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The comparison of densitometric measurement with gravimetric measurement of wet-on-wet ink trapping

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THE COMPARISON OF DENSITOMETRIC MEASUREMENT WITH GRAVIMETRIC MEASUREMENT OF WET-ON-WET INK TRAPPING

by

Kongsak Lawphongpanich

A thesis submitted in partial fulfillment of the
requirements for the degree of Master of Science in the
School of Printing in the College of Graphic Arts and Photography
of the Rochester Institute of Technology

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CERTIFICATE OF APPROVAL

MASTER'S THESIS

This is to certify that the Master's Thesis of

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has been approved by the Thesis Committee as satisfactory for the thesis requirement for the Master of Science degree at the convocation of

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An Abstract

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ABSTRACT

Densitometric and gravimetric methods are employed to measure the percent of ink trapped in a wet-on-wet transfer. This study seeks to discover the relationship of the two methods and if there is a significant difference between the percent gravimetric and percent apparent trapping. An IGT Printability tester (Model-A2) with two printing discs is used to apply ink onto paper. When using the gravimetric method the amount of ink on the disc and ink transferred onto paper are weighed with an analytical balance. The author then calculates the percent ink trapping. When using the densitometric methods densities are measured by polarizing and non-polarizing reflection densitometers which are used to generate data to determine percent trapping. The author also applies an antilogarithm trapping formula to determine percent trapping. The experimental factors studied here are the three tack levels of both ink layers, which are defined as low, medium, and high; the three different types of paper, which are newsprint, uncoated paper, and coated paper; and the four measurement methods. These methods are gravimetric, apparent, polarizing apparent, and antilogarithm trapping.

The study results reveal that there is significant difference between the four measurement methods. The regression analysis establishes the relationship equations between percent gravimetric and apparent trapping. All of the equations are rejected because they lack accur-

acy in predicting percent gravimetric trapping using percent apparent trapping.

Improved equations for percent trapping on newsprint, uncoated paper, and coated paper are presented. These equations estimate the percent gravimetric trapping on the basis of densitometric data but they are complex, and therefore only useful in the laboratory at this time. The author also determines percent gravimetric trapping using densities in terms of the exponential function and establishes prediction equations of percent trapping for each type of experimental paper. Further study will be needed to develop equations useful to the printing industry.

Abstract approved: ____________________________ thesis advisor

____________________________ title and department

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I. INTRODUCTION

With the exception of the first color, which is transferred to paper, multicolor printing by lithography and letterpress is accomplished by transferring wet ink to each preceding ink which is still "wet". This is called "wet-on-wet ink trapping". One of the problems which occurs on a regular basis is that the second ink is not trapped on the first ink in a predictable manner. Inks which overprint may transfer a thinner layer than expected. As a result, the color of the reproduction may shift.

The author conducted a literature research to discover factors affecting the wet-on-wet trapping. The major factors are ink tack differences, the time interval between the ink transfer, and the types of paper. Other factors mentioned in the literature are ink viscosity, temperature, ink pigment, ink film thickness, and printing speed.

The literature indicates three different techniques have been used to measure the percentage of ink trapping. Densitometry, gravimetry, and X-ray spectroscopy have been applied. The most commonly used, probably because it is convenient, is the densitometric approach. The gravimetric method can be said to give more accurate measurement than the densitometric one, because it measures the physical amount of ink which is transferred. The X-ray spectroscopy is the newest method which was made available in 1978 by Abdel Ghany Saleh. This method

is limited by the type of paper and ink used. At the present time, its application appears to be strictly a research tool.

When the gravimetric method is applied, it shall be referred to as "gravimetric trapping". The amount of ink transferred is expressed in terms of the weight or ink film thickness. Using the densitometric method, the amount of ink transferred is measured in terms of reflection densities. If a standard, non-polarizing densitometer is used, the trapping is called "apparent trapping". When a polarizing densitometer is used, trapping is called "polarizing apparent trapping". "Antilogarithm trapping" is densitometric trapping. It is calculated with a formula different from the apparent and polarizing apparent trapping. This formula is expressed in terms of antilogarithms of densities.

The Problem

In the printing industry, a densitometric method is widely used to measure percent ink trapping. But in the laboratory, the gravimetric method is used to measure the physical amount of ink trap. In the densitometric method, a standard reflection densitometer is normally used to measure the densities. Harry Hull stated that "the polarizing densitometer is more accurate than the standard densitometer for the measurement of either wet or dry trapping."[1] In 1980 Warren Childers suggested that apparent trapping should be expressed in terms of antilogarithms of densities. He indicated that "the errors occur because the currently used numbers represent ratios of logarithms. The accurate answer for percent ink trap must be calculated from antilogarithms

instead."[2] The problem is that the comparison of these methods has not been studied. So we do not know whether or not each method provides equivalent values of trapping. The logical question will seem to be: "Do the densitometric approaches provide information about trapping which relates to gravimetric measurement?"

In this study, the comparison of these trapping methods is studied. The author expects the results to show a significant difference between these methods. The relationship between percent gravimetric and apparent trapping will be established if this is possible. In addition, the author studies whether percent gravimetric trapping can be predicted using reflection densities.

FOOTNOTES FOR CHAPTER ONE

[1]Harry H. Hull, "The Polarizing Reflection Densitometer." LTF Reports of Progress During 1963, p. 10.

[2]Warren Childers, "Expert Shows Math Path to Avoid Ink Trap Trap." Graphic Arts Monthly, Dec. 1980, p. 64.

II. REVIEW OF LITERATURE AND THEORETICAL BASIS

Review of the Factors that Affect Ink Trapping

When printing process color by lithography or letterpress, trapping has been a common problem. The amount of ink transferred onto paper is different from the amount transferred onto an already printed ink film layer. Normally less ink transfers onto an ink layer than onto paper. The trapping problem seems to be more critical in wet-on-wet printing than in the wet-on-dry printing process. In the wet-on-dry process, the ink layer has been set or partially dried before the over-printing color ink is printed onto it. Using the wet-on-wet process on multi-color presses, the ink does not have time to set before the next color ink is printed over it. The phenomenon of reduced ink transfer is called "undertrapping." Sometimes a pressman will change the printing sequence to correct or improve the problem of undertrapping. If one can understand the causes and factors which affect it, the change of color sequence can be eliminated.

The disadvantages of undertrapping are variation in color and saturation of secondary colors on the printed sheets. To understand these problems, the ink transfer process and other physical properties of ink and paper must be investigated.

The author conducted a literature search to discover factors affecting wet-on-wet printing. Ink tack differences and the time interval

between the ink transfer are important. Other factors mentioned in the literature are viscosity, ink film thickness, temperature, type of substrate, and printing speed.

The Effect of Ink Tack on Ink Trapping

The tack of ink can be defined as its stickiness. Stickiness develops when ink film splits during ink transfer. Splitting is defined as the division of the ink into two sections, each section adhering to different surfaces. In offset lithography, printers are primarily concerned with splitting between the paper and the blanket. After the ink transfers to paper, the tack starts to increase to its maximum value. How tack increases was described by Tollenear and Ernst as follows:

> First of all part of the ink is pressed into the surface cavities of the paper during the printing process, causing immobilization of this part of the ink so that it can no longer be subjected to splitting of ink. Moreover, the thickness of the layer decreases after the ink is applied, due to absorption by the paper.
>
> This implies that the thickness of the layer playing a part in subsequent splitting decreases in the time interval between the printing of the first and the second layers ...
>
> A third cause for the increase in tackiness is stiffening of the ink, either by its thixotropic character, or by a filtering-out process. The latter, however, can become so pronounced with inks containing a vehicle of low viscosity, that the ink loses too much of this vehicle after printing ... When such a case of drastic filtering out occurs, the tackiness does not increase after printing, but decreases.[1]

From Carlson and Lindberg's study[2] of the effect of ink tack, the percentage of ink trapping versus the different tackiness was plotted. The ink was transferred to three types of paper: newsprint, machine coated paper, and grease proof paper. The curves showed the

same characteristics in that the percentages of ink transferred increased about 0.40-0.50 percent, when the tack increased by one unit.

In wet-on-dry printing, ink has enough time to set and to complete at least partial drying. In wet-on-wet printing, the time interval between the first layer and the second layer is very short. The result is that transfer of the second ink varies and usually the amount of ink transferred is less than to a dry ink layer or to paper. This phenomenon is called "undertrapping".

The usual practice in wet-on-wet printing is to manipulate the tack of the preceding color, or the first color down, to be higher than the tack of the following color. The reason for this is to produce a surface strong enough to resist the separating force of the second ink layer. Eight to ten units spread in tack between the colors is thought to be a good condition to get high trapping.

The Effect of Time Intervals on Ink Trapping

The shorter the time interval, the less ink is transferred to the preceding layer of ink in the wet-on-wet printing. Tollenear and Ernst[3] tested and proved this statement using an IGT printability tester. The first print was made with a colorless polyisobutylene (Oppanol B3), while the second layer was black ink. The time interval between the first and second layers was varied. A minimal amount of black ink was trapped on the Oppanol B3 layer using the shortest time interval of six seconds. As the time interval increased, more black ink transferred onto the Oppanol B3 layer, showing better trapping.

The tack as well as the amount of ink absorbed by the paper increases when the time interval is increased. Tollenear and Ernst[4] studied this on grease-proof paper, coated paper, and cast-coated paper. The conclusion was that the ink absorption of the coated paper, as well as the cast-coated paper, showed a signficant increase in ink transfer when the time interval was extended.

The Effect of Ink Viscosity on Ink Transfer

Not only the tack but also the viscosity affect the ink transfer. The tack and viscosity are independent ink properties. An ink of high tack can be of low viscosity and vice versa. According to Lindberg's investigation,[5] the viscosity affected the amount of ink transferred. Yellow, cyan and magenta news inks were studied. They had different viscosities and tack. The best ink transfer was obtained when the first ink printed had the highest tack and viscosity values. But for letterpress inks, the highest ink transfer occurred with an ink combination where the first ink had higher tack but lower viscosity than the second one. For both types of inks, lithography and letterpress, the curves showed that the difference in viscosity influenced the ink transfer, mainly for low amounts of the second color ink.

Karttunen and Oittinen[6] also studied the effect of the viscosity in the ink transfer. In the case of single ink transfer, the ink transfer increased at lower ink viscosity. In two-color transfer, the viscosity of both inks affected the amount of the ink transfer. The smaller the viscosity differences, the more the second ink was transferred. When the viscosity of the second ink was raised, a higher proportion of the

first ink took part in the transfer of the second ink. The result was less second ink transfer and more pronounced back trapping, the picking up of a previously printed ink by an overprinted ink.

The Effect of Ink Film Thickness on Ink Transfer

It has been established that the resistance to ink splitting becomes lower as the thickness of the layers increase. Tollenear and Ernst[7] studied this effect using an IGT printability tester. The first layer was a colorless polyisobutylene (Oppanol B3). The second layer was black ink. The ink film thickness on the disc of the black ink was constant while the amount of the Oppanol B3 was varied. As the thickness of the Oppanol increased, less of the black ink was transferred.

Carlson and Lindberg[8] studied the influence of the amount of ink on newsprint, machine coated paper, and grease proof paper. The relationship between the ink transfer percentages and amount of ink on the plates were plotted. When the amount of ink on both plates was equal, the curves did not show much influence on ink transfer although the amount of ink was increased. If the amount of ink on the second plate only was increased, the percentages of ink transfer were increased. This may be due to an increase in the flexibility and plasticity of the paper by wetting, which made it more acceptable to ink.

The Effect of Temperature on Ink Transfer

Oittinen and Karttunen[9] studied the effect of temperature on the wet-on-wet ink transfer. Their conclusion was that the transfer of the first ink increased when the temperature was raised from 23°C to 30°C.

This result was due to increased immobilization, which was immediate absorption of ink pressed into the surface cavities of the paper during the ink transfer. The transfer of the second ink was decreased because of decreased immobilization. When both inks were printed at higher temperature and lower viscosity, the trapping of the second ink was clearly improved by the "cold-setting effect" of the first ink. The cold-setting effect is that the ink sets faster during the printing when the ink has a higher temperature than the paper. This cold-setting effect might be used instead of tack or viscosity differences to eliminate the undertrapping problem.

The Effect of Ink Pigments on Ink Transfer

Preucil[10] investigated the effect of yellow pigment on ink transfer. He demonstrated poor trapping onto yellow ink, with the tack of yellow ink higher than the tack of succeeding colors. It was found that low cost yellow ink, slightly orange or chrome in color, contained some lead chromate pigment. It was suspected that the heavy lead pigment, as used with much less vehicle was more affected by water. This produced the poor trapping.

Testing with special yellow ink, which consisted totally of lead chromate pigments, showed a trapping decrease to about 60 percent at a yellow ink density of 1.00 and a high-low tack relationship of 22-13 Gm-m.

Review of the Correction for Undertrapping

The problem of the wet-on-wet undertrapping is associated with an unpredictable density drop when compared to normal or dry trapping. One way to solve this problem is by increasing the ink film thickness of the second- and third-printed inks to compensate for wet-on-wet undertrapping. The gray balance must be considered. For a given set of process inks, the gray balance is usually expressed as a relationship of the equivalent neutral density (END) of the individual ink densities. If these END values are known to apply to normal trapping, it is possible to calculate how much the single ink densities must be increased so as to still satisfy the gray balance with a certain amount of undertrapping.

Kurt Schlapfer and Jaree Keretho[11] derived an equation to calculate the wet-on-wet (WOW) densities from the wet-on-dry (WOD) densities.

For the solid prints:

$$D_{2w} = D_i \left[1 - \left(1 - \frac{D_{2d}}{D_i}\right)^{2-c}\right]$$

$$D_{3w} = D_i \left[1 - \left(1 - \frac{D_{3d}}{D_i}\right)^{3-2c}\right]$$

D_{2w} = the density of second ink on the wet ink (WOW).
D_{3w} = the density of third ink on the wet ink (WOW).
D_{2d} = the density of second ink on the dry ink (WOD).
D_{3d} = the density of third ink on the dry ink (WOD).
D_i = the saturation density attained at high levels of ink film thickness
c = the degree of the first ink which is involved in ink splitting.

In the case of $c = 1$, the first ink film behaves as in wet-on-dry printing. When $c = 0.5$, the film of the first ink merges completely with the second ink film on the plate and splitting takes place in the middle of the united ink film. This might be considered to be the extreme condition in wet-on-wet printing.

For half-tone printing, the correction terms k and ck are introduced.

$$D_{2w} = \frac{D_i}{k} \left[1 - \left(1 - \frac{kD_{2d}}{D_i}\right)^{2-ck}\right]$$

$$D_{3w} = \frac{D_i}{k} \left[1 - \left(1 - \frac{kD_{3d}}{D_i}\right)^{3-2ck}\right]$$

k = a constant which affects the total density of two-color printing overlap. It is similar to the n factor in the Yule-Neilson equation.

ck = a correction of c for half-tone prints.

Under-Color Removal and Undertrapping

Although it is well known that under-color removal (UCR) reduces the problem of undertrapping of wet-on-wet ink transfer, the required amount of UCR is still unknown. The dot area in half-tones and separations must be increased to compensate for undertrapping. It is not possible to compensate in the solid area by increasing the area. UCR is only effective in the three color overlap area. UCR cannot improve the problem of undertrapping in two-color overlaps.

If UCR is applied, a reduction in the black printer is not necessary because undertrapping in black cannot affect the color reproduction and gray balance. Conversely, a reduction of the black printer reduces the density range in printing, which is more disadvantageous than undertrapping in the shadows. In most cases UCR is sufficient when the total dot percentage of cyan, magenta, and yellow is equal to 180 percent and the black is 100 percent dot area. The preferred printing sequence is with the color having the lowest overlapping dot area printed first in order to allow the subsequent colors to be printed on the paper.

There are some disadvantages in the case of UCR. The maximum attainable density range in printing is reduced. Also, it requires a fuller black printer and this produces a desaturation of pure-colored areas.

Review of Trapping Measurement Methods

At the present time, there are three methods of measuring the percent ink trapping. They are as follows:

I. The gravimetric method,

II. The densitometric method,

III. The X-ray fluorescence spectroscopy method.

The Gravimetric Method

In the gravimetric method, the amount of ink transfer is measured in terms of ink film thickness or the weight of ink per unit area. The

formula of ink trapping is the ratio of amount of ink transfer on the preceding ink layer and directly on the substrate. Normally, ink trapping is expressed in terms of trapping percentage; the formula is as follows:

$$\text{percent gravimetric trapping} = (W_2/W_1)\ (100)$$

where W_1 = the amount of ink transferred on paper. The unit of W_1 is ink film thickness or weight per unit area.

W_2 = the amount of ink transferred on the preceding ink layer when printed on the same substrate. The unit of W_2 is ink film thickness or weight per unit area.

An IGT printability tester with two printing discs or a two-color proofing press can be used for testing percent gravimetric trapping. The IGT tester was designed to print solid image although half-tone image can be accomplished on it. A two or more-color proofing press is more appropriate for half-tone work.

On the IGT tester, two printing discs are used and each is inked with a different ink. The first disc transfers the underprinting layer and the second disc transfers the overprinting layer. A strip of the desired substrate is printed in one turn of the printing sector of the IGT tester by both discs. The amount of ink transferred from each disc is weighed for trapping calculation.

The Densitometric Methods

Ink trapping determined by a non-polarizing reflection densitometer is called "apparent trapping". Reflection density is thought to be

approximately proportional to its ink film thickness. The percent apparent trapping is calculated from the reflection densities of three printed areas. These three areas are: the solid area of the first color printed on the substrate, the solid area of the second color printed on the substrate, and the overlapping area of the two colors. A color filter is chosen which gives the highest density reading for the overlapping area. The equation used for determining the percent apparent trapping is:

Percent apparent trapping[12] =

$$100\ \frac{(\text{Density of overlapping area - Density of first color})}{\text{Density of second color}}$$

The equations used for determining the trapping for color combinations are summarized by Harry Hull.

> In the following formulas, "D" refers to reflection density and the subscripts refer to the color over which the density is read. The letters y, m, c, and k refer to yellow, magenta, cyan, and black. All densities refer to readings made with the densitometer set to read zero on the paper. The slash in the formula should read as 'over'. Thus, m/y refers to magenta overprinting yellow.
>
> (1) In the following trap formulas, density measurements are made with the green filter (on most densitometers the position of the green filter is indicated by a magenta-colored dot).
>
> (a) % trap m/y = 100 $(D_{my} - D_y)/D_m$
>
> (b) % trap m/c = 100 $(D_{mc} - D_c)/D_m$
>
> (c) % trap m/cy = 100 $(D_{mcy} - D_{cy})/D_m$
>
> (d) % trap k/cy* = 100 $(D_{kcy} - D_{cy})/D_k$
>
> (2) In the following trap formulas, density measurements are made with the red filter (the position of the red filter is commonly indicated by a cyan-colored dot).

(e) % trap c/m = 100 $(D_{cm} - D_m)/D_c$

(f) % trap c/y = 100 $(D_{cy} - D_y)/D_c$

(g) % trap c/my = 100 $(D_{cmy} - D_{my})/D_c$

(h) % trap k/m* = 100 $(D_{km} - D_m)/D_k$

(j) % trap k/y* = 100 $(D_{ky} - D_y)/D_k$

(k) % trap k/my* = 100 $(D_{kmy} - D_{my})/D_k$

(3) In the following trap formulas, density measurements are made with the blue filter (the position of the blue filter is commonly indicated by a yellow-colored dot).

(m) % trap y/m = 100 $(D_{ym} - D_m)/D_y$

(n) % trap y/c = 100 $(D_{yc} - D_c)/D_y$

(p) % trap y/mc = 100 $(D_{ymc} - D_{mc})/D_y$

(q) % trap k/c* = 100 $(D_{kc} - D_c)/D_k$

(r) % trap k/mc* = 100 $(D_{kmc} - D_{mc})/D_k$

*These trap measurements are more accurate if a polarizing densitometer is used.[13]

According to Hull, a polarizing densitometer is more accurate than a non-polarizing densitometer for the measurement of either wet or dry ink trapping. This is due to the effect of the gloss of the overprinting ink. Gloss affects the reading of the non-polarizing densitometer but not the polarizing one. From the study by Hull[14], with measurement of trapping of black on cyan with a blue filter, the black was much higher in gloss on the overprinted area and received higher readings with the standard densitometer. With the polarizing densitometer, the density reading of black was approximately the same as the density reading on paper.

Warren Childers[15] suggested another trapping formula. He stated that "the errors occur because the currently used numbers represent ratios of logarithms. The accurate answer for percent ink trap must be calculated from antilogarithms instead." The equation is as follows:

percent antilogarithm trapping = 100 $[10^{(D_{2/1} - D_1 - D_2)}]$

$D_{2/1}$ = the density of the overlapping area of two inks.
D_1 = the density of the first ink layer on paper.
D_2 = the density of the second ink layer on paper.

These densities are measured with the same filter which gives the highest density of the overlapping area.

Ink trapping measurement by the densitometric method is less accurate than measurement by the gravimetric method because the optical properties of the inks printed on paper cause the error in trapping determination. The main advantages of the densitometric method are that it is extremely convenient and fast. It is used to measure percent ink trapping on a printed sheet from a press while it is running.

The X-ray Fluorescence Spectroscopy Method (XRFS)

The principle of this method is the same as the gravimetric method, but the method of measuring the amount of ink transfer is different. Ink film thickness is measured by using the X-ray fluorescence spectrometer which counts the amount of heavy metal in the ink. The counting number is converted to the ink film thickness by a calibration chart or curve. The trapping is the ratio of the ink film thickness on the ink layer and on the paper. The brief outline of the XRFS was explained by Abdel Saleh.

> When an electron from an inner shell of an atom is ejected, i.e., the shell becomes ionized, readjustment of the other higher level electrons in the atom takes place to fill this deficiency and render the atom to its round state,

> resulting in the emission of X-ray quanta whose wavelengths are characteristic of the element itself.
>
> Ejection of an inner electron can occur when an X-ray beam of sufficient energy is incident upon a sample containing the element in question. In a conventional wavelength dispersive X-ray spectrometer, the characteristic radiation of the elements present in a sample are excited the incident primary radiation emitted by an X-ray tube and then dispersed by an analysing crystal and measured in a detector. The intensity of the measured characteristic radiation is related to the concentration of the element in question present in the sample. In the case of the thin films, the relationship is linear over a thickness ranged defined by the sample matrix and the wavelength being measured.[16]

To use this method, there must be a heavy metal in the testing ink. One way to introduce it to the ink is by choosing ink pigments which contain heavy metals in their molecular structure, e.g., phthalocyanine pigment in the cyan ink. Another way is by mixing a suitable quantity of barium sulphate extender white pigment into the process ink. The types of paper used in the test are limited because some heavy metals, e.g., copper, calcium, aluminum, iron, and barium, are present in paper coating material. The heavy metal in paper can interfere with the counting and characteristic line. The ideal case for the XRFS is the use of heavy elements present in the ink formulas and absent from the paper. The types of paper which are suitable and contain no heavy metal are starglass art, superstar art, and poladin.

The advantages of the XRFS are:

a) The capability of measuring a wide range of half-tone tests and solid prints.

b) Overprints of two or more layers of inks, thin or thick, solid or half-tone, printed on different substrates.

However, the disadvantages are the limitations of the type of paper and ink components and the high cost of instruments and testing.

Summary of Literature Review

The printing industry has used the apparent trapping formula many years to measure percent trapping of ink. This formula is an empirical equation which is not accurate and is not scientifically verified. It is based on the assumption that ink density is proportionally related to ink film thickness. This assumption is only true when the ink film thickness is thinner than one micron. A polarizing densitometer eliminates the gloss effect on ink densities but it does not improve the accuracy of the apparent trapping measurement. The antilogarithm formula is introduced to more accurately estimate the apparent trapping measurements. However, there is no evidence that the antilogarithm formula is a better formula to estimate trapping. The gravimetric method is the only technique measuring the physical amount of ink trapped. Because it measures the physical amount of ink transferred, the author believes it is the most accurate method for estimating percent trapping.

Research Questions

The literature review indicates that the "apparent trapping" measuring method of ink trapping is widely used at the present time but does not provide accurate estimations. However, new techniques and new instruments have been developed to improve the accuracy of appar-

ent trapping measurements. Harry Hull recommended a polarizing densitometer to improve the sensitivity of the apparent trapping measurement while Warren Childers suggested an antilogarithm formula to more accurately measure trapping. The gravimetric, in contrast to the apparent, method determines the physical amount of ink transferred onto another ink layer for calculating the percent trapping. In order to discover the best method of measuring ink trapping, the author investigates how accurate densitometric results are when they are compared to gravimetric results. This question can be restated as whether there is any significant difference between the gravimetric and densitometric percent trapping, including apparent, polarizing apparent, and antilogarithm trapping. The results are expected to show that there are significant differences between the results obtained using the two methods.

The author also investigates whether there is any relationship between apparent trapping and gravimetric trapping, so that an apparent trapping value can be converted into a gravimetric value. If there is no relationship between the two methods, the next research question is whether it is possible to determine percent gravimetric trapping using a set of ink densities.

Hypotheses

Statistical analyses will be applied to answer research questions. The statistical techniques are: the analysis of variance, multiple range test, graphical analysis, and regression analysis. The hypotheses are stated as null hypotheses for statistical purposes.

The question of whether there are any significant difference between gravimetric, apparent, polarizing apparent, and antilogarithm trapping contained a null hypothesis stating, "There is no difference between the gravimetric method and densitometric methods, including apparent, polarizing apparent, and antilogarithm trapping."

The second question investigates the relationship between percent gravimetric and percent apparent trapping. The null statement, for this study, is: "There is no relationship between the percent gravimetric and percent apparent trapping". The author expects that the relationship will be the first, second, or third order regression equation model.

The first order equation model is:

$$y = b_0 + b_1(x)$$

The second order equation model is:

$$y = b_0 + b_1(x) + b_2(x^2)$$

The third order equation model is:

$$y = b_0 + b_1(x) + b_2(x^2) + b_3(x^3)$$

Where y, for all equations, is the percent gravimetric trapping.
x is the percent apparent trapping.
b_0 is the value of y at the intercept., and
b_1, b_2, and b_3 are regression coefficients.

The third question investigating the percent gravimetric trapping determined by the reflection densities uses a null hypothesis stating, "The percent gravimetric trapping and densities have no relationship so

the percent gravimetric trapping cannot be determined by using densities." It is expected that one of the following regression equation models will predict the percent gravimetric trapping in terms of densities:

The first order equation model is:

$$G = b_0 + b_1(D_1) + b_2(D_2) + b_3(D_3) + b_4(D_4)$$

The second order equation model is:

$$G = b_0 + b_1(D_1) + b_2(D_2) + b_3(D_3) + b_4(D_4) + b_{12}(D_1D_2) + b_{13}(D_1D_3) + b_{14}(D_1D_4) + b_{23}(D_2D_3) + b_{24}(D_2D_4) + b_{34}(D_3D_4) + b_{11}(D_1^2) + b_{22}(D_2^2) + b_{33}(D_3^2) + b_{44}(D_4^2)$$

The third order equation model is:

$$G = b_0 + b_1(D_1) + b_2(D_2) + b_3(D_3) + b_4(D_4) + b_{12}(D_1D_2) + b_{13}(D_1D_3) + b_{14}(D_1D_4) + b_{23}(D_2D_3) + b_{24}(D_2D_4) + b_{34}(D_3D_4) + b_{11}(D_1^2) + b_{22}(D_2^2) + b_{33}(D_3^2) + b_{44}(D_4^2) + b_{123}(D_1D_2D_3) + b_{124}(D_1D_2D_4) + b_{134}(D_1D_3D_4) + b_{234}(D_2D_3D_4) + b_{111}(D_1^3) + b_{222}(D_2^3) + b_{333}(D_3^3) + b_{444}(D_4^3)$$

The fourth order equation model is:

$$G = b_0 + b_1(D_1) + b_2(D_2) + b_3(D_3) + b_4(D_4) + b_{12}(D_1D_2) + b_{13}(D_1D_3) + b_{14}(D_1D_4) + b_{23}(D_2D_3) + b_{24}(D_2D_4) + b_{34}(D_3D_4) + b_{11}(D_1^2) + b_{22}(D_2^2) + b_{33}(D_3^2) + b_{44}(D_4^2) + b_{123}(D_1D_2D_3) + b_{124}(D_1D_2D_4) + b_{134}(D_1D_3D_4) + b_{234}(D_2D_3D_4) + b_{111}(D_1^3) + b_{222}(D_2^3) + b_{333}(D_3^3) + b_{444}(D_4^3) + b_{1234}(D_1D_2D_3D_4) + b_{1111}(D_1^4) + b_{2222}(D_2^4) + b_{3333}(D_3^4) + b_{4444}(D_4^4)$$

Where G is the percent gravimetric trapping.

D_1 is the density of the overlapping area of two inks. The filter used is the one giving the highest density reading.

D_2 is the density of the first color ink printed on paper. The filter used is the same one as D_1.

D_3 is the density of the second color ink printed on paper. The filter used is the same one as D_1.

D_4 is the density of the first color ink printed on paper. The filter used is the one giving the highest density for just the D_4 density.

b is a constant.

b_1 b_2, b_3.... and b_{4444} are regression coefficients.

FOOTNOTES FOR CHAPTER TWO

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[2]G.E. Carlson and B. Lindberg, "Quantitative Studies of Ink Transfer in High Wet-On-Wet Printing." Fourth Graphic Arts Conference, Rochester, N.Y. Oct. 25-27, 1967, p. 8.

[3]Tollenear and Ernst, p. 7.

[4]Tollenear and Ernst, p. 8.

[5]Carlson and Lindberg, p. 8.

[6]Pirkko Oittinen and Simo Karttunen, "Effect of Temperature on Wet-On-Wet Transfer." Graphic Arts in Finland, No. 2, 1972, pp. 11-22.

[7]Tollenear and Ernst, p. 6.

[8]Carlson and Lindberg, p. 67.

[9]Oittinen and Kurttunen, p. 28.

[10]Frank M. Preucil, "Other Photo and Color Reproduction Projects." GATF Reports of Progress During 1958, p. 1.

[11]Kurt Schlapfer and Jaree Keretho, "Undertrapping in Wet-on-Wet Printing and Undercolor Removal (UCR)." Advances in Printing Science and Technology, 1977, p. 204.

[12]Frank M. Preucil, "Multicolor Press Problems - Trapping and Dot Spread." GATF Reports of Progress During 1957, p. 2.

[13]Harry H. Hull, "How to Calculate Ink Trap." Graphic Arts Monthly and the Printing Industry, Vol. 45, No. 10, Oct. 1973, p. 87.

[14]Harry H. Hull, "The Polarizing Reflection Densitometer." GATF Reports Of Progress During 1963, p. 10.

[15]Warren Childers, "Expert Shows Math Path to Avoid Ink Trap Trap." Graphic Arts Monthly, Dec. 1980, p. 64.

[16]Abdel G. Saleh, "Wet-On-Wet Printing: An Analysis of Trapping Problems." Professional Printer, Vol. 23, No. 4, July/Aug., 1979, p. 12.

III. METHODOLOGY

The variables of this study were: tack of the first ink layer, tack of the second ink layer, and different types of paper, and four measurement methods. The levels of tack of both ink layers were termed low, medium, and high. Both layers of ink were printed by the same set of lithographic process color inks. Cyan ink exhibited the highest tack and was equal to an average of 21.386 Gm-m. Magenta ink exhibited a medium tack at an average of 15.75 Gm-m. Yellow ink had the lowest tack equal to an average of 11.275 Gm-m. Additional ink properties are shown in Table 1. Figure 1 shows the relationship between the change in ink tack and the amount of time on the inkometer.

Newsprint, uncoated paper, and coated paper represented the types of paper. The paper properties are shown in Table 2. The best expected densities for cyan, magenta, and yellow ink were specified individually for each paper. The assumption was that each specified combination of the selected densities would result in the achievement of gray balance in each case.

The measurement methods of trapping were the gravimetric and densitometric methods. The densitometric methods included apparent trapping, polarizing apparent trapping, and antilogarithm trapping.

The experimental design included four factors resulting in nine different combinations of ink tack for each of the three different paper stocks. The design also took into account four different methods to

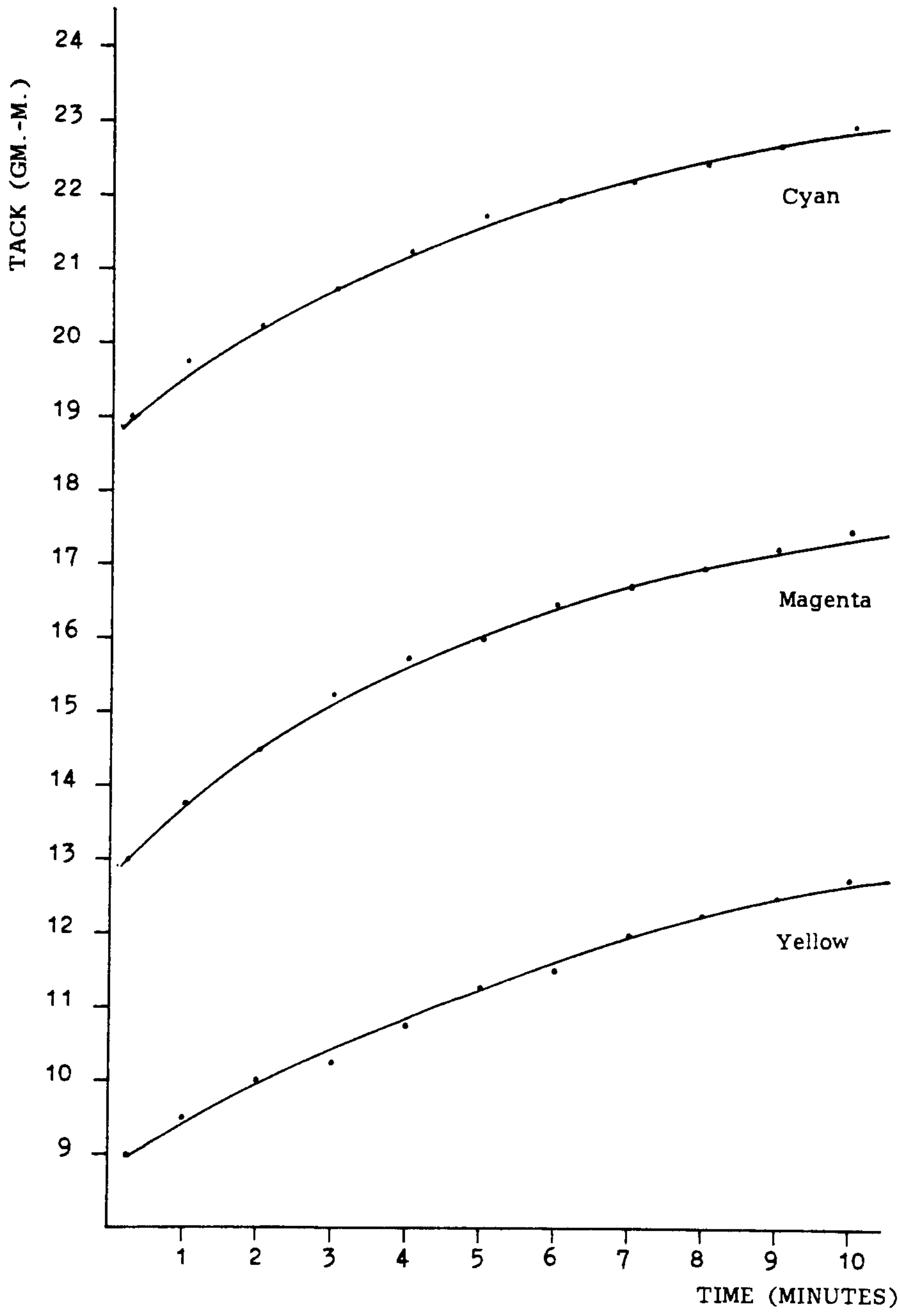

Figure 1. Relationship between tack and time of ink on the inkometer

Table 1

Ink Properties

TYPE - Offset lithography inks
BRAND - Superior
COMPANY - Superior Printing Ink Co., Inc.
COMPANY COLOR NAME:
Cyan - Offset S-C N-S Process Blue MBA 8421
Magenta - Offset N-S Process Red DRA-6085-xHG
Yellow - Offset Super Gloss Process Yellow YA 9715

Ink	Tack† (GM-M)	Physical Density (Gm/cm^3)
Cyan	21.386	1.033975
Magenta	15.75	1.02555
Yellow††	11.275	0.9765

Notes:

† Tack was measured by a THWING-ALBERT inkometer. The inkometer was set at a temperature of 90°F and the speed at 620 ft/min. The average of tack was taken from eleven readings. The readings were taken at one minute intervals for ten minutes, with the first reading being taken after twenty seconds.

†† Yellow ink was mixed with 20 percent of the "LES TAC" to reduce the tack from 20.364 to 11.275. Les tac is the tack reducer manufactured by the Capital Printing Ink Company.

Table 2

Paper Properties

Paper	Basis Weight (gm/m^2)	Thickness (cm)	Roughness† (cm^3/m^2)	Oil Absorption† (mm^{-1})
Coated	101	0.009	0.82	8.86
Uncoated	45	0.010	7.08	37.38
Newsprint	75	0.009	9.95	27.36

†Surface roughness and oil absorption were tested on an IGT printability tester with 40 kilograms of pressure.

evaluate trapping for each combination. Table 3 shows the experimental design. The ink tack combinations were: 1) low-low, 2) low-medium, 3) low-high, 4) medium-low, 5) medium-medium, 6) medium-high, 7) high-low, 8) high-medium, and 9) high-high. Each combination was replicated three times.

Measurement of Gravimetric Trapping

An IGT (Instituut voor Grafishe Techniek, "Research Institute for the Printing and Allied Industries TNO") printability tester, model A2, was the instrument used for transferring ink onto paper. The spring tensions were set at 40 kilograms. Two of the two-centimeter aluminum printing discs were used to print two inks sequentially on paper to simulate wet-on-wet trapping. Before the discs were inked, they were weighed carefully in an analytical balance, the Mettler H15, which has a measurement accuracy of 0.0001 gram. A small piece of tape was used to prevent total coverage of the ink on each disc. This was necessary to measure densities for determining the percent trapping by the densitometric methods. By properly positioning uninked areas of both discs, two distinct printed areas resulted (see Figure 2).

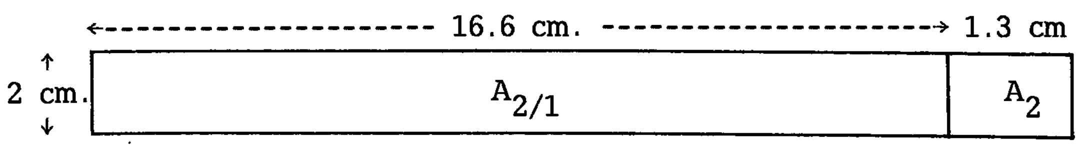

Figure 2. Diagram of printed strip

Tack of Second Ink Layer	Measurement Methods	Newsprint: Yel den = 75, Mag den =1.0, Cy den = 95									Uncoated Paper: Yel den = 80, Mag. den.=1.05, Cy den =1.0									Coated Paper: Yel den.=1.0, Mag den.=1.30, Cy den.=1.25								
		Tack of First Ink Layer									Tack of First Ink Layer									Tack of First Ink Layer								
		Low			Medium			High			Low			Medium			High			Low			Medium			High		
		1st	2nd	3rd	1st	2nd	3rd	1st	2nd	3rd	1st	2nd	3rd	1st	2nd	3rd	1st	2nd	3rd	1st	2nd	3rd	1st	2nd	3rd	1st	2nd	3rd
High	Grav.																											
	Appar.																											
	Anti																											
	Pola																											
Medium	Grav																											
	Appar.																											
	Anti.																											
	Pola.																											
Low	Grav																											
	Appar																											
	Anti																											
	Pola.																											

Table 3 Experimental Design

The major printed section, $A_{2/1}$, was the one where the overprinting occurred. The minor area, A_2, consisted of a small section of the second ink printed directly on the paper. A specific amount of each ink was applied on an inking unit by means of an ink pipette. Six minutes was allocated for each inking up to achieve good ink distribution. Each printing disc was inked with a different color and was allowed one and a half minutes for inking up. At this stage each disc (with the tape removed) was weighed for a second time. This method provided an accurate determination of the amount of ink on each disc before transfer.

After the two discs were mounted on the IGT tester the ink was transferred to a two-inch by ten-inch strip of test paper. Then each disc was weighed for a third time. The amount of ink lost from the first disc represented the weight of the first ink printed on the test paper. The weight lost from the second disc represented the amount of the second ink trapped by the ink layer of the first disc.

Percent ink trapping was calculated by dividing the ink film thickness of the second ink printed on the first ink layer by the ink film thickness of the second ink printed directly on the paper. The amount of the second ink trapped on the first ink was calculated by the total ink weight lost from the second disc after ink transfer. The amount of the second ink printed directly on the paper was estimated by an ink transfer graph. The graph shows the relationship between the amount of ink on the printing disc and the amount of ink transferred to the paper (see Figures 3A-C). The data and results of the gravimetric trapping are shown in Appendices A1 to A3.

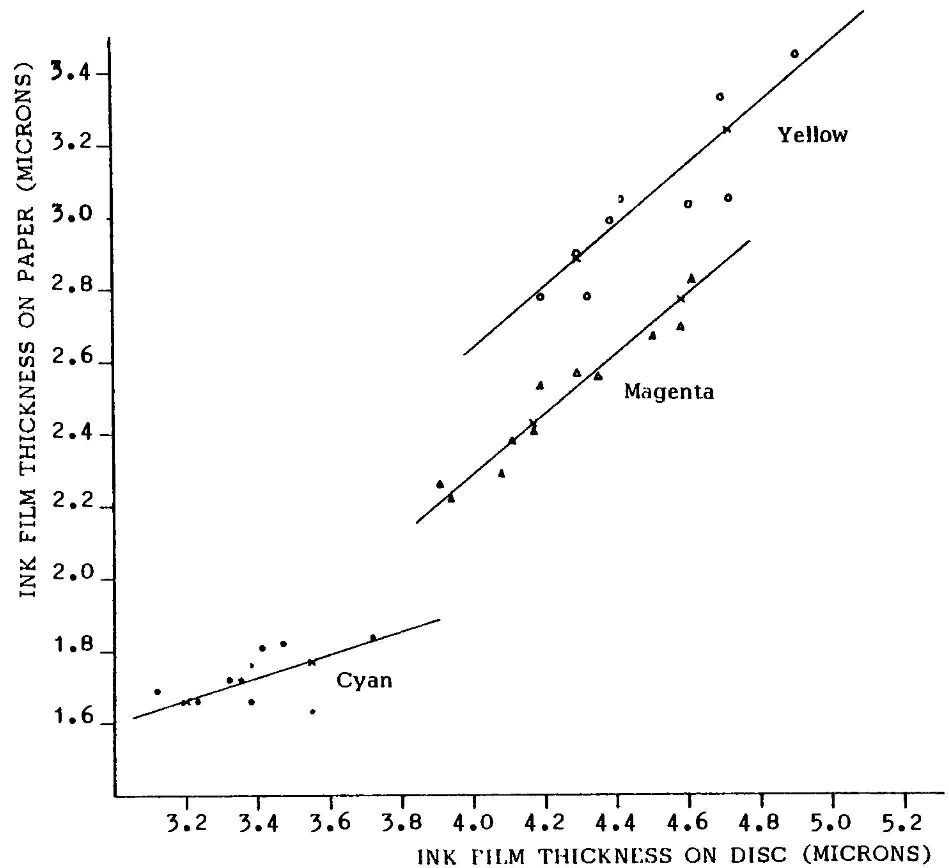

Figure 3A. Ink transfer from the printing disc to newsprint

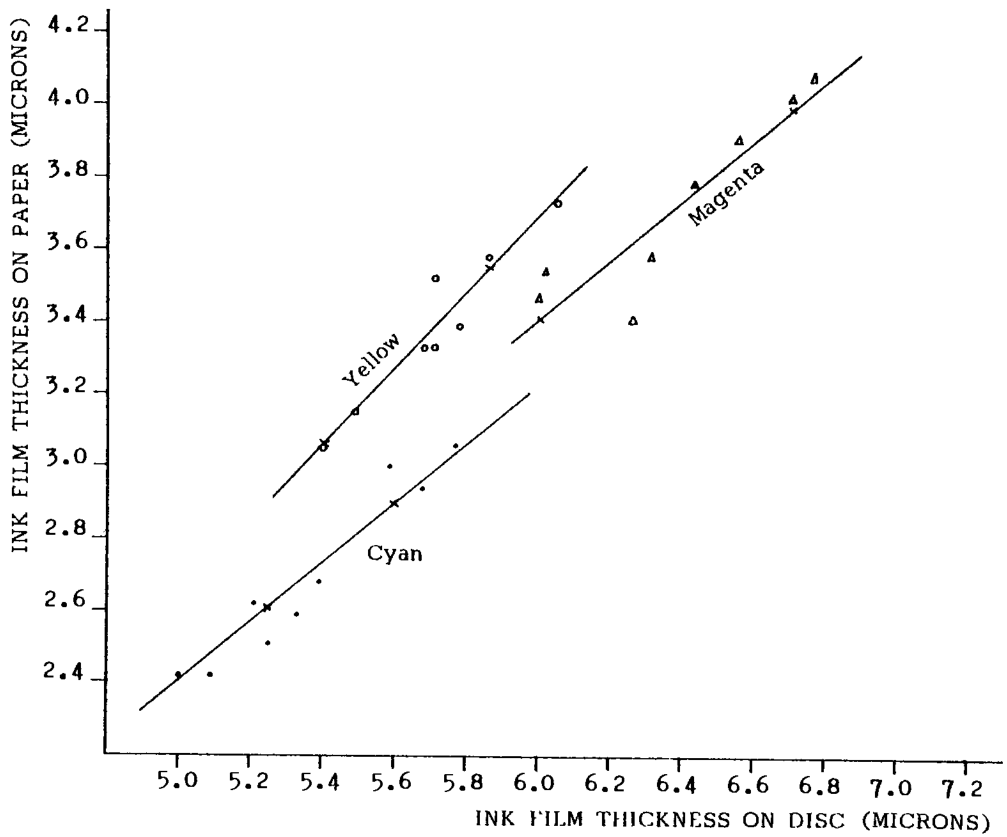

Figure 3B. Ink transfer from the printing disc to uncoated paper

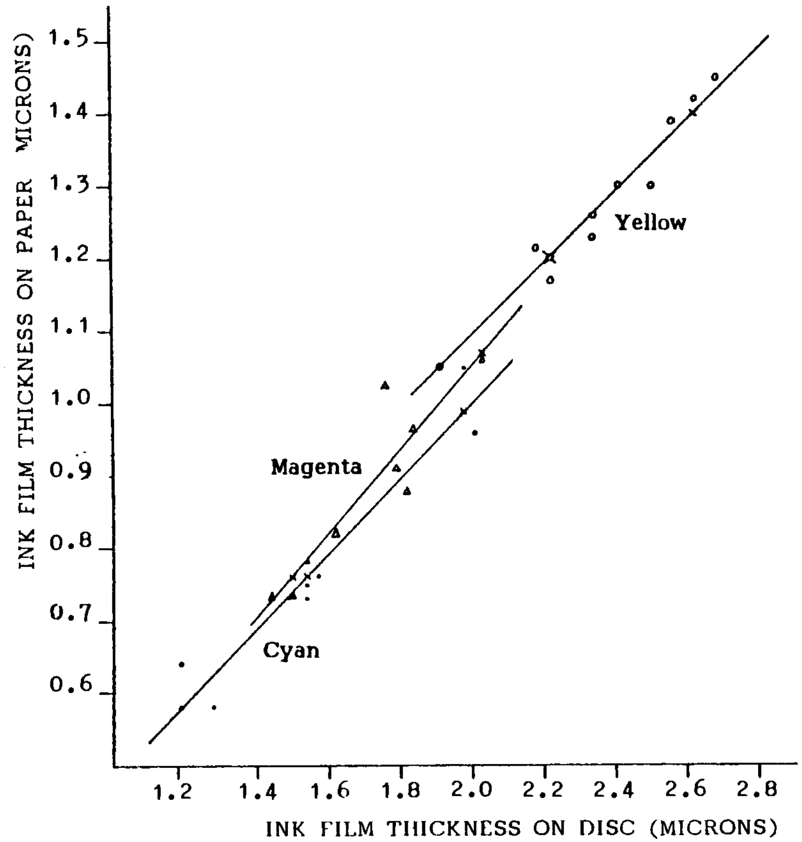

Figure 3C. Ink transfer from the printing disc to coated paper

Sample Calculation of Gravimetric Trapping

The following calculation sample was from the experimental number 3 from Appendix A1. In this case cyan ink trapped yellow ink on newsprint.

Calculation of the cyan ink film thickness on the first disc was:

before the ink transfer,
the disc weight = 84.4600 gm.
the weight of disc and cyan ink = 84.4713 gm.
the weight of ink on the disc = 84.4713 - 84.4600
= 0.0113 gm.

After the ink transfer,
the weight of the disc and ink = 84.4654 gm.
the weight of ink transferred to paper = 84.4713 - 84.4654
= 0.0059 gm.

The weight of ink on paper was converted to its ink film thickness.
The printed area of the cyan ink = 16.6 x 2
= 33.2 cm^2
The physical density of the cyan ink = 1.033975 gm/cm^3

The ink film thickness = weight ÷ (density x area)
= 0.0059 ÷ (1.033975 x 33.2)
= 0.000171871 cm.
= 1.72 microns

Calculation of the yellow ink film thickness on the second disc was:
before the ink transfer,
the disc weight = 85.5884 gm.
the weight of the disc and yellow ink = 85.6044 gm.
the weight of ink on the disc = 85.6044-85.5884
= 0.016 gm.

The physical density of yellow ink = 0.9765 gm/cm^3
The area of ink on the disc = 17.9 x 2
= 35.8 cm^2
The ink film thickness of yellow ink on disc = 0.0160 ÷ [(0.9765)(35.8)]
= 4.58 microns

After the ink transfer,
the weight of the disc and ink = 85.5936 gm.
the weight of ink transferred = 85.6044 - 85.5936
= 0.0108 gm.

A small part of the second ink was transferred directly to the paper. This is the area "A_2" in Figure 2. The amount of ink of this area was subtracted from the total amount of ink transferred from the disc to improve the accuracy of the percent gravimetric trapping determination. It was assumed that the ink split 50 percent when it transferred to the paper. The weight of ink transferred directly to the paper was half of the total weight of ink on the disc multiplied by a ratio of the area of ink on the paper and total area of ink on the disc.

The weight of yellow ink on paper = ½(0.0160)(2.45/35.8)
= 0.0006 gm.

The actual weight of yellow ink trapped on the cyan ink
= 0.0108 -0.0006
= 0.0102 gm.

The physical density of yellow ink = 0.9765 gm/cm^3
The area of ink transferred = 33.2 cm^2
The ink film thickness of yellow ink = 0.0102/[(0.9765)(33.2)]
= 0.003146225 cm.
= 3.15 microns.

The ink film thickness of the yellow ink transferred directly to the newsprint was determined from the graph in Figure 3A. When the ink film thickness of yellow ink on the disc was 4.58 microns, the ink film thickness transferred on newsprint was 3.12 microns.

The percent of gravimetric trapping = (3.15/3.12)100 = 100.96%.

Measurement of Densitometric Trapping

After determining the gravimetric trapping, densities of ink printed by the IGT were measured for the densitometric trapping determination. The densities were measured 24 hours after the ink transfer.

Three different techniques of densitometric trapping were included for this study: apparent trapping, polarizing apparent trapping, and antilogarithm trapping. For apparent trapping, reflection densities of inks were measured by a non-polarizing "MACBETH RD-514" reflection densitometer. A polarizing densitometer, GRETAG D142, measured the densities for the determination of polarizing apparent trapping. The density data of the apparent trapping was also used to determine the antilogarithm trapping. But the formula of the antilogarithm trapping was different from both apparent and polarizing apparent trapping.

The procedure of measuring the densities was the same for both densitometers. First, the densitometer had to be calibrated following the manufacturer's instructions. Second, it had to be set to read zero on the paper surface for each filter. Third, a proper filter was selected to read the highest density of the overlapping area (area A1 in Figure 2). Then densities of five locations of the overlapping area were measured. The average of these densities represented the density of the area. The same filter was also used to measure the density of the second ink which was transferred directly on the paper surface (area A2 in Figure 2).

The density of the first ink cannot be measured because the entire area was overprinted by the second ink. To estimate the density of the first ink, graphs of the ink film thickness and density had been plotted. The graphs showing ink density with individual filter against the ink film thickness were plotted for each type of paper. The graphs are found in Appendix B.

To estimate the first ink density, its ink film thickness which was transferred to the paper must be known. The ink film thickness had already been calculated when calculating the gravimetric trapping. They are shown in Appendix A. Then its density using the same filter as the overlapping area was determined from the graph.

In Appendix B, each set of graphs shows the relationship between density and ink film thickness for each combination of ink, filter, and paper. Also they show the difference between densities, at the same ink film thickness, measured with non-polarizing and polarizing densitometers. The lines representing the relationship were estimated by regression analysis to get the equations that represented the best relationship.

From the three densities, 1) the overlapping density, 2) the first ink density, and 3) the second ink density, the percent trapping was calculated by the formula for each technique. The details of the calculation and formula of each technique were shown in samples of the calculation. The summary of the densities and percent trapping of the apparent and antilogarithm trapping are shown in Appendix C. Polarizing apparent trapping is shown in Appendix D.

<u>Sample Calculation of Apparent Trapping</u>.

The formula for the apparent trapping was as follows:

$$\text{the percent apparent trapping} = 100[(D_{2/1} - D_1)/D_2]$$

where

$D_{2/1}$ is the density of the overlapping area.
D_1 is the density of the first ink printed directly on paper.
D_2 is the density of the second ink printed directly on paper

The following sample is taken from Appendix C1, number 3. In this case cyan ink trapped yellow ink on newsprint. The data was as follows:

$$D_{2/1} = 0.84; \qquad D_1 = 0.15; \qquad \text{and } D_2 = 0.78.$$

A blue filter was used to measure the densities. The density "D_1" was estimated from the cyan ink film thickness which was equal to 1.72 microns from experiment number 3 of newsprint found in Appendix A1. From the graph of cyan printed on newsprint in Appendix B, the density was equal to 0.75 when it was measured by the non-polarizing densitometer with the blue filter. The "D_1" was 0.15.

$$\text{The percent apparent trapping} = 100[(0.84 - 0.15)/0.78]$$
$$= 88.46\ \%$$

<u>Sample Calculation of Antilogarithm Trapping</u>

The density data for the apparent trapping were required for determininination of the percent antilogarithm trapping. A different formula was used to determine the antilogarithm trapping. The antilogarithm formula was:

$$\text{the percent antilogarithm trapping} = 100[10^{(D_{2/1} - D_1 - D_2)}]$$

To illustrate calculation of antilogarithm trapping the data shown by the apparent trapping is used once again.

$$D_{2/1} = 0.84; \qquad D_1 = 0.15; \qquad \text{and } D_2 = 0.78.$$

The percent antilogarithm trapping $= 100[10^{(0\ 84\ -\ 0\ 15\ -\ 0\ 78)}]$

$= 81.28\%$

Sample Calculation of Polarizing Apparent Trapping

The formula for the polarizing apparent trapping was the same as for apparent trapping. The polarizing densitometer was, however, required for measurment of the densities for polarizing apparent trapping. The formula was:

$$\text{the percent trapping} = 100[(D_{2/1} - D_1)/D_2]$$

The sample calculation again used experiment number 3 shown in Appendix D1. The data for the experiment (yellow trapping on cyan ink on newsprint) is the following:

$$D_{2/1} = 1.25; \qquad D_1 = 0.16; \qquad \text{and } D_2 = 1.21.$$

A blue filter was used to measure the densities.

The Density "D_1" was estimated from the cyan ink film thickness which was equal to 1.72 microns from experiment number 3 using newsprint (see Appendix A1). Then the ink film thickness was converted to a density using the graph of cyan printed on newsprint (see Appendix B). The polarizing density of D_1 was equal to 0.16.

The percent polarizing apparent trapping

$= 100\ [(1.25 - 0.16)/1.21]$

$= 90.08\%$

IV. RESULTS

Effects of Experimental Factors on Ink Trapping

Table 4 is a summary of the experimental results. Each value represented the percent trapping for both ink tacks, each type of paper, and each measurement method. The table shows the percent trapping difference for each combination of factors. In addition, the percent trapping for the three replications of each factor combination was different, because of experimental error.

To analyze whether the result differences were caused by the influence of the factors or by experimental error, statistical analysis was applied. The approach applied is called "Analysis of Variance (ANOVA)".[1] The ANOVA was done on an XDS SIGMA 9 computer at the Rochester Institute of Technology. The program was called "ANOVA" and resided in the LMNLIB2 account. Appendix F contains a program listing of the ANOVA. The ANOVA (Table 5A) showed that all of the factors, including the tack of both ink layers, the types of paper, and the measurement methods, as well as their interactions, were significant and influenced the percent ink trapping. The ANOVA tested these factors with 99.0 percent of confidence. The interaction of the ink tack of both layers was one of these significant interactions. This interaction explained why percent trapping was different when the tack of both ink layers were changed.

NEWSPRINT — Yel den = .75, Mag den.=1.0, Cy. den.=.95

Tack of second ink layer	Measurement methods	Tack of first ink layer: Low 1st	Low 2nd	Low 3rd	Medium 1st	Medium 2nd	Medium 3rd	High 1st	High 2nd	High 3rd
High	Grav	86.81	83.13	84.73	91.47	98.99	93.67	100.96	97.95	105.70
	Appar	13.16	13.92	17.51	38.09	45.45	35.06	88.46	84.84	87.34
	Anti	21.88	20.89	23.17	36.31	38.02	31.62	81.28	79.43	79.43
	Pola	15.25	14.57	23.73	50.05	55.74	48.60	90.08	91.67	94.34
Medium	Grav	55.73	54.55	54.47	75.49	77.14	84.10	94.24	94.84	98.91
	Appar	72.82	75.24	71.15	16.11	13.37	15.05	59.58	60.40	59.62
	Anti	52.48	55.59	50.12	14.45	13.10	13.34	38.02	39.81	38.02
	Pola	67.23	68.55	63.33	32.74	29.41	28.93	71.38	73.73	75.00
Low	Grav	14.11	27.33	34.94	56.80	51.53	53.57	75.74	80.24	78.53
	Appar.	65.17	64.00	64.36	60.78	64.36	59.22	18.63	22.11	20.41
	Anti	48.98	43.65	51.08	39.81	46.24	30.02	14.79	18.20	16.68
	Pola	66.67	62.70	69.17	73.60	73.13	68.70	38.28	34.39	38.52

UNCOATED PAPER — Yel den.=.80, Mag. den =1.05, Cy den =1.1

Tack of second ink layer	Measurement methods	Tack of first ink layer: Low 1st	Low 2nd	Low 3rd	Medium 1st	Medium 2nd	Medium 3rd	High 1st	High 2nd	High 3rd
High	Grav	103.14	101.68	104.41	114.77	111.37	117.22	117.70	115.63	113.98
	Appar	20.25	19.75	17.72	35.06	31.54	37.82	88.46	82.93	82.93
	Anti	23.47	22.19	23.39	31.62	29.85	32.13	81.28	72.44	72.44
	Pola	22.07	28.45	26.75	45.28	46.02	50.0	90.65	82.5	85.22
Medium	Grav	81.81	77.42	70.52	101.10	97.61	95.72	105.23	102.94	105.48
	Appar	81.48	82.24	78.85	14.62	16.82	14.42	50.88	54.81	56.19
	Anti	63.10	64.53	60.26	12.45	12.88	12.79	36.31	33.88	34.67
	Pola	74.36	72.27	71.93	35.98	34.87	38.02	69.33	71.05	60.97
Low	Grav	51.69	56.41	55.43	79.62	67.21	79.23	99.18	97.46	93.80
	Appar.	75.00	80.61	76.53	67.28	67.02	66.02	25.23	20.67	23.0
	Anti	55.59	64.57	50.88	42.17	48.90	44.67	15.85	14.96	16.98
	Pola	77.27	83.19	77.12	80.47	81.20	84.02	40.16	37.60	42.15

COATED PAPER — Yel den =1.0, Mag den.=1.30, Cy den.=1.25

Tack of second ink layer	Measurement methods	Tack of first ink layer: Low 1st	Low 2nd	Low 3rd	Medium 1st	Medium 2nd	Medium 3rd	High 1st	High 2nd	High 3rd
High	Grav	48.97	35.66	41.64	93.09	87.02	89.76	102.16	[illegible]	112.80
	Appar	16.19	10.73	12.50	88.24	87.5	89.5	96.16	95.96	95.54
	Anti	13.18	12.02	13.34	75.86	74.11	78.52	91.20	91.20	90.16
	Pola	18.60	12.42	15.38	73.42	71.41	72.61	96.18	96.75	94.16
Medium	Grav	-37.43	-37.04	-50.56	46.56	32.58	45.19	66.29	75.25	77.16
	Appar	4.88	6.28	3.94	18.83	24.22	23.78	76.38	87.88	71.90
	Anti	6.76	6.17	6.03	8.51	10.72	9.33	50.12	69.18	44.16
	Pola	4.17	4.84	4.07	18.42	32.52	38.06	68.44	79.69	65.19
Low	Grav	-59.86	-42.29	-43.79	11.24	26.29	25.81	42.66	33.72	46.69
	Appar.	1.74	2.24	3.50	32.41	31.09	31.86	17.69	41.22	[illegible]
	Anti	7.41	4.90	4.17	18.62	11.75	14.45	20.89	16.98	12.62
	Pola	0.81	1.41	3.87	33.48	32.55	33.51	47.10	[illegible]	36.88

Table 4 Summary of Experimental Results

TABLE 5A

Analysis of Variance

Factors	Levels of Factors
A (Types of Paper)	3
B (Tack of first ink layer)	3
C (Tack of second ink layer)	3
D (Measurement methods)	4
R (Replication)	3

Source of Variation	Sum of Squares	Degree of Freedom	Mean Square	F-ratio	F(0.01, α, 216)	Significant
A	32094.55078	2	16047.27344	998.03	4.6052	***
B	45054.82813	2	22527.41406	1401.05	4.6052	***
AB	42785.10156	4	10696.27344	665.23	3.3192	***
C	26721.64453	2	13360.82031	830.95	4.6052	***
AC	23758.91797	4	5939.72656	369.41	3.3192	***
BC	43948.96484	4	10981.23828	683.33	3.3192	***
ABC	8574.46484	8	1071.80811	66.66	2.5113	***
D	39749.62891	3	13249.87500	824.05	3.7816	***
AD	22491.96484	6	3748.66064	233.14	2.8020	***
BD	13741.85938	6	2290.30981	142.44	2.8020	***
ABD	875.94434	12	72.99536	4.54	2.1848	***
CD	16486.60938	6	2747.76807	170.89	2.8020	***
ACD	630.82568	12	52.56840	3.27	2.1848	***
BCD	32162.91797	12	2680.24316	166.67	2.1848	***
ABCD	3993.58569	24	166.39940	10.35	1.7908	***
ERROR	3473.06061	216	16.07898	---	---	---
Total	356543.93750	323				

TABLE 5B

The Ranked List of Significant Experimental Factors

Rank	Factor
1	B (Tack of first ink layer)
2	A (Types of paper)
3	C (Tack of second ink layer)
4	D (Measurement methods)
5	BC (Interaction between tack of both ink layers)
6	AB (Interaction between paper and first ink tack)
7	AC (Interaction between paper and second ink tack)
8	AD (Interaction between paper and measurement methods)
9	CD (Interaction between tack of second ink layer and measurement methods)
10	BCD (Interaction between tack of both ink layers and measurement methods)
11	BD (Interaction between tack of first ink layer and measurement methods)
12	ABC (Interaction between paper, tack of the first ink layer, and tack of second ink layer)
13	ABCD (Interaction between paper, both ink tacks, and measurement methods)
14	ABD (Interaction between paper, tack of first ink layer, and measurement methods)

Table 5B contains a ranked list[2] of the factors and their interactions. Factors at the top of the list were the most significant, while those at the bottom were least significant. The first four significant factors were: the tack of the first ink layer, the types of paper, the tack of the second layer, and the measurement methods. Among the interactions, the tack of both ink layers influenced the percent trapping the most. This interaction agreed with the industrial practice of changing the printing sequence in order to improve ink trapping.

The Effects of Individual Factors on Ink Trapping

The multiple range test[3] is a statistical technique which studies levels of an individual factor. It determines whether there is a significant difference of percent trapping among levels of the significant factors. If there is no significant difference between tested levels these levels of the factor are grouped into the same level. This test reduces the number of factor levels which allows concentrated study of the important levels of significant factors. The multiple range test compares differences between the difference of statistical means of factor levels with the statistic, which is the Significant Studentized Range (SSR). If the difference of the means is less than the SSR value, a line is drawn under the two means to signify that they are not significantly different. If the difference exceeds the value of SSR, no line is drawn.

Table 6A shows the multiple range test of the first ink layer tack with 95 percent confidence. No line was drawn under the two mean or three mean comparison of tack levels. The conclusion was that signifi-

TABLE 6A

Multiple Range Test of First Ink Layer Tack

The formula of the multiple range test is:

$$LSD_{0.05} = \sqrt{2}\ t_{0.025},\ \nu \sqrt{\frac{S_e^2}{n}}$$

where

$\sqrt{2}\ t_{0.025}$ = the significant studentized range.
ν = the degrees of freedom associated with the Mean Square For Error from the ANOVA table.
S_e^2 = the Mean Square For Error in the ANOVA table.
n = the number of observations involved in each treatment mean.

The data for the first ink layer tack was:

$n = 108$, $S_e^2 = 16.07898$, $\sqrt{\frac{S_e^2}{n}} = 0.386$

The percent trapping mean of the low tack ($\bar{x}_l$) = 39.12%
The percent trapping mean of the medium tack ($\bar{x}_m$) = 49.42%
The percent trapping mean of the high tack ($\bar{x}_h$) = 66.25%

g	2	3
SSR	1.07	1.28

where

g = the number of means in a group to be compared.
SSR = the significant studentized range.

The means were arranged in order according to size:

$\bar{x}_l$	$\bar{x}_m$	$\bar{x}_h$
39.12	49.42	66.25

If the difference of the means was less than the SSR value, a line was drawn under the two means or three means to signify that they were not significantly different. If the difference exceeded the SSR value, no line was drawn.

Conclusion: there was significant difference between each level of the first ink tack because no line was drawn.

TABLE 6B

Multiple Range Test of Second Ink Layer Tack

The data was:

$n = 108$, $S_e^2 = 16.07898$, $\sqrt{\frac{S_e^2}{n}} = 0.386$.

The percent trapping mean of the low tack ($\bar{x}_l$) = 41.88%
The percent trapping mean of the medium tack ($\bar{x}_m$) = 47.88%
The percent trapping mean of the high tack ($\bar{x}_h$) = 63.32%

g	2	3
SSR	1.07	1.28

The result of the comparison was as follows:

$\bar{x}_l$	$\bar{x}_m$	$\bar{x}_h$
41.88	47.88	63.32

Conclusion: there was significant difference between each tack level of the second ink because no line was drawn under the means of the tack.

TABLE 6C

Multiple Range Test of three Paper Types

The data was:

$n = 108$, $S_e^2 = 16.07898$, $\sqrt{\frac{S_e^2}{n}} = 0.386$

The percent trapping mean of the coated paper ($\bar{x}_C$) = 37.60%
The percent trapping mean of the uncoated paper ($\bar{x}_{UC}$) = 61.13%
The percent trapping mean of the newsprint ($\bar{x}_{NP}$) = 54.03%

g	2	3
SSR	1.07	1.28

The result of the comparison was as follows:

$\bar{x}_C$	$\bar{x}_{NP}$	$\bar{x}_{UC}$
37.60	54.03	61.13

Conclusion: there was significant difference between coated paper, uncoated paper, and newsprint because no line was drawn under the means of the paper types.

TABLE 6D

Multiple Range Test of Measurement Methods

The data was:

$n = 81, \quad S_e^2 = 16.07898, \quad \sqrt{\frac{s_e^2}{n}} = 0.7717$

The percent trapping mean of the gravimetric trapping ($\bar{x}_G$) = 67.54%
The percent trapping mean of the apparent trapping ($\bar{x}_A$) = 47.14%
The percent trapping mean of the antilogarithm trapping ($\bar{x}_L$) = 37.52%
The percent trapping mean of the polarizing apparent trapping ($\bar{x}_p$) = 53.32%

g	2	3	4
SSR	2.14	2.55	2.80

The result of the comparison was as follows:

$\bar{x}_L$	$\bar{x}_A$	$\bar{x}_p$	$\bar{x}_G$
37.52	47.14	53.32	67.54

Conclusion: there was significant difference between each measurement method because no line was drawn under the mean of trapping of different measurement methods.

cant differences between low, medium, and high tack of the first ink layer existed. Table 6B shows the multiple range test of the second ink layer tack. There was a significant difference between each level of the second ink layer tack. Consequently, the three levels could not be grouped. The test for the types of paper is shown in Table 6C. There was significant difference between coated paper, uncoated paper, and newsprint. The results presented in Table 6D confirmed that there were significant differences between measurement of gravimetric, apparent, polarizing apparent, and antilogarithm trapping.

Graphical Analysis of the Individual Effect of Experimental Factors

Graphical Analysis[4] is a statistical technique used to describe the pattern of trapping variability caused by each experimental factor. The graphs do not suggest functional relationships between the percent trapping and the experimental factors. The average percent trapping was on the vertical axis while the factors being studied were on the horizontal axis.

The graphical analysis of the tack of the first ink layer and the measurement methods is shown in Figure 5. A comparison of tack levels indicated that a high tack of the first ink gave a higher percent trapping, while a low tack gave a low percent trapping. The graph also showed difference between the gravimetric and densitometric measuring methods. Using the gravimetric method, the graph showed a rapid increase in average percent trapping when the tack changed from low to medium or high. The apparent, polarizing apparent, and antilogarithm trapping all result in similar shaped graphs. The average percent

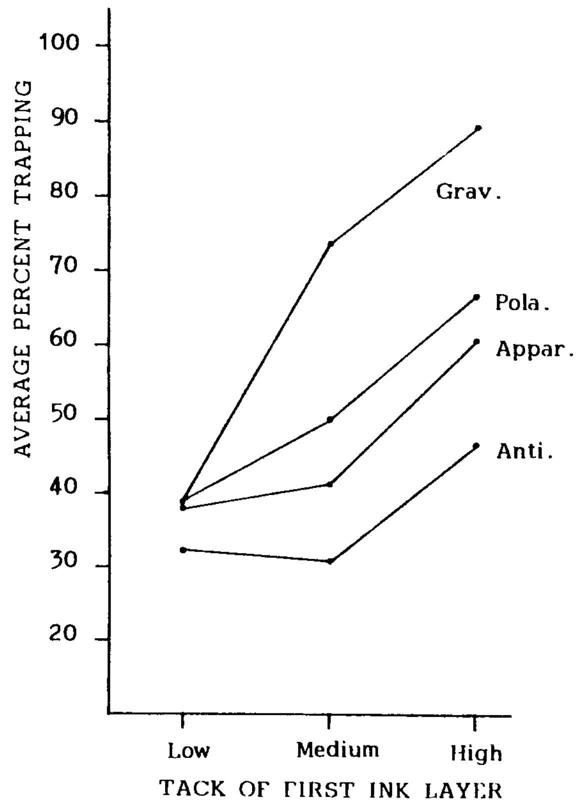

Figure 5.

Graphical analysis of the first layer ink tack and measurement methods

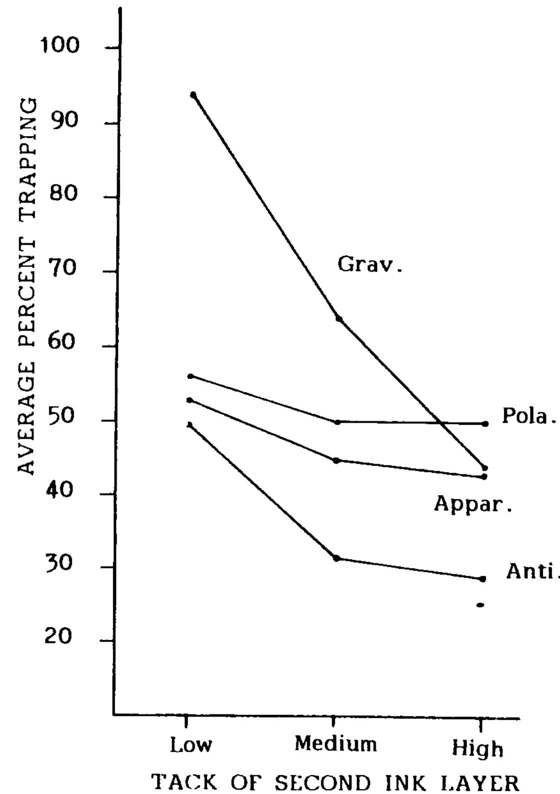

Figure 6.

Graphical analysis of the second layer ink tack and measurement methods

trapping for these methods did not increase much when the tack changed from low to medium, but they did change significantly at high tack level.

The graphical analysis of the tack of the second ink layer and the measurement methods is shown in Figure 6. Using the gravimetric method, the tack levels influenced the percent trapping significantly. Average percent trapping increased while the second ink tack decreased. However, densitometric trapping results were not significantly influenced by the second ink layer tack: not much difference in percent trapping occurred between the medium and high tack. However, the three densitometric measuring methods showed similar lines. The general conclusion may state that the gravimetric method measured higher percent trapping than the densitometric methods. A comparison of the three densitometric methods suggested that polarizing apparent trapping measured higher percent trapping than either the apparent or antilogarithm measuring methods, with the antilogarithm formula giving the lowest percent trapping.

The graphical analysis of paper types with the measurement methods is shown in Figure 7. Uncoated paper showed the best percent trapping while coated paper showed the least. Thus, certain paper properties caused the difference in the amount of percent trapping.

The paper surface roughness and oil absorption might have explained why there was a greater percent trapping on the uncoated paper and newsprint compared to the coated paper. Since the ink could be caught more easily between the microscopic valleys located on the paper surface, thus the rougher the paper surface, the more val-

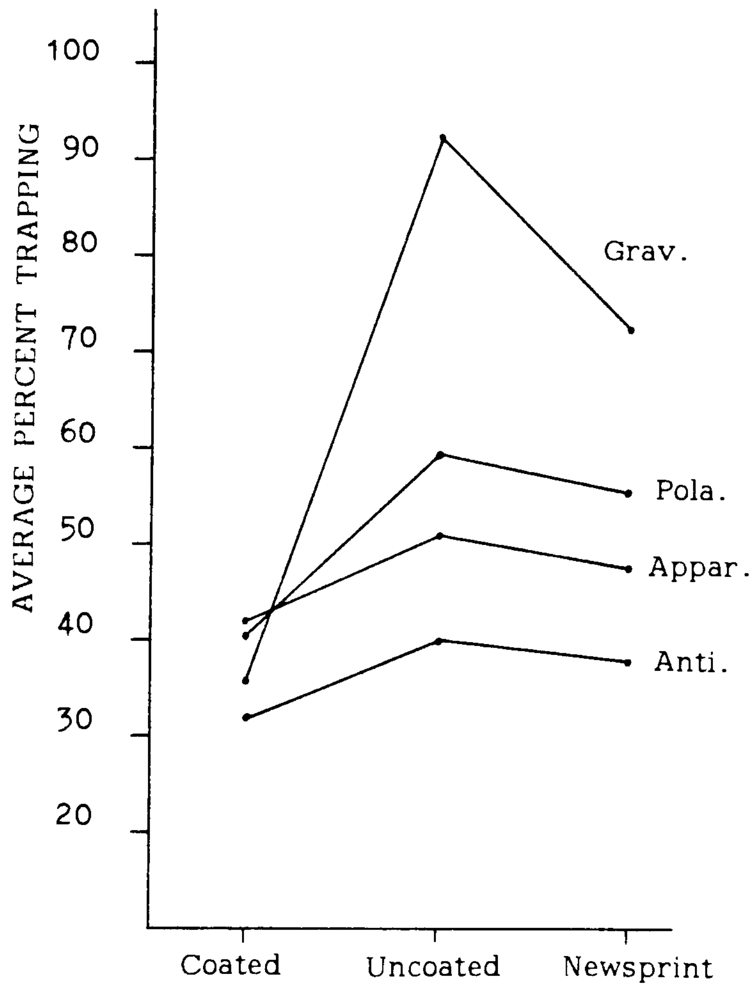

Figure 7.
Graphical analysis of types of paper and measurement methods

leys they have. In addition, the greater the oil absorption property the more ink will be absorbed into the paper surface. As indicated by Table 2 these characteristics were measured by an IGT technique. The surface roughness of uncoated paper, newsprint, and coated paper was 9.95, 7.08, and 0.82 cm^3/m^2, respectively, with higher numbers indicating a rougher paper surface. The oil absorption of uncoated paper, newsprint, and coated paper was 37.38, 27.36, and 8.86 mm^{-1}, respectively, where higher numbers indicating greater absorbency. Uncoated paper had the highest oil absorbency and surface roughness, while coated paper absorbed the least amount of oil and was the smoothest.

The tack of both ink layers was one of the major factors which caused percent trapping variation. The graphical analysis of the tack values and the measurement methods is shown in Figure 8. The graphs of gravimetric and densitometric measuring methods were different. For both methods, the best percent trapping occurred when the tack of the first ink layers was high while the tack of the second ink layer was low. However, using the gravimetric method poor trapping resulted when the first ink layer tack was low and the second ink layer tack was high. When the tack of both layers was the same, the percent trapping ranged between 70 percent and 80 percent. Using the gravimetric response this is a relatively high level of trap. The response of apparent trapping was different when the tacks of both ink layers were the same. These showed low percent trapping. The reason may have been the additivity failure of ink density. The ink transfer curve describing the relationship between ink density and ink film thickness

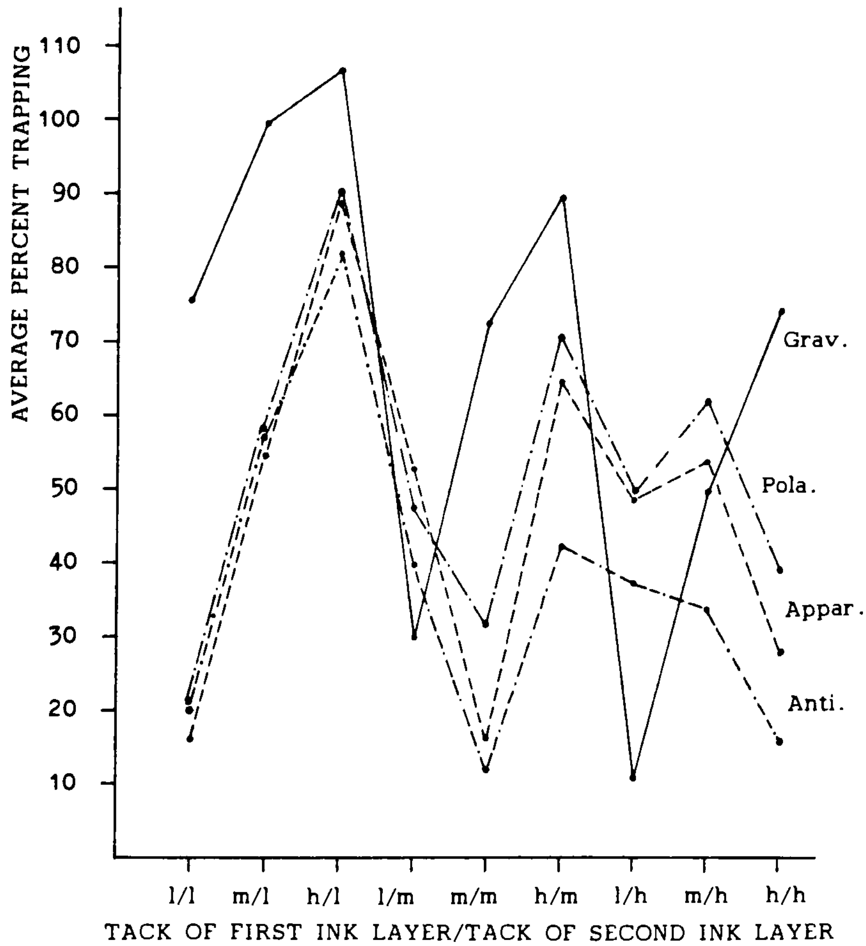

Figure 8. Graphical analysis of the ink tack combination and measurement methods

showed that ink density would not change after it had reached its maximum density. The density would remain the same even when the ink film thickness was increased. In this experiment both ink layers were the same color when the tack of both ink layers were the same. As a result, their combined density, which was the maximum, meant the subsequent increases in ink film thickness affected the density readings very slightly. Because of this, the percent densitometric trapping was low.

Figure 9 shows the graphical analysis of the tack of both ink layers, three types of paper, and gravimetric and apparent trapping measurement methods. The graphs of the three papers showed similarities between them when the gravimetric method was applied. On the coated paper back trapping occurred when the tack of the first ink was low and the second ink tack was either medium or high. The graph of apparent trapping on coated paper was different compared to the uncoated paper and newsprint graphs. They responded oppositely when both ink layers had the same tack. When the low tack ink trapped over medium tack ink, the average apparent trapping was 87 percent on the coated paper. However, the average percent trapping for this combination using apparent trapping was 35 and 40 percent for the uncoated paper and newsprint, respectively. When medium tack ink trapped over low tack ink, the coated paper showed low percent trapping, while the uncoated paper and newsprint showed high average percentages. Similar contradicting results occurred again when high tack ink was printed over low tack ink. These contradicting results showed the failure of the densitometric methods.

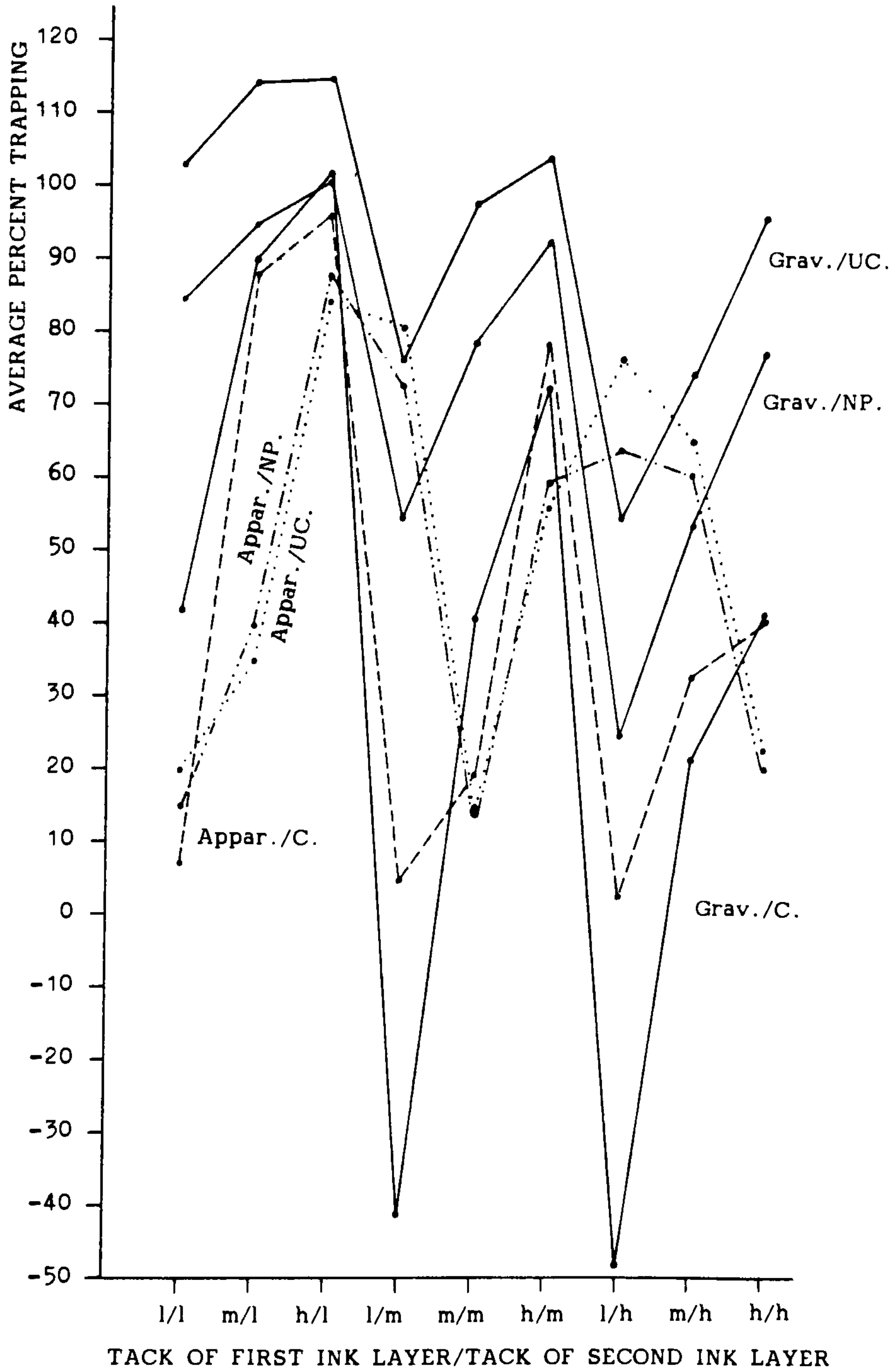

Figure 9. Graphical analysis between ink tack combination, types of paper, and measurement methods

Gravimetric and apparent trapping had similar graphs when coated paper was used. Although apparent trapping cannot measure back trapping, there could have been a relationship between the gravimetric and apparent trapping percentage on coated paper.

Discussion of the Effects of the Experimental Factors

From the Analysis of Variance (ANOVA), the multiple range test, and the graphical analysis, the author concluded that there was a significant difference between the gravimetric and three densitometric trapping methods. Comparison of the three densitometric methods suggests that polarizing apparent trapping resulted in higher percent trapping response than the response of apparent and antilogarithm methods. Among these densitometric methods the antilogarithm formula provides the lowest percent trapping response. Graphs of the response of gravimetric trapping and three densitometric trapping methods (see Figure 8) were different. The author concluded from this difference that the polarizing apparent and antilogarithm measurements were not as effective as the gravimetric method in measuring trapping.

Tack of both ink layers proved to be one of the major factors affecting percent trapping. Results of this study showed that, in order to achieve high trapping, the first ink tack should be as high as possible while the tack of the overprinted ink should be as low as possible. This study therefore suggested that proper ink tack in the printing units of a multicolor press would be critical in achieving good trapping. Of the three types of paper, uncoated stock gave the high-

est percent trapping. Coated paper showed back trapping, which did not occur with uncoated paper or newsprint. This suggests that paper characteristics such as surface roughness and oil absorption need to be studied in more detail to determine how these properties affect percent trapping.

Relationship of Gravimetric and Apparent Trapping Percentage

The second research question investigated was whether there was any relationship between percent gravimetric and percent apparent trapping. This experiment which was described more fully in Chapter 3 considered the tack of both the first and second layers of ink, three different paper substrates, and four measurement methods. The author analyzed the relationship between the gravimetric and apparent trapping percentage using the XDS SIGMA 9 Computer at the Rochester Institute of Technology and a program entitled, "Statistical Package for the Social Sciences (SPSS)". This software performed stepwise regression analysis.[1] Appendix G is a listing of this program.

The computer calculated the regression coefficients of independent variables x, x^2, and x^3. These variables were used in the equation models (see Chapter 2, page 21) testing the second hypothesis, which sought to discover a relationship between gravimetric and apparent trapping. The programmer had to state statistical inclusion criteria at the start of the stepwise regression. The inclusion criteria were: the maximum number (n) of variables included in the equation; the statistical F-ratio which was the test for significance of regression coefficients; and the tolerance (T) of independent variables. If the tolerance select-

ed is 0.001 then the independent variable explained at least 0.1 percent variation in the relationship between gravimetric trapping and apparent trapping. The values given to the inclusion criteria for this study were: n (maximum number of variables in the equation) = 80; F-ratio = 0.01; and T (tolerance) = 0.001. Independent variables were included in the final regression equations if they met these criteria.

The author used the R^2 and standard error statistics in judging how well the regression equation fit the data in order to determine the accuracy of the equation. The R^2 statistic indicated the proportion of variation explained and unexplained, respectively, and the standard error was the standard deviation of the experimental values from the predicted values. The ideal case of the best-fit equation would occur when R^2 was equal to 1.0 and the standard error was equal to zero. In this ideal case, the regression equation would estimate a prediction value which would be the same as the experimental value. The author stated criteria for accepting or rejecting the validity of the regression equations describing the relationship between percent gravimetric and apparent trapping. These criteria stated that the R^2 must be more than 0.85 and the standard error must be less than 10.0. An acceptable equation would therefore explain at least 85 percent of the variation in the relationship between the two trapping methods while the average difference between the calculated values and the experimental values would be less than 10 percent.

The computer results of the regression equations for three types of paper are shown in Table 7A. The equation estimated the percent gravimetric trapping (y) using percent apparent trapping (x) for the three papers used in the experiment. The best-fit equation was the

TABLE 7A

Regression Equations of Relationship Between Gravimetric and Apparent Trapping for Newsprint, Uncoated Paper and Coated Paper

Inclusion criteria: n (maximum number of variables) = 80

F-ratio = 0.01

T (tolerance) = 0.001

Notation: y is the percent gravimetric trapping

x is the percent apparent trapping.

The first order equation is:

$$y = 41.41622 + 0.5577924(x)$$

$R^2 = 0.15740$ Standard error = 38.48977

The second order equation is:

$$y = 20.14744 + 1.961092(x) - 0.01452672(x^2)$$

$R^2 = 0.20812$ Standard error = 37.55185

The third order equation* is:

$$y = -49.92513 + 9.909369(x) - 0.2132077(x^2) + 0.001347830(x^3)$$

$R^2 = 0.51778$ Standard error = 29.49345

* the best fit equation

third-order equation, where R^2 was equal to 0.51778 and the standard error was equal to 29.49345. The best-fit equation was:

$$y = -49.92513 + 9.909369(x) - 0.2132077(x^2) + 0.001347830(x^3)$$

The author rejected this equation since R^2 was less than 0.85 and the standard error was greater than 10.0, and concluded that there was no relationship between the gravimetric and apparent trapping for these three types of paper.

Because the ANOVA indicated that paper was a significant factor, the author analyzed the data of each type of paper separately using stepwise regression. Table 7B shows the equations for newsprint. The best-fit equation for newsprint was:

$$y = 102.7176 - 1.118427(x) + 0.0001238833(x^3)$$

R^2 was equal to 0.20046 and the standard error was equal to 22.26117. The author did not accept this equation for the newsprint, since R^2 and standard error statistics did not meet the necessary criteria. Table 7C shows the regression equations for uncoated paper. The best-fit equation was:

$$y = 112.2225 - 0.495925(x) - 0.000021101754(x^3)$$

R^2 was equal to 0.16709 and the standard error was equal to 18.64816. The relationship equation between the gravimetric and apparent trap-

TABLE 7B

Regression Equations of
Relationship Between Gravimetric and
Apparent Trapping for Newsprint

Inclusion criteria: n (maximum number of variables) = 80

F-ratio = 0.01

T (tolerance) = 0.001

Notation: y is the percent gravimetric trapping

x is the percent apparent trapping.

The first order equation is:

$$y = 82.64952 - 0.1795452(x)$$

$R^2 = 0.03769$ Standard error = 23.92874

The second order equation is:

$$y = 108.0482 - 1.720939(x) + 0.01647467(x^2)$$

$R^2 = 0.15406$ Standard error = 22.89789

The third order equation* is:

$$y = 102.7176 - 1.118427(x) + 0.0001238833(x^3)$$

$R^2 = 0.20046$ Standard error = 22.26117

* the best fit equation

TABLE 7C

Regression Equations of
Relationship Between Gravimetric and
Apparent Trapping for Uncoated Paper

Inclusion criteria: n (maximum number of variables) = 80

F-ratio = 0.01

T (tolerance) = 0.001

Notation: y is the percent gravimetric trapping

x is the percent apparent trapping.

The first order equation is:

$$y = 108.2223 - 0.2924821(x)$$

$R^2 = 0.16201$ Standard error = 18.32699

The second order equation is:

The coefficient of x^2 is not significant so the equation is the same as the first order equation.

The third order equation* is:

$$y = 112.2225 - 0.4695925(x) - 0.000021101754(x^3)$$

$R^2 = 0.16709$ Standard error = 18.64816

* the best fit equation

ping for the uncoated paper was rejected because the R^2 and the standard error statistics did not meet the criteria for acceptance.

Table 7D shows the regression equation for coated paper. The R^2 and standard error of the third order equation was 0.84706 and 21.24476, respectively. The R^2 was a good approximation of the criteria value 0.85. However, the standard error was more than double the criteria value of 10.0. The equation representing the best fit for coated paper was:

$$y = -51.24160 + 6.006516(x) - 0.1183149(x^2) + 0.0007635232(x^3)$$

R^2 showed that 84.7 percent of the variables in the relationship between the gravimetric and apparent trapping were included in this equation. The calculated standard error of 21.25% was higher than the criteria value of 10.0. The author therefore concluded that there was no relationship between gravimetric and apparent trapping for the coated paper. If the regression analysis included a higher-order equation or more variables, a relationship between the percent gravimetric and percent apparent trapping might be established. However, the resulting equations would be so complex as to be unusable by industry.

Determination of Percent Gravimetric Trapping Using Densities

Because experimental data did not establish a relationship between gravimetric and apparent trapping the author attempted to predict percent gravimetric trapping in terms of densities. Four different densities were measured for this study. The first density (D_1) was

TABLE 7D

Regression Equations of
Relationship Between Gravimetric and
Apparent Trapping for Coated Paper

Inclusion criteria: n (maximum number of variables) = 80

F-ratio = 0.01

T (tolerance) = 0.001

Notation: y is the percent gravimetric trapping

x is the percent apparent trapping

The first order equation is:

$$y = -16.33267 + 1.240702(x)$$

$R^2 = 0.74314$ Standard error = 26.408

The second order equation is:

$$y = -31.00977 + 2.476105(x) - 0.01250109(x^2)$$

$R^2 = 0.77934$ Standard error = 24.98139

The third order equation* is:

$$y = -51.24160 + 6.006516(x) - 0.1183149(x^2) + 0.0007635232(x^3)$$

$R^2 = 0.84706$ Standard error = 21.24476

* the best fit equation

measured from the overlapping area of two inks. The second density (D_2) was the density of the first ink printed directly on paper. The third density (D_3) was the density of the second ink printed directly on paper. The filter which gave the highest density reading for D_1 was also used to measure D_2 and D_3. The final density (D_4) was, like D_2, the first ink printed directly on paper. However, a different filter was used, while D_1, D_2, and D_3 used the filter giving the highest density reading for D_1, D_4 used the filter giving the highest reading for just D_4.

The author used the stepwise regression analysis to determine percent gravimetric trapping using densities. As in earlier experiments, the author used the XDS SIGMA 9 computer and the program, "Statistical Package for the Social Sciences (SPSS)". Determining gravimetric trapping using densities suggested four regression equation models (which were listed in Chapter 2, page 22). In this experiment the inclusion criteria values were: n (the maximum number of variables in equation) = 10; the F-ratio = 4.0; and T (tolerance) = 0.05. The criteria used by the author to accept or reject the equations were that R^2 was equal to 0.85 and the standard error was equal to 10.0. Appendix H contains a listing of the computer program.

The experimental data is shown in Appendices E. The experimental factors were the tack of both ink layers and the three types of paper. The experiment used high, medium, and low tack for both ink layers. The types of paper used were newsprint, uncoated paper, and coated paper. The percent trapping was determined by the gravimetric method and the densities were measured by a MACBETH RD-514, which is a standard non-polarizing densitometer.

The percent gravimetric trapping data used for this study was the same as the data used to establish the relationship between the gravimetric and apparent trapping. This data is listed in Appendices A. In addition, the densities D_1, D_2, and D_3 were obtained from the apparent trapping data listed in Appendices C. However, the density, D_4, was estimated from a graph of the density and ink film thickness with different filters in Appendix B. Appendices E shows the data for determining the percent gravimetric trapping using densities.

Table 8A shows the regression equations for newsprint, uncoated paper, and coated paper. The third order equation was the best-fit equation to predict the percent gravimetric trapping for these three types of paper. The equation was:

$$G = -0.2734142 + 143.6512(D_1) - 36.40506(D_3^2) - 20.74427(D_1^3) - 14.97150\,(D_2^3)$$

where G was the percent gravimetric trapping and D_1, D_2, D_3, and D_4 were densities. The R^2 was equal to 0.86163 (86.163%). Although the equation should have been rejected because the standard error was 15.87202, which was higher than the criteria value of 10.0, the equation included 86.163% of the gravimetric variation, a value higher than the required criteria value of 85.0. Because of this higher F-ratio, and since this equation predicted gravimetric trapping for all three types of paper, ignoring the standard error, would be useful and not provide inaccurate information.

TABLE 8A

Regression Equations For Determining Gravimetric Trapping Using Densities For Newsprint, Uncoated Paper, and Coated Paper

Inclusion criteria: n (maximum number of variables) = 10
F-ratio = 4.0
T (tolerance) = 0.05

Notation: G = the percent gravimetric trapping
D_1 = the density of the overlapping area of two inks
D_2 = the density of the first ink layer
D_3 = the density of the second ink layer
D_4 = the density of the first ink layer but the filter is chosen to give the highest density

The first order equation is:

$$G = 149.2871 + 87.03033(D_1) - 27.15685(D_2) - 150.7733(D_3)$$

$R^2 = 0.79010$ Standard error = 19.42138

The second order equation is:

$$G = 51.68571 + 100.8584(D_1) - 28.06811(D_1D_2) - 61.75933(D_3^2)$$

$R^2 = 0.81687$ Standard error = 18.14087

The third order equation* is:

$$G = -0.2734142 + 143.6512(D_1) - 36.40506(D_3^2) - 20.74427(D_1^3) - 14.97150(D_2^3)$$

$R^2 = 0.86163$ Standard error = 15.87202

The fourth order equation is:

$$G = -1.674448 + 140.1440(D_1) - 34.60313(D_3^2) - 16.04897(D_1^3) - 13.90704(D_1D_2D_3D_4)$$

$R^2 = 0.85575$ Standard error = 16.20568

* the best-fit equation

The author then analyzed each substrate independently. The regression equations for newsprint appear in Table 8B. The best-fit equation determining the percent gravimetric trapping using densities was:

$$G = -119.1700 + 286.8951(D_1) - 40.904(D_1D_2D_4) - 29.58042(D_1^4) - 35.92690(D_3^4).$$

For newsprint R^2 was equal to 0.90629 while the standard error was equal to 7.86325. In other words, this equation explained 90.6 percent of the total variation in the gravimetric trapping and showed that there was a 7.8 percent average difference between the experimental trapping values and the calculated ones.

For uncoated paper (Table 8C) the best-fit equation was the second-order equation where R^2 was equal to 0.86865 and the standard error was equal to 7.73473. The equation was:

$$G = 231.4337 - 0.03714963(D_2) - 282.3184(D_3) + 144.8207(D_1D_3) - 26.53772(D_2^2).$$

The equation predicted 86.86 percent of the gravimetric trapping variation while the average estimated results would have a 7.7 percent average difference.

Table 8D shows the regression equations for coated paper. The second order equation was the best-fit, since R^2 was equal to 0.95252 and the standard error was equal to 12.10285. The equation was:

TABLE 8B
Regression Equations For Determining Gravimetric Trapping Using Densities For Newsprint

Inclusion criteria: n (maximum number of variables) = 10
F-ratio = 4.0
T (tolerance) = 0.05

Notation: G = the percent gravimetric trapping
D_1 = the density of the overlapping area of two inks
D_2 = the density of the first ink layer
D_3 = the density of the second ink layer
D_4 = the density of the first ink layer but the filter is chosen to give the highest density

The first order equation is:

$$G = 59.84629 + 175.9142(D_1) - 43.76016(D_2) - 141.4072(D_3)$$

$R^2 = 0.83456$ Standard error = 10.21824

The second order equation is:

$$G = 33.24642 + 198.5368(D_1) - 132.2279(D_3) - 47.2048(D_1D_2)$$

$R^2 = 0.87287$ Standard error = 8.95724

The third order equation is:

$$G = 16.52071 + 211.5332 - 125.2784(D_3) - 52.99735(D_1D_2D_4)$$

$R^2 = 0.8948$ Standard error = 8.14805

The fourth order equation* is:

$$G = -119.1700 + 286.8951(D_1) - 40.904(D_1D_2D_4) - 29.58042(D_1^4) - 35.92690(D_3^4)$$

$R^2 = 0.90629$ Standard error = 7.86325

* the best-fit equation

TABLE 8C

Regression Equations For Determining Gravimetric Trapping Using Densities For Uncoated Paper

Inclusion criteria: n (maximum number of variables) = 10
F-ratio = 4.0
T (tolerance) = 0.05

Notation:
G = the percent gravimetric trapping
D_1 = the density of the overlapping area of two inks
D_2 = the density of the first ink layer
D_3 = the density of the second ink layer
D_4 = the density of the first ink layer but the filter is chosen to give the highest density

The first order equation is:

$$G = 95.01301 + 151.7852(D_1) - 31.21760(D_2) - 148.1049(D_3)$$

$R^2 = 0.82411$ Standard error = 8.75386

The second order equation* is:

$$G = 231.4337 - 0.03714963(D_2) - 282.3184(D_3) + 144.8207(D_1D_3) - 26.53772(D_2^2)$$

$R^2 = 0.86865$ Standard error = 7.73473

The third order equation is the same as the second order equation.

The fourth order equation is the same as the second order equation.

* the best-fit equation

TABLE 8D

Regression Equations For Determining Gravimetric Trapping Using Densities For Coated Paper

Inclusion criteria: n (maximum number of variables) = 10
F-ratio = 4.0
T (tolerance) = 0.05

Notation:
- G = the percent gravimetric trapping
- D_1 = the density of the overlapping area of two inks
- D_2 = the density of the first ink layer
- D_3 = the density of the second ink layer
- D_4 = the density of the first ink layer but the filter is chosen to give the highest density

The first order equation is:

$$G = 85.30951 + 104.1255(D_1) - 68.36436(D_2) - 99.58426(D_3)$$

$R^2 = 0.91326$ Standard error = 15.99942

The second order equation* is:

$$G = 60.25103 + 101.5277(D_1) - 151.4329(D_3) - 47.49919(D_1D_2) + 65.52299(D_3D_4)$$

$R^2 = 0.95252$ Standard error = 12.10285

The third order equation is:

$$G = -51.15205 + 145.4638(D_1) - 49.35735(D_2) - 67.68843(D_1D_2D_3) + 48.63919(D_2^3)$$

$R^2 = 0.94371$ Standard error = 13.17843

The fourth order equation is:

$$G = -52.00485 + 134.5622(D_1) - 68.67055(D_1D_2D_3) + 22.05573(D_2^4)$$

$R^2 = 0.93360$ Standard error = 13.99838

* the best-fit equation

$$G = 60.25103 + 101.5277(D_1) - 151.4329(D_3) - 47.49919(D_1D_2) + 65.52299(D_3D_4)$$

In comparison with the R^2 value for coated paper, this equation predicted the percent gravimetric trapping very well, since the R^2 value suggested that 95.25 percent of the gravimetric trapping variation was included in the equation. However, the standard error was 12.10285, which was over the acceptable criteria of 10.0, even though it was the lowest error value calculated from any of the coated regression equations. The author concluded that experimental errors were a major factor causing the standard error to be so high.

The best-fit equation for each paper was more accurate than the equation for all three types of paper, indicating that separate equations were needed to determine the percent gravimetric trapping. In addition, the density variables included in the best-fit equations were different for each type of paper, making one comprehensive equation impossible to establish.

To increase the accuracy of the equations, the author reduced the F-ratio value and T (tolerance) to 0.01 and 0.001, respectively. In addition, n (the maximum number of variables) in the equation were increased from 10 to 80. Table 9 shows that the R^2 values for the second set equation, where n = 80, F-ratio = 0.01, and T = 0.001, increased while the standard error decreased. Appendices I show the regression equations for all three types as well as for each individual type of paper. In those equations, n (maximum number of variables) = 80, F-ratio = 0.01, and T (tolerance) = 0.001. Table 10 summarizes the

TABLE 9

Comparison of R^2 and Standard Error Between Two Sets of Inclusion Criteria For Determining Gravimetric Trapping Using Densities

	The inclusion criteria			
	n = 10 F-ratio = 4.0 Tolerance = 0.05		n = 80 F-ratio = 0.01 Tolerance = 0.001	
Paper	R^2	Std. error	R^2	Std. error
newsprint	0.90629	7.86325	0.95825	6.15442
uncoated	0.86865	7.73473	0.97333	4.08726
coated	0.95252	12.10285	0.99095	6.6224

Note: The values of R^2 and standard error were from the best-fit equation for each type of paper.

best-fit equations for all three papers as well as for each type of paper.

The author concluded that these more sensitive criteria resulted in much more accurate, though complicated, regression equations: the R^2 values were above 95 percent while the standard errors were less than 7.0 percent for each type of paper. The best-fit equation (Table 10) for newsprint established that R^2 was 0.95825 and the standard error was 6.154472, which meant that this equation included 95.8 percent of the gravimetric trapping variation for newsprint. The uncoated paper equation (Table 10) established that R^2 was equal to 0.97333, which meant the equation was 97.33 percent accurate in determining the percent trapping on uncoated paper. The standard error explained that there was 4.09 percent of average differences between experimental values and calculated values of percent trapping on uncoated paper. For the coated paper, R^2 was equal to 0.99095 and the standard error was equal to 6.6224. Here the R^2 value approached the ideal case, which was defined as $R^2 = 1.0$. Therefore, the coated paper equation was very accurate, accounting for 99.10 percent of the gravimetric trapping variation, while also having the standard error of 6.6 percent, which was below the criterion value of 10.0. As with the gravimetric and apparent trapping relationship, each type of paper needed separate, complex equations, which included many variables, to predict percent gravimetric trapping accurately.

TABLE 10
Summary of Regression Equations (F-ratio = 0.01)
For Determining Gravimetric Trapping
Using Densities

Inclusion criteria: n (maximum number of variables = 80
F-ratio = 0.01
T (tolerance) = 0.001

Notation: G = the percent gravimetric trapping
D_1 = the density of the overlapping area of two inks
D_2 = the density of the first ink layer
D_3 = the density of the second ink layer
D_4 = the density of the first ink layer but the filter is chosen to give the highest density

These following equations are the best-fit equations which have the highest R^2 and the lowest standard error from the computer results.

The equation for the newsprint, uncoated paper, and coated paper is:

$$G = -225.8657 + 477.2871(D_1) + 153.0985(D_2) + 168.3707(D_4) - 139.1358(D_1D_3) - 158.1360(D_1D_4) - 152.0179(D_2^2) - 20.25123(D_3^2) - 99.30195(D_1D_2D_4) - 216.4647(D_2D_3D_4) + 50.02378(D_1^3) - 83.54683(D_2^3) + 177.6226(D_1D_2D_3D_4) - 39.09424(D_1^4) + 15.05495(D_3^4)$$

$R^2 = 0.92468$ Standard error = 12.56622

The equation for the newsprint is:

$$G = 473.6443 - 524.9047(D_1) + 82.11858(D_3) - 876.7252(D_4) + 1178.132(D_1D_4) + 111.5904(D_2D_4) + 210.5982(D_2^2) - 477.7634(D_1D_2D_4) + 56.4296(D_2D_3D_4) -60.38354(D_1^3) - 86.80699(D_3^3)$$

$R^2 = 0.95825$ Standard error = 6.15442

TABLE 10 (continued)

The equation for the uncoated paper is:

$$G = 531.0696 - 204.5510(D_2) - 865.0237(D_3) + 103.8759(D_1D_3) + 420.7120(D_1D_4) + 322.5885(D_2D_3) + 33.27951(D_2^2) - 186.6137(D_4^2) - 186.8209(D_2D_3D_4) - 113.5982(D_1^3) + 219.54(D_3^3)$$

$R^2 = 0.97333$ Standard error = 4.08726

The equation for the coated paper is:

$$G = 178.0405 + 286.2175(D_1) - 104.6036(D_3) - 14.22634(D_1D_2) - 97.53547(D_1D_3) - 135.6808(D_2D_4) - 15.62728(D_1^2) - 254.1412(D_4^2) + 70.89942(D_1D_2D_3) - 10.92884(D_1^4) - 1.647462(D_2^4) + 17.14313(D_3^4) + 107.1476(D_4^4)$$

$R^2 = 0.99095$ Standard error = 6.6224

Determination of Percent Gravimetric Trapping Using Exponential Function of Densities

The prediction equations resulting from the study of percent gravimetric trapping using densities were complicated and included many variables. In addition, each type of paper needed an individual equation to estimate the percent trapping for that paper. These equations could be useful for research studies, but could not be practical in a production situation. What pressmen need is a simple equation for determining the percent trapping which can also apply to different types of paper.

The author proposes equations which can be developed to a general equation for estimating the percent trapping. The author suggests that the percent gravimetric trapping be estimated by an exponential function of densities. Percent gravimetric trapping is expressed using densities expressed as $1\text{-}e^{-D}$, where e is equal to 2.71828 and D represents the density.

The same set of data (see Appendices E) used to determine percent gravimetric trapping in terms of densities was used in this study. The four densities: D_1, D_2, D_3, and D_4, were converted to the exponential functions, $1\text{-}e^{-D_1}$, $1\text{-}e^{-D_2}$, $1\text{-}e^{-D_3}$, and $1\text{-}e^{-D_4}$, respectively. D_1 was the reflection density of the overlapping area. D_2 and D_3 were the densities of the first ink layer and second ink layer printed directly on paper. The filter giving the highest density reading for the overlapping area (D_1) was used in measuring D_1, D_2, and D_3. D_4 was measured using the filter providing the highest density for just the D_4 density.

The regression analysis established the equations for determining the percent gravimetric trapping as a function of the exponential densities. The equation model was:

$$G = b_0 + b_1 F(D_1) + b_2 F(D_2) + b_3 F(D_3) + b_4 F(D_4)$$

where G = the percent gravimetric trapping

$F(D_1) = 1 - e^{-D_1}$

$F(D_2) = 1 - e^{-D_2}$

$F(D_3) = 1 - e^{-D_3}$

$F(D_4) = 1 - e^{-D_4}$

b_0 = a constant

b_1, b_2, b_3, and b_4 = regression coefficients

The XDS SIGMA 9 computer at the Rochester Institute of Technology analyzed the stepwise regression. The computer program was entitled, "Statistical Programs for Social Sciences (SPSS)." A listing of the program is shown in Appendix J.

The inclusion criteria values for determining which variables should be included in the equation were: n (the maximum number of variables in an equation) = 10; F-ratio = 4.0; and T (Tolerance) = 0.05. The criteria for accepting or rejecting the equation were: R^2 must be more than 0.85 and the standard error must be less than 10.0. These criteria determine the accuracy of the equation. Table 11 shows the regression equations for determining the percent gravimetric trapping using an exponential function of densities. The author concluded that the exponential term of density D_4 was not significant and influenced the

TABLE 11

Regression Equations for Determining Gravimetric Trapping Using Densities in Terms of Exponential Function.

Inclusion criteria: n (maximum number of variables) = 10
F-ratio = 4.0
T (Tolerance) = 0.05

Notation: D_1 = the density of the overlapping area
D_2 = the density of the first ink layer on paper
D_3 = the density of the second ink layer on paper
$F(D_1) = 1-e^{-D_1}$
$F(D_2) = 1-e^{-D_2}$
$F(D_3) = 1-e^{-D_3}$

The equation for newsprint, uncoated paper, and coated paper is:

$$G = 175.5434 + 186.1842[F(D_1)] - 30.41148[F(D_2)] - 329.2199[F(D_3)]$$

$R^2 = 0.82196$ Standard error = 17.88671

The equation for newsprint is:

$$G = 33.34853 + 435.1165[F(D_1)] - 66.50078[F(D_2)] - 332.4320[F(D_3)]$$

$R^2 = 0.85867$ Standard error = 9.44424

The equation for uncoated paper is:

$$G = 67.96646 + 422.9188[F(D_1)] - 50.09665[F(D_2)] - 369.2511[F(D_3)]$$

$R^2 = 0.84331$ Standard error = 8.26238

The equation for coated paper is:

$$G = 90.72696 + 234.9237[F(D_1)] - 119.5472[F(D_2)] - 209.1781[F(D_3)]$$

$R^2 = 0.97043$ Standard error = 9.34179

percent trapping less than five percent because it did not meet the inclusion criteria.

The equation for the newsprint, uncoated paper, and coated paper was:

$$G = 175.5434 + 186.1842[F(D_1)] - 30.41148[F(D_2)] - 329.2199[F(D_3)]$$

where R^2 and the standard error were equal to 0.82196 and 17.88671, respectively. The author rejected this equation as an estimate of the percent gravimetric trapping because the R^2 value was lower than the criteria value of 0.85 and because the standard error was greater than the criteria value of 10.0.

The regression equation for the newsprint was:

$$G = 33.34853 + 435.1165[F(D_1)] - 66.50078[F(D_2)] - 3332.4320[F(D_3)]$$

R^2 was equal to 0.85867 while the standard error was equal to 9.44424. The author concluded that this equation could determine the percent gravimetric trapping on the newsprint with 86 percent of the gravimetric trapping variation included in the equation. For this equation, the average difference between the predicted values and experimental values was about 9.4 percent.

The regression equation for uncoated paper was:

$$G = 67.96646 + 422.9188[F(D_1)] - 50.09665[F(D_2)] - 369.2511[F(D_3)]$$

where R^2 was equal to 0.84331 and the standard error was equal to 8.26238. The R^2 value was less than the criteria value of 0.85, but the standard error was less than the criteria value of 10.0. As a result, the author concluded that the equation could estimate the percent gravimetric trapping on the uncoated paper. This equation included 84.33 percent of the variation of the gravimetric trapping. The average difference between the predicted values and the experimental values was 8.26 percent.

The regression equation for coated paper was:

$$G = 90.72696 + 234.9237[F(D_1)] - 119.5472[F(D_2)] - 209.1781[F(D_3)]$$

R^2 was equal to 0.97043 which meant the equation predicted 97.04 percent of the gravimetric trapping variation on coated paper. Comparison of this R^2 value with the newsprint and uncoated paper R^2 values showed that the coated paper equation was the most accurate of the equations estimating percent gravimetric trapping. However, the standard error for coated paper was equal to 9.34179, which was quite high, and explained a 9.34 percent average difference between the predicted values and the experimental values.

Although each paper type required an individual equation for estimating the percent trapping, the author believes that a general equation, one applicable to any paper, can be established if a numerical characteristic of paper is introduced into the equation. Further study is needed to determine what paper characteristic will be the best one to include in the equation.

FOOTNOTES FOR CHAPTER FOUR

[1]Albert D. Rickmers, and Hallis N. Todd, Statistics: An Introduction,. McGraw-Hill Book Company, Chapter 8, 1967.

[2]Albert D. Rickmers. Private discussion.

[3]Albert D. Rickmers, and Hallis N. Todd, Statistics: An Introduction. McGraw-Hill Book Company, Chapter 8, 1967.

[4]*Ibid*., p. 220.

[5]Norman H. Nie; C. Hadlai Hull; Jean G. Jenkins; Karin Steinbrenner; and Dale H. Bent; Statisical Package for the Social Sciences, McGraw-Hill Book Company, Chapter 20.

V. CONCLUSIONS AND RECOMMENDATIONS

This thesis demonstrated that the tack of both ink layers, the type of paper, and the measurement method influence percent ink trapping. The results showed that when the tack of the first ink layer was increased, the percent ink trapping also increased. In addition, the percent trapping decreased when the tack of the second ink layer increased. The combinations of tack of both ink layers were also significant. The best combination, which caused the highest percent trapping, was a high tack of the first ink and a low tack of the second ink.

The results also showed that uncoated paper exhibited the highest percent trapping while coated paper exhibited the lowest. The percent trapping for newsprint occurred in between.

The percent trapping using the gravimetric method was significantly different from measurements using the three densitometric methods. These densitometric methods included, apparent, polarizing apparent, and antilogarithm trapping. There were also significant differences among these three densitometric methods. Generally, the polarizing apparent trapping method produced higher percent trapping than the apparent and antilogarithm methods. The percent antilogarithm trapping response was lower than the percent apparent trapping. The author believes that the gravimetric approach for measuring percent ink trapping is the best available method because it measures the physical amount of ink transferred.

This study also showed that the gravimetric and three densitometric methods did not provide identical trapping values. At the same time, the author did not find any evidence that the antilogarithm formula for estimating ink trapping was better than the apparent one when compared to the percent gravimetric trapping. There was also no evidence that using a polarizing densitometer improved the estimation of the percent trapping when compared to the percent gravimetric method.

The relationship between the percent gravimetric trapping and the percent apparent trapping could not be established with a third-order regression equation. The relationship may be established if it is expressed using a higher order equation, but such an equation will be too complex for printing industry uses.

The determination of the percent gravimetric trapping using densities provided equations that estimated the percent gravimetric trapping with high accuracy. However, each paper type required an individual equation in order to estimate percent trapping. These paper-type equations will be useful in research studies, but they need to be made simpler and easier to be useful to printers.

Finally, this thesis showed that the percent gravimetric trapping could be expressed in terms of the exponential function of densities. The densities required were the densities of the overprint area, and the densities of both the first and second ink layers transferred directly onto paper. Although the results showed that each type of paper used in this study required an individual equation, the author believes that a general equation for differerent types of paper can be established to predict percent trapping using the exponential function of densities.

To derive the general equation for measuring percent gravimetric trapping, the author recommends that a numerical characteristic of paper, such as surface roughness or oil absorption, be included in the equation of exponential function of densities. Further study is needed to discover the critical paper characteristic that should be present in the general equation of ink trapping. The author suggests two equation models which may be the general equation for determining the percent gravimetric trapping. They are:

I) $$G = b_0 + b_1(1-e^{-KD_1}) + b_2(1-e^{-KD_2}) + b_3(1-e^{-KD_3}).$$

II) $$G = b_0[1-e^{-K(D_1-D_2)}]/[1-e^{-KD_3}]$$

where
- G is the percent gravimetric trapping.
- D_1 is the highest density reading (among three filter readings) of the overlapping area of two inks.
- D_2 is the density of the first ink layer printed directly on paper.
- D_3 is the density of the second ink layer printed directly on paper.
- K is the paper characteristic variable.
- e is a constant equal to 2.71828
- b_0, b_1, b_2 and b_3 are regression coefficients.

The regression analysis can be applied to calculate the coefficients b_0, b_1, b_2, and b_3. In both equations, the author introduces a new variable, K, which represents the paper characteristic. This paper variable will cause the equation to be useful for different types of

paper. If a general equation is established, it should be easy to use by pressmen in determining percent trapping. The best solution would be a densitometer programmed to calculate the percent gravimetric trapping using the general equation.

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APPENDICES A1 to A3

The Experimental Data of Gravimetric Trapping

How to Read the Tables in Appendix A

- The first column lists the experimental number
- The second column lists the colors of the first and second ink layers. The letters Y, M, and C refer to yellow, magenta, and cyan inks, respectively. The slash should read as "over". Thus, Y/M refers to yellow ink printed over magenta ink.
- The third column lists the tack of the first and second ink layers. The letters l, m, and h refer to low, medium, and high tack, respectively. The slash should read as "over". Thus, l/m refers to low tack ink printed over medium tack ink.
- The fourth column lists the disc number and the color of ink which is applied on the disc. The disc number one transfers the first ink layers directly on paper. The disc number two transfers the second ink layer on the first ink layers.
- The fifth column lists the weight of the disc.
- The sixth column lists the weight of the disc and ink before the ink transfer.
- The seventh column lists the ink film thickness on the disc before the ink transfer.
- The eighth column lists the weight of the disc and ink after the ink transfer.
- The ninth column lists the film thickness of ink transferred from the disc.
- The tenth column lists the film thickness of the second ink layer transferred directly to paper.
- The eleventh column lists the percent gravimetric trapping.

Appendix A1

The Experimental Data of Gravimetric Trapping on Newsprint

The first replication

Exp. No.	Combination of Ink	Combination of Tack	Disc No.	Disc Color	Disc Wt. (gm)	Before transferring Ink & Disc Wt. (gm)	Before transferring Ink on Disc (μm)	After transferring Disc & Ink (gm)	After transferring Ink trans. (μm)	2nd ink on paper (μm)	Grav. Trap (%)
1	Y/Y	l/l	1	Y	84.4599	84.4744	4.47	84.4643	3.12	-	-
			2	Y	85.5881	85.6031	4.29	85.5945	2.50	2.88	86.81
2	Y/M	l/m	1	M	84.4601	84.4745	4.23	84.4660	2.50	-	-
			2	Y	85.5886	85.6038	4.35	85.5946	2.68	2.93	91.47
3	Y/C	l/h	1	C	84.4600	84.4713	3.29	84.4654	1.72	-	-
			2	Y	85.5884	85.6044	4.58	85.5936	3.15	3.12	100.96
4	M/Y	m/l	1	Y	84.4599	84.4748	4.60	84.4646	3.15	-	-
			2	M	85.5886	85.6040	4.30	85.5986	1.41	2.53	55.73
5	M/M	m/m	1	M	84.4597	84.4740	4.20	84.4654	2.53	-	-
			2	M	85.5880	85.6038	4.30	85.5967	1.89	2.53	75.49
6	M/C	m/h	1	C	84.4596	84.4704	3.15	84.4651	1.58	-	-
			2	M	85.5884	85.6030	3.98	85.5952	2.14	2.27	94.27
7	C/Y	h/l	1	Y	84.4620	84.4759	4.29	84.4657	3.15	-	-
			2	C	85.5886	85.6001	3.11	85.5989	0.23	1.63	14.11
8	C/M	h/m	1	M	84.4606	84.4745	4.08	84.4659	2.53	-	-
			2	C	85.5885	85.6007	3.30	85.5970	0.96	1.69	56.80
9	C/C	h/h	1	C	84.4599	84.4708	3.18	84.4653	1.60	-	-
			2	C	85.5880	85.6002	3.30	85.5954	1.28	1.69	75.74

Appendix A1 (continued)

The second replication

Exp. No.	Combination of Ink	Combination of Tack	Disc No.	Disc Color	Disc Wt. (gm)	Before transferring Ink & Disc Wt. (gm)	Before transferring Ink on Disc (μm)	After transferring Disc & Ink (gm)	After transferring Ink trans. (μm)	2nd ink on paper (μm)	Grav. Trap (%)
10	Y/Y	l/l	1	Y	84.4575	84.4727	4.69	84.4622	3.24	-	-
			2	Y	85.5854	85.6020	4.75	85.5926	2.71	3.26	83.13
11	Y/M	l/m	1	M	84.4578	84.4715	3.99	84.4637	2.27	-	-
			2	Y	85.5859	85.6012	4.38	85.5912	2.93	2.96	98.99
12	Y/C	l/h	1	C	84.4575	84.4682	3.19	84.4630	1.55	-	-
			2	Y	85.5854	85.6006	4.35	85.5907	2.87	2.93	97.95
13	M/Y	m/l	1	Y	84.4576	84.4720	4.19	84.4619	2.94	-	-
			2	M	85.5854	85.6012	4.30	85.5959	1.38	2.53	54.55
14	M/M	m/m	1	M	84.4576	84.4725	4.38	84.4630	2.79	-	-
			2	M	85.5857	85.6016	4.33	85.5942	2.00	2.56	78.13
15	M/C	m/h	1	H	84.4575	84.4684	3.11	84.4629	1.57	-	-
			2	M	85.5855	85.6012	4.28	85.5924	2.39	2.52	94.84
16	C/Y	h/l	1	Y	84.4578	84.4722	4.44	84.4622	3.08	-	-
			2	C	85.5857	85.5976	3.40	85.5956	0.47	1.72	27.33
17	C/M	h/m	1	M	84.4578	84.4717	4.08	84.4633	2.47	-	-
			2	C	85.5860	85.5975	3.11	85.5942	0.84	1.63	64.86
18	C/C	h/h	1	C	84.4576	84.4688	3.26	84.4630	1.69	-	-
			2	C	85.5857	85.5977	3.24	85.5927	1.34	1.67	80.24

Appendix A1 (continued)

The third replication

Exp. No.	Combination of Ink	Combination of Tack	Disc No.	Disc Color	Disc Wt. (gm)	Before transferring Ink & Disc Wt. (gm)	Before transferring Ink on Disc (μm)	After transferring Disc & Ink (gm)	After transferring Ink trans. (μm)	2nd ink on paper (μm)	Grav. Trap (%)
19	Y/Y	l/l	1	Y	84.4619	84.4755	4.23	84.4663	2.84	-	-
			2	Y	85.5921	85.6074	4.38	84.5988	2.47	2.95	83.73
20	Y/M	l/m	1	M	84.4648	84.4792	4.23	84.4706	2.53	-	-
			2	Y	85.5948	85.6110	4.63	85.6008	2.96	3.16	93.67
21	Y/C	l/h	1	C	84.4660	84.4772	3.26	84.4708	1.86	-	-
			2	Y	85.5961	85.6116	4.4	85.6009	3.15	2.97	106.06
22	M/Y	m/l	1	Y	84.4654	84.4799	4.47	84.4697	3.15	-	-
			2	M	85.5959	85.6118	4.33	85.6064	1.40	2.57	54.47
23	M/M	m/m	1	M	84.4623	84.4783	4.70	84.4682	2.97	-	-
			2	M	85.5926	85.6077	4.11	85.6003	2.01	2.39	84.10
24	M/C	m/h	1	C	84.4655	84.4772	3.41	84.4708	1.86	-	-
			2	M	85.5959	85.6116	4.28	85.6031	2.30	2.53	90.91
25	C/Y	h/l	1	Y	84.4648	84.4796	4.57	84.4697	3.05	-	-
			2	C	85.5956	85.6074	3.19	85.6050	0.58	1.66	39.94
26	C/M	h/m	1	M	84.4657	84.4809	4.46	84.4714	2.79	-	-
			2	C	85.5956	85.6080	3.35	85.6042	0.99	1.68	53.57
27	C/C	h/h	1	C	84.4624	84.4733	3.18	84.4676	1.66	-	-
			2	C	85.5926	85.6041	3.11	85.5993	1.28	1.63	78.53

Appendix A2

The Experimental Data of Gravimetric Trapping
on Uncoated Paper

The first replication

Exp. No.	Combination of Ink	Combination of Tack	Disc No.	Disc Color	Disc Wt. (gm)	Before transferring Ink & Disc Wt. (gm)	Before transferring Ink on Disc (μm)	After transferring Disc & Ink (gm)	After transferring Ink trans. (μm)	2nd ink on paper (μm)	Grav. Trap (%)
1	Y/Y	l/l	1	Y	84.4578	84.4771	5.95	84.4658	3.49	-	-
			2	Y	85.5860	85.6063	5.81	85.5939	3.61	3.5	103.14
2	Y/M	l/m	1	M	84.4583	84.4797	6.29	84.4675	3.58	-	-
			2	Y	85.5865	84.6060	5.58	85.5932	3.73	3.25	114.77
3	Y/C	l/h	1	C	84.4583	84.4759	5.13	84.4672	2.53	-	-
			2	Y	85.5864	85.6058	5.55	85.5928	3.79	3.22	117.70
4	M/Y	m/l	1	Y	84.4581	84.4772	5.89	84.4660	3.45	-	-
			2	M	85.5862	85.6098	6.43	85.5985	3.08	3.765	81.81
5	M/M	m/m	1	M	84.4581	84.4796	6.31	84.4672	3.64	-	-
			2	M	85.5863	85.6093	6.26	85.5960	3.67	3.63	101.10
6	M/C	m/h	1	C	84.4582	84.4758	5.13	84.4675	2.42	-	-
			2	M	85.5862	85.6092	6.26	85.5954	3.82	3.63	105.23
7	C/Y	h/l	1	Y	84.4583	84.4764	5.58	84.4661	3.18	-	-
			2	C	85.5864	85.6051	5.05	85.5999	1.31	2.44	53.69
8	C/M	h/m	1	M	84.4581	84.4790	6.14	84.4674	3.41	-	-
			2	C	85.5862	85.6056	5.24	85.5978	2.07	2.6	79.62
9	C/C	h/h	1	C	84.4580	84.4757	5.16	84.4669	2.56	-	-
			2	C	85.5861	85.6052	5.16	85.5961	2.51	2.535	99.01

Appendix A2 (continued)

The second replication

Exp. No.	Combination of Ink	Combination of Tack	Disc No.	Disc Color	Disc Wt. (gm)	Before transferring Ink & Disc Wt. (gm)	Before transferring Ink on Disc (μm)	After transferring Disc & Ink (gm)	After transferring Ink trans. (μm)	2nd ink on paper (μm)	Grav. Trap (%)
10	Y/Y	l/l	1	Y	85.5938	85.6129	5.89	85.6019	3.39	-	-
			2	Y	84.4644	84.4850	5.89	84.4725	3.64	3.58	101.68
11	Y/M	l/m	1	M	84.4639	84.4854	6.31	84.4731	3.61	-	-
			2	Y	85.5931	85.6133	5.78	85.6001	3.86	3.466	111.37
12	Y/C	l/h	1	C	84.4639	84.4815	5.13	84.4260	2.59	-	-
			2	Y	85.5931	85.6141	6.01	85.5995	4.29	3.71	115.63
13	M/Y	m/l	1	Y	84.4644	84.4828	5.68	84.4717	3.42	-	-
			2	M	85.5736	85.6170	6.37	85.6064	2.88	3.72	77.42
14	M/M	m/m	1	M	84.4645	84.4862	6.37	84.4740	3.58	-	-
			2	M	85.5936	85.6173	6.46	85.6040	3.67	3.79	97.61
15	M/C	m/h	1	C	84.4641	84.4824	5.33	84.4732	2.68	-	-
			2	M	85.5933	85.6168	6.40	85.6029	3.85	3.74	102.94
16	C/Y	h/l	1	Y	84.4719	84.4897	5.49	84.4780	3.61	-	-
			2	C	85.5938	85.6129	5.16	85.6073	1.43	2.535	56.41
17	C/M	h/m	1	M	84.4632	84.4846	6.29	84.4728	3.47	-	-
			2	C	85.5927	85.6115	5.08	85.6051	1.66	2.47	67.21
18	C/C	h/h	1	C	84.4641	84.4835	5.79	84.4737	2.93	-	-
			2	C	85.5935	85.6136	5.43	85.6039	2.69	2.76	97.46

Appendix A2 (continued)

The third replication

Exp. No.	Combination of Ink	Combination of Tack	Disc No.	Disc Color	Disc Wt. (gm)	Before transferring Ink & Disc Wt. (gm)	Before transferring Ink on Disc (μm)	After transferring Disc & Ink (gm)	After transferring Ink trans. (μm)	2nd ink on paper (μm)	Grav. Trap (%)
19	Y/Y	l/l	1	Y	84.4632	84.4820	5.80	84.4701	3.67	-	-
			2	Y	85.5932	85.6132	5.72	85.6010	3.55	3.40	104.41
20	Y/M	l/m	1	M	84.4623	84.4840	6.37	84.4720	3.52	-	-
			2	Y	85.5926	85.6137	6.04	85.5997	4.01	3.74	107.22
21	Y/C	l/h	1	C	84.4624	84.4805	5.27	84.4715	2.62	-	-
			2	Y	85.5927	85.6138	6.04	85.5993	4.26	3.74	113.90
22	M/Y	m/l	1	Y	84.4629	84.4820	5.89	84.4701	3.67	-	-
			2	M	85.5930	85.6160	6.26	85.6065	2.53	3.63	70.52
23	M/M	m/m	1	M	84.4629	84.4838	6.14	84.4716	3.58	-	-
			2	M	85.5933	85.6168	6.40	85.6038	3.58	3.74	95.72
24	M/C	m/h	1	C	84.4622	84.4804	5.30	84.4722	2.39	-	-
			2	M	85.5927	85.6159	6.29	85.6020	3.85	3.65	105.48
25	C/Y	h/l	1	Y	84.4626	84.4819	5.95	84.4704	3.55	-	-
			2	C	85.5932	85.6125	5.21	85.6069	1.43	2.58	55.43
26	C/M	h/m	1	M	84.4630	84.4844	6.29	84.4720	3.64	-	-
			2	C	85.5932	85.6126	5.24	85.6050	2.06	2.60	79.23
27	C/C	h/h	1	C	84.4627	84.4807	5.24	84.4720	2.53	-	-
			2	C	85.5932	84.6128	5.21	84.6038	2.42	2.58	93.80

Appendix A3

The Experimental Data of Gravimetric Trapping
on Coated Paper

The first replication

Exp. No.	Combination of Ink	Combination of Tack	Disc No.	Disc Color	Disc Wt. (gm)	Before transferring Ink & Disc Wt. (gm)	Before transferring Ink on Disc (μm)	After transferring Disc & Ink (gm)	After transferring Ink trans. (μm)	2nd ink on paper (μm)	Grav. Trap (%)
1	Y/Y	l/l	1	Y	84.4578	84.4656	2.41	84.4615	1.26	-	-
			2	Y	85.5853	85.5948	2.72	85.5922	0.71	1.45	48.97
2	Y/M	l/m	1	M	84.4577	84.4633	1.64	84.4608	0.73	-	-
			2	Y	85.5855	85.5940	2.43	85.5900	1.23	1.31	93.89
3	Y/C	l/c	1	C	84.4577	84.4627	1.46	84.4604	0.67	-	-
			2	Y	85.5855	85.5937	2.35	85.5892	1.30	1.27	102.36
4	M/Y	l/m	1	Y	84.4577	84.4649	2.22	84.4611	1.17	-	-
			2	M	85.5855	85.5916	1.66	85.5927	-0.32	0.855	-37.43
5	M/M	m/m	1	M	84.4573	84.4645	2.11	84.4607	1.12	-	-
			2	M	85.5852	85.5919	1.82	85.5902	0.44	0.945	46.56
6	M/C	m/h	1	C	84.4573	84.4624	1.49	84.4602	0.64	-	-
			2	M	85.5851	85.5914	1.72	85.5892	0.59	0.89	66.29
7	C/Y	h/l	1	Y	84.4578	84.4655	2.38	84.4614	1.26	-	-
			2	C	85.5855	85.5910	1.49	85.5925	-0.44	0.735	-59.86
8	C/M	h/m	1	M	84.4577	84.4634	1.67	84.4608	0.76	-	-
			2	C	85.5855	85.5906	1.38	85.5901	0.09	0.68	13.24
9	C/C	h/h	1	C	84.4578	84.4630	1.51	84.4607	0.67	-	-
			2	C	85.5854	85.5915	1.65	85.5901	0.35	0.82	42.68

Appendix A3 (continued)

The second replication

Exp.	Combination of		Disc		Disc Wt.	Before transferring Ink & Disc Wt.	Ink on Disc	After transferring Disc & Ink	Ink trans.	2nd ink on paper	Grav. Trap
10	Y/Y	l/l	1	Y	84.4570	84.4657	2.68	84.4611	1.42	-	-
			2	Y	85.5855	85.5939	2.4	85.5921	0.46	1.29	35.66
11	Y/M	l/m	1	M	84.4574	84.4635	1.79	84.4608	0.79	-	-
			2	Y	85.5857	85.5942	2.43	85.5902	1.14	1.31	87.02
12	Y/C	l/h	1	C	84.4572	84.4622	1.46	84.4599	0.67	-	-
			2	Y	85.5855	85.5936	2.32	85.5895	1.17	1.25	93.60
13	M/Y	m/l	1	Y	84.4574	84.4653	2.41	84.4613	1.20	-	-
			2	M	85.5853	85.5920	1.82	85.5932	-0.35	0.945	-37.04
14	M/M	m/m	1	M	84.5855	84.5918	1.85	84.5890	0.82	-	-
			2	M	85.4576	85.4639	1.72	85.4626	0.29	0.89	32.58
15	M/C	m/h	1	C	84.4575	84.4627	1.51	84.4605	0.64	-	-
			2	M	85.5855	85.5926	1.93	85.5897	0.76	1.01	75.25
16	C/Y	h/l	1	Y	84.4576	84.4655	2.44	84.4616	1.20	-	-
			2	C	85.5855	85.5920	1.76	85.5933	-0.37	0.875	-42.29
17	C/M	h/m	1	M	84.4574	84.4634	1.76	84.4608	0.76	-	-
			2	C	85.5854	85.5919	1.76	85.5909	0.23	0.875	26.29
18	C/C	h/h	1	C	84.4576	84.4632	1.63	84.4605	0.79	-	-
			2	C	85.5855	85.5919	1.73	85.5907	0.29	0.86	33.72

Appendix A3 (continued)

The third replication

Exp. No.	Combination of Ink	Combination of Tack	Disc No.	Disc Color	Disc Wt. (gm)	Before transferring Ink & Disc Wt. (gm)	Before transferring Ink on Disc (μm)	After transferring Disc & Ink (gm)	After transferring Ink trans. (μm)	2nd ink on paper (μm)	Grav. Trap (%)
19	Y/Y	l/l	1	Y	84.4643	84.4723	2.47	84.4680	1.33	-	-
			2	Y	85.5942	85.6030	2.51	85.6009	0.56	1.345	41.64
20	Y/M	l/m	1	M	84.4643	84.4698	1.62	84.4672	0.76	-	-
			2	Y	85.5939	85.6021	2.35	85.5981	1.14	1.27	89.76
21	Y/C	l/h	1	C	84.4641	84.4693	1.51	84.4668	0.73	-	-
			2	Y	85.5941	85.6025	2.32	85.5976	1.41	1.25	112.80
22	M/Y	m/l	1	Y	84.4643	84.4723	2.47	84.4685	1.17	-	-
			2	M	85.5941	85.6004	1.72	85.6019	-0.45	0.89	-50.56
23	M/M	m/l	1	M	84.4643	84.4702	1.73	84.4673	0.85	-	-
			2	M	85.5943	85.6016	1.99	85.5997	0.47	1.04	45.19
24	M/C	m/h	1	C	84.4642	84.4694	1.51	84.4670	0.70	-	-
			2	M	85.5941	85.6015	2.02	86.5984	0.82	1.06	77.36
25	C/Y	h/l	1	Y	84.4643	84.4720	2.38	84.4678	1.30	-	-
			2	C	85.5941	85.6004	1.70	85.6016	-0.37	0.845	-43.79
26	C/M	h/m	1	M	84.4643	84.4706	1.85	84.4677	0.85	-	-
			2	C	85.5940	85.5999	1.59	85.5990	0.21	0.775	25.81
27	C/C	h/h	1	C	84.4641	84.4701	1.75	84.4673	0.82	-	-
			2	C	85.5940	85.6008	1.8	85.5991	0.44	0.90	48.89

APPENDIX B

The Graphs of Ink Densities vs. Ink Film Thickness for Each Combination of Ink, Filter, and Paper

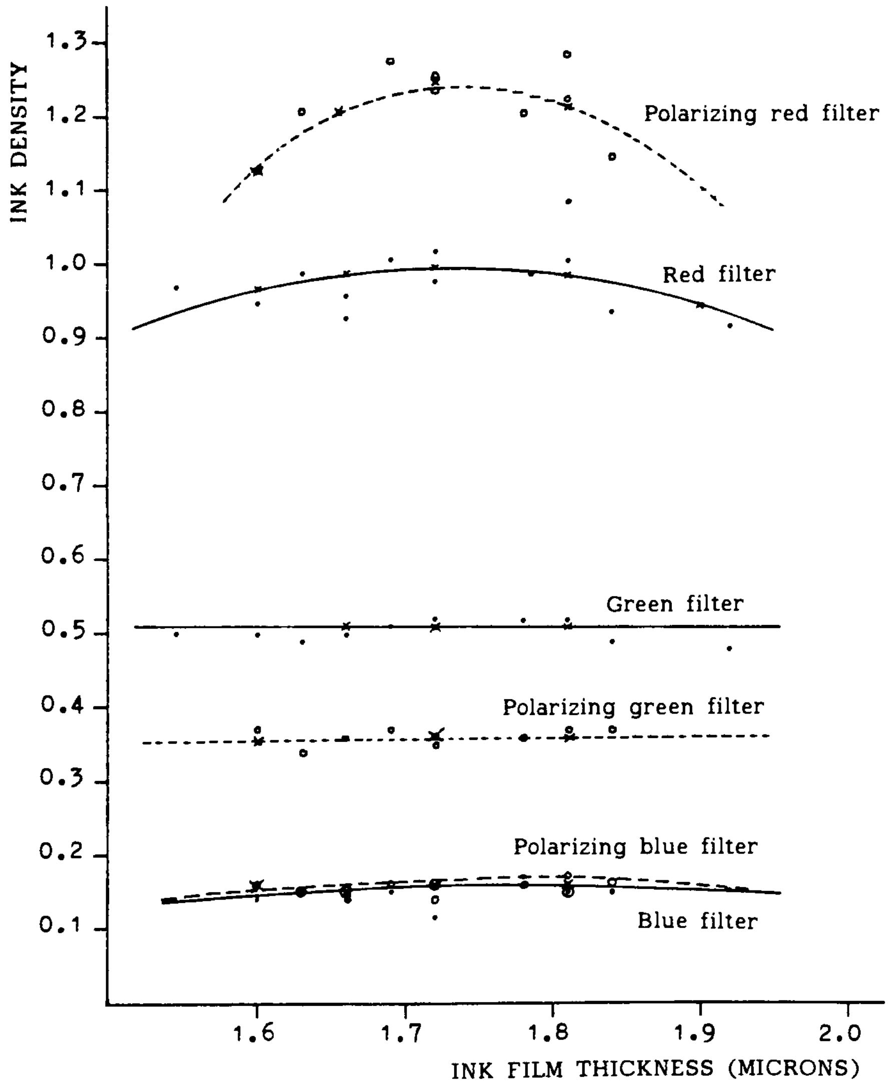

Figure 4A. Cyan ink density vs. ink film thickness on newsprint

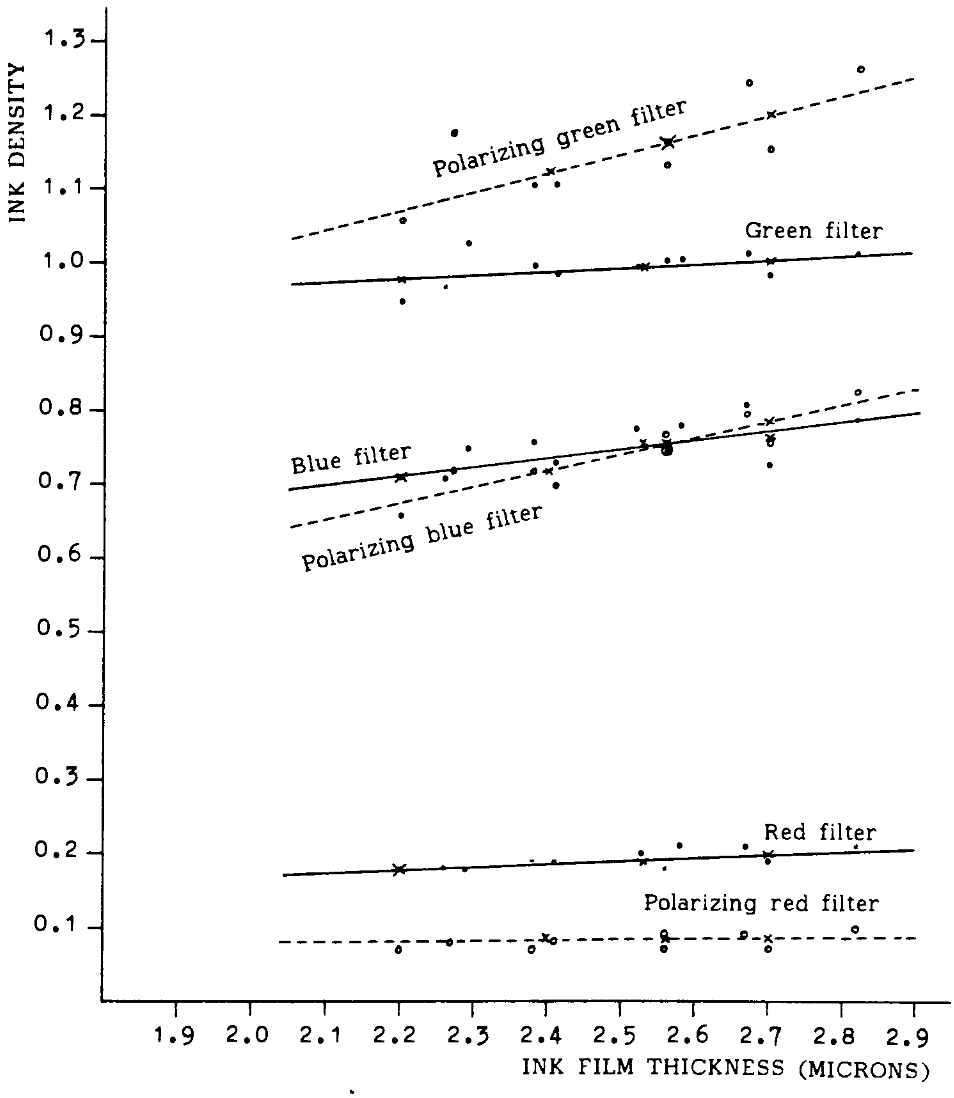

Figure 4B. Magenta ink density vs. ink film thickness on newsprint

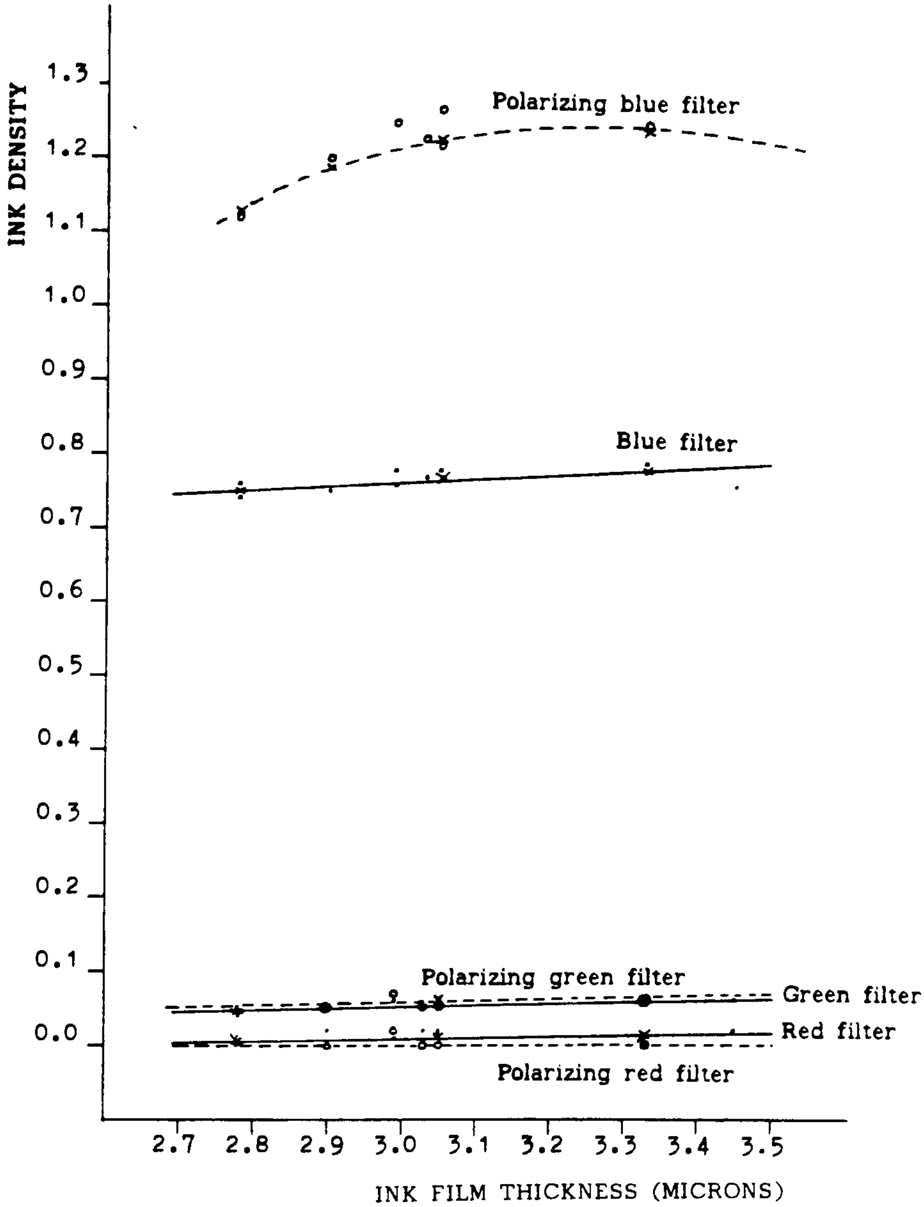

Figure 4C. Yellow ink density vs. ink film thickness on newsprint

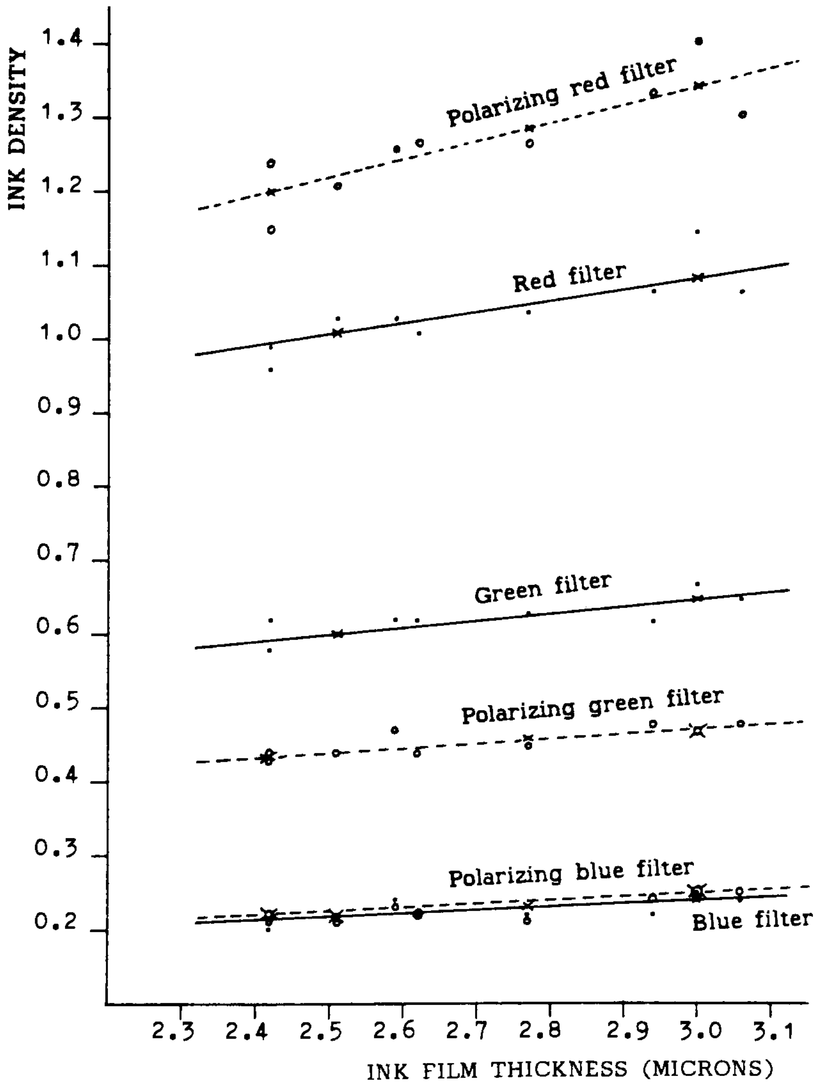

Figure 4D. Cyan ink density vs. ink film thickness on uncoated paper

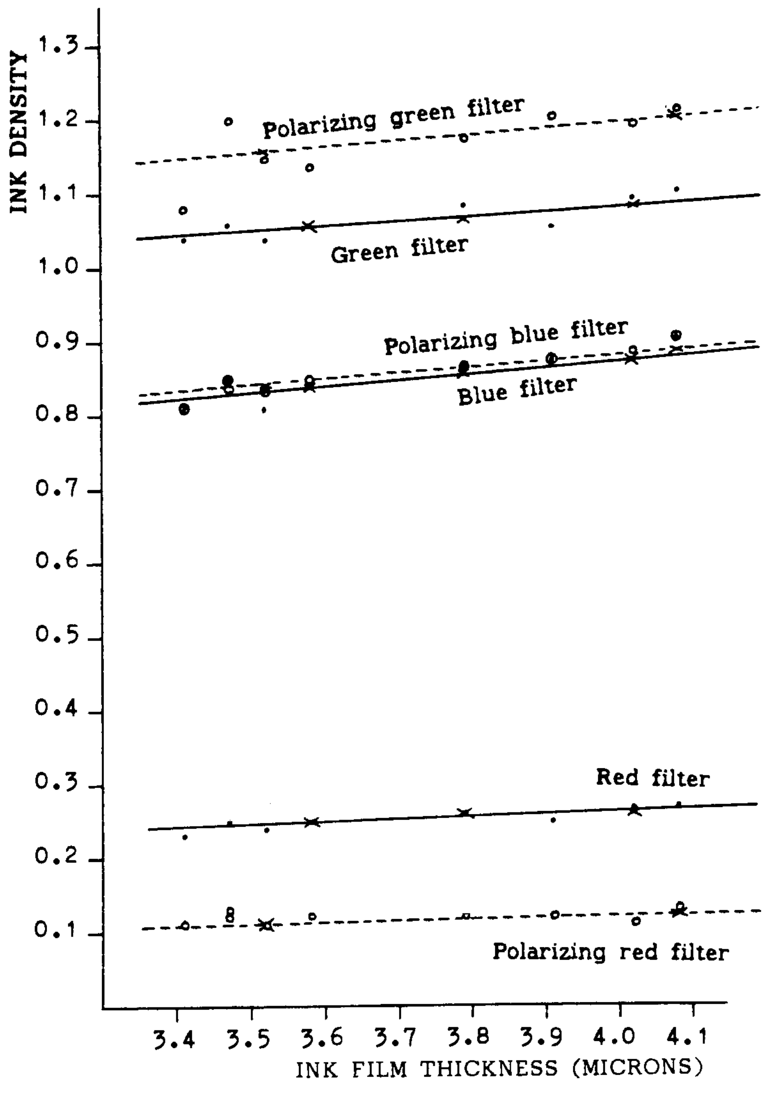

Figure 4E. Magenta ink density vs. ink film thickness on uncoated paper

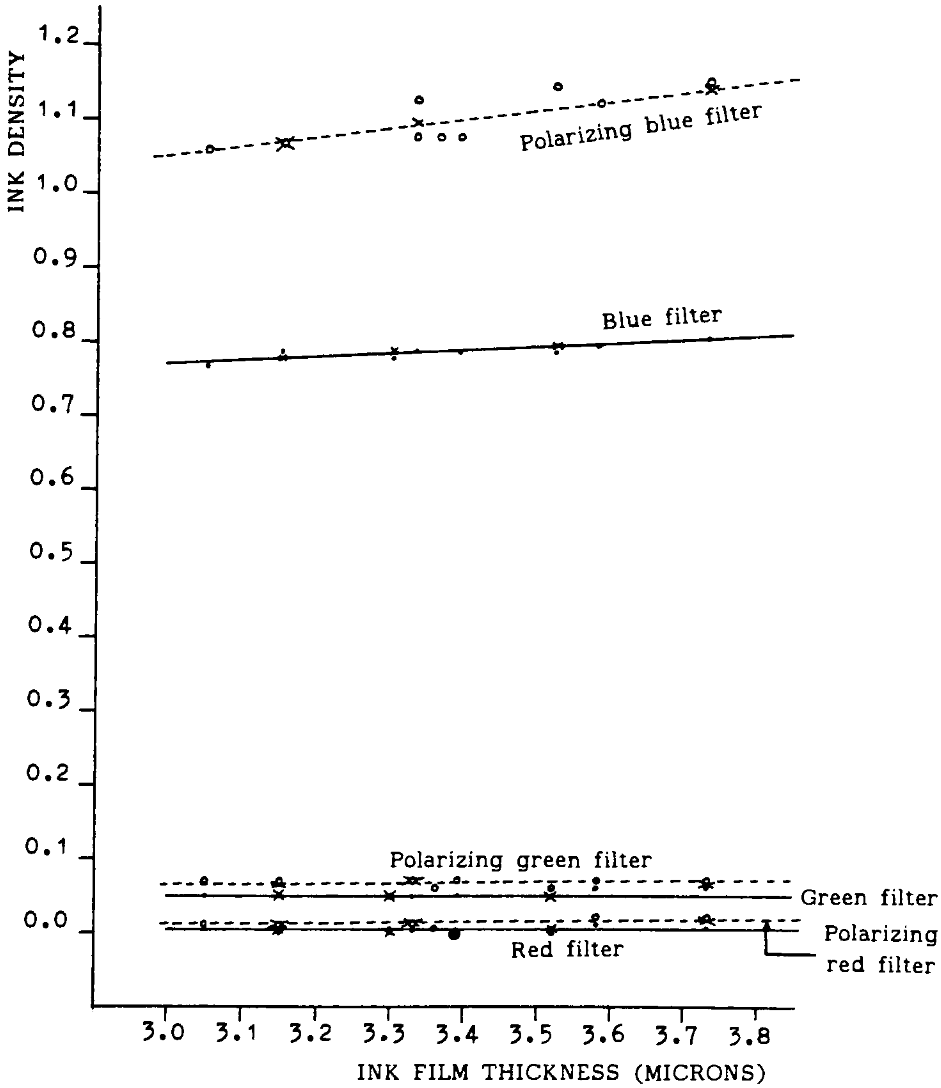

Figure 4F. Yellow ink density vs. ink film thickness on uncoated paper

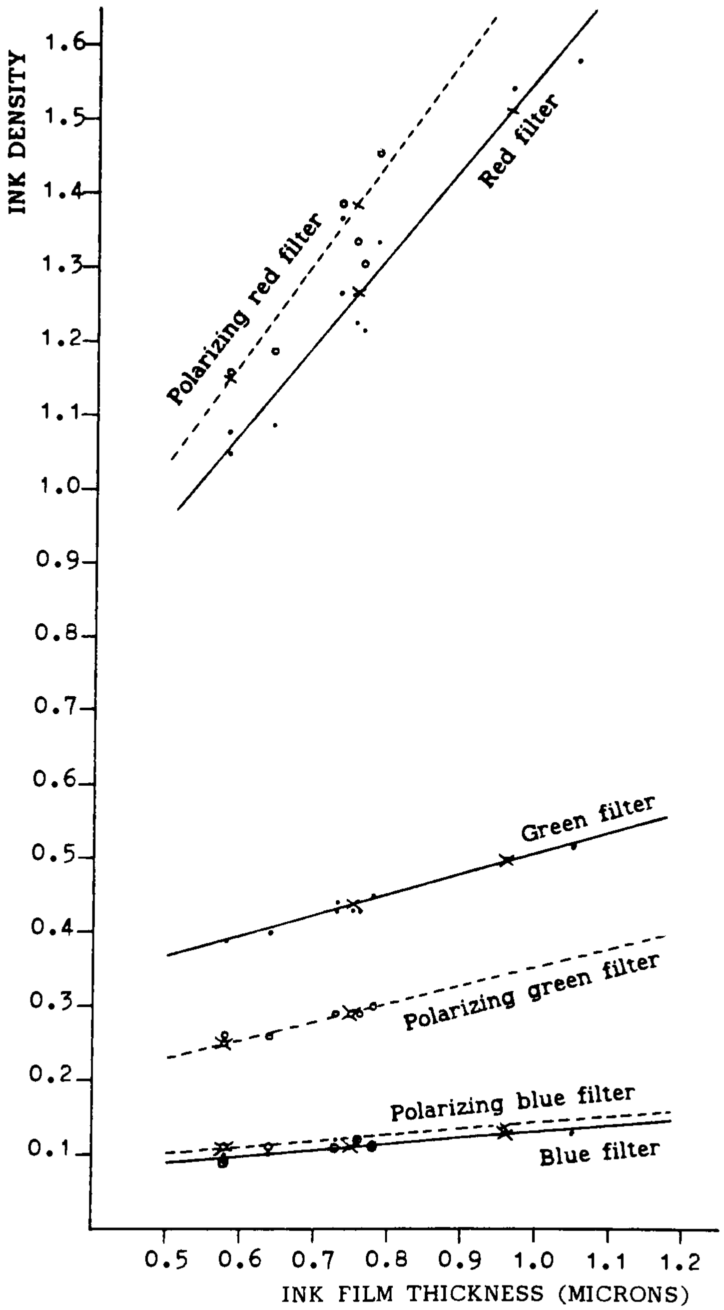

Figure 4G. Cyan ink density vs. ink film thickness on coated paper

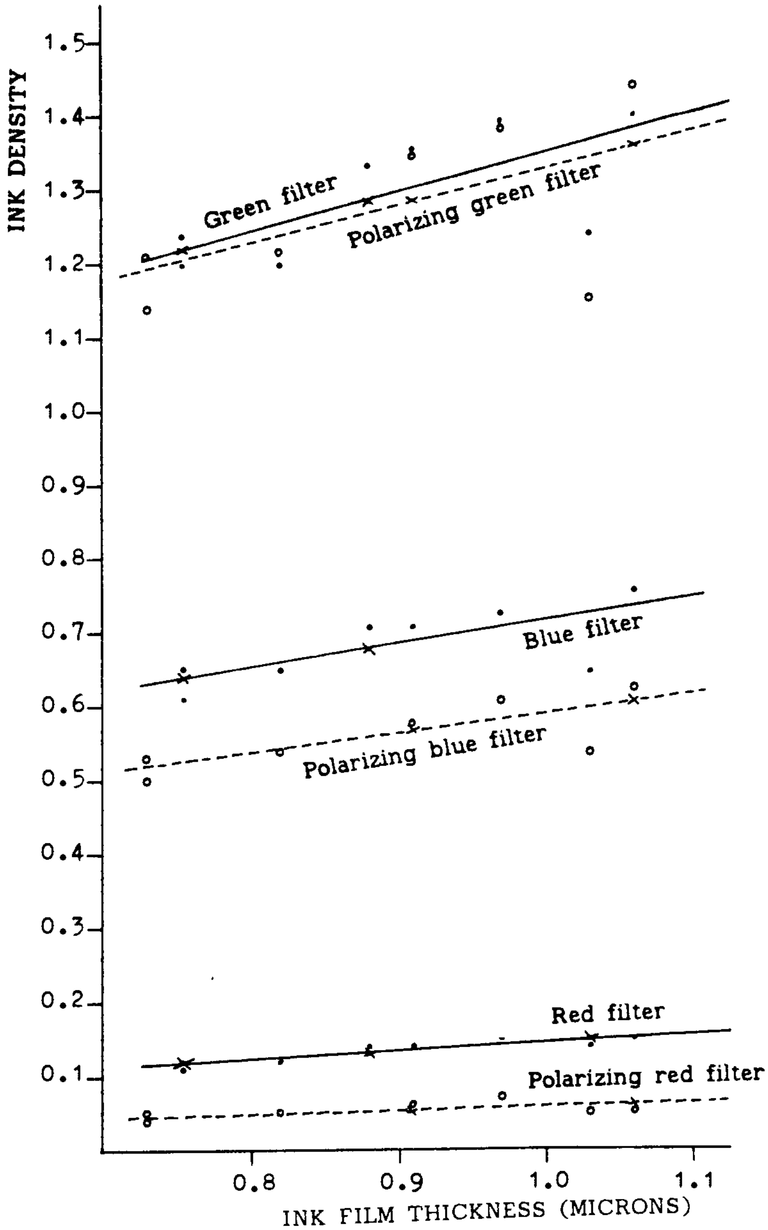

Figure 4H. Magenta ink density vs. ink film thickness on coated paper

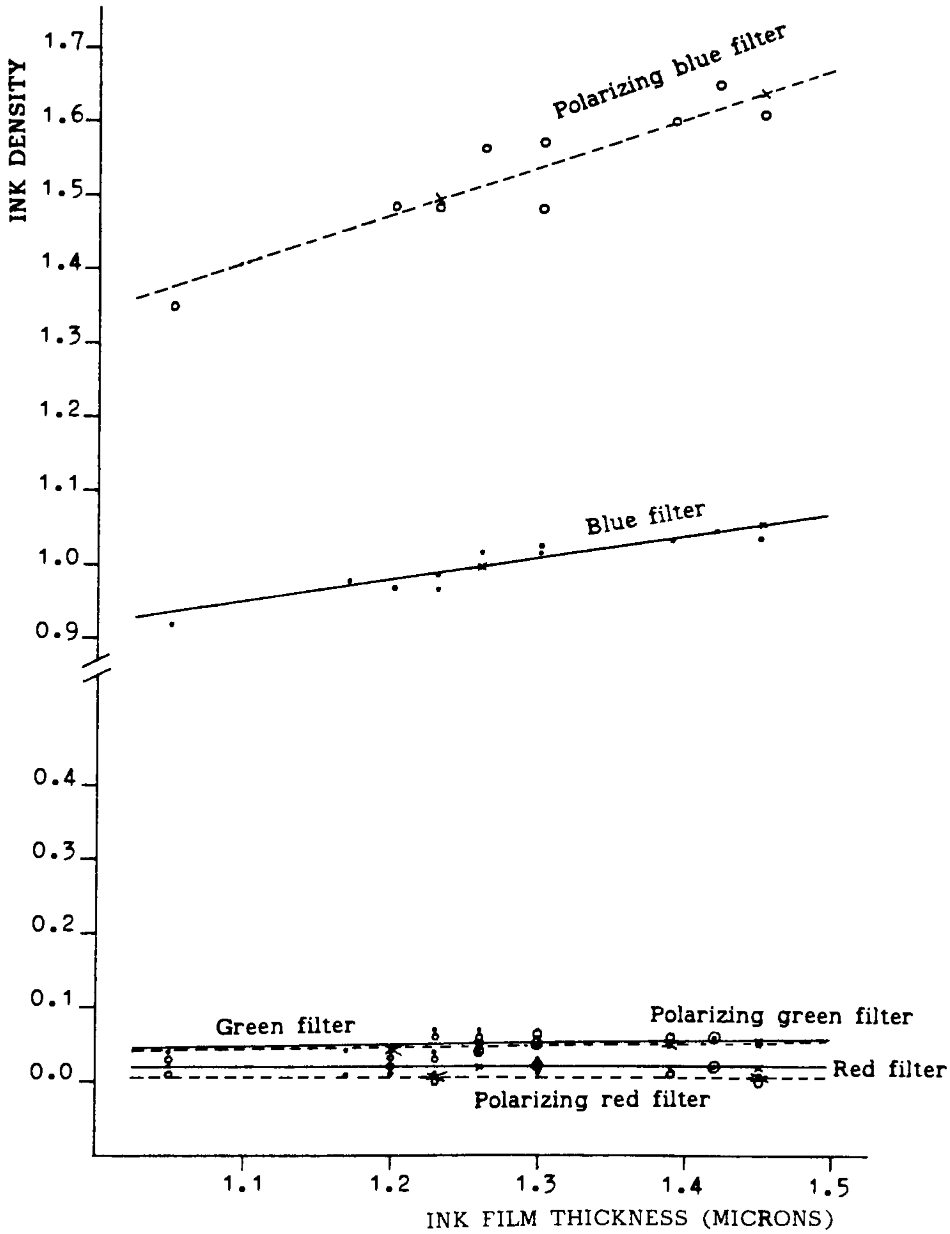

Figure 4I. Yellow ink density vs. ink film thickness on coated paper

APPENDICES C1 to C3

The Experimental Data of Apparent and Antilogarithm Trapping

How to Read the Tables in Appendix C

- The first column lists the experimental number
- The second column lists the ink colors of the first and second ink layers. The letters Y, M, and C refer to yellow, magenta, and cyan inks, respectively. The slash should read as "over". Thus, Y/M refers to yellow ink printed over magenta ink.
- The third column lists the colors of the filters used to measure ink densities. The letters B, G, and R refer to blue, green, and red filters.
- The fourth column lists the reflection density ($D_{2/1}$) of the overlapping area of the two inks.
- The fifth column lists the reflection density (D_1) of the first ink layer printed directly on paper.
- The sixth column lists the reflection density (D_2) of the second ink printed directly on paper.
- The seventh column lists the percent apparent trapping.
- The eighth column lists the percent antilogarithm trapping.

Appendix C1

The Experimental Data of Apparent and Antilogarithm Trapping on Newsprint

The first replication

Exp. No.	Color Combin-ation	Filter Color	Density $D_{2/1}$	D_1	D_2	Apparent Trapping(%)	Antilog. Trapping (%)
1	Y/Y	B	0.87	0.77	0.76	13.16	21.88
2	Y/M	B	1.03	0.75	0.72	38.89	36.31
3	Y/C	B	0.84	0.15	0.78	88.46	81.28
4	M/Y	G	0.81	0.06	1.03	72.82	52.48
5	M/M	G	1.16	1.00	1.00	16.00	14.45
6	M/C	G	1.10	0.51	1.01	59.50	38.02
7	C/Y	R	0.59	0.01	0.89	65.17	48.98
8	C/M	R	0.81	0.20	1.02	60.78	39.81
9	C/C	R	1.16	0.97	1.02	18.63	14.79

The second replication

Exp. No.	Color Combin-ation	Filter Color	$D_{2/1}$	D_1	D_2	Apparent Trapping(%)	Antilog. Trapping (%)
10	Y/Y	B	0.89	0.78	0.7	13.92	20.89
11	Y/M	B	1.07	0.72	0.77	45.45	38.02
12	Y/C	B	0.81	0.15	0.76	86.84	79.43
13	M/Y	G	0.83	0.055	1.03	75.24	55.59
14	M/M	G	1.16	1.02	1.02	13.73	13.18
15	M/C	G	1.12	0.51	1.01	60.40	39.81
16	C/Y	R	0.65	0.01	1.00	64.00	43.65
17	C/M	R	0.80	0.195	0.94	64.36	46.24
18	C/C	R	1.21	1.00	0.95	22.11	18.20

The third replication

Exp. No.	Color Combin-ation	Filter Color	$D_{2/1}$	D_1	D_2	Apparent Trapping(%)	Antilog. Trapping (%)
19	Y/Y	B	0.89	0.77	0.755	17.53	23.17
20	Y/M	B	1.03	0.76	0.77	35.06	31.62
21	Y/C	B	0.84	0.15	0.79	87.34	79.43
22	M/Y	G	0.80	0.06	1.04	71.15	50.12
23	M/M	G	1.60	1.28	1.35	23.70	9.33
24	M/C	G	1.13	0.51	1.04	59.62	38.02
25	C/Y	R	0.62	0.015	0.94	64.36	51.88
26	C/M	R	0.82	0.21	1.03	59.22	38.02
27	C/C	R	1.88	1.36	1.46	35.62	12.02

Appendix C2

The Experimental Data of Apparent and Antilogarithm Trapping on Uncoated Paper

The first replication

Exp. No.	Color Combin-ation	Filter Color	Density $D_{2/1}$	D_1	D_2	Apparent Trapping(%)	Antilog. Trapping (%)
1	Y/Y	B	0.96	0.80	0.79	20.25	23.44
2	Y/M	B	1.11	0.84	0.77	35.06	31.62
3	Y/C	B	0.91	0.22	0.78	88.46	81.28
4	M/Y	G	0.93	0.05	1.08	81.48	63.10
5	M/M	G	1.22	1.065	1.06	14.62	12.45
6	M/C	G	1.22	0.59	1.07	58.88	36.31
7	C/Y	R	0.77	0.005	1.02	75.00	55.59
8	C/M	R	0.95	0.245	1.08	65.28	42.17
9	C/C	R	1.29	1.02	1.07	25.23	15.85

The second replication

Exp. No.	Color Combin-ation	Filter Color	Density $D_{2/1}$	D_1	D_2	Apparent Trapping(%)	Antilog. Trapping (%)
10	Y/Y	B	0.95	0.79	0.81	19.75	22.39
11	Y/M	B	1.11	0.845	0.79	33.54	29.85
12	Y/C	B	0.90	0.22	0.82	82.93	72.44
13	M/Y	G	0.93	0.05	1.07	82.24	64.57
14	M/M	G	1.24	1.06	1.07	16.82	12.88
15	M/C	G	1.19	0.62	1.04	54.81	33.88
16	C/Y	R	0.80	0.01	0.98	80.61	64.57
17	C/M	R	0.88	0.25	0.94	67.02	48.98
18	C/C	R	1.29	1.075	1.04	20.67	14.96

The third replication

Exp. No.	Color Combin-ation	Filter Color	Density $D_{2/1}$	D_1	D_2	Apparent Trapping(%)	Antilog. Trapping (%)
19	Y/Y	B	0.95	0.81	0.79	17.72	22.39
20	Y/M	B	1.13	0.835	0.78	37.82	32.73
21	Y/C	B	0.90	0.22	0.82	82.93	72.44
22	M/Y	G	0.87	0.05	1.04	78.85	60.26
23	M/M	G	1.21	1.06	1.04	14.42	12.74
24	M/C	G	1.18	0.59	1.05	56.19	34.67
25	C/Y	R	0.76	0.01	0.98	76.53	58.88
26	C/M	R	0.93	0.25	1.03	66.02	44.67
27	C/C	R	1.25	1.02	1.00	23.0	16.98

Appendix C3

The Experimental Data of Apparent and Antilogarithm Trapping on Coated Paper

The first replication

Exp. No.	Color Combin-ation	Filter Color	Density $D_{2/1}$	D_1	D_2	Apparent Trapping(%)	Antilog. Trapping (%)
1	Y/Y	B	1.17	1.00	1.05	16.19	13.18
2	Y/M	B	1.53	0.63	1.02	88.24	75.86
3	Y/C	B	1.08	0.11	1.01	96.04	91.20
4	M/Y	G	0.11	0.05	1.23	4.88	6.76
5	M/M	G	1.54	1.41	1.20	10.83	8.51
6	M/C	G	1.37	0.40	1.27	76.38	50.12
7	C/Y	R	0.04	0.02	1.15	1.74	7.41
8	C/M	R	0.47	0.12	1.08	32.41	18.62
9	C/C	R	1.80	1.18	1.30	47.69	20.89
				The second replication			
10	Y/Y	B	1.16	1.05	1.03	10.73	12.02
11	Y/M	B	1.56	0.65	1.04	87.50	74.13
12	Y/C	B	1.06	0.11	0.99	95.96	91.20
13	M/Y	G	0.13	0.05	1.29	6.20	6.17
14	M/M	G	1.57	1.26	1.28	24.22	10.72
15	M/C	G	1.57	0.41	1.32	87.88	69.18
16	C/Y	R	0.05	0.02	1.34	2.24	4.90
17	C/M	R	0.58	0.12	1.39	33.09	11.75
18	C/C	R	1.86	1.32	1.31	41.22	16.98
				The third replication			
19	Y/Y	B	1.14	1.015	1.00	12.50	30.90
20	Y/M	B	1.54	0.645	1.00	89.50	78.52
21	Y/C	B	1.08	0.115	1.01	95.54	90.16
22	M/Y	G	0.10	0.05	1.27	03.94	6.03
23	M/M	G	1.60	1.28	1.35	23.70	9.33
24	M/C	G	1.43	0.425	1.36	73.90	44.16
25	C/Y	R	0.07	0.02	1.43	3.50	4.17
26	C/M	R	0.56	0.13	1.27	33.86	14.45
27	C/C	R	1.88	1.36	1.46	35.62	12.02

APPENDICES D1 to D3

The Experimental Data of Polarizing Apparent Trapping

How to Read the Tables in Appendix D

- The first column lists the experimental number
- The second column lists the ink colors of the first and second ink layers. The letters Y, M, and C refer to yellow, magenta, and cyan inks, respectively. The slash should read as "over". Thus, Y/M refers to yellow ink printed over magenta ink.
- The third column lists the tack of the first and second ink layers. The letters l, m, and h refer to low, medium, and high tack, respectively. The slash should reads as "over". Thus l/m refers to low tack ink printed over medium tack ink.
- The fourth column lists the colors of the filters used to measure densities. The letters B, G, and R refer to blue, green, and red filter, respectively.
- The fifth column lists the polarizing density ($D_{2/1}$) of the overlapping area of the two inks.
- The sixth column lists the polarizing density (D_1) of the first ink layer printed directly on paper.
- The seventh column lists the polarizing density (D_2) of the second ink layer printed directly on paper.
- The eighth column lists the percent polarizing apparent trapping.

Appendix D1

The Experimental Data of Polarizing Apparent Trapping on Newsprint

The first replication

Exp. No.	Coordination Color	Coordination Tack	Filter Color	Density $D_{2/1}$	Density D_1	Density D_2	Polarizing Trapping(%)
1	Y/Y	l/l	B	1.44	1.26	1.18	15.25
2	Y/M	l/m	B	1.35	0.75	1.18	50.85
3	Y/C	l/h	B	1.25	0.16	1.21	90.08
4	M/Y	m/l	G	0.86	0.06	1.19	67.23
5	M/M	m/m	G	1.54	1.17	1.13	32.74
6	M/C	m/h	G	1.18	0.36	1.15	71.30
7	C/Y	h/l	R	0.74	0.00	1.11	66.67
8	C/M	h/m	R	1.00	0.08	1.25	73.60
9	C/C	h/h	R	1.62	1.13	1.28	38.28
				The second replication			
10	Y/Y	l/l	B	1.45	1.265	1.27	14.57
11	Y/M	l/m	B	1.37	0.69	1.22	55.74
12	Y/C	l/h	B	1.20	0.155	1.14	91.67
13	M/Y	m/l	G	0.90	0.05	1.24	68.55
14	M/M	m/m	G	1.59	1.24	1.19	29.41
15	M/C	m/h	G	1.23	0.36	1.18	73.73
16	C/Y	h/l	R	0.79	0.00	1.26	62.70
17	C/M	h/m	R	0.95	0.08	1.18	73.73
18	C/C	h/h	R	1.68	1.235	1.19	37.39
				The third replication			
19	Y/Y	l/l	B	1.44	1.16	1.18	23.73
20	Y/M	l/m	B	1.37	0.755	1.24	49.60
21	Y/C	l/h	B	1.25	0.16	1.22	89.34
22	M/Y	m/l	G	0.82	0.06	1.20	63.33
23	M/M	m/m	G	1.64	1.29	1.21	28.93
24	M/C	m/h	G	1.25	0.365	1.18	75.00
25	C/Y	h/l	R	0.83	0.00	1.20	69.17
26	C/M	h/m	R	0.99	0.09	1.31	68.70
27	C/C	h/h	R	1.68	1.21	1.22	38.52

Appendix D2

The Experimental Data of Polarizing Apparent Trapping on Uncoated Paper

The first replication

Exp. No.	Coordination Color	Coordination Tack	Filter Color	Density $D_{2/1}$	Density D_1	Density D_2	Polarizing Trapping(%)
1	Y/Y	l/l	B	1.44	1.12	1.14	28.07
2	Y/M	l/m	B	1.33	0.85	1.06	45.28
3	Y/C	l/h	B	1.19	0.22	1.07	90.65
4	M/Y	m/l	G	0.94	0.07	1.17	74.36
5	M/M	m/m	G	1.59	1.17	1.17	35.90
6	M/C	m/h	G	1.27	0.445	1.19	69.33
7	C/Y	h/l	R	0.94	0.005	1.21	72.27
8	C/M	h/m	R	1.14	0.11	1.28	80.47
9	C/C	h/h	R	1.75	1.24	1.27	40.16

The second replication

Exp. No.	Coordination Color	Coordination Tack	Filter Color	Density $D_{2/1}$	Density D_1	Density D_2	Polarizing Trapping(%)
10	Y/Y	l/l	B	1.44	1.11	1.16	28.45
11	Y/M	l/m	B	1.37	0.85	1.13	46.02
12	Y/C	l/h	B	1.21	0.22	1.20	82.50
13	M/Y	m/l	G	0.93	0.07	1.19	72.27
14	M/M	m/m	G	1.58	1.165	1.19	34.87
15	M/C	m/h	G	1.27	0.46	1.14	71.05
16	C/Y	h/l	R	0.95	0.01	1.13	83.19
17	C/M	h/m	R	1.06	0.11	1.17	81.20
18	C/C	h/h	R	1.82	1.335	1.29	37.60

The third replication

Exp. No.	Coordination Color	Coordination Tack	Filter Color	Density $D_{2/1}$	Density D_1	Density D_2	Polarizing Trapping(%)
19	Y/Y	l/l	B	1.45	1.145	1.14	26.75
20	Y/M	l/m	B	1.41	0.845	1.13	50.00
21	Y/C	l/h	B	1.20	0.22	1.15	85.22
22	M/Y	m/l	G	0.89	0.07	1.14	71.93
23	M/M	m/m	G	1.59	1.16	1.13	38.05
24	M/C	m/h	G	1.24	0.44	1.16	68.97
25	C/Y	h/l	R	0.92	0.01	1.18	77.12
26	C/M	h/m	R	1.14	0.115	1.22	84.02
27	C/C	h/h	R	1.74	1.23	1.21	42.15

Appendix D3

The Experimental Data of Polarizing Apparent Trapping on Coated Paper

The first replication

Exp. No.	Coordination Color	Coordination Tack	Filter Color	Density $D_{2/1}$	Density D_1	Density D_2	Polarizing Trapping(%)
1	Y/Y	l/l	B	1.83	1.525	1.64	18.60
2	Y/M	l/m	B	1.68	0.52	1.58	73.42
3	Y/C	l/h	B	1.58	0.10	1.54	96.10
4	M/Y	m/l	G	0.10	0.05	1.20	4.17
5	M/M	m/m	G	1.61	1.40	1.14	18.42
6	M/C	m/h	G	1.10	0.265	1.22	68.44
7	C/Y	h/l	R	0.03	0.020	1.24	0.81
8	C/M	h/m	R	0.43	0.045	1.15	33.48
9	C/C	h/h	R	1.93	1.28	1.38	47.10

The second replication

Exp. No.	Coordination Color	Coordination Tack	Filter Color	Density $D_{2/1}$	Density D_1	Density D_2	Polarizing Trapping(%)
10	Y/Y	l/l	B	1.83	1.63	1.61	12.42
11	Y/M	l/m	B	1.69	0.54	1.61	71.43
12	Y/C	l/h	B	1.59	0.10	1.54	96.75
13	M/Y	m/l	G	0.11	0.05	1.24	4.84
14	M/M	m/m	G	1.65	1.25	1.23	32.52
15	M/C	m/h	G	1.28	0.26	1.28	79.69
16	C/Y	h/l	R	0.05	0.02	1.46	1.41
17	C/M	h/m	R	0.54	0.055	1.49	32.55
18	C/C	h/h	R	1.99	1.455	1.47	36.39

The third replication

Exp. No.	Coordination Color	Coordination Tack	Filter Color	Density $D_{2/1}$	Density D_1	Density D_2	Polarizing Trapping(%)
19	Y/Y	l/l	B	1.81	1.57	1.56	15.38
20	Y/M	l/m	B	1.67	0.53	1.57	72.61
21	Y/C	l/h	B	1.59	0.10	1.57	94.90
22	M/Y	m/l	G	0.10	0.05	1.23	4.07
23	M/M	m/m	G	1.77	1.26	1.34	38.06
24	M/C	m/h	G	1.16	0.28	1.35	65.19
25	C/Y	h/l	R	0.08	0.02	1.55	3.87
26	C/M	h/m	R	0.54	0.05	1.38	3.87
27	C/C	h/h	R	2.08	1.49	1.60	36.88

APPENDICES E1 to E3

The Experimental Data for Determining Gravimetric Trapping Using Densities

How to Read the Tables in Appendix E

- The first column lists the experimental number
- The second column lists the ink colors of the first and second ink layers. The letters Y, M, and C refer to yellow, magenta, and cyan inks, respectively. The slash should read as "over". Thus, Y/M refers to yellow ink printed over magenta ink.
- The third column lists the colors o the filters used to measure the densities, D_1, D_2, and D_3. The letters B, G, and R refer to blue, green, and red filters.
- The fourth column lists the density (D_1) of the overlapping area of the two inks.
- The fifth column lists the density (D_2) of the first ink printed directly on paper.
- The sixth column lists the density (D_3) of the second ink printed directly on paper.
- The seventh column lists the color of filters used to measure the density, D_4.
- The eighth column lists the density (D_4) of the first ink printed directly on paper. But the density is measured by using the filter in column seven.
- The ninth column lists the percent gravimetric trapping.

Appendix E1

The Experimental Data for Determining Gravimetric Trapping Using Density for Newsprint

Exp. No.	Color Combin-ation	Filter Color for D_1, D_2, D_3	Density D_1	Density D_2	Density D_3	Filter Color for D_4	Density D_4	Gravi. Trapping (%)
				The first replication				
1	Y/Y	B	0.87	0.77	0.76	B	0.77	86.81
2	Y/M	B	1.03	0.75	0.72	G	1.00	91.47
3	Y/C	B	0.84	0.15	0.78	R	1.00	100.96
4	M/Y	G	0.81	0.06	1.03	B	0.77	55.73
5	M/M	G	1.16	1.00	1.00	G	1.00	75.49
6	M/C	G	1.10	0.51	1.01	R	0.96	94.27
7	C/Y	R	0.59	0.01	0.89	B	0.77	14.11
8	C/M	R	0.81	0.20	1.02	G	1.00	56.80
9	C/C	R	1.16	0.97	1.02	R	0.97	75.74
				The second replication				
10	Y/Y	B	0.89	0.78	0.70	B	0.78	83.13
11	Y/M	B	1.07	0.72	0.77	G	0.985	98.99
12	Y/C	B	0.81	0.15	0.76	R	0.93	79.95
13	M/Y	G	0.83	0.055	1.03	B	0.76	54.55
14	M/M	G	1.16	1.02	1.02	G	1.02	77.34
15	M/C	G	1.12	0.51	1.01	R	0.95	98.84
16	C/Y	R	0.65	0.01	1.00	B	0.77	27.33
17	C/M	R	0.80	0.195	0.94	G	1.00	51.53
18	C/C	R	1.21	1.00	0.95	R	1.00	80.24
				The third replication				
19	Y/Y	B	0.89	0.77	0.755	B	0.755	83.93
20	Y/M	B	1.03	0.76	0.77	G	1.00	93.67
21	Y/C	B	0.84	0.15	0.79	R	0.97	105.70
22	M/Y	G	0.80	0.06	1.04	B	0.77	54.47
23	M/M	G	1.16	1.025	1.03	G	1.025	84.10
24	M/C	G	1.13	0.51	1.04	R	0.99	90.91
25	C/Y	R	0.62	0.015	0.94	B	0.765	34.94
26	C/M	R	0.82	0.21	1.03	G	1.05	53.51
27	C/C	R	1.19	0.99	0.98	R	0.99	78.53

Appendix E2
The Experimental Data for Determining Gravimetric Trapping Using Density for Uncoated Paper

Exp. No.	Color Combin-ation	Filter Color for D_1, D_2, D_3	Density D_1	Density D_2	Density D_3	Filter Color for D_4	Density D_4	Gravi. Trapping (%)
				The first replication				
1	Y/Y	B	0.96	0.80	0.79	B	0.80	103.14
2	Y/M	B	1.11	0.84	0.77	G	1.06	114.77
3	Y/C	B	0.91	0.22	0.78	R	1.01	117.70
4	M/Y	G	0.93	0.05	1.08	B	0.80	81.81
5	M/M	G	1.22	1.065	1.06	G	1.065	101.10
6	M/C	G	1.22	0.59	1.07	R	0.995	105.23
7	C/Y	R	0.77	0.005	1.02	B	0.79	53.69
8	C/M	R	0.95	0.245	1.08	G	1.05	79.62
9	C/C	R	1.29	1.02	1.07	R	1.02	99.01
				The second replication				
10	Y/Y	B	0.95	0.79	0.81	B	0.79	101.68
11	Y/M	B	1.11	0.845	0.79	G	1.06	111.37
12	Y/C	B	0.90	0.22	0.82	R	1.02	115.63
13	M/Y	G	0.93	0.05	1.07	B	0.795	77.42
14	M/M	G	1.24	1.06	1.07	G	1.06	97.61
15	M/C	G	1.19	0.62	1.04	R	1.035	102.94
16	C/Y	R	0.80	0.01	0.98	B	0.81	56.41
17	C/M	R	0.88	0.25	0.94	G	1.05	67.21
18	C/C	R	1.29	1.075	1.04	R	1.075	93.80
				The third replication				
19	Y/Y	B	0.95	0.81	0.79	B	1.06	114.77
20	Y/M	B	1.13	0.835	0.78	G	1.05	117.22
21	Y/C	B	0.90	0.22	0.82	R	1.025	113.90
22	M/Y	G	0.87	0.05	1.04	B	0.81	70.52
23	M/M	G	1.21	1.06	1.04	G	1.06	95.72
24	M/C	G	1.18	0.59	1.05	R	0.99	105.43
25	C/Y	R	0.76	0.01	0.98	B	0.805	55.43
26	C/M	R	0.93	0.25	1.03	G	1.065	79.23
27	C/C	R	1.25	1.02	1.00	R	1.02	93.80

Appendix E3
The Experimental Data for Determining Gravimetric Trapping Using Density for Coated Paper

Exp. No.	Color Combin- ation	Filter Color for D_1, D_2, D_3	Density D_1	Density D_2	Density D_3	Filter Color for D_4	Density D_4	Gravi. Trapping (%)
				The first replication				
1	Y/Y	B	1.17	1.00	1.05	B	1.00	48.97
2	Y/M	B	1.53	0.63	1.02	G	1.22	93.89
3	Y/C	B	1.08	0.11	1.01	R	1.18	102.36
4	M/Y	G	0.11	0.05	1.23	B	0.97	-37.43
5	M/M	G	1.54	1.41	1.20	G	1.41	46.56
6	M/C	G	1.37	0.40	1.27	R	1.14	66.29
7	C/Y	R	0.04	0.02	1.15	B	1.00	-59.86
8	C/M	R	0.47	0.12	1.08	G	1.23	13.24
9	C/C	R	1.80	1.18	1.30	R	1.18	42.68
				The second replication				
10	Y/Y	B	1.16	1.05	1.03	B	1.05	35.60
11	Y/M	B	1.56	0.65	1.04	G	1.24	87.02
12	Y/C	B	1.06	0.11	0.99	R	1.18	93.60
13	M/Y	G	0.13	0.05	1.29	B	0.97	-50.56
14	M/M	G	1.57	1.26	1.28	G	1.26	32.58
15	M/C	G	1.57	0.41	1.32	R	1.14	75.25
16	C/Y	R	0.05	0.02	1.34	B	0.98	-42.29
17	C/M	R	0.58	0.12	1.39	G	1.23	26.29
18	C/C	R	1.86	1.32	1.31	R	1.32	33.72
				The third replication				
19	Y/Y	B	1.14	1.015	1.00	B	1.015	41.64
20	Y/M	B	1.54	0.645	1.00	G	1.24	89.76
21	Y/C	B	1.08	0.115	1.01	R	1.19	112.80
22	M/Y	G	0.10	0.05	1.27	B	0.98	-37.04
23	M/M	G	1.60	1.28	1.35	G	1.28	45.19
24	M/C	G	1.43	0.425	1.36	R	1.22	77.36
25	C/Y	R	0.07	0.02	1.43	B	1.01	-43.79
26	C/M	R	0.56	0.13	1.27	G	1.27	25.81
27	C/C	R	1.88	1.36	1.46	R	1.36	48.89

APPENDIX F

The Program Listing of the Analysis of Variance

Appendix F

The Program Listing of Analysis of Variance

```
!JOB 662C5HDT,F660709HLACH(ROSS),7.KONGSAK
!LIMIT (TIME,1),(CORE,6)
!RUN (LMN,ANOVA,LMNLTB2)
!DATA
TRAP  05        A0003B0003C0003D0004R0003
48.97 103.1486.81 93.89 114.7791.47 102.36117.70100.96-37.4381.81 55.73
46.56 101.1075.49 66.29 105.2394.27 -59.8653.69 14.11 13.24 79.62 56.80
42.68 99.01 75.74 16.19 20.25 13.16 88.24 35.06 38.89 96.04 88.46 88.46
04.88 81.48 72.82 10.83 14.62 16.00 76.38 58.88 59.50 1.74  75.00 65.17
32.41 65.28 60.78 47.69 25.23 18.63 13.18 23.47 21.88 75.86 31.62 36.31
91.20 81.28 81.28 06.76 63.10 52.48 08.51 12.45 14.45 50.12 36.31 38.02
07.41 55.59 48.98 18.62 42.17 39.81 20.89 15.85 14.79 18.60 28.07 15.25
73.42 45.28 50.85 96.10 90.65 90.08 04.17 74.36 67.23 18.42 35.90 32.74
68.44 69.33 71.30 00.81 77.27 66.67 33.48 80.47 73.60 47.10 40.16 38.28
35.60 101.6883.13 87.02 111.3798.99 93.60 115.6397.95 -37.0477.42 54.55
32.59 97.61 77.34 75.25 102.9494.84 -42.2956.41 27.33 26.29 67.21 51.53
33.72 97.46 80.24 10.73 19.75 13.92 87.50 33.54 45.45 95.96 82.93 84.84
06.20 82.24 75.24 24.22 16.82 13.37 87.68 54.81 60.40 2.24  80.61 64.00
33.09 67.02 64.36 41.22 20.69 22.11 12.02 22.39 20.89 74.13 29.85 38.02
91.20 72.44 79.43 06.17 64.57 55.59 10.72 12.88 13.18 69.18 33.88 39.81
04.90 64.57 43.65 11.75 48.96 46.24 16.99 14.96 18.20 12.42 28.45 14.57
71.43 46.02 55.74 96.75 82.50 91.67 04.84 72.27 68.55 32.52 34.87 29.41
79.69 71.05 73.73 01.41 93.19 62.70 32.55 81.20 73.73 36.39 37.60 39.39
41.64 104.4183.73 89.76 117.2293.67 112.80113.90105.70-50.5670.52 54.47
45.19 95.72 84.10 71.36 105.4890.91 -43.7955.43 34.94 25.81 79.23 53.57
48.89 93.80 73.53 12.50 17.72 17.53 89.50 37.82 35.06 95.54 82.93 87.34
3.94  78.65 71.15 23.70 14.42 15.05 73.90 56.19 59.62 -3.50 76.53 64.36
33.08 60.02 59.22 35.62 23.00 20.41 13.34 23.39 23.17 78.52 32.73 31.62
90.16 72.44 79.43 06.03 60.26 50.12 09.33 12.74 13.34 44.16 34.67 38.02
04.17 58.88 51.88 14.45 44.67 38.02 12.02 16.98 16.60 15.38 26.75 23.73
72.61 50.00 48.60 94.90 85.22 89.34 04.07 71.93 67.23 38.06 38.02 28.93
65.19 68.97 75.00 03.87 77.12 69.17 33.51 84.02 68.70 36.88 42.15 38.52
!END
```

APPENDIX G

The Program Listing for the Relationship of Gravimetric and Apparent Trapping

The Program Listing for Newsprint, Uncoated Paper, and Coated Paper is:

```
!JOB 662C5HDT,F660709HIACH(PDSS),7.KONGSAK
!LIMIT (CORE,24),(TIME,1),(ORDER)
!RUN (LMN,SPSS,LMNIB2)
!DATA
VARIABLE = 10
RUN NAME          REGRESSION OF TRAPPING BETWEEN GRAV. AND DENST.
VARIABLE LIST     GRAV,DENST
N OF CASES        UNKNOWN
INPUT MEDIUM      CARD
INPUT FORMAT      FREEFIELD
VAR LABELS        GRAV      %TRAPPING OF GRAVIMETRIC METHOD
                  DENSI      % TRAPPING OF DENSITOMETRIC METHOD
REGRESSION        VARIABLES = GRAV,DENST/
                  REGRESSION = GRAV WITH DENST(1) RESID = 0/
OPTIONS           11,12,15
STATISTICS        ALL
READ INPUT DATA
86.81 13.16
83.13 13.97
83.73 17.53
91.47 38.69
98.99 45.45
93.67 35.06
100.96 88.46
97.95 94.84
105.7 87.34
55.73 72.67
54.59 75.24
54.47 71.15
75.49 16.0
77.34 13.37
84.1 15.05
94.27 59.5
94.84 60.4
90.91 59.62
14.11 65.17
77.33 64.
34.94 64.36
56.80 60.78
51.53 64.36
53.57 59.27
75.74 19.63
80.24 22.11
78.53 20.41
103.14 20.75
101.68 19.75
104.41 17.72
114.77 35.06
111.37 33.54
117.22 37.82
117.7 88.46
115.63 92.93
113.90 32.93
81.81 81.48
77.42 92.24
70.52 78.85
101.10 14.62
97.61 16.82
95.72 14.47
105.73 58.88
102.94 54.81
105.48 56.19
53.69 75.0
56.41 80.61
55.43 76.53
79.62 65.28
67.21 67.02
79.23 66.02
99.01 75.23
97.46 20.67
93.80 23.0
48.97 10.19
35.60 10.73
41.64 12.50
93.89 98.24
87.02 97.5
89.76 99.5
107.36 96.60
093.60 095.96
112.80 095.54
```

```
-37.43 4.88
-37.04 6.20
-50.56 3.94
046.56 10.83
032.58 24.22
045.19 073.70
66.29 76.38
75.25 87.88
77.36 73.90
-59.86 1.74
-42.29 2.24
-043.79 3.50
13.24 32.41
26.29 33.09
25.81 33.86
42.68 47.69
33.72 41.27
48.89 35.67
!END
REGRESSION          VARIABLES = GRAV,DENSI,X2/
                    REGRESSION = GRAV WITH DENSI TO X2(3) RESID = 0/
OPTIONS             11,12,15
STATISTICS          ALL
COMPUTE             X2 = (DENSI**2)/100
COMPUTE             X3 = (DENSI**3)/1000
REGRESSION          VARIABLES = GRAV,DENSI,X2,X3/
                    REGRESSION = GRAV WITH DENSI TO X3(5) RESID = 0/
OPTIONS             11,12,15
STATISTICS          ALL
FINISH
```

The Program Listing for Newsprint is:

```
!JOB 6620SHCT,F660709HLACH(ROSS),7.KONGSAK
!LIMIT (CORE,24),(TIME,1),(ORDER)
!RUN (LMN,SPSS,LMNLIB2)
!DATA
VARIABLE = 10
RUN NAME          REGRESSION OF TRAPPING BETWEEN GRAV. AND DENST.
VARIABLE LIST     GRAV,DENST
N OF CASES        UNKNOWN
INPUT MEDIUM      CARD
INPUT FORMAT      FREEFIELD
VAR LABELS        GRAV      $TRAPPING BY GRAVIMETRIC METHOD
                  DENSI      $ TRAPPING OF DENSITOMETRIC METHOD
REGRESSION        VARIABLES = GRAV,DENST/
                  REGRESSION = GRAV WITH DENST(1) RESTD = 0/
OPTIONS           11,12,15
STATISTICS        ALL
READ INPUT DATA
80.81 13.16
83.13 13.92
83.73 17.53
91.47 39.69
93.99 45.45
93.07 35.06
100.06 58.46
97.95 94.84
105.7 87.34
55.73 72.62
54.55 75.24
54.47 71.15
75.49 16.0
77.34 13.37
84.1 15.05
94.27 59.5
94.64 60.4
90.91 59.62
14.11 65.17
27.33 64.
34.94 64.36
56.60 60.78
51.53 64.36
53.57 59.22
75.74 18.63
80.24 22.11
78.53 20.41
!END
REGRESSION        VARIABLES = GRAV,DENST,X2/
                  REGRESSION = GRAV WITH DENST TO X2(3) RESTD = 0/
OPTIONS           11,12,15
STATISTICS        ALL
COMPUTE           X2 = (DENSI**2)/100
COMPUTE           X3 = (DENSI**3)/1000
REGRESSION        VARIABLES = GRAV,DENST,X2,X3/
                  REGRESSION = GRAV WITH DENST TO X3(5) RESTD = 0/
OPTIONS           11,12,15
STATISTICS        ALL
FINISH
```

The Program Listing for Uncoated Paper is:

```
!JOB 662C5HCT,F660709HIACH(ROSS),7.KONGSAK
!LIMIT (CORE,24),(TIME,1),(ORDER)
!RUN (LMN,SPSS,LMNLIB2)
!DATA
VARIABLE = 10
RUN NAME          REGRESSION OF TRAPPING BETWEEN GRAV. AND DENST.
VARIABLE LIST     GRAV,DENST
N OF CASES        UNKNOWN
INPUT MEDIUM      CARD
INPUT FORMAT      FREEFIELD
VAR LABELS        GRAV      %TRAPPING BY GRAVIMETRIC METHOD
                  DENSI      % TRAPPING OF DENSITOMETRIC METHOD
REGRESSION        VARIABLES = GRAV,DENST/
                  REGRESSION = GRAV WITH DENST(1) RESID = 0/
OPTIONS           11,12,15
STATISTICS        ALL
READ INPUT DATA
103.14 20.25
101.68 19.75
104.41 17.72
114.77 35.06
111.37 33.54
117.22 37.82
117.7 98.46
115.63 52.93
113.90 87.93
91.81 91.49
77.42 82.24
70.52 78.85
101.10 14.62
97.01 16.87
95.72 14.47
105.23 59.98
102.94 54.91
105.38 56.19
53.80 75.0
56.41 80.01
55.43 76.53
79.07 65.28
67.21 67.02
79.23 66.02
99.01 25.23
97.46 20.67
33.80 23.0
!END
REGRESSION        VARIABLES = GRAV,DENSI,X2/
                  REGRESSION = GRAV WITH DENST TO X2(3) RESID = 0/
OPTIONS           11,12,15
STATISTICS        ALL
COMPUTE           X2 = (DENSI**2)/100
COMPUTE           X3 = (DENSI**3)/1000
REGRESSION        VARIABLES = GRAV,DENST,X2,X3/
                  REGRESSION = GRAV WITH DENST TO X3(5) RESID = 0/
OPTIONS           11,12,15
STATISTICS        ALL
FINISH
```

The Program Listing for Coated Paper is:

```
!JOB 662C5HDT,F660709HIACH(RCSS),7.KONGSAK
!LIMIT (CORE,24),(TIME,1),(ORDER)
!RUN (LMN,SPSS,LMNLIB2)
!DATA
VARIABLE = 10
RUN NAME          REGRESSION OF TRAPPING BETWEEN GRAV. AND DENST.
VARIABLE LIST     GRAV,DENST
N OF CASES        UNKNOWN
INPUT MEDIUM      CARD
INPUT FORMAT      FREEFIELD
VAR LABELS        GRAV       *TRAPPING BY GRAVIMETRIC METHOD
                  DENSI       % TRAPPING OF DENSITOMETRIC METHOD
REGRESSION        VARIABLES = GRAV,DENST/
                  REGRESSION = GRAV WITH DENST(1) RESTD = 0/
OPTIONS           11,12,15
STATISTICS        ALL
READ INPUT DATA
48.97 16.19
35.60 10.73
41.54 12.50
93.89 98.24
87.02 97.5
99.76 99.5
102.36 96.60
093.60 095.96
112.90 095.54
-37.43 4.88
-37.04 6.20
-50.56 3.94
046.56 10.83
032.58 24.22
045.19 073.70
66.29 76.38
75.25 97.89
77.36 73.90
-59.86 1.74
-42.29 2.24
-043.79 3.50
13.22 32.41
26.29 33.09
25.81 33.86
42.69 47.69
33.72 41.22
44.69 35.62
!EOD
REGRESSION        VARIABLES = GRAV,DENST,Y2/
                  REGRESSION = GRAV WITH DENST TO X2(3) RESTD = 0/
OPTIONS           11,12,15
STATISTICS        ALL
COMPUTE           X2 = (DENSI**2)/100
COMPUTE           X3 = (DENSI**3)/1000
REGRESSION        VARIABLES = GRAV,DENST,Y2,X3/
                  REGRESSION = GRAV WITH DENST TO X3(5) RESTD = 0/
OPTIONS           11,12,15
STATISTICS        ALL
FINISH
```

APPENDIX H

The Program Listing for Determination of Gravimetric Trapping Using Densities

The Program Listing for Newsprint, Uncoated Paper, and Coated Paper is:

```
!JOB 66205HDT,F660709HLACH(ROSS),7.KONGSAK
!LIMIT (CORE,28),(TIME,1)
!RUN (LMN,SPSS,LMNLIB2)
!DATA
RUN NAME        DETERMINATION OF GRAV. TRAPPING USING DENSITIES
VARIABLE = 75
VARIABLE LIST   X1,X2,X3,Y4,Y
N OF CASES      UNKNOWN
INPUT MEDIUM    CARD
INPUT FORMAT    FREEFIELD
VAR LABELS      X1           DENSITY OF OVERLAPPING AREA
                X2           DENSITY OF FIRST INK LAYER ON PAPER
                X3           DENSITY OF SECOND INK LAYER ON PAPER
                X4           HIGHEST DENSITY OF FIRST INK ON PAPER
                Y            PERCENT GRAVIMETRIC TRAPPING
COMPUTE         X1X2 = X1*X2
COMPUTE         X1X3 = Y1*X3
COMPUTE         X1X4 = X1*X4
COMPUTE         X2X3 = Y2*X3
COMPUTE         X2X4 = Y2*X4
COMPUTE         X3X4 = Y3*X4
COMPUTE         SQX1 = Y1**2
COMPUTE         SQX2 = Y2**2
COMPUTE         SQX3 = Y3**2
COMPUTE         SQX4 = Y4**2
COMPUTE         X1X2X3 = Y1*X2*X3
COMPUTE         X1X2X4 = Y1*X2*Y4
COMPUTE         X1X3X4 = Y1*X3*Y4
COMPUTE         X2X3X4 = Y2*X3*Y4
COMPUTE         TY1 = X1**3
COMPUTE         TY2 = X2**3
COMPUTE         TY3 = X3**3
COMPUTE         TY4 = X4**3
COMPUTE         X1X2X3X4 = Y1*X2*Y3*X4
COMPUTE         FY1 = X1**4
COMPUTE         FY2 = X2**4
COMPUTE         FY3 = X3**4
COMPUTE         FY4 = X4**4
REGRESSION      VARIABLES = Y, X1 TO Y4, X1X2 TO FX4/
                REGRESSION = Y WITH X1 TO X4(19) RESID=0/
OPTIONS         6,11,12,15
STATISTICS      4
READ INPUT DATA
.87,.77,.76,.77,80.61
.89,.78,.79,.79,83.13
.89,.755,.77,.755,93.93
1.03,.75,.72,1.0,91.47
1.07,.72,.77,.985,98.90
1.03,.76,.77,1.0,93.67
.84,.15,.78,1.0,100.96
.81,.15,.76,.93,79.95
.84,.15,.79,.97,105.70
.81,.06,1.03,.77,55.73
.83,.055,1.03,.76,54.55
.80,.06,1.04,.77,54.47
1.16,1.0,1.0,1.0,75.49
1.16,1.02,1.02,1.02,77.34
1.16,1.025,1.03,1.025,84.1
1.10,.51,1.01,.96,94.27
1.12,.51,1.01,.95,98.64
1.13,.51,1.04,.99,90.91
.59,.01,.89,.77,14.11
.65,.01,1.0,.77,27.33
.67,.015,.84,.765,34.94
.81,.19,1.02,1.0,56.80
.8,.195,.94,1.0,51.53
.85,.21,1.03,1.05,53.57
1.16,.97,1.07,.97,75.74
1.21,1.0,.95,1.0,80.24
1.19,.99,.98,.99,79.53
.96,.8,.76,.8,103.14
.95,.76,.81,.79,101.68
.95,.81,.79,.81,104.41
1.11,.84,.77,1.06,114.77
1.11,.845,.79,1.06,111.37
1.13,.835,.78,1.05,117.27
.91,.27,.78,1.01,117.7
.9,.72,.82,1.07,115.63
.9,.72,.82,1.025,113.90
.93,.08,1.08,.8,61.81
.93,.05,1.07,.795,77.42
.87,.05,1.04,.81,70.52
1.22,1.065,1.06,1.065,101.1
1.24.1.06,1.07.1.06.97.61
```

```
1.21,1.06,1.04,1.06,95.72
1.22,.59,1.07,.905,105.23
1.19,.62,1.04,1.035,102.94
1.18,.59,1.05,.90,105.48
.77,.005,1.02,.70,53.60
.8,.01,.68,.81,56.41
.76,.01,.96,.805,55.43
.95,.245,1.05,1.05,70.62
.88,.25,.94,1.05,67.21
.93,.25,1.03,1.065,70.23
1.29,1.02,1.02,1.02,99.01
1.29,1.075,1.04,1.075,67.46
1.25,1.07,1.0,1.02,93.80
1.17,1.,1.05,1.,48.97
1.16,1.05,1.03,1.05,35.6
1.14,1.015,1.0,1.015,41.64
1.53,.63,1.02,1.22,93.80
1.56,.65,1.04,1.24,87.02
1.54,.645,1.,1.24,89.76
1.08,.11,1.01,1.18,102.36
1.06,.11,1.01,1.18,93.6
1.08,.115,1.01,1.18,112.8
.11,.05,1.23,.67,-37.43
.1,.05,1.27,.95,-37.04
.13,.05,1.29,.97,-50.56
1.54,1.41,1.2,1.41,46.58
1.57,1.26,1.26,1.26,32.59
1.6,1.28,1.35,1.28,45.19
1.37,.4,1.27,1.14,66.29
1.57,.41,1.32,1.14,75.25
1.43,.425,1.36,1.22,77.35
.04,.02,1.15,1.0,-59.86
.05,.02,1.34,.96,-42.29
.07,.02,1.43,1.01,-43.79
.47,.12,1.08,1.23,13.24
.58,.12,1.39,1.23,26.29
.56,.13,1.27,1.27,25.81
1.8,1.16,1.3,1.18,42.68
1.86,1.32,1.31,1.32,33.72
1.88,1.36,1.46,1.36,48.89
!END
REGRESSION        VARIABLES = Y, X1 TO X4, X1X2 TO FX4/
                  REGRESSION = Y WITH X1 TO SQX4(29) RESID=0/
OPTIONS           6,11,12,16
STATISTICS        4
REGRESSION        VARIABLES = Y, X1 TO X4, X1X2 TO FX4/
                  REGRESSION = Y WITH X1 TO TX4(11) RESID=0/
OPTIONS           6,11,12,16
STATISTICS        4
REGRESSION        VARIABLES = Y, X1 TO X4, X1X2 TO FX4/
                  REGRESSION = Y WITH X1 TO FX4(30) RESID=0/
OPTIONS           6,11,12,16
STATISTICS        4
REGRESSION        VARIABLES = Y, X1 TO X4, X1X2 TO FX4/
                  REGRESSION = Y(10,4,.05) WITH X1 TO X4(11) RESID=0/
OPTIONS           6,11,12,16
STATISTICS        4
REGRESSION        VARIABLES = Y, X1 TO X4, X1X2 TO FX4/
                  REGRESSION = Y(10,4,.05) WITH X1 TO SQX4(15) RESID=0/
OPTIONS           6,11,12,16
STATISTICS        4
REGRESSION        VARIABLES = Y, X1 TO X4, X1X2 TO FX4/
                  REGRESSION = Y(10,4,.05) WITH X1 TO TX4(45) RESID=0/
OPTIONS           6,11,12,16
STATISTICS        4
REGRESSION        VARIABLES = Y, X1 TO X4, X1X2 TO FX4/
                  REGRESSION = Y(10,4,.05) WITH X1 TO FX4(55) RESID=0/
OPTIONS           6,11,12,16
STATISTICS        4
FINISH
```

The Program Listing for Newsprint is:

```
!JOB 662C5HDT,F660709HIACH(ROSS),7.KONGSAK
!LIMIT (CORE,29),(TIME,1)
!RUN (LMN,SESS,LMNLIB2)
!DATA
RUN NAME          DETERMINATION OF GRAV. TRAPPING USING DENSITIES
VARIABLE = 75
VARIABLE LIST     X1,Y2,X3,Y4,Y
N OF CASES        UNKNOWN
INPUT MEDIUM      CARD
INPUT FORMAT      FREEFIELD
VAR LABELS        X1        DENSITY OF OVERLAPPING AREA
                  X2        DENSITY OF FIRST INK LAYER ON PAPER
                  X3        DENSITY OF SECOND INK LAYER ON PAPER
                  X4        HIGHEST DENSITY OF FIRST INK ON PAPER
                  Y         PERCENT GRAVIMETRIC TRAPPING
COMPUTE           X1X2 = Y1*X2
COMPUTE           X1X3 = Y1*X3
COMPUTE           X1X4 = Y1*X4
COMPUTE           X2X3 = Y2*X3
COMPUTE           X2X4 = X2*X4
COMPUTE           X3X4 = Y3*X4
COMPUTE           SQX1 = Y1**2
COMPUTE           SQX2 = Y2**2
COMPUTE           SQX3 = Y3**2
COMPUTE           SQX4 = Y4**2
COMPUTE           X1X2X3 = Y1*X2*Y3
COMPUTE           X1X2X4 = Y1*X2*X4
COMPUTE           X1X3X4 = Y1*X3*X4
COMPUTE           X2X3X4 = Y2*X3*X4
COMPUTE           TX1 = X1**3
COMPUTE           TX2 = X2**3
COMPUTE           TX3 = X3**3
COMPUTE           TX4 = X4**3
COMPUTE           X1X2X3X4 = Y1*X2*Y3*X4
COMPUTE           FX1 = X1**4
COMPUTE           FX2 = X2**4
COMPUTE           FX3 = X3**4
COMPUTE           FX4 = X4**4
REGRESSION        VARIABLES = Y, X1 TO Y4, Y1X2 TO FX4/
                  REGRESSION = Y WITH X1 TO X4(19) RESID=0/
OPTIONS           6,11,12,16
STATISTICS        4
READ INPUT DATA
.87,.77,.76,.77,86.81
.89,.79,.79,.79,83.13
.89,.755,.77,.755,93.93
1.03,.75,.72,1.0,91.47
1.07,.72,.77,.98,98.90
1.03,.76,.77,1.0,63.67
.84,.15,.79,1.0,100.96
.81,.15,.76,.93,79.95
.84,.15,.79,.97,105.70
.81,.06,1.03,.77,55.73
.83,.055,1.03,.76,54.55
.80,.06,1.04,.77,54.47
1.16,1.0,1.0,1.0,75.49
1.16,1.07,1.02,1.07,77.34
1.18,1.075,1.03,1.025,84.1
1.10,.51,1.01,.96,94.27
1.12,.51,1.01,.95,98.84
1.13,.51,1.04,.99,90.91
.59,.01,.89,.77,14.11
.65,.01,1.0,.77,27.33
.67,.015,.94,.765,34.94
.81,.19,1.02,1.0,56.80
.8,.195,.94,1.0,51.53
.85,.21,1.03,1.05,53.57
1.16,.97,1.05,.97,75.74
1.21,1.0,.95,1.0,80.74
1.19,.99,.98,.99,78.53
!EOD
REGRESSION        VARIABLES = Y, X1 TO Y4, Y1X2 TO FX4/
                  REGRESSION = Y WITH X1 TO SQX4(29) RESID=0/
OPTIONS           6,11,12,16
STATISTICS        4
REGRESSION        VARIABLES = Y, X1 TO X4, Y1X2 TO FX4/
                  REGRESSION = Y WITH X1 TO TX4(11) RESID=0/
OPTIONS           6,11,12,15
STATISTICS        4
REGRESSION        VARIABLES = Y, X1 TO X4, X1X2 TO FX4/
                  REGRESSION = Y WITH X1 TO FX4(39) RESID=0/
OPTIONS           6,11,12,16
STATISTICS        4
REGRESSION        VARIABLES = Y, X1 TO Y4, Y1X2 TO FX4/
```

```
                REGRESSION = Y(10,4,.05) WITH X1 TO X4(11) RESID=0/
OPTIONS         6,11,12,16
STATISTICS      4
REGRESSION      VARIABLES = Y, X1 TO X4, X1X2 TO FX4/
                REGRESSION = Y(10,4,.05) WITH X1 TO SQX4(15) RESID=0/
OPTIONS         6,11,12,16
STATISTICS      4
REGRESSION      VARIABLES = Y, X1 TO X4, X1X2 TO FX4/
                REGRESSION = Y(10,4,.05) WITH X1 TO TX4(45) RESID=0/
OPTIONS         6,11,12,16
STATISTICS      4
REGRESSION      VARIABLES = Y, X1 TO X4, X1X2 TO FX4/
                REGRESSION = Y(10,4,.05) WITH X1 TO FX4(55) RESID=0/
OPTIONS         6,11,12,16
STATISTICS      4
FINISH
```

The Program Listing for Uncoated Paper is:

```
!JOB 662C5HDT,F660709HLACH(ROSS),7.KONGSAK
!LIMIT (CORE,28),(TIME,1)
!RUN (LMN,SPSS,LMNLIB2)
!DATA
RUN NAME          DETERMINATION OF GRAV. TRAPPING USING DENSITIES
VARIABLE = 75
VARIABLE LIST     X1,X2,X3,X4,Y
N OF CASES        UNKNOWN
INPUT MEDIUM      CARD
INPUT FORMAT      FREEFIELD
VAR LABELS        X1          DENSITY OF OVERLAPPING AREA
                  X2          DENSITY OF FIRST INK LAYER ON PAPER
                  X3          DENSITY OF SECOND INK LAYER ON PAPER
                  X4          HIGHEST DENSITY OF FIRST INK ON PAPER
                  Y           PERCENT GRAVIMETRIC TRAPPING
COMPUTE           X1X2 = X1*X2
COMPUTE           X1X3 = X1*X3
COMPUTE           X1X4 = X1*X4
COMPUTE           X2X3 = X2*X3
COMPUTE           X2X4 = X2*X4
COMPUTE           X3X4 = X3*X4
COMPUTE           SQX1 = X1**2
COMPUTE           SQX2 = X2**2
COMPUTE           SQX3 = X3**2
COMPUTE           SQX4 = X4**2
COMPUTE           X1X2X3 = X1*X2*X3
COMPUTE           X1X2X4 = X1*X2*X4
COMPUTE           X1X3X4 = X1*X3*X4
COMPUTE           X2X3X4 = X2*X3*X4
COMPUTE           TX1 = X1**3
COMPUTE           TX2 = X2**3
COMPUTE           TX3 = X3**3
COMPUTE           TX4 = X4**3
COMPUTE           X1X2X3X4 = X1*X2*X3*X4
COMPUTE           FX1 = X1**4
COMPUTE           FX2 = X2**4
COMPUTE           FX3 = X3**4
COMPUTE           FX4 = X4**4
REGRESSION        VARIABLES = Y, X1 TO X4, X1X2 TO FX4/
                  REGRESSION = Y WITH X1 TO X4(19) RESID=0/
OPTIONS           6,11,12,16
STATISTICS        4
READ INPUT DATA
.96,.8,.79,.8,103.14
.95,.76,.81,.79,101.68
.95,.81,.79,.81,104.41
1.11,.94,.77,1.06,114.77
1.11,.845,.76,1.06,111.37
1.13,.835,.78,1.05,117.22
.91,.27,.78,1.01,117.7
.9,.72,.82,1.02,115.63
.9,.72,.82,1.075,113.90
.93,.05,1.08,.8,81.81
.93,.05,1.07,.795,77.42
.87,.05,1.04,.81,70.52
1.22,1.065,1.06,1.065,101.1
1.24,1.06,1.07,1.06,97.61
1.21,1.06,1.04,1.06,95.72
1.22,.59,1.07,.995,105.23
1.19,.62,1.04,1.035,102.94
1.18,.59,1.05,.99,105.48
.77,.005,1.02,.79,53.69
.8,.01,.68,.81,56.41
.76,.01,.98,.805,55.43
.95,.245,1.08,1.05,79.62
.88,.25,.94,1.05,67.21
.93,.25,1.03,1.065,79.73
1.29,1.02,1.02,1.02,89.01
1.29,1.075,1.04,1.075,87.46
1.25,1.02,1.0,1.02,93.80
!END
REGRESSION        VARIABLES = Y, X1 TO X4, X1X2 TO FX4/
                  REGRESSION = Y WITH X1 TO SQX4(29) RESID=0/
OPTIONS           6,11,12,16
STATISTICS        4
REGRESSION        VARIABLES = Y, X1 TO X4, X1X2 TO FX4/
                  REGRESSION = Y WITH X1 TO TX4(11) RESID=0/
OPTIONS           6,11,12,16
STATISTICS        4
REGRESSION        VARIABLES = Y, X1 TO X4, X1X2 TO FX4/
                  REGRESSION = Y WITH X1 TO FX4(30) RESID=0/
OPTIONS           6,11,12,16
STATISTICS        4
REGRESSION        VARIABLES = Y, X1 TO X4, X1X2 TO FX4/
```

```
                    REGRESSION = Y(10,4,.05) WITH X1 TO X4(11) RESID=0/
OPTIONS             6,11,12,16
STATISTICS          4
REGRESSION          VARIABLES = Y, X1 TO X4, X1X2 TO FX4/
                    REGRESSION = Y(10,4,.05) WITH X1 TO SQX4(15) RESID=0/
OPTIONS             6,11,12,16
STATISTICS          4
REGRESSION          VARIABLES = Y, X1 TO X4, X1X2 TO FX4/
                    REGRESSION = Y(10,4,.05) WITH X1 TO TX4(45) RESID=0/
OPTIONS             6,11,12,16
STATISTICS          4
REGRESSION          VARIABLES = Y, X1 TO X4, X1X2 TO FX4/
                    REGRESSION = Y(10,4,.05) WITH X1 TO FX4(55) RESID=0/
OPTIONS             6,11,12,16
STATISTICS          4
FINISH
```

The Program Listing for Coated Paper is:

```
!JOB 662C5HDT,F660709HLACH(PCSS),7.KONGSAK
!LIMIT (CORE,28),(TIME,1)
!RUN (LMN,SPSS,LMNLIP2)
!DATA
RUN NAME          DETERMINATION OF GRAV. TRAPPING USIND DENSITIES
VARIABLE = 75
VARIABLE LIST     X1,X2,X3,X4,Y
N OF CASES        UNKNOWN
INPUT MEDIUM      CARD
INPUT FORMAT      FREEFIELD
VAR LABELS        X1          DENSITY OF OVERLAPPING AREA
                  X2          DENSITY OF FIRST INK LAYER ON PAPER
                  X3          DENSITY OF SECOND INK LAYER ON PAPER
                  X4          HIGHEST DENSITY OF FIRST INK ON PAPER
                  Y           PERCENT GRAVIMETRIC TRAPPING
COMPUTE           X1X2 = X1*X2
COMPUTE           X1X3 = X1*X3
COMPUTE           X1X4 = X1*X4
COMPUTE           X2X3 = X2*X3
COMPUTE           X2X4 = X2*X4
COMPUTE           X3X4 = X3*X4
COMPUTE           SQX1 = X1**2
COMPUTE           SQX2 = X2**2
COMPUTE           SQX3 = X3**2
COMPUTE           SQX4 = X4**2
COMPUTE           X1X2X3 = X1*X2*X3
COMPUTE           X1X2X4 = X1*X2*X4
COMPUTE           X1X3X4 = X1*X3*X4
COMPUTE           X2X3X4 = X2*X3*X4
COMPUTE           TX1 = X1**3
COMPUTE           TX2 = X2**3
COMPUTE           TX3 = X3**3
COMPUTE           TX4 = X4**3
COMPUTE           X1X2X3X4 = X1*X2*X3*X4
COMPUTE           FX1 = X1**4
COMPUTE           FX2 = X2**4
COMPUTE           FX3 = X3**4
COMPUTE           FX4 = X4**4
REGRESSION        VARIABLES = Y, X1 TO X4, X1X2 TO FX4/
                  REGRESSION = Y WITH X1 TO X4(19) RESID=0/
OPTIONS           6,11,12,16
STATISTICS        4
READ INPUT DATA
1.17,1.,1.05,1.,48.97
1.16,1.05,1.03,1.05,35.6
1.14,1.015,1.0,1.015,41.64
1.53,.63,1.02,1.22,93.99
1.56,.65,1.04,1.24,87.02
1.54,.645,1.,1.24,89.76
1.08,.11,1.01,1.18,102.36
1.08,.11,1.01,1.18,93.6
1.08,.115,1.01,1.16,112.9
.11,.05,1.23,.97,-37.43
.1,.05,1.27,.99,-37.04
.13,.05,1.29,.97,-50.56
1.54,1.41,1.2,1.41,46.56
1.57,1.26,1.28,1.26,32.59
1.6,1.28,1.35,1.28,45.19
1.37,.4,1.27,1.14,66.29
1.57,.41,1.32,1.14,75.25
1.43,.425,1.36,1.22,77.36
.04,.02,1.15,1.0,-59.86
.05,.02,1.34,.98,-42.29
.07,.02,1.43,1.01,-43.79
.47,.12,1.08,1.23,13.24
.58,.12,1.39,1.23,26.29
.56,.13,1.27,1.27,25.81
1.8,1.18,1.3,1.18,42.68
1.86,1.32,1.31,1.32,33.72
1.88,1.36,1.46,1.36,48.89
!END
REGRESSION        VARIABLES = Y, X1 TO X4, X1X2 TO FX4/
                  REGRESSION = Y WITH X1 TO SQX4(29) RESID=0/
OPTIONS           6,11,12,16
STATISTICS        4
REGRESSION        VARIABLES = Y, X1 TO X4, X1X2 TO FX4/
                  REGRESSION = Y WITH X1 TO TX4(11) RESID=0/
OPTIONS           6,11,12,16
STATISTICS        4
REGRESSION        VARIABLES = Y, X1 TO X4, X1X2 TO FX4/
                  REGRESSION = Y WITH X1 TO FX4(39) RESID=0/
OPTIONS           6,11,12,16
STATISTICS        4
REGRESSION        VARIABLES = Y, X1 TO X4, X1X2 TO FX4/
```

```
                  REGRESSION = Y(10,4,.05) WITH X1 TO X4(11) RESID=0/
OPTIONS           6,11,12,16
STATISTICS        4
REGRESSION        VARIABLES = Y, X1 TO X4, X1X2 TO FX4/
                  REGRESSION = Y(10,4,.05) WITH X1 TO SQX4(15) RESID=0/
OPTIONS           6,11,12,16
STATISTICS        4
REGRESSION        VARIABLES = Y, X1 TO X4, X1X2 TO FX4/
                  REGRESSION = Y(10,4,.05) WITH X1 TO IX4(45) RESID=0/
OPTIONS           6,11,12,16
STATISTICS        4
REGRESSION        VARIABLES = Y, X1 TO X4, X1X2 TO FX4/
                  REGRESSION = Y(10,4,.05) WITH X1 TO FX4(55) RESID=0/
OPTIONS           6,11,12,16
STATISTICS        4
FINISH
```

APPENDICES I1 to I4

Regression Equations (F-ratio = 0.01) for

Determination of Gravimetric Trapping Using Densities

APPENDIX I1

Regression Equations (F-ratio = 0.01) for Determination Gravimetric Trapping Using Densities for Newsprint, Uncoated Paper, and Coated Paper

Inclusion criteria: n = (maximum number of variables) = 80
F-ratio = 0.01
T (tolerance) = 0.001

Notation:

G = the percent gravimetric trapping.

D_1 = the density of the overlapping area using a filter that gives the highest density reading.

D_2 = the density of the first ink layer printed directly on paper. The filter is the same as the one used to measure D_1.

D_3 = the density of the second ink layer printed directly on paper. The filter is the same as the one used to measure D_1.

D_4 = the density of the first ink layer printed directly on paper. The filter is the one which gives the highest density reading.

The first order equation is:

$$G = 146.0235 + 85.70029(D_1) - 27.43079(D_2) - 154.2237(D_3) + 8.189555(D_4)$$

$R^2 = 0.79061$ Standard error = 19.52502

APPENDIX I1 (continued)

The second order equation is:

$$G = 24.39786 + 482.2081(D_1) - 78.41643(D_2) - 358.0146(D_3) + 46.79202(D_4) - 55.1346(D_1D_2) - 162.3664(D_1D_3) - 129.0475(D_1D_4) + 143.1296(D_2D_3) - 38.513(D_2D_4) - 27.33038(D_3D_4) - 24.66775(D_1^2) + 4.667305(D_2^2) + 187.5249(D_3^2) + 52.38092(D_4^2)$$

$R^2 = 0.90565$ Standard error = 14.06458

The third order equation is:

$$G = -121.5997 + 436.0642(D_1) + 91.87769(D_2) + 49.01776(D_4) - 173.2184(D_1D_3) - 71.68513(D_1D_4) - 236.2968(D_2^2) - 89.63020(D_3^2) + 91.07108(D_1D_2D_3) - 57.4979(D_1D_2D_4) + 48.72001(D_2D_3D_4) - 23.07277(D_1^3) + 129.5520(D_2^3) + 70.78144(D_3^3) + 9.287952(D_4^3)$$

$R^2 = 0.91574$ Standard error = 13.29076

The fourth order equation* is:

$$G = -225.8657 + 447.2871(D_1) + 153.0985(D_2) + 168.3707(D_4) - 139.1358(D_1D_3) - 158.1360(D_1D_4) - 152.0179(D_2^2) - 20.25123(D_3^2) - 99.30195(D_1D_2D_4) - 216.4647(D_2D_3D_4) + 50.02378(D_1^3) + 83.54683(D_2^3) + 177.6226(D_1D_2D_3D_4) - 39.09424(D_1^4) + 15.05495(D_3^4)$$

$R^2 = 0.92468$ Standard error = 12.56622

* the best-fit equation

APPENDIX I2

Regression Equations (F-ratio = 0.01) for Determination Gravimetric Trapping Using Densities for Newsprint

Inclusion criteria: n = (maximum number of variables) = 80
F-ratio = 0.01
T (tolerance) = 0.001

Notation:

G = the percent gravimetric trapping.

D_1 = the density of the overlapping area using a filter that gives the highest density reading.

D_2 = the density of the first ink layer printed directly on paper. The filter is the same as the one used to measure D_1.

D_3 = the density of the second ink layer printed directly on paper. The filter is the same as the one used to measure D_1.

D_4 = the density of the first ink layer printed directly on paper. The filter is the one which gives the highest density reading.

The first order equation is:

$$G = 66.62225 + 185.1727(D_1) - 46.06241(D_2) - 143.5593(D_3) - 13.49358(D_4)$$

$R^2 = 0.8366$ Standard error = 10.38309

The second order equation is:

$$G = -285.0145 + 684.9535(D_1) - 82.50674(D_2) + 182.3519(D_3) - 25.93503(D_4) - 257.5213(D_1D_2) - 458.8379(D_1D_3) + 337.8642(D_2D_3) - 39.59758(D_2D_4) + 28.65879(D_2^2)$$

$R^2 = 0.94531$ Standard error = 6.83327

APPENDIX I2 (continued)

The third order equation* is:

$$G = 473.6443 - 524.9047(D_1) + 82.11858(D_3) - 876.7252(D_4) + 1178.132(D_1D_4) + 111.5904(D_2D_4) + 210.5982(D_2^2) - 477.7634(D_1D_2D_4) + 56.40296(D_2D_3D_4) - 60.38354(D_1^3) - 86.80699(D_3^3)$$

$R^2 = 0.95825$ Standard error = 6.15442

The fourth order equation is:

$$G = 119.8314 - 568.3903(D_1) + 277.1903(D_2) + 950.6644(D_1D_4) - 728.5611(D_3D_4) + 451.3859(D_3^2) - 504.9610(D_1D_2D_4) + 112.7498(D_1D_2D_3D_4) - 2.118248(D_1^4) + 63.08970(D_2^4) - 88.15729(D_3^4)$$

$R^2 = 0.95793$ Standard error = 6.17808

* the best-fit equation

APPENDIX I3

Regression Equations (F-ratio = 0.01) for Determination
Gravimetric Trapping Using Densities for Uncoated Paper

Inclusion criteria: n = (maximum number of variables) = 80
F-ratio = 0.01
T (tolerance) = 0.001

Notation:

G = the percent gravimetric trapping.

D_1 = the density of the overlapping area using a filter that gives the highest density reading.

D_2 = the density of the first ink layer printed directly on paper. The filter is the same as the one used to measure D_1.

D_3 = the density of the second ink layer printed directly on paper. The filter is the same as the one used to measure D_1.

D_4 = the density of the first ink layer printed directly on paper. The filter is the one which gives the highest density reading.

The first order equation is:

$$G = 93.66345 + 149.464(D_1) - 30.73869(D_2) - 147.2212(D_3) + 2.750589(D_4)$$

$R^2 = 0.82425$ Standard error = 8.94706

The second order equation is:

$$G = 172.3355 + 96.28618(D_2) - 233.0268(D_3) - 328.6195(D_1D_2) + 169.7369(D_1D_3) + 141.72(D_1D_4) + 124.8423(D_2D_3) - 131.1731(D_3D_4) + 88.73451(D_2^2)$$

$R^2 = 0.9500$ Standard error = 5.27592

APPENDIX I3 (continued)

The third order equation* is:

$$G = 531.0696 - 204.5510(D_2) - 865.0237(D_3) + 103.8759(D_1D_3) + 420.7120(D_1D_4) + 322.5885(D_2D_3) + 33.27951(D_2^2) - 186.6137(D_4^2) - 186.8209(D_2D_3D_4) - 113.5982(D_1^3) + 219.54(D_3^3)$$

$R^2 = 0.97333$ Standard error = 4.08726

The fourth order equation is:

$$G = 633.4273 - 278.6023(D_2) - 587.2590(D_3) - 454.3137(D_4) - 23.70084(D_1D_3) + 452.2846(D_1D_4) + 454.2751(D_2D_3) + 39.10709(D_2^2) - 258.9036(D_2D_3D_4) - 54.18243(D_1^4) + 117.8855(D_3^4) + 27.9419(D_4^4)$$

$R^2 = 0.97485$ Standard error = 4.09901

* the best-fit equation

APPENDIX I4

Regression Equations (F-ratio = 0.01) for Determining Gravimetric Trapping Using Densities for Coated Paper

Inclusion criteria: n = (maximum number of variables) = 80
F-ratio = 0.01
T (tolerance) = 0.001

Notation:

G = the percent gravimetric trapping.

D_1 = the density of the overlapping area using a filter that gives the highest density reading.

D_2 = the density of the first ink layer printed directly on paper. The filter is the same as the one used to measure D_1.

D_3 = the density of the second ink layer printed directly on paper. The filter is the same as the one used to measure D_1.

D_4 = the density of the first ink layer printed directly on paper. The filter is the one which gives the highest density reading.

The first order equation is:

$$G = 40.78578 + 94.51249(D_1) - 66.01887(D_2) - 107.9761(D_3) + 54.66835(D_4)$$

$R^2 = 0.92238$ Standard error = 15.47566

The second order equation is:

$$G = 195.4347 + 197.2974(D_1) - 235.2529(D_3) - 277.7278(D_4) - 121.6996(D_1D_2) - 168.4679(D_1D_3) + 121.7459(D_1D_4) + 156.3581(D_2D_3) - 167.7829(D_2D_4) + 257.6559(D_3D_4) + 80.30859(D_2^2)$$

$R^2 = 0.98764$ Standard error = 7.24226

APPENDIX I4 (continued)

The third order equation is:

$$G = 246.6373 + 366.1270(D_1) - 151.5838(D_3) - 347.8138(D_4) - 182.0249(D_1D_2) - 152.2673(D_1D_3) - 15.30315(D_2D_3) + 105.6885(D_3D_4) - 38.81159(D_1^2) + 17.32061(D_2^2) + 108.9856(D_1D_2D_3) + 18.76984(D_3^3) + 61.13868(D_4^3)$$

$R^2 = 0.98935$ Standard error = 7.18669

The fourth order equation* is:

$$G = 178.0405 + 286.2175(D_1) - 104.6036(D_3) - 14.22634(D_1D_2) - 97.53547(D_1D_3) - 135.6808(D_2D_4) - 15.62728(D_1^2) - 254.1412(D_4^2) + 70.89942(D_1D_2D_3) - 10.92884(D_1^4) - 1.647462(D_2^4) + 17.14313(D_3^4) + 107.1476(D_4^4)$$

$R^2 = 0.99095$ Standard error = 6.6224

* the best-fit equation

APPENDIX J

The Program Listing for Determination of Gravimetric Trapping Using Densities in Exponential Function

The Program Listing for Newsprint, Uncoated Paper, and Coated Paper is:

```
!JOB 662C5HCT,F660709HLACH(ROSS),7.KONGSAK
!LIMIT (CORE,24),(TIME,1)
!RUN (LMN,SPSS,LMNLIB2)
!DATA
RUN NAME            DETERMINATION OF GRAV. TRAPPING USING EXP. OF DENSI
VARIABLE = 40
VARIABLE LIST       X1,X2,X3,X4,Y
N OF CASES          UNKNOWN
INPUT MEDIUM        CARD
INPUT FORMAT        FREEFIELD
VAR LABELS          X1          DENSITY OF OVERLAPPING AREA
                    X2          DENSITY OF FIRST INK LAYER ON PAPER
                    X3          DENSITY OF SECOND INK LAYER ON PAPER
                    X4          HIGHEST DENSITY OF FIRST INK ON PAPER
                    Y           PERCENT GRAVIMETRIC TRAPPING
COMPUTE             D1 = -X1
COMPUTE             D1 = EXP(D1)
COMPUTE             D2 = -X2
COMPUTE             D2 = EXP(D2)
COMPUTE             D3 = -X3
COMPUTE             D3 = EXP(D3)
COMPUTE             D4 = -X4
COMPUTE             D4 = EXP(D4)
COMPUTE             EXPX1 = 1-D1
COMPUTE             EXPX2 = 1-D2
COMPUTE             EXPX3 = 1-D3
COMPUTE             EXPX4 = 1-D4
REGRESSION          VARIABLES = Y, X1 TO X4, EXPX1 TO EXPX4/
                    REGRESSION = Y(10,4,.05) WITH EXPX1 TO EXPX4(19)/
OPTIONS             11,12,15,16
STATISTICS          ALL
READ INPUT DATA
.87,.77,.76,.77,86.81
.89,.78,.79,.78,83.13
.89,.755,.77,.755,83.93
1.03,.75,.72,1.0,91.47
1.07,.72,.77,.985,98.99
1.03,.76,.77,1.0,93.67
.84,.15,.78,1.0,100.96
.81,.15,.76,.93,79.95
.84,.15,.79,.97,105.70
.81,.06,1.03,.77,55.73
.83,.055,1.03,.76,54.55
.80,.06,1.04,.77,54.47
1.16,1.0,1.0,1.0,75.49
1.16,1.02,1.02,1.02,77.34
1.18,1.025,1.03,1.025,84.1
1.10,.51,1.01,.96,94.27
1.12,.51,1.01,.95,98.84
1.13,.51,1.04,.99,90.91
.59,.01,.89,.77,14.11
.65,.01,1.0,.77,27.33
.67,.015,.94,.765,34.94
.81,.19,1.02,1.0,56.80
.8,.195,.94,1.0,51.53
.82,.21,1.03,1.05,53.57
1.16,.97,1.02,.97,75.74
1.21,1.0,.95,1.0,80.24
1.19,.99,.98,.99,78.53
.96,.8,.79,.8,103.14
.95,.76,.81,.79,101.68
.95,.81,.79,.81,104.41
1.11,.84,.77,1.06,114.77
1.11,.845,.78,1.06,111.37
1.13,.835,.79,1.05,117.27
.91,.22,.78,1.01,117.7
.9,.22,.82,1.02,115.63
.9,.22,.82,1.025,113.90
.93,.05,1.08,.8,81.81
.93,.05,1.07,.795,77.42
.87,.05,1.04,.81,70.52
1.22,1.065,1.06,1.065,101.1
1.24,1.06,1.07,1.06,97.61
1.21,1.06,1.04,1.06,95.72
1.22..59.1.07,.995,105.23
1.19,.62,1.04,1.035,107.94
1.16,.59,1.05,.99,105.48
.77,.005,1.02,.79,53.69
.8,.01,.98,.81,56.41
.76,.01,.98,.805,55.43
.95,.245,1.09,1.05,79.62
.88,.25,.94,1.05,67.71
.93,.25,1.03,1.065,79.23
1.29.1.02.1.02.1.02.99.01
```

```
1.79,1.075,1.04,1.075,97.46
1.75,1.07,1.0,1.02,93.80
1.17,1.,1.05,1.,48.97
1.16,1.05,1.03,1.05,35.6
1.14,1.015,1.0,1.015,41.64
1.53,.63,1.02,1.22,93.89
1.50,.65,1.04,1.24,87.02
1.54,.645,1.,1.24,89.76
1.06,.11,1.01,1.18,102.36
1.06,.11,1.01,1.16,93.6
1.08,.115,1.01,1.16,112.8
.11,.05,1.23,.67,-37.43
.1,.05,1.27,.98,-37.04
.13,.05,1.29,.97,-50.56
1.54,1.41,1.2,1.41,46.56
1.57,1.26,1.28,1.26,32.59
1.6,1.26,1.35,1.26,45.19
1.37,.4,1.27,1.14,66.29
1.57,.41,1.32,1.14,75.75
1.43,.425,1.36,1.22,77.35
.04,.02,1.15,1.0,-59.86
.05,.02,1.34,.98,-42.29
.07,.02,1.43,1.01,-43.79
.47,.12,1.08,1.23,13.24
.58,.12,1.39,1.23,26.29
.56,.13,1.27,1.27,25.81
1.8,1.18,1.3,1.18,42.68
1.86,1.32,1.31,1.32,33.72
1.88,1.36,1.46,1.36,48.89
!END
FINISH
```

The Program Listing for Newsprint is:

```
!JOB 662C5HDT,F660709HLACH(ROSS),7,KONGSAK
!LIMIT (CORE,24),(TIME,1)
!RUN (LMN,SPSS,LMNLIB2)
!DATA
RUN NAME        DETERMINATION OF GRAV. TRAPPING USING EXP. OF DENSI
VARIABLE = 40
VARIABLE LIST   X1,X2,X3,X4,Y
N OF CASES      UNKNOWN
INPUT MEDIUM    CARD
INPUT FORMAT    FREEFIELD
VAR LABELS      X1       DENSITY OF OVERLAPPING AREA
                X2       DENSITY OF FIRST INK LAYER ON PAPER
                X3       DENSITY OF SECOND INK LAYER ON PAPER
                X4       HIGHEST DENSITY OF FIRST INK ON PAPER
                Y        PERCENT GRAVIMETRIC TRAPPING
COMPUTE         D1 = -X1
COMPUTE         D1 = EXP(D1)
COMPUTE         D2 = -X2
COMPUTE         D2 = EXP(D2)
COMPUTE         D3 = -X3
COMPUTE         D3 = EXP(D3)
COMPUTE         D4 = -X4
COMPUTE         D4 = EXP(D4)
COMPUTE         EXPX1 = 1-D1
COMPUTE         EXPX2 = 1-D2
COMPUTE         EXPX3 = 1-D3
COMPUTE         EXPX4 = 1-D4
REGRESSION      VARIABLES = Y, X1 TO X4, EXPX1 TO EXPX4/
                REGRESSION = Y(10,4,.05) WITH EXPX1 TO EXPX4(19)/
OPTIONS         11,12,15,16
STATISTICS      ALL
READ INPUT DATA
.87,.77,.76,.77,86.81
.89,.78,.79,.78,83.13
.89,.755,.77,.755,83.93
1.03,.75,.72,1.0,91.47
1.07,.72,.77,.985,98.99
1.03,.76,.77,1.0,93.67
.84,.15,.78,1.0,100.96
.81,.15,.76,.93,79.95
.84,.15,.79,.97,105.70
.81,.06,1.03,.77,55.73
.83,.055,1.03,.76,54.55
.80,.06,1.04,.77,54.47
1.16,1.0,1.0,1.0,75.49
1.16,1.02,1.02,1.07,77.34
1.18,1.075,1.03,1.025,84.1
1.10,.51,1.01,.96,94.27
1.12,.51,1.01,.95,98.84
1.13,.51,1.04,.98,90.91
.59,.01,.80,.77,14.11
.65,.01,1.0,.77,27.33
.62,.015,.94,.765,34.94
.81,.19,1.02,1.0,56.80
.8,.195,.94,1.0,51.53
.87,.21,1.03,1.05,53.57
1.16,.97,1.07,.97,75.74
1.21,1.0,.95,1.0,80.74
1.19,.99,.98,.99,79.53
!END
FINISH
```

The Program Listing for Uncoated Paper is:

```
!JOB 662C5HDT,F660709HLACH(POSS),7.KONGSAK
!LIMIT (CORE,24),(TIME,1)
!RUN (LMN,SPSS,LMNTI82)
!DATA
RUN NAME          DETERMINATION OF GRAV. TRAPPING USING EXP. OF DENSI
VARIABLE = 40
VARIABLE LIST     X1,X2,X3,X4,Y
N OF CASES        UNKNOWN
INPUT MEDIUM      CARD
INPUT FORMAT      FREEFIELD
VAR LABELS        X1          DENSITY OF OVERLAPPING AREA
                  X2          DENSITY OF FIRST INK LAYER ON PAPER
                  X3          DENSITY OF SECOND INK LAYER ON PAPER
                  X4          HIGHEST DENSITY OF FIRST INK ON PAPER
                  Y           PERCENT GRAVIMETRIC TRAPPING
COMPUTE           D1 = -X1
COMPUTE           D1 = EXP(D1)
COMPUTE           D2 = -X2
COMPUTE           D2 = EXP(D2)
COMPUTE           D3 = -X3
COMPUTE           D3 = EXP(D3)
COMPUTE           D4 = -X4
COMPUTE           D4 = EXP(D4)
COMPUTE           EXPX1 = 1-D1
COMPUTE           EXPX2 = 1-D2
COMPUTE           EXPX3 = 1-D3
COMPUTE           EXPX4 = 1-D4
REGRESSION        VARIABLES = Y, X1 TO X4, EXPX1 TO EXPX4/
                  REGRESSION = Y(10,4,.05) WITH EXPX1 TO EXPX4(19)/
OPTIONS           11,12,15,16
STATISTICS        ALL
READ INPUT DATA
.96,.8,.79,.9,103.14
.95,.76,.61,.79,101.68
.95,.81,.79,.81,104.41
1.11,.84,.77,1.06,114.77
1.11,.945,.76,1.06,111.37
1.13,.835,.78,1.05,117.27
.91,.27,.78,1.01,117.7
.9,.72,.82,1.02,115.63
.9,.72,.82,1.025,113.90
.93,.05,1.08,.9,81.81
.93,.05,1.07,.795,77.42
.87,.05,1.04,.81,70.52
1.22,1.065,1.06,1.065,101.1
1.24,1.06,1.07,1.06,97.61
1.21,1.06,1.04,1.06,95.72
1.22,.59,1.07,.995,105.23
1.19,.62,1.04,1.035,107.94
1.18,.59,1.05,.99,105.48
.77,.005,1.02,.79,53.69
.8,.01,.98,.81,56.41
.76,.01,.96,.865,55.43
.95,.245,1.08,1.05,79.62
.88,.25,.94,1.05,67.21
.93,.25,1.03,1.065,72.73
1.29,1.02,1.02,1.02,99.01
1.29,1.075,1.04,1.075,97.46
1.25,1.02,1.0,1.02,93.80
!EOD
FINISH
```

The Program Listing for Coated Paper is:

```
!JOB 662C5HDT,F660709HLACH(ROSS),7.KONGSAK
!LIMIT (CORE,24),(TIME,1)
!RUN (LMN,SPSS,LMNLIB2)
!DATA
RUN NAME        DETERMINATION OF GRAV. TRAPPING USING EXP. OF DENSI
VARIABLE = 40
VARIABLE LIST   X1,X2,X3,X4,Y
N OF CASES      UNKNOWN
INPUT MEDIUM    CARD
INPUT FORMAT    FREEFIELD
VAR LABELS      X1          DENSITY OF OVERLAPPING AREA
                X2          DENSITY OF FIRST INK LAYER ON PAPER
                X3          DENSITY OF SECOND INK LAYER ON PAPER
                X4          HIGHEST DENSITY OF FIRST INK ON PAPER
                Y           PERCENT GRAVIMETRIC TRAPPING
COMPUTE         D1 = -X1
COMPUTE         D1 = EXP(D1)
COMPUTE         D2 = -X2
COMPUTE         D2 = EXP(D2)
COMPUTE         D3 = -X3
COMPUTE         D3 = EXP(D3)
COMPUTE         D4 = -X4
COMPUTE         D4 = EXP(D4)
COMPUTE         EYPX1 = 1-D1
COMPUTE         EYPX2 = 1-D2
COMPUTE         EYPY3 = 1-D3
COMPUTE         EYPY4 = 1-D4
REGRESSION      VARIABLES = Y, X1 TO X4, EXPX1 TO EYPX4/
                REGRESSION = Y(10,4,.05) WITH EYPX1 TO EXPX4(19)/
OPTIONS         11,12,15,16
STATISTICS      ALL
READ INPUT DATA
1.17,1.,1.05,1.,48.97
1.16,1.05,1.03,1.05,35.6
1.14,1.015,1.0,1.015,41.64
1.53,.63,1.02,1.22,93.89
1.56,.65,1.04,1.24,87.02
1.54,.645,1.,1.24,84.76
1.08,.11,1.01,1.19,102.36
1.06,.11,1.01,1.18,93.6
1.05,.115,1.01,1.19,112.8
.11,.05,1.23,.97,-37.43
.1,.05,1.27,.98,-37.04
.13,.05,1.29,.97,-50.56
1.54,1.41,1.2,1.41,46.56
1.57,1.26,1.26,1.26,32.59
1.6,1.28,1.35,1.28,45.19
1.37,.4,1.27,1.14,66.29
1.57,.41,1.32,1.14,75.25
1.43,.425,1.36,1.22,77.36
.04,.02,1.15,1.0,-59.86
.05,.02,1.34,.96,-42.29
.07,.02,1.43,1.01,-43.79
.47,.12,1.08,1.23,13.24
.58,.12,1.39,1.23,26.29
.56,.13,1.27,1.27,25.81
1.8,1.18,1.3,1.19,42.69
1.86,1.32,1.31,1.32,33.72
1.88,1.36,1.46,1.36,48.89
!END
FINISH
```