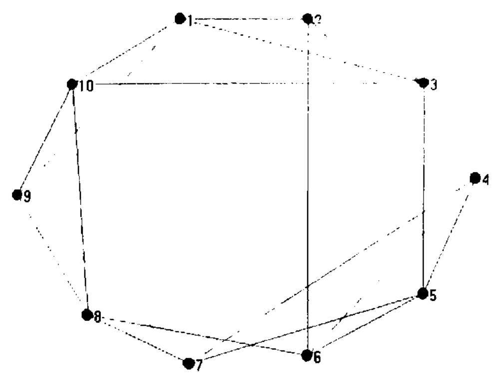


Figure 3.4

The previous example is a network with only a small number of nodes, and therefore, not extremely realistic. So, let's consider the case of 50 nodes with numerous connectivities and transmission rates, as well as numerous differing properties such as clustering coefficient, average path length and centrality.



Small World Network

Figure 3.5

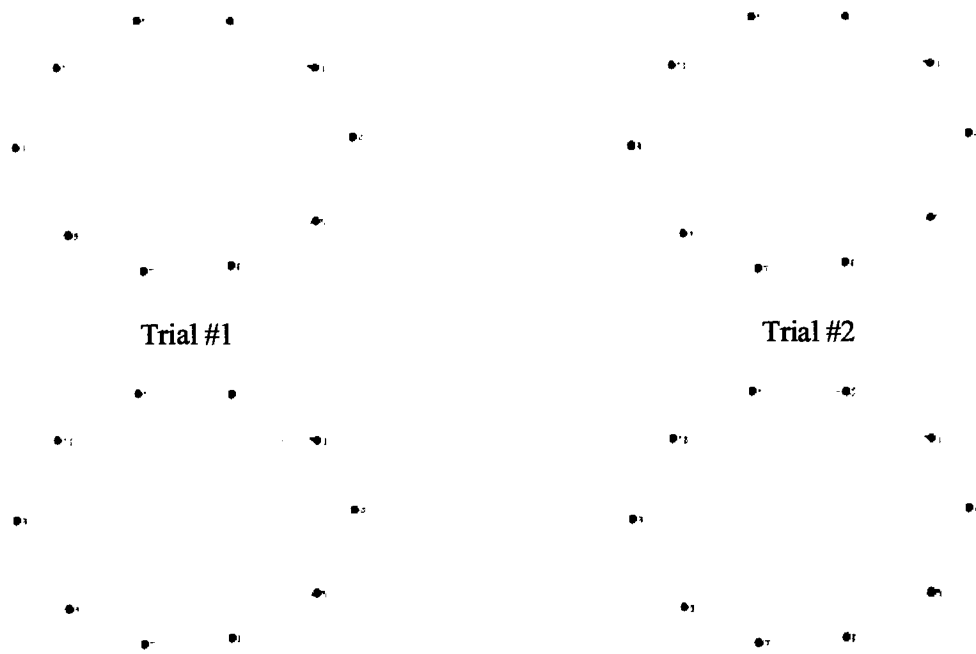
Now, let's consider the case of the small world network. Recall the network generated by rewiring the ten-node ring. In applying the same method of rumor transmission to this network, we should hope to see a quicker spread, because of the number of connections and where they occur. Figure 3.6 shows the binary outputs given after running the time trials on this network. Notice that only four trials were necessary in order to infect the whole network after infecting node 1.

	Nodes ***1 2 3 4 5 6 7 8 9 10*** →
1	[1 0 0 0 0 0 0 0 0 0]
2	[1 1 0 0 0 0 0 0 1 0]
Trials 3	[1 1 1 1 0 1 0 0 1 1]
4 ↓	[1 1 1 1 1 1 1 1 1 1]

The above set of binary sequences displays the spread of the rumor over the 10 nodes in the small world network. In the initial state, or trial 1, node 1 has been infected with the rumor, as was done in the random case. The small world network shows a quicker transmission, as only 4 trials were needed here, opposed to the 6 need in the random network. Every infected node has a 1 in its place while those still susceptible to the rumor have 0's. The last state, or trial 4, shows the rumor in its final stage, where the entire network has become infected.

Figure 3.6

The following networks depict this rumor spread by coloring the infected nodes red. It is important to note that the number of nodes in the small world network is in fact less than that of the random, so the hastiness of the transmission of the rumor here is significant.

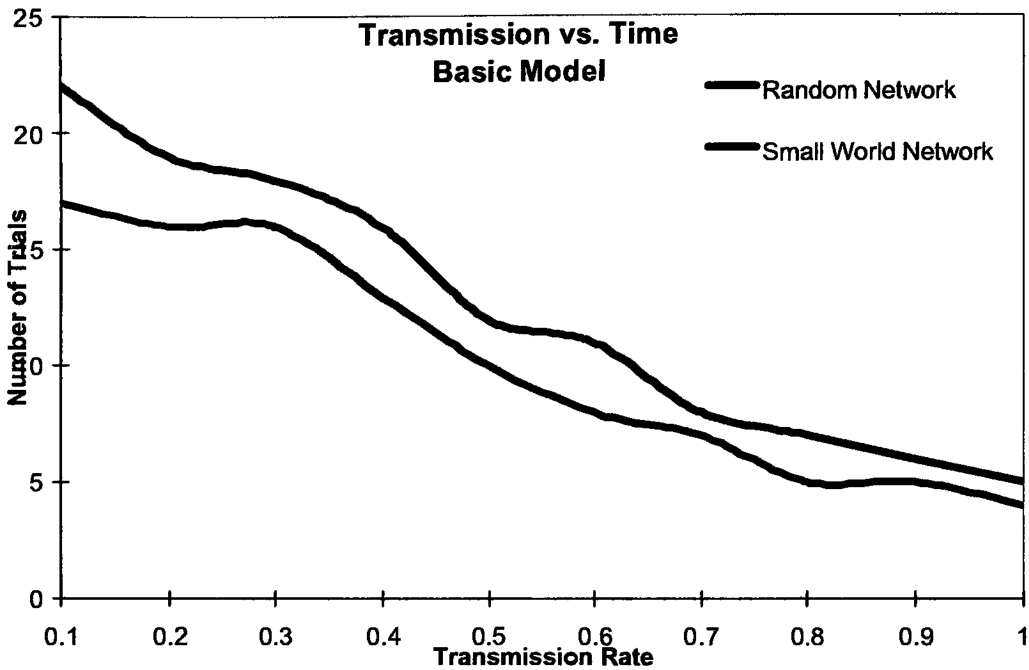


Trial #1 Trial #2

Trial #3 Trial #4

Figure 3.7

A rigorous study involving 100 networks of each type yielded the following results. Comparing the transmission rates assigned to each of the networks' rumor flows with the number of time trials required to infect the whole network shows that the assumptions made about small world networks versus random networks are actually valid. As seen below in Figure 3.8, the number of trials to infect a small world network was consistently lower than that of the random networks, regardless of the transmission rate assigned to the nodes in the networks.



The above results have been obtained from a sample of 10 random and small world networks of 50 nodes with each transmission rate, totaling 100 random and 100 small world networks. The average number of trials was calculated over the 10 networks at each transmission rate to obtain the smooth curve seen for both types of networks. Most importantly, only the transmission rates viewed along the axis were considered, which may have contributed to the unsteady decrease. Observe the transmission rate of 0.6 in the random network case. Here, we have a slight jump in the curve which may indicate that not only should we probably add more experiments, but we should also include experiments on transmissions other than those seen above.

Figure 3.8

3.2 Maki-Thompson Model

The Maki-Thompson Model has been studied very frequently in mathematical modeling, especially within the area of graph theory. The application of this model to our networks and their propagation of rumors will give us the opportunity to compare the results to those of the Basic Model. While in the Basic Model, we introduced only spreaders into the network, the Maki-Thompson Model considers another type of individual known as the stifler. The stifler's role in rumor propagation doesn't begin until the rumor has been passed to that individual. They will then take no action, meaning that they will not spread the rumor to anyone else they might be connected to in the network, yet simply ignore the fact that they had heard anything at all. When considering the possible identification of a stifler in a real social network, one might think of a neutral individual, or someone who takes no side in the belief of the rumor. Below is the flow chart describing the algorithm for the flow of rumor based on the Maki-Thompson Model, with the stifler's role being introduced to make the operation a bit more complicated.

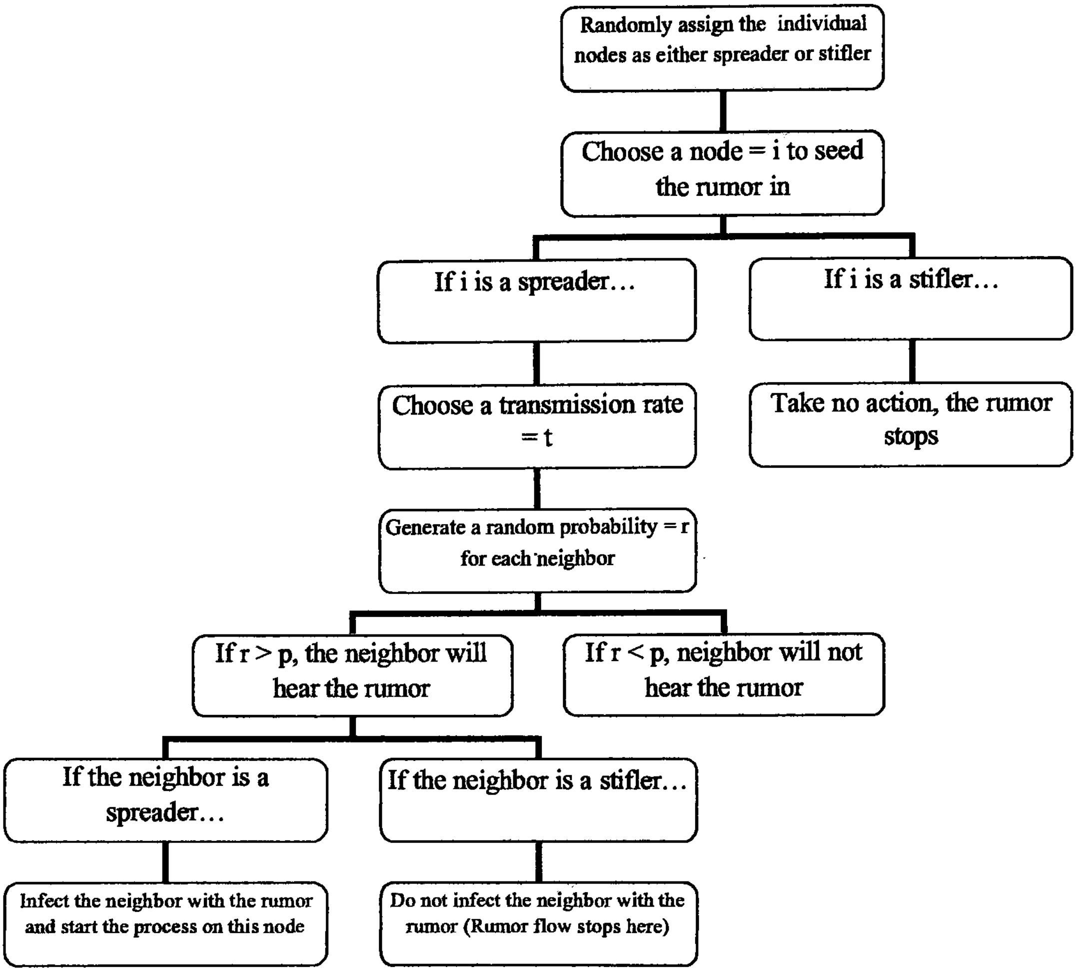
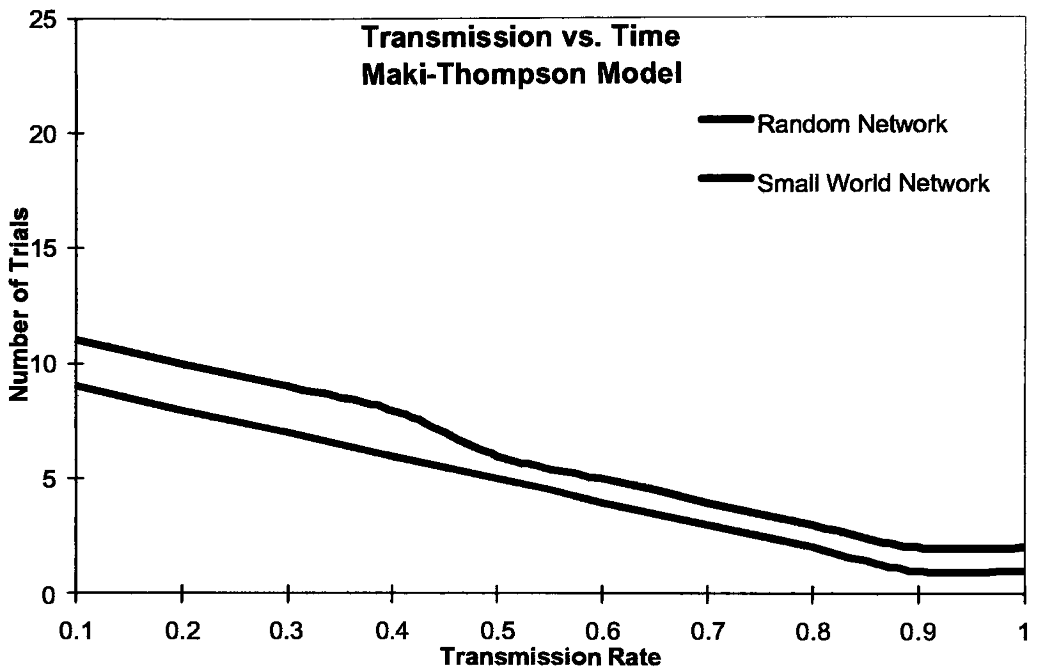


Figure 3.9

The role of the spreader has not changed from the Basic Model to the Maki-Thompson Model, but their affect on the flow of the rumor over the network will somewhat weaken. In the Basic Model, everyone a speader spoke to would eventually believe the rumor and then, in turn, pass it on to their connections. Here, the stifler will slow down the rumor spread, and also restrict it from reaching everyone in the network.

The assumption should remain that the small world network will spread the rumor quicker than the random network. However, there is a slight modification in this model.

Neither type of network will allow the rumor to be spread completely over its nodes. Below are the results of the Maki-Thompson Model applied to the same 200 networks considered in the Basic Model in the previous section.



The above results have been obtained from a sample of 10 random and small world networks of 50 nodes with each transmission rate, totaling 100 random and 100 small world networks. The average number of trials was calculated over the 10 networks at each transmission rate to obtain the smooth curve seen for both types of networks. In this model, we have smoother curves than in the Basic Model. This may be due to the fact that not as many trials were necessary to cease the spread of the rumor, thereby allowing for the average trials to have a more normal distribution.

Figure 3.10

Again, as the transmission rate increases the number of trials required to spread the rumor as much as possible will decrease. More significantly, the small world network does, again, allow for the quicker rumor propagation, and also in a more linear manner.

3.3 Daley-Kendall Model

The Daley-Kendall Model adds yet another individual type to the network, called an ignorant. This individual is simply unknowing of the rumor and its validity. Now, we must consider the ignorant, spreader, and stifler. The ignorant's job is simply to quicken the rumor spread by having to make a decision about the truth of the rumor, and thus, whether to believe it or not, then transforming into either a stifler or spreader. In this paper, we will only examine the case of allowing the ignorant to become a spreader. There are many more parameters to consider in order to transform an individual into another state, which will be examined at a later time. For the purpose of simplicity, this paper examines the case where an ignorant must hear the rumor a random number of times before becoming a spreader, by way of believing the rumor. Once this modification has been made, the rumor flow proceeds as in the Maki-Thompson Model, with stifler and spreader dispersion throughout the network. Figure 3.11 is an explicit flow chart demonstrating the process of transmitting a rumor through a network using the Daley-Kendall Model. While it is more complex and requires more steps to proceed with the rumor flow, the rumor will actually spread quicker in this model than the other two previously considered.

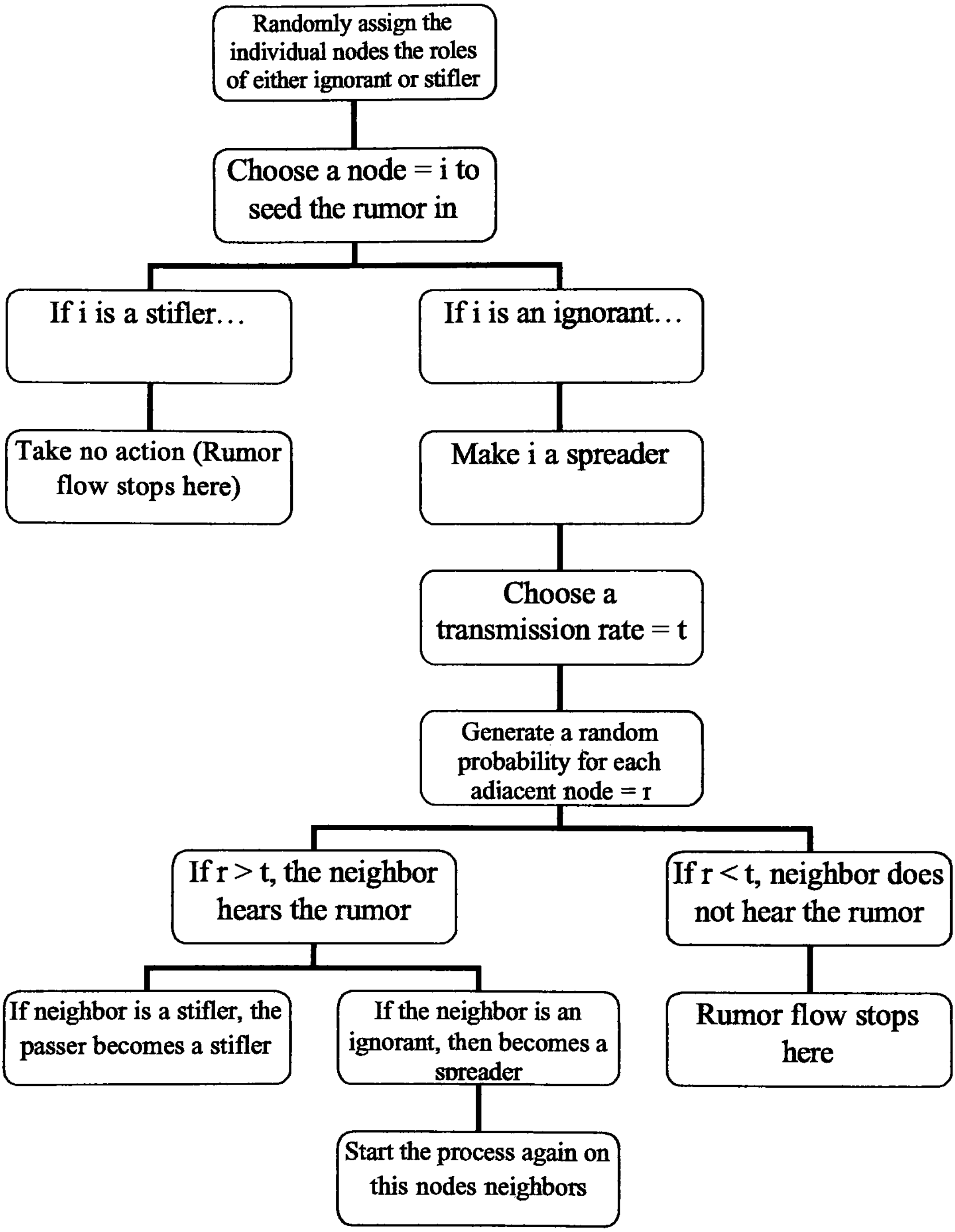
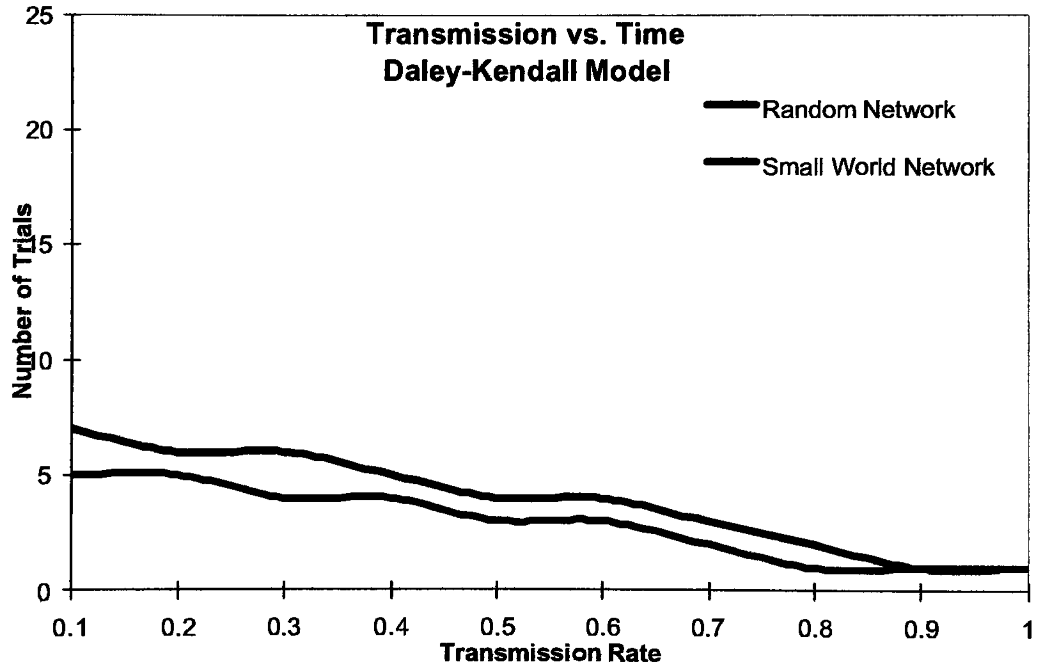

Randomly assign the individual nodes the roles of either ignorant or stifler
Choose a node = i to seed the rumor in
If i is a stifler…
If i is an ignorant…
Take no action (Rumor flow stops here)
Make i a spreader
Choose a transmission rate = t
Generate a random probability for each adjacent node = r
If r > t, the neighbor hears the rumor
If r < t, neighbor does not hear the rumor
If neighbor is a stifler, the passer becomes a stifler
If the neighbor is an ignorant, then becomes a spreader
Rumor flow stops here
Start the process again on this nodes neighbors

Figure 3.11

The results of the Daley-Kendall Model on numerous random and small world networks of different connectivities, is shown below in Figure 3.12. It is imperative to be consistent withthe increments along the time trials axis is imperative as it should be clear now that the Daley-Kendall Model has allowed for a quicker rumor spread over both random and small world networks. What is significant, yet again, is the validity in the assumption about small world networks allowing quicker rumor propagation than random networks.



The above results have been obtained from a sample of 10 random and small world networks of 50 nodes with each transmission rate, totaling 100 random and 100 small world networks. The average number of trials was calculated over the 10 networks at each transmission rate to obtain the smooth curve seen for both types of networks. Now, we can see a slight change in the consistency, as in the Basic Model.

Figure 3.12

The leveling off of the two curves at around 1 trial requirement is interesting since both random and small world networks allow for the rumor to transmit in only one time increment. However, it should be noted that this is only in the case where both random and small world networks are completely connected, and are thus, in fact, the same network. Consider two 50-node networks, random and small world. When connected with a probability of 1.0, the two become complete networks of size 50, and therefore, the same network.

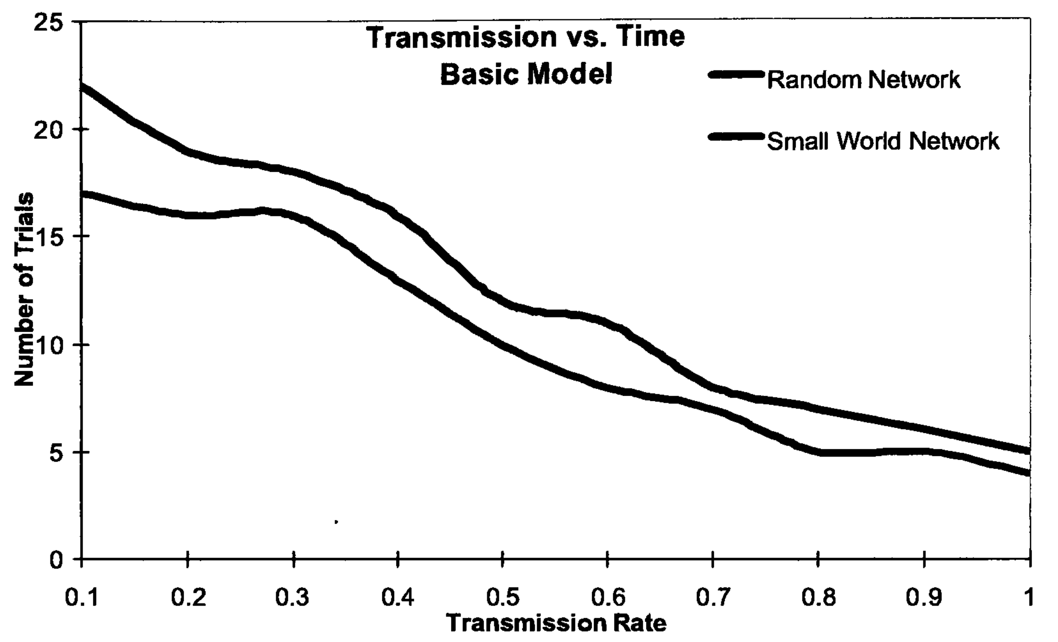
The Daley-Kendall Model also demonstrates a more fluid transmission of the rumor as connectivities increase, which may have to do with the fact that the networks of less connectivity already allowed an extremely fast transmission of the rumor, which allows for less variability in trial times as connectivity increases. By requiring that the ignorant node becomes a spreader at some point, we have set the standards for eventually more spreaders within the network than the previous Maki-Thompson Model, and therefore more individuals who will believe and pass the rumor on to their neighbors.

4 Comparing Random and Small World Networks

As the purpose of this paper is to explain the difference in the flow over the two types of networks, random and small world, there is a need to compare the results of the different models. It has already been shown that in each of the cases the random networks require more time to spread a rumor over the network. Now, these results will be displayed more specifically, comparing the models in both cases, as well as some of the other properties of the networks as discussed in Section 2.

4.1 Transmission rate

A transmission rate is assigned each time a rumor is introduced into a social network; in order to determine the amount of time it takes for that rumor to completely infect the network. The most intuitive result would be that, as the transmission rates increase, the amount of time necessary for rumor flow will decrease. In both the random and small world cases, this is generally the case. However; there is always a possibility of the appearance of an outlier. There are many factors that might contribute to such a jump in the curve, such as in the random networks at around 0.6 or small world networks at around 0.3, including the properties of the random network, which node was initially infected and the infected node's role in the social network.

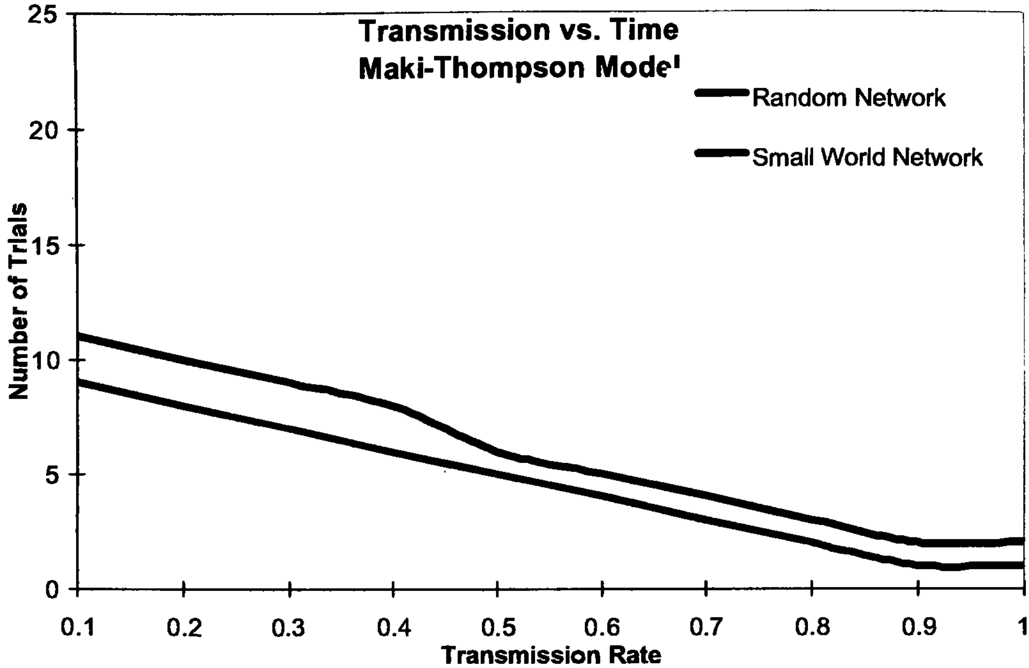


The above results have been obtained from a sample of 10 random and small world networks of 50 nodes with each transmission rate, totaling 100 random and 100 small world networks. The average number of trials was calculated over the 10 networks at each transmission rate to obtain the smooth curve seen for both types of networks. Most importantly, only the transmission rates viewed along the axis were considered, which may have contributed to the unsteady decrease. Observe the transmission rate of 0.6 in the random network case. Here, we have a slight jump in the curve which may indicate that not only should we probably add more experiments, but we should also include experiments on transmissions other than those seen above.

Figure 4.1

In Figure 4.1, notice how the two networks at transmission 1.0 do not have the same result from the Basic Model Even with the 100% chance of passing the rumor from node to node, the random network still does not have the capability of passing the rumor as quickly as the small world network. Again, this is a result of the differing constructs of the two networks. The small world network is more capable of allowing the quicker propagation of the rumor due to its structure and the notion of "six degrees of freedom".

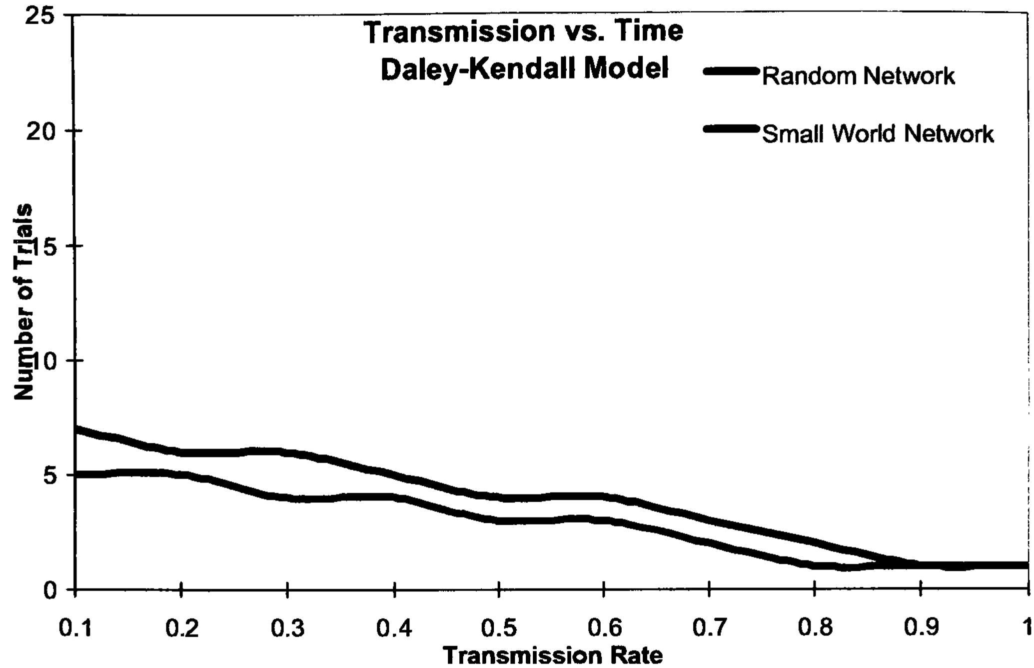
The Maki-Thompson Model's results are extremely interesting as well. Figure 4.2 show a much smoother transmission of the rumor over the networks. Recall, the Maki-Thompson Model introduces stiflers into the network which stops the spread of the rumor quickly. Here, we can see the random and small world networks time trials are decreasing, as they should, but not as steeply as in the Basic Model's results. This is due to the fact that the with transmission rate being 0.1, not as many trials are required in this model as in the previous model. Whereas in the Basic Model, the random and small world networks require 22 and 17 trials respectively at a transmission of 0.1, here they only required merely 11 and 9 trials respectively. The small world network consistently requires fewer trials than the random network, even in the case where the random network allows somewhat fast rumor propagation.



The above results have been obtained from a sample of 10 random and small world networks of 50 nodes with each transmission rate, totaling 100 random and 100 small world networks. The average number of trials was calculated over the 10 networks at each transmission rate to obtain the smooth curve seen for both types of networks. In this model, we have smoother curves than in the Basic Model. This may be due to the fact that not as many trials were necessary to cease the spread of the rumor, therefore, allowing for the average trials to have a more normal distribution.

Figure 4.2

Again, as we introduce yet another individual, the ignorant, into the network, the Daley-Kendall Model proves our assumptions to be true. Figure 4.3 illustrates the even quicker propagation of the rumor over both network types, as supposed. Now, the random and small world networks only require about seven and five trials respectively at a 0.1 transmission rate, which is extremely fast when considering that these networks are both of size 50. The Daley-Kendall Model shows, again, the quicker transmission of a rumor over a small world network than a random network.



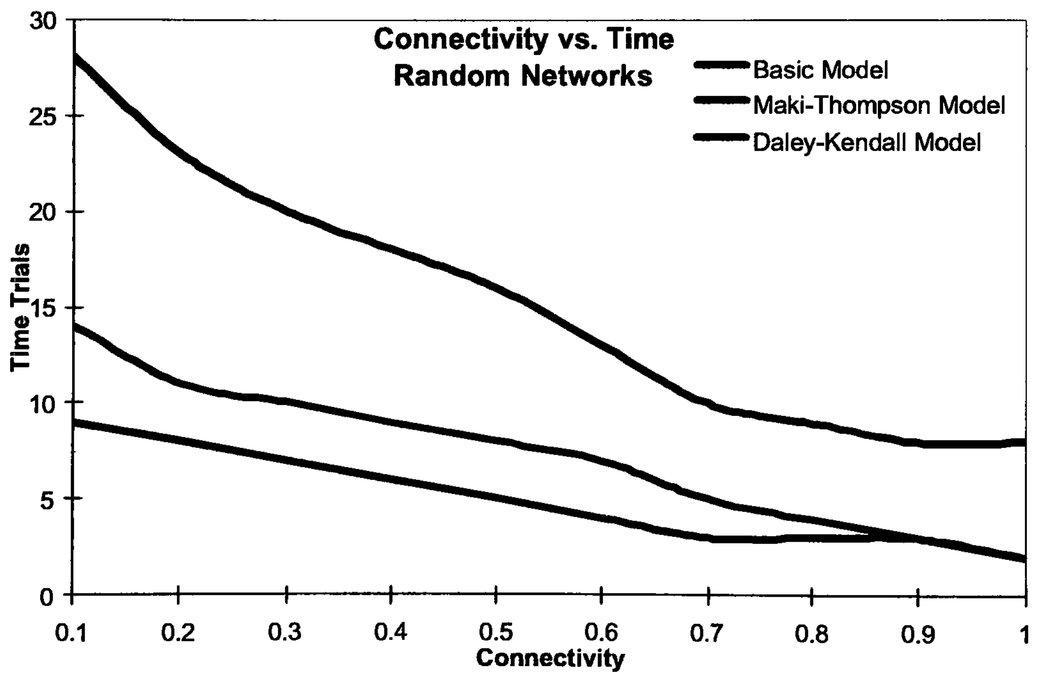
The above results have been obtained from a sample of 10 random and small world networks of 50 nodes with each transmission rate, totaling 100 random and 100 small world networks. The average number of trials was calculated over the 10 networks at each transmission rate to obtain the smooth curve seen for both types of networks. Now we can see a slight change in the consistency, as in the Basic Model.

Figure 4.3

As we introduce new and different individual types into these networks, we find that the spread of the rumor stops sooner. The appearance of the stifler indicates that not everyone in the network will actually hear the rumor before it is ended. The ignorant simply speeds the flow over those who do in fact hear the rumor, as this individual is automatically transformed into a spreader. Future study should consider the cases where either the ignorant automatically becomes a stifler, or using parameters to determine which of those two identities the ignorant becomes. A new comparison could then be made against the already studied ignorant to spreader case, as shown above.

4.2 Connectivity

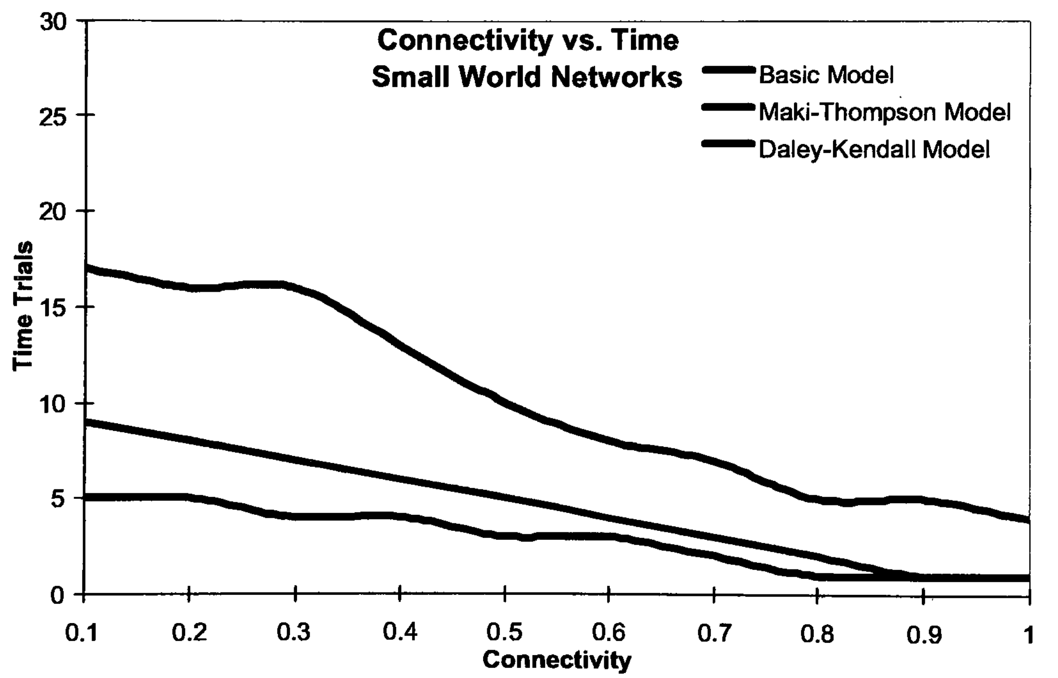
The most obvious realization that can be made concerning the connectivity is that the more relationships there are in a network, the quicker the rumor will propagate. In both the random and small world cases, this holds true. Moreover, the different models of rumor flow present us with the proof that their properties will also result from such a change. The random networks' results are below, displaying all three of the models in comparison with each other.



The above display illustrates the comparison of the three models on random networks. 10 random networks of 50 nodes were considered at each connectivity, so no two networks are exactly the same. All 10 networks trial requirements were averaged at each respective connectivity to obtain the three smooth curves you see above. The consistent decrease in the number of time trials as connectivity increases is clear with all three models, and the assumptions made about the three with respect to each other are valid. The Basic Model requires more time to allow propagation than the Maki-Thompson and Daley Kendall, even in the complete case of connectivity = 1.0.

Figure 4.4

As would have been guessed, the Basic Model required the highest number of trials in order to allow the rumor flow over every individual node in the network, while the Maki-Thompson and Daley-Kendall models take less time, not requiring every node to hear or spread the rumor. What's more, as shown in the next figure, the small world networks' have the same properties. However; those small world networks with connectivity of 0.10 only need a maximum of about sixteen trials to spread the rumor, instead of the twenty-three needed in the random case. (This comparison made on the Basic Model, but is consistent with the others).



The above display illustrates the comparison of the three models on small world networks. 10 small world networks of 50 nodes were considered at each connectivity, so no two networks are exactly the same. All 10 networks trial requirements were averaged at each respective connectivity to obtain the three smooth curves you see above. While we again see the comparison between the models, what's more is that the small world networks' average trials are much less than the random networks'. Compare this figure to the previous and notice the difference between those averaging at a connectivity of 0.1. Here, we have 17, 9, and 5 trials required for the three models whereas the random case required 28, 14, and 9 trials at that same connectivity.

Figure 4.5

All three of the models presented show that the assumptions are true regarding the speed of rumor flow over random and small world networks. The assumptions made at the start of the study can be supposed to be true, considering that an extremely large sample of 200 networks of each type was considered. Therefore, the sample is more representative of the whole population of networks of the same size. Moreover, the importance of dealing with large networks is apparent, as we must consider such sizes as being representative of an entire population, or real social network.

4.3 Centrality

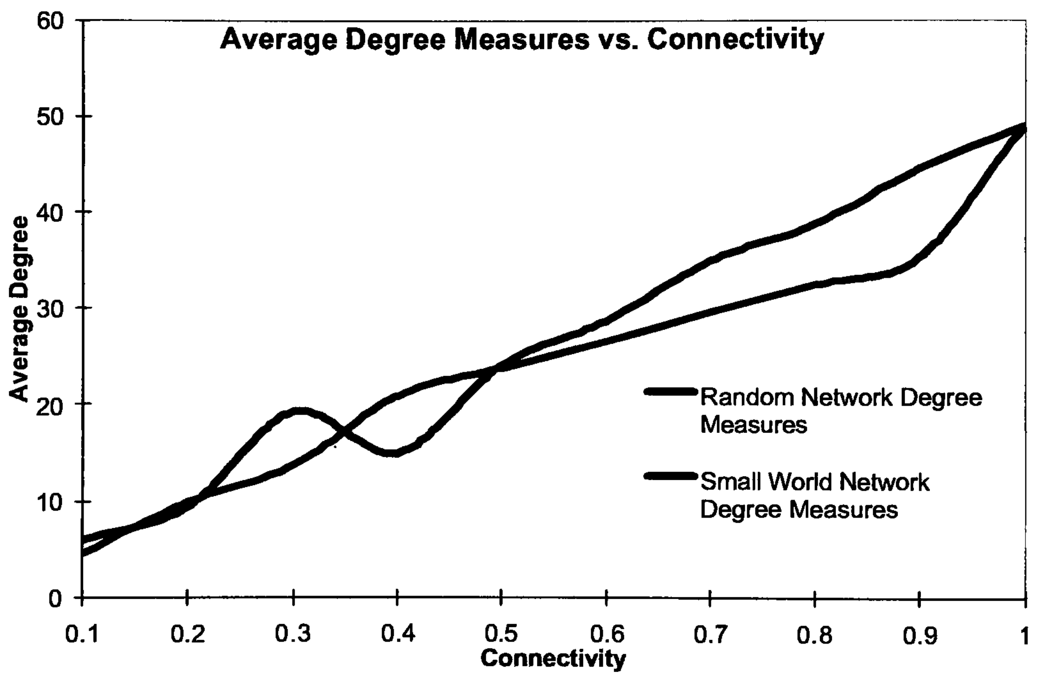
The measure of centrality is such that a network can be classified by its parts. A central node will allow for a quicker rumor spread through its own neighbors, but with the remaining nodes being less capable of transmitting that rumor. As random networks have sporadic dispersion of significance over their nodes, they are more likely to contain one or more central nodes. However, small world networks have a more even dispersion of nodes and edges, yielding a weaker possibility of a central node existing. What is interesting is that the networks without central nodes will have a more consistent flow of the rumor over its individuals, allowing for a more fluid spread of the rumor. Those networks with central nodes require such individuals to have enough connections to have an effect on the flow of the rumor over the entire network.

A method to understanding networks and their individual nodes is to evaluate the location of these individuals in the network. Measuring the network's central location equates to finding the centrality of a node. These measurements help determine the significance, or prominence, of an individual in the network. Not all network paths are created equal, and therefore, not all measures will lead us to a central node. More and more research shows that the shorter paths in the network are more important, but when the shortest path is not extremely obvious, we can consider other modes of centrality measuring. Again, in order to compare the difference in centralities between the random and small world network, we should consider the generated graphs of fifty nodes and their centrality measures.

4.3.1 Degree

Social network researchers measure network activity for a node by using the concept of degrees, or the number of direct connections a node has. In personal networks, "the more connections, the better" is the general method of finding the central node based on degree measurement. A random network may have a wide range of degree values, since the connections are made randomly, but the degrees of the individual nodes in the small world networks should be somewhat uniform, verifying that the more strategic method for edge placement confirms the notion of "six degrees of freedom".

Three measures of centrality were proposed to be considered in section 2. The measures of Degree, Closeness and Betweenness are some of the most widely used measures of centrality. Considering the measure of degree, it is obvious that those individuals with the highest degree will have the largest number of relationships within any given network, thus; being considered a central node. The comparison to make within a real social network would be that the most popular individual in a network represents the central node. A random network will more than likely contain at least one central node, whereas in the small world network, the "six degrees of freedom" concept tells us that the majority of individuals will have the same number of connections and thus, having no central nodes. Below are the results of the two networks comparisons in average degree measures. Recall that the average degree of a network is the mean of all of the nodes' degree measures.



The average degrees illustrated here are actually the mean of the average degrees over 10 random and 10 small world networks of 50 nodes, separately, for each connectivity. The comparison to be made here is with the intent of showing that as connectivity increases, average degree will also increase. However, some interesting observations to be made here are in the jumps of the random network curve at around 0.3 and 0.4 connectivities. The jump at 0.3 to a degree of 20 is interesting because we might claim that this difference was made by one outlying network, so when averaged with the remaining networks degrees, caused this change. This is supported by the fact that when the curve jumps back down at connectivity 0.4, it seems to be a more reasonable average degree measure. Also, the overlapping of the small world and random network curves is significant. Some questions to ask might be 'why does this occur?' and 'what affect does this have on the propagation of a rumor over these networks?'.

Figure 4.6

As can be seen above, complete networks of 50 nodes should have average degrees of 49, no matter what type of network they were derived from. A complete network, as discussed previously, is one such that all nodes are connected to all other nodes in the network. In a network of 50 nodes, each individual has 49 others to be connected to, and thus, degree 49. While all nodes have degree 49, it is clear that the network's average degree measure will be 49. What is interesting about these results is that for the majority of the time, the small world network has an average degree less than that of the random network. This may be drawn from the fact that the degree distribution over the small world network is actually more uniform, which might also have something to do with the shape of the curve. Notice that the curve representing the average degrees of the random networks is not quite as smooth as that of the small world networks. Random networks assume one or more central nodes, not allowing for as uniform a degree distribution. This has an effect on the mean degree of such a distribution. The overlap in the two curves may be due to the fact that both networks are of the same size and connectivity, meaning that their *average* degrees should actually not be quite so different.

Since the degree measures of the small world networks are more comparable than those of the random network, the flow of a rumor over such a network should be more fluid due to the fact that each node holds the same power to hear and spread the rumor. No one individual is required to keep the rumor spreading and, therefore, there are fewer ruptures in the flow. However, as the degree measures in the random network are not as analogous, the network will require one or more powerful individuals (or central nodes) to keep transmitting the rumor.

4.3.2 Closeness

Closeness centrality states how close an individual is to the others in the network. Closeness and betweenness measure global centrality according to the length (shortness) and the number of multiple-step path roles each entity has in the network. It identifies the entities that play significant roles as intermediaries. The ubiquity of an entity's ties make the actor a globally-relevant one, and the shortness of the *paths* make the actor a central one. Closeness refers to how often an entity is in a shortest path. It reflects the ability to access information through the "grapevine" of network members.

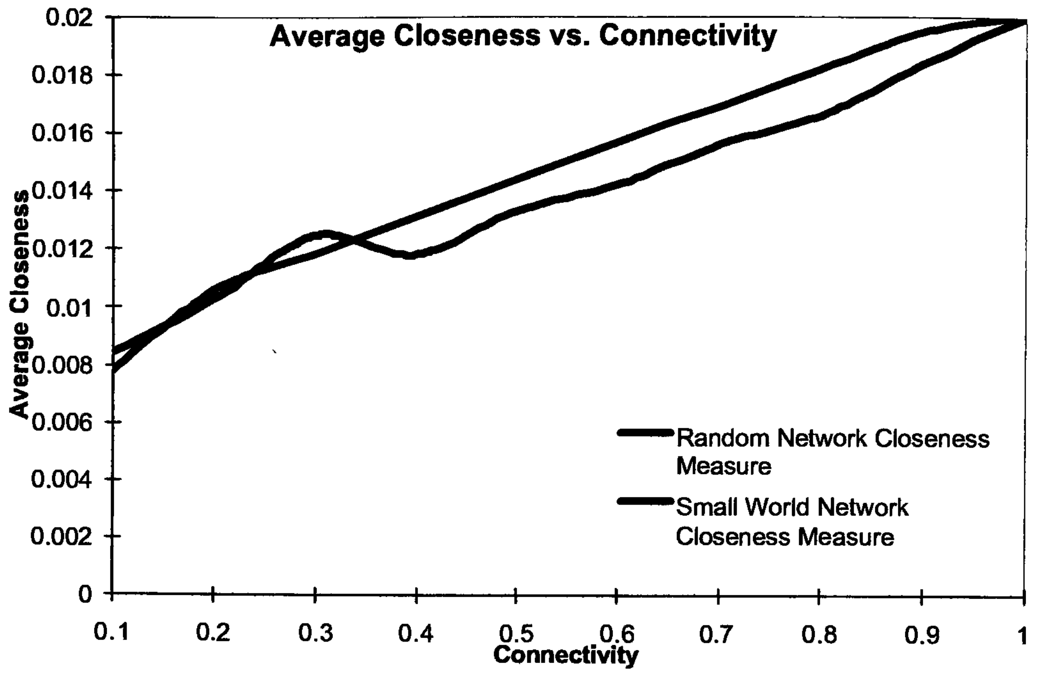
Nodes having the shortest paths to all others are considered closer to everyone else. They are in an excellent position to monitor the information flow in the network and have the best visibility into what is happening in the network. Considering the differences concerning random and small world networks, it should be quite obvious that small world networks would have, on average, shorter path lengths between nodes. Closeness measures are that of proportion and those smaller in value have shorter path lengths associated with them. When we consider an entire network and average the closeness values over it, we should hopefully see that a small world network would have a smaller closeness measure associated with it. Figure 4.7 displays the average closeness values of 50 nodes in each of the networks generated. The random networks have average closeness measurements ranging anywhere from around 42% to 100%, meaning that the less connected networks require almost half of the networks nodes in a path in order to pass a rumor from one node to another. Obviously, a completely connected network will have a closeness measurement of 2% meaning that no other nodes are required between two individuals as all nodes are connected to all others. Let's look a the example of the complete network to determine the closeness measurement. A 50 node complete network means that every node is connected to every other node in the network, measuring a path length to each of them of 1. So, we have from (6),

$$C_c(v) = \frac{1}{\sum_{t \in V} d_G(v,t)} = \frac{1}{1+1+\cdots+1} = \frac{1}{49} \qquad \forall v \in V$$

as the individual closeness measures for each node. Thus, the average of these closeness measures is as follows,

$$\overline{C_c(v)} = \frac{\sum_{v=1}^{50}\left(\frac{1}{\sum_{t \in V} d_G(v,t)}\right)}{50} = \frac{\left(\frac{50}{49}\right)}{50} = \frac{1}{49}$$

If we examine the closeness measure of the random and small world networks, we would not only see that the average closeness measure of the random network is normally smaller than that of the small world network, but also that the small world network again has a more uniform curve. This shows that the closeness measures over the small world network are much more consistent than in the random networks.



The average closeness measures are shown above, comparing random and small world of 50 nodes each. Again, we must consider the inconsistency in the random networks curve at the connectivity 0.3. The remainder of the curve is more reasonable, showing a steady increase in closeness as the connectivity increases, once reaching completeness, having a closeness of 0.02 (when rounded). Most significantly, is to again state that the small world network supersedes the random network in its closeness measure, showing us that the small world network has shorter path lengths than the random at each of the connectivities, disregarding the possible outlier at 0.3.

Figure 4.7

Above, the average closeness measures show such a comparison made on the 50 node networks. At a connectivity of 0.10, the average closeness of the random network is about 41, while the small world is about 39. It is interesting that the closeness of the small world network is actually less than that of the random network at this point. However, it is easy to see that as the connectivities are increased, the assumptions about the comparison are actually shown to be true, as small world networks have a higher closeness for the majority of the time. At a connectivity of 1.0 (a complete network), both types should have a closeness of 100, since they are in fact the same 50 node network at this point.

With a greater closeness measure, there are more frequent occurrences of individuals being "close" to each other, or within the shortest paths of one another. When the closeness measure of an entire network is very small, there are fewer nodes who are able to communicate with many others. This leaves the network in a weaker state for spreading information, or rumors. Since the small world networks have larger closeness measures for the majority of the time, there are additional relationships and therefore added possibility for rumor transmission.

4.3.3 Betweenness

Betweenness centrality aims at evaluating the strategic importance of an individual related to the information paths. Its calculation, for an individual, is based on the sum of the shortest paths linking every couple of individuals in the network, this individual is apart. Betweenness refers to how often it functions as an intermediary. Therefore, it's the number of people who a person is connected to indirectly through their direct links.

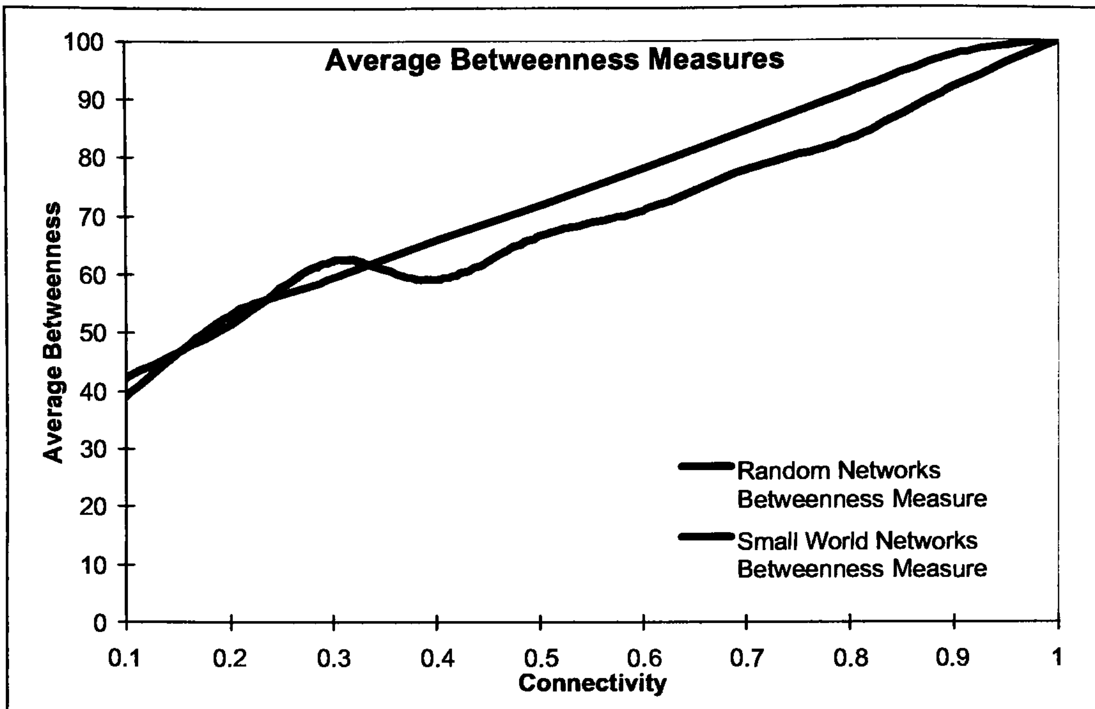
A node with high betweenness has great influence over what flows in the network, so a more connected network should have a larger average betweenness measure. A random network should have an unpredictable pattern of betweenness measures where a small world network should have a more uniform pattern, obviously due to the fact that a small world network is generated by manipulation. A lower betweenness measure represents a weaker relationship between two nodes while a higher measure represents an extremely strong relationship. The betweenness measures as represented in the following figures are in terms of percents, and another way to consider this measure is in a proportion as discussed in earlier sections of this paper. Consider the possible differences between random and small world networks. The complete networks are easy to consider because, in this case, the path lengths and conditional path lengths are easy to find. Recall (7) where

$$C_B(v) = \sum_{s \neq t \neq v \in V} \frac{\sigma_{st}(v)}{\sigma_{st}} = \frac{2}{1} + \frac{2}{1} + \cdots + \frac{2}{1} = 49(2) = 98$$

which is clear since the shortest path between any two nodes in a complete graph is 1, and the shortest path between any two nodes passing through a third is 2. Then, we have the average betweenness of the complete graph being

$$\overline{C_B(v)} = \frac{\sum_{v=1}^{50} \left(\sum_{s \neq t \neq v \in V} \frac{\sigma_{st}(v)}{\sigma_{st}} \right)}{50} = \frac{50(98)}{50} = 98$$

The betweenness measure of any type of network should decrease as the number of edges in the network increases, as shown below in the comparison of the betweenness measures of random and small world networks. Again, the consistency of the small world networks' betweenness measures should allow for a more uniform average betweenness curve. The random network, however, does not show such a behavior.



Above, the betweenness measures both increase towards 98 as the connectivity increases to 1.0, as supposed. We would expect to see a larger betweenness measure for all small world networks rather than random networks, which is proven everywhere except for at the connectivity of 0.3. Referring back to the section regarding average degrees, we see that there was a discrepancy on the average degree of random networks at connectivity of 0.3. The discrepancy here, in the average betweenness so follows. There may exist some outlier with an extremely large betweenness measure to cause such a quick jump.

Figure 4.8

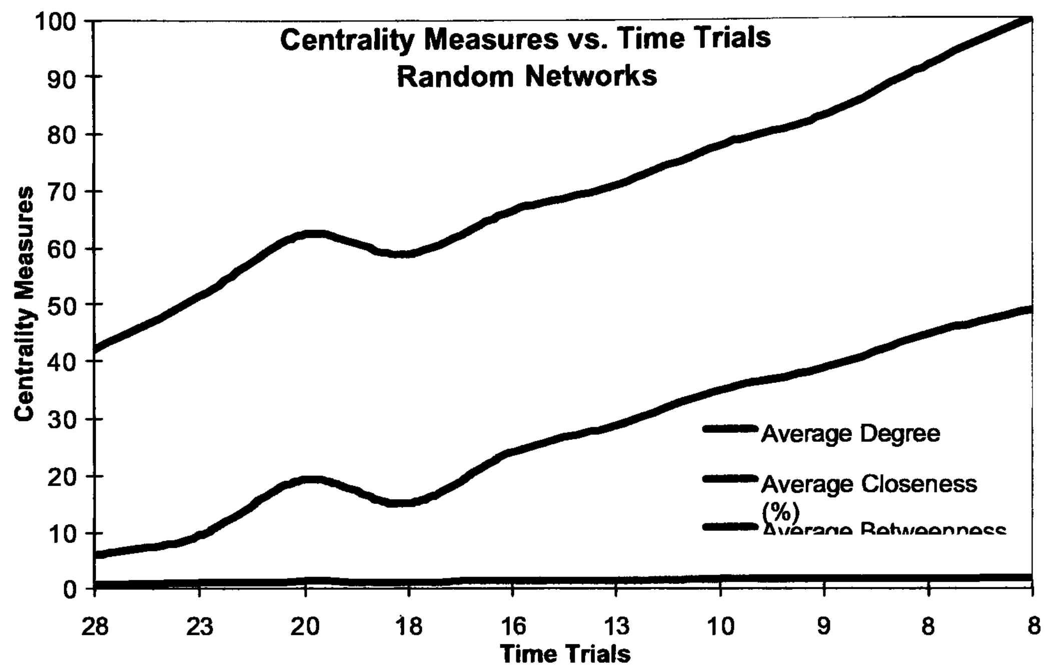
Above, we can see that at a connectivity of between 0.30 and 0.40, there is some interesting behavior on the part of the random network as far as the average betweenness goes. The continual decreasing motion is interrupted here, and gives way to questioning the central node and its location with respect to the others in the network.

The flow a rumor over the random network may be slightly interrupted at the point of this change in this inconsistency. But in the small world network, the rumor flow should be much more fluid because of the consistency in the decrease in the betweenness measures as the connectivity of the network increases. The complete networks where the betweenness is 0 have a much quicker transmission of rumors. But the networks with a lesser connectivity, having larger betweenness measures, have a slower transmission of rumors.

4.3.4 Centrality on the models

In this section, we will discuss the comparison of the time required to spread a rumor with the measures of centrality. Only the Basic Model is considered, as we have already concluded that the Maki-Thompson and Daley-Kendall models have stronger results in the same manner. By observing the three measures of centrality and their results, on one graph against the time trials, it can be shown that these measures actually have strong relationships to not only the rumor transmission, but to each other.

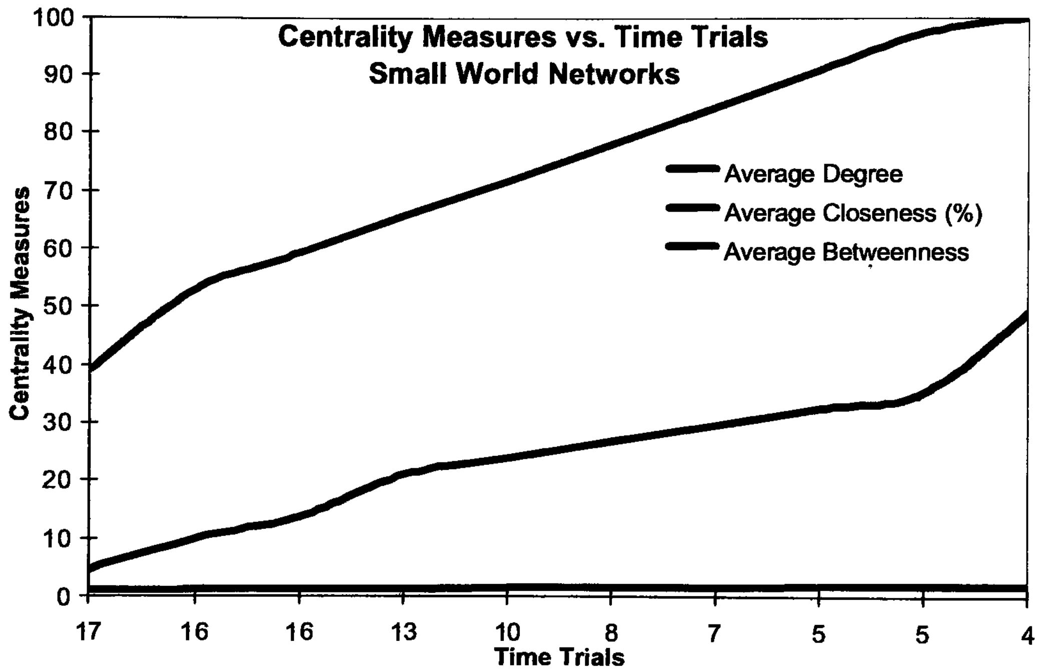
Below, the random networks are considered and the rumor transmission over them using the Basic Model. As the time required completing the transmission of the rumor over the entire network decreases from 28 trials to 8, the measures of centrality behave interestingly as discussed in the previous section. The average degree increases from about 5 to 49, where the random network becomes a complete network. The rumor flow only requires 8 time trials to completely finish its procedure here, where at the connectivity of 0.10 (degree of about 5), the rumor transmission took 28 time trials. At the highest closeness measure, we have the quickest rumor transmission of 8 trials and, at the lowest; we have the slowest transmission of 28 trials. The betweenness measure, at its largest, has an associated requirement of 28 trials, when at a betweenness of 0, there are only 8 trials. Recall that we still have 8 trials, even in the complete case, since not everyone will believe the rumor the first time they hear it. The rumor must be reinforced in order for the node to become *infected.*



The comparison of the measures of centrality on time required for rumor propagation to cease is important. As the number of trials decreases from 28 to 8 (in the random case), the centrality measures are increasing. This has a direct correlation with the fact that as a network becomes more connected, the centrality measures increase and the number of trials required to spread the rumor decreases. This graph simply displays the results obtained from the 100 random networks and their average centrality measures at every connectivity ranging from 0.1 to 1.0.

Figure 4.9

Comparing the random network results to that of the small world network, we have to consider the uniformity of the nodal distribution over the network. Because the degree, closeness, and betweenness measures are so consistent in a small world network, the transmission of the rumor does not require as much time to complete its flow. As seen in the case of the small world network, the average degree curve is quite similar to that of the random network, yet smoother. This difference contributes to the quicker spread of the rumor. In the case of closeness and betweenness, the more rapid increase and decreases (respectively) are associated with the quicker spread of the rumor. Below, the greatest number of trials required in the small world case is 17, and the lowest is 4. Already, it is obvious that the small world network will allow for quicker rumor spread for the comparable measures of centrality.



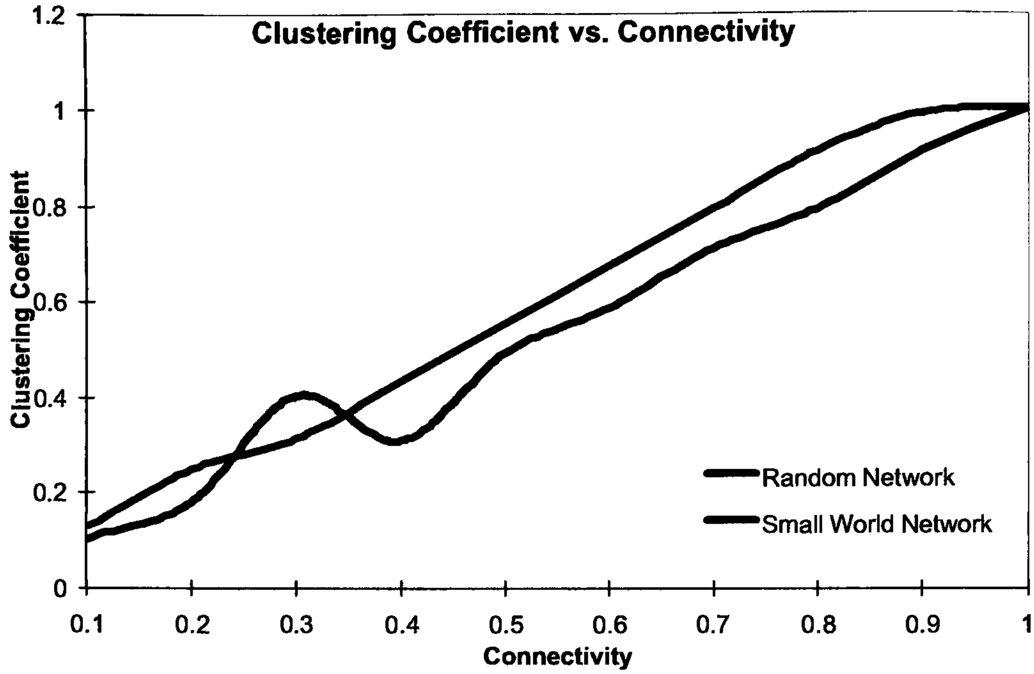
The same comparison can be made with the small world case. As the number of trials required for the rumor to cease decreases, the centrality measures will increase, as discussed in Figure 4.9. What's more, the small world networks centrality measures seem to increase at a quicker rate and more smoothly. This suggests a more structured network with more capability of spreading a rumor. Note that the closeness measure has been converted to a percentage, rather than a proportion for the purpose of a better view.

Figure 4.10

The two types of networks do, in fact, rely on the properties of centrality to maintain the properties of rumor transmission. Small world networks, those with no central nodes, have a quicker transmission of rumors, while the networks in the random case require a central node to spread the rumor as quickly as possible, not quite as fast as the small world network.

4.4 Clustering Coefficient

The clustering coefficient of a network is an important property to discuss in order to make comparisons between the random and small world networks. Recall the idea of the "six degrees of freedom" in a small world network. This concept coincides with the fact that the small world networks should have larger clustering coefficients that their respective random networks, meaning that the random network has fewer clusters among its individual nodes. Consider the 50 node networks discussed earlier and their clustering coefficients. Below, Figures 4.11 displays the different average clustering coefficients for each generated network of 50 nodes. In both cases, the lower the connectivity of the network, the lower the clustering coefficient.



As a network becomes more connected, its clustering coefficient will increase reaching 1 when the graph is completely connected. In this figure, we see such a trend. However, the most important comparison to make is the difference between random and small world network clustering coefficients. As previously mentioned, we will see that random networks have smaller clustering coefficients than small world. That is the case everywhere but at the point of 0.3 connectivity. Yet again, this jump in the random networks' average clustering coefficient is most likely due to an outlier of higher degree and therefore clustering than should be for 0.3 connectivity.

Figure 4.11

The difference in the range of values of clustering coefficients is not quite so obvious. However, with a longer examination of the two plots, we see that the small world networks' coefficients do have larger values than their corresponding random networks' values. This suggests that the clustering coefficient is more highly dependent on connectivity in the random network than in the small world network. The clustering coefficient increases quicker with connectivity in the small world network.

5 Conclusion

The propagation of a rumor over a social network is a complicated and intricate process. As discussed previously, there are numerous models for demonstrating the flow of a rumor over such networks. The three models referred to in this paper are just a subset of this faction. Their conditions and characteristics are somewhat simple when compared to more complex models. Yet, the study of their results is relevant to further studies of other models. These three models proved, beyond a doubt, the assumptions made regarding the flow of a rumor over random and small world networks. In all above circumstances, the resulting time for a rumor to spread over a small world network was consistently less than that of the random network.

The basic case started with a simple group of individuals, all prepared to spread information to their neighbors. Conditions were absent in this model to show the basic flow of a rumor over both random and small world networks. The result was a complete spread of the rumor over an entire network, which is suspected to take much longer than if a rumor stops before covering the entire network. The Basic Model did, in fact, show an evident difference between the flows over the two networks, proving the claim that small world ne tworks are better structured to allow rumor propagation.

Another individual, known as the stifler, was introduced into the network in the Maki-Thompson Model. The stifler's job was simply to neglect to spread the rumor to its neighbors. The rumor does not necessarily reach all individuals in the network in this case. Thus, the rumor flow is ended quicker than in the Basic Model. Yet again, the random network required more time in order to completely conclude the process. The results of the Maki-Thompson study showed us two very important details. Not only does the rumor flow relentlessly quicker over the small world networks, but it also ceases its progress over that type of network sooner than in the random networks.

Although we did not consider all the possible cases of the Daley-Kendall Model, that which we did consider also proved the assumptions to be true. Introducing an ignorant into the network, and forcing the transformation to spreader, also sped up the flow of the rumor over both networks. Yet again, the rumor spread more fluidly and quickly over the small world network than the random network. Recall, the suggestion has been made to consider the remaining cases of the Daley-Kendall Model, where the ignorant must either become a stifler or have a parameter that assists in the modification to either spreader or stifler.

Following the results of the Basic, Maki-Thompson and Daley-Kendall models, determining what role the properties of these two types of networks might play in the rumor propagation. The method for generating these networks, their measures of centrality and clustering coefficients, all proved to be examples of properties that influenced the results of the stochastic models. Small world networks assumed larger clustering coefficients, degree, closeness, and betweenness measures than random networks. These details were evidently the foundation for the claim that small world networks are networks of higher structure and thus, better resemble a social network. The random network type holds too many inconsistencies in all of the above mentioned properties. While the clustering coefficient is very low over the entire network, the existence of group relationships is lacking. The degree, closeness, and betweenness measures are also smaller, suggesting to us that the relationship distribution is not stable, and acquaintances are more numerous between any two nodes. The rumor flow between even only two individuals takes longer in the random networks than in the small world networks. Consequently, this paper has proven that a small world network maintains the properties which will allow for a quicker propagation of a rumor.

6 Glossary of Terms

Adjacency Matrix – The n by n matrix in which entry $a_{i,j}$ is the number of edges in the graph with endpoints $\{v_i, v_j\}$. Each entry is denoted by either a 1 or a 0, where a 1 indicates an edge between the i^{th} and j^{th} vertices, and 0 indicates no edge.

Adjacent - Two vertices are adjacent if they are connected by an edge, in other words, the vertices are the endpoints of a common edge.

Average Degree – The average number of adjacent vertices to each node in the graph.

Average Path Length – The average length, or number of edges, between any two vertices in a graph.

Bipartite Graph – A graph G is bipartite iff the vertex set is the union of two disjoint independent sets. None of the vertices can be connected to any other vertices within their partition.

Clustering Coefficient - The clustering coefficient C_i for a vertex v_i is the proportion of links between the vertices within its actual neighborhood to the number of links that could possibly exist between them.

Complete Graph – A graph whose vertices are adjacent to all other vertices in the graph, therefore; having degree n -1 for a graph of size n.

Complete Subgraph – A subgraph whose vertices are adjacent to all other vertices within that subgraph.

Connected Graph – A graph G is connected if each pair of vertices in G belongs to a path. In other words, each vertex has degree greater than or equal to 1, being connected to at least one other vertex in the graph.

Cut-Edge – An edge, so that when removed, increases the number of components in the graph.

Cut-Vertex - An edge, so that when removed, increases the number of components in the graph.

Cycle – A graph with an equal number of vertices and edges whose vertices can be placed around a circle so that two vertices are adjacent iff they appear consecutively around the circle.

Degree – The number of incident edges to any vertex, or the number of adjacent vertices to any vertex.

Directed Graph – A directed graph, or digraph, is one whose edges can only be traveled in one direction. There are usually two or more endpoints, a start and a finish.

Disconnecting Set – A set $F \subseteq E(G)$ such that $G-F$ has more than one component. This is the set of any cut-edges in a graph.

Distance – The number of vertices or edges between two endpoints of a path.

Edge – a line connecting two vertices, which are then called the endpoints of the edge. An edge is representative of a connection made between two individuals or objects in a graph or network.

Empty Graph – The graph whose vertex set and edge set are empty, therefore; nonexistent.

Flow – A flow f assigns a value $f(e)$ to each edge e.

Incidence Matrix – The n by m matrix in which entry $m_{i,j}$ is 1 if v_i is an endpoint of e_j, and zero otherwise.

Incident – Any edge connected to a vertex is incident to that vertex.

Independent Set – A set of pairwise nonadjacent vertices.

Isolated Vertices – Any vertex with no incident edges or adjacent vertices.

Isomorphism – An isomorphism from a simple graph G to a simple graph H is a bijection $f: V(G) \rightarrow V(H)$ such that $uv \in E(G)$ iff $f(u)f(v) \in E(H)$. In other words, any graph whose edges take different route to connect the same vertices. Isomorphisms must have equivalent adjacency and incidence matrices.

Loop – An edge whose endpoints are equal. In other words, a loop occurs when a vertex is connected, or adjacent to itself.

Neighborhood – All vertices adjacent to one vertex in a graph. Each adjacent vertex is known as a neighbor.

Network A network is a digraph (or directed graph) with weighted edges. This paper considers undirected graphs.

Node – A node is representative of a vertex, however; has some value in name or size.

Path – A simple graph whose vertices can be ordered so that two vertices are adjacent iff they are consecutive in the list.

Random Graph – A graph whose edges are randomly placed between two vertices.

Regular Graph – A graph whose vertices have equal degree. A k-regular graph is a graph where all vertices have degree k.

Separating set A set $U \subseteq V(G)$ such that $G - U$ has more than one component. This is the set of any cut-vertices in a graph.

Small World Network - A small-world network is a class of random graphs where most nodes are also neighbors of one another, but every node can be reached from every other by a small number of hops or steps. A small world network, where nodes represent people and edges connect people that know each other, captures the small world phenomenon of strangers being linked by a mutual acquaintance.

Sink Vertex – The endpoint of the network flow. This vertex is the final node visited in the flow of a network.

Social Network A social network is a social structure made of nodes which are generally individuals or organizations. It indicates the ways in which they are connected through various social familiarities ranging from casual acquaintance to close familial bonds.

Source Vertex – The starting point of the network flow. This node is the source or beginning of the flow.

Subgraph – A graph H such that the vertex set of H is a subset of the vertex set of the original graph G and the edge set assignment of these vertices is the same as in G.

Torus – The surface obtained by adding a "handle" or "3-dimensional branch" to a sphere.

Triangle – The complete graph of 3 vertices.

Vertex – A node representative of an individual or object within a graph or network.

7 Codes

7.1 Graph Generation

```
/*
 * graph.c
 *
 * generate and store/manage the graph
 *
 */

#include <stdio.h>
#include <stdlib.h>
#include <string.h>
#include <time.h>

int *graph = NULL;
int graph_size;

/* initialize an empty nxn graph */
/* let rxc indicate row by columns == r*n+c */
int initialize_graph(int n) {
	srand((unsigned)time(NULL));
	graph = malloc(n*n*sizeof(int));

	if (graph == NULL)
		return -1;

	graph_size = n;
	memset(graph,0,sizeof(graph));

	return 1;
}

int free_graph() {
	free(graph);
	graph = NULL;

	return 1;
}

int generate_random_graph(double cr) {
```

```
	int i,j;
	if (graph == NULL) return -1;

	for (i=0;i<graph_size;i++) {
		for(j=i+1;j<graph_size;j++) {
			double r = rand()/(double)RAND_MAX;
			if (cr > r) {
				graph[i*graph_size+j] = 1;
				graph[j*graph_size+i] = 1;
			}
		}
	}
	return 1;
}

int print_matrix() {
	int i,j;

	if (graph == NULL) return -1;

	for(i=0;i<graph_size;i++) {
		printf("[ ");
		for(j=0;j<graph_size;j++)
			printf("%d ",graph[i*graph_size+j]);
		printf("]\n");
	}

	return 1;
}

int adjacent(int a, int b) {
	if (graph == NULL || a >= graph_size || b >= graph_size)
		return -1;

	return graph[a*graph_size + b];
}
```

7.2 Infect Node

```
/*
 * infect.c
 *
 * infection progression code
 */

#include <stdio.h>
#include <stdlib.h>
#include <string.h>
#include "extern.h"


int will_hear(int);
```

```
double *infect = NULL;

int graph_size;

int initialize_infect(int n, int first) {
        infect = malloc(n*sizeof(double));

        if (infect == NULL)
        return -1;
        graph_size = n;
        memset(infect,0,sizeof(infect));
        infect[first] = 1;

        return 1;
}

int print_infect(int time) {
        int i;
        printf("Time %d: \t[ ",time);
                for(i=0;i<graph_size;i++)
                        printf("%.2f ",infect[i]);
        printf("]\n");

        return 1;
}


/*
 *  This will mutate the rumor as a floating point with the following rules:
 *
 *  - every time increment, increment all >0 rumors by 0.1
 *  - each transmitted rumor increments by .1 from the parent
 *  - if an infected node talks to another infected node, both are set to the mean plus the difference
 *  - if an infected node talks to an infected node with difference >= 1.0, both become stiflers (-1)
 *  - if any infected node talks to a stifler, it also becomes a stifler
 *
 */
int run_infect_mutate(double prtn) {
        int run=1;
        int time=0;

        print_infect(time);
        while(run) {
                double *newinfect = malloc(graph_size*sizeof(double));
                int i,j;
                double lastval;

                time++;

                for(i=0;i<graph_size;i++) {
                        newinfect[i] = infect[i];
                        if (newinfect[i] > 0)
                                newinfect[i] += 0.08;
```

```
		}

		for(i=0;i<graph_size;i++) {
			if (infect[i] >= 1) {
				for(j=0;j<graph_size;j++) {
					double r;
					if (j==i) continue;
					r = rand()/(double)RAND_MAX;

					if (prtn > r && adjacent(i,j)) {
						double delta = infect[i] - infect[j];
						//printf("Delta: %.2f %.2f -
%.2f\n",newinfect[i],newinfect[j],delta);
						if (delta < 0) delta *= -1.0;

						if(newinfect[j] == 0)
							newinfect[j] = infect[i] + 0.1;
						else if (newinfect[j] == -1) {
							newinfect[i] = -1;
							break;
						}
						else if (delta > 1) {
							newinfect[i] = -1;
							newinfect[j] = -1;
							break;
						}
						else if (delta > 0) {
							double mean =
(newinfect[i]+newinfect[j])/2.0;

							newinfect[i] = mean + delta;
							newinfect[j] = mean + delta;
						}
					}
				}
			}
		}
		for(i=0;i<graph_size;i++)
			infect[i] = newinfect[i];

		free(newinfect);

		print_infect(time);


		/* check if this is the last necessary round */
		lastval = 0;
		run = 0;
		for(i=0;i<graph_size;i++) {
			if (will_hear(i)) {
				if (lastval == 0)
					lastval = infect[i];
				if (lastval != infect[i]) {
					if (lastval == -1 && infect[i] == 0)
						continue;
					run = 1;
					break;
```

```
                                }
                        }
                }
        }

        return 1;
}

int run_infect_basic(double prtn) {
        int run=1;
        int time=0;

        print_infect(time);
        while(run) {
                double *newinfect = malloc(graph_size*sizeof(double));
                int i,j;
                double lastval;

                time++;

                for(i=0;i<graph_size;i++)
                        newinfect[i] = infect[i];

                for(i=0;i<graph_size;i++) {
                        if (infect[i] >= 1) {
                                for(j=0;j<graph_size;j++) {
                                        double r;
                                        if (j==i) continue;
                                        r = rand()/(double)RAND_MAX;

                                        if (prtn > r && adjacent(i,j)) {
                                                if(newinfect[j] == 0)
                                                        newinfect[j] = 1.0;
                                        }
                                }
                        }
                }
                for(i=0;i<graph_size;i++)
                        infect[i] = newinfect[i];

                free(newinfect);

                print_infect(time);


                /* check if this is the last necessary round */
                lastval = 0;
                run = 0;
                for(i=0;i<graph_size;i++) {
                        if (will_hear(i)) {
                                if (lastval == 0)
                                        lastval = infect[i];
                                if (lastval != infect[i]) {
                                        run = 1;
                                        break;
                                }
```

```
                    }
                }
//              run = 1;
//              if (time >= 10) run = 0;
        }

        return 1;
}


/*
 * This will spider the graph and determine which nodes are connected
 */
int *final_round() {
        int *final = malloc(graph_size*sizeof(int));
        int i,j;
        int change = 1;

        for(i=0;i<graph_size;i++) {
                if (infect[i] == 0)
                        final[i] = 0;
                else
                        final[i] = 1;
        }

        while (change) {
                change = 0;
                for(i=0;i<graph_size;i++) {
                        if (final[i] >= 1)
                                for(j=0;j<graph_size;j++) {
                                        if (j==i)
                                                continue;
                                        if (final[j] == 0 && adjacent(i,j)) {
                                                change = 1;
                                                final[j] = 1;
                                        }
                                }
                }

        }

        return final;
}

int will_hear(int n) {
        static int *final = NULL;

        if (n >= graph_size)
                return 0;

        if (final == NULL)
                final = final_round();


        return final[n];
```

7.3 Default Settings File

```
#ifndef _RUMOR_H
#define _RUMOR_H

#define RUMOR_VERSION               "0.1-devel"

#define DEFAULT_GRAPH_SIZE10
#define DEFAULT_CONN_RATE0.25
#define DEFAULT_FIRST_INF    0
#define DEFAULT_TRANS_RATE          0.25

#endif
```

7.4 Display Result File

```
/*
 * rumor.c
 */
#include <stdio.h>
#include <stdlib.h>
#include <string.h>

#include "rumor.h"
#include "extern.h"

int show_help(char *);

int main(int argc, char *argv[]) {
        int i;
        int graph_type = 1;         /* 0 random */
        int print = 0;
        double conn_rate = DEFAULT_CONN_RATE;
        int graph_size = DEFAULT_GRAPH_SIZE;
        int first = DEFAULT_FIRST_INF;
        double trans_rate = DEFAULT_TRANS_RATE;
        int mutate=0;

        printf("Rumor Simulation engine v%s\n\n", RUMOR_VERSION);

        /* argument parsing */

        for(i=1;i<argc;i++) {
                if (strlen(argv[i])>1) {
                        if (argv[i][0] == '-') {
                                switch(argv[i][1]) {
                                case '?':
                                case 'h':
                                        show_help(argv[0]);
                                        break;
                                case 'n':
                                        if (i<(argc+1)) {
                                                i++;
                                                graph_size = (int)strtol(argv[i],(char **)NULL,10);
                                                if (graph_size <=0) {
```

```
									printf("Invalid graph size:
%d\n",graph_size);
									graph_size = 0;
								}
							}
							else
								i--;
							break;
						case 'c':
							if (i<(argc+1)) {
								i++;
								conn_rate = strtod(argv[i],(char **)NULL);
								if (conn_rate>1 || conn_rate <0) {
									printf("Invalid connection rate:
%f\n",conn_rate);
									conn_rate=DEFAULT_CONN_RATE;
								}
							}
							break;
						case 't':
							if (i<(argc+1)) {
								i++;
								trans_rate = strtod(argv[i],(char **)NULL);
								if (trans_rate>1 || trans_rate <0) {
									printf("Invalid transmission rate:
%f\n",conn_rate);
									conn_rate=DEFAULT_TRANS_RATE;
								}
							}
							break;
						case 'r':
							graph_type = 1;
							break;
						case 'm':
							mutate = 1;
							break;
						case 'p':
							print = 1;
							break;
						case 'f':

						default:
							printf("Unknown option '%s'\n",argv[i]);
						}
				}
		}
	}
	printf("DEBUG: graphsize %d, graphtype %d, cr %f\n",graph_size,graph_type,conn_rate);


	initialize_graph(graph_size);

	switch(graph_type) {
	case 1:
		generate_random_graph(conn_rate);
```

```
			break;
		}

		if(print)
			print_matrix();
		initialize_infect(graph_size,first);
		if (mutate)
			run_infect_mutate(trans_rate);
		else
			run_infect_basic(trans_rate);

		free_graph();

		return 0;
}

int show_help(char *self) {
		printf("Usage: %s [-h] [-p] [-n gsize] [-c rate] [-t rate] [-r] [-m]\n\n",self);
		printf("\t-h\tDisplay this help\n");
		printf("\t-p\tDisplay the adjacency matrix\n");
		printf("\t-n\tSet the size of the graph to gsize (Default: %d)\n",DEFAULT_GRAPH_SIZE);
		printf("\t-c\tSet the connectivity rate (between 0 and 1, default:
%f)\n",DEFAULT_CONN_RATE);
		printf("\t-t\tSet the transmission rate (between 0 and 1, default:
%f)\n",DEFAULT_TRANS_RATE);
		printf("\t-r\tSpecifies a random graph generation (default)\n");
		printf("\t-r\tAllow rumor mutation\n");

		exit(0);

		return -1;
}
```

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