Development of a technique to measure the MTF of a transducer used in an ultrasonic fingerprint scanner

Laura Blair
Development of a Technique to Measure the MTF of a Transducer used in an Ultrasonic Fingerprint Scanner

by

Laura Blair

B.S. Alfred University, 1999

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in the Chester F. Carlson Center for Imaging Science Rochester Institute of Technology

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Signature of the Author __________________

Laura Blair

Accepted by ____________________________

Name Illegible

Coordinator, M.S. Degree Program

July 18, 2003
M.S. DEGREE THESIS

The M.S. Degree Thesis of Laura Blair has been examined and approved by the thesis committee as satisfactory for the thesis required for the M.S. degree in Imaging Science.

Navalgund Rao
Dr. Navalgund Rao, Thesis Advisor

Maria Helguera
Dr. Maria Helguera

Daniel Phillips
Dr. Daniel Phillips

July 18, 2003
Date
Title of Thesis:
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used in an Ultrasonic Fingerprint Scanner

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Development of a Technique to Measure the MTF of a Transducer used in an Ultrasonic Fingerprint Scanner

by

Laura Blair

Submitted to the
Chester F Carlson Center for Imaging Science
in partial fulfillment of the requirements
for the Master of Science Degree
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Abstract

Beam characteristics of a transducer play a major role in defining the resolution of any ultrasound imaging system. Direct experimental estimation of the beam profile is often desirable, however, one has to carefully correct for effects that are attributed to the measurement process itself. We report here our experience of using a measurement technique developed for circular disk focused transducers. When the anticipated beam size is much smaller than 0.5 mm the use of a hydrophone to scan the monochromatic beam is not adequate. A more suitable approach is to make a measurement in pulse-echo mode using a thin (125 micron or less) wire target. In this case, a short pulse excitation is used from which the monochromatic information is derived via Fourier transform. However, two measurement artifacts need to be corrected. Multiple reflections due to the finite wire size show up as periodic spikes in the spectrum. These were corrected for using Cepstrum domain filtering method. Second, the wire target measurement essentially represents a one-dimensional projection of the two-dimensional beam pattern in a plane perpendicular to the beam axis. Smoothing effects were investigated by comparing experimental results and theoretical predictions based on Lommel diffraction formulation. The apodization of the transducer to remove the side-lobes would have also caused an increase in width of the main lobe. Line Spread Function and Modulation Transfer Function, with respect to frequency and distance from the target to the transducer, were characterized and reported.
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Without the help of a few key people this thesis would not have been possible.

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Chapter 1

Introduction

The beam characteristics of a transducer play a major role in defining the resolution of any ultrasound imaging system. Quality metrics typically measured for imaging systems include the Point Spread Function (PSF), Line Spread Function (LSF), Edge Spread Function (ESF), and Modulation Transfer Function (MTF). Often transducers are not defined in terms of these metrics but instead by physical dimensions, resonant frequency, and electrical characteristics. Determination of the image quality created by a particular transducer requires knowledge beyond the typically provided parameters. Quantitative methods of characterization should be implemented.

The objective of the work that follows is to measure the LSF and MTF of a transducer used in a fingerprint imaging system. In Chapter 2 we define a linear imaging system model for the experimental setup being studied. Since diffraction governs the variation of the beam profile with distance, it is an important factor in our analysis. Discussion of the use of Lommel functions to establish solutions for pressure field distribution is presented with emphasis on the focused transducer case.
CHAPTER 1. INTRODUCTION

In any direct experimental estimation, one has to carefully correct for the effects that are attributed to the measurement process itself. Signal distortion can be eliminated by improving the processing technique to remove the cause of the distortion or by post-processing signal restoration. In our case, the signal distortion observed was believed to be caused by the diameter of the wire target utilized. Replacement with a thinner wire was not possible so restoration of the signal after data collection was the chosen method. Three techniques for noise filtering are introduced in Chapter 2. Motivation behind choosing one of the three is discussed along with specific parameters for that technique.

Discussion of previous circular disk focused transducer measurements identified in the literature is also presented in Chapter 2. High frequency transducers (greater than 10MHz) resulting in smaller spot sizes have become more prevalent. This makes direct measurement of the beam profile more difficult since commonly available hydrophones have an active element diameter larger than the spot size of the high frequency transducers. Therefore techniques similar to the one used in this study should be implemented with appropriate post processing to achieve adequate results.

In Chapter 3 we introduce the experimental procedure and data processing implemented in order to measure the LSF and MTF of the transducer. A thin (125 micron) nylon wire target was scanned by the given transducer in a plane perpendicular to the beam direction. The two dimensional monochromatic beam profile of the transducer, \( P(x,y) \) is referred to as the Point Spread Function (PSF). It is reasonable to assume that scanning the wire target in any direction yields a one dimensional projection of \( P(x,y) \), which is referred to as the Line Spread
CHAPTER 1. INTRODUCTION

Function (LSF). Since \( P(x,y) \) is circularly symmetric, it is sufficient to scan in any one direction and obtain one LSF for every frequency. The MTF is determined by calculating the Fourier transform of the LSF.

The Cepstrum domain filtering procedure used to remove multiple reflection artifacts in the data is outlined. The shape and width of the filter used are detailed. A comparison between Rectangular and Gaussian filtering is presented. Elimination of noise without reducing amplitude can be achieved by optimizing filter width. This is also described in detail in the chapter. Within the same chapter, misalignment analysis due to the transducer and wire not being completely perpendicular is presented. The motivation of full-width half maximum calculations is discussed.

An alternative method for processing the wire signal is outlined. This method involves calculation of the envelope of the signal and is of interest to us because it is the technique currently used by the manufacturer of the imaging system. The goal is to compare this processing method to our technique.

Finally, Chapter 3 outlines the theoretical calculation of the Lommel diffraction for comparison to the measured beam profiles.

Chapter 4 includes the experimental results and data analysis. Since our method uses a short pulse excitation rather than monochromatic excitation, the different frequency components are separated out via Fourier transform in post processing operations. Ten frequencies are identified and beam profiles for each of these frequencies are presented. Beam profile characteristics, such as near and far field regions, are discussed. The characteristics were observed to change with frequency. Discussion of full width half maximum calculations for each frequency
and Z-distance reveal relationships between width and frequency as well as width and Z-distance.

Significant differences between our data processing method at nominal frequency and the envelope measurement technique utilized by the manufacturer were not evident. Higher frequencies evaluated in our data processing method did exhibit improved beam profiles. The results of this analysis are included in Chapter 4.

Issues observed when calculating the theoretical beam profiles are presented. Although the results do not match experimental findings general observations are discussed including two main differences between theoretical and experimental results. Side-lobes that were not observed experimentally are apparent in the calculated result. This discrepancy may be explained if the transducer was apodized. Secondly, the full width half maximum of the theoretical calculations is much smaller than the experimental results. Two possibilities are suggested for this observation. Apodization would increase the width of the main lobe. Also, the diameter of the nylon wire target may be a contributing factor. Therefore a microscope image of the nylon wire target was analyzed to find the effective diameter, where specular reflection takes place. This is also presented in Chapter 4.

Finally, conclusions and suggestions for future work related to this project are presented in Chapter 5.
Chapter 2

Theory

2.1 Ultrasound and Ultrasound Modalities

Ultrasound is used in many imaging situations where an object has to be examined non-destructively. Obvious applications are medical (obstetrics, gynecology, cardiovascular, ophthalmology, etc) [30] but ultrasound is also used in other applications including industrial in-service inspection of parts to identify defects [30]. In any case, the ultrasonic transducer generates a sound wave as a result of electrical excitation which interacts with the object of interest. Typically ultrasound is operated in pulse-echo mode meaning the transducer generates a pulse for a finite duration. The sound wave is scattered during the interaction with the test object and some of the scattered energy is reflected back to the transducer. At this point the transducer is no longer generating a pulse but is ready to receive the reflected signal. The reflected wave is converted into a voltage which varies as a function of time, also known as A-line, or amplitude. A graphic demonstrating the A-scan process is displayed in Figure 2.1 [30].

It is often desirable to display information from the object beyond the line of sight and beam width of the transducer. One mode of data acquisition that
Figure 2.1: Illustration of the A-scan Process.
involve scanning the ultrasound beam over the object plane is C-scan mode [30]. This method is also known as constant depth scanning because the transducer is scanned laterally in the x and y plane resulting in a two-dimensional image of a thin slice of the object at a particular depth. Since the transducer is focused at a specific depth, uniform resolution is achieved across the image. Figure 2.2 illustrates C-scan mode[30]. As shown in the figure, only information from a particular x,y plane that is located a distance $z_0$ from the transducer face is acquired. A time gate is used to specify the depth, $z_0$, of signal amplitudes measured. The length of the time gate will determine the thickness of the x,y plane slice.

The measurement of the transducer beam characteristics becomes an important factor in establishing the resolution of the overall imaging system. Quality metrics of the transducer include Point Spread Function (PSF), Line Spread
Figure 2.3: Near field, True-Focus and Far Field Regions of Transducer Beam. A, B, C, D, and E indicate the locations of Measurement for this Experiment (0.25 mm apart).

Function (LSF), Edge Spread Function (ESF) and Modulation Transfer Function (MTF) [20]. The LSF and MTF will be defined in further detail in Section 2.6. First transducer measurement techniques will be discussed.

The research presented is for a focused transducer. High spatial resolution and good signal-to-noise ratios are often the motivation for using a focused transducer[34]. These qualities are achieved because the focusing increases the power density along the transducer’s central axis. Therefore the beam is concentrated in a smaller area which causes a reduction in diameter or spot size[12]. Figure 2.3 illustrates the focusing of the beam profile. Next the imaging system being studied needs to be described.
CHAPTER 2. THEORY

2.2 Fingerprint Imaging

Ultra-Scan Corporation [Buffalo, New York] developed a method for imaging fingerprints using ultrasound. Unlike optical fingerprint capture devices, the ultrasound technique is not affected by dirt, grease, and surface contamination on fingers being measured. Therefore the accuracy of an ultrasound system is much higher than optical fingerprint scanners. Applications of this technology include healthcare and security. It can easily be used for access control and identity screening.

The goal of this research was to experimentally determine imaging system parameters for the transducer used in the ultrasound fingerprint scanner. Figure 2.4 shows a simple schematic of the system. Since the transducer is the focus of this analysis it is the only system component identified in this figure. The container holding the transducer is filled with Klearol oil. A finger is placed on the top of the container contacting a plexiglass surface and the transducer is operated in C-mode scanning.

Ultrasonic properties of the medium through which the pulse is travelling will affect the sound wave. Two important properties that will be defined here are propagation speed, c, and characteristic acoustic impedance, Z [30]. Shear or transverse waves will not be observed in Klearol oil because it is a fluid. A constant compressional wave propagation speed is assumed in conventional pulse echo imaging. For Klearol oil the value for c is 1570 m/s. For comparison, propagation speeds for other materials are listed in Table 2.1.

Acoustic Impedance, Z, is defined in Equation 2.1 where \( \rho_0 \) is the equilibrium
Figure 2.4: Simple Schematic of the Ultrasound Fingerprint Scanner.
Table 2.1: Propagation Speeds and Acoustic Impedances for Various Materials [30].

<table>
<thead>
<tr>
<th>Material</th>
<th>Speed m/s</th>
<th>Acoustic Impedance kg/m²s x 10⁻⁶</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>1497</td>
<td>1.49</td>
</tr>
<tr>
<td>Air</td>
<td>330</td>
<td>.0004</td>
</tr>
<tr>
<td>Aluminum</td>
<td>6419</td>
<td>17.33</td>
</tr>
<tr>
<td>Glass, Pyrex</td>
<td>5640</td>
<td>12.63</td>
</tr>
<tr>
<td>Blood</td>
<td>1580</td>
<td>1.67</td>
</tr>
<tr>
<td>Plexiglass</td>
<td>1484</td>
<td>3.2</td>
</tr>
<tr>
<td>Klearol Oil</td>
<td>1570</td>
<td>?</td>
</tr>
</tbody>
</table>

medium density and $K$ is the compressibility [30]. Table 2.1 also includes acoustic impedance values for various materials.

$$Z = \rho_o c = (\rho_o K)^{\frac{1}{2}}$$ (2.1)

The echo signal that is returned to the transducer is generated by reflection and scattering in the medium. These processes occur at an interface or boundary across which there is a change in the acoustic properties such as density, compressibility or acoustic impedance. A sound wave is not only scattered by a solid object but also by a region with acoustic properties that differ from the rest of the medium such as a bubble in water [25].

The angle between the direction of propagation of the wave meeting the interface and the normal to the interface is θ₁. In general the wave will be reflected at angle θᵢ and refracted at angle θᵣ. Reflection and transmission coefficients, R and T, are defined in Equations 2.2 and 2.3, respectively [30].

$$R = \frac{Z_2 \cos \theta_i - Z_1 \cos \theta_t}{Z_2 \cos \theta_i + Z_1 \cos \theta_t}$$ (2.2)
\[ R = \frac{2Z_2 \cos \theta_i}{Z_2 \cos \theta_i + Z_1 \cos \theta_t} \quad (2.3) \]

Therefore an echo generating boundary can be defined as a surface across which a difference in acoustic impedance, \( Z \), exists. A large acoustic impedance mismatch results in a high amplitude echo. The above discussion has been based on reflection and refraction. Scattering can also be observed and is illustrated in Figure 2.5. Scattering occurs when a surface is rough or if the boundary is small compared to the wavelength. This results in the incident wave being redirected in many directions instead of speculally [30]. The scattering phenomena will depend on the scale of the object with respect to the wavelength of the transducer. If the object is much larger than the wavelength, the surface appears smooth because the roughness is small in comparison. The beam is specularly reflected and only a weak frequency dependence is observed. For the opposite situation, when the object size is much smaller than the wavelength, the scattering strength is observed to be weak unless the acoustic impedance mismatch is very large. The scattering is termed Rayleigh scattering and has a fourth power frequency dependence. Finally, when the object size and wavelength are comparable the scattering strength is moderate and the frequency dependence is linear.

Figure 2.6 is a schematic of the surfaces of reflection for the sound wave pulse in the fingerprint imaging system. At each of these surfaces an echo will be reflected back to the transducer due to acoustic impedance mismatch and will be observed as a peak in the data. The first peak will be a return from the surface of the rigid container. If the container wall has a sizeable thickness there may be two peaks; one from the exterior surface and one from the interior surface. If
the wall is not very thick the two peaks will not be resolved. After the plastic container surface signal will come the air gap or contamination signal. Following this will be the signal from the fingerprint. Of course, this is the peak of interest and should be windowed for analysis. To focus on a particular peak a time gate can be applied.
CHAPTER 2. THEORY

2.3 Linear System Modelling of the Fingerprint Scanner

The imaging system being studied physically scans the object in a plane with an ultrasound beam. The system can be assumed to be linear and shift invariant (LSI) and therefore can be described by a collection of LSI systems connected in series. The general definition of a linear system is a system in which the output must be proportional to the input whereas shift invariance is identified when a certain structure in an image appears the same regardless of where it is placed within the image[9]. The imaging chain model for the fingerprint scanner is shown in Figure 2.7. The other components of the fingerprint scanner (waveform generator and receiver) can also be defined as LSI systems. The focus of the research for this study was the transducer so those components will not be evaluated at this time.

The echo time signal from a wire target, in response to a short pulse excitation of the transducer, can be written as a time convolution of multiple terms as shown in Equation 2.4.

\[
s(t, z, \rho) = p(t) * p(t) * r(t) * h(t, z, \rho) * h(t, z, \rho)
\]  

(2.4)

In Equation 2.4, \( p(t) \) is the transducer's impulse response, \( r(t) \) is the wire target's reflection impulse response, and \( h(t, z, \rho) \) is the transducer's diffraction impulse response[10]. Terms \( h(t, z, \rho) \) and \( p(t) \) are duplicated in the equation because the same transducer was used to transmit and receive. In other words, the transducer was operated in pulse-echo mode. The transducer impulse response and diffraction impulse response terms must be incorporated for both transmis-
Figure 2.7: Imaging Chain Model for the Fingerprint Scanner.
sion and reception. Variable \( z \) represents the perpendicular distance between the plane of the transducer and the plane that contains the wire target whereas variable \( \rho \) represents the distance that the transducer is moved laterally as a part of the scanning process. Note that in this experiment \( \rho \) is perpendicular to \( z \) and the wire target is perpendicular to the plane that contains \( \rho \) and \( z \).

We will assume that the Fourier transform of all the terms in Equation 2.4 exists. Although the exact nature of the wire target's impulse response, \( R(f) \), is unknown, its Fourier transform is the frequency dependent scattering by a wire target and is generally assumed to have a power law dependence. Taking the Fourier transform of Equation 2.4 with respect to time results in the multiplication of the frequency terms, via the filter theorem, as shown in Equation 2.5

\[
S(f, z, \rho) = P(f) \cdot P(f) \cdot R(f) \cdot H(f, z, \rho) \cdot H(f, z, \rho) \quad (2.5)
\]

For this particular application, the diffraction term, \( H(f, z, \rho) \), is of interest and the other terms can be eliminated by a normalization process. Let \( z = z_0 \) be the distance where the maximum on-axis (\( \rho = 0 \)) amplitude signal, \( s(t, z_0, 0) \), is observed. Let \( S(f, z_0, 0) \) be the magnitude spectrum of time signal \( s(t, z_0, 0) \) given by 2.6

\[
S(f, z_0, 0) = P(f) \cdot P(f) \cdot R(f) \cdot H(f, z_0, 0) \cdot H(f, z_0, 0) \quad (2.6)
\]

A normalized beam profile for any frequency, \( f_0 \), can be expressed as Equation 2.7 by dividing Equation 2.5 by 2.6.
\[
B(f_o, z, \rho) = \frac{S(f_o, z, \rho)}{S(f_o, z_o, 0)} = \frac{H(f_o, z, \rho)}{H(f_o, z_o, 0)} \frac{H(f_o, z, \rho)}{H(f_o, z_o, 0)}
\] (2.7)

Note that \(B(f_o, z, \rho)\) depends only on the diffraction terms that are responsible for the transducer beam pattern but it is actually calculated from the Fourier transform of the experimentally measured echo time signals \(s(t, z, \rho)\).

### 2.4 Diffraction

The resolution of an ultrasound image can be correlated to the transducer field pattern. Diffraction governs the variation of the beam profile with \(Z\) distance, or depth [30]. Diffraction occurs with any wave propagation and can be viewed as a linear filter. In order to determine the acoustic field pressure or intensity in front of the transducer as a function of distance and time, diffraction must be considered. There are many approaches to solving the transducer's pressure field distribution: double integration of the Rayleigh expression[26], reduction to a single integral or expressed as a series[8], implementation of Bessel function expansions[1]. This study will implement Bessel function expansion expressions to create solutions in terms of Lommel functions. The use of Lommel functions to establish solutions for pressure field distribution has been demonstrated by Chen et al.[5] and Daly[10]. An analytical closed form solution under the Fresnel approximation results in field distribution in the far and near-field regions. The far-field is defined as a large distance from the transducer whereas near-field is close to the transducer face. In terms of transducer diameter, \(D\), and wavelength, \(\lambda_o\), the near-field region, ranges from a few wavelengths from the face of the transducer to a \(Z\) distance of \(\frac{D^2}{2\lambda_o}\). The far-field is located for \(Z\) distances greater...
than $\frac{D^2}{2\lambda_o}$ In Figure 2.3, region $E$ is far-field, $A$ is near-field, and $C$ is true focus. Note that the Fresnel approximation is valid for any distance from the transducer beyond a few wavelengths. Therefore the calculations based on this approximation in region extremely close to the face of the transducer are not accurate. The Fresnel approximation is preferred over Fraunhofer however which is only valid for far field. The Fresnel region contains the Fraunhofer region whereas the Fraunhofer region is the portion of the Fresnel region where the Fraunhofer approximation is true [10].

In any plane perpendicular to the beam axis, the field of a circular disk transducer is circularly symmetric. The field can be defined as a function of $z$ and $\rho$ where $\rho$ is the off-axis distance as defined in Equation 2.8. Figure 2.8 is included to display diffraction from a circular transducer.

$$\rho = \sqrt{x^2 + y^2}$$ (2.8)

The amplitude of the diffraction pattern for the geometry shown in Figure 2.8 is written as 2.9 [30][1] where $r$ is the distance from an area on the face of the transducer to the object, $\sigma_0$ is the area of the aperture of the transducer, $f(\sigma_0)$ is the spatial excitation profile on the transducer face which is a constant due to spatial uniformity, and $k$ is the spatial wave number. The wave number is defined as the temporal frequency, $\omega$, divided by the speed of sound, $c$. The source plane is denoted by the subscript "o". Integration is performed over the transducer face and is called the Rayleigh-Sommerfeld integral. $H(\rho, z, w)$ is also described as the velocity potential transfer function[1][10].
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Figure 2.8: Diffraction from a Circular Transducer.

\[
H(\rho, z, \omega) = \frac{1}{2\pi} \int_{\sigma_0} f(\sigma_0) e^{\frac{-jk\rho}{r}} d\sigma_0 \quad (2.9)
\]

Assuming \( z \) and \( \rho \) are fixed, diffraction can be described as a linear low-pass spatial filter. The impulse response and transfer function must exist as Fourier Transform pairs. An estimate of the Rayleigh-Sommerfeld integral allowing a closed form solution can be defined if the Fresnel approximation is made and the transducer is circularly symmetric:

\[
\hat{H}(\rho, z, \omega) = \frac{1}{z} e^{-jk(z + \frac{\rho^2}{2z})} \int_{0}^{a} e^{-jk\frac{\rho^2}{2z}} J_{0}\left(\frac{kp}{z} \rho_0\right) \rho_0 d\rho_0 \quad (2.10)
\]

where the off-axis distance to the transducer’s face is defined in Equation 2.11
Expression 2.10 is an estimate as indicated by the hat notation \((\hat{H})\). The solution to the Rayleigh-Sommerfeld integral can be described by alternating infinite series of integer order Bessel functions[19]. More generally, a closed-form expression of 2.10 can be created if Lommel functions are used but the result will be the formulation for an unfocused transducer. The research was done with a focused transducer and therefore the derivation must be made for the focused case. Closed-form solutions for the focused transducer have been derived by Luukkala et al[27], Madsen[24] and Daly[10]. We will follow Daly’s derivation which extends Lommel based results developed for unfocused transducers to the focused case by assuming that focusing introduces a time delay. The result for a focused transducer of radius \(a\) with distance \(z\) from the face of the transducer to the plane of the scatterer is:

\[
\hat{H}(\rho, z, \omega) = \frac{e^{-j(ka^2 + \frac{k^2 z^2}{2} + \frac{k a^2}{2})}}{k z} \left[U_1(u, v) + jU_2(u, v)\right]
\]  

(2.12)

Variables \(u\) and \(v\) are defined in Equations 2.13 and 2.14 where the time delay is described by \(1/\epsilon = 1/z-1/A\) and \(A\) is the focal distance of the transducer. \(U_1\) and \(U_2\) are Lommel functions defined by summations of Bessel functions as shown in Equation 2.15 where \(J_n(v)\) is a Bessel function of the first kind of order \(n\).
\[ v = \frac{kap}{z} \]  

(2.14)

\[ U_n(u, v) = \sum_{s=0}^{\infty} (-1)^s \left( \frac{u}{v} \right)^{n+2s} J_{n+2s}(v) \]  

(2.15)

As A approaches infinity, the transducer is considered unfocused because the focal length becomes infinite. Therefore \( \epsilon = z \) and the Lommel diffraction formulation for unfocused transducers is defined. When the on-axis case is evaluated (\( \rho = 0 \)), the equation simplifies to 2.16.

\[ \tilde{H}(0, z, \omega) = \frac{2\epsilon}{kz} e^{-j\omega(z + \frac{a^2}{4z})} \sin \left( \frac{ka^2}{4\epsilon} \right) \]  

(2.16)

At the focus, when the distance from the face of the transducer to the plane of the scatterer equals A, \( 1/\epsilon = 0 \). The Lommel function becomes:

\[ \tilde{H}(\rho, A, \omega) = \frac{a^2}{A} e^{-j(kA + \frac{ka^2}{2A})} J_1(kap/A) \frac{J_1(kap/A)}{kap/A} \]  

(2.17)

The magnitude squared of expression 2.17 is recognized as the Airy Function[10][26].

### 2.5 Noise Filtering

Signal distortion due to experimental, instrumental or computational errors or internal or external noise is often observed [2]. For accurate data analysis it is crucial to correct the data for effects that are due to the measurement process itself. In order to understand the measurement process and effects that occur, a detailed knowledge of the medium characteristics or the measurement system
characteristics should be available [38]. In this case the medium characteristics were understood whereas the measurement system was being studied.

Two ways to eliminate distortions include improving the processing techniques to remove the cause of the distortion or restoring signal after processing. In our experiment the noise observed in the spectrum is believed to be due to the diameter of the wire. Since it was not possible to use a significantly thinner wire, attempts were made to restore the signal after data collection. Two filtering techniques were chosen for distortion correction: Homomorphic and Cepstrum Filtering. Both can easily be implemented as a digital algorithm and should result in the restoration of the pure signal without noise.

Homomorphic processing has been used in ultrasound applications previously [2][33]. The magnitude spectrum is calculated by Fourier transforming the measured signal. This signal can be modelled as a convolution of the object characteristic function and the impulse response of the transducer. Therefore the Fourier transform is a multiplicative relationship between the transfer function of the transducer and the transfer function of the object. The result of the Fourier transform is considered to be in the Cepstrum domain. Typically the transfer function of the transducer is a bandpass Gaussian shaped function [33]. In our study the object is a wire which can be described as a line of delta functions. To convert the multiplicative relationship to a summation a logarithmic operator is used. Now, the Gaussian shaped transducer function and the noise in the data are additive. The noise is generally high frequency whereas the transducer transfer function is low frequency. Application of a low-pass filter to this additive function will remove the noise and allow the signal to be recovered. The filter
must be of a duration equal to the transfer function but narrower than the noise. After the noise has been filtered out the inverse Fourier transform is computed resulting in the log of the impulse response of the transducer convolved with the object function. The exponential of the result is calculated to eliminate the log function and the signal without noise is recovered.

Cepstrum filtering is similar to Homomorphic filtering but requires the transducer spectrum and noise to be additive instead of multiplicative. Since the components are assumed to be additive, the logarithmic operator is not used. Again, the noise is high frequency whereas the Gaussian transducer transfer function is low frequency so a filter can be used to eliminate the noise. The low-pass filter isolates the signal. An inverse Fourier transform returns the data from Cepstrum domain to frequency domain and the noise is no longer observed in the spectrum.

The proper choice of a suitable low-pass filter is imperative for accurate removal of noise with either the Cepstrum or Homomorphic technique. These filters pass all information which have a frequency less than some cutoff frequency. The information is passed with partial or no attenuation whereas the data above the cutoff frequency is completely attenuated[16]. A short duration low-pass filter will truncate essential data whereas a broad low-pass filter will result in a distorted spectrum because the noise will not be entirely removed. Therefore the best results are achieved when the duration of the low-pass filter varies directly with the width of the spectrum[33].

The shape of the filter is also a key parameter in determining the performance of the filtering technique. Although a rectangular filter may be simple to implement and may not degrade the amplitude of the spectrum, alternative windows
may perform more satisfactorily. The Sinc function (defined in Equation 2.18 and displayed in Figure 2.9) is the Fourier Transform pair of the Rectangular function [16][3]. This means that the Fourier transform of a Rectangle is a Sinc and the Fourier transform of a Sinc is a Rectangle. When a Rectangular filter is multiplied by the data the result of the inverse transform is equivalent to convolving the signal with a Sinc function. A Sinc function has a number of fluctuations or ripples which are observed in the signal as a result of the filtering. Changing the width or amplitude of the Rectangular function filter does not reduce the ripple amplitudes. The fluctuations are present because the filter suddenly completely attenuates high frequency components[23].

\[ Sinc(x) = \frac{\sin(x)}{x} \quad (2.18) \]

Therefore a filter which is tapered at the edges will give a better result. A Gaussian window, for example, doesn’t abruptly cut off and therefore ripples are not observed in the inverse Fourier transformed data. Actually the Gaussian function has an identical Fourier transform pair meaning a Gaussian transforms to a Gaussian. An example of a Gaussian function is shown in Figure 2.10.

2.6 Line Spread Function and Modulation Transfer Function

Characterization of the transducer is essential in defining the resolution of the overall imaging system. Often, transducers are defined by physical dimensions, resonant frequency, and electrical characteristics but other factors are necessary to determine the image quality created by a particular transducer[36]. Quantitative
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Figure 2.9: The Sinc Function.

Figure 2.10: The Gaussian Function.
methods of characterizing this component include determining the Modulation Transfer Function (MTF) and the Line Spread Function (LSF).

The modulation transfer function is used to describe the imaging performance of the system in the spatial frequency domain [9]. The MTF defines the degradation of contrast of an image with respect to the object and is used for characterization over the entire spatial frequency range. Simply defined it is the ratio of the output modulation to the input modulation and therefore describes the reduction in contrast of spectral components as they pass through the system [16]. Equation 2.19 defines the imaging system in which \( g(x,y) \) is the image created by convolution of \( f(x,y) \), the target, and \( h(x,y) \), the impulse response of the system.

\[
g(x, y) = f(x, y) * h(x, y) \quad (2.19)
\]

In the frequency domain, Equation 2.20 defines the relation between the Fourier transforms of the image, target and impulse response of the system. \( H(\xi, \eta) \) is the transfer function of the system and its magnitude is defined as the Modulation Transfer Function.

\[
G(\xi, \eta) = F(\xi, \eta) \cdot H(\xi, \eta) \quad (2.20)
\]

Typically the MTF shows a gradual decay of amplitude with increasing spatial frequency [35]. The resolution limit of the system is identified by the cut-off frequency [34]. Historically, Modulation Transfer Functions were evaluated by directly measuring masks of different spatial frequencies. More recently, the MTF can be determined by calculating the Fourier transform of the Line Spread Func-
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Measurement of the Line Spread Function is done by scanning the image plane of a target of finite width. Ideally the target would be infinitely narrow. To correctly characterize the transducer, the LSF should be measured along the beam axis at the focal plane and in the far-field. Since the target is a line it can be described as a line mass of delta functions, Equation 2.21 [16].

\[ f(x, y) = \delta(x) \cdot 1(y) \]  

(2.21)

The impulse response, \( h(x,y) \), and transfer function, \( H(\xi,\eta) \), of the system are unknown. The output is measured and can be expressed as 2.22 [16]. Note that the output depends only on \( x \) because the input to the system was located along the \( y \)-axis only. If the one dimensional Fourier transform is calculated on Equation 2.22 the result is the \( \xi \)-axis profile of the modulation transfer function as shown in Equation 2.23. Expanding Equation 2.23 to include the definition of a Fourier transform is useful in demonstrating this relationship. The result is Equation 2.24 where the term in parenthesis is the equation for a projection along lines with a constant \( x \) value. A projection is formed by combining a collection of parallel line integrals [21] as shown in Figure 2.11. Line integrals are defined as integrations taken along straight lines through the object. The Fourier Slice Theorem indicates that the one-dimensional Fourier transform of parallel projections is equal to the slice of the two-dimensional Fourier transform of the original function [21].
Figure 2.11: Parallel Projections of a Rect Function at 0 degrees and 45 degrees.

\[(\delta(x) \cdot 1(y)) * h(x, y) = \int h(x, \beta) d\beta \quad (2.22)\]

\[\Im \left( \int h(x, \beta) d\beta \right) = H(\xi, 0) \quad (2.23)\]

\[\int_{-\infty}^{\infty} \left( \int_{-\infty}^{\infty} h(x, \beta) d\beta \right) e^{-i2\pi\xi x} dx = H(\xi, 0) \quad (2.24)\]

We can call the term in the parenthesis of Equation 2.24 \(P_{\theta=0}(x)\) for projection at zero degrees from the \(x\)-axis and write Equation 2.25. This can be substituted into 2.24 to result in Equation 2.26. Therefore the Fourier transform of the line
integral projection is equal to the slice of the two-dimensional Fourier transform of the initial function. This result is independent of the orientation between the object and coordinate system. If the parallel projections are rotated by an angle, \( \theta \), the Fourier transform of the projection is equal to the slice of the two-dimensional Fourier transform at angle \( \theta \) [21]. If the projections at every angle are Fourier transformed and combined; the result is the two-dimensional Fourier transform of the original function.

\[
\int_{-\infty}^{\infty} h(x, \beta) d\beta = P_{\theta=0}(x) \tag{2.25}
\]

\[
\int_{-\infty}^{\infty} P_{\theta=0}(x) e^{-i2\pi\xi x} dx = H(\xi, 0) \tag{2.26}
\]

The Fourier slice theorem can be demonstrated with a simple example using the \( \text{Rect}(x,y) \) function. The line integral projection of the \( \text{Rect}(x,y) \) function onto the x-axis is \( \text{Rect}(x) \). \( \text{Sinc}(\xi) \) is the Fourier transform of \( \text{Rect}(x) \). The two-dimensional Fourier transform of the original function, \( \text{Rect}(x,y) \) is \( \text{Sinc}(\xi, \eta) \). A slice of \( \text{Sinc}(\xi, \eta) \) in the \( \xi \)-axis results in \( \text{Sinc}(\xi) \) illustrating the theorem.

In our case, circular symmetry is observed because we are utilizing a circular disk focused transducer. The two-dimensional Fourier transform of a function that has circular symmetry can be expressed as a Hankel transform as derived in reference [4]. The Hankel transform of \( f(r) \) results in \( F(q) \) as shown in Equation 2.27 where \( r = (x^2 + y^2)^{\frac{1}{2}} \) and \( q = (\xi^2 + \eta^2)^{\frac{1}{2}} \). The zero-order Bessel function, \( J_0(z) \) is defined in Equation 2.28 [3].
The Abel function is the line integral projection for a two-dimensional function that has circular symmetry. This projection would be the same in all directions. The equation for the Abel transform is 2.29 [3].

\[ f_A(x) = \int_x^\infty \frac{f(r)rdr}{\sqrt{r^2 - x^2}} \]  

(2.29)

The Fourier Slice theorem also exists for circularly symmetric functions. In this situation the Fourier transform of the Abel transformed original function is equal to the Hankel transform of the original function [4].

The Fourier Slice Theorem proves that the LSF in one axis is the Fourier transform pair of a profile of the MTF. If a different orientation of the line mass had been chosen, the MTF profile would be for that orientation. To calculate the entire MTF every orientation of the LSF would have to be calculated. If the impulse response is circularly symmetric, however, the transfer function is also circularly symmetric and can be calculated with only one profile. It is also possible to reverse the procedure and calculate the LSF from the inverse transform of the MTF, if known.

The MTF can also be calculated from the ESF as was done by Shiloh et al. [35]. They utilized a plate glass edge after finding that the surface curvature of commercially available needles created a limitation [35]. The glass plate was
stated to form a 'perfectly sharp' edge, ideal for Edge Spread Function (ESF) measurement. In this case the edge of the glass can be described as a step function input to the system. A derivative of the edge response output results in the LSF (see [16] for the derivation). Again, the LSF can be Fourier transformed to determine the MTF and if the system is circularly symmetric only one edge-response measurement is required to define the entire MTF.

For calculation of the MTF it is recommended that the measured LSF be extrapolated beyond the location where it was 1% of its peak value [9]. In reality the LSF has infinite spatial extent but measurement becomes difficult as it approaches zero. The results presented in this study are not measured to zero amplitude. Erroneous oscillations may be observed if the abruptly ending LSF is used [11] to calculate the MTF. The forced smooth transition of the LSF curve improves the overall MTF.

2.7 Circular Disk Focused Transducer Characterization

For a circular disk focused transducer, theory predicts that the beam spot size in water is given by (in mm) [10]:

$$D \approx \frac{1.22 \cdot 1.5 \cdot f\#}{f_o (in MHz)}$$  \hspace{1cm} (2.30)

where $f\#$ stands for f-number and $f_o$ for center frequency of the transducer. The circular disk focused transducer measurement technique implemented by Barbu-McInnis [1] [31], used a 0.5 mm diameter hydrophone to scan the monochromatic beam in a plane perpendicular to the beam axis of a 3.5 MHz transducer. The finite size of the hydrophone’s sensing element was concluded to
be minimally responsible for smoothing of the beam profile. The lommel diffraction formulation [10] was used to calculate the theoretically correct beam profile values and a model was developed to correct for smoothing. Transducers with frequencies greater than 10 MHz are increasingly being used in commercial systems. The anticipated spot size can be below a few hundred microns. A direct measurement of the beam will require expensive hydrophones with an active element diameter much smaller than this anticipated spot size. Radulescu et al. [28] identified some limitations when characterizing ultrasound hydrophone probes at high frequencies: the diameter of the reference and test hydrophones caused spatial averaging. It was concluded that uncertainty of the characterization increased with increasing frequency [29]. Therefore if a high frequency transducer is to be characterized by a hydrophone, a correction model must be developed to minimize uncertainty. Cherin et al. [6] used two methods to characterize a high-frequency ultrasound transducer: (1) signal detection in transmission mode by a 4 micron diameter hydrophone and (2) a pulse echo mode characterization of a 50 micron diameter glass fiber. Larger than theoretical beam width and depth of field values were concluded to be caused, in part, by an averaging effect over the surface. Reflection mode measurements off of a small point target have proven difficult to perform experimentally in our laboratory. Gottlieb et al. [18] and Raum et al. [32] describe use of tungsten wire targets and standard pulse echo analysis.

In the current study the anticipated beam size is much smaller than 0.5 mm, therefore scanning the beam with a 0.5 mm diameter hydrophone will not be adequate. We have used a simpler technique with associated post processing that
provides a one-dimensional projection of the two-dimensional beam profile for several frequencies within the bandwidth of the transducer.

Since our technique utilizes a wire target, the theory of scattering from a cylinder must be noted. As mentioned in Section 2.2, scattering defines the effects that arise when an obstacle is placed in the path of a wave [37]. Rayleigh was the first to investigate scattering and did so for the limiting case where scatterers are small compared to the wavelength of the sound wave [13]. Morse and Ingard investigated the solution for scattering by rigid circular cylinders that were smaller than and equivalent in size to the wavelength [13][25]. Since it is not typical for the scatterer to be large compared to the wavelength for sound waves (it is typical for light), this case is not examined. Sound waves will penetrate the scatterer therefore the rigid and immovable assumption is invalid stated Faran. He provided experimental results which were compared to scattering patterns calculated based on theory [13].

The equations for scattering from a cylinder of radius \( a \) will be presented below following Morse and Ingrad. First, the pressure wave resulting from a plane wave with intensity \( I \) will be defined in Equation 2.31 [25]:

\[
P_p = P_o e^{i(kr \cos \phi - ct)}
\]  

(2.31)

Where \( k \), the wave number is \( \frac{2\pi}{\lambda} \), \( c \) is the velocity of the wave, \( t=\text{time} \) and \( P_o \) is defined in equation 2.32 [25]. In this equation, \( \rho \) is the mean density of the medium.

\[
P_o = \sqrt{\rho cl}
\]  

(2.32)
The scattered wave at a large distance from the cylinder has a pressure and radial velocity defined in Equations 2.33 and 2.34 [25]. \( \psi_s(\phi) \) is explained by Equation 2.35 [25].

\[
P_s \simeq -\sqrt{\frac{2\rho c I_a}{\pi r}} \psi_s(\phi) e^{ik(r-ct)}
\]

\[
u_s \simeq \frac{P_s}{\rho c}
\]

\[
\psi_s(\phi) = \frac{1}{\sqrt{ka}} \sum_{m=0}^{\infty} \epsilon_m \sin(\gamma_m)e^{-im\cos(m\phi)}
\]

In the above equations, \( r \) is the radial distance from the origin, \( \phi \) is the axial angle for cylindrical coordinates, \( \gamma_m \) is the phase shift for cylindrical scattering, and \( \epsilon_m \) is equal to 1 when \( m \) equals 0 and equal to 2 when \( m \) does not equal zero.

2.8 Summary

We have described how an ultrasonic transducer generates a sound wave and receives the reflected signal after interaction with the object. We have defined pulse-echo and C-scan mode ultrasound imaging and have discussed focused transducers. This background information is useful in understanding the basic experiment undertaken in this paper. The fingerprint scanner was also introduced including advantages of its use over optical systems. The system was defined as linear and shift invariant. The echo time signal from the wire target was presented as a convolution. The Fourier transform of this signal was calculated and then modified to isolate the diffraction term.
The importance of diffraction evaluation was discussed. Lommel functions were used to establish solutions for pressure field distribution of focused transducers. Fresnel approximation was assumed allowing analysis in the far and near-field regions.

A discussion of filtering methods to remove signal distortion included Homomorphic and Cepstrum filtering. Either method requires a careful choice of filter shape and width so as to avoid undesired artifacts.

Issues of characterizing high frequency transducers and attempts by other researchers to do the characterization were presented. The advantage of our method is the use of a short pulse excitation signal and post processing to achieve a one-dimensional projection of the two-dimensional beam profile for multiple frequencies within the bandwidth of the transducer. The Line Spread Function and Modulation Transfer Function were defined and their use in characterizing the transducer was expressed.
Chapter 3

Methods and Materials

3.1 Experimental Setup

In order to measure the transducer beam characteristics for our circular disk focused transducer, a 125 micron diameter nylon wire target (shown in Figure 3.1) was attached to the interior of a plastic container filled with Klearol (mineral) oil. The transducer being studied was suspended in the oil from a Parker Hannifin [Cleveland, Ohio] M4004M micrometer stage. Motion in the $Z$ and $\rho$ directions were possible with the micrometer stage as shown in Figure 3.2. The $\rho$-micrometer allowed a measurement accuracy of 0.001mm whereas the $Z$ micrometer was reliable to 0.01mm.

A short pulse signal was transmitted by the transducer using a Panametrics 5077PR Pulser. The response from the wire target, originating due to the acoustic impedance mismatch between the wire and the Klearol oil, was measured with the same transducer. The ratio of the effective diameter of the wire target to the wavelength of the transducer is 0.2. A parameter that indicates scattering pattern is $ka$ where $k$ is the wave number and $a$ is the radius of the transducer. If $ka$ is less than 1 the scatterer is considered a point scatterer whereas larger than
Figure 3.1: A 200X Magnification Microscope Image of the Wire used in this Study. The distance between the lines on the scale represent 25 microns.
Figure 3.2: Experimental Setup
CHAPTER 3. METHODS AND MATERIALS

Table 3.1: Initial Experiments

<table>
<thead>
<tr>
<th>#</th>
<th>Experiment Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Find Maximum Amplitude $\rho$ and $Z$ Location</td>
</tr>
<tr>
<td>2</td>
<td>Vary $\rho$ but Hold $Z$ Constant at Maximum Amplitude</td>
</tr>
<tr>
<td>3</td>
<td>Vary $Z$ but Hold $\rho$ Constant at Maximum Amplitude</td>
</tr>
<tr>
<td>4</td>
<td>Vary $\rho$ but Hold $Z$ Constant at 75% of Maximum Amplitude</td>
</tr>
<tr>
<td>5</td>
<td>Vary $\rho$ but Hold $Z$ Constant at 50% of Maximum Amplitude</td>
</tr>
<tr>
<td>6</td>
<td>Identify How the Time Arrival of the Signal Changes as $Z$ or $\rho$ Changes</td>
</tr>
</tbody>
</table>

one indicates a specular scatterer. For the nylon wire target, $ka$ is calculated to be $\approx 5$.

Identifying the location of the maximum amplitude signal was the first step of the study. A Hameb 60 MHz Oscilloscope model HM604 was used to display the response from the wire target for this step. The amplitude and micrometer readings were recorded for the six experiments listed in Table 3.1. All experiments were conducted at ambient temperature (20-25°C).

After the $\rho$ and $Z$ locations were optimized to find the maximum amplitude signal, $\rho$ was varied and $Z$ maintained at maximum amplitude. The results are shown in Figure 3.3. The amplitude decreases as the distance from the nominal $\rho$ position increases. As expected, the same trend was observed when $Z$ was varied and $\rho$ maintained at nominal, as shown in Figure 3.4. Figures 3.3 and 3.4 were used to determine how quickly the amplitude decreases so appropriate $\rho$ and $Z$ data collection ranges could be identified. A range of $\rho$ measurements with $Z$ held at 75\% maximum, Figure 3.5, and at 50\% maximum, Figure 3.6, were collected to verify that a full set of off-axis data could be measured when $Z$ wasn’t at nominal. Finally, the time between the signal from the wire and the transducer signal was measured as a function of $Z$ position. A linear relationship was observed as
shown in Figure 3.7. As anticipated, the time separation increases as the distance between the transducer and wire increases. For the off axis measurement range used in this study, the time of the signal did not change as distance increased.

Based on the results discussed above, the experiment was set up to measure the response from the nylon wire target at 5 vertical (Z) distances from the wire, each 0.25 mm apart. At each Z distance 85-100 measurements were collected at 0.005 mm lateral (ρ) incremental steps across a total distance of 0.43-0.50mm. The vertical and lateral data collection locations were chosen so the midpoint corresponded with the maximum amplitude signal. Table 3.2 summarizes the data collection locations. The signal was digitized at a rate of 1 gigasample/sec using a NI-Scope 5201 digitizer and Labview 5.1.1 software. The total number of samples taken was 20,000 and the digitization scale depended on the signal to
Figure 3.4: Vary Z but Hold ρ Constant at Maximum Amplitude.

Figure 3.5: Vary ρ but Hold Z Constant at 75% of Maximum Amplitude.
noise ratio of the measurement. Measurements with low noise were recorded at 0.5 V/div whereas low amplitude signals used 0.2 V/div. All experiments were conducted at ambient temperature (20-25°C).

<table>
<thead>
<tr>
<th>Label</th>
<th>Vertical Distance Z (mm)</th>
<th>Distance between $\rho$ Measurements (mm)</th>
<th>Range of $\rho$ Measurements (mm)</th>
<th>Number of Measurements</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-0.5</td>
<td>0.005</td>
<td>$\pm$ 0.250</td>
<td>100</td>
</tr>
<tr>
<td>B</td>
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<td>0.005</td>
<td>$\pm$ 0.245</td>
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<tr>
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<td>0.005</td>
<td>$\pm$ 0.225</td>
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<tr>
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<td>0.005</td>
<td>$\pm$ 0.213</td>
<td>85</td>
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<tr>
<td>E</td>
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<td>0.005</td>
<td>$\pm$ 0.225</td>
<td>90</td>
</tr>
</tbody>
</table>
Figure 3.7: Identify how the Time Arrival of the Signal Changes as Z Changes.
3.2 Data Processing

A flowchart demonstrating the data processing is displayed in Figure 3.8. The signals at each \( \rho \) and \( Z \) location were digitized and windowed prior to data processing. Figure 3.9 shows the raw data after digitization but prior to windowing. The first peak can be ignored as it is of the transducer-oil interface. Of interest is the second peak which is the response from the wire. To focus on the second peak a time-gate or window was applied. The width of the window was set to 2048 for Fourier transform calculations. Measurements at different \( Z \) locations resulted in peaks requiring different time-gates. In other words, the distance between the transducer and the wire affected the time at which the signal was recorded. The further the transducer from the wire, the longer the time period. The time-gate had to be chosen for each \( Z \)-position to ensure all data were captured. Figure 3.10 shows an example of data that have been windowed to just the signal from the wire. All further processing will be conducted on these data.

Next, a Fourier transform and magnitude were calculated on the time-gated signal. Periodic noise was observed as shown in Figure 3.11. This noise was thought to be caused by overlapping multiple reflections from the edges of the nylon wire and therefore is undesirable and must be removed. Three different attempts were made to remove the noise. The simplest was to use a one dimensional convolution kernel on the spectrum. This was determined to be ineffective because a kernel appropriately sized to remove noise resulted in a smoothed spectrum with lost data. Two more complex attempts were Cepstrum domain filtering and Homomorphic filtering. Cepstrum filtering is useful if the spectrum is made
Figure 3.8: Data Processing Method for Experiment.
Figure 3.9: Entire Digitized Signal.

Figure 3.10: Data Windowed to Signal from Wire.
up of two additive components. In this case the two components would be a Gaussian-like transducer spectrum and the periodic noise. Assuming the components are additive, the forward Fourier transform can be taken of the time-gated spectrum data. This is known as the cepstrum domain. The two additive components separate out because the Gaussian has low frequency components whereas the noise has high frequency components. At this point a filter is applied to isolate the slowly varying frequency components and the inverse Fourier transform is computed resulting in the spectrum data without noise. Homomorphic filtering, on the other hand, is useful if the spectrum is composed of multiplicative components. Homomorphic filtering removes the noise by calculating the log of the data prior to computing the forward Fourier transform. By taking the log the multiplicative components are now additive. Therefore the forward Fourier transform will be computed on the additive components and a filter can be used to remove the high frequencies which correspond to the noise. After the inverse Fourier transform is calculated the exponential is taken to return to the spectrum domain [17].

The most successful technique used on the data in this study was the Cepstrum domain filtering. Figure 3.12 shows the result of forward Fourier transforming the noisy spectrum displayed in 3.11. Both Rect and Gaussian functions with varying widths were used in attempts to filter the data shown in 3.12.

The width of the filter was optimized to eliminate the noise but minimize the reduction in amplitude of the spectrum. In other words, use of a filter with a small width would result in elimination of the noise but would also reduce the amplitude of the spectrum by removing data. A filter with a large width would
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Figure 3.11: Periodic Noise observed in the Spectrum.

Figure 3.12: Forward Fourier Transform of Noisy Data.
produce a spectrum with the full amplitude but some noise would remain. The following figures demonstrate this point. A Rectangular filter with a larger width, Figure 3.13, results in a higher amplitude than the calculation done with the small width Rectangular filter, Figure 3.14, but some noise is still visible.

Use of a Rectangular function as a filter results in an abrupt cutoff of the data prior to inverse Fourier transforming. Therefore ringing is observed in the spectrum. The Gaussian filter involves a smooth transition to zero and ringing is not found. Figure 3.15 shows the data filtered with a Rectangular function whereas Figure 3.16 is data filtered with a Gaussian function. The difference between the abrupt and smooth transition is easily observed. Figure 3.17 can be compared with 3.14, above, to conclude that the ringing resulting from filter-
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Figure 3.14: Inverse Fourier Transform of Data Filtered with a Rectangular Function of Small Width.
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Figure 3.15: Data Filtered with a Rectangular Function of Small Width.

ing with a Rectangular function is not observed when filtering with a Gaussian function. Therefore Gaussian filters of appropriate width were employed when Cepstrum filtering. An example of a Gaussian function used in the Cepstrum filtering process is displayed in Figure 3.18.

After Cepstrum filtering, average amplitude values over specific frequency bins were calculated for 10 frequency values. For proprietary reasons the frequencies will not be stated but will be described in reference to the nominal frequency. The frequencies chosen included the nominal and the following variations from nominal: ±2MHz, ±4MHz, ±6MHz, -7MHz, +8MHz, +9MHz. The width of each frequency bin was 2MHz. This means that to identify the amplitude at the nominal frequency for a specific $\rho$ and $Z$ location an average of amplitude values
from -1MHz to +1MHz was calculated.

A map of $B(f_o, z, \rho)$ was generated within the focal zone for each of the ten frequencies. The true beam pattern in any plane at a distance $Z$ is a two dimensional circularly symmetric function $P(x,y)$. Since our measurements have been performed using a wire target, we will tacitly assume that $B(f_o, z, \rho )$ constitutes a one dimensional projection of the two dimensional function $P(x,y)$. Therefore we will refer to it as system $LSF(f_o, z, \rho )$.

The amplitudes at each $\rho$ for a specific $Z$ location and frequency were compiled and plotted as shown in Figure 3.19. Next, figures were created displaying all five $Z$ locations for each frequency. An example is shown in Figure 3.20. A top-down view of the same information is displayed in 3.21 for a different perspective.
Figure 3.17: Inverse Fourier Transform of Data Filtered with a Gaussian Function of Appropriate Width.
Figure 3.18: An Example of a Gaussian Function used in Cepstrum Filtering.

Figure 3.19: Spectrum for nominal frequency and Z position C.
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Figure 3.20: Spectrum for nominal frequency and all five Z positions.

Figure 3.21: Top-Down View of Beam Profiles for five Z Distances at the Nominal Frequency.
Finally, the value of the Modulation Transfer Functions at each frequency and Z location were calculated by taking the Fourier transform of the Line Spread Functions.

3.3 Full-width Half Maximum and Misalignment Analysis

The location of the maximum was measured for different Z positions and is shown in Table 3.3. A shift was observed because the transducer and wire are slightly misaligned. Figure 3.22 illustrates the location of the maximum for all five Z positions. Vertical Distances B, C, and D have the same maximum locations whereas the maximum locations of A and E are shifted. The transducer is furthest from the wire for E and the maximum is shifted to left with respect to the origin. Location A is closest to the wire resulting in the maximum amplitude being shifted from the origin to the right. Figure 3.23 shows the regression line for A, C, and E with a correlation of 0.9979. The transducer was calculated to be 4.85 degrees misaligned from the wire. As discussed in sections 2.7 and 3.1, the nylon wire target is a cylindrical scatterer with \( ka \approx 5 \). Faran [13] calculated the amplitude of the pressure wave scattered by a rigid cylinder for \( ka=5.0 \) which indicates nominally constant pressure amplitude for \( \pm 30 \) degrees from the direction of incident sound. The pressure amplitude decreases to 75 percent of maximum at \( \pm 85 \) degrees and decreases to zero amplitude at \( \pm 130 \) degrees from the direction of incident sound. Our measured misalignment of \( \sim 5 \) degrees would have the same pressure amplitude as signal scattered at 0 degrees. Therefore the misalignment observed was determined to be inconsequential.

The angle between the wire target and the transducer for the furthest off-axis \( p \)
Table 3.3: Location of Maximum as a function of the Z Position.

<table>
<thead>
<tr>
<th>Z-Distance from Nominal (mm)</th>
<th>Location of Maximum (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.5</td>
<td>0.03</td>
</tr>
<tr>
<td>-0.25</td>
<td>-0.02</td>
</tr>
<tr>
<td>0</td>
<td>-0.015</td>
</tr>
<tr>
<td>0.25</td>
<td>-0.02</td>
</tr>
<tr>
<td>0.5</td>
<td>-0.055</td>
</tr>
</tbody>
</table>

Figure 3.22: Shift (mm) of Location of Maximum for All Five Z Locations.

measurement was calculated for each Z position and compared to Faran’s results for a rigid cylinder when $ka=5.0$. The largest angle, observed for Z position $A$, was 2.1 degrees. Based on the nominally constant results for $\pm 30$ degrees calculated by Faran this 2.1 degree angle is insignificant.

The full width half maximum can be defined as the distance between points on the curve at which the function reaches half of its maximum value. Full width half maximum calculations were made for all ten frequencies for Z distances $A$,
Figure 3.23: Linear Relationship of Shift (mm) and Z location (mm).

B, and C. The goal was to determine if the full width half maximum changes as the distance between the transducer and the wire is increased. Results of this analysis will be discussed in Chapter 4.

3.4 Measurement of Envelope

An alternative method for processing the digitized data measures the envelope of the signal. The goal was to compare our experimental method described above to the technique currently utilized by the manufacturer of the transducer. The envelope of the raw data is detected by summing the phase shifted and original signals after taking the magnitude of each independently.

Figure 3.24 shows the data processing steps for this alternative method. The
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Collect and Digitize Data for each $Z, \rho$ position

1. Window Data
2. Calculate Envelope of the Signal
3. Identify Maximum Amplitude
4. Average Values Immediately Surrounding Maximum Amplitude
5. Compile all $\rho$ Results for a Particular $Z$ and Plot

Figure 3.24: Envelope Measurement Method of Data Processing.

Figure 3.25: Envelope Measurement of Windowed Data.
envelope of the original data was measured resulting in Figure 3.25. Beam profiles were created by locating the maximum amplitude for each \( \rho \) measurement. An average of values immediately surrounding the maximum amplitude was used instead of just using the peak value to overcome noise in the data. The beam profiles were then compared to those calculated by the previously described method. Full width half maximum calculations were computed for analysis. Results will be presented in Section 4.3

3.5 Theoretical Calculation

A goal of the study was to calculate the theoretical beam profile. The radius of curvature and diameter of the transducer were provided by the manufacturer. The focal length can be assumed to be equivalent to the radius of curvature because the transducer is highly focused.

In order to calculate the theoretical beam profile, the distance between the transducer and wire had to be calculated. This can be determined by identifying the time of the wire/target signal and calculating the distance based on knowledge of the speed of sound in Klearol oil. Plots similar to 3.9 were analyzed and the time between the transducer/oil peak and the transducer/wire peak recorded. The actual time is half of this value since two-way travel of pulse-echo mode must be accounted for. Time was then converted to a distance by multiplying by the velocity of sound in Klearol oil (1570 meters/sec). This process was repeated for each \( Z \) location to determine the distance between the wire and the transducer for \( A, B, C, D \) and \( E \). The manufacturer’s specified diameter and focal length, along with the nominal frequency identified previously in the experiment were
used in the theoretical calculation of Lommel diffraction (described in Section 2.4). Beam profiles were then plotted for the distances calculated based on the time of the signal. These will be displayed and discussed in section 4.4.

3.6 Summary

Response from a wire target that was exposed to a short pulse signal was measured at multiple ρ and Z locations. An initial experiment was conducted to determine the nominal ρ and Z locations, the decrease in amplitude as the distance from nominal increased, and the time shift of the signal for the different Z locations. Based on these results an experiment was defined and conducted which involved collecting signals at 85-100 ρ locations for each of 5 Z positions. The digitized signals were windowed so further processing could be done focusing only on the signal from the wire. A Fourier transform was then calculated to yield the frequency spectrum information contained in the short pulse. A Cepstrum domain filter was applied to the frequency spectrum data to remove some periodic noise that was evident. The process involved taking a forward Fourier transform of the magnitude spectrum, applying a Gaussian filter of optimum width and then inverse transforming the filtered data. This resulted in the original spectrum without the periodic noise. When choosing the width of the Gaussian filter there was a tradeoff between the elimination of the noise and a reduction in amplitude of the spectrum. Care was taken to minimize the amplitude reduction but still successfully remove the periodic noise.

Amplitude values at ten frequencies were identified for each spectrum at each Z and ρ value. System LSF(f0, z, ρ) were displayed by compiling the amplitudes
at each $\rho$ for a specific $Z$ location and frequency. Images were also created
displaying all five $Z$ locations for each frequency.

Misalignment analysis was presented indicating that the location of the max-
imum is different for each $Z$ location. A calculation indicated the transducer is
4.85 degrees misaligned from being perpendicular to the wire target.

An alternative method for processing the data, measuring the envelope of the
signal, was presented. Data processing steps for this method were described.
The results of this analysis and a comparison to our experimental method will be
presented in the next chapter.

Finally a discussion of the theoretical calculations was presented. Theoretical
and experimental results will be compared in Section 4.4
Chapter 4

Results

4.1 Transducer Beam Characteristics

This chapter summarizes the results of data acquisition and processing described in Chapter 3. Figures 4.1 through 4.10 illustrate the beam profiles for the ten different frequencies from the nominal: $\pm 2\text{MHz}$, $\pm 4\text{MHz}$, $\pm 6\text{MHz}$, $-7\text{MHz}$, $+8\text{MHz}$, $+9\text{MHz}$. Ten frequencies were required to characterize how the MTF and LSF varied with frequency. For all five $Z$ positions the amplitude of the beam profile at these ten frequencies was adequate for analysis. The top plot in each figure was created to display the image, a surface plot of the image data, and a contour plot of the image data in a tri-level display. The beam profiles are displayed such that the near field, $Z$-distance $A$, is observed first and the far field, $Z$-distance $E$, is at the back of the plot. The bottom plot in each figure is a top-down view of the pressure amplitude map where white represents large pressure amplitude values and black indicates pressure amplitude values near zero. $Z$ distance $A$ is located toward the bottom of the page and $E$ toward the top in these figures.

Observations from Figures 4.1 through 4.10 need to be summarized. For all frequencies the maximum amplitude is observed on-axis. Amplitude decreases as
Figure 4.1: Tri-Level Display and Top-Down view for all Five $Z$ Locations at Frequency -7MHz from Nominal.
Figure 4.2: Tri-Level Display and Top-Down view for all Five $Z$ Locations at Frequency -6MHz from Nominal.
Figure 4.3: Tri-Level Display and Top-Down view for all Five $Z$ Locations at Frequency -4MHz from Nominal.
Figure 4.4: Tri-Level Display and Top-Down view for all Five Z Locations at Frequency -2MHz from Nominal.
Figure 4.5: Tri-Level Display and Top-Down view for all Five Z Locations at Nominal Frequency.
Figure 4.6: Tri-Level Display and Top-Down view for all Five Z Locations at Frequency +2MHz from Nominal.
Figure 4.7: Tri-Level Display and Top-Down view for all Five Z Locations at Frequency +4MHz from Nominal.
Figure 4.8: Tri-Level Display and Top-Down view for all Five Z Locations at Frequency +6MHz from Nominal.
Figure 4.9: Tri-Level Display and Top-Down view for all Five Z Locations at Frequency +8MHz from Nominal.
Figure 4.10: Tri-Level Display and Top-Down view for all Five Z Locations at Frequency +9MHz from Nominal.
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the radial distance increases because less signal is received from the target. As the Z distance was increased from the nominal, the amplitude of the frequency spectrum decreased. Therefore as the distance between the transducer and the wire increases, less signal is returned to the transducer. The signal that is received has a similar profile as signal measured at the nominal location although the width is larger. Attenuation is the cause of this decrease in amplitude. The intensity of a wave propagating through a medium decreases with the distance of the propagation due to two types of losses: absorption of energy by the medium and loss due to scattering [30]. Both mechanisms result in loss of amplitude termed attenuation. Attenuation and its frequency dependence are parameters of the material through which the pulse is propagating. In general, the signal amplitude will decrease exponentially with distance and attenuation increases with frequency.

When the transducer was moved from nominal closer to the wire target the amplitude decreased and near field diffraction effects were observed. The near-field diffraction effects are indicated by the amplitude varying greatly within this region as compared to little variation in the far field. Refer back to Figure 2.3 to observe the Z locations A, B, C, D, and E with respect to the transducer.

The relationship between radial location, Z location, and peak shape, discussed above, is true for all frequencies. There were, however, differences observed for the different frequency plots. To aid in comparison, Figure 4.11 shows the beam profile at three different frequencies; -7MHz from nominal, at nominal, +9MHz from nominal. Figure 4.12 is also helpful for comparison. In this figure, pressure amplitude maps for five frequencies (-7MHz from nominal, -4MHz from
nominal, at nominal, +4MHz from nominal and +9MHz from nominal) are displayed. It can be concluded from these figures that the broadness of the beam profile is related to frequency. Lower frequencies result in broader profiles. Higher frequencies exhibit a tighter central lobe with a more definite form. Note that the amplitudes have been normalized in these figures. If the figures were not normalized a change in amplitude with frequency would mask the relationship between broadness and frequency. The highest amplitude is observed at nominal and amplitude decreases as the frequency varies from nominal.

Figures 4.13, 4.14, 4.15, 4.16, and 4.17 are included to compare the beam profiles for all ten frequencies at Z locations A, B, C, D and E, respectively. Figures 4.15, 4.16 and 4.17 are amplitude normalized to ease in analysis. In other words, instead of displaying the actual amplitude at each frequency which would show a drop off in maximum as the frequency increased or decreased from nominal, the normalized amplitudes were computed resulting in overlayed data. Figures 4.13 and 4.14 were not amplitude normalized because of near field diffraction effects observed. These figures have many profiles with three maximums instead of the one observed at Z-distances C, D, and E. Notice that in Figure 4.14 frequencies -7MHz, -6MHz and -4MHz only have one maximum. This indicates that the Z-position is still in focus and not in the near field region. Therefore the identification of near-field, true-focus and far-field must be referenced to both Z-distance and frequency.

Figures 4.15, 4.16, and 4.17 also indicate that the width of the frequency spectrum is inversely proportional to frequency. To emphasize this a zoomed in version of the graphs is shown below the full plot. The smallest width is
Figure 4.11: Tri-Level Display for all Five Z Locations at Frequencies -7 MHz, Nominal and +9 MHz.
CHAPTER 4. RESULTS

Figure 4.12: Top-Down view for all Five Z locations at Frequencies -7MHz, -4MHz, Nominal +4MHz, and +9MHz.
observed for the +9MHz frequency from nominal profile whereas -7MHz frequency from nominal had the largest. The spread of beam profiles is also related to Z location. For nominal Z location the beam profiles are more spread out for the varying frequencies. The relationship between frequency and width is apparent for Z location C but not as obvious as at Z location E which has tightly spaced beam profiles. Therefore a variation in frequency will have a greater impact at true-focus than in the far-field region. Full-width half maximum analysis will be discussed in Section 4.2.

The misalignment discussed in Section 3.3 is apparent. Figure 4.17 illustrates that as the distance between the transducer and the wire is increased, the beam profiles move to the left. On the other hand, as the distance is decreased the beam profiles move the right as observed in Figure 4.13.
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4.2 Full-Width Half Maximum

Full-width half maximum calculations were performed to determine how the width of the beam profiles varied with Z location. Table 4.1 summarizes the results of this calculation. The lowest full-width half maximum values were found for Z location C, or nominal. As the distance from nominal increased so did the full-width half maximum. Therefore Z location E had the largest value. Figure 4.18 illustrates this visually. For all frequency values, Z location C has the lowest full-width half maximum values. The beam profiles of C, D, and E at the nominal frequency are overlayed in Figure 4.19. Again, it is apparent that location C has the tightest beam profile whereas E is the broadest. If the beam profiles were overlayed for each frequency, the same trend would be observed. Z position C
Figure 4.15: Beam Profiles of all Frequencies Overlayed and Normalized for $Z$ location $C$. 
Figure 4.16: Beam Profiles of all Frequencies Overlayed and Normalized for Z location $D$. 
Figure 4.17: Beam Profiles of all Frequencies Overlayed and Normalized for Z location $E$. 
Table 4.1: Full-Width Half Maximum (mm) Results

<table>
<thead>
<tr>
<th>Freq. (MHz)/Z</th>
<th>-7</th>
<th>-6</th>
<th>-4</th>
<th>Nominal</th>
<th>+2</th>
<th>+4</th>
<th>+6</th>
<th>+8</th>
<th>+9</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.12</td>
<td>0.11</td>
<td>0.10</td>
<td>0.09</td>
<td>0.08</td>
<td>0.07</td>
<td>0.07</td>
<td>0.07</td>
<td>0.06</td>
</tr>
<tr>
<td>D</td>
<td>0.17</td>
<td>0.16</td>
<td>0.15</td>
<td>0.13</td>
<td>0.13</td>
<td>0.12</td>
<td>0.11</td>
<td>0.11</td>
<td>0.11</td>
</tr>
<tr>
<td>E</td>
<td>0.22</td>
<td>0.21</td>
<td>0.19</td>
<td>0.17</td>
<td>0.16</td>
<td>0.14</td>
<td>0.14</td>
<td>0.13</td>
<td>0.14</td>
</tr>
</tbody>
</table>

Figure 4.18: Full-Width Half Maximum comparison of Z locations C, D, and E.

is located in the true focus region whereas D and E are in the far field region. The pressure amplitude in the true focus region is the highest and the peak the narrowest. Notice that full-width half maximum calculations were not made for locations A and B because they are in near field region as indicated by the near field diffraction effects (varying pressure amplitudes).

Full width half maximum calculations can also be used to quantitatively analyze how the width varies with frequency. Figure 4.18 illustrates that the full-
CHAPTER 4. RESULTS

Figure 4.19: Nominal Frequency Beam Profiles Superimposed for Z locations C, D, and E.
width half maximum decreases with frequency. Frequency $+9\text{MHz}$ had the smallest full width half maximum whereas $-7\text{MHz}$ had the largest indicating full width half maximum has an inverse relationship to frequency. Returning to Figures 4.15, 4.16 and 4.17 are useful in observing the relationship between frequency and beam profile width. The higher frequency beam profiles are found to be narrow whereas lower frequencies exhibit broader profiles.

### 4.3 Measurement of Envelope

A comparison between the method of analysis used in this study and that used by the manufacturer was desired. The measurement of envelope technique, as discussed in Section 3.4 was used to determine the beam profile for all five $Z$ locations. These results can be compared to the beam profiles calculated by the method used in this study. Figure 4.20 illustrates the normalized beam profiles for each calculation technique at $Z$ location $C$. It appears that the envelope measurement technique results in a beam with a broader profile than the method used in this study. The same trend is observed in Figures 4.21 and 4.22 which show the normalized beam profiles for $Z$ positions $D$ and $E$. Again, locations $A$ and $B$ could not be normalized due to near field diffraction effects. Therefore the two techniques for calculating profiles at these locations will be plotted independently in Figures 4.23 and 4.24.

Full width half maximum calculations were made on the envelope measurement technique results. They are compared to the full width half maximum results for our experimental method at nominal frequency and at $+9\text{MHz}$ from nominal in Table 4.2 and Figure 4.25. The results in the table indicate that al-
though the experimental technique nominal frequency has smaller values for all three \( Z \) locations, the values may not be significantly different. The same can be said for the figure. At first glance it appears that the experimental technique nominal frequency results in a smaller full width half maximum but realistically the difference in values may not be significant. However, the +9MHz from nominal frequency has significantly lower FWHM values than the envelope measurement technique. We previously concluded that as frequency increases full width half maximum decreases. Therefore use of high frequency information from the signal may improve resolution compared to the currently utilized envelope measurement method. Figure 4.26 emphasizes this conclusion by comparing the envelope measurement method to the nominal frequency, +9MHz, and -7MHz. It is ap-
Figure 4.21: Comparison of Beam Profile Calculated by Envelope Measurement and Our Experimental Method for a $Z$ of $D$. 
Figure 4.22: Comparison of Beam Profile Calculated by Envelope Measurement and Our Experimental Method for a Z of E.

parent that -7MHz has a much higher full width half maximum than the envelope technique and therefore would degrade the resolution. The +9MHz frequency results in a visually tighter profile which would be advantageous in comparison to measuring the envelope. Implementation of +9MHz frequency in the Ultra-Scan imaging system would require more data processing than the measurement of envelope technique. It would also be necessary to determine if the attenuation of signal in tissue would make measurement of the fingerprint at this high frequency unfeasible.
Figure 4.23: Comparison of Beam Profile Calculated by Envelope Measurement (top) and Our Experimental Method (bottom) for a Z of A.
Figure 4.24: Comparison of Beam Profile Calculated by Envelope Measurement (top) and Our Experimental Method (bottom) for a $Z$ of $B$. 
Figure 4.25: Full-Width Half Maximum Comparison between Experimental and Envelope Measurement Methods.

Table 4.2: Comparison of Full Width Half Maximum Values for Envelope Measurement and Our Experimental Techniques.

<table>
<thead>
<tr>
<th>Location</th>
<th>Measurement of Envelope (mm)</th>
<th>Experimental Nominal Frequency (mm)</th>
<th>Experimental +9 MHz Frequency (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>.085</td>
<td>.080</td>
<td>.06</td>
</tr>
<tr>
<td>D</td>
<td>.135</td>
<td>.130</td>
<td>.105</td>
</tr>
<tr>
<td>E</td>
<td>.175</td>
<td>.160</td>
<td>.135</td>
</tr>
</tbody>
</table>
4.4 Theoretical Calculations

Theoretical calculations of beam profiles were computed for comparison to the experimentally measured beam profiles. Figures 4.27 through 4.31 show that the shape of the theoretical calculations are not similar to what was observed experimentally. The near-field diffraction effects predicted theoretically for $Z$ position $A$ are not the same as what is observed in the experimental results. Three peaks were apparent at $Z$ position $A$ experimentally but only two are apparent in the theoretical results. $Z$ positions $B$ and $C$ have similar theoretical and experimental results. Locations $D$ and $E$ are completely different from what was observed in our study.

In section 3.5 the method used to calculate the theoretical results was pre-
CHAPTER 4. RESULTS

sented. The focal length and diameter used in the calculations were provided by the manufacturer of the transducer. There is a 1mm difference between the experimental and theoretical nominal Z location (focal length). The manufacturer's focal length does not match the distance at which we observe the maximum peak. This 1mm difference would cause non-matching theoretical and experimental results.

Although the theoretical results do not match what was observed experimentally, general observations can be made based on Z position C. Side-lobes that were not observed experimentally are apparent in the calculated theoretical results. The transducer may have been apodized to reduce the side-lobes. Apodization would cause the width of the main lobe to increase however, a negative result. Visual analysis indicates that the main lobe is much tighter for the theoretical results.

Another possible cause of the the larger width beam profile observed experimentally is the width of the wire target. Analysis of the microscope image was conducted to determine the effective diameter of the wire. Profiles of the wire were evaluated based on the pixel intensity. Twelve pixel profiles were measured, as shown in Figure 4.32, and averaged, shown in Figure 4.33. The noise apparent in individual pixel profiles was eliminated by the averaging. To be able to determine the effective width, the average was zoomed and displayed in Figure 4.34. The full-width half maximum of this result is 75 microns but it could be argued that the effective width is less than 75 microns. Visual estimation indicates an effective width, where a specular reflection takes place, of approximately 20-30 microns. The ratio of object size to the wavelength of the transducer would be
Figure 4.27: Theoretical Calculation of Beam Profile for Z Position A.
Figure 4.28: Theoretical Calculation of Beam Profile for Z Position B.
Figure 4.29: Theoretical Calculation of Beam Profile for the Nominal Z Location, C.
Figure 4.30: Theoretical Calculation of Beam Profile for Z Position $D$. 
Figure 4.31: Theoretical Calculation of Beam Profile for Z Position E.
0.2 for this effective width.

Finally, it should be pointed out that the apodization could change the effective diameter of the transducer. The diameter value provided by the manufacturer was used in the theoretical calculation. Therefore the theoretical calculation will be erroneous if the diameter used was not accurate. Based on the information provided by the manufacturer, the f-number (ratio of the radius of curvature and diameter) is less than one which is hard to imagine in a physical design sense again leading to the conclusion that the effective diameter may be less than the value provided. We can estimate the diameter that would result in an f-number
Figure 4.33: Average of the Twelve Pixel Intensity Profiles of the Wire Target.
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Figure 4.34: Zoom in of the Average Intensity Profile of the Wire Target.
of one and plot the theoretical results.

Figure 4.35 though 4.39 display the theoretical calculations if the diameter is chosen to produce and f-number of one. Again, the results do not correlate with our experimental data. It is possible that our estimation of the diameter is incorrect and the f-number is not one. Since the apodization is an unknown parameter we would only be guessing if we tried to correct for it in our calculations.

4.5 Modulation Transfer Function

As stated in Section 2.6, the Fourier transform of the Line Spread Function (LSF) in one axis results in the Modulation Transfer Function (MTF) profile. MTF calculations were made for the true-focus and far-field Z locations C, D, and E. The results are displayed in Figures 4.40, 4.41 and 4.42, respectively. The cutoff frequency is the resolution limit of the transducer and is defined as the location where the MTF reaches the x-axis. Far-field measurements have a lower cutoff frequency than the true-focus results. Therefore if the transducer is measuring an object located in far-field the resolution limit will be lower and high frequency information will not be captured. The true-focus location has the optimum resolution limit and will capture data across a larger range of frequencies. The transducer being evaluated in this study indicated over 70 percent higher cutoff frequency for true-focus than far-field.

All three Z positions indicate that higher frequency corresponds with higher resolution as anticipated. High frequency transducers are used in industries and medical applications requiring high resolution ultrasonic imaging. The cutoff frequency for +9MHz is considerably larger than for -7MHz.
Figure 4.35: Theoretical Calculation of Beam Profile for Z Position A using Estimated Transducer Diameter.
Figure 4.36: Theoretical Calculation of Beam Profile for Z Position B using Estimated Transducer Diameter.
Figure 4.37: Theoretical Calculation of Beam Profile for the Nominal Z Location, C using Estimated Transducer Diameter.
Figure 4.38: Theoretical Calculation of Beam Profile for Z Position \( D \) using Estimated Transducer Diameter.
Figure 4.39: Theoretical Calculation of Beam Profile for Z Position E using Estimated Transducer Diameter.
Figure 4.40: Modulation Transfer Function for Z Location C.
Figure 4.41: Modulation Transfer Function for Z Location D. The top plot displays the 0=20 cycles/mm range whereas the bottom figure is for 0-12 cycles/mm.
Figure 4.42: Modulation Transfer Function for Z Location E. The top plot displays the 0=20 cycles/mm range whereas the bottom figure is for 0-12 cycles/mm.
It is difficult to visually resolve the high frequency MTF curves (+6MHz, +8MHz, and +9MHz) in Figures 4.40 though 4.42. Therefore the area under the MTF curves was calculated and is displayed in Figure 4.43 and Table 4.3. This calculation also allows comparison of the different Z positions. The area under the MTF curve can be related to the quality of the imaging system. A curve with a larger area indicates more information will be captured by the imaging system when compared to a curve with a smaller area. In this study the nominal Z location has the largest MTF area for all frequencies measured. Therefore measurements at nominal Z will retain the most information through the imaging process. For all Z positions, the low frequency data shows the smallest area. As frequency increases so does the area under the curve. It appears to reach a limit, however, at which the area levels out and increasing the frequency does not increase area.

The spread of frequencies at nominal Z location is apparent. This was also observed in Section 4.1 for the beam profiles. Z locations D and E overlay more closely than the nominal.
Figure 4.43: Area Under Modulation Transfer Function Curves for Z Locations C, D, and E.

Table 4.3: Area Under MTF Curves for Z Locations C, D, and E.

<table>
<thead>
<tr>
<th>Frequency (MHz)</th>
<th>Z Position C</th>
<th>Z Position D</th>
<th>Z Position E</th>
</tr>
</thead>
<tbody>
<tr>
<td>-7</td>
<td>5.8</td>
<td>4.7</td>
<td>4.0</td>
</tr>
<tr>
<td>-6</td>
<td>6.4</td>
<td>5.1</td>
<td>4.2</td>
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<tr>
<td>-4</td>
<td>6.7</td>
<td>5.3</td>
<td>4.2</td>
</tr>
<tr>
<td>-2</td>
<td>7.1</td>
<td>5.8</td>
<td>4.4</td>
</tr>
<tr>
<td>Nominal</td>
<td>7.4</td>
<td>6.1</td>
<td>4.5</td>
</tr>
<tr>
<td>+2</td>
<td>7.6</td>
<td>6.4</td>
<td>4.6</td>
</tr>
<tr>
<td>+4</td>
<td>7.8</td>
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</tr>
<tr>
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<td>7.9</td>
<td>7.1</td>
<td>4.8</td>
</tr>
<tr>
<td>+9</td>
<td>7.8</td>
<td>7.2</td>
<td>4.8</td>
</tr>
</tbody>
</table>
Chapter 5

Conclusions

The overall metrics (LSF and MTF) of the system currently being studied will strongly depend on the transducer beam profile. The profile has been shown to be both frequency and depth dependent. Higher frequencies result in tighter profiles. The width of the beam profile increases as the depth of target increases from nominal focus. Near and far field diffraction effects were observed as the beam profile was varied from nominal $Z$ distance.

Measuring signal from a thin nylon wire target in pulse-echo mode proved to be a reasonable alternative to using a hydrophone when the beam size was much smaller than 500 microns. Whether a significant amount of smoothing has occurred due to the 125 micron diameter of the wire target is yet to be determined. One approach would be to repeat the measurement on 10 or 20 micron tungsten wire. However it is our contention that due to the curvature of the 125 micron wire, the effective diameter from where a specular reflection takes place is perhaps much smaller, maybe on the order of 20-30 microns as shown in our microscope image analysis. Evaluation of Edge Spread Function (ESF) as described by Shiloh et al [35] is recommended for future experiments. The
CHAPTER 5. CONCLUSIONS

derivative of the edge response can be taken to yield the LSF. The LSF can be Fourier transformed resulting in the MTF.

Theoretical calculations based on Lommel diffraction formulations were made for our finite circular transducer and a cylindrical nylon wire target with an essentially infinite length. Analysis of the theoretical predictions proved inconclusive with regards to the question of whether the wire target caused significant smoothing and increased the full width half maximum of the beam profile. The discrepancies observed in the comparison between experimental and theoretical results could be due to the wire target width or the apodization of the transducer which would also increase the width of the main lobe. The values used to calculate the theoretical predictions (focal length and diameter) were brought into question making the results of the calculation unreliable for comparison to actual data.

The Line Spread Function (LSF) was defined for the focused transducer being studied. Monochromatic information was derived by Fourier transforming the short pulse echo time signal and removing periodic noise via Cepstrum domain filtering. The periodic noise was most probably from multiple reflections on the finite sized wire. Calculation of the MTFs were made by Fourier transforming the LSF. Analysis of these results indicated that MTF varied with frequency. Higher frequency beam profiles resulted in MTFs with higher cutoff frequencies. As Z-position increased from nominal, cut-off frequency decreased.

Confirmation of the MTFs calculated in this study could be made by experimental measurement using a phantom with appropriate sinusoidal patterns.

Future work will be done to measure the LSF and MTF for the entire imaging
system and then compare the results to those found for the transducer. This will yield information about the relationship between the characteristics of the transducer and the overall image quality. An advantage of MTF analysis is that if the MTF of each subsystem can be identified, the MTF of the entire system is calculated by cascading the subsystem MTFs.
Bibliography


