9-21-1972

Stability of Large Flexible Satellite

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STABILITY OF LARGE FLEXIBLE SATELLITE

by

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A Thesis Submitted

in

Partial Fulfillment

of the

Requirements for the Degree of

MASTER OF SCIENCE

in

Mechanical Engineering

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ROCHESTER, NEW YORK

July, 1972

A B S T R A C T

The stability of a gravity gradient satellite with a rigid main body and elastic antennas is studied using Liapunov's Direct method. The complete conditions for equilibrium and stability for a particular class of two dimensional models are determined. Previous work restrained the analysis to elastic stability in the small, and only for equilibrium positions corresponding to zero initial elastic deformation. Although the work presented here is for two dimensional motion, the intent is to bring forth an approach to the determination of the complete equilibrium and stability criteria in the large as well as the small. The effect of different parameters on the equilibrium and stability is also determined. It is shown that: (1) a satellite will always be in equilibrium at 0° and 90° attitude angle positions, (2) a satellite if stable at 0°, will always be unstable at 90° orientation and vice versa, and (3) there can exist only one more equilibrium position between 0° and 90°, and if so, 0° and 90° will be unstable equilibrium positions and the stability of the third position must be ascertained. A truncated power series is used to approximate the shape of the elastic antennas. The results are compared to those obtained by a more conventional method using the eigenfunctions of the freely vibrating antennas as comparison function. It is found that using the power series yields a more conservative stability criteria.

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NOMENCLATURES

A,B,C	Principle moments of inertia of the main body about the x,y and z axes respectively.
A',B',C'	Principle moments of inertia of the "complete" undeformed satellite about the x,y and z axes respectively.
EI	Flexural rigidity of the elastic antennas.
H	Hamiltonian
K	Earth gravity constant
k	Curvature of the beam
L	Lagrangian
$\mathcal{L}_E$	Lagrangian density
M	Mass of the rigid satellite
m	Mass of the elastic antennas
Q_j	Generalized forces
R,ψ,Z	Orbiting particle coordinate system with respect to inertial coordinate system
T	Kinetic Energy
U	Dynamic Potential
u	Elastic displacement of antenna in x direction
v	Elastic displacement of antenna in y direction
V	Potential
V_L	Liapunov function
V_m	Absolute velocity of particle dm
V_M	Absolute velocity of particle dM
V_{mxy}	Velocity of partical dm relative to x,y,z coordinate system

V_G	Gravity potential
V_E	Elastic Potential
$V_G)_M$	Gravity potential of rigid satellite
$V_G)_E$	Elastic potential of elastic antenna
θ	Attitude angle
ξ	Generalized coordinates

CHAPTER I

EQUILIBRIUM EQUATIONS

1.1 Introduction :

Budynas and Poli (References 1 & 2) analyzed the equilibrium of three dimensional models and established the stability for 0°, 90°, 180° and 270° orientations.

In this study, a particular class of two dimensional models are examined. Particular attention is given to the behaviour of a satellite in positions other than those mentioned above. The equilibrium positions and stability requirements for a given satellite are sought and the effect of change in parameters on the stability studied.

For this analysis it is assumed that : the centre of the mass of the satellite follows a Keplerian orbit and is unaffected by the attitude motion of the spacecraft; and that damping in the elastic antennas is negligible.

Let the centre of the earth (considered fixed) be the origin of the reference cylindrical co-ordinate system $R\psi Z$ (see figure 1.1). The satellite's reference cartesian co-ordinate system is such than when the antennas are undeformed, xyz is the principal axis system with the centre of mass of the composite satellite as origin. Let θ be the attitude angle of the satellite, relating the xyz and $R\psi Z$ systems.

The first equilibrium equation is obtained from the Lagrangian analysis of the equations of motion and the second one is derived from the virtual work method using the Hamiltonian pronciple.

1.2 Lagrangian Analysis:

For an elastic system, the Hamiltonian is an integral which is a function of system's generalized elastic positions, ς_j, and velocities, $\dot{\varsigma}_j$, and spatial derivatives.

$$H_E = H_E\left(\varsigma_j, \varsigma_j, \frac{\partial \varsigma_j}{\partial x_k}, \frac{\partial^2 \varsigma_j}{\partial x_k \partial x_l}, \ldots\right)$$

$$= \int_{x_o}^{x_1} \left(\sum_{j=0}^{n} \dot{\varsigma}_j \frac{\partial \mathcal{L}_1}{\partial \dot{\varsigma}_j} - \mathcal{L} \right) dx \tag{1}$$

Where $\mathcal{L}$, called the Lagrangian density, is the Lagrangian per unit lenth of infinitesimal element located at x. The Lagrangian of a rigid and elastic coupled body can be expressed as,

$$L = L_1(q_1, \dot{q}_1) + \int_{x_o}^{x_1} \mathcal{L}_1\left(q_1, \dot{q}_1, \varsigma_j, \dot{\varsigma}_j, x, \frac{\partial \varsigma}{\partial x}, \frac{\partial^2 \varsigma}{\partial x^2}, \cdots\right) \tag{2}$$

where L_1 is the part of Lagrangian expressible as a funcion of rigid body terms alone; and $\mathcal{L}_1$ is the part that can be expressed as a funciton of coupled rigid-elastic terms. The Hamiltonian in this case can be written as,

$$H = \sum_{i=1}^{n} \dot{q}_i \frac{\partial L_1}{\partial \dot{q}_i} + \int_{X_o}^{X_1} \left(\sum_{i=1}^{n} \dot{q}_i \frac{\partial \mathcal{L}_1}{\partial \dot{q}_i} + \sum_{j=1}^{m} \dot{\varsigma}_j \frac{\partial \mathcal{L}_1}{\partial \dot{\varsigma}_j} \right) dx - L \tag{3}$$

Defining generalized coordinates ϕ_i as

$$\phi_i = q_i \qquad i = 1,2,3,\ldots\ldots\ldots.n$$

$$\phi_{i+n} = q_i \qquad i = 1,2,3,4, \ ,\ldots\ldots$$

thus above equation can be rewritten as:

$$H = \sum_{j=1}^{n} \dot{\phi}_j \frac{\partial L_1}{\partial \dot{\phi}_j} + \int_{x_0}^{x_1} \left(\sum_{i=1}^{n+m} \dot{\phi}_i \frac{\partial \mathcal{L}_1}{\partial \dot{\phi}_i} \right) dx - L \tag{4}$$

the kinetic energy takes the form;*

$$T = \tfrac{1}{2}\left\{ \sum_{i,j=1}^{n} \alpha_{ji} \dot{\phi}_j \dot{\phi}_i + \sum_{j=1}^{n} 2\beta_j \dot{\phi}_j + \gamma \right\}$$
$$+ \tfrac{1}{2}\left\{ \sum_{i,j=1}^{n+m} \bar{\alpha}_{ji} \dot{\phi}_j \dot{\phi}_i + \sum_{j=1}^{n+m} 2\bar{\beta}_j \dot{\phi}_j + \bar{\gamma} \right\} \tag{5}$$

where the first term is total kinetic energy of the rigid elements and second term is the total kinetic energy of the elastic elements. The coefficients $\alpha_{ji}, \beta_j, \gamma$ are functions of rigid body coordinates ϕ_i (i=1,2,.....n) only, and $\bar{\alpha}_{ji}, \bar{\beta}_j, \bar{\gamma}$ are funcions of spatial coordinates x and generalized coordinates ϕ_i (i=1,2,..........n+m).

Denoting the potential energy of the system by V, and substituting equation (5) into the Lagrangian, L, where

$$L = T - V$$

equation (4) can be written in the form

$$H = T_2 + V - T_0$$

* - For the detailed derivation see reference 2.

Where $T_2 = \frac{1}{2}\sum_{j=1}^{n} \alpha_{j1} \dot{\phi}_j \dot{\phi}_i + \frac{1}{2}\int_{x_o}^{x_1} \sum_{j=1}^{n+m} \bar{\alpha}_{j1} \dot{\phi}_j \dot{\phi}_i \, dx$

and $T_o = \frac{1}{2} \gamma + \frac{1}{2} \int_{x_o}^{x_1} \bar{\gamma} \, dx$

$V - T_o$ is referred as dynamic potential 'U', hence

$$H = T_2 + U \tag{6}$$

The kinetic energy, (K.E.), of the satellite, is,

$$T = \frac{1}{2}\int_M (\underline{V}_M \cdot \underline{V}_M) \, dM + \frac{1}{2}\int_m (\underline{V}_m \cdot \underline{V}_m) \, dm + \frac{1}{2}\int_m (\underline{V}_m \cdot \underline{V}_m) \, dm \tag{7}$$

where the first term is the K.E. of the rigid satellite and the remaining terms are the K.E. of the two antennas.

$$\underline{V}_M = \dot{\underline{R}}_c + \underline{\Omega} \times \underline{r}_M$$

$$\text{and} \quad \underline{V}_m = \dot{\underline{R}}_c + \underline{\Omega} \times \underline{r}_m + \underline{v}_{mxy} \tag{8}$$

where $\Omega = \dot{\Psi} + \dot{\theta}$, the total angular velocity of the satellite.

However $\dot{R}_c$ and r_m can be rewritten as

$$\dot{\underline{R}}_c = (\dot{R}_c \cos\theta + R_c \dot{\Psi} \sin\theta) \, \underline{i} + (R_c \dot{\Psi} \cos\theta - \dot{R}_c \sin\theta) \, \underline{j} \tag{9}$$

$$\underline{r}_m = x \, \underline{i} + y \, \underline{j} + z \, \underline{k} \tag{10}$$

substituting equations (9) and (10) in to the equation (7)

$$\underline{V}_M = \left[\dot{R}_c \cos\theta + R_c \dot{\Psi} \sin\theta - y(\dot{\Psi} + \dot{\theta})\right] \underline{i} + \left[R_c \dot{\Psi} \cos\theta - \dot{R}_c \sin\theta + x(\dot{\Psi} + \dot{\theta})\right] \underline{j} \tag{11}$$

Thus the first term on the right hand side of equation (7) becomes,

$$\tfrac{1}{2}\int_M (\underline{V}_M \cdot \underline{V}_M)\,dM = \tfrac{1}{2}\int_M \left[\dot{R}_c + R^2\,\dot{\Psi}^2 + (x^2+y^2)(\dot{\Psi}+\dot{\theta})^2\right] dM \qquad (12)$$

since
$$\int_M x\,dM = \int_M y\,dM = 0$$
and
$$\int_M (x^2+y^2)\,dM = C$$

Equation (12) takes the form

$$\tfrac{1}{2}\int_M (\underline{V}_M \cdot \underline{V}_M)\,dM = \tfrac{1}{2}M\,(\dot{R}_c^2 + R_c^2\,\dot{\Psi}) + \tfrac{1}{2}C\,(\dot{\Psi}+\dot{\theta})^2$$

however
$$\underline{r}_m = (x+u)\,\underline{i} + y\,\underline{j}$$

and
$$\underline{v}_{mxy} = u\,\underline{i} + v\,\underline{j}$$

substituting the above in equation (11),

$$\underline{V}_m = \left[\dot{R}_c\cos\theta + R_c\,\dot{\Psi}\sin\theta + \dot{u} - v(\dot{\Psi}+\dot{\theta})\right]\underline{i} + \left[R_c\,\dot{\Psi}\cos\theta - \dot{R}_c\sin\theta + \dot{v} + (x+u)(\dot{\Psi}+\theta)\right]\underline{j}$$

Thus the total kinetic energy is,

$$\begin{aligned} T = {} & \tfrac{1}{2}\,(M+2m)(\dot{R}_c + R_c^2\,\dot{\Psi}) + \tfrac{1}{2}\,C'(\dot{\Psi}+\dot{\theta})^2 \\ & + \tfrac{1}{2}\int_L \Big[\dot{u}^2 + \dot{v}^2 + (2ux + u^2 + v^2)(\dot{\Psi}+\dot{\theta})^2 \\ & + 2\dot{R}_c\left\{(\dot{u} - v(\dot{\Psi}+\dot{\theta}))\cos\theta - (\dot{v} + u(\dot{\Psi}+\dot{\theta}))\right\}\sin\theta \\ & + 2R_c\dot{}\left\{(\dot{u} - v(\dot{\Psi}+\dot{\theta}))\sin\theta + (\dot{v} + u(\dot{\Psi}+\dot{\theta}))\right\}\cos\theta \\ & + 2(\dot{\Psi}+\dot{\theta})(\dot{v}(x+u) - \dot{u}v)\Big]\,dm \end{aligned} \qquad (13)$$

Neglecting all external forces except the gravitational force of earth, the total potential energy V is,

$$V = V_G + V_E;$$

where V_G and V_E are gravity and elastic potentials respectively.

$$V_G = (V_G)_M + (V_G)_m = -\int_M \frac{K}{R_M}\, dM - \int_{\mathbb{L}} \frac{K}{R_M}\, dM^{*} \qquad (14)$$

and

$$\underline{R}_M = \underline{R}_c + \underline{r}_M = (\underline{R}_M \cdot \underline{R}_M)^{\frac{1}{2}}$$

$$\underline{R}_M^{-1} = \left[(\underline{R}_c + \underline{r}_m)\cdot(\underline{R}_c + \underline{r}_m)\right]^{-\frac{1}{2}}$$

$$= \left[R_c^{-1}\ 1 + \frac{r_M^2}{R_c^2} + \frac{2}{R_c^2}(R_c \cdot r_M)\right]^{-\frac{1}{2}}$$

Representing $\underline{R}_c$ and $\underline{r}_m$ with respect to x, y axis,

$$\underline{R}_c \cdot \underline{r}_M = (R_c\cos\theta\ \underline{i} - R_c\sin\theta\ \underline{j})\cdot(x\ \underline{i} + y\ \underline{j} + z\ \underline{k})$$

$$= R_c x\cos\theta - R_c y\sin\theta$$

hence $$R_M^{-1} = R_c^{-1}\left[1 + \frac{R_M^2}{R_c^2} + \frac{2}{R_c}(x\cos\theta - y\sin\theta)\right]^{-\frac{1}{2}}$$

$$* - \int_{\mathbb{L}} (\quad)\, dm = \int_{\ell_o}^{\ell} (\quad)\, dm + \int_{-\ell_o}^{-\ell} (\quad)\, dm$$

$$= R_c^{-1}\left[1 - \tfrac{1}{2}\frac{x^2 + y^2 + z^2}{R_c^2} - \frac{1}{R_c}(x\cos\theta\right.$$

$$\left. - y\sin\theta) + \frac{3}{2R_c}(x^2\cos^2\theta + y^2\sin^2\theta - 2xy\sin\theta\cos\theta)\right]$$

since xyz is a principal axis system,

$$\int_M x\, dM = \int_M y\, dM = \int_M xy\, dM = 0$$

and

$$V_G)_M = -\frac{K}{R_c}\int_M \left[1 - \tfrac{1}{2}\frac{x^2 + y^2 + z^2}{R_c^2} + \frac{3}{2R_c^2}(x^2\cos^2\theta + y^2\sin^2\theta)\right]dM \tag{15}$$

now $A = \int_M (y^2 + z^2)\, dM$

$$B = \int_M (x^2 + z^2)\, dM$$

$$C = \int_M (x^2 + y^2)\, dM$$

Substituting these in equation (15), yields,

$$V_G)_M = -\frac{KM}{R_c} + \frac{1}{4}\frac{K}{R_c^3}(A + B + C)$$

$$- \frac{3K}{4R_c^3}\left[(C + B - A)\cos^2\theta + (C + A - B)\sin^2\theta\right] \tag{16}$$

$V_G)_m$ may be determined in a similar fashion. The final form of the potential is;

$$V_G = -\frac{K}{R_c}(M + 2m) + \frac{1}{4}\frac{K}{R_c^3}(A + B + C) - \frac{3K}{4R_c^3}\Big[(C' + B - A)\cos^2\theta + (C + A - B)\sin^2\theta\Big] + \frac{\frac{1}{2}K}{R_c^3}\int_L \Big[(2ux + u^2)(1 - 3\cos^2\theta) + v^2(1 - 3\sin^2\theta) + 3v(x + u)\sin^2\theta + 2\dot{R}_c(u\cos\theta - v\sin\theta)\Big]\,dm \tag{17}$$

The elastic potential V_E for an inextensible beam due to the bending alone is

$$V_E = \frac{1}{2}\int_L EI\left[\left(\frac{\partial^2 u}{\partial x^2}\right)^2 + \left(\frac{\partial^2 v}{\partial x^2}\right)^2\right] dx \tag{18}$$

where $\left[\left(\frac{\partial^2 u}{\partial x^2}\right)^2 + \left(\frac{\partial^2 v}{\partial x^2}\right)^2\right] = \kappa^2$ is the curvature of the beam.

Since the Lagrangian, L is

$$L = T - V \tag{19}$$

and considering the orbit of satellite to be circular, that is,

$$R_c = \text{constant}$$
$$\dot{R}_c = 0$$
$$\dot{\varphi} = \dot{\varphi}_o = \left(\frac{K}{R_c^3}\right)^{\frac{1}{2}} = \text{constant}$$

then equations (13), (17), (18), and (19) when substituted into equation (1) yields,

$$
\begin{aligned}
H = {} & \tfrac{1}{2} C'\dot{\theta}^2 - \frac{3K}{4R_c^2} (C'+B'-A)\cos^2\theta + (C'+A-B')\sin^2\theta \\
& + \tfrac{1}{2}\int_{\mathbb{L}} \Bigg\{ \frac{m}{\ell-\ell_o} \Big[\dot{u}^2 + v^2 + (2ux+u^2+v^2)(\dot{\Psi}^2+\dot{\theta}^2) \\
& + 2R_c\dot{\Psi}_o^2 (u \sin\theta - v\cos\theta) + 2\dot{\theta}\left(\dot{v}(x+u) - \dot{u}v\right) \\
& + \frac{K}{R_c^3} \Big[(2ux+u^2)(1-3\cos^2\theta) + v^2(1-3\sin^2\theta) + 3v(x+u)\sin 2\theta \\
& - 2R_c(u\cos\theta - v\sin\theta) \Big]\Big] + EI\kappa^2 \Bigg\} dx
\end{aligned}
\qquad (20)
$$

Since H = T + U, the dynamic potential U can be obtained simply by excluding all terms in equation (20) which involves u,v, and $\dot{\theta}$. After further simplification, the form of U can be shown to be,

$$
\begin{aligned}
U = {} & -\tfrac{3}{4}\dot{\psi}^2(B'-A)\cos 2\theta + \tfrac{1}{2}\int_{\mathbb{L}} \Bigg\{ -\frac{3m}{\ell-\ell_o}\dot{\psi}^2 \Big[(2ux+u^2)\cos^2\theta \\
& + v^2\sin^2\theta - v(x+u)\sin^2\theta \Big] + EI\kappa^2 \Bigg\} dx
\end{aligned}
\qquad (21)
$$

The axial deformation of an in extensionable beam can be written in terms of the lateral deformation (figure 1.2)

$$dx + du = \left\{(dx)^2 - (dv)^2\right\}^{\frac{1}{2}}$$

$$= dx \left\{1 - \left(\frac{dv}{dx}\right)^2\right\}^{\frac{1}{2}}$$

expanding binomially and neglecting the fourth and higher order terms,

$$du = -\tfrac{1}{2}\left(\frac{dv}{dx}\right)^2 dx$$

the axial deformation,u, along the beam is obtained by integrating du from 0 to x. Thus,

$$u = \int_0^x du = -\tfrac{1}{2}\int_0^x \left\{\left(\frac{dv}{dx}\right)^2\right\} dx$$

since $\frac{dv}{dx} = 0$ for $0 \leqslant x \leqslant \ell_o$,

$$u = -\tfrac{1}{2}\int_{\ell_o}^{x} \left\{\left(\frac{dv}{dx}\right)^2\right\} dx.$$

since u is a function of the square of the slope, $\frac{\partial v}{\partial x}$, the terms u^2,vu, and $\frac{\partial^2 u}{\partial x^2}$ can be neglected as they are much smaller than the other termsin equation (21). Thus only one term involving u is left to be considered, xu. Thus,

$$\int_{\ell_o}^{\ell} xu\, dx = -\tfrac{1}{2}\int_{\ell_o}^{\ell} x\left\{\int_{\ell_o}^{x}\left(\frac{\partial v}{\partial x'}\right)^2 dx'\right\} dx$$

$$= -\tfrac{1}{2}\int_{\ell_o}^{\ell} \tfrac{1}{2}\left\{(\ell^2 - x'^2)\left(\frac{\partial v}{\partial x'}\right)^2\right\} dx$$

Substituting these, equation (21) reduces to,

$$U = -\frac{3}{4}\dot{\psi}^2(B'-A)\cos 2\theta + \frac{1}{2}\int_{L}\left\{\frac{3m}{\ell-\ell_o}\dot{\varphi}^2\left[\frac{1}{2}(\ell^2-x^2)\left(\frac{\partial v}{\partial x}\right)^2\cos^2\theta - v^2\sin^2\theta + vx\sin 2\theta\right] + EI\left(\frac{\partial^2 v}{\partial x^2}\right)^2\right\}dx \quad (22)$$

Differentiating the above with respect to θ and equating to zero yields one of the equilibrium equations.

1.3 Virtual Work Principle :

From Hamilton's principle, the equation of motion for a nonconservative system is written as (Reference 9)

$$\delta I = \delta\int_1^2 (T + W)\,dt = 0$$

where T is the total kinetic energy and W is the work done by applied forces in going from state 1 to 2. The work W can be expressed in two parts, (1) work done by conservative forces derivable from generalized potential, -V, and (2) work done by nonconservative forces, Q_j. If the virtual displacement is expressed as δq_j, force per unit length $-Q_x$, and virtual elastic displacement δn, then the above equation can be rewritten as:

$$\delta\int_1^2 L\,dt + \int_1^2 \sum_{j=1}^{m} Q_j\,\delta q_j\,dt + \int_1^2\int_{x_o}^{x_1} Q_x\delta n\,dx\,dt = 0 \quad (23)$$

The Lagrangian L can be written as the sum of the rigid body component L_R, and elastic component $\mathcal{L}_E$ (see reference 9).

$$\delta\int_1^2 L_R dt + \delta\int_1^2\int_{x_o}^{x_1} \mathcal{L}_E dx\, dt + \int_1^2 \sum_{j=1}^{m} Q_j\, \delta q_j dt + \int_1^2\int_{x_o}^{x_1} Q_x\, \delta n \; dx \; dt = 0 \tag{24}$$

It has been shown that (reference 9),

$$\delta\int_1^2 L_R(q_j, \dot{q}_i, t)\, dt = \int_1^2 \sum_{j=1}^{m} \left[\frac{L_R}{q_j} - \frac{d}{dt}\left(\frac{\partial L_R}{\partial \dot{q}_j} \right)\right] \delta q_j dt$$

and

$$\delta\int_1^2\int_{x_o}^{x_1} \mathcal{L}_E \; dx \; dt = \int_1^2\int_{x_o}^{x_1} \left\{ \left[\frac{\partial \mathcal{L}_E}{\partial q_j} - \frac{d}{dt}\left(\frac{\partial \mathcal{L}_E}{\partial \dot{q}_j}\right)\right] \delta q_j + \left[\frac{\partial \mathcal{L}_E}{\partial n} - \frac{d}{dt}\left(\frac{\partial \mathcal{L}_E}{\partial \dot{n}} \right)\right] + \frac{\partial \mathcal{L}_E}{\left(\frac{\partial n}{\partial x}\right)} \delta\left(\frac{\partial n}{\partial x}\right) + \frac{\partial \mathcal{L}_E}{\left(\frac{\partial^2 n}{\partial x^2}\right)} \delta\left(\frac{\partial^2 n}{\partial x^2}\right) \right\} dx \; dt \tag{25}$$

Integrating by parts, simplifying and equating the coefficients of δq_j and δn to zero,

$$\frac{\partial U}{\partial q_j} = 0$$

$$\frac{\partial}{\partial x}\left\{\frac{\partial \mathcal{U}}{\partial\left(\frac{\partial \xi}{\partial x}\right)}\right\} - \frac{\partial^2}{\partial x^2}\left\{\frac{\partial \mathcal{U}}{\partial\left(\frac{\partial^2 \xi}{\partial x^2}\right)}\right\} - \frac{\partial \mathcal{U}}{\partial \xi} = 0 \qquad (26)$$

where $\mathcal{U}$ is the dynamic potential of the elastic components. Differentiating equation (22) with respect to Θ and substituting the value of $\mathcal{L}_E$ yields first of the equilibrium equation, and substituing the value of $\mathcal{L}_E$ in equation (26) yields second equilibrium equation.

The two governing equilibrium equations with the corresponding boundary conditions are:

$$(B'-A)\sin 2\Theta - \frac{m}{\ell}\left\{\sin 2\Theta \int_{\mathbb{L}} \left[\tfrac{1}{2}(\ell^2-x^2)\left(\frac{dv}{dx}\right)^2 + v^2\right] dx - 2\cos 2\Theta \int_{\mathbb{L}} xv\, dx\right. = 0 \qquad (27)$$

$$EI\frac{d^4v}{dx^4} - \frac{3}{4}\frac{m}{\ell}\dot{\varphi}^2\left(\cos^2\Theta \frac{d}{dx}\left[(\ell^2-x^2)\frac{dv}{dx}\right] + 2v\sin^2\Theta - x\sin 2\Theta\right) = 0 \qquad (28)$$

With the Boundary Conditions :

$$v = \frac{dv}{dx} = 0 \text{ at } x = \ell_c$$

$$\frac{d^2v}{dx^2} = \frac{d^3v}{dx^3} = 0 \text{ at } x = \ell \qquad (29)$$

The above equations can be reduced to the following nondimensional form:

$$\frac{d^4\tilde{v}}{d\tilde{x}^4} - K_1\left[\cos^2\theta \frac{d}{d\tilde{x}}\left\{(1-\tilde{x}^2)\frac{d\tilde{v}}{d\tilde{x}}\right\} + 2\tilde{v}\sin^2\theta - \tilde{x}\sin 2\theta\right] = 0 \qquad (30)$$

where $K_1 = \frac{3}{2}\frac{m\,\dot{\Psi}^2 \ell^3}{EI}$

$$\sin 2\theta - K_2\left[\sin 2\theta \int_{\mathbb{L}} \left\{\tfrac{1}{2}(1-\tilde{x}^2)\left(\frac{d\tilde{v}}{d\tilde{x}}\right)^2 + \tilde{v}^2\right\} d\tilde{x} - 2\cos 2\theta \int_{\mathbb{L}} \tilde{x}\tilde{v}\, d\tilde{x}\right] = 0 \qquad (31)$$

where $K_2 = \frac{m\ell^2}{(B'-A)}$

with the boundary conditions:

$$\tilde{v} = \frac{d\tilde{v}}{d\tilde{x}} = 0; \quad \text{at } \tilde{x} = 0$$

$$\frac{d^2\tilde{v}}{d\tilde{x}^2} = \frac{d^3\tilde{v}}{d\tilde{x}^3} = 0; \quad \text{at } \tilde{x} = 1. \qquad (32)$$

CHAPTER II

SOLUTION OF EQUILIBRIUM EQUATIONS

2.1. Method of Solution :

Equations (30) and (31) represent a set of two nonlinear, homogenous, simultaneous, differential, integral equations with two unknowns θ and $\tilde{v}(x)$. These are solved as follows:

Using the boundary conditions (32), for a given orientation θ_o, equation (30) is solved for deflection $\tilde{v}$ as function of $\tilde{x}$. $\tilde{v}$ being known, equation (31) is solved for the attitude angle θ. If θ is the same as θ_o, then there exists a valid equilibrium position. The procedure is then repeated for different values of θ_o.

2.2 Procedure:

Assuming deflection $\tilde{v}$ as a function of an infinite power series in $\tilde{x}$ with constant coefficients,

$$\tilde{v}(\tilde{x}) = \sum_{n=0}^{\infty} a_n \tilde{x}^n$$

$$\frac{d\tilde{v}}{d\tilde{x}} = \sum_{n=0}^{\infty} n a_n \tilde{x}^{n-1}$$

$$\frac{d^2\tilde{v}}{d\tilde{x}^2} = \sum_{n=0}^{\infty} n(n-1)\tilde{x}^{n-2}$$

$$\frac{d^3\tilde{v}}{d\tilde{x}^3} = \sum_{n=0}^{\infty} n(n-1)(n-2)a_n \tilde{x}^{n-2}$$

$$\frac{d^4\tilde{v}}{d\tilde{x}^4} = \sum_{n=0}^{\infty} n(n-1)(n-2)(n-3)a_n \tilde{x}^{n-3}$$

substituting these in equation (30),

$$\sum_{n=0}^{\infty} n(n-1)(n-2)(n-3)a_n\tilde{x}^{n-4} - K_1\left[\cos^2\theta \frac{d}{d\tilde{x}}\left\{(1-\tilde{x}^2)\sum_{n=0}^{\infty} na_n\tilde{x}^{n-1}\right\}\right.$$
$$\left. + 2\sin^2\theta \sum_{n=0}^{\infty} a_n\tilde{x}^n - \tilde{x}\sin 2\theta\right] = 0$$

simplifying,

$$\sum_{n=0}^{\infty} n(n-1)(n-2)(n-3)a_n\tilde{x}^{n-4} - K_1\left[\cos^2\theta \left\{(1-\tilde{x}^2)\sum_{n=0}^{\infty} n(n-1)a_n\tilde{x}^{n-2}\right.\right.$$
$$\left.\left. - 2x\sum_{n=0}^{\infty} na_n\tilde{x}^{n-1}\right\} + 2\sin^2\theta\sum_{n=0}^{\infty} a_n\tilde{x}^n - \tilde{x}\sin 2\theta\right] = 0$$

rearranging and simplifying;

$$\sum_{n=0}^{\infty} n(n-1)(n-2)(n-3)a_n\tilde{x}^{n-4} - K_1\cos^2\theta\sum_{n=0}^{\infty} n(n-1)a_n\tilde{x}^{n-2}$$
$$+\sum_{n=0}^{\infty}\left\{K_1\cos^2\theta\, n(n-1)a_n + 2K_1\cos^2\theta . na_n - 2K_1\sin^2\theta . a_n\right\}\tilde{x}^n$$
$$+ K_1\tilde{x}\sin 2\theta = 0 \qquad (33)$$

The set of series in the above equation are expanded and the coefficients of each power of $\tilde{x}$ are arranged together. This represents an infinite power series in $\tilde{x}$ with coefficients as function of parameters of satellite and attitude angle θ.

The right hand side of the above series being zero, the coefficients of each power of $\bar{x}$ must identically be equal to zero. Hence an infinite number of simultaneous equations in a_n $(n=0,1,2,3,.......)$ are obtained. As the solution of a set of infinite simultaneous equations is not feasible, the series is truncated. The computer time involved in using a larger series and the fact that the contribution of 21st term and onwards is of the order of 10^{-13}, which is beyond the accuracy of the method used resulted in truncating the series after 20 terms.

The Boundary conditions (32) are applied as follows;

$$v = \frac{dv}{dx} = 0 \quad \text{at } x = 0; \quad \frac{d^2v}{dx^2} = \frac{d^3v}{dx^3} = 0 \text{ at } x = 1$$

Substituting equation (28) in above,

$$\begin{aligned} a_o &= 0 \\ a_1 &= 0 \\ 2a_2 + 6a_3 + \ldots\ldots\ldots\ldots + 380\, a_{20} &= 0 \\ 6a_3 + 24a_4 + \ldots\ldots\ldots\ldots + 380.18a_{20} &= 0 \end{aligned} \qquad (32a)$$

Thus equations (32a) in conjunction with the set of simultaneous equations (obtained as explained above) can be solved for unknowns a_o, a_1, a_2, ... a_{20}.

CHAPTER III

RESULTS

It has been shown (References 1 & 2) that for a gravity-gradient type satellite, there are at least four possible equilibrium positions. These occur at 0°, 90°, 180°, and 270° attitude angles. If stability exists at 0° and 180° attitude angles, there will be instability at 90° and 270° orientation and vice versa.

Figure 3.1 illustrates the region of asymptotic stability and instability for 0° and 180° positions for this type of satellite. The curves are plots of orbital spin rate $\dot{\psi}/\omega_1$ versus dimensionless inertia parameter $(B'-A) / B'$. Figure 3.2 illustrates the region of asymptotic stability and instability for 90° and 270° orientations.

Consider three points P_1, P_2, and P_3, in three different regions in figure 3.1. The corresponding points in figure 3.2 are represented by P_1', P_2', and P_3' respectively.

P_1' represents Elastic and coupled elastic-rigid stability; Rigid body instability region.

P_2' represents Elastic Stability; coupled elastic-rigid and Rigid body instability region.

P_3' represents Elastic, coupled elastic-rigid and Rigid body instability region.

3.1 Elastic and Coupled Elastic-Rigid Stability; Rigid Body Instability.

From figure 3.2, it is obvious that for a satellite to be in elastic and coupled elastic-rigid instability and rigid body instability region, orbital spin rate $\dot{\psi}/\omega_1$ should be in the neighborhood of 0.0 to 0.4 - the exact value depending on h_2, $(m\ell^2/A)$, where ω_1 is the fundamental natural frequency of vibration of the antenna.

Consider a satellite with the following parameters:

$m = 0.3269$ slugs

$EI = 15.277$ lb-ft^2

$\dot{\psi} = 0.667$ rev./hr.

$\dfrac{\dot{\psi}}{\omega_1} = 0.2$

$$\text{Hence } \omega_1 = \left[\frac{12.362\ EI}{m\ell^3}\right]^{1/2} = \frac{\dot{\psi}}{0.2} = \frac{.001164134}{0.2}$$

$$= 0.00582$$

Solving for ℓ, $\qquad \ell = 258$ ft.

The method described in section 2.1 gives all possible equilibrium positions for a particular satellite. However, for a given attitude angle θ, equation (31) can also be solved for the ratio λ, where $\lambda = \dfrac{m\ell^2}{B'-A}$, instead of θ as outlined previously. The advantage of this method being that for a satellite to have an equilibrium position at a desired attitude angle θ', other parameters being known, the moment of inertia can be made such that:

$(B'-A) = \frac{m l^2}{\lambda}$. In this case, the given satellite will always have an equilibrium position at θ' orientation.

Typical results are tabulated in table I.

Attitude Angle θ	Maximum Deflection v_{max}	Ratio λ
0	0.0	8.641
10	-0.020576	9.2639
20	-0.039477	11.684
30	-0.054943	19.48
40	-0.065113	107.40
50	-0.068171	-28.25
60	-0.062714	-12.9257
70	-0.048367	-8.9227
80	-0.026408	-7.4786
90	0.0	-7.0715

Figures 3.3 through 3.12 illustrate the deflection curves for elastic antennas at various attitude angles. Figure 3.13 shows the graph for λ vs. θ for a particular value of EI, while figure 3.14 shows the effect of EI on λ.

From figure 3.13 it can be seen that,

(1) The 0° and 90° orientations will always be equilibrium positions. Depending on the value of λ, there may or may

not exist a third equilibrium position. Thus, there can exist a maximum of only three equilibrium positions in the range of 0°-90°. For example, if $8.62 > \lambda > -7.07$, then $0^{\circ}$and 90° are the only possible equilibrium positions. However, if $\lambda = 11.684$ then equilibrium exist for $\theta = 0^{\circ}$, 20°, and 90° attitude angles.

2) If the moment of inertia about the y axis (B') is greater than the moment of inertia about the x axis, (A), then the equilibrium position will occur between 0° and 42° ; and if A is greater then B', the equilibrium will occur at an attitude angle between 42° and 90°.

3) The effect of λ on the attitude angles diminishes in the region of 30° and 60° orientation and increases in the rest of the region.

3.2 Elastic Stability; Coupled Elastic-Rigid and Rigid Body Instability.

As explained in section 3.1, for a satellite to be in "elastic stability and coupled and rigid body instability region", the orbital spin rate $\dot{\psi}/\omega_1$ should be in the neighborhood of 0.4 to 0.55. For the orbital spin rate to be equal to 0.5, the length of antennas is calculated to be 474 ft.

Substituting this value of ℓ and other parameters, equation (30) is solved for deflection $\tilde{v}$. However unlike section 3.1, the deflection for this case becomes too large such that the linear theory of elasticity can no longer be used.

The curvature of the deflected beam ρ, as derived from the theory of elasticity:

$$\frac{1}{\rho} = \frac{M}{EI} \tag{34}$$

from the geometry of the deflected beam,

$$\frac{1}{\rho} = \frac{-d^2y/dx^2}{\left[1 + (dy/dx)^2\right]^{3/2}} \tag{35}$$

For small deflections $(dy/dx)^2$ is very small and can be neglected. Thus the above equations are reduced to

$$\frac{1}{\rho} = -\frac{M}{EI} = -\frac{d^2y}{dx^2}$$

$$M = EI\frac{d^2y}{dx^2}$$

Differentiating this expression twice with respect to x yields the force intensity, w(x),

$$w(x) = EI\frac{d^4y}{dx^4}$$

However, when deflections are large, the slope and $(dy/dx)^2$ can no longer be neglected. Hence from equations (34) and (35),

$$M = EI\,\frac{d^2y/dx^2}{\left[1 + (dy/dx)^2\right]^{3/2}}$$

Differentiating the above equation twice with respect to 'x', and simplifying and rearranging the terms, an expression for force for large deflections is obtained.

$$\left[1 + \left(\frac{dv}{dx}\right)^2\right]^2 \frac{d^4v}{dx^4} - 9\left[1 + \left(\frac{dv}{dx}\right)^2\right] \frac{dv}{dx}\,\frac{d^2v}{dx^2}\,\frac{d^3v}{dx^3}$$
$$- 3\left[1 + \left(\frac{dv}{dx}\right)^2\right]\left(\frac{d^2v}{dx^2}\right)^3 + 15\left(\frac{dv}{dx}\right)^2\left(\frac{d^2v}{dx^2}\right)^3$$
$$= \frac{3}{2}\,\frac{m}{\ell}\,\frac{\dot{\psi}^2}{EI}\left[1 + \left(\frac{dv}{dx}\right)^2\right]^{\frac{7}{2}} \left\{\cos^2\theta\,\frac{d}{dx}\left[(\ell^2 - x^2)\frac{dv}{dx}\right] + 2v\sin^2\theta - x\sin 2\theta\right\} \tag{36}$$

where the right hand side of the above equation is the force intensity w(x).

As explained in chapter 2, a set of simultaneous equations in $a_n (n=0,1,2,....)$ is obtained. These equations turn out to be nonlinear in nature and their solution is extreamely difficult to obtain.

As a result "elastic stability; coupled and rigid body instability" and "elastic, coupled and rigid body instability" regions are not investigated in detail.

CHAPTER IV

STABILITY TEST

4.1 Liapunov Function :

As derived in Chapter I,

$$H = T_2 + U$$

The total energy of the system being $T + V$, Hamiltonian in general is not the total energy. In order to make Liapunov function zero at equilibrium, V_L is defined as $V_L = H - H_o$, where H_o is Hamiltonian evaluated at equilibrium. Thus

$$V_L = T_2 + U - H_o$$

since T_2 is zero at equilibrium, $H_o = U_o$ and the above expression can be rewritten as

$$V_L = T_2 + U - U_o$$

4.2 Stability Matrix:

It has been shown (references 2,4,6) that for a satellite to be stable in the region about equilibrium, V_L must be positive definite (if $V_L > 0$ in a neighborhood about an equilibrium point except that it may possibly be zero at equilibrium, then V_L is defined as positive definite). Given the Liapunov function $V_L = T_2 + U - U_o$; T_2 being known to be positive definite, for V_L to be positive definite, $U-U_o$ must be positive definite.

If matrix $S_{ij} = \dfrac{\partial^2 U(q_i, q_j)}{\partial q_i \, \partial q_j}$ is positive definite, then $U - U_o$ will be positive definite, and the system will be stable in the region about equilibrium. If the matrix S_{ij} is not positive definite, then the system will be unstable in the neighborhood of the equilibrium position.

By Sylvester's theorom (reference 8), the matrix S_{ij} will be positive definite only if the principal determinants of the matrix are greater than zero. That is,

$$D_1 = S_{11} > 0$$

$$D_2 = \begin{vmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{vmatrix} > 0$$

$$D_3 = \begin{vmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{vmatrix} > 0$$

and so on. It can be shown that to satisfy the above inequalities all the principle diagonal elements must be greater than zero.

The dynamic potential :

$$U(\theta) = -\frac{3}{4}\dot{\psi}^2(B'-A)\cos 2\theta + \frac{1}{2}\int_{L}\left\{\frac{3m}{\ell}\dot{\psi}^2\left[\frac{1}{2}(\ell^2-x^2)\left(\frac{\partial v}{\partial x}\right)^2\cos^2\theta - v^2\sin^2\theta + vx\sin 2\theta\right] + EI\left(\frac{\partial^2 v}{\partial x^2}\right)^2\right\}dx$$

and $$v(x) = \sum_{n=0}^{\infty} a_i x^i$$

To ease the computation the above series was further truncated after six terms without any appreciable loss. Thus

$$v(x,q_i) = \sum_{n=0}^{6} a_i\, x^i$$

$$= a_o + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5$$

Boundary conditions imply that $a_o = a_1 = 0$

therefore $\quad v(x) = a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5$

Differentiating U successively with respect to θ, a_2, a_3, a_4, and a_5, stability matrix S_{ij} is formed as outlined Above.

4.3 Stability Check for 0^o Attitude Angle :

Substituting the values of a_2, a_3, a_4, a_5, and θ in the stability matrix, and simplifying stability matrix for 0^o attitude angle is reduced to

$$\begin{bmatrix}
3\dot{\psi}^2(B'-A) & \frac{\eta\ell^4}{2} & \frac{2\eta\ell^5}{5} & \frac{\eta\ell^6}{3} & \frac{2\eta\ell^7}{7} \\
\frac{\eta\ell^4}{2} & \frac{8}{15}\eta\ell^5 + 8EI\ell & \frac{\eta\ell^6}{2} + 12EI\ell^2 & \frac{16\eta\ell^7}{35} + 16EI\ell^3 & \frac{5}{12}\eta\ell^8 + 20EI\ell^4 \\
\frac{2\eta\ell^5}{5} & \frac{\eta\ell^6}{2} + 12EI\ell^2 & \frac{18\eta\ell^7}{35} + 24EI\ell^3 & \frac{\eta\ell^8}{2} + 36EI\ell^4 & \frac{10\eta\ell^9}{21} + 48EI\ell^5 \\
\frac{\eta\ell^6}{3} & \frac{16}{35}\eta\ell^7 + 16EI\ell^3 & \frac{\eta\ell^8}{2} + 36EI\ell^4 & \frac{32}{63}\eta\ell^9 + \frac{288EI\ell^5}{5} & \frac{\eta\ell^{10}}{2} + 80EI\ell^6 \\
\frac{2\eta\ell^7}{7} & \frac{5}{12}\eta\ell^8 + 20EI\ell^4 & \frac{10\eta\ell^9}{21} + 48EI\ell^5 & \frac{\eta\ell^{10}}{2} + 80EI\ell^6 & \frac{50}{99}\eta\ell^{11} + \frac{800EI\ell^7}{7}
\end{bmatrix}$$

where $\eta = \dfrac{3\, m \dot{\psi}^2}{l}$

Substituting the values of m, EI, $\dot{\psi}$, and l for a given satellite, its stability can be determined as outlined in section 4.2.

4.4 Stability Check for 90° Attitude Angle :

Substituting the values of a_2, a_3, a_4, a_5, and θ in the stability matrix and simplifying, stability matrix for 90° attitude angle is reduced to:

$$\begin{bmatrix} -3\dot{\psi}^2(B'-A) & -\frac{\eta l^4}{4} & -\frac{2\eta l^5}{5} & -\frac{\eta l^6}{3} & -\frac{2\eta l^7}{7} \\ -\frac{\eta l^4}{2} & -\frac{2\eta l^5}{5}+8EIl & -\frac{\eta l^6}{3}+12EIl^2 & -\frac{2\eta l^7}{7}+16EIl^3 & -\frac{\eta l^8}{4}+20EIl^4 \\ -\frac{2\eta l^5}{5} & -\frac{\eta l^6}{5}+12EIl^2 & -\frac{2\eta l^7}{7}+24EIl^3 & -\frac{\eta l^8}{4}+36EIl^4 & -\frac{2\eta l^9}{9}+48EIl^5 \\ -\frac{\eta l^6}{5} & -\frac{2\eta l^7}{7}+16EIl^3 & -\frac{\eta l^8}{4}+36EIl^4 & -\frac{2\eta l^9}{9}+288EIl^5 & -\frac{\eta l^{10}}{5}+80EIl^6 \\ -\frac{2\eta l^7}{7} & -\frac{\eta l^8}{8}+20EIl^4 & -\frac{2\eta l^9}{9}+48EIl^5 & -\frac{\eta l^{10}}{5}+80EIl^6 & -\frac{2\eta l^{11}}{11}+\frac{800}{7}EIl^7 \end{bmatrix}$$

where $\eta = \dfrac{3m\dot{\psi}^2}{l}$ as before.

Substituting the values of m, EI, $\dot{\psi}$, and l for a given satellite, its stability can be determined as explained earlier.

Examining the first term in the above stability matrix and the first term in the stability matrix for 0°, it can be noted that both terms are the same except for the sign. Hence, if any satellite if stable at 0^o will be unstable at 90^o orientation and vice versa.

4.5 Stability Check for 20^0 Attitude Angle :

As explained in Chapter 3, parameters m and (B'-A) are selected such that there will be an equilibrium at 20^o attitude angle. From Table 1, to have equilibrium at 20^o, $\frac{m\ell^2}{(B'-A)}$ = 11.684, m being equal to .3269 slugs, ℓ = 258 ft, (B'-A) can be calculated. For these values of parameters the stability matrix is reduced to:

$$\begin{bmatrix} -658\times10^{10} & 116\times10^{6} & 315\times10^{8} & 813\times10^{10} & 206\times10^{13} \\ 116\times10^{6} & 340\times10^{2} & 128\times10^{5} & 434\times10^{7} & 139\times10^{10} \\ 315\times10^{8} & 128\times10^{5} & 646\times10^{7} & 248\times10^{10} & 849\times10^{12} \\ 813\times10^{10} & 434\times10^{7} & 248\times10^{10} & 102\times10^{13} & 724\times10^{18} \\ 206\times10^{13} & 139\times10^{10} & 849\times10^{12} & 724\times10^{18} & 134\times10^{18} \end{bmatrix}$$

Since all the principle diagonal elements are not greater than zero, there will be instability at 20^o attitude angle.

4.6 Stability Check for 30° Attitude Angle :

As in the previous section, parameters are selected for equilibrium at 30° orientation. For these values of parameters, the stability matrix is reduced to :

$$\begin{bmatrix} -890\times10^{10} & 226\times10^{6} & 610\times10^{8} & 158\times10^{11} & 399\times10^{13} \\ 226\times10^{6} & 333\times10^{2} & 126\times10^{5} & 430\times10^{7} & 138\times10^{10} \\ 610\times10^{8} & 126\times10^{5} & 640\times10^{7} & 247\times10^{10} & 846\times10^{12} \\ 158\times10^{11} & 430\times10^{7} & 247\times10^{10} & 101\times10^{13} & 375\times10^{23} \\ 399\times10^{13} & 138\times10^{10} & 846\times10^{12} & 375\times10^{23} & 133\times10^{18} \end{bmatrix}$$

Once again as all the principle diagonal elements are not greater than zero there will be instability at 30° attitude angle.

4.7 Stability :

Comparing the curves 3.1 and 3.13, it is noted that the upper half of the curve 3.13 corresponds to the curve in figure 3.1 for a particular value of h_1(h_1 = 1 in this case). The stable region on the right hand side of the curve for h_1 = 1 in figure 3.1 corresponds to the curves OF and O'E in figure 3.13. The unstable region on the left hand side of the curve for h_1 = 1 in figure 3.1 corresponds to the curves FC, FG and ED in figure 3.13.

From the analysis of figures 3.1 and 3.13, for a given set of parameters, we can conclude that : (1) for values of λ in the range of 0-F there will only be two equilibrium positions, namely the 0° and 90° orientations, of which 0° position will be stable and 90° position unstable. (2) For values of λ in the range F to $+\infty$ there will be three equilibrium positions, where 0° and 90° positions will always be unstable, and the third equilibrium position must be checked to ascertain stability as outlined in section 4.2. For the examples studied here, the third position is found to be unstable.

A similar analogy exists between a curve in figure 3.2, and the lower half of the curve 3.13.

CHAPTER V

CONCLUSIONS

The classical problem of the stability of a rigid satellite was dealt with in references 3 and 4. Whereas in references 1 and 2, the equilibrium equations were set up for a large flexible satellite, and the equilibrium positions and stability were determined for 0°, 90°, 180°, and 270° attitude angles. The effect of flexibility of a satellite on its stability was discussed, and it was shown that it had no effect on its equilibrium positions. These studies, however, did not discuss all the possible equilibrium positions.

The present study solves the equilibrium equations for any given attitude angle by assuming a truncated power series solution. Equilibrium positions for a particular satellite are determined and its stability ascertained. The relation between different parameters is set up to have equilibrium at a desired position.

The discussion on stability up to this point has been limited to the possibility of an equilibrium position being asymptotically stable for only small disturbances about it. Hence, stability in the large has not been determined.

However, in this study it has been shown that in the range of 0°-90° attitude angles, 0° and 90° will always be equilibrium positions, and depending on the value of λ, there may exist one more equilibrium position. It has also been shown that if there are only two equilibrium positions, one of them will be stable. However, if three equilibrium positions exist, 0° and 90° will definitely be unstable, and the third position should be checked for stability. Thus one can trace the behaviour of a particular class of satellite and can successfully prodict its equilibrium positions and stability.

Compared with Reference (2), which used the eigenfunctions of the freely vibrating antennas to solve the equilibrium equations, the method used in this analysis is in close agreement for low values of the orbital spinrate, and starts deviating in a more conservative direction as the spin rate increases. The results are plotted in figure 5.1, where curve A represents the method using eigenfunctions while curve B represents the truncated power series method. However it should be noted that for the particular class of satellites consired in this study, it is necessary that the spinrate be low, hence one can assure tha analysis to be quire accurate.

It should be noted that the results obtained are approximate solutions, rather than exact. Also the "matrix inversion" method is used to solve the simultaneous equations and hence an eigenvalue solution (which has been shown to exist in references 1 and 2) is not possible. A numerical iterative procedure would be much more suited for solving these equations, particularly for 90° and 270° positions and their neighborhood.

Since the equilibrium equations are set up in a linear fashion, only elastic and coupled stability could be determined. Thus satellites of only one class is studied in detail.

Coupled and rigid body instability could not be checked. However, the nonlinear equations for these cases are set up, and if one desires to persue this further, one could very well start from sect. 3.2. If this is done, the complete behaviour of any given satellite can be predicted.

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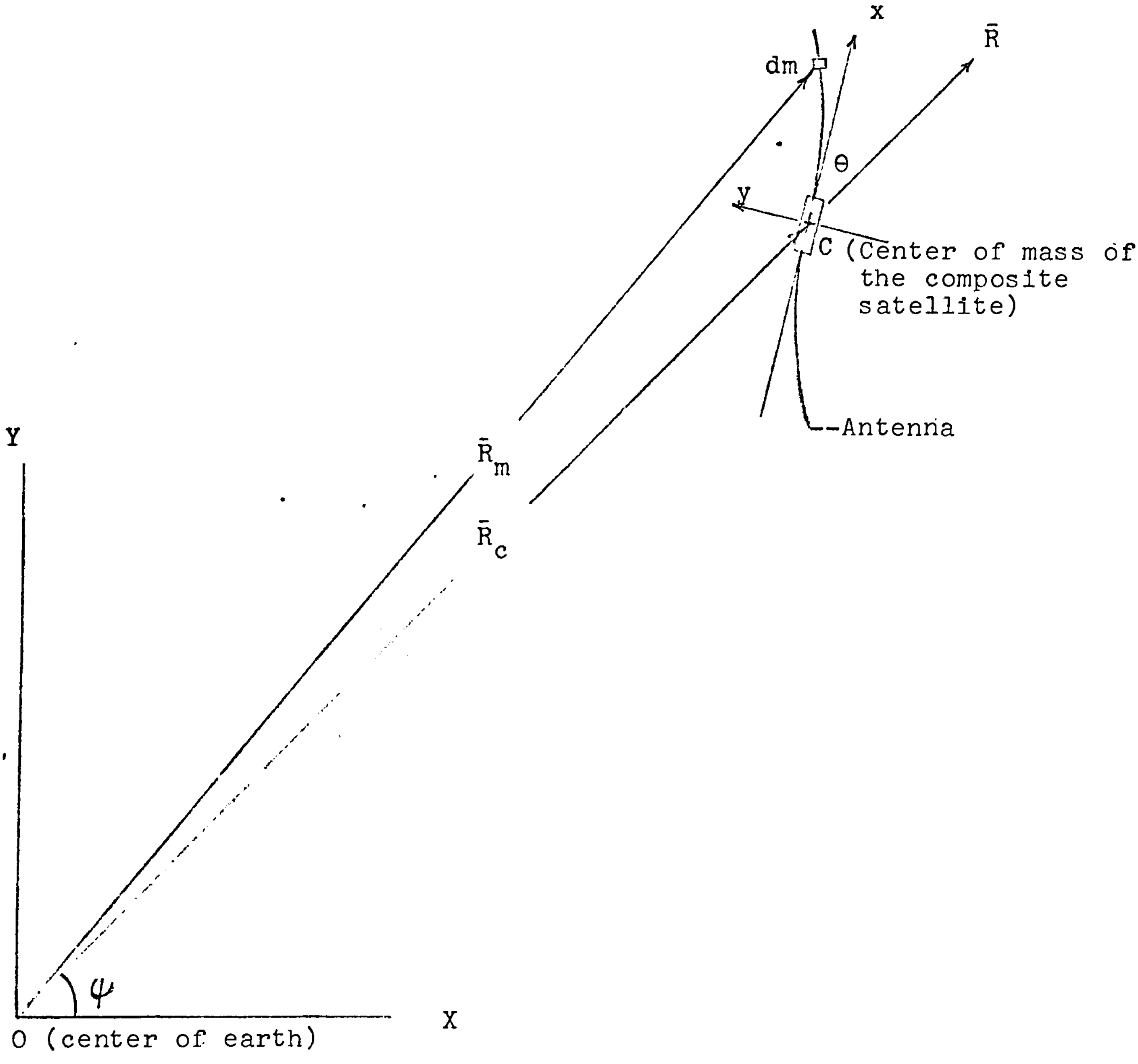

Fig. 1.1. The Two Dimensional Model.

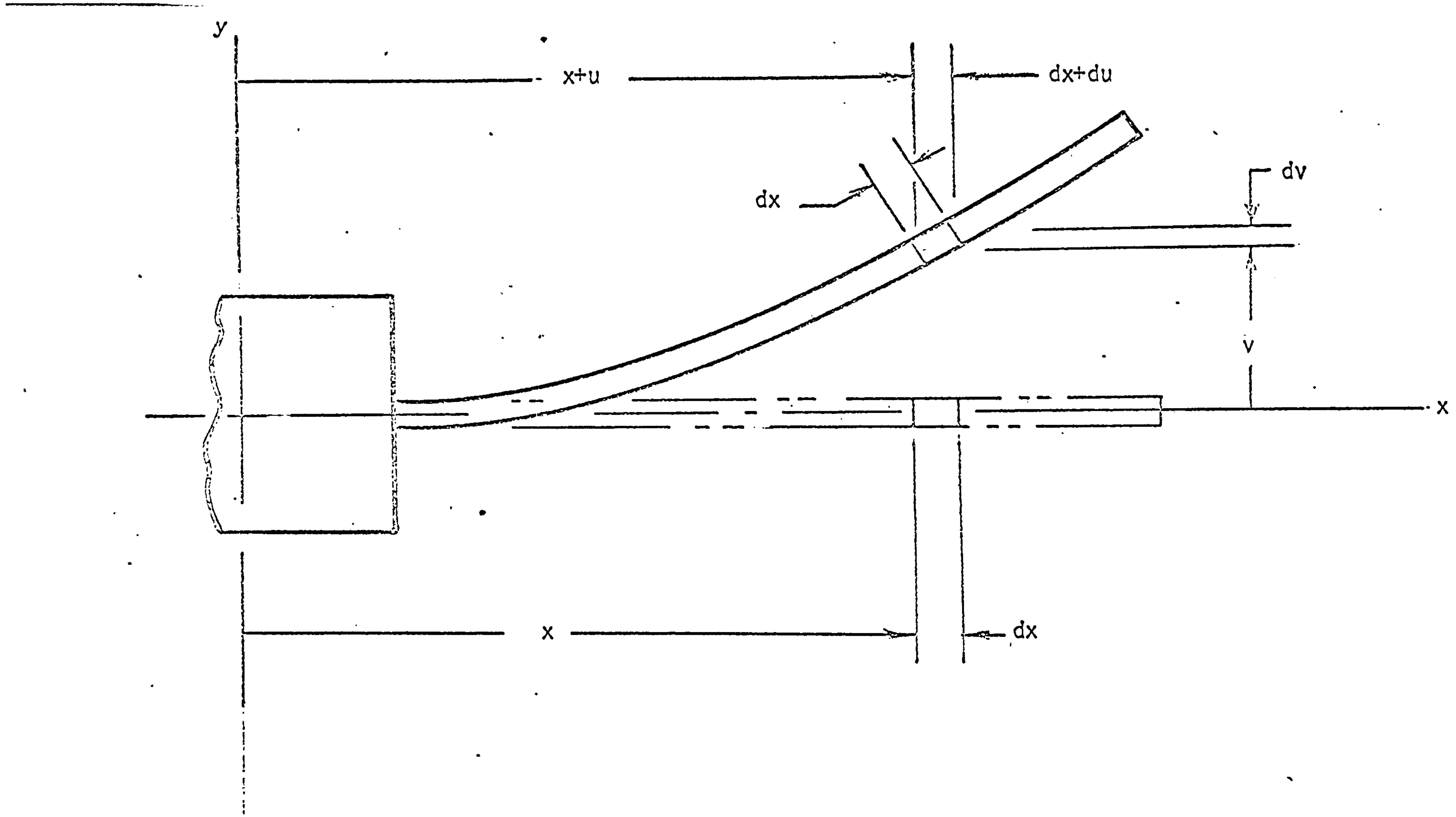

Fig. 1.2. The Deformation of an Inextensionable Beam

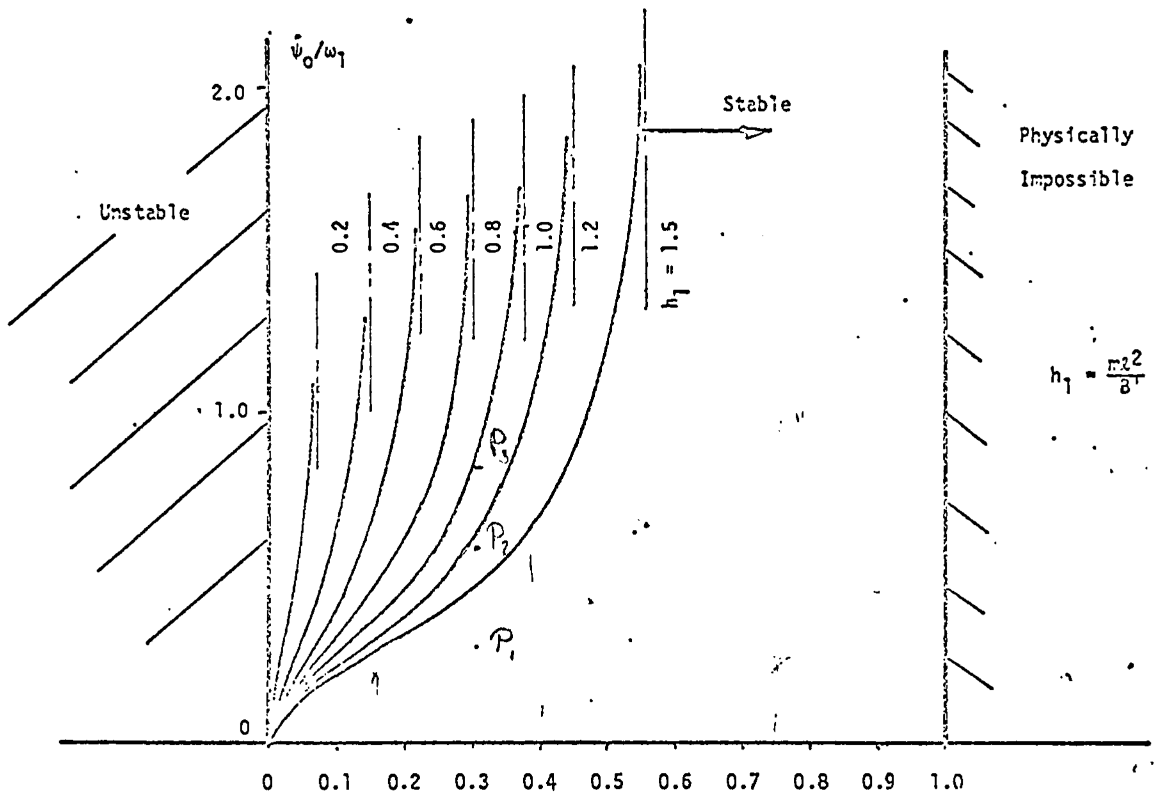

Fig. 3.1 Stability Plot for 0° Position.

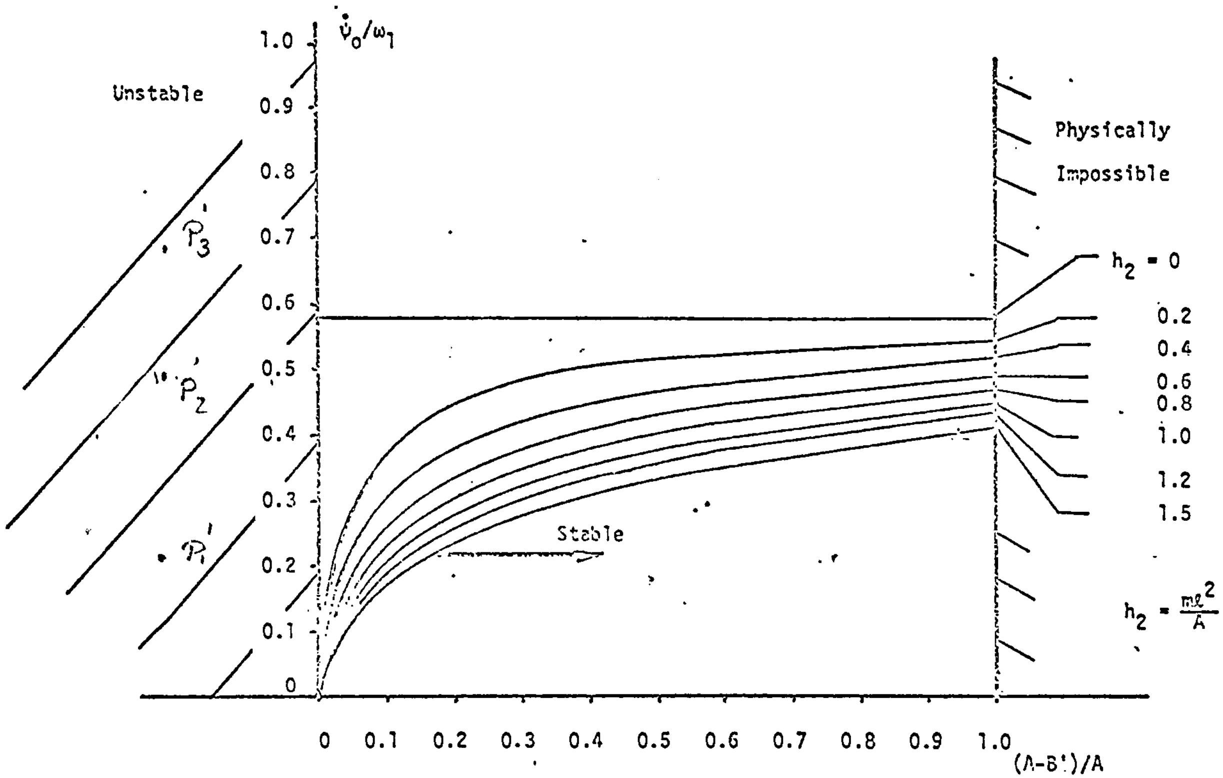

Fig. 3.2 Stability Plot for 90° position.

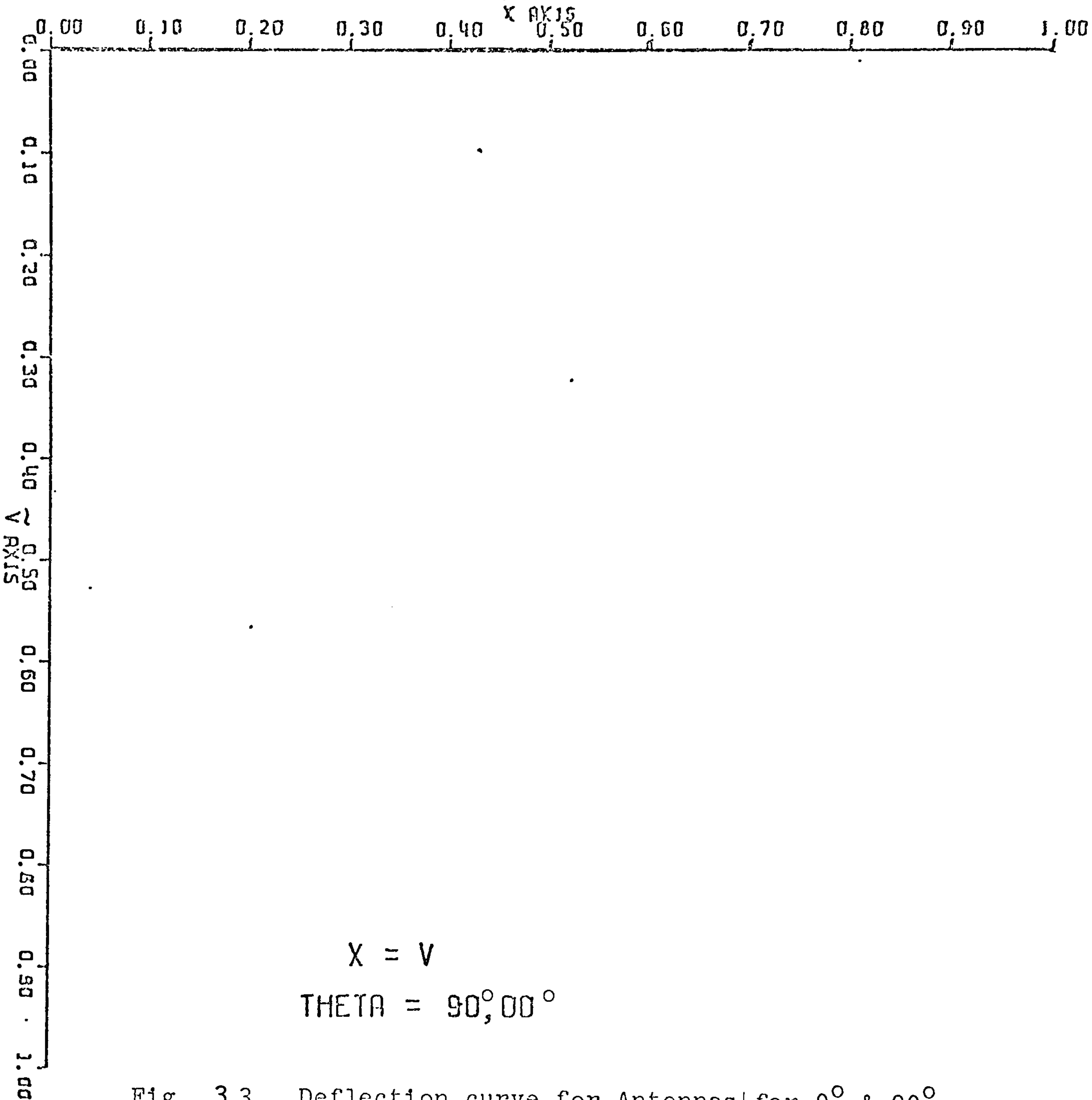

Fig. 3.3. Deflection curve for Antennas for 0° & 90° Attitude Angle.

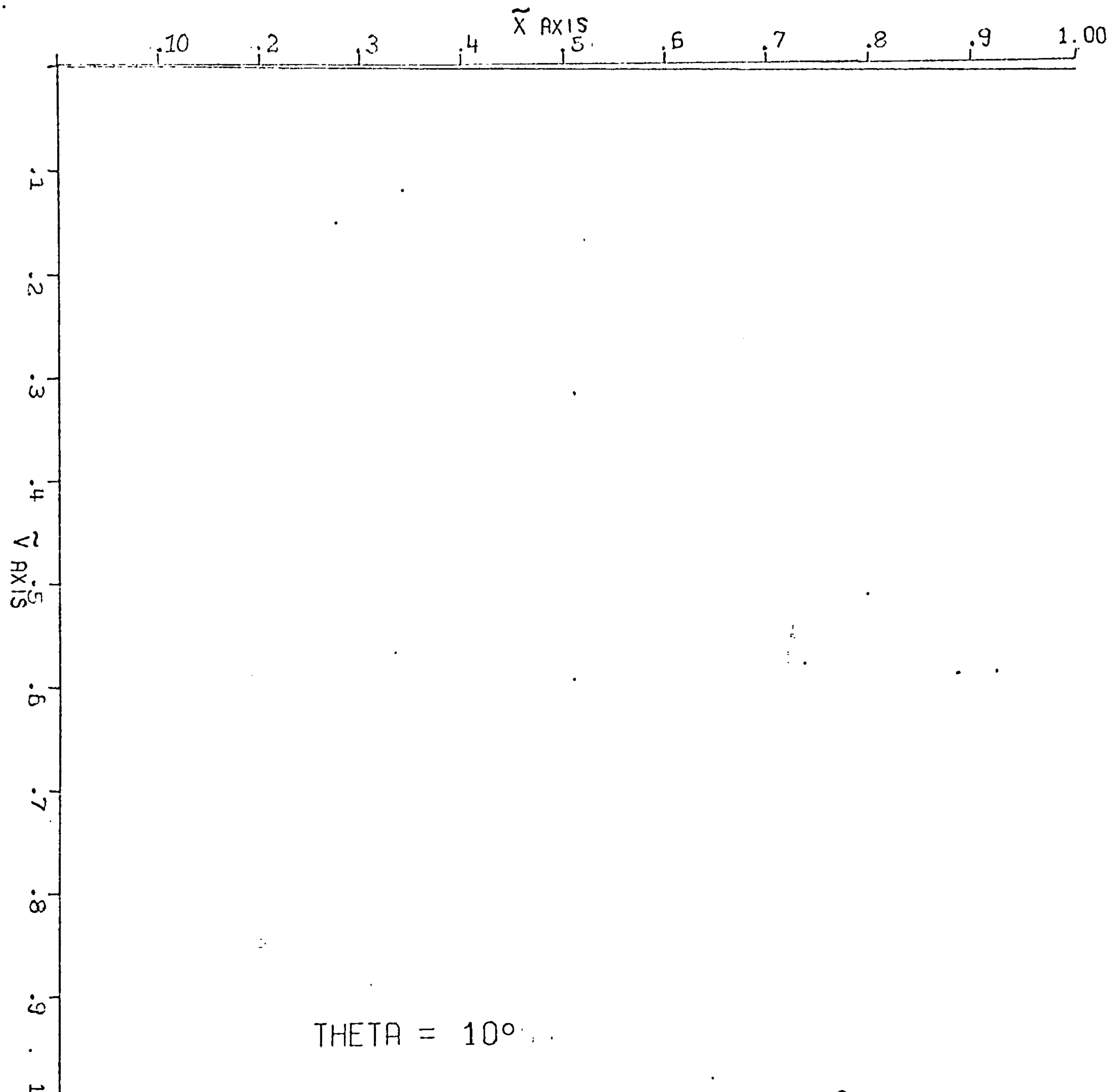

Fig. 3.4. Deflection Curve for Antennas for 20° Attitude Angle.

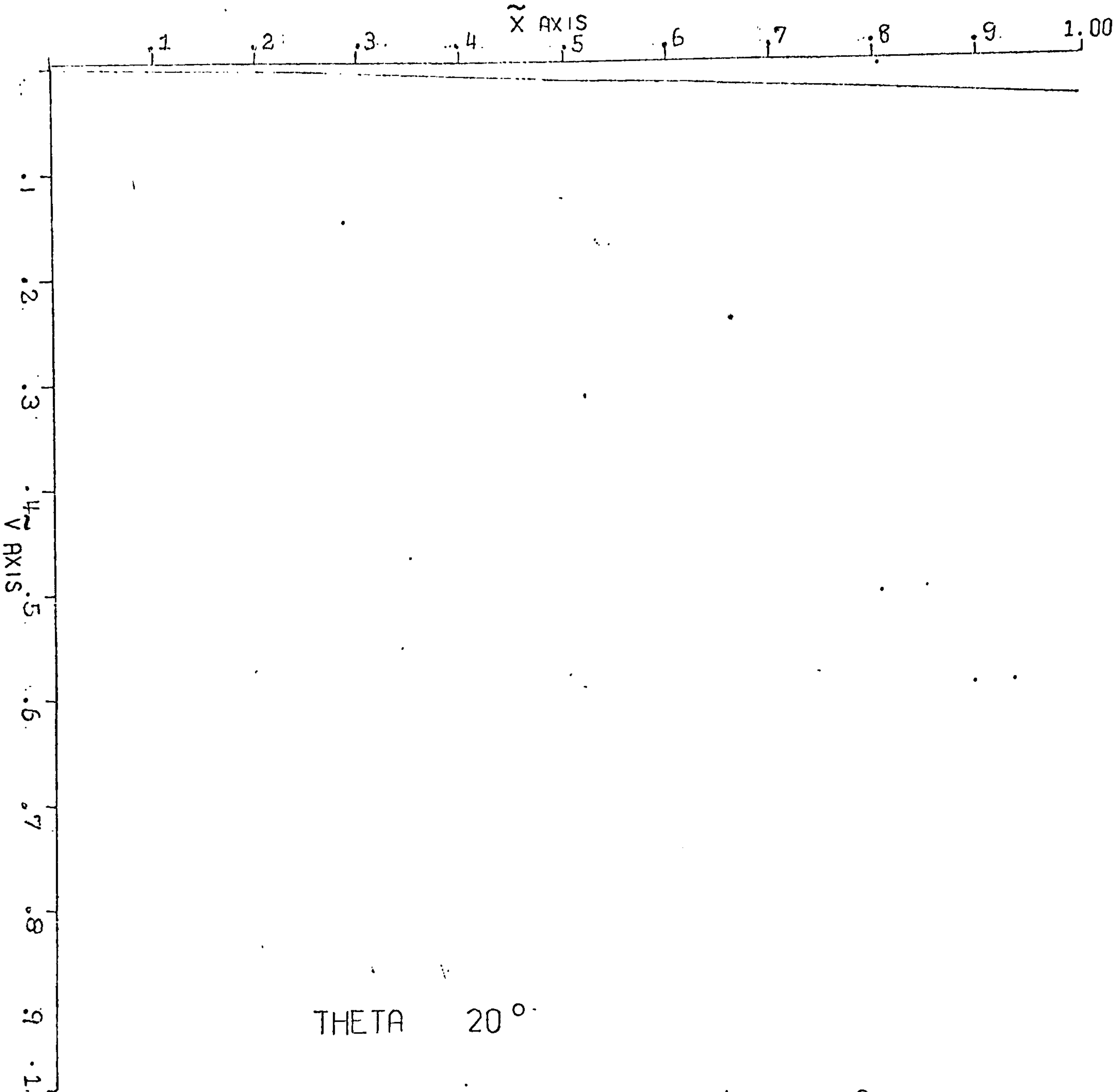

Fig. 3.5. Deflection Curve for Antennas for 20° Attitude Angle.

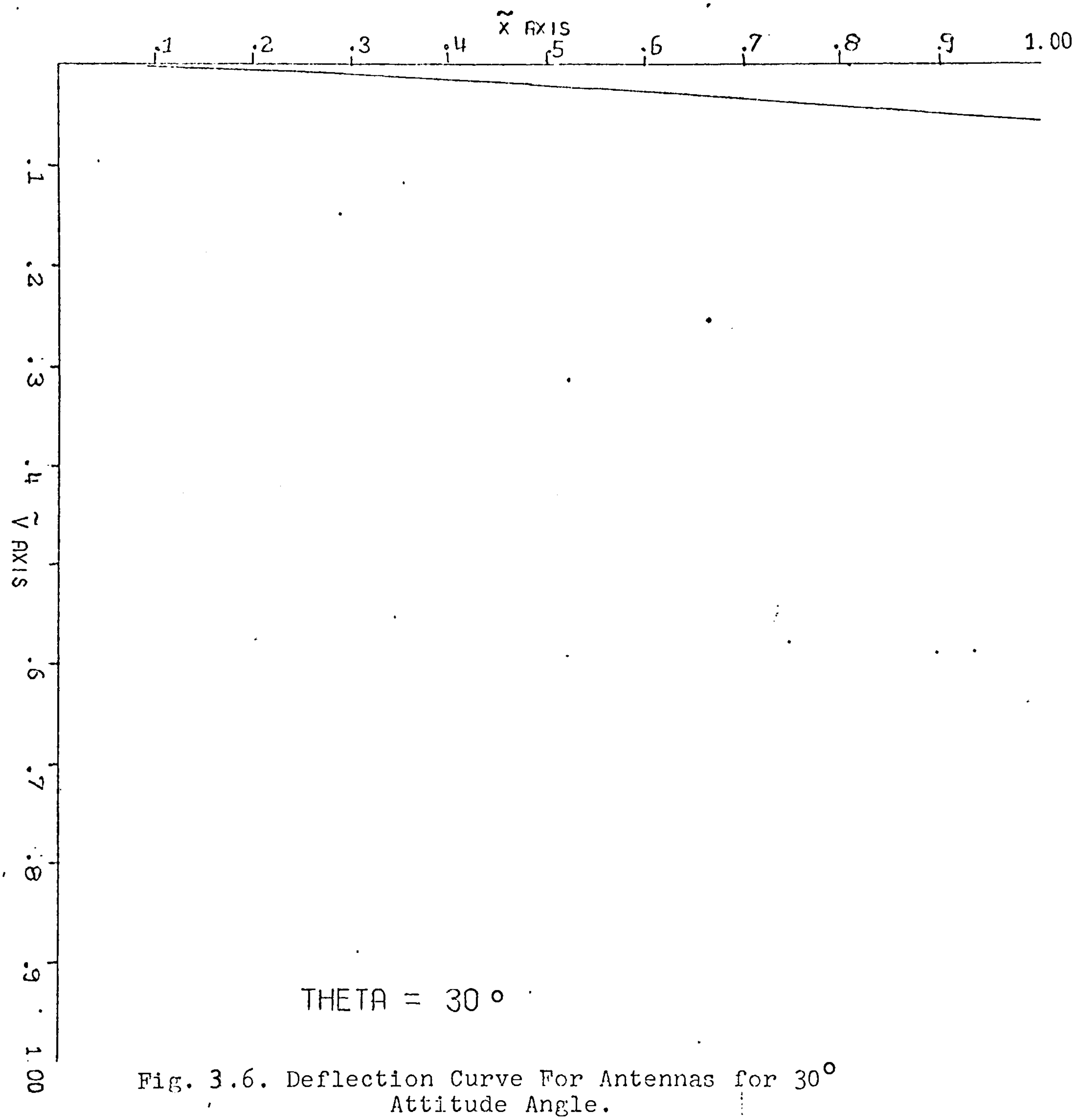

Fig. 3.6. Deflection Curve For Antennas for 30° Attitude Angle.

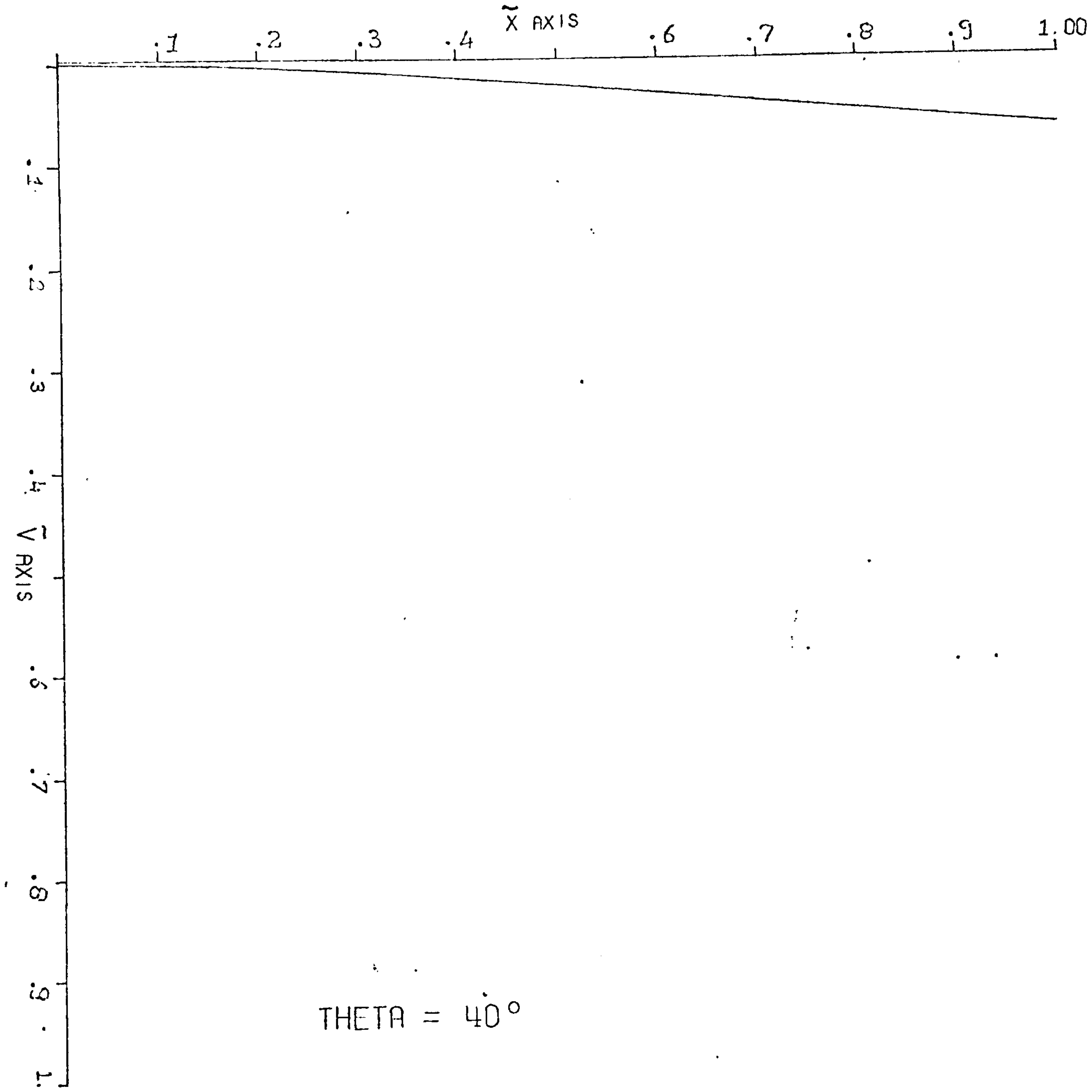

Fig. 3.7. Deflection Curve for Antennas for 40° Attitude Angle.

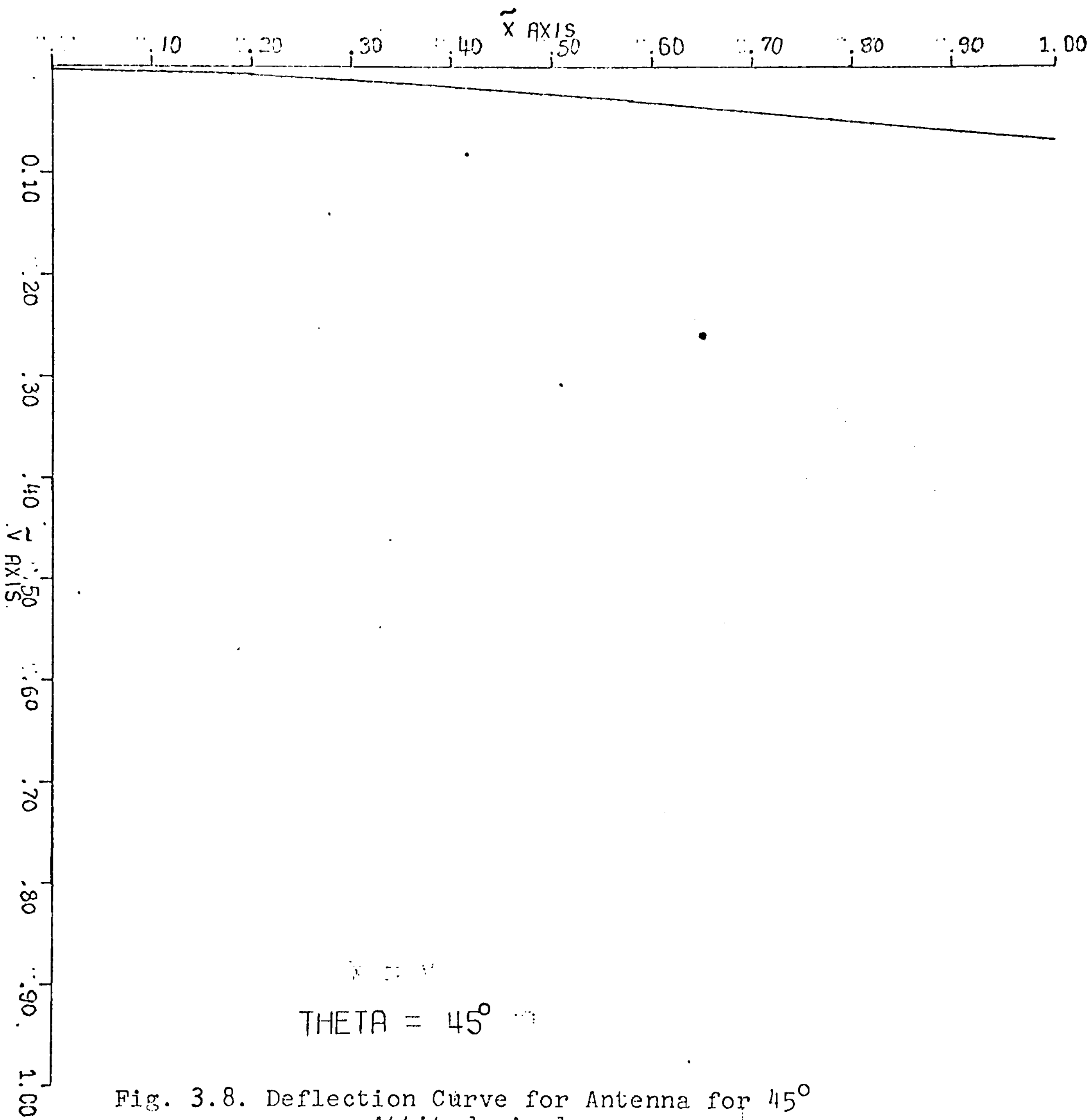

Fig. 3.8. Deflection Curve for Antenna for 45° Attitude Angle.

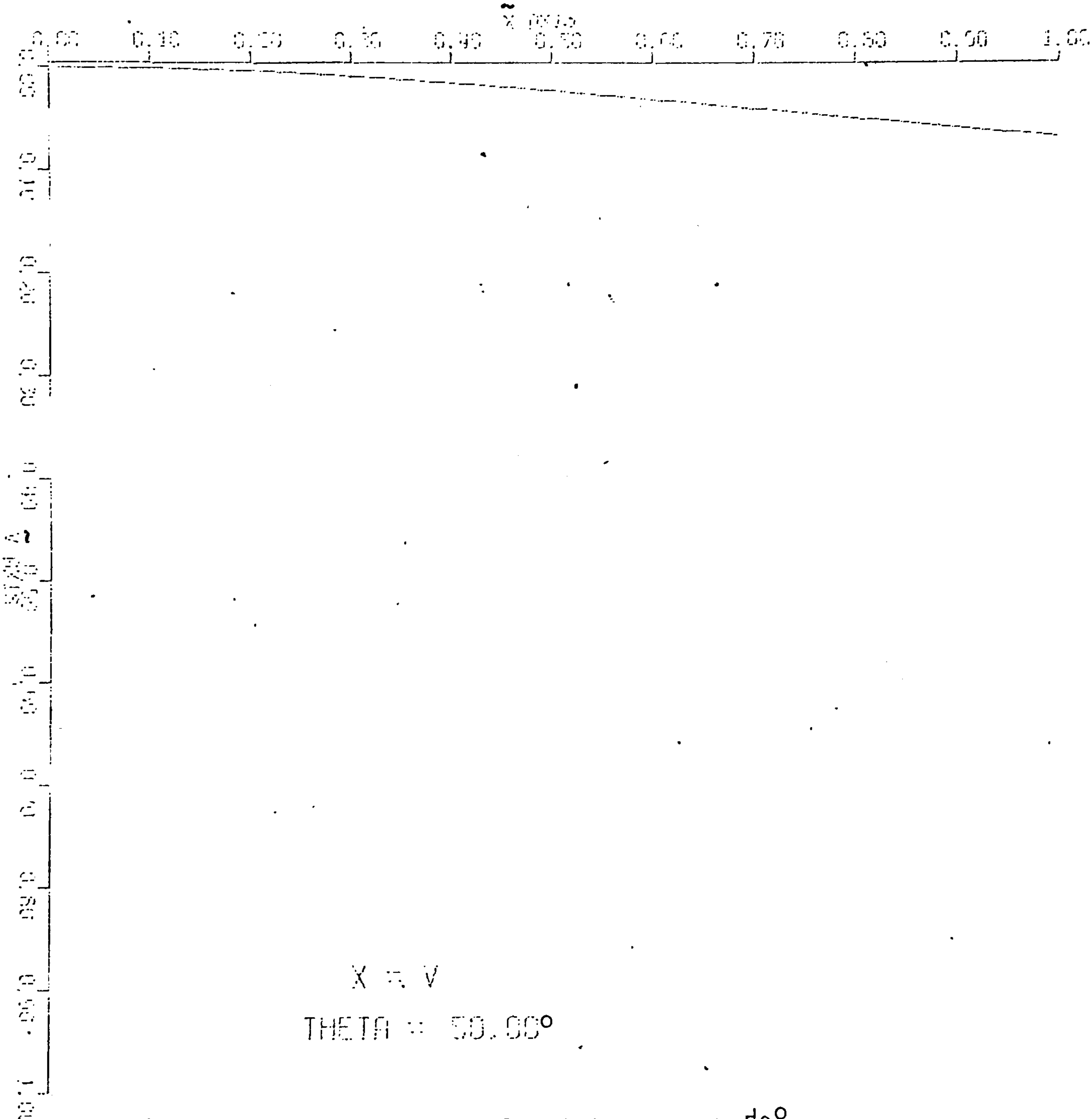

Fig. 3.9. Deflection Curve for Antennas at 50°
Attitude Angle.

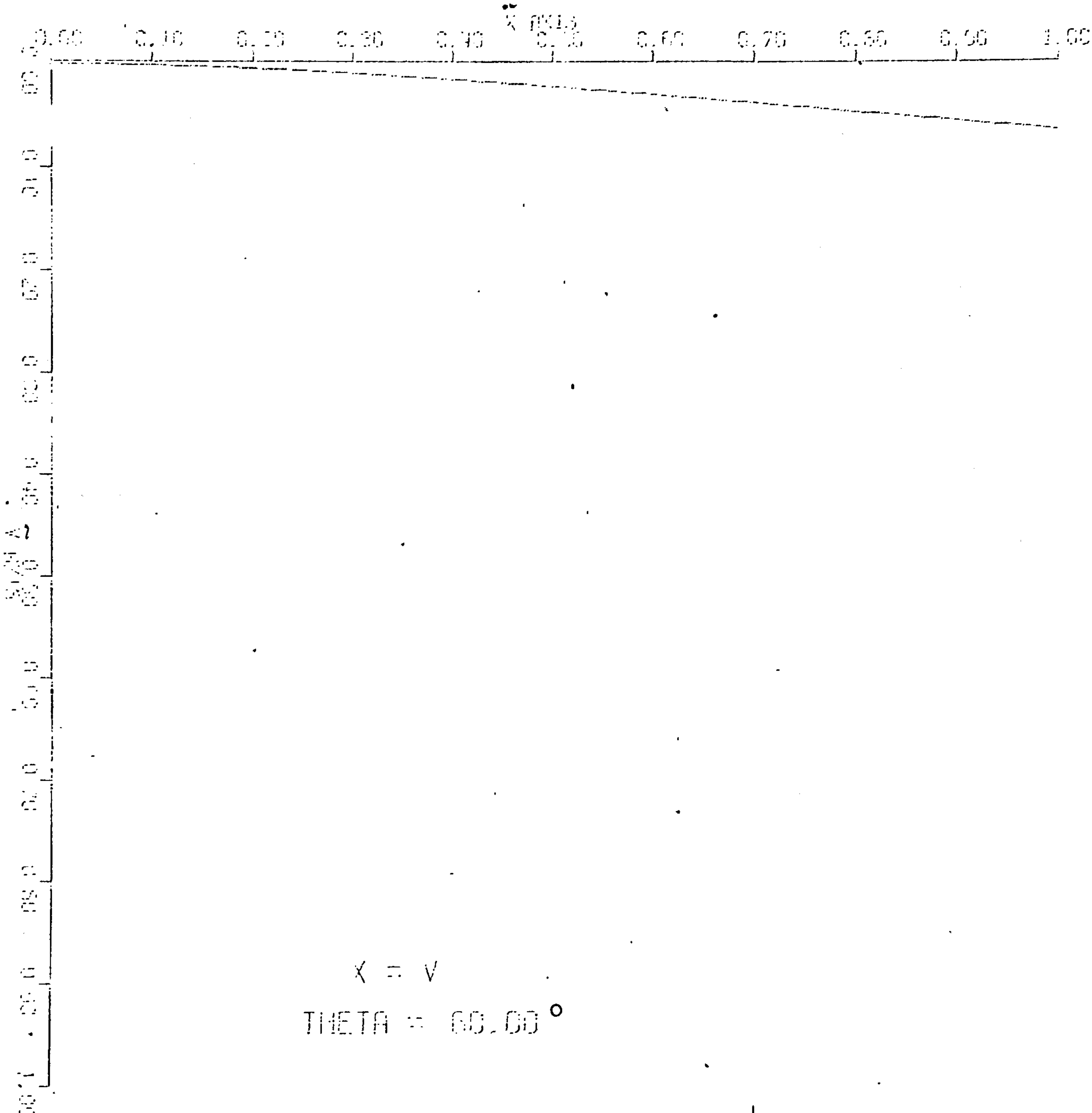

Fig. 3.10. Deflection Curve For Antenna At 60°
Attitude Angle.

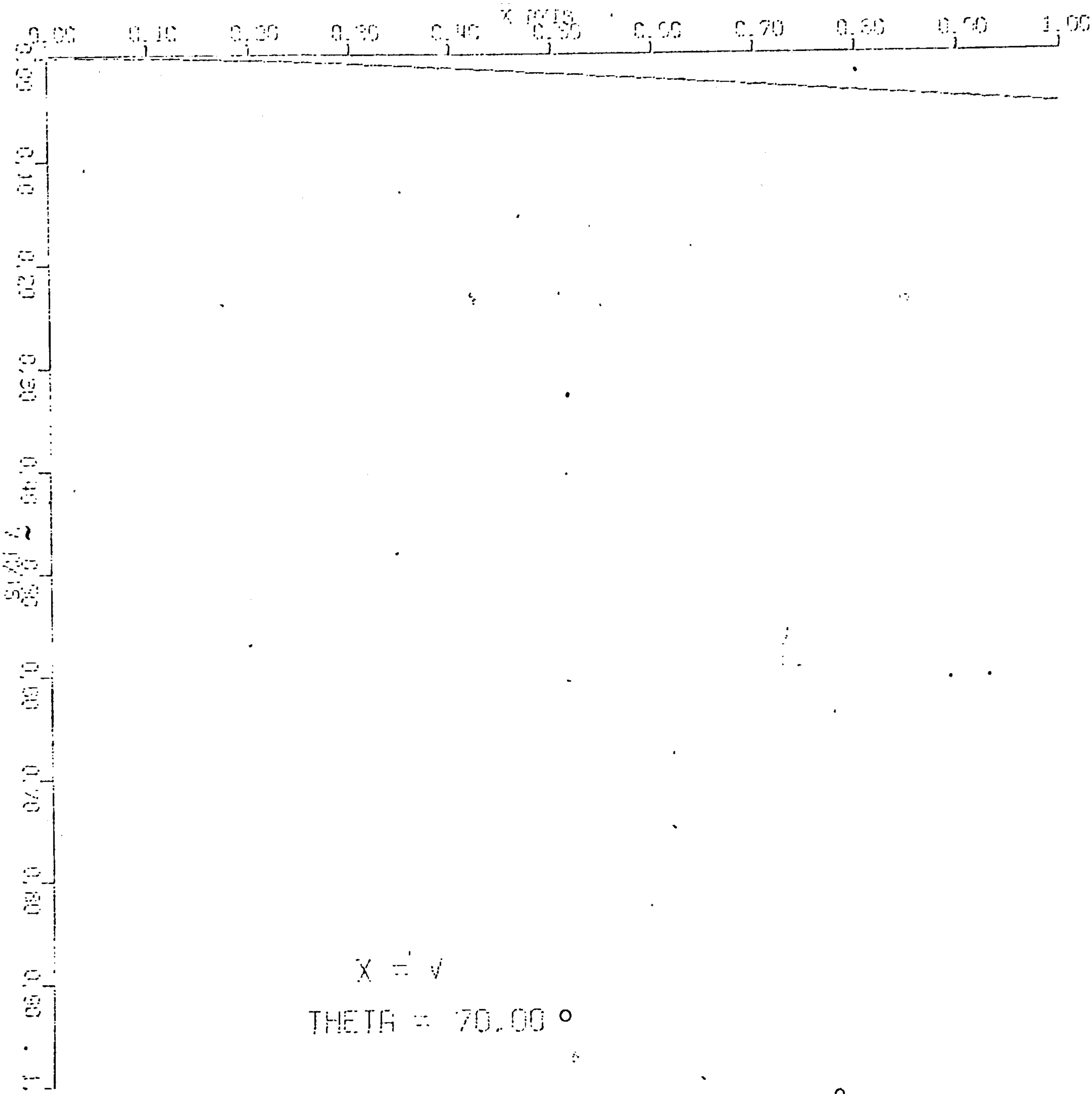

Fig. 3.11. Deflection Curve for Antennas at 70^0
Attitude Angle.

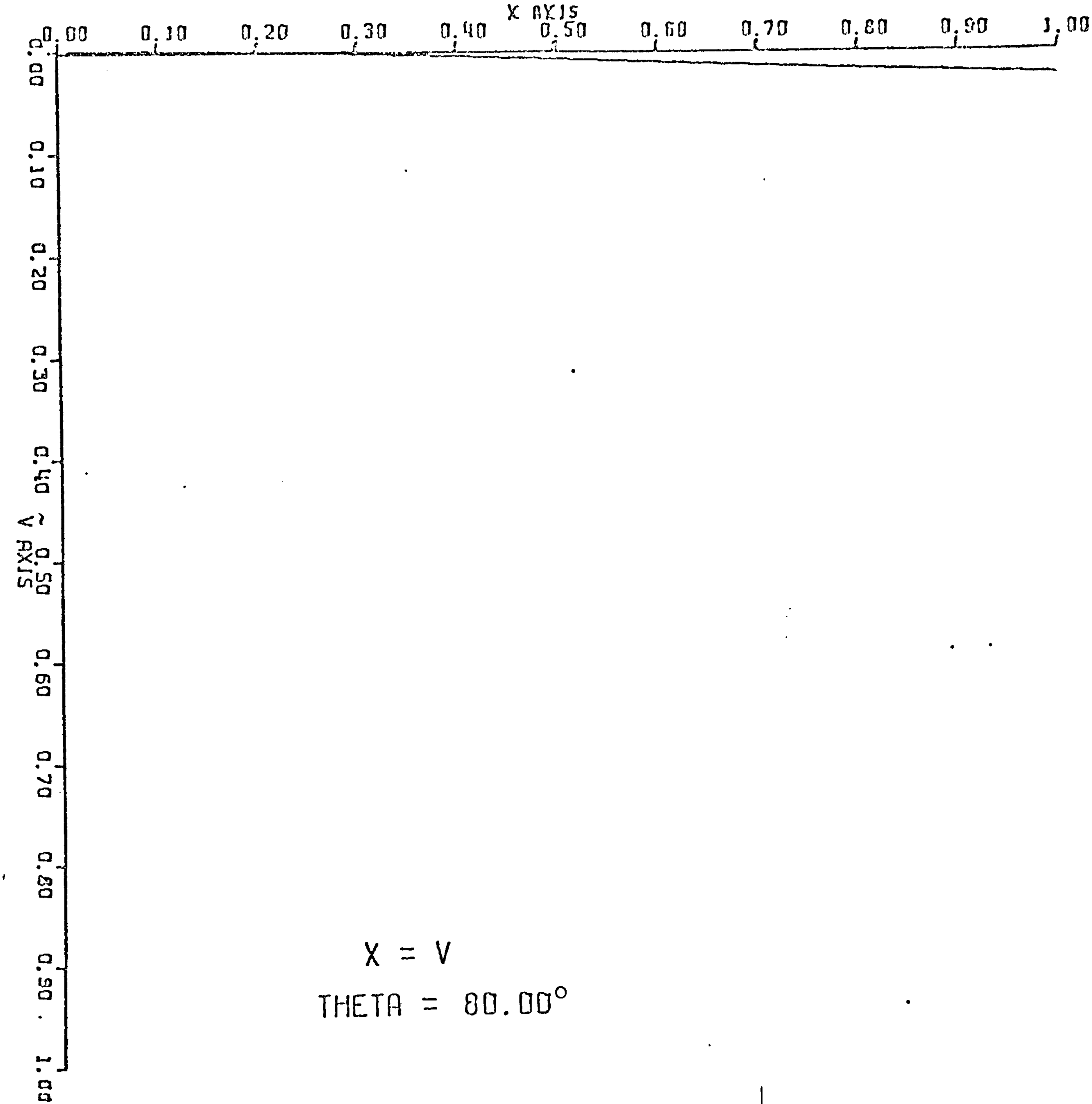

Fig. 3.12. Deflection Curve For Antennas at 80° Attitude Angle.

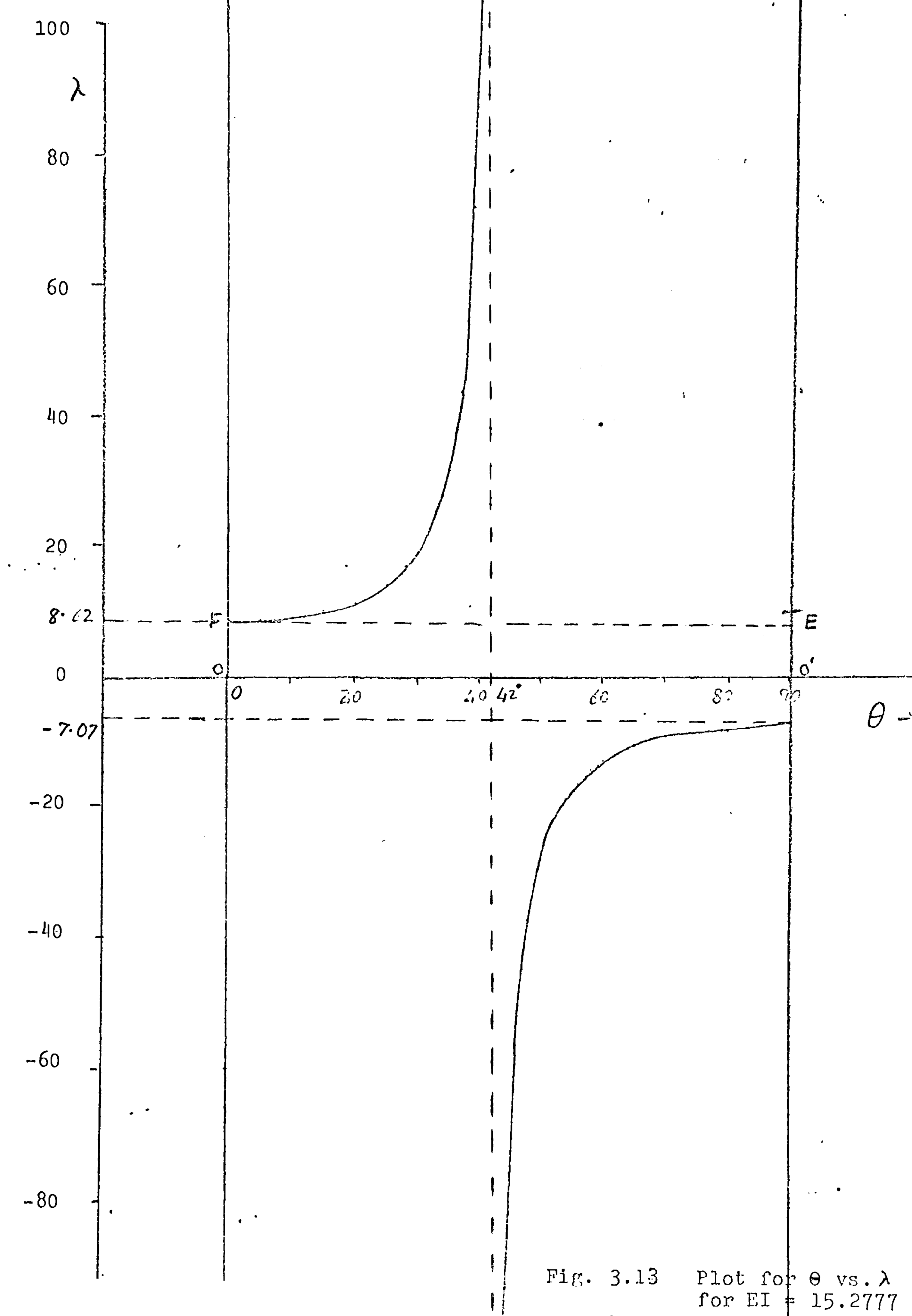

Fig. 3.13 Plot for θ vs. λ for EI = 15.2777

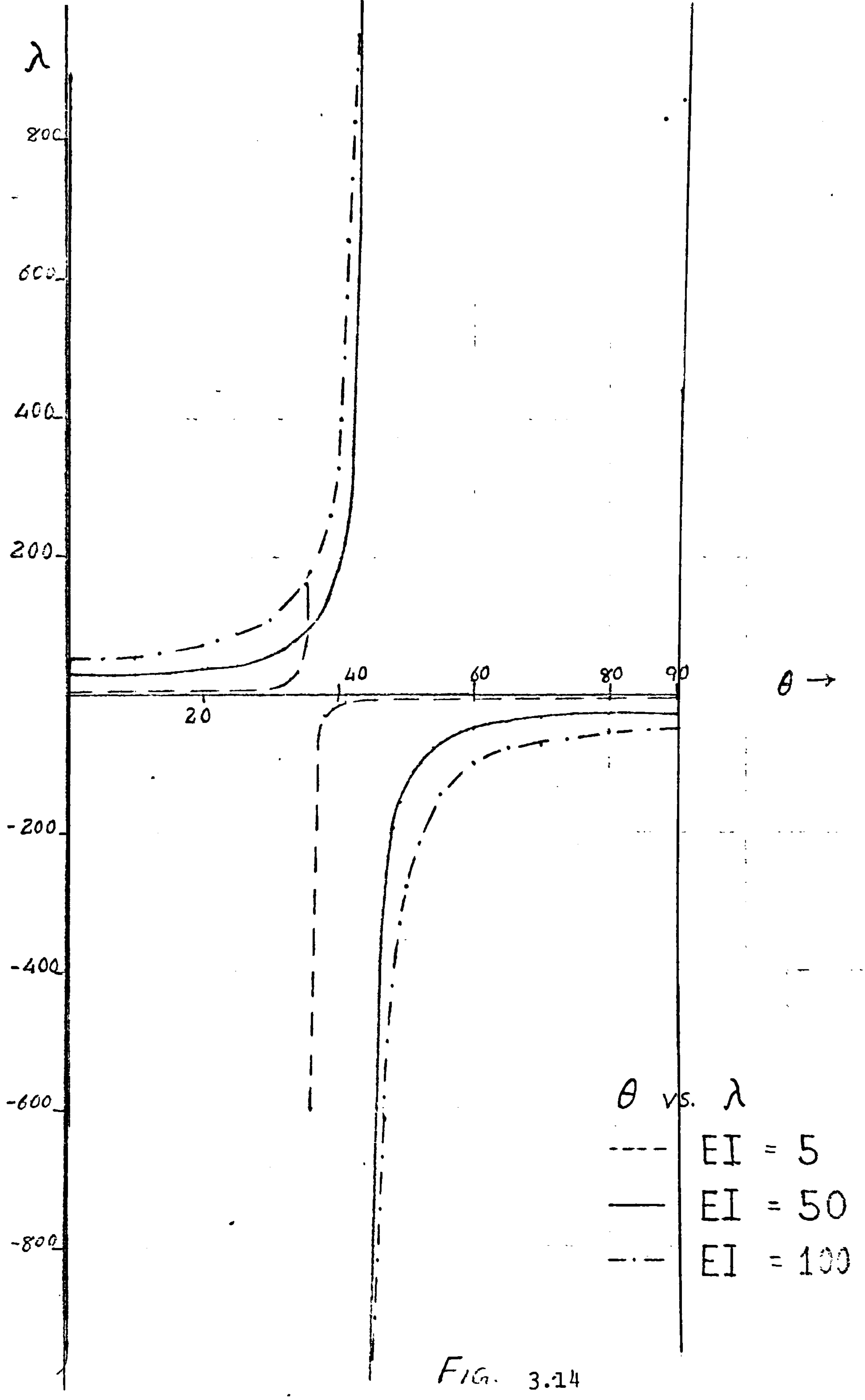
λ
800
600
400
200
-200
-400
-600
-800
20
40
60
80
90
θ →
θ vs. λ
EI = 5
EI = 50
EI = 100

FIG. 3.14

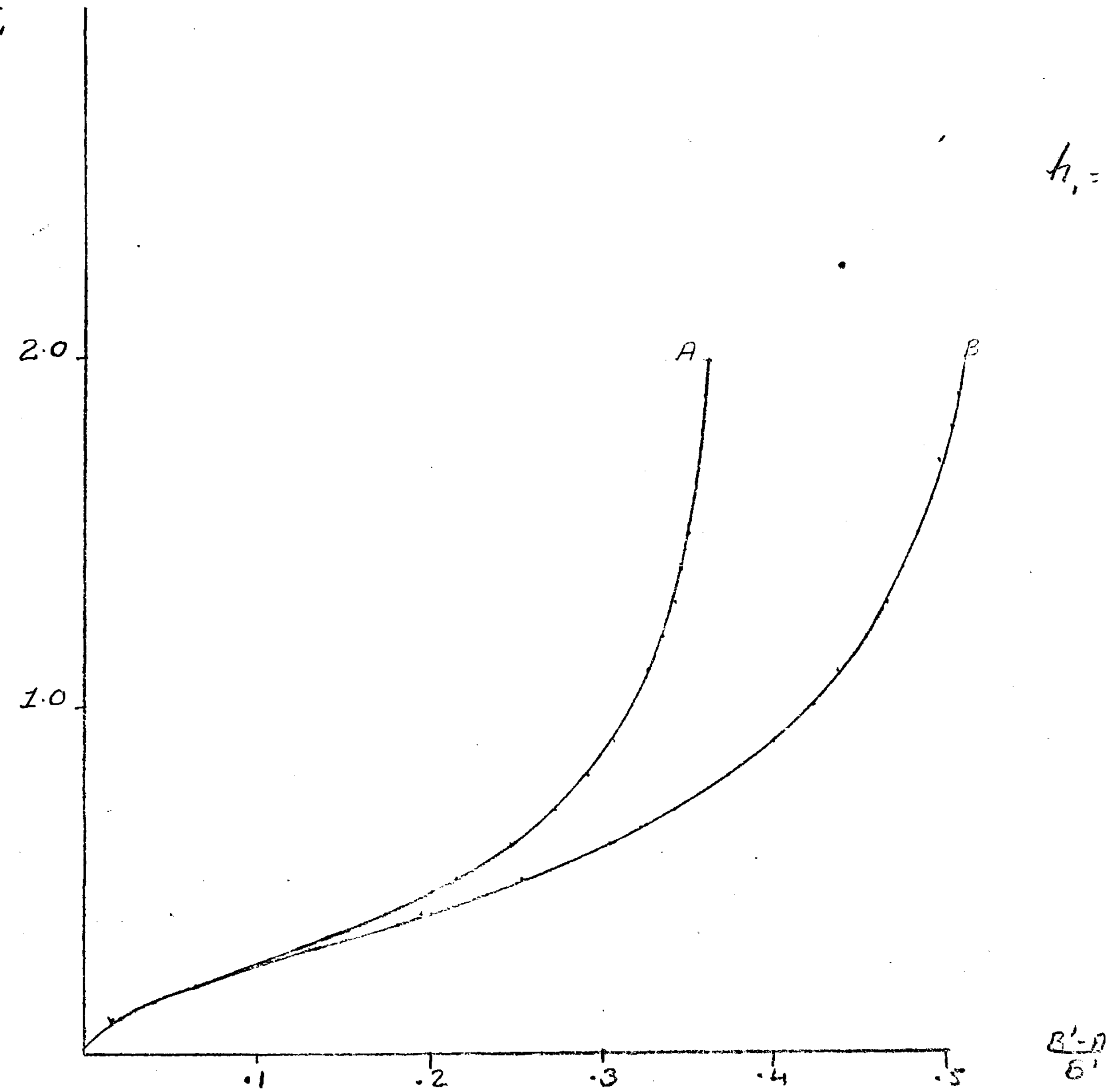

Fig. 5.1. Comparison for accuracy,