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Department of Photographic Science and Instrumentation

SUBJECT: Undergraduate Research Thesis

MEASUREMENT OF VOLUME SCATTERING FUNCTION IN TURBID WATER AND ITS CORRELATION WITH MTF DEGRADATION

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ABSTRACT:

Scattering of light in turbid water is a major source of image degradation. The intention of this experiment was to construct a small angle scattering meter and measure the volume scattering functions for specific turbid conditions. These experimentally derived volume scattering functions were used to correlate a mathematical modulation transfer function derivation with previously measured modulation transfer functions for a specific turbid medium. Due to design shortcomings, errors were produced in the production of the volume scattering functions. This produced a lack of correlation between our mathematically derived MTF's and those measured in the past. The validity of the mathematical transformation from the VSF to MTF cannot be attested to until more accurate data and sophisticated analysis are obtained

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INTRODUCTION:

Scattering of light is a major factor in determining image degradation when photographing objects through a turbid water medium. This problem of preserving image quality is significant because if turbid mediums exhibiting specific properties can be objectively defined, then correlations and prediction of image degradation may be developed which enables a photographer to more intelligenly design an appropriate system of film, camera, lens and other factors in order to maximize the amount of information he can obtain in a given turbid medium.

In-water vision and photography is vitally dependent on the light transmission characteristics of the medium. Even under the most ideal conditions, in-water visibility is limited to less than a hundred meters. In severe cases water turbidity may prevent a diver from seeing his hands directly in front of his mask.

Two events occur that can alter the way a beam of light travels through water. The photons may be absorbed and converted to other forms of energy (principally heat) or they may be scattered and diverted away from the beam. Absorbed light is effectively removed from the optical spectrum and does not contribute to the formation of images, however, scattered light energy is not lost but merely deflected and may add to the nonscattered image forming light energy.

Image quality is very adversely affected by scattered light. If the amount of scattering is small then only the fine details in the image will be disturbed, but if the medium is more turbid, as is often the case in in-water imaging, then the details and the object itself will become blurred and indistinct.

If one is to evaluate this phenomenon in a quantitative way, it is of utmost importance to be able to describe turbidity meaningfully. This is a problem of vast magnitude in that the conditions that designate a medium as turbid are infinite in variety and scope. Particles in the water can range in size from 10^{-10} meter (the water molecules themselves) to 10^{-2} meter (living organisms). Absorption characteristics can be almost identical with pure water as is the case with small biological organisms. Complex decomposition products of living tissues exhibit radically different spectral absorption as compared to other contaminants like clay and silt particles which absorb almost completely in the visible spectrum.

It is precisely this infinite variety of these components and their concentrations that leads one to abandon the quantitative study of turbidity from the approach of its causes and to shift to the study of its effects on image degradation due to scattering.

BACKGROUND THEORY:

Many of the optical properties of a turbid water medium as well as many of the principles which govern its propagation of light can be studied by injecting a highly collimated beam of monochromatic light into otherwise unlighted water and measuring all aspects of the resulting distribution of flux. Any optical input to the water may be represented by an appropriate superposition of highly collimated monochromatic beams.

Scattering is caused by fine particles, by molecules of water and by various solutes, but these contributions are usually quite minor and often difficult to detect. Even in very clear, blue ocean water scattering by water molecules produces only 7% of the total scattering coefficient and is dominant only at scattering angles near 90°, where it provides more than 2/3 of the scattered intensity[1]. Although the magnitude of this small component varies inversely as the fourth power of wavelength, it is so heavily masked by nonselective scattering due to large particles that total scattering in a turbid water medium is virtually independent of wavelength. Most of the scattering in natural waters is caused by transparent organisms and particles large compared with the wavelength of light. The scattering is believed to result chiefly from refraction and reflection at the surfaces of these scatterers. As a consequence, scattering at small forward angles predominates and polarized light tends to preserve its polarization. To the extent that all scattering curves have nearly identical shapes, the scattering by natural waters can be specified in terms of some single number, such as the total volume scattering coefficient, s , or the volume scattering function at some selected angle.

Figure 1[2]

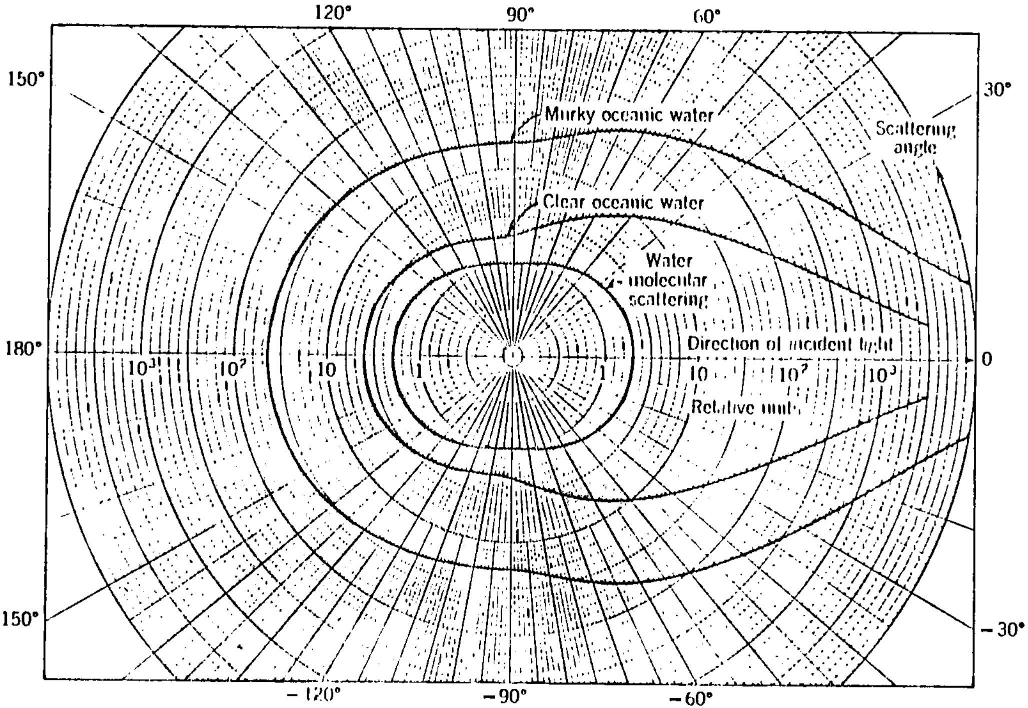


Figure 2.26 Polar plot of the volume scattering functions $\sigma(\theta)$ of pure water (water-molecule scattering only) and oceanic water.

No single number can adequately specify the clarity of any natural water because two independent mechanisms, absorption and scattering, govern water clarity. Even for monochromatic light, at least two coefficients, such as α and K are required, and a more complete specification requires data on the volume scattering function $\sigma(\theta)$.

The volume scattering function $\sigma(\theta)$ is defined by the equation:

$$dJ(\theta) = \sigma(\theta)\, H dv$$

where H is the irradiance produced by the collimated lamp on the volume dv, and $dJ(\theta)$ is the radiant intensity. The dimension of $\sigma(\theta)$ is reciprocal length; typical units are reciprocal steradian-meters. Very few measurements have been made of the volume scattering function in the nearly forward direction (i.e., 0-1° region). Consequently there is no assurance that the similarity in the shape of $\sigma(\theta)$ for various natural waters will also hold for very-small-angle scattering.

Multiple scattering of light by a turbid medium will also cause degraded optical resolution. If most of the light energy is scattered through small angles, as is the case in most natural situations, it would be possible to use the formulas derived for this case by Wells[3] which convert the volume scattering function to an image modulation transfer function. This conversion allows calculation of image degradation given the scattering properties of the medium.

It would be desirable to compare MTF's generated by use of Well's derived transformations from the volume scattering function of a turbid medium to MTF's that are obtained photographically in the same medium. At present no such known correlation has been published since no set of data has been compiled which gives both the scattering parameters and the measured MTF's for a specific hydrosol.

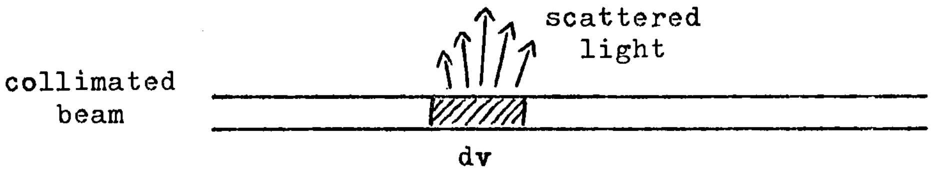
MTF's have been photographically obtained by two research studies at R.I.T. for a variable, well defined and repeatable turbid medium[4]. It is the intention of this experiment to construct a small-angle scattering meter and measure the volume scattering functions for the turbid conditions described by Greenburg and Fulmer. These experimentally derived VSF's* will be used to test a correlation of Well's mathematical MTF derivations with the experimentally derived MTF curves of Greenburg, Fulmer and Worzel.

PROCEDURE:

Meter Function:

The purpose of the volume scattering meter is to measure the amount of radiation scattered in a given direction by a small segment (dv) of a collimated light beam.

Figure 2:



The energy scattered is symmetric axially around the beam so that complete information can be had by measuring from 0 to 180 in one plane which incorporates the axis of the beam. In this experiment it is necessary to measure from 0 to 30 since only small-angle forward scattering is of interest. The method of measuring is illustrated in Figure 3.

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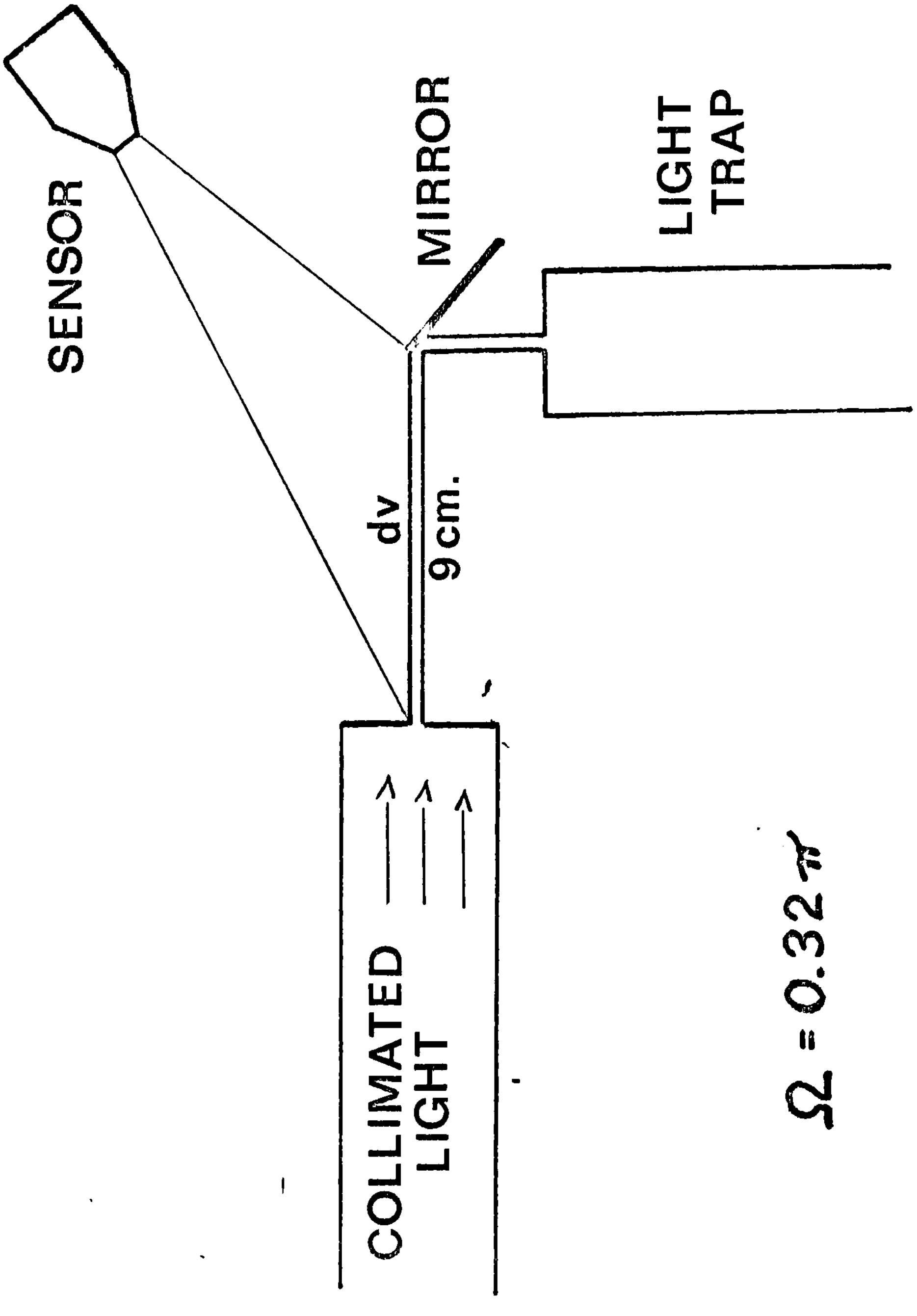
* Volume Scattering Function

Figure 3;


SENSOR
MIRROR
LIGHT TRAP
dv
9 cm.
COLLIMATED LIGHT
$\Omega = 0.32\pi$

Meter Specifications:

The Volume Scattering Meter was designed and constructed from 0.5 inch thick plywood. (see Figure 4) The tank shape chosen allows the sensor arm to swing freely over its range and also minimizes the volume of water necessary to fill the tank to the appropriate level. The sensor arm length measured from pivot point to sensor tube was two meters. Four inch metal bracket bracings were used generously to reinforce all areas of stress.

A black, opaque polyethylene sheet (4 mil) was used as a means of retaining water in the tank. A system of matte-black vinyl tiles and black sheets of expanded aluminum honey-combs covered all inside surfaces to act as light baffles. In addition, a floating top of these light-baffling materials was also used. Such a system proved to be quite efficient in reducing and virtually eliminating total internal reflections and flare.

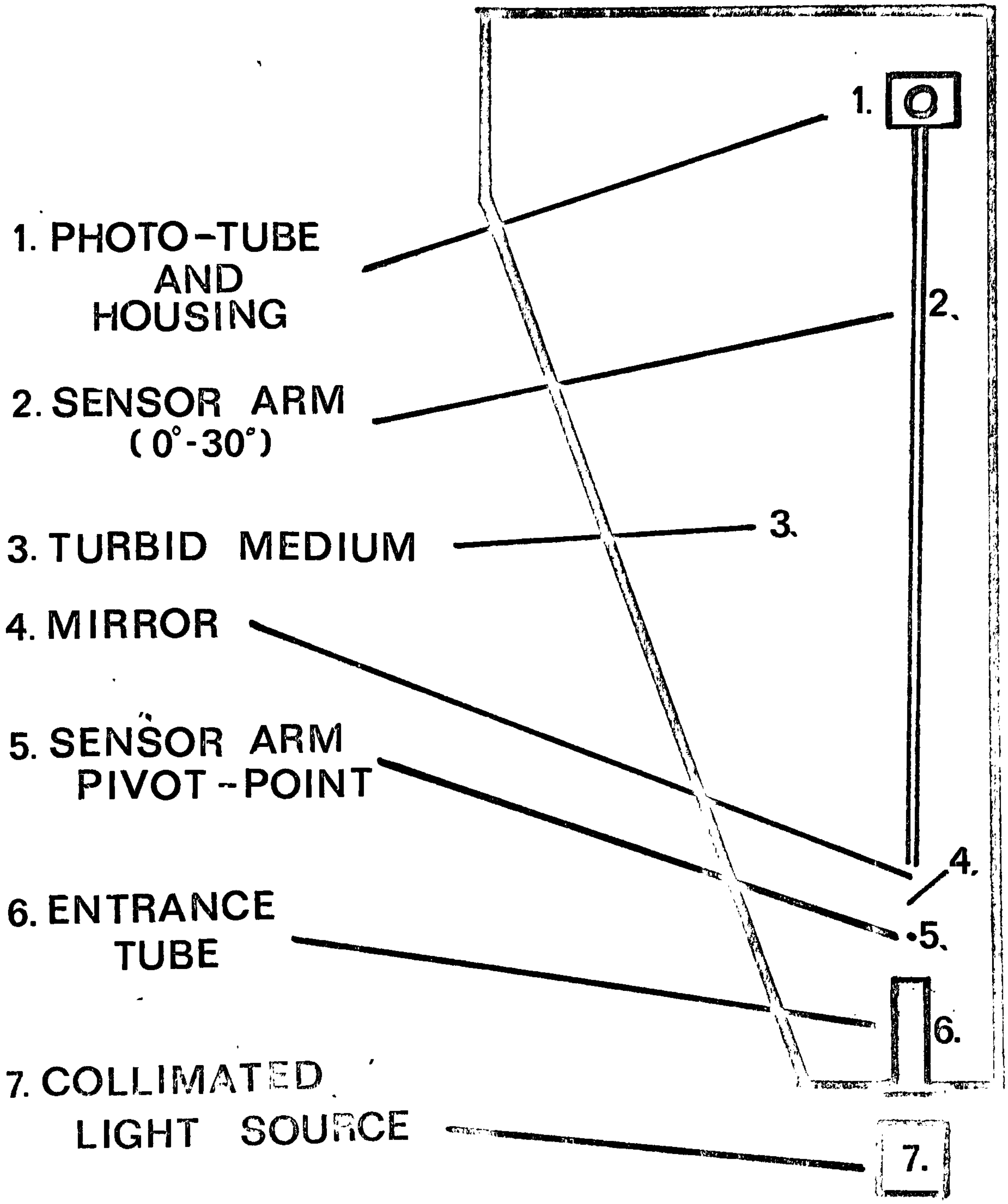
Providing a specified volume of water to the meter was accomplished by using a graduated stainless steel 'lightning-mixer' (capacity: 25 gallons). Cold tap water was pumped into the graduated mixer; this specific water volume was next filtered using diatomaceous earth and then pumped into the meter. The final working volume of the meter was two-hundred gallons.

Light Source:

A Spectra-Physics Model 155, 0.5mW Helium-Neon laser (633nm) was used as the collimated light source. The light beam diameter was determined by using an entrance aperture of 0.09375 inches

Figure 4:

VOLUME SCATTERING METER:



Sensor:

A Gamma Scientific Model 700 Log/Linear Photmultiplier photometer was used with a GE-931A photomultiplier tube mounted in a Gamma-Scientific Universal Photomultiplier Head and then secured in the submersible sensor housing.

Chemistry:

A series of aliquots of a concentrated calcium oxalate precipitate in suspension were added to the filtered tap water to produce a series of five controlled levels of turbidity. (see Appendix I) Measurements were recorded of light scattering magnitude as a function of angular displacement for each of the five levels of turbidity.

CALCULATIONS:

Volume Scattering Function $\sigma(\theta)$:

The scattering function $\sigma(\theta)$ is defined as

$$\sigma(\theta) = \frac{dJ(\theta)}{Hdv} \qquad (1)$$

where $dJ(\theta)$ is an element of radiant intensity scattered light in the direction θ, dv is an element of volume and H is the irradiance received by the sample volume. It is imperative to understand the basic importance of $\sigma(\theta)$ when attempting to transform $\sigma(\theta)$ to a modulation transform function by the use of equations derived by Wells[3].

Although the volume scattering function definition operates mathematically with infintesimal elements of radiant intensity and volume, any workable instrument will have a finite sample volume and a sensor which collects energy scattered at an angle θ over some solid angle. And thus Eq(1) may be rewritten as:[5]

$$\sigma(\theta) = \frac{\Delta J(0)}{H\,\Delta V}$$

$$= \frac{\dfrac{P(\theta)}{\Omega}}{\dfrac{P(0)}{A} \cdot A \cdot L}$$

$$\sigma(\theta) = \frac{P(\theta)}{P(0)} \cdot \frac{1}{\Omega \cdot L} \qquad (2)$$

where

$P(\theta)$ = the power scattered into the solid angle (0) in the direction θ

$P(0)$ = the total power entering the sample volume

ΔV = the sample volume

Ω = the solid angle over which P(0) is collected and measured

A = the projected area of the sample volume, V, as seen in the direction of P(0)

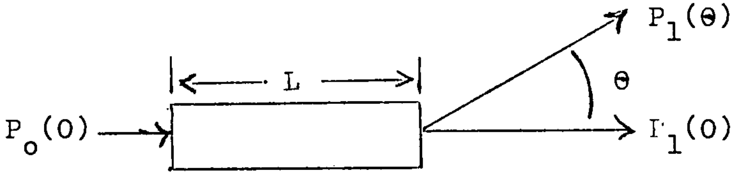
L = the distance through the sample volume in the direction of P(0). (A photon has a chance of being scattered into the solid angle, , while traveling this distance.)

Construction of our volume scattering meter was performed in such a way as to allow measurements to be taken such that the ratio P(θ)/P(0) could be computed. It is not necessary to know either P(θ) or P(0) in absolute terms. The length, L, and the solid angle, Ω , are determined by the geometry of the instrument. The scattered component P(θ) is measured leaving the sample volume; the irradiating power P(0) can be determined either entering or leaving the sample volume.

The question is which value should be used for P(0) in order to calculate the value of the ratio P(θ)/P(0), used to determine $\sigma(\theta)$. Neither limiting value for P(0) is correct. The desired ratio falls somewhere between these limits, and the error involved depends on the properties of the water and the sample path length used. When, by necessity, a sample path length of finite length is used, the signals measured are affected by properties of the water other than those we are attempting to measure, and we are not conforming to the definition of the property we are measuring. This error is small if the sample path length is short or the water very clear, but it can be significant when the instrument has a long path length and the water is turbid.

To analyse this problem we will make use of the terms α, a, and s, where α is the beam volume attenuation coefficient, a is the volume absorption coefficient, s is the total volume scattering coefficient, and $\alpha = a + s$. Calculation of the ratio $P(\theta)/P(0)$ will require a correction factor involving absorption and path length. A sample volume of length L, is illustrated below.

Figure 5:



where

$P_o(0)$ = the light flux entering the volume

$P_1(0)$ = the flux leaving the volume and traveling undeviated from the direction of the entering flux $P_o(0)$ after traversing the distance L. It is the flux remaining which has been neither scattered nor absorbed

$P_1(\theta)$ = the flux leaving the sample volume which has been scattered into a small solid angle Ω in a direction with respect to the entering flux $P_o(0)$.

The relative values for $P_o(0)$, $P_1(\theta)$ and $P_1(0)$ are measurable quantities. The component $P_1(\theta)$ has been attenuated while traveling the distance, L, and therefore the ratio $P_1(\theta)/P_o(0)$ is too small. The component $P_1(0)$ has also been attenuated while traveling the distance, L, but part of this loss is due to scattering. A proportionate part of this scattered light is contributed to $P_1(\theta)$ and it seems that the ratio $P_1(\theta)/P_1(0)$ will be too large. The desired ratio $P(\theta)/P(0)$ will fall somewhere between these limits:

$$\frac{P_1(\theta)}{P_o(0)} < \frac{P(\theta)}{P(0)} < \frac{P_1(\theta)}{P_1(0)}$$

The desired ratio, $P(\theta)/P(0)$, is that which makes $P(\theta)/P(0) \cdot 1/L$ equal to the limit that $P_1(\theta)/P_o(0) \cdot 1/L$ would approach as the sample path length is decreased to zero. The $P_o(0)$ is constant and we can write,

$$\frac{P(\theta)}{L} = \lim_{L \to 0} \frac{P_1(\theta)}{L}$$

where $P_1(\theta)$ is the power leaving the sample volume of length, L, and entering the solid angle, Ω, in the direction θ. The limit $P(\theta)/L$ is the power which would be scattered in the direction θ within the solid angle Ω as a result of $P_o(0)$ traversing a unit length of water if no absorption occurred. To account for this absorption, we can use the volume absorption coefficient, a, and write,

$$P(\theta)e^{-aL} = P_1(\theta) \qquad (3)$$

The light leaving the volume, $P_1(0)$, is that remaining portion of $P_o(0)$ which has been neither scattered nor absorbed. The total volume attenuation coefficient, α, relates these two:

$$P(0)e^{-\alpha L} = P_1(0) \qquad (4)$$

Dividing Eq(3) by Eq(4) gives

$$\frac{P(\theta)e^{-aL}}{P(0)e^{-\alpha L}} = \frac{P_1(\theta)}{P_1(0)}$$

$$\frac{P(\theta)}{P(0)} = \frac{e^{-\alpha L}P_1(\theta)}{e^{-aL}\ P_1(0)}$$

$$= e^{-(\alpha - a)L} \cdot \frac{P_1(\theta)}{P_1(0)}$$

$$\frac{P(\theta)}{P(0)} = e^{-sL} \cdot \frac{P_1(\theta)}{P_1(0)} \qquad (5)$$

When a scattering measurement is made using a finite volume of water, an unavoidable error is caused by absorption in the sample volume. If the instrument used has a sample path length that is small relative to the attenuation length, ($1/\alpha$), of the water, this error is small and is probably less than the instrumental errors and time-varying fluctuations. If the measurement is made using a path length that is not small relative to the attenuation length, the results can be corrected by applying the factors indicated in Eq(5).

Transformation of VSF to MTF:

By using the derived equations by Wells[3] in the small-angle approximation ($\sin\theta \approx \tan\theta \approx \theta$) the theory of the MTF permits the result of all orders of multiple scattering to be expressed in closed form. The specific transformation from s(0) the VSF, to F(v,R), the MTF is expressed as:

$$F(\nu,R) = \exp\Big[\big(Q(\nu R)-s_t\big)R\Big] \qquad (6)$$

where

$$Q(\psi) = 2\pi\int_{\theta}^{\Theta} u(\theta)J_o(2\pi\psi\theta)d\theta, \qquad (7)$$

$$u(\theta) = \int_{\theta}^{\Theta} s(t)dt \qquad (8)$$

$$s_t = \int_{0}^{\Omega} s(\theta)dw = 2\pi\int_{0}^{\Theta} s(\theta)\sin\theta d\theta \qquad (9)$$

Equation (9) expresses the total small-angle scattering constant s_t as the integral of the VSF over the solid angle dw up to a limit Ω defined by a cone of half-angle Θ. The constant s_t is the fraction of power scattered at all angles ($<\Theta$) per unit length along the beam. The symbol ψ is the angular spatial frequency in cycles per radian which is equal to the product of spatial frequency times range (νR). The function $F(\nu, R)$ is the modulation transfer function.

.RESULTS:

Due to design shortcomings, the volume scattering meter produced errors in measuring the scattered intensity between 0° and 1°. For this reason, an extrapolation from 1° to 0° was necessary and was accomplished by using the equation of Petzold[5].:

$$\log \sigma(\theta) = m\log \theta + \log C$$

where m and C are constants determined by neighboring data points as θ approaches zero.

Numerical integration of the volume scattering function, $s(\theta)$ produced $u(\theta)$. Both $s(\theta)$ and $u(\theta)$ are apparently singular at $\theta=0°$. That is, they are singular to the best of our ability to extrapolate. Although either $u(\theta)$ or both $s(\theta)$ and $u(\theta)$ may

Figure 6

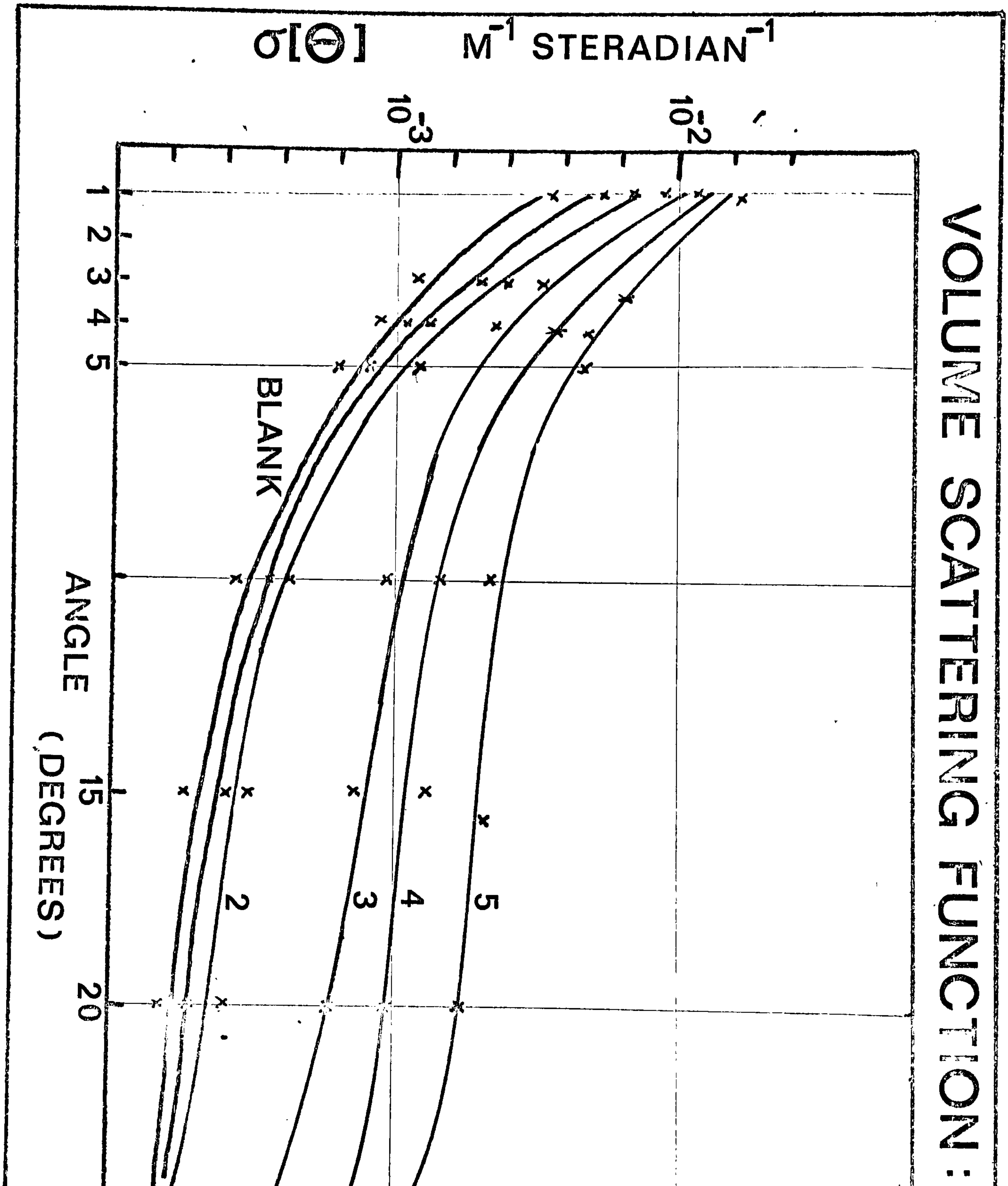

VOLUME SCATTERING FUNCTION :
σ[Θ] M^{-1} STERADIAN^{-1}
10^{-3}
10^{-2}
1
2
3
4
5
15
20
25
ANGLE
(DEGREES)
BLANK
2
3
4
5

Figure 7

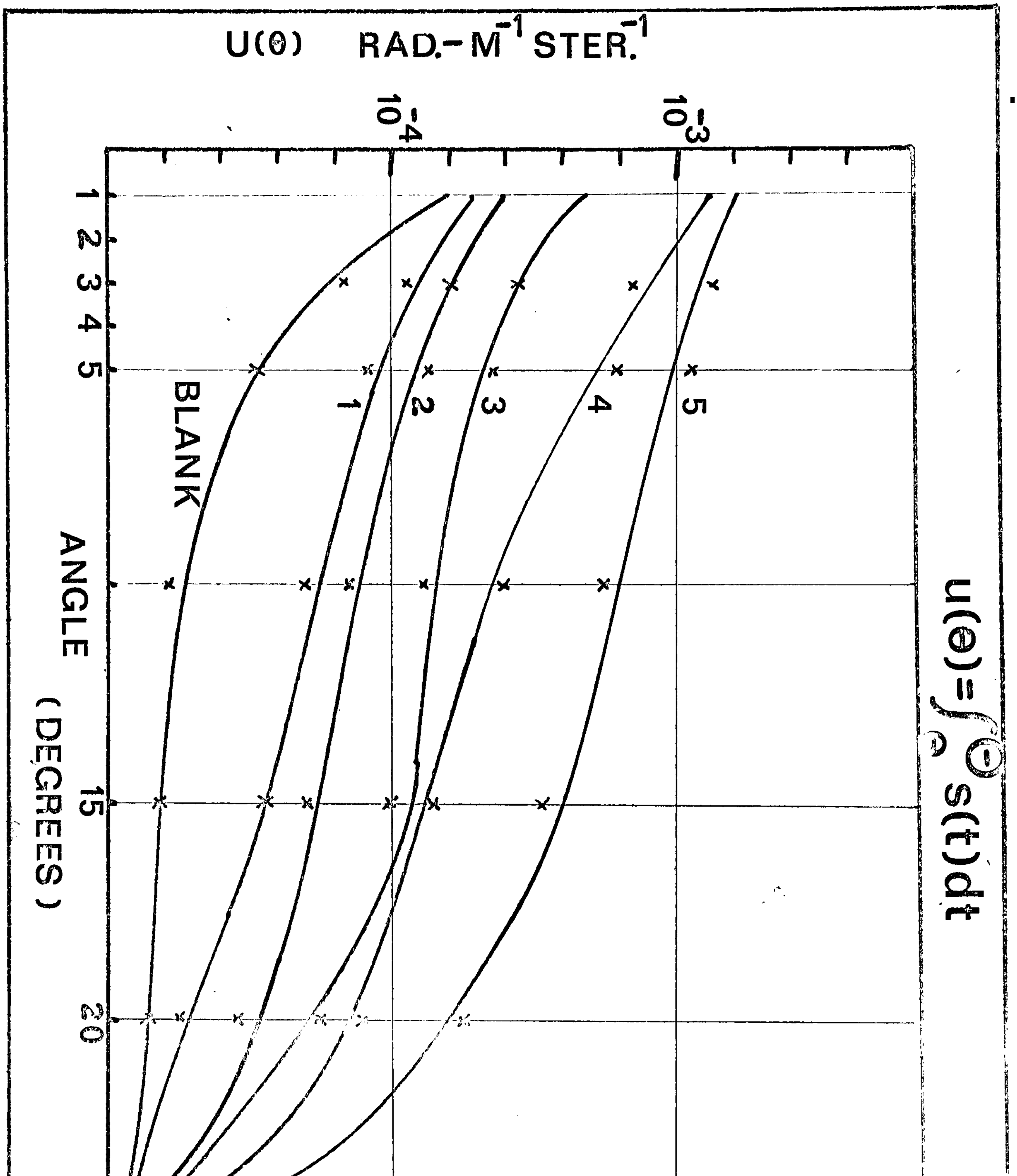
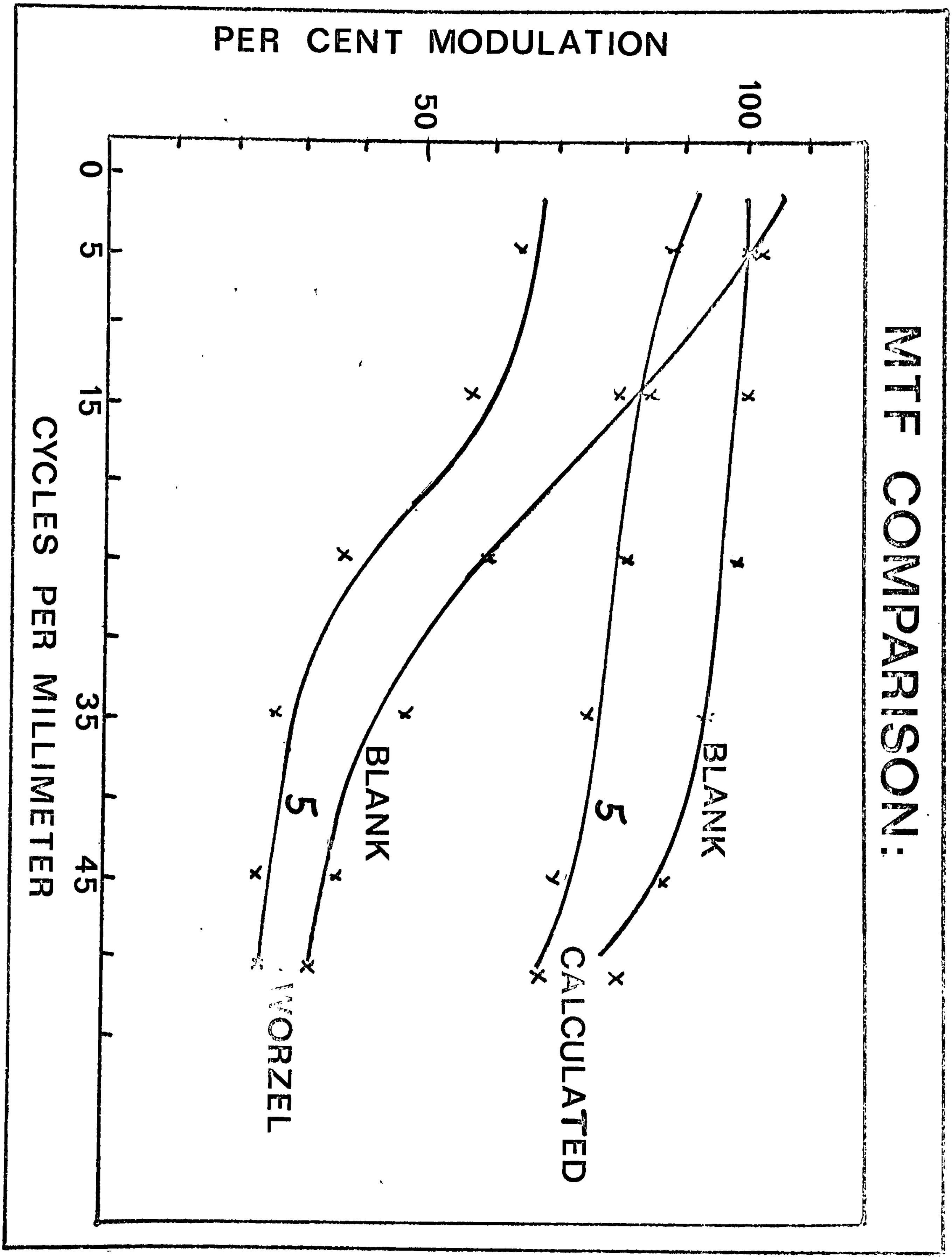

U(θ) RAD.-M^{-1} STER.$^{-1}$
10^{-4}
10^{-3}
1
2
3
4
5
15
20
25
ANGLE
(DEGREES)
BLANK
1
2
3
4
5
u(θ)=∫ θ s(t)dt

Figure 8


MTF COMPARISON:
PER CENT MODULATION
100
50
0
5
15
35
45
CYCLES PER MILLIMETER
BLANK
5
CALCULATED
BLANK
5
WORZEL

turn at extremely small angles to approach a finite value. Such singularities create no special problem so long as they are integrable to give finite total cross sections. At small angles both s(θ) and u(θ) can be approximated as

$$s(\theta) \sim C_1\theta^m \qquad u(\theta) \sim C_2\theta^n$$

The singularity in u(θ) can be separated by defining a new function such that;

$$V(\theta) = \theta^{-n}u(\theta) = \theta^{-n}C_2\theta^n = C_2$$

Using this function, the integrable singularity produced in Eq(7) can be handled by the use of the general formula,

$$\int_0^x f(\theta)\,\frac{d\theta}{\theta^m} = \int_0^{x(1-m)} \left[f(t^{1/1-m}) \,/\, 1-m \right] dt$$

The volume scattering functions are graphed in Figure 6 and Eq(8) is illustrated in Figure 7. The final MTF result, Eq(6), is plotted for two levels of turbidity and a comparison is made with the data from Worzel[4] (see Figure 6 in his report)

DISCUSSION OF RESULTS:

The dissimilarity of the MTF comparison indicates that the approximation used for extrapolation of s(θ) from 1° to 0° was inaccurate. Since s(θ) and u(θ) are singular when $\theta = 0^\circ$ then as the function approaches zero, the function precisely describing this area must be determined because any errors in this extrapolation will produce large sources of error when integrating. It is therefore imperative that some means should be developed to secure scattering data for extremely small angles. (See Recommendations for Further Work)

CONCLUSIONS:

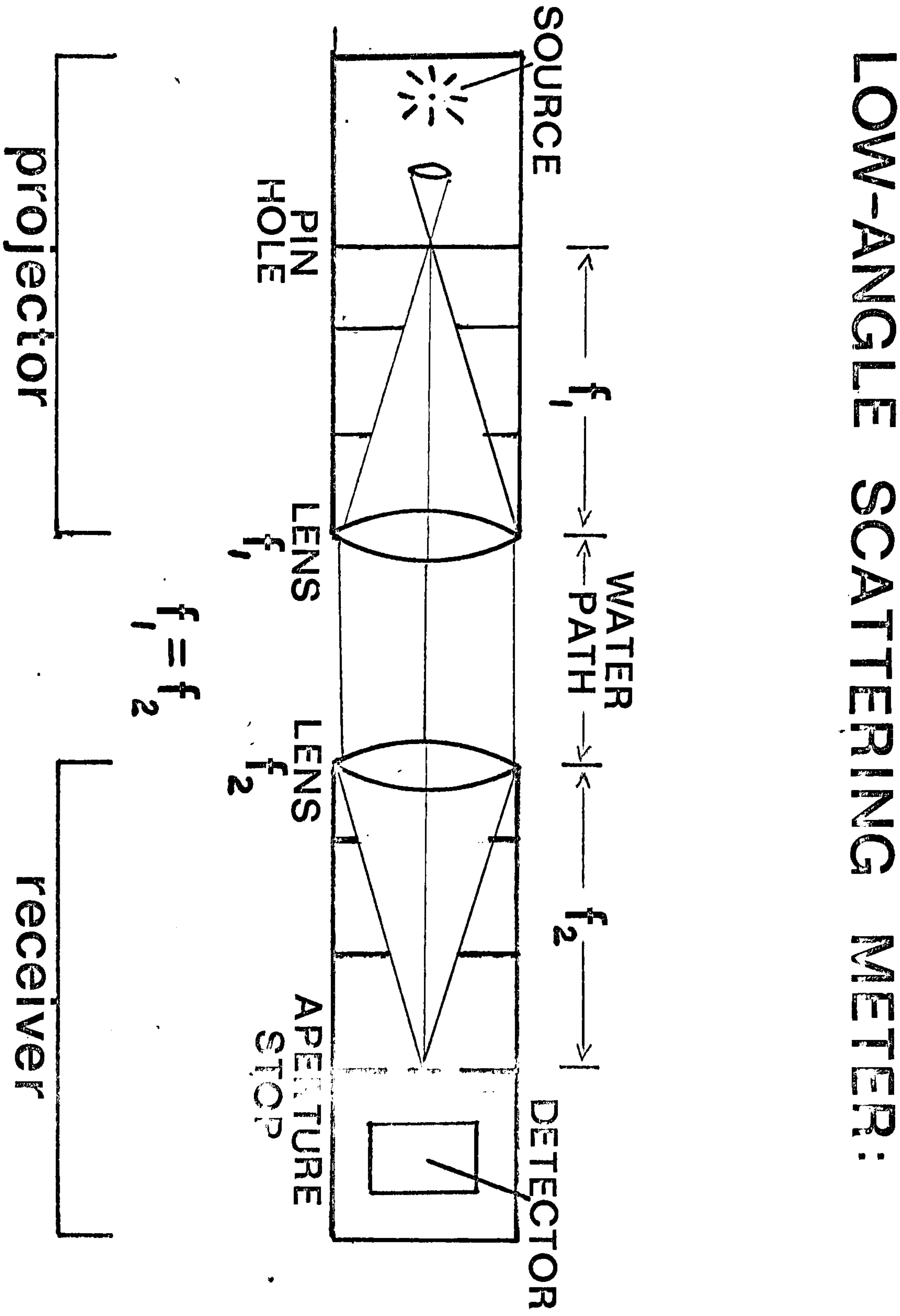
A set of equations that mathematically predict MTF from the volume scattering function of turbid water has been tested using five turbidity level variations of a calcium oxalate suspension. While these equations seem feasible, more critically accurate data for angles between 0° and 1° are needed as well as computer analysis before any statement can be drawn concerning the validity of these transform equations.

RECOMMENDATIONS FOR FURTHER WORK:

Because of the singularity involved with the volume scattering function $\sigma(\theta)$, it seems apparent that a low-angle volume scattering meter must be constructed that exclusively measures scattered radiation over the angular interval of 0° to 1°. By more precisely defining data throughout this range, extrapolations can become more meaningfully valid and thus the cascading series of integrations involved in transforming the volume scattering function $\sigma(\theta)$ to the MTF will increase in accuracy.

A recommended design for a low-angle scattering meter would be based on the principle of the coronograph whereby an opaque stop is used to block the central unscattered beam and annulus shaped field stops of various sizes are used to select scattered radiation at incremental angular units (milliradians). A low angle scattering meter optical schematic is illustrated in Figure 9.

It seems evident that both a low angle scattering meter as well as a general angle scattering meter are needed to adequately describe the scattering properties of a turbid medium.


LOW-ANGLE SCATTERING METER:
SOURCE
PIN HOLE
f_1
LENS f_1
WATER PATH
LENS f_2
f_2
APERTURE STOP
DETECTOR
$f_1 = f_2$
projector
receiver

Further work may also include a redesigning of the method by which the dv is formed by the mirror. A more repeatable and accurate method is needed to limit the errors at small angles as a result of a poorly aligned mirror.

The in situ capability of the general angle scattering meter should be preserved but by designing a more accurate method of swinging the sensor arm, the liquid volume of the tank could be decreased and thus ease of handling would be enhanced.

But more importantly, after mechanical considerations are solved, more in depth investigations of various particle sizes and mixtures should be investigated. Combinations of latex, polystyrene and teflon particles along with various dyes are being investigated by the chemistry department at R.I.T. Such mixtures can theoretically be formed to approximate certain basic natural water conditions and as a result many standardized laboratory models may be developed.

Current trends that show promise are the use of silver halide suspensions to investigate scattering models. The use of silver halide is significant in that because of the vast amount of knowledge in emulsion chemistry, particle shape, size and size distribution can be accurately determined. This can be of utmost importance when comparing changes in a scattering model using multiple-scatter analysis or radiative transfer.

ACKNOWLEDGEMENTS:

We wish to thank Dr. T.P. Wallace, Dean of the College of Science at R.I.T. (We greatly appreciated his enthusiasm.) Dr. Peter L. Sachs of Woods Hole Oceanographic Institute was also very helpful in providing us with valuable references and needed insights. Countless others have also cooperated by

allowing us to borrow everything from fish-tank filters to scotch tape and calculators. Without considerate and helpful people such as these, projects such as ours would become exceedingly difficult to complete.

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Petzold, Theodore J.
Scripps Institution of Oceanography
Contract No. N62269-71-C-0676
October 1972

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Light Scattering by Small Particles
Van de Hulst, H.C.
Wiley & Sons,Inc. (1957)

APPENDIX I:

Calcium Oxalate was chosen as the crystal material to use for the turbidity production because:

1) Ease of preparation
2) Low solubility once the precipitate is formed (0.0054 grams/liter at 24°C)
3) Particle size (20-40 microns) is large when compared to the wavelength of light

Preparation:*

1) Dissolve 1.2126 grams of calcium chloride (reagent grade and desiccated) in six-hundred milliliters of distilled water.
2) Dissolve 1.3776 grams of oxalic acid (reagent grade) in 250 milliliters of distilled water
3) Add oxalic acid solution to calcium chloride solution (magnetic stirrer going as the solution is added from a separatory funnel with a drain time of five minutes.)
4) Allow solutions to stir at moderate rate for 20 min.
5) Add 2.7 milliliters of 28% ammonium hydroxide to adjust pH level to approximately seven.
6) Dilute the entire solution to one liter with distilled water.
7) This procedure produces a suspension of approximately 1400 parts per million (by weight)
8) Each 50 ml. diluted in 70 liters of water produces a suspension of about one part per million. (by weight)

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* Worzel, Howard (Unpublished B.S. Thesis, 1967) 'Image Quality Degradation From The Transmission of Light Through Turbid Water'

APPENDIX I: (continued)

The specific chemical reactions for production of this precipitate are:

$$CaCl_2 + C_2H_2O_4 \longrightarrow CaC_2O_4 + 2HCl$$

$$2HCl + 2NH_4OH \longrightarrow 2NH_4Cl + 2H_2O$$

The calcium oxalate is produced as a finely divided suspension of particles. The ammonium chloride produced is completely soluble. (It possesses a solubility of 29.7 grams per 100cc. of water at 0°C)

Worzel used 20 milliliter aliquots diluted in seventy liters of water. To maintain this same concentration in a larger volume of water, only the mathematics of a simple proportion need be used.

Specifically,

X = volume of calcium oxalate diluted in 70 liters

Y = proportionate volume of calcium oxalate diluted in two hundred gallons

Units: liters

X	Y	Code
0.02	0.216	1
0.04	0.432	2
0.06	0.648	3
0.08	0.864	4
0.09	0.972	5
0.00	0.000	Blank

APPENDIX II:

Tabular Data for the Volume Scattering Function $\sigma(\theta)$:

Angle	Blank	1	2	3	4	5
0.5	2.55-3	4.19-3	6.18-3	6.46-3	7.90-3	1.05-2
1.0	4.25-3	6.54-3	8.34-3	1.35-2	1.98-3	2.38-2
1.5	2.55-3	3.01-3	4.18-3	7.56-3	1.38-2	1.98-2
2.0	1.92-3	2.35-3	3.01-3	5.39-3	1.11-2	1.25-2
2.5	1.81-3	2.09-3	2.66-3	5.14-3	9.08-3	1.14-2
3.0	1.34-3	1.57-3	2.01-3	3.77-3	7.09-3	9.05-3
4	1.12-3	1.16-3	1.58-3	3.23-3	6.31-3	7.36-3
5	8.73-4	9.90-4	1.15-3	2.17-3	4.54-3	5.67-3
10	4.25-4	4.95-4	6.78-4	1.35-3	2.38-3	3.39-3
15	3.35-4	3.67-4	4.07-4	8.00-4	1.38-3	2.28-3
20	2.45-4	2.02-4	2.49-4	6.85-4	1.17-3	1.69-3
25	1.56-4	1.57-4	2.03-4	5.48-4	7.83-4	1.14-3

2.55-3 = 2.55 x 10^{-3}