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# An Optimization-Based Approach for Vaccine Prioritization

by

Timothy J. Schmoke

A Thesis Submitted in Partial Fulfillment of the Requirements for the Degree of Master of  
Science in Industrial Engineering

Department of Industrial and Systems Engineering

Kate Gleason College of Engineering

Rochester Institute of Technology

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# **An Optimization-Based Approach for Vaccine Prioritization**

**Thesis**

**Advisor: Dr. Rubén A. Proaño**

**By: Timothy J. Schmoke (2013)**

Committee Members:

**Dr. Rubén A. Proaño**

Assistant Professor, Industrial and Systems Engineering

**Dr. Brian K. Thorn**

Associate Professor, Industrial and Systems Engineering

Approved By:

Dr. Rubén A. Proaño \_\_\_\_\_ Date \_\_\_\_\_

Dr. Brian K. Thorn \_\_\_\_\_ Date \_\_\_\_\_

## **Abstract**

An effective vaccine prioritization process is essential to prevent the many issues that currently weaken global vaccination efforts. Identifying challenges associated with vaccine development is important when considering which initiatives will provide immunization that is effective, affordable, and easy to administer. The process of establishing priorities for vaccine development is complicated, though, by the conflicting interests of multiple stakeholders involved in the vaccine market. Additionally, uncertainties exist regarding: (1) the resources and time required for vaccine development, (2) the expected benefits of development, and (3) the anticipated demand for vaccination, further complicating the prioritization process.

This study proposes a decision-support tool for prioritizing vaccine initiatives through the use of mathematical optimization models. The tool will allow a panel of decision makers to assess vaccine candidates over multiple criteria with information that is both quantitative and qualitative. This assessment will be the result of a methodology that integrates Data Envelopment Analysis and the Analytic Hierarchy Process. Ultimately, the decision-support tool will allow researchers and funding agencies to determine which vaccine initiatives should be: more effective, affordable, profitable, reliable, easier to use and store, and more suitable to the needs of multiple populations from diverse locations and having multiple logistic needs.

## **1. Introduction**

With numerous vaccine initiatives in various stages of development around the world, government funding agencies, foundations, vaccine producers, and researchers are faced with the challenge of determining which initiatives should be fostered and receive additional attention and financial support. For these organizations, a renewed focus on developing priority-setting

strategies for new vaccine development is timely and critical. This is evident in the 2010 National Vaccine Plan, a report by the U.S. government with goals for enhancing all aspects of vaccines and vaccination. In the report, strategies are listed to guide disease prevention and improve vaccination, including: “develop and implement a process for prioritizing and evaluating new vaccine targets of domestic and global public health importance” [1]. The Institute of Medicine (IOM) has also targeted vaccine prioritization as an integral part in the design of a national and global vaccine development strategy. Despite a few efforts by the IOM to address the vaccine prioritization problem, to date, no universally accepted method or model exists to guide these important decisions [2].

Prioritizing vaccine initiatives can provide a framework for organizations or stakeholders in the vaccine market to discuss investment alternatives and converge toward solutions that satisfy most parties. Additionally, allocating proper resources to a vaccine initiative can reduce the time it spends in development, allowing it to provide immunization to the public sooner. Increased interests, efforts, and collaborations related to vaccine-preventable diseases are adding to the need for vaccine prioritization. Figure 1 shows the market attractiveness of vaccine investments over the last 30 years. A renewed interest in vaccine development is evident, with a number of organizations and foundations focusing on the advancement of new vaccines.

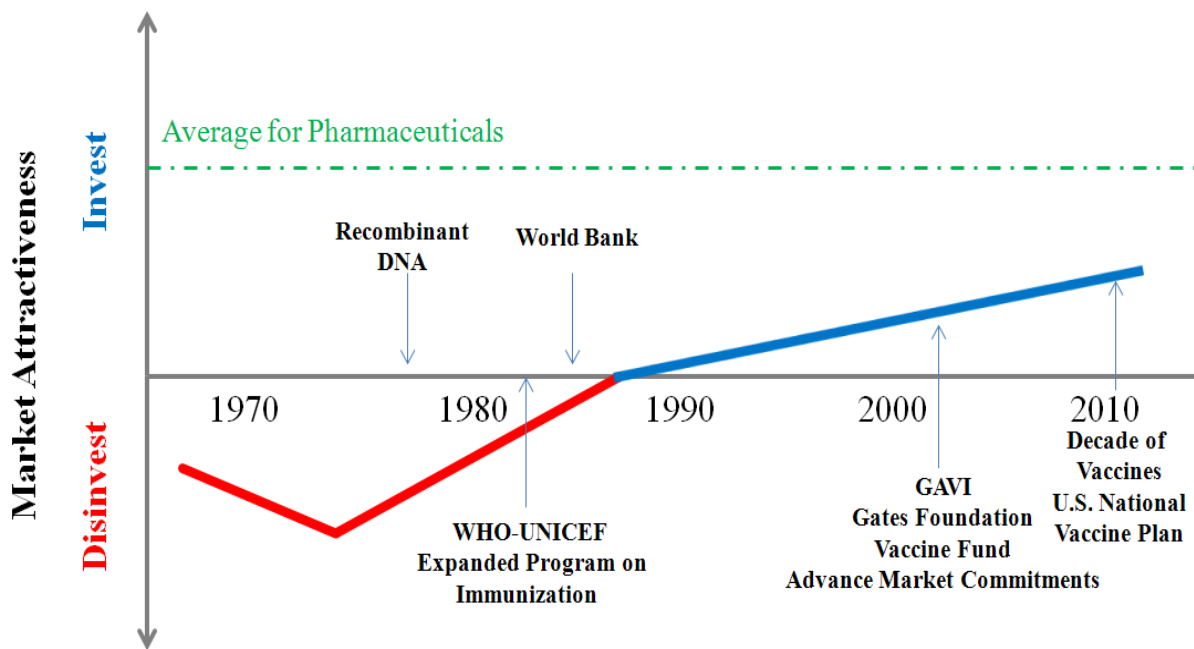


Figure 1: Historical attractiveness of vaccine investments.  
SOURCE: Institute of Medicine, 2012 [2].

For any organization choosing to invest in the development of a vaccine initiative, there are a number of factors that make the decision to select one project over another challenging. Varying public health environments, social and economic infrastructures, political conditions, and climates, for example, can have a significant impact on the effectiveness and profitability of an initiative. These circumstances can vary significantly around the world and can heavily influence a vaccine initiative's likelihood of success. Additionally, decision makers can face uncertainty regarding: the expected length of time for the vaccine to become licensed for use, the vaccine initiative's financial viability, and the logistic challenges associated with vaccine delivery. Other aspects that influence vaccine prioritization include: the magnitude of disease burden, public perception of the disease and the need for its control, whether the disease has the

potential to cause epidemics and pandemics, whether the vaccine has characteristics that are attractive for use in developing countries, and the cost-effectiveness of the vaccine [3].

Key stakeholders of the vaccine market assess the importance of vaccine criteria differently, and while organizations' internal mechanisms to set priorities are not well known or publicized, information has been gathered about the varying priorities of the public sector, private sector, and non-governmental groups. Manufacturers, for example, favor the development of vaccines that promise high returns and involve the use of currently available technologies. Non-governmental organizations such as UNICEF, GAVI, and PAHO, on the other hand, are interested in expanding immunization in developing countries and developing vaccines that are inexpensive, easy to distribute, and do not require the use of expensive cold chains. Furthermore, different governments (from developing and industrialized nations) are interested in the development of vaccines that target diseases specific to their regions.

An effective vaccine prioritization process will bring together government agencies, vaccine manufacturers, humanitarian groups, and other organizations invested in the development of vaccines, allowing them to collectively identify vaccine priorities that best represent everyone's interests or provide the least level of conflict. In this thesis, the development of an optimization based heuristic is proposed that considers the interests of multiple stakeholders to prioritize vaccine candidates that are at different developmental stages and target different diseases. Moreover, a heterogeneous group of decision makers is assumed with different interests and levels of expertise. Therefore, vaccine prioritization is addressed as a multiple criteria decision making (MCDM) problem, involving multiple decision makers who base their decisions on both quantitative and qualitative information. The next section describes previous attempts at prioritizing vaccine initiatives.

## 2. Previous Vaccine Prioritization Efforts

The Institute of Medicine, aware of the difficulty associated with vaccine prioritization, established three committees over the last 30 years to address the vaccine prioritization problem. In 1985, the Institute of Medicine (IOM) published its first report on vaccine prioritization: “New Vaccine Development: Establishing Priorities: Volume 1, Diseases of Importance in the United States” [4]. Part one of a two-part study, this report aimed to help the National Institute of Allergy and Infectious Diseases (NIAID) establish priorities for accelerated vaccine development. The committee behind the report was charged with developing a decision-making framework to prioritize vaccine candidates for the US population. In addition, the committee was asked to evaluate the model’s ability to set priorities for vaccines needed by technologically less developed nations, and modify the model to rank potential vaccines for international use. The committee’s findings relative to the international aspects of vaccine development appeared in part two of the study: “New Vaccine Development: Establishing Priorities: Volume 2, Diseases of Importance in Developing Countries” [5].

The method used by the committee to rank vaccine initiatives was based on a quantitative model in which vaccine candidates were ranked according to two criteria: (1) expected health benefits measured by the reduction of morbidity and mortality, and (2) expected net savings of health care resources. A measurement system based on infant mortality equivalents (IMEs)<sup>1</sup> was used to compare the health impacts of a disease versus the potential benefits of a vaccine. Estimates and judgments by experts were also used when information was incomplete. The

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<sup>1</sup> The number of acute morbidity days or chronic cases considered to be equal in undesirability to an infant death.



committee adopted a flexible format so that new candidate vaccines could be assessed similarly or current candidate vaccines could be reassessed with new data.

Since the time the report was published, analytical techniques have advanced and other metrics have proven to be better measures of health valuation. In addition, the epidemiological data used to compare diseases was variable in quality and in some cases, completely absent. This created a serious impediment to the development of a comprehensive prioritization scheme, leading the IOM to create a second committee in 2000 to address the same problem.

The 2000 report, “Vaccines for the 21st Century” [6], used an efficiency measure for deriving its priorities based on the incremental cost per incremental quality adjusted life year gained by vaccination (\$/QALY). Using this measure, initiatives were grouped into one of four categories. The highest priority, Level I, was designated for vaccine programs projected to save money and increase the number of QALYs. Vaccine programs that did not save money were grouped into the remaining three categories based on the efficiency of the investment. Level II included candidates whose \$/QALY was less than \$10,000, Level III included candidates for which \$/QALY ranged between \$10,000 and \$100,000, and Level IV was for candidates whose \$/QALY was greater than \$100,000.

Like the 1985-1986 IOM model, one weakness of the 2000 IOM model was that it only considered a single attribute to propose vaccine priorities. Additionally, the 2000 model did not provide a method for choosing between vaccine initiatives with equal QALYs when one targeted a minor form of illness affecting a large portion of the population, and the other targeted a disease with a small number of cases but high mortality and the potential for large social disruption. An additional shortcoming of the 2000 model was that it only focused on vaccine candidates for the US market.

The IOM's most recent attempt at addressing the vaccine prioritization problem is currently in development and is the focus of the report: "Ranking Vaccines: A Prioritization Framework: Phase I: Demonstration of Concept and a Software Blueprint" [2]. For this recent modeling strategy, the committee limited its scope to models that consider multiple attributes; the committee recognized that the narrow range of attributes used to prioritize vaccines in the previous IOM studies significantly limited their value and applications. The three multi-attribute approaches that the committee reviewed were: mathematical programming, multi-attribute utility theory, and the analytic hierarchy process. These approaches were evaluated against four criteria: transparency, axiomatic foundation, priority scaling, and sensitivity analysis. Ultimately, multi-attribute utility theory was chosen for the foundation of the committee's work because it provides weights and data that are available for all users to see and use, independence from irrelevant alternatives (IIA), scaling that can be used for an ordinal ranking, and the ability to conduct sensitivity analyses on results [2].

The current committee's model improves upon the previous two by including multiple attributes that address the varied interests of the public sector, private sector, and non-governmental groups. However, as mentioned in the "2010 National Vaccine Plan" [1], collaboration between stakeholders is an important part of establishing and understanding priorities for development. This is a major weakness of the new model, which rather than promote collaboration, allows users to create priorities that result solely from the application of their own criteria or interests. The fact that the model only considers a single decision maker's preferences ignores the idea that vaccine prioritization should reflect the perspectives of multiple stakeholders. Therefore, despite the IOM's efforts, there is still a need for a prioritization model that considers the individual interests of stakeholders but also encourages collaboration between

them. The prioritization model, in accompaniment with stakeholder input, should be able to generate a vaccine ranking that best addresses the conflicting interests of all stakeholders. This is the focus of the model proposed in the following section.

### **3. Methodology**

#### **3.1 Overview**

The overall goal of this project is to develop a mathematical optimization model that derives a vaccine priority ranking using quantitative and qualitative criteria and the preferences of multiple stakeholders. The model is designed to complement the IOM's current prioritization effort. The proposed algorithmic methodology integrates Data Envelopment Analysis (DEA) with the Analytic Hierarchy Process (AHP). This method has each individual in a panel of decision makers compare: the qualifications of their peers for assessing vaccine initiatives, how well each criteria satisfies the goal, and how well each initiative performs with respect to each criteria. Pairwise comparison matrices for the decision makers, criteria of evaluation, and initiatives with respect to each criterion are derived using the AHP framework. DEA is then used to calculate the optimal weight of each decision maker, the optimal weight of each criteria of evaluation, and the optimal weight of each initiative with respect to each criteria of evaluation. The optimal weights are then used to calculate the relative priority of each initiative.

#### **3.2 AHP**

Developed by Saaty in the 1970s, the Analytic Hierarchy Process (AHP) helps decision makers identify alternatives that best suit their interests [7]. The ranking/selection of a set of alternatives is done with respect to an overall goal, which is broken down into a set of criteria. Figure 1 shows the basic hierarchical structure of an AHP problem.

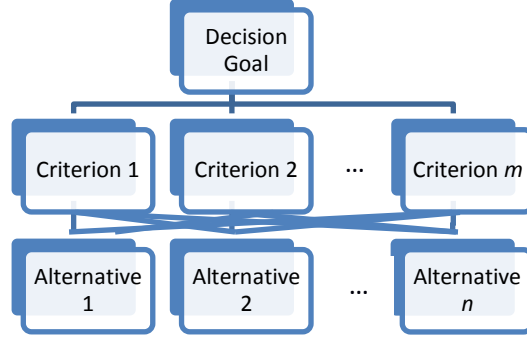


Figure 2: AHP hierarchical structure

The AHP provides a comprehensive and rational framework for representing and quantifying the elements of decision problems, relating those elements to overall goals, and evaluating alternative solutions. Applying the AHP to a decision problem involves four steps: 1) structuring the problem into a hierarchical model, 2) comparing elements of the problem to one another to generate pairwise comparison matrices, 3) calculating the weight or priority of each element in the hierarchy, and 4) aggregating the weights across various levels to obtain the final weights of the alternatives. (See [8].)

Pairwise comparison matrices are an integral part of the AHP, indicating how much more important one objective might be than another. Let

$$A = (a_{ij})_{n \times n} = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix} \quad (1)$$

be a pairwise comparison matrix for a single decision maker. The entry in row  $i$  and column  $j$  of  $A$  indicates how much more important objective  $i$  is than objective  $j$ , with  $a_{ii} = 1$  for all  $i$  and  $a_{ji} = 1/a_{ij}$  for  $j \neq i$ . How to calculate priorities from a pairwise comparison matrix has been the focus of several studies, with techniques aimed at deriving a priority vector including: the eigenvector method [9], the weighted least-square method (WLSM) [10], the logarithmic least square method (LLSM) [11], and the fuzzy programming method [12].

### **3.3 DEAHP**

In 2006, Ramanathan [13] developed a method that uses data envelopment analysis (DEA) for generating local weights of alternatives from pairwise comparison matrices in the AHP. DEA is a linear programming based approach used to determine the productive efficiency of a system or decision-making-unit (DMU) (e.g., a university, hospital, or restaurant) by comparing how well the DMU converts inputs into outputs. The DMU that produces the largest amount of outputs by consuming the least amount of inputs is considered to have an efficiency score of one. The efficiencies of the other DMUs are obtained relative to the efficient DMU, and are assigned efficiency scores between zero and one [8]. Ramanathan's method, referred to as DEAHP, considers each criterion or alternative in a pairwise comparison matrix as a DMU. The row elements of the pairwise comparison matrix are viewed as the outputs of the DMUs, and a dummy input with a value of one is used to build a model that calculates the efficiency score for each DMU. The efficiency scores are then used as the local priorities of the DMUs, whether they are decision criteria or alternatives.

The DEAHP method succeeds in producing true weights for perfectly consistent pairwise comparison matrices, but was criticized by Wang et al [14] for not being able to produce rational weights for inconsistent pairwise comparison matrices. In addition, Wang et al [14] proved that the DEAHP may also produce illogical results for pairwise comparison matrices with satisfactory consistency. As a result, Wang and Chin [15] introduced a new DEA methodology in 2009.

### **3.4 DEA Methodology for Priority Determination in the Group AHP**

Wang and Chin's DEA methodology [15] is aimed at deriving the best local priority vector from a pairwise comparison matrix or, for the group AHP, the best local priority vectors

from a group of pairwise comparison matrices, regardless of whether they are perfectly consistent or inconsistent.

Let

$$A^{(k)} = (a_{ij})^{(k)}_{n \times n} = \begin{bmatrix} a_{11}^{(k)} & \cdots & a_{1n}^{(k)} \\ \vdots & \ddots & \vdots \\ a_{n1}^{(k)} & \cdots & a_{nn}^{(k)} \end{bmatrix} \quad (2)$$

be a pairwise comparison matrix provided by the  $k$ th decision maker ( $DM_k$ ) ( $k = 1, \dots, m$ ), where  $(a_{ij})^{(k)}$  is the  $k$ th decision maker's assessment of how important objective  $i$  is relative to objective  $j$ , and  $m$  is the number of decision makers. In addition, let  $h_k > 0$  be the decision maker's relative importance weight satisfying  $\sum_{k=1}^m h_k = 1$ , and  $z_1, \dots, z_n$  be the decision variables. The following model is solved for each  $w_i$  ( $i = 1, \dots, n$ ) to obtain the best relative local priorities of the  $n$  criteria or alternatives under group decision making. Subscript zero represents the decision criterion or alternative under evaluation, namely  $DMU_0$ :

$$\begin{aligned} & \text{Maximize } w_0 = \sum_{j=1}^n (\sum_{k=1}^m h_k a_{0j}^{(k)}) z_j, \\ & \text{Subject to } \begin{cases} \sum_{j=1}^n (\sum_{k=1}^m \sum_{l=1}^n h_k a_{lj}^{(k)}) z_j = 1, \\ \sum_{j=1}^n (\sum_{k=1}^m h_k a_{ij}^{(k)}) z_j \geq n z_i, \quad i = 1, \dots, n, \\ z_j \geq 0, \quad j = 1, \dots, n. \end{cases} \end{aligned} \quad (3)$$

After the best local priorities for both criteria and alternatives have been derived by the DEA methodology, the final weight of each decision alternative can be computed using the simple additive weighting (SAW) method [9]. Let  $w_1, \dots, w_m$  be the best local priorities of the  $m$  decision criteria and  $w_{1j}, \dots, w_{nj}$  be the best local priorities of the  $n$  decision alternatives with respect to the  $j$ th criterion ( $j = 1, \dots, m$ ). Equation (4) can be used to calculate the final weight of each decision alternative.

$$\text{Final weight of Alternative } A_n = \sum_{j=1}^m w_{nj}w_j \quad (4)$$

The above methodology is effective in assessing the value of different alternatives, given the assignment of the decision maker weights is correct. Decision maker weights are a) determined by an outside decision maker, or b) agreed upon by the decision makers involved in the decision making process. For certain group decision making problems, though, there might not be any individuals outside of the problem who are qualified to assign decision maker weights, or if a group decision making problem is highly classified, having an outside decision maker assign decision maker weights is not an option. When decision makers must work together to identify their weights, they must discuss their qualifications for making the decision with each other. This can be an impractical and even inconvenient conversation to have, especially when members of the group have to identify who among them is least qualified to address the problem. A method for calculating decision maker weights using the members of the group in an anonymous manner is essential to preserving the integrity of the group decision making process.

We propose to adapt the DEA methodology proposed by Wang and Chin [15] to address the vaccine prioritization problem. To do this, we will expand Wang and Chin's model for group decision making to calculate not only the local priorities of the criteria and alternatives, but also the weights of the decision makers, whose inputs will be used to provide feedback about stakeholder preferences and priorities. The following section reviews the proposed methodology.

### **3.5 Comprehensive DEA Methodology for Priority Determination in the Group AHP**

Consider a problem where  $d$  decision makers must establish the relative priority of  $p$  projects or initiatives, while considering their performance over  $c$  different attributes or criteria.

Let

$$R^{(k)} = (r_{ij})^{(k)}_{d \times d} = \begin{bmatrix} r_{11}^{(k)} & \cdots & r_{1d}^{(k)} \\ \vdots & \ddots & \vdots \\ r_{d1}^{(k)} & \cdots & r_{dd}^{(k)} \end{bmatrix} \quad (5)$$

be a pairwise comparison matrix provided by the  $k$ th decision maker ( $x_k$ ) ( $k = 1, \dots, d$ ), where  $(r_{ij})^{(k)}$  is the  $k$ th decision maker's assessment of how qualified decision maker  $i$  is relative to decision maker  $j$  for assessing the given problem. The linear programming model in (6) is solved for each  $x_i$  ( $i = 1, \dots, d$ ) to obtain the weights of the  $d$  decision makers involved in group decision making:

$$\begin{aligned} & \text{Maximize } x_i = \sum_{j=1}^d (\sum_{k=1}^d h r_{ij}^{(k)}) u_j, \quad i = 1, \dots, d \\ & \text{Subject to } \begin{cases} \sum_{j=1}^d (\sum_{k=1}^d \sum_{i=1}^d h r_{ij}^{(k)}) u_j = 1, \\ \sum_{j=1}^d (\sum_{k=1}^d h r_{ij}^{(k)}) u_j \geq d u_i, \quad i = 1, \dots, d, \\ u_j \geq 0, \quad j = 1, \dots, d. \end{cases} \end{aligned} \quad (6)$$

For the model (6),  $x_i$  is the relative weight or score for decision maker  $i$  and  $u_j$  is the score of decision maker  $j$  with respect to decision maker  $i$ . In addition,  $h$  is the initial weight of each decision maker and is equal to  $1/d$ . The first constraint requires the sum of the output values with respect to each decision maker to equal one. The second constraint is a product of Saaty's eigenvector method [9].

Next, let

$$S^{(k)} = (s_{ij})^{(k)}_{c \times c} = \begin{bmatrix} s_{11}^{(k)} & \cdots & s_{1c}^{(k)} \\ \vdots & \ddots & \vdots \\ s_{c1}^{(k)} & \cdots & s_{cc}^{(k)} \end{bmatrix} \quad (7)$$

be a pairwise comparison matrix provided by the  $k$ th decision maker where  $(s_{ij})^{(k)}$  is the  $k$ th decision maker's assessment of how important criteria  $i$  is relative to criteria  $j$  in satisfying the



goal. The linear programming model in (8) is solved for each  $CR_i$  ( $i = 1, \dots, c$ ) to obtain the weights of the  $c$  criteria related to the given problem:

$$\begin{aligned} & \text{Maximize } CR_i = \sum_{j=1}^c (\sum_{k=1}^d x_k s_{ij}^{(k)}) v_j, \quad i = 1, \dots, c \\ & \text{Subject to } \begin{cases} \sum_{j=1}^c (\sum_{k=1}^d \sum_{i=1}^c x_k s_{ij}^{(k)}) v_j = 1, \\ \sum_{j=1}^c (\sum_{k=1}^d x_k s_{ij}^{(k)}) v_j \geq c v_i, \quad i = 1, \dots, c, \\ v_j \geq 0, \quad j = 1, \dots, c. \end{cases} \end{aligned} \quad (8)$$

For the model (8),  $CR_i$  is the relative weight or score for criterion  $i$  and  $v_j$  is the score of criterion  $j$  with respect to criterion  $i$ . In addition,  $x_i$  is the weight of each decision maker as calculated by model (6). The first constraint requires the sum of the output values with respect to each criterion to equal one. The second constraint is a product of Saaty's eigenvector method [9].

Now, let

$$T^{(k)} = (t_{lij})^{(k)}_{p \times p} = \begin{bmatrix} t_{l11}^{(k)} & \cdots & t_{l1p}^{(k)} \\ \vdots & \ddots & \vdots \\ t_{lp1}^{(k)} & \cdots & t_{lpp}^{(k)} \end{bmatrix} \quad (9)$$

be a pairwise comparison matrix provided by the  $k$ th decision maker where  $(t_{lij})^{(k)}$  is the  $k$ th decision maker's assessment of how project  $i$  compares to project  $j$  with respect to criteria  $l$ . The linear programming model in (10) is solved for each  $PCR_{il}$  ( $i = 1, \dots, p$ ,  $l = 1, \dots, c$ ) to obtain the weight of each project with respect to the each criteria:

$$\begin{aligned} & \text{Maximize } PCR_{il} = \sum_{j=1}^p (\sum_{k=1}^d x_k t_{lij}^{(k)}) w_{jl}, \quad i = 1, \dots, p, \quad l = 1, \dots, c \\ & \text{Subject to } \begin{cases} \sum_{j=1}^p (\sum_{k=1}^d \sum_{i=1}^p x_k t_{lij}^{(k)}) w_{jl} = 1, \quad l = 1, \dots, c, \\ \sum_{j=1}^p (\sum_{k=1}^d x_k t_{lij}^{(k)}) w_{jl} \geq p w_{il}, \quad i = 1, \dots, p, \quad l = 1, \dots, c, \\ w_{jl} \geq 0, \quad j = 1, \dots, p, \quad l = 1, \dots, c. \end{cases} \end{aligned} \quad (10)$$

For the model (10),  $PCR_{il}$  is the relative weight or score for project  $i$  with respect to criterion  $l$  and  $w_{jl}$  is the score of project  $j$  with respect to criterion  $l$  with respect to the project-criterion being evaluated (project  $i$  with respect to criterion  $l$ ). In addition,  $x_i$  is the weight of each decision maker as calculated by model (6). The first constraint requires the sum of the output values with respect to each project-criterion to equal one. The second constraint is a product of Saaty's eigenvector method [9].

Lastly, the final weight of each initiative can be computed using the simple additive weighting (SAW) method [9]. Let  $CR_1, \dots, CR_c$  be the best local priorities of the  $c$  decision criteria and  $PCR_{1l}, \dots, PCR_{il}$  be the best local priorities of the  $i$  decision alternatives with respect to the  $l$ th criterion ( $l = 1, \dots, c$ ). Equation (4) can be used to calculate the final weight of each decision alternative.

$$\text{Final weight of Alternative } A_i = \sum_{l=1}^c PCR_{il} CR_l \quad (11)$$

### 3.6 Arrow's Impossibility Theorem

According to Arrow's impossibility theorem [16], when voters have three or more distinct alternatives, no rank order voting system can convert the ranked preferences of individuals into a community-wide ranking while also meeting the following criteria:

- If every voter prefers alternative A to alternative B, then the group prefers A to B.
- If every voter prefers alternative A to alternative B, then any change in preferences that does not affect this relationship must not affect the group preference for A over B. This requirement is also known as "independence from irrelevant alternatives" (IIA).

- There are no dictators; no single voter possesses the power to always determine the group's preference.

The Comprehensive DEA Methodology for Priority Determination in the Group AHP (CDEAGAHP) is a rank-order system that was designed to satisfy the first two criteria while allowing decision makers to have different weights of influence over the decision-making process. If every voter prefers alternative A to alternative B, then the group ranking will have alternative A ranked above alternative B. If an alternative is eliminated from consideration, then the new ordering for the remaining alternatives will be equivalent to the original ordering minus the eliminated alternative (IIA). Lastly, the CDEAGAHP allows a decision maker to have more weight than other decision makers by applying the AHP and DEA to derive weights based on peer evaluations.

An illustrative example that demonstrates an application of the Comprehensive DEA Methodology for Priority Determination in the Group AHP (CDEAGAHP) is explained in Appendix A. The following section applies the methodology to a vaccine prioritization problem.

#### **4. Vaccine Prioritization Using the Comprehensive DEA Methodology for Priority Determination in the Group AHP**

Data was gathered to apply the Comprehensive DEA Methodology for Priority Determination in the Group AHP (CDEAGAHP) to a vaccine prioritization problem with five decision makers evaluating ten vaccine candidates against ten criteria. For the five decision makers, two were chosen to represent the interests of the public sector, two were chosen to

represent the interests of the private sector, and one was chosen to represent the interests of non-governmental organizations. These decision makers are listed in Table 10.

Table 10: Vaccine Example Decision Makers

<b>Decision Maker</b>	<b>Sector</b>
Health Agency Representative	Public
Public Health Unit Representative	Public
Vaccine Industry Representative	Private
Biopharmaceutical Industry Representative	Private
International Vaccine Initiative Representative	Non-governmental Organization

Ten criteria were chosen for the experiment that are similar to some of the 29 criteria used in the 2012 IOM model. Criteria were chosen to capture the health, economic, demographic, scientific, business, and programmatic considerations associated with vaccine development. The ten criteria are listed in Table 11.

Table 11: Vaccine Example Criteria

<b>Criterion</b>	<b>Definition</b>
Target Population	Vaccine targets a disease that affects a population of interest
Cost-Effectiveness	\$/QALY gained
Incident Cases Prevented Per Year Through Vaccination	The number of incident cases of disease prevented in one year
Total Development Costs	Sum of development costs

Potential to Improve Delivery Methods	Vaccine development has the potential to improve delivery methods or stimulate novel approaches to deliver vaccines
Premature Deaths Averted Per Year Through Vaccination	The number of deaths due to disease prevented in one year
QALYs Gained	Net increase in QALYs gained in the population vaccinated
Reduces Challenges Relating to Cold Chain Requirements	Vaccine development has the potential to stimulate novel approaches to mitigate the challenges relating to cold-chain storage and related packaging.
Healthcare Cost Reduction	Health care costs saved
Time to Licensure	The estimated length of time until successful licensure

For the vaccine candidates, ten were chosen as a subset of the 26 vaccine candidates that were used for evaluation in “Vaccines for the 21<sup>st</sup> Century” [6]. The ten vaccine candidates are: ‘Chlamydia’, ‘Group A Streptococcus’, ‘Hepatitis C’, ‘Human Papillomavirus’, ‘Influenza’, ‘Melanoma’, ‘Multiple Sclerosis’, ‘Neisseria Gonorrhea’, ‘Rheumatoid Arthritis’, and ‘Rotavirus’.

In an ideal situation, a decision like this would involve actual representatives from various organizations who could assess each other, the criteria, and the vaccine candidates with respect to the criteria. First, each decision maker would rank all of the decision makers, including

him or herself, in order of ability to prioritize vaccine candidates. For the ranking, the first position would be for the decision maker with the most ability to prioritize vaccine candidates, while the last position would be for the decision maker with the least ability to prioritize vaccine candidates. If someone thought that two or more decision makers were equal in rank, they could rank them in the same position. Before these rankings would occur, though, each decision maker would have the opportunity to review the CV's of the other decision makers to gain an understanding of their experience with vaccine development.

After ranking the decision makers, each decision maker would then rank the criteria of evaluation in order of importance for prioritizing vaccine candidates. For the ranking, the first position would be for the criterion that the decision maker thinks is most important to consider when prioritizing vaccine candidates, while the last position would be for the criterion that the decision maker thinks is least important to consider when prioritizing vaccine candidates. If a decision maker considered two or more criteria to be equal in performance, they could rank them in the same position.

Lastly, in an ideal situation, the decision makers would rank the vaccine candidates in order of preference for the qualitative criteria ("Target Population", "Potential to Improve Delivery Methods", and "Reduces Challenges Relating to Cold Chain Requirements"). For these rankings, the first position would be for the vaccine candidate that the decision maker thought best satisfied the criterion, while the last position would be for the vaccine candidate that the decision maker thought least satisfied the criterion. If a decision maker felt that two or more vaccine candidates were equal in satisfying a certain criterion, they could rank them in the same position.

Given that this is a derived experiment, assumptions were made about how the decision makers might assess each other, the criteria, and the vaccine candidates with respect to the qualitative criteria. The rankings that were created for each decision maker were used to generate the pairwise comparison matrices for the model. The pairwise comparison matrices were created according to the following rules: 1)  $a_{ij}$  is an integer valued 1-10, 2)  $a_{ij} = 1/a_{ji}$ , 3)  $a_{ii} = 1$  for all  $i$ , 3) if the rank of option  $i$ , ( $r_i$ ), and the rank of option  $j$ , ( $r_j$ ), are equal, then elements  $a_{ij}$  and  $a_{ji}$  equal one, 4) if  $r_i > r_j$ , then  $a_{ij}$  equals  $(r_j - r_i + 1)$ .

Typically, decision makers would create pairwise comparison matrices themselves rather than record rankings of the decision makers, criteria, and alternatives, but constructing pairwise comparison matrices is a cumbersome task for decision makers when more than a few criteria are considered. In the case where actual decision makers would be using this model, it was decided that it would be easier for decision makers to rank their preferences so pairwise comparison matrices could be generated from those rankings.

For a quantitative criterion, alternatives are ranked according to how they perform with respect to that criterion. For the quantitative criteria “QALYs Gained”, “Savings of Vaccine Use”, “Incident Cases Prevented Per Year”, and “Premature Deaths Averted Per Year”, the vaccine candidates were ranked in descending order. For the quantitative criteria “Time to Licensure”, “Total Development Costs”, and “Cost-Effectiveness”, the vaccine candidates were ranked in ascending order. The resulting rankings were then used to generate the appropriate pairwise comparison matrices for all of the decision makers. Table 12 shows the values associated with each vaccine initiative for the seven quantitative criteria.

Table 12: Vaccine Example Quantitative Data

	Cost- Effectiveness	Incident Cases Prevented Per Year	Development Costs (millions)	Premature Deaths Averted Per Year	QALYs Gained	Savings of Vaccine Use	Time to Licensure
Chlamydia	-\$350	101,000,000 <sup>e</sup>	\$360	4,373 <sup>a</sup>	110,000	\$175 million	15
Hepatitis C	\$3,000	4,000,000 <sup>f</sup>	\$360	69,027 <sup>a</sup>	41,000	\$67.9 million	15
Human Papillomavirus	\$4,000	22,389,339 <sup>i</sup>	\$360	276,961 <sup>a</sup>	48,000	\$140 million	7
Influenza	-\$4,000	1,209,024,324 <sup>i</sup>	\$360	500,000 <sup>b</sup>	125,000	\$1 billion	7
Melanoma	\$800	220,000 <sup>g</sup>	\$360	77,496 <sup>a</sup>	14,000	\$36.1 million	7
Multiple Sclerosis	-\$12,000	179,114 <sup>i</sup>	\$360	17,084 <sup>a</sup>	15,000	\$180 million	15
Neisseria Gonorrhea	\$2,300	62,000,000 <sup>h</sup>	\$360	299 <sup>a</sup>	47,000	\$92.1 million	15
Rheumatoid Arthritis	-\$4,000	1,455,307 <sup>i</sup>	\$360	37,670 <sup>a</sup>	60,000	\$300 million	15
Rotavirus	\$30,000	78,362,687 <sup>i</sup>	\$120	527,000 <sup>c</sup>	14,000	\$120 million	3
Group A Streptococcus	\$14,000	89,893,197 <sup>i</sup>	\$400	517,000 <sup>d</sup>	6,200	\$185 million	15



*Note.* Data for ‘Cost-Effectiveness’, ‘Development Costs’, ‘QALYs Gained’, ‘Savings of Vaccine Use’, and ‘Time to Licensure’ from “Vaccines for the 21<sup>st</sup> Century” [6].

a. "Global Health Observatory Data Repository." *Cause-specific Mortality, 2008: WHO Region by Country*. N.p., n.d. Web. <<http://apps.who.int/gho/data/node.main.887?lang=en>>.

b. "Influenza (Seasonal)." *WHO*. N.p., n.d. Web. <<http://www.who.int/mediacentre/factsheets/fs211/en/>>.

c. Parashar, Umesh D., et al. "Global mortality associated with rotavirus disease among children in 2004." *Journal of Infectious Diseases* 200.Supplement 1 (2009): S9-S15.

d. Carapetis, Jonathan R. "The current evidence for the burden of group A streptococcal diseases." *Geneva: World Health Organization* (2004): 1-57.

e. "Prevalence and Incidence of Selected Sexually Transmitted Infections." *WHO*. N.p., n.d. Web. <[http://whqlibdoc.who.int/publications/2011/9789241502450\\_eng.pdf](http://whqlibdoc.who.int/publications/2011/9789241502450_eng.pdf)>.

f. "Hepatitis C." *WHO*. N.p., n.d. Web. <<http://www.who.int/mediacentre/factsheets/fs164/en/>>.

g. "Global Health Observatory Data Repository." *Incidence: WHO Region by Country*. N.p., n.d. Web. <<http://apps.who.int/gho/data/node.main.903?lang=en>>.

h. "Sexually Transmitted Diseases." *WHO*. N.p., n.d. Web. <[http://www.who.int/vaccine\\_research/diseases/soa\\_std/en/index2.html](http://www.who.int/vaccine_research/diseases/soa_std/en/index2.html)>.

i. Incident Cases Prevented Per Year = (U.S. Incident Cases Prevented Per Year / U.S. Population Estimate) \* World Population Estimate

Table 13 shows the weight assigned to each decision maker as determined by the linear programming model in (6).

Table 13: Vaccine Example Decision Maker Weights

<b>Decision Maker</b>	<b>Weight</b>
Public Health Unit Representative	0.247393
Health Agency Representative	0.217927
Vaccine Industry Representative	0.1999291
International Vaccine Initiative Representative	0.19927
Biopharmaceutical Industry Representative	0.190454

Table 14 shows the weight assigned to each criterion as determined by the linear programming model in (8).

Table 14: Vaccine Example Criterion Weights

<b>Criterion</b>	<b>Weight</b>
Prevented Deaths	0.154938
Incident Cases	0.138406
Cost Effectiveness	0.124149
Cold Chain	0.113343
QALYs Gained	0.111973
Development Costs	0.108845
Savings	0.103035

Time to Licensure	0.0862401
Delivery Methods	0.0726194
Priority Population	0.0552014

Table 15 shows the ranking of vaccine candidates produced by the linear programming model in (10).

Table 15: Model Vaccine Candidate Ranking

<b>Vaccine Initiative</b>	<b>Weight</b>
Influenza	0.2199
Rotavirus	0.1484
Chlamydia	0.1293
Group A Strep	0.106
HPV	0.1032
Arthritis	0.097
Hepatitis C	0.09299
Multiple Sclerosis	0.08339
Melanoma	0.06271
Gonorrhea	0.05956

The ranking suggests that the top three vaccine candidates are ‘Influenza’, ‘Rotavirus’, and ‘Chlamydia’. This ranking is consistent with the quantitative data associated with these vaccine candidates and the rankings provided by the decision makers.

Lastly, the k-means clustering algorithm [17] was used to sort the vaccine candidates into clusters and identify which vaccine candidates are similar in overall performance with respect to the final ranking. Tables 16 through 19 show the clusters that result from applying the k-means algorithm with  $k = 2$  to  $k = 5$ .

Table 16: Vaccine Example k-means Clustering:  $k=2$

Vaccine Initiative	Weight
Influenza	0.2199
Rotavirus	0.1484
Chlamydia	0.1293
Group A Strep	0.106
HPV	0.1032
Arthritis	0.097
Hepatitis C	0.09299
Multiple Sclerosis	0.08339
Melanoma	0.06271
Gonorrhea	0.05956

Table 17: Vaccine Example k-means Clustering: k=3

Vaccine Initiative	Weight
Influenza	0.2199
Rotavirus	0.1484
Chlamydia	0.1293
Group A Strep	0.106
HPV	0.1032
Arthritis	0.097
Hepatitis C	0.09299
Multiple Sclerosis	0.08339
Melanoma	0.06271
Gonorrhea	0.05956

Table 18: Vaccine Example k-means Clustering: k=4

Vaccine Initiative	Weight
Influenza	0.2199
Rotavirus	0.1484
Chlamydia	0.1293
Group A Strep	0.106
HPV	0.1032
Arthritis	0.097
Hepatitis C	0.09299
Multiple Sclerosis	0.08339

Melanoma	0.06271
Gonorrhea	0.05956

Table 19: Vaccine Example k-means clustering: k=5

Vaccine Initiative	Weight
Influenza	0.2199
Rotavirus	0.1484
Chlamydia	0.1293
Group A Strep	0.106
HPV	0.1032
Arthritis	0.097
Hepatitis C	0.09299
Multiple Sclerosis	0.08339
Melanoma	0.06271
Gonorrhea	0.05956

Sorting vaccine candidates into clusters is consistent with the idea that there is not enough precision to decisively state that one vaccine candidate is better than another. For this reason, vaccine candidates should be sorted into groups of similar performance, similar to how hotels are grouped according to star ratings or countries are grouped according to credit ratings. Review of the clusters can be used to identify where significant and insignificant differences exist among vaccine candidates. For instance, the clustering of the vaccine candidates above suggests that there is not a significant difference between the rankings of ‘Chlamydia’ and

‘Rotavirus’, but there is a significant difference between the rankings of these two vaccine candidates and ‘Influenza’.

## 5. Sensitivity Analysis

The goal of sensitivity analysis is to understand how changes in a problem’s parameters affect the problem’s solution. In the case of the vaccine prioritization problem, it is important to understand how changes in the decision maker’s preferences might affect the final ranking of alternatives. Consider the objective function of model (6):

$$x_i = \sum_{j=1}^d (\sum_{k=1}^d h r_{ij}^{(k)}) u_j, \quad i = 1, \dots, d$$

For this problem, it is important to know how a decision maker’s assessment of his/her fellow decision makers can change without affecting the final ranking of alternatives. Using sensitivity analysis, the objective function coefficient ranges were determined; within these ranges, the current basis remains optimal. For this problem, that provides the range of  $\sum_{k=1}^d h r_{ij}^{(k)}$  for each  $u_j$ . This range must be divided by  $h$  to determine how the summation of the decision makers’ assessments  $(\sum_{k=1}^d r_{ij}^{(k)})$  can vary when comparing decision maker  $i$  to decision maker  $j$ . Table 12 shows the range in which  $\sum_{k=1}^d r_{ij}^{(k)}$  can vary for each  $i, j$ .

Table 12: Objective Function Coefficient Ranges

Decision Makers	j = Health Agency		j = Public Health		j = Vaccine Industry	
	$\sum_{k=1}^d r_{ij}^{(k)}$ Lower Bound	$\sum_{k=1}^d r_{ij}^{(k)}$ Upper Bound	$\sum_{k=1}^d r_{ij}^{(k)}$ Lower Bound	$\sum_{k=1}^d r_{ij}^{(k)}$ Upper Bound	$\sum_{k=1}^d r_{ij}^{(k)}$ Lower Bound	$\sum_{k=1}^d r_{ij}^{(k)}$ Upper Bound
i = Health Agency	4.8725	5.78	-5.00E+100	4.624	6.675	21.195
i = Public Health	5.415	6.685	-5.00E+100	5.57	7.925	32.235
i = Vaccine Industry	4.631	31.815	4.347	35.01	-5.00E+100	5.77
i = BiopharmIndustry	3.6505	44.575	3.4355	52.25	4.4285	5.00E+100
i = InternationalVaclnit	4.427	5.19	4.6505	26.7	6.605	24.77

Table 12: Objective Function Coefficient Ranges (cont.)

Decision Makers	j = BiopharmIndustry		j = InternationalVaclnit	
	$\sum_{k=1}^d r_{ij}^{(k)}$ Lower Bound	$\sum_{k=1}^d r_{ij}^{(k)}$ Upper Bound	$\sum_{k=1}^d r_{ij}^{(k)}$ Lower Bound	$\sum_{k=1}^d r_{ij}^{(k)}$ Upper Bound
i = Health Agency	7.415	21.555	5.67	23.9
i = Public Health	8.845	29.855	6.77	87.4
i = Vaccine Industry	5.64	8.19	5.11	45.865
i = BiopharmIndustry	-5.00E+100	5.64	3.993	68.65
i = InternationalVaclnit	7.365	21.33	-5.00E+100	5.625

Although  $\sum_{k=1}^d r_{ij}^{(k)}$  can vary within these ranges, the final ranking of alternatives can also change. Within the ranges shown in Table 12, the values of the decision variables ( $u_j$ 's) remain unchanged, but the value of  $x_i$  may change, depending on whether the variables associated with the coefficients are basic or nonbasic. If a variable is nonbasic, then the value of the



coefficient can vary within the range shown in Table 12 and not affect the value of  $x_i$  or the final ranking of alternatives. The ranges highlighted in Table 12 are associated with nonbasic variables. For these ranges, the summation of the decision makers' preferences can vary within this range and not change the value of  $x_i$  or the final ranking of alternatives.

To understand how the final ranking of alternatives might also be affected by the weight of the decision makers, sensitivity analysis was performed to determine how a single decision maker's weight can vary such that the ranking of a single vaccine candidate will remain the same. As an example, the weight of 'Health Agency Representative' was analyzed to see how it can vary such that the ranking of 'Chlamydia' will remain the same. A vaccine candidate's ranking will remain the same if the weight associated with it is greater than the weight of the vaccine candidate ranked below it and less than the weight of the vaccine candidate ranked above it. In this case, the weight of 'Chlamydia' must be greater than or equal to 0.1061 and less than or equal to 0.1483 to remain the same.

To determine the range for which the weight of 'Health Agency Representative' can vary such that the ranking of 'Chlamydia' will remain the same, equations 12 and 13 were solved.

$$\sum_{l=1}^c PCR_{Chlamydia,l} CR_l \leq 0.1483 \quad (12)$$

$$\sum_{l=1}^c PCR_{Chlamydia,l} CR_l \geq 0.1061 \quad (13)$$

The functions for each criterion ( $CR_l$ ) and for 'Chlamydia' with respect to each criterion ( $PCR_{Chlamydia,l}$ ) were rewritten in terms of the values associated with those functions and  $x_{Health\ Agency}$ . Each equation was solved for  $x_{Health\ Agency}$  which represents the value that can be added to or subtracted from the weight of 'Health Agency Representative' such that the ranking of 'Chlamydia' will remain the same. The calculations associated with solving these equations

can be found in Appendix K. The results of the analysis suggest that the weight of 'Health Agency Representative' can vary between 0 and 0.321654 without affecting the final ranking of 'Chlamydia'. A large range would suggest that 'Health Agency Representative' has little to no influence over the ranking of 'Chlamydia', while a small range would suggest that 'Health Agency Representative' has a significant influence over the ranking of 'Chlamydia'. In this case, 'Health Agency Representative' has a small influence over the ranking of 'Chlamydia'.

## **6. Conclusions**

The Comprehensive DEA Methodology for Priority Determination in the Group AHP (CDEAGAHP) does an effective job of calculating decision maker weights, criterion weights, and weights for each project with respect to each criterion to come up with a final weight for each project. The CDEAGAHP provides a methodology for calculating decision maker weights using the members of the group in an anonymous manner which is essential to preserving the integrity of the group decision making process. The methodology can be applied to any multiple criteria decision making (MCDM) problem that involves multiple decision makers evaluating multiple alternatives and assessing them according to a variety of quantitative and qualitative criteria. Examples of MCDM problems the methodology can be applied to include choosing a new vehicle for a company to purchase or identifying which vaccine candidates should receive increased attention and funding around the world. The rankings generated by the model are frequently consistent with the data related to the projects and the rankings provided by the decision makers.

Future work related to this problem includes evaluating alternative methods for incorporating quantitative data into the model. Presently, quantitative data is gathered about each project with respect to a particular criterion and the projects are ranked according to how they perform with respect to that criterion. That ranking is then used to generate the pairwise comparison matrix for all of the decision makers. Alternative methods could be used to incorporate this data.

For the vaccine prioritization problem specifically, testing the model with more decision makers, criteria, and vaccine candidates will provide a better understanding of how the model behaves when these conditions change. Additionally, collaboration with committee members of the 2012 IOM model committee could provide insight to other aspects of the vaccine prioritization problem which should be included in the model.

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## **Appendix A: Vehicle Experiment**

The purpose of this experiment was to ensure that the DEA methodology for priority determination in the group AHP can be extended to include the determination of decision maker weights. This experiment serves as an illustrative example of the proposed methodology and demonstrates how the methodology can be applied to other problems with multiple criteria and decision makers. The methodology explained in Section 3.4 is used to calculate the weights of the decision makers, criteria, and alternatives.

For the experiment, six groups of 5-9 individuals were asked to participate in a study that involved ranking vehicle choices. Each person was told that they were an employee working for a company that was considering the purchase of a vehicle for its sales team to use for business trips. Each person was randomly assigned a role in the company and was given a packet of information with eight vehicles to choose from. They were told that the company wanted to include members from all levels of the organization, so the roles within the company included: Chief Financial Officer, Human Resources Manager, Assembly Line Worker, Assembly Line Supervisor, Sales Team Manager, and multiple Sales Team Representatives. In the packet of information, the following information was provided about each vehicle: price, mileage, gas mileage, body style, exterior color, features, and pictures of the exterior and interior of the car. The eight vehicles were: a 2009 Ford F150 XLT, a 2013 Chrysler Town & Country Limited, a 2012 Jeep Grand Cherokee Laredo, a 2012 Toyota Camry SE, a 2002 Nissan Altima 2.5 S, a 2006 Dodge Grand Caravan SE, a 2012 Chevrolet Silverado 1500 LT, and a 2006 Honda CR-V SE. Table 1 shows some of the characteristics for each vehicle.

Table 1: Vehicle Characteristics

	2009 Ford F150 XLT	2013 Chrysler Town & Country Limited	2012 Jeep Grand Cherokee Laredo	2012 Toyota Camry SE	2002 Nissan Altima 2.5 S	2006 Dodge Grand Caravan SE	2012 Chevrolet Silverado 1500 LT	2006 Honda CR-V SE
Price	\$17,994	\$38,646	\$25,991	\$22,700	\$5,995	\$7,988	\$27,869	\$13,995
Mileage	49,832 mi.	12 mi.	29,288 mi.	7,658 mi.	97,176 mi.	60,500 mi.	7,225 mi.	85,000 mi.
Gas Mileage	14-15 mpg (city) / 18 -20 mpg (highway)	17 mpg (city) / 25 mpg (highway)	12-17 mpg (city) / 18-23 mpg (highway)	21-25 mpg (city) / 30-35 mpg (highway)	19-23 mpg (city) / 26-29 mpg (highway)	19-20 mpg (city) / 26 mpg (highway)	12-15 mpg (city) / 18-22 mpg (highway)	21-23 mpg (city) / 26- 29 mpg (highway)
Body Style	Pickup	Minivan	SUV	Sedan	Sedan	Minivan	Pickup	SUV

After reviewing the details about each car for a certain period of time, the individuals were asked to, as a group, rank the vehicles in order of which they thought the company should consider purchasing them. The individuals were informed that in addition to cost, the company would like them to consider comfort, durability, and the impression the vehicle will make when choosing which one to purchase. The first position in the ranking was for the vehicle the company should give the most consideration to for purchasing, while the last position in the ranking was for the vehicle the company should give the least consideration to for purchasing. If the group decided that two or more vehicles were equal in rank, they were able to rank them in the same position. Table 2 shows the resulting ranking for Group 1. The rankings for the rest of the groups can be found in Appendix B.

Table 2: Group 1 Vehicle Ranking

1) 2012 Toyota Camry SE
2) 2012 Jeep Grand Cherokee Laredo
3) 2006 Honda CR-V SE
4) 2002 Nissan Altima 2.5 S
5) 2006 Dodge Grand Caravan SE
6) 2013 Chrysler Town & Country Limited
7) 2012 Chevrolet Silverado 1500 LT, 2009 Ford F150 XLT

Following this activity, each member of the group was asked to independently fill out a packet that had them rank the decision makers, criteria of evaluation, and vehicles in comparison to the criteria. These rankings were requested to provide the information necessary to generate the pairwise comparison matrices for the model.

First, each decision maker had to rank all of the decision makers, including him or herself, in order of ability to choose a vehicle for the company to purchase. For the ranking, the first position was for the decision maker with the most ability to choose a vehicle for the company to purchase, while the ninth position was for the decision maker with the least ability to choose a vehicle for the company to purchase. If someone thought that two or more decision makers were equal in rank, they could rank them in the same position. Before the ranking, each individual had to explain their experience with purchasing vehicles, so when they had to record their ranking of decision makers, they had to consider an individual's experience with purchasing vehicles in addition to an individual's position in the company.

After ranking the decision makers, each person had to rank the criteria of evaluation in order of importance in choosing a vehicle for the company to purchase. The criteria of evaluation were: gas mileage, color, cost, body style, make, appearance, features, and mileage. For the ranking, the first position was for the criterion that was most important to consider when choosing a vehicle for the company to purchase, while the eighth position was for the criterion that was least important to consider when choosing a vehicle for the company to purchase. If two or more criteria were considered equal in importance, a decision maker could rank them in the same position.

Lastly, the decision makers were asked to rank the vehicles in order of preference for the qualitative criteria (color, body style, make, appearance, and features). For these rankings, the first position was for the vehicle that the decision maker thought best satisfied that criterion, while the eighth position was for the vehicle that the decision maker thought least satisfied that criterion. If two or more vehicles were considered equal in satisfying a certain criterion, a decision maker could rank them in the same position.

The rankings provided by each decision maker were used to generate the pairwise comparison matrices for the model. Typically, decision makers would create pairwise comparison matrices themselves rather than record rankings of the decision makers, criteria, and alternatives, but constructing pairwise comparison matrices is a cumbersome task for decision makers when more than a few criteria are considered. In an effort to provide decision makers with an easy and quick method for identifying their preferences, the decision makers ranked their preferences, (as described above), so pairwise comparison matrices could be generated later. The pairwise comparison matrices were created according to the following rules: 1)  $a_{ij}$  is an integer



valued 1-9, 2)  $a_{ij} = 1/a_{ji}$ , 3)  $a_{ii} = 1$  for all  $i$ , 3) if the rank of option  $i$ ,  $r_i$ , and the rank of option  $j$ ,  $r_j$ , were equal, then element  $a_{ij}$  and  $a_{ji}$  equaled one, 4) if  $r_i > r_j$ , then  $a_{ij}$  equaled  $(r_i - r_j + 1)$ .

For the quantitative criteria ‘cost’ and ‘mileage’, the vehicles were ranked in ascending order, and for the quantitative criteria ‘gas mileage’, the vehicles were ranked in descending order. The resulting rankings were then used to generate the appropriate pairwise comparison matrices using the method described above.

Table 3 shows the weight assigned to each decision maker for Group 1, as determined by the model and the rankings provided by all of the decision makers. The weights assigned to each decision maker for the rest of the groups can be found in Appendix C.

Table 3: Group 1 Decision Maker Weights

<b>Decision Maker</b>	<b>Weight</b>
Sales Team Representative	0.205
Sales Team Manager	0.161
Sales Team Representative	0.134
Assembly Line Supervisor	0.122
Assembly Line Worker	0.110
Sales Team Representative	0.103
Chief Financial Officer	0.100
Sales Team Representative	0.070
Human Resources Manager	0.049

When the decision makers were ranking the decision makers in the group, they were considering an individual’s role within the company along with their experience with purchasing

vehicles. This is evident by the resulting decision maker weights. In most cases, the individuals with the most weight were a) part of the sales team, the group that would be using the new vehicle, or b) the ones who seemed to have the most experience with vehicles based on the group discussions. The individuals with the least weight seemed to have a lesser knowledge of vehicles based on the group discussions, causing their peers to rank them low on their individual assessments, regardless of their position in the company. This in turn resulted in them receiving the least amount of influence in determining which vehicle the company should purchase.

Table 4 shows the weight assigned to each criterion for Group 1, as determined by the rankings provided by all of the decision makers. The weights assigned to each criterion for the rest of the groups can be found in Appendix D.

Table 4: Group 1 Criterion Weights

<b>Criterion</b>	<b>Weight</b>
Gas Mileage	0.235
Cost	0.185
Mileage	0.173
Body Style	0.158
Make	0.112
Features	0.106
Appearance	0.075
Color	0.037

For most of the groups, ‘gas mileage’, ‘cost’, and ‘mileage’ were given higher consideration to than the other criteria, while ‘color’, ‘features’, and ‘make’ were given less consideration to than the other criteria.

Table 5 shows the ranking of vehicles produced by the model for Group 1. The ranking of vehicles produced by the model for the rest of the groups can be found in Appendix E.

Table 5: Group 1 Model Vehicle Ranking

1) 2012 Toyota Camry SE
2) 2006 Honda CR-V SE
3) 2002 Nissan Altima 2.5 S
4) 2013 Chrysler Town & Country Limited
5) 2012 Jeep Grand Cherokee Laredo
6) 2006 Dodge Grand Caravan SE
7) 2012 Chevrolet Silverado 1500 LT
8) 2009 Ford F150 XLT

For most groups, a few differences existed between the ranking that resulted from the model and the ranking that resulted from the group discussions. Changes in a vehicle’s ranking occurred when the model needed to identify which vehicles performed best according to the criteria that the decision makers determined to be significant. The ranking of the 2002 Nissan Altima usually changed between the two rankings for this reason. For the group rankings, decision makers would rank the Altima low due to its age and appearance. When it came to the model ranking, though, decision makers frequently chose ‘cost’ and ‘gas mileage’ as top criteria. Because the Altima is number one in terms of cost and number three in terms of gas mileage, it

frequently moved up in rank between the group and model rankings. In these instances, the model was simply accounting for the criteria that the decision makers determined to be significant.

Lastly, the k-means clustering algorithm [16] was used to sort the vehicles into clusters and identify which vehicles were similar in overall performance according to the final ranking. Tables 6 through 9 show the clusters that result from applying the k-means algorithm to Group 1's results with  $k = 2$  to  $k = 5$ . The results of k-means clustering for the other groups can be found in Appendices F-J.

Table 6: Group 1 K-means clustering:  $k = 2$

1) 2012 Toyota Camry SE
2) 2006 Honda CR-V SE
3) 2002 Nissan Altima 2.5 S
4) 2013 Chrysler Town & Country Limited
5) 2012 Jeep Grand Cherokee Laredo
6) 2006 Dodge Grand Caravan SE
7) 2012 Chevrolet Silverado 1500 LT
8) 2009 Ford F150 XLT

Table 7: Group 1 K-means clustering:  $k = 3$

1) 2012 Toyota Camry SE
2) 2006 Honda CR-V SE
3) 2002 Nissan Altima 2.5 S
4) 2013 Chrysler Town & Country Limited
5) 2012 Jeep Grand Cherokee Laredo
6) 2006 Dodge Grand Caravan SE
7) 2012 Chevrolet Silverado 1500 LT
8) 2009 Ford F150 XLT

Table 8: Group 1 K-means clustering:  $k = 4$

1) 2012 Toyota Camry SE
2) 2006 Honda CR-V SE
3) 2002 Nissan Altima 2.5 S
4) 2013 Chrysler Town & Country Limited
5) 2012 Jeep Grand Cherokee Laredo
6) 2006 Dodge Grand Caravan SE
7) 2012 Chevrolet Silverado 1500 LT
8) 2009 Ford F150 XLT

Table 9: Group 1 K-means clustering:  $k = 5$

1) 2012 Toyota Camry SE
2) 2006 Honda CR-V SE
3) 2002 Nissan Altima 2.5 S
4) 2013 Chrysler Town & Country Limited
5) 2012 Jeep Grand Cherokee Laredo
6) 2006 Dodge Grand Caravan SE
7) 2012 Chevrolet Silverado 1500 LT
8) 2009 Ford F150 XLT

The clustering of the Group 1 vehicles shows that there is not a significant difference between the ranking of the 2006 Honda CR-V and 2002 Nissan Altima, but there is a significant difference between the ranking of these two vehicles and the 2012 Toyota Camry SE.

## **Appendix B: Vehicle Example Group Vehicle Rankings**

Table B1: Group 2 Vehicle Ranking

1) 2012 Toyota Camry SE
2) 2012 Jeep Grand Cherokee Laredo
3) 2013 Chrysler Town & Country Limited
4) 2006 Honda CR-V SE, 2006 Dodge Grand Caravan SE
5) 2012 Chevrolet Silverado 1500 LT
6) 2002 Nissan Altima 2.5 S
7) 2009 Ford F150 XLT

Table B2: Group 3 Vehicle Ranking

1) 2012 Toyota Camry SE
2) 2006 Honda CR-V SE
3) 2006 Dodge Grand Caravan SE
4) 2013 Chrysler Town & Country Limited
5) 2012 Jeep Grand Cherokee Laredo
6) 2002 Nissan Altima 2.5 S
7) 2012 Chevrolet Silverado 1500 LT, 2009 Ford F150 XLT

Table B3: Group 4 Vehicle Ranking

1) 2012 Toyota Camry SE
2) 2012 Jeep Grand Cherokee Laredo
3) 2006 Honda CR-V SE
4) 2002 Nissan Altima 2.5 S

5) 2006 Dodge Grand Caravan SE
6) 2013 Chrysler Town & Country Limited
7) 2009 Ford F150 XLT
8) 2012 Chevrolet Silverado 1500 LT

Table B4: Group 5 Vehicle Ranking

1) 2012 Toyota Camry SE
2) 2012 Jeep Grand Cherokee Laredo
3) 2006 Honda CR-V SE
4) 2006 Dodge Grand Caravan SE
5) 2002 Nissan Altima 2.5 S
6) 2013 Chrysler Town & Country Limited
7) 2012 Chevrolet Silverado 1500 LT
8) 2009 Ford F150 XLT

Table B5: Group 6 Vehicle Ranking

1) 2012 Toyota Camry SE
2) 2012 Jeep Grand Cherokee Laredo
3) 2013 Chrysler Town & Country Limited
4) 2006 Honda CR-V SE
5) 2002 Nissan Altima 2.5 S
6) 2006 Dodge Grand Caravan SE
7) 2012 Chevrolet Silverado 1500 LT, 2009 Ford F150 XLT

### Appendix C: Vehicle Example Decision Maker Weights

Table C1: Group 2 Decision Maker Weights

Decision Maker	Weight
Sales Team Representative	0.203857
Assembly Line Supervisor	0.178611
Sales Team Representative	0.147251
Sales Team Representative	0.139284
Human Resources Manager	0.137797
Sales Team Manager	0.105846
Sales Team Representative	0.0490293
Assembly Line Worker	0.0430622
Chief Financial Officer	0.0348999

Table C2: Group 3 Decision Maker Weights

Decision Maker	Weight
Chief Financial Officer	0.219524
Sales Team Representative	0.173396
Sales Team Representative	0.165258
Assembly Line Supervisor	0.122575
Human Resources Manager	0.119295
Sales Team Manager	0.108542
Assembly Line Worker	0.101119
Sales Team Representative	0.0881727



Table C3: Group 4 Decision Maker Weights

<b>Decision Maker</b>	<b>Weight</b>
Sales Team Manager	0.166367
Chief Financial Officer	0.164945
Human Resources Manager	0.160456
Sales Team Representative	0.13028
Sales Team Representative	0.12123
Sales Team Representative	0.118721
Sales Team Representative	0.113491
Assembly Line Supervisor	0.113004
Assembly Line Worker	0.0497484

Table C4: Group 5 Decision Maker Weights

<b>Decision Maker</b>	<b>Weight</b>
Sales Team Manager	0.288403
Chief Financial Officer	0.192078
Human Resources Manager	0.150424
Sales Team Representative	0.134789
Sales Team Representative	0.121699
Sales Team Representative	0.107403
Assembly Line Supervisor	0.0461601
Assembly Line Worker	0.0315077

Table C5: Group 6 Decision Maker Weights

<b>Decision Maker</b>	<b>Weight</b>
Chief Financial Officer	0.319642
Sales Team Representative	0.281548
Sales Team Manager	0.200677
Sales Team Representative	0.191439
Human Resources Manager	0.0945378

## Appendix D: Vehicle Example Criterion Weights

Table D1: Group 2 Criterion Weights

Criterion	Weight
Mileage	0.211369
Cost	0.206324
Appearance	0.16657
Gas Mileage	0.153486
Body Style	0.114927
Make	0.0996222
Features	0.0741825
Color	0.0687389

Table D2: Group 3 Criterion Weights

Criterion	Weight
Cost	0.211628
Gas Mileage	0.193846
Mileage	0.181939
Appearance	0.125923
Body Style	0.125665
Make	0.0947138
Features	0.0880294
Color	0.0545618

Table D3: Group 4 Criterion Weights

<b>Criterion</b>	<b>Weight</b>
Appearance	0.225753
Cost	0.214722
Gas Mileage	0.185236
Mileage	0.176551
Body Style	0.157368
Features	0.0885038
Make	0.0627282
Color	0.0287769

Table D4: Group 5 Criterion Weights

<b>Criterion</b>	<b>Weight</b>
Gas Mileage	0.235707
Mileage	0.205662
Cost	0.203233
Appearance	0.154181
Body Style	0.0982986
Features	0.0919879
Make	0.057545
Color	0.0251336

Table D5: Group 6 Criterion Weights

<b>Criterion</b>	<b>Weight</b>
Body Style	0.261357
Mileage	0.202141
Appearance	0.173748
Gas Mileage	0.159869
Cost	0.102853
Make	0.0711132
Color	0.0506744
Features	0.0374448

## Appendix E: Vehicle Example Model Vehicle Ranking

Table E1: Group 2 Model Vehicle Ranking

Vehicle	Weight
1) Toyota	0.21226
2) Jeep	0.162489
3) Nissan	0.1478
4) Chrysler	0.145391
5) Honda	0.137473
6) Chevy	0.125838
7) Dodge	0.116254
8) Ford	0.0961831

Table E2: Group 3 Model Vehicle Ranking

Vehicle	Weight
1) Toyota	0.234231
2) Nissan	0.167109
3) Honda	0.159574
4) Chrysler	0.147843
5) Dodge	0.138073
6) Jeep	0.117067
7) Chevy	0.102084
8) Ford	0.0865146

Table E3: Group 4 Model Vehicle Ranking

<b>Vehicle</b>	<b>Weight</b>
1) Toyota	0.233732
2) Nissan	0.176567
3) Jeep	0.16396
4) Chrysler	0.158372
5) Honda	0.157053
6) Chevy	0.120783
7) Dodge	0.117781
8) Ford	0.0891504

Table E4: Group 5 Model Vehicle Ranking

<b>Vehicle</b>	<b>Weight</b>
1) Toyota	0.241221
2) Nissan	0.161906
3) Honda	0.149379
4) Chrysler	0.139058
5) Jeep	0.126731
6) Dodge	0.114109
7) Chevy	0.110166
8) Ford	0.0876489

Table E5: Group 6 Model Vehicle Ranking

<b>Vehicle</b>	<b>Weight</b>
1) Toyota	0.220474
2) Nissan	0.152795
3) Jeep	0.148616
4) Chrysler	0.136919
5) Honda	0.136383
6) Chevy	0.135783
7) Dodge	0.0940066
8) Ford	0.0933311



## Appendix F: Vehicle Example Group 2 k-means Clustering

Table F1: Group 2 K-means clustering:  $k = 2$

1) 2012 Toyota Camry SE
2) 2012 Jeep Grand Cherokee Laredo
3) 2002 Nissan Altima 2.5 S
4) 2013 Chrysler Town & Country Limited
5) 2006 Honda CR-V SE
6) 2012 Chevrolet Silverado 1500 LT
7) 2006 Dodge Grand Caravan SE
8) 2009 Ford F150 XLT

Table F2: Group 2 K-means clustering:  $k = 3$

1) 2012 Toyota Camry SE
2) 2012 Jeep Grand Cherokee Laredo
3) 2002 Nissan Altima 2.5 S
4) 2013 Chrysler Town & Country Limited
5) 2006 Honda CR-V SE
6) 2012 Chevrolet Silverado 1500 LT
7) 2006 Dodge Grand Caravan SE
8) 2009 Ford F150 XLT

Table F3: Group 2 K-means clustering:  $k = 4$

1) 2012 Toyota Camry SE
2) 2012 Jeep Grand Cherokee Laredo
3) 2002 Nissan Altima 2.5 S
4) 2013 Chrysler Town & Country Limited
5) 2006 Honda CR-V SE
6) 2012 Chevrolet Silverado 1500 LT
7) 2006 Dodge Grand Caravan SE
8) 2009 Ford F150 XLT

Table F4: Group 2 K-means clustering:  $k = 5$

1) 2012 Toyota Camry SE
2) 2012 Jeep Grand Cherokee Laredo
3) 2002 Nissan Altima 2.5 S
4) 2013 Chrysler Town & Country Limited
5) 2006 Honda CR-V SE
6) 2012 Chevrolet Silverado 1500 LT
7) 2006 Dodge Grand Caravan SE
8) 2009 Ford F150 XLT

## Appendix G: Vehicle Example Group 3 k-means Clustering

Table G1: Group 3 K-means clustering:  $k = 2$

1) 2012 Toyota Camry SE
2) 2002 Nissan Altima 2.5 S
3) 2006 Honda CR-V SE
4) 2013 Chrysler Town & Country Limited
5) 2006 Dodge Grand Caravan SE
6) 2012 Jeep Grand Cherokee Laredo
7) 2012 Chevrolet Silverado 1500 LT
8) 2009 Ford F150 XLT

Table G2: Group 3 K-means clustering:  $k = 3$

1) 2012 Toyota Camry SE
2) 2002 Nissan Altima 2.5 S
3) 2006 Honda CR-V SE
4) 2013 Chrysler Town & Country Limited
5) 2006 Dodge Grand Caravan SE
6) 2012 Jeep Grand Cherokee Laredo
7) 2012 Chevrolet Silverado 1500 LT
8) 2009 Ford F150 XLT

Table G3: Group 3 K-means clustering:  $k = 4$

1) 2012 Toyota Camry SE
2) 2002 Nissan Altima 2.5 S
3) 2006 Honda CR-V SE
4) 2013 Chrysler Town & Country Limited
5) 2006 Dodge Grand Caravan SE
6) 2012 Jeep Grand Cherokee Laredo
7) 2012 Chevrolet Silverado 1500 LT
8) 2009 Ford F150 XLT

Table G4: Group 3 K-means clustering:  $k = 5$

1) 2012 Toyota Camry SE
2) 2002 Nissan Altima 2.5 S
3) 2006 Honda CR-V SE
4) 2013 Chrysler Town & Country Limited
5) 2006 Dodge Grand Caravan SE
6) 2012 Jeep Grand Cherokee Laredo
7) 2012 Chevrolet Silverado 1500 LT
8) 2009 Ford F150 XLT

## Appendix H: Vehicle Example Group 4 k-means Clustering

Table H1: Group 4 K-means clustering:  $k = 2$

1) 2012 Toyota Camry SE
2) 2002 Nissan Altima 2.5 S
3) 2012 Jeep Grand Cherokee Laredo
4) 2013 Chrysler Town & Country Limited
5) 2006 Honda CR-V SE
6) 2012 Chevrolet Silverado 1500 LT
7) 2006 Dodge Grand Caravan SE
8) 2009 Ford F150 XLT

Table H2: Group 4 K-means clustering:  $k = 3$

1) 2012 Toyota Camry SE
2) 2002 Nissan Altima 2.5 S
3) 2012 Jeep Grand Cherokee Laredo
4) 2013 Chrysler Town & Country Limited
5) 2006 Honda CR-V SE
6) 2012 Chevrolet Silverado 1500 LT
7) 2006 Dodge Grand Caravan SE
8) 2009 Ford F150 XLT

Table H3: Group 4 K-means clustering:  $k = 4$

1) 2012 Toyota Camry SE
2) 2002 Nissan Altima 2.5 S
3) 2012 Jeep Grand Cherokee Laredo
4) 2013 Chrysler Town & Country Limited
5) 2006 Honda CR-V SE
6) 2012 Chevrolet Silverado 1500 LT
7) 2006 Dodge Grand Caravan SE
8) 2009 Ford F150 XLT

Table H4: Group 4 K-means clustering:  $k = 5$

1) 2012 Toyota Camry SE
2) 2002 Nissan Altima 2.5 S
3) 2012 Jeep Grand Cherokee Laredo
4) 2013 Chrysler Town & Country Limited
5) 2006 Honda CR-V SE
6) 2012 Chevrolet Silverado 1500 LT
7) 2006 Dodge Grand Caravan SE
8) 2009 Ford F150 XLT

## Appendix I: Vehicle Example Group 5 k-means Clustering

Table I1: Group 5 K-means clustering:  $k = 2$

9) 2012 Toyota Camry SE
10) 2002 Nissan Altima 2.5 S
11) 2006 Honda CR-V SE
12) 2013 Chrysler Town & Country Limited
13) 2012 Jeep Grand Cherokee Laredo
14) 2006 Dodge Grand Caravan SE
15) 2012 Chevrolet Silverado 1500 LT
16) 2009 Ford F150 XLT

Table I2: Group 5 K-means clustering:  $k = 3$

1) 2012 Toyota Camry SE
2) 2002 Nissan Altima 2.5 S
3) 2006 Honda CR-V SE
4) 2013 Chrysler Town & Country Limited
5) 2012 Jeep Grand Cherokee Laredo
6) 2006 Dodge Grand Caravan SE
7) 2012 Chevrolet Silverado 1500 LT
8) 2009 Ford F150 XLT

Table I3: Group 5 K-means clustering:  $k = 4$

1) 2012 Toyota Camry SE
2) 2002 Nissan Altima 2.5 S
3) 2006 Honda CR-V SE
4) 2013 Chrysler Town & Country Limited
5) 2012 Jeep Grand Cherokee Laredo
6) 2006 Dodge Grand Caravan SE
7) 2012 Chevrolet Silverado 1500 LT
8) 2009 Ford F150 XLT

Table I4: Group 5 K-means clustering:  $k = 5$

1) 2012 Toyota Camry SE
2) 2002 Nissan Altima 2.5 S
3) 2006 Honda CR-V SE
4) 2013 Chrysler Town & Country Limited
5) 2012 Jeep Grand Cherokee Laredo
6) 2006 Dodge Grand Caravan SE
7) 2012 Chevrolet Silverado 1500 LT
8) 2009 Ford F150 XLT



## Appendix J: Vehicle Example Group 6 k-means Clustering

Table J1: Group 6 K-means clustering:  $k = 2$

17) 2012 Toyota Camry SE
18) 2002 Nissan Altima 2.5 S
19) 2012 Jeep Grand Cherokee Laredo
20) 2013 Chrysler Town & Country Limited
21) 2006 Honda CR-V SE
22) 2012 Chevrolet Silverado 1500 LT
23) 2006 Dodge Grand Caravan SE
24) 2009 Ford F150 XLT

Table J2: Group 6 K-means clustering:  $k = 3$

1) 2012 Toyota Camry SE
2) 2002 Nissan Altima 2.5 S
3) 2012 Jeep Grand Cherokee Laredo
4) 2013 Chrysler Town & Country Limited
5) 2006 Honda CR-V SE
6) 2012 Chevrolet Silverado 1500 LT
7) 2006 Dodge Grand Caravan SE
8) 2009 Ford F150 XLT

Table J3: Group 6 K-means clustering:  $k = 4$

1) 2012 Toyota Camry SE
2) 2002 Nissan Altima 2.5 S
3) 2012 Jeep Grand Cherokee Laredo
4) 2013 Chrysler Town & Country Limited
5) 2006 Honda CR-V SE
6) 2012 Chevrolet Silverado 1500 LT
7) 2006 Dodge Grand Caravan SE
8) 2009 Ford F150 XLT

Table J4: Group 6 K-means clustering:  $k = 5$

1) 2012 Toyota Camry SE
2) 2002 Nissan Altima 2.5 S
3) 2012 Jeep Grand Cherokee Laredo
4) 2013 Chrysler Town & Country Limited
5) 2006 Honda CR-V SE
6) 2012 Chevrolet Silverado 1500 LT
7) 2006 Dodge Grand Caravan SE
8) 2009 Ford F150 XLT

## Appendix K: Sensitivity Analysis Calculation

$$\text{Final weight of Alternative } A_i = \sum_{l=1}^c PCR_{il} CR_l$$

$$A_{Chlamydia} = 0.1293$$

$$0.1061 \leq A_{Chlamydia} \leq 0.1483$$

$$\begin{aligned} A_{Chlamydia} \geq & (PCR_{Chlamydia, \text{ Cold Chain}})(CR_{\text{Cold Chain}}) + \\ & (PCR_{Chlamydia, \text{ Costeffectiveness}})(CR_{\text{Costeffectiveness}}) + \\ & (PCR_{Chlamydia, \text{ Delivery Methods}})(CR_{\text{Delivery Methods}}) + (PCR_{Chlamydia, \text{ Incident Cases}})(CR_{\text{Incident Cases}}) \\ & + (PCR_{Chlamydia, \text{ Licensure}})(CR_{\text{Licensure}}) + (PCR_{Chlamydia, \text{ Development Costs}})(CR_{\text{Development Costs}}) + \\ & (PCR_{Chlamydia, \text{ Prevented Deaths}})(CR_{\text{Prevented Deaths}}) + \\ & (PCR_{Chlamydia, \text{ Priority Population}})(CR_{\text{Priority Population}}) + \\ & (PCR_{Chlamydia, \text{ \$/QALY Gained}})(CR_{\text{\$/QALY Gained}}) + (PCR_{Chlamydia, \text{ Savings}})(CR_{\text{Savings}}) \end{aligned}$$

$$0.1483 \geq$$

$$\begin{aligned} & (0.174248776 + (x_{\text{Health Agency}})(0.20745582))(0.094918693 + (x_{\text{Health Agency}})(0.08454348)) + \\ & (0.100617054 + (x_{\text{Health Agency}})(0.120296914))(0.105894635 + (x_{\text{Health Agency}})(0.08376367)) + \\ & (0.027392656 + (x_{\text{Health Agency}})(0.038151967))(0.052969646 + (x_{\text{Health Agency}})(0.09016668)) + \\ & (0.17421749 + (x_{\text{Health Agency}})(0.20829227))(0.120315309 + (x_{\text{Health Agency}})(0.08301262)) + \\ & (0.052596999 + (x_{\text{Health Agency}})(0.062884365))(0.076275375 + (x_{\text{Health Agency}})(0.04572506)) + \\ & (0.075880705 + (x_{\text{Health Agency}})(0.0907221))(0.09001065 + (x_{\text{Health Agency}})(0.08642504)) + \\ & (0.01607623 + (x_{\text{Health Agency}})(0.01922052))(0.136847309 + (x_{\text{Health Agency}})(0.08301262)) + \\ & (0.052125907 + (x_{\text{Health Agency}})(0.054869718))(0.049243159 + (x_{\text{Health Agency}})(0.02734054)) + \\ & (0.173912778 + (x_{\text{Health Agency}})(0.207928445))(0.093882309 + (x_{\text{Health Agency}})(0.08301262)) + \\ & (0.064371765 + (x_{\text{Health Agency}})(0.076962172))(0.084057963 + (x_{\text{Health Agency}})(0.08707979)) \end{aligned}$$

$$0.1483 \geq$$

$$0.0165394661 + (0.0147315979) (x_{Health\ Agency}) + (0.0196914353) (x_{Health\ Agency}) + (0.017539037) (x_{Health\ Agency})^2 +$$

$$0.0106548062 + (0.0084280537) (x_{Health\ Agency}) + (0.0127387978) (x_{Health\ Agency}) + (0.010076511) (x_{Health\ Agency})^2 +$$

$$0.0014509793 + (0.0024699048) (x_{Health\ Agency}) + (0.0020208962) (x_{Health\ Agency}) + (0.0034400362) (x_{Health\ Agency})^2 +$$

$$0.0209610311 + (0.0144622503) (x_{Health\ Agency}) + (0.0250607488) (x_{Health\ Agency}) + (0.0172908871) (x_{Health\ Agency})^2 +$$

$$0.0040118558 + (0.0024050009) (x_{Health\ Agency}) + (0.0047965285) (x_{Health\ Agency}) + (0.0028753914) (x_{Health\ Agency})^2 +$$

$$0.0068300716 + (0.006557993) (x_{Health\ Agency}) + (0.0081659552) (x_{Health\ Agency}) + (0.0078406611) (x_{Health\ Agency})^2 +$$

$$0.0021999888 + (0.00133453) (x_{Health\ Agency}) + (0.0026302764) (x_{Health\ Agency}) + (0.0015955457) (x_{Health\ Agency})^2 +$$

$$0.0025668443 + (0.0014251504) (x_{Health\ Agency}) + (0.0027019582) (x_{Health\ Agency}) + (0.0015001677) (x_{Health\ Agency})^2 +$$

$$0.0163273332 + (0.0144369554) (x_{Health\ Agency}) + (0.0195208025) (x_{Health\ Agency}) + (0.017260685) (x_{Health\ Agency})^2 +$$

$$0.0054109594 + (0.0056054798) (x_{Health\ Agency}) + (0.0064692834) (x_{Health\ Agency}) + (0.0067018498) (x_{Health\ Agency})^2$$

$$0.1483 \geq 0.0869533358 + (0.0718569162) (x_{Health\ Agency}) + (0.1037966823) (x_{Health\ Agency}) + (0.086120772) (x_{Health\ Agency})^2$$

$$0.1483 \geq 0.0869533358 + (0.1756535985) (x_{Health\ Agency}) + (0.086120772) (x_{Health\ Agency})^2$$

$$(0.086120772) (x_{Health\ Agency})^2 + (0.1756535985) (x_{Health\ Agency}) - 0.0613466642 \leq 0$$

$$x_{Health\ Agency} = -2.343571315, 0.3039519014$$

$$0.1061 \leq 0.0869533358 + (0.1756535985) (x_{Health\ Agency}) + (0.086120772) (x_{Health\ Agency})^2$$

$$(0.086120772) (x_{Health\ Agency})^2 + (0.1756535985) (x_{Health\ Agency}) - 0.0191466642 \geq 0$$

$$x_{Health\ Agency} = \mathbf{-2.1433466462, 0.1037272324}$$

$$x_{Health\ Agency} = 0.217927$$

$$0.217927 - 2.34357 < x_{HA} < 0.217927 + 0.103727$$

$$\mathbf{0 < x_{Health\ Agency} < 0.321654}$$