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Product clustering for focused factories and cellular manufacturing

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Product Clustering for Focused Factories and Cellular Manufacturing

by

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Abstract

In the past, clustering analysis techniques have been broadly utilized to segment markets. Recent studies (Berry et al. 1991) have used market-based clustering to examine manufacturing operations required to be competitive in different market segments. These authors proposed a methodology to categorize a set of products based on their similarity across a set of key variables, such as order-winning criteria, time considerations and volume levels. Their method incorporated the Agglomerative Hierarchy with k-Means Refinement Model to identify groups of similar products to be manufactured in a focused-factory.

While many other multivariate clustering methods exist in the literature, the application of these methods to focused factory development is missing. Therefore, this thesis explores the use of other clustering methods for this purpose and incorporates the application of multivariate clustering analysis to support the formation of manufacturing cells. In the case of cellular manufacturing, existing methods rely solely on process similarity to define which products should be grouped together and manufactured in the same cell. Additional attributes, such as volume, setup time, quality and delivery lead times, have been ignored in the literature.

Motivated by the need to consider multiple criteria in the formation of focused factories and manufacturing cells, and the lack of analytical methods to support the decision-maker, this research has explored the performance of five clustering methods, each with differing clustering strategies: the Agglomerative Hierarchy with k-Means Refinement, the Plant Location Model, the Covering Model, the Average-Weighted Distance Model and the Fuzzy Set Method.

The use of these five methods has been illustrated using two cases drawn from the literature and results illustrate how different clustering strategies may produce different solutions. Findings in this research suggest that some sources of variability in results come from the choices made throughout the analysis process. However, since this thesis has only initiated the extension of multivariate clustering analysis to the definition of focused factories and manufacturing cells, there is a significant opportunity for future research.

Dedication

This thesis is dedicated to my parents Manuel Batista and Rosa Liriano de Batista, who used to tell me that a good education is the best heritage one can get. They motivated me to pursue my master's degree and have always supported my decisions throughout my career.

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Chapter 1

Introduction

Extensive literature has talked about the gains associated with adopting a focused manufacturing strategy, a concept that was first implemented to reverse the process in companies whose diverse product mix had provided confusion and lack of focus to their overall corporate strategy. What conventional factories needed was a manufacturing strategy to be consistent with those business requirements necessary to achieve success and remain competitive; the idea of focus provides this.

Focused Factory Systems (or focused manufacturing as some authors refer to it) call for a simplicity of tasks so that companies will employ their efforts and allocation of assets on those things that they do well. In spite of some reported benefits like low costs, increase in productivity, quality levels, reliability and speed, not a lot has been said about a specific method to help managers decide on how to regroup products in a manner that will assure the acquisition of those benefits.

However, recent studies (Berry et al. 1991) came to our attention that analyzed market-based clustering methods to group products into focused factories based on operation variables, such as order-winning criteria, that would best support different markets. They proposed a two-stage methodology in a case study about a circuit board company. In the first stage, a hierarchical clustering model was used as a guide to get an idea of similarities between products and the number of group products suggested by a tree diagram called a dendogram. The second stage used a k-Means Refinement Model to group products based on similarities (or dissimilarities) across the different variables.

The data from this circuit board company was used to explore the performance of other clustering algorithms, such as the Plant Location Model, the Covering Model, the Fuzzy Set Method and the Average-Weighted Distance Model. All can be found in the literature of cluster analysis, except for the Average-Weighted Distance Model, which was created by this research. Although many clustering algorithms are available, these methods were selected because of their different approach on how to assign objects into clusters. In a second case study, these methods are also used to examine the grouping of similar machined parts into manufacturing cells.

Chapter 2 presents in the Literature Review information relevant to the main topics used in this research: Focused Factories, Cluster Analysis and Cellular Manufacturing. Chapter 3 describes each of these clustering models, including the methodology proposed by Berry et al., as an extension to the circuit board example used in Berry's paper. The chapter ends with conclusions and observations of this study's findings. Chapter 4 explains an interesting case in the cellular manufacturing environment in which other real-world considerations, besides processing similarities, are taken into account when grouping parts into cells. Some observations are made regarding the results. Chapter 5 draws general conclusions about the different clustering methods described and explains some possibilities for the variability in the outcomes. The various appendices complement the information presented in this thesis, such as input and output data files used to run the models among several computer programs. In addition, some information regarding the original experimental study used in the cellular manufacturing chapter is shown in this section. Throughout this thesis, the terms groups and clusters, as well as objects and members, have been used interchangeably.

Chapter 2

Literature Review

This chapter is divided in three parts: focused factories, cluster analysis and cellular manufacturing. The first part uncovers main issues that gave origin to the focused factory concept, from its early beginnings to date. The second examines the essence of cluster analysis, as well as the various algorithms and approaches used in this thesis. In addition, this second part concludes with an explanation of the links between cluster analysis and focused factories. The third part gives a brief background on how cellular manufacturing came into play as a philosophy adopted by companies, including U.S. industries. Further, the cell formation problem is described along with a list of the work done in this area. Part three of this chapter also refers to some links between part one of this chapter, that is, the so-called cellular manufacturing and focused-factory systems.

I. Focused Factories

- **Why focus? A new management approach: focused manufacturing**

The Focused Factory concept was first introduced by Wickham Skinner whose experience in U.S. companies since the 1960's led him to observe that the productivity crisis present in the 1970's called for a revision of the manufacturing task. In his article "The Focused Factory," Skinner points out that "while the economy had moved toward an era of more advanced technologies and shorter product lives, we have not readjusted our concepts of production to keep up with these changes" (121). The problem is that "guided by the principles

of economies of scale…too many companies attempt to do too many things with one plant…..” , which can only increase complexity and confusion (116).

He defines Focused Manufacturing as a new management approach that can help companies manage the manufacturing requirements created by a broad mix of products, volumes, specifications, and customer demand patterns (117). According to Skinner, “ such a plant can become a competitive weapon because its entire apparatus is focused to accomplish the particular manufacturing task demanded by the company’s overall strategy and marketing objective” (114). Therefore, “one of the first tasks towards acquiring focus is to decide on how to compete,” for example, based on short delivery cycles, superior quality and reliability, rate of new product introduction, flexibility in adjustments to volume changes, low investment, and low costs, among other things (115).

Moreover, “repetition and concentration in one area allow its managers to become effective and experienced in the task required for success. Its problems are demanding, but limited in scope. Therefore, the focused factory is manageable and controllable” (116). It is based on the concept that simplicity, repetition, experience and homogeneity breed competence (115).

In the words of Skinner: “Even if the focused plant is a rarity, seeing the problem as one of “how to compete” can broaden management’s horizon. In fact, resources for simplifying the focus of a manufacturing complex are not hard to acquire when the expected payoff is the ability to compete successfully, using manufacturing as a competitive weapon” (121).

In referring to conventional plants he thinks that “the plant’s manufacturing policies are not designed, tuned and focused as a whole on that one key strategic manufacturing task essential to the company’s overall success in its industry” (117). “Conventional factories, indeed, produce

many products for numerous customers in a variety of markets, thereby demanding the performance of a multiplicity of tasks all at once from one set of assets and people" (114). Hayes and Wheelwright support this argument. When talking about the different dimensions in which a company can compete they argue that "it s difficult to offer superior performance along all of these dimensions simultaneously, since it will only be a second best to some other company that devotes more of its resources to developing that competitive advantage" (41).

In 1989, Terrence Hill took on the concept of focused factories first discussed by Skinner and argued that "being competitive in different types of markets is a prerequisite for a company today and the concept of focus offers principles to achieve this" (148). Hill defined focused manufacturing as a philosophy that "concerns linking an organization's manufacturing facilities to the appropriate competitive factors of its business" (142).

- **Principles and Concepts**

The focus factory idea can be contemplated as a return to the early days of manufacturing when plants were small and produced a single product. Because of its size, managing and keeping track of the costs becomes an easy and efficient task. When interviewed for the article 'Focus' plants sharpen profit, Nabil Nasr stated that the concept of focus factory is really simple: "We're focusing a specific group of people and equipment together into making a certain product" (qtd. in Zelickson, 1996).

At the core of the first part of this thesis lies the notion reported by Hill that "origins of existing plants are often driven by the concepts of economies of scale and control through specialists; thus the resulting manufacturing plants are typically large, with process layouts based on functional similarity and centralized infrastructure." That is, "processes are often shared to

facilitate higher levels of equipment utilization, and the firm's organization combines the line/executive function with appropriate specialists undertaking advisory and support roles" (157). "The rationale behind this," according to Hill, "stems from the principle of economies of scale, which for many years has been a sound and appropriate way of organizing and managing businesses" (142).

One tradeoff caused by economies of scale is that despite the gains associated with reducing certain costs, it enables other costs to increase. These costs are known as diseconomies of scale, such as distribution diseconomies that result from shipments to a larger geographic area; diseconomies of bureaucratization; diseconomies of confusion that are the consequence of the increase in the number of products, processes, and specialists in a given plant; and finally, diseconomies of vulnerability to risk (Hayes and Wheelwright 63-64).

"The discussion of the economies and diseconomies of scale," as Hayes and Wheelwright further point out, "leads to the notion of an 'optimal plant size'." "Management's task is then to select the size that makes it possible to manufacture its products at a low total delivered cost, or the one that promises the highest total profits". However, "determining the best size for a facility is difficult because it depends on a number of situational factors, such as the facility's age and its general environment - labor, government, market, competition, and technology – all of which change over time". One of the reasons managers prefer smaller facilities is that it counteracts the confusion and complexity of larger plants, which reflects management's need to balance the economies and diseconomies of scale (65).

In Schroeder's words: "with markets characterized by difference rather than similarity, it is essential that firms learn to effectively compete in market segments that are different from one another and the concept of focus offers principles by which to achieve this" (qtd. in Hill 154).

Moreover, "economies of scale are most appropriate for high-volume and steady state markets," where in many cases the opposite applies: markets are low-volume and dynamic in nature. The key reasons why the conditions have changed are discussed by Hill and summarized below (143-144):

"Marketing-led strategies are usually based on principles of growth through extending the product range" by introducing new ones on existing processes and within the infrastructure. "Over time," Hill continues, "the incremental nature of these marketing changes will invariably alter the manufacturing task, causing complexity, confusion and a production organization that lacks focus".

- "When capacity is released due to a falloff in demand for a product (s), companies typically reutilize the spare capacity," supporting the argument of high plant utilization, "by introducing new products." "However, when evaluating the suitability of processes for a product, it is necessary for the companies to check not only that technical specifications are met, but the consistency of the business requirements over time, as products go through their life cycles and relevant order-winners accordingly change". Order-winners are explained in the next section.

- **What Focus Means According to Skinner and Hill**

The assumption that underlies the notion of facilities focus is that a factory or any organizational entity will perform better when assigned a narrower, more clearly defined set of tasks than it would when required to perform a broader and ambiguous set of tasks (Hayes and Wheelwright 111). Moreover, focusing a facility is one approach to reducing the diseconomies

of scale and the good news is that a growing body of empirical evidence is helping to clarify the potential value of such focus, as will be described later.

Hill comments about this argument: "the emphasis here is on a limited and consistent set of tasks, which will often be far from a layman's definition of narrow. So to avoid confusion, the dimension of narrow should be omitted" (142).

On the other hand, as outlined by Skinner, the emphasis on focused manufacturing must be on building competitive strength that can be achieved by "centralizing the factory's focus on relative competitive ability; avoiding the common tendency to add staff and overhead in order to save on direct labor and capital investment; and letting each manufacturing unit work on a limited task instead of the usual complex mix of conflicting objectives, products and technologies" (119). Table 1 provides a means to distinguish between the two authors' opinions:

Table 1

Skinner vs. Hill Focus Idea

Skinner	Hill
"A factory that focuses on a narrow product mix for a particular market niche will outperform the conventional plant, which attempts a broader mission. Such plant can become a competitive weapon because its entire apparatus is focused to accomplish the particular manufacturing task demanded by the company's overall strategy and marketing objective" (114).	"It is the homogeneity of tasks and the repetition and experience involved in completing these that form the basis of focused manufacturing. Thus, focusing the demands placed on manufacturing will enable resources, efforts, and attention to be concentrated on a defined and homogeneous set of activities, allowing management to prioritize the key tasks necessary to achieve a better performance."
"Learning to focus each plant on a limited, concise, manageable set of product, technologies, volumes and markets".	Focus should not be seen as the preferred dogma on which to base manufacturing arrangements.
"An ideal focused factory has its own facilities, processes and infrastructure dedicated to producing products to a rationalized market criteria".	Not all parts of the business can be successfully focused, but the unfocused part will be smaller and easier to improve by other methods.
"Learning to structure basic manufacturing policies and supporting services so that they focus on one explicit manufacturing task, instead of on many inconsistent, conflicting, implicit tasks."	The underlying theme when focusing a business must be pragmatism. Thus, if one approach to focus does not bring improvement in a particular situation, that facet should not be applied. The process of change needs to be ongoing.

In achieving focus, companies go through the following steps: process review, identification of order-winners and qualifiers, clustering products based on order-winners and qualifiers process rearrangement and infrastructure rearrangement (160-161). From all of these, Hill especially stresses the need to identify order-winners and qualifiers as a way to understand differences between markets. What Hill refers to as qualifiers, summarized in Berry's article, are those customer requirements for which a certain level of performance is necessary from all companies competing for the business, such as reliable delivery and minimum quality levels. Order-winners, on the other hand, are those requirements of superior performance which would differentiate one firm from the others (365).

The ideal will be that the manufacturing task would provide the appropriate conditions to support order-winners.

- **Alternative Approaches to Focused Manufacturing**

Thus far, some of the advantages of focused manufacturing have been pointed out. As Hill says: "Narrowing the range of demands placed on a manufacturing facility will lead to better performance because management attention can be concentrated on a few key tasks and priorities" (142). In reality, there are some tradeoffs associated with downsizing, like the duplication of certain processes or parts of the infrastructure. Hill suggests that companies adopt the approach or combination of approaches that best fit the different alternatives proposed (146).

1. <u>Focus Based on Products/Markets</u>

This orients relevant parts of the manufacturing facilities toward a particular customer, generic group of products or industrial sector. This approach is often referred to as a customer-

driven approach, because products are divided depending on market segmentation. A major strength of this approach is that it can turn the development of a facilities strategy into a proactive process, creating and capitalizing on opportunities rather than simply reacting to the changing requirements for existing products and markets.

2. Focus Based on Processes

This results from grouping similar products on the basis of similarity of manufacturing technology. The goal is to improve utilization of the manufacturing processes while taking advantage of the concentrated expertise. Two characteristics of this approach are related to one another: it minimizes plant duplication, but at the same time results in underutilized facilities in terms of capacity for smaller plants.

3. Focus Based on Order-Winners

The main objective of this approach is to help companies arrive at strategic decisions by reviewing its market order-winning criteria: price, delivery reliability, delivery speed, quality, demand increases, and product range (47-61). This approach can also be viewed from a benchmarking perspective, which offers an important dimension within the domain of order-winners by ensuring that corporate performance is measured against externally derived best-practice norms (Hill 71).

According to Hayes and Wheelwright, what makes the focus concept so powerful and demanding is that it can incorporate the specific demands on, and capabilities of, each facility (90). Some arguments for and against different approaches to focus are listed in Table 2.

Table 2

Advantages and Disadvantages of Alternative Approaches to Focusing Facilities

Advantages	Disadvantages
Volume Split (high volume vs. low volume)	
Exploits economies of scales, where appropriate	Duplication of production processes, overhead, and inventories
Permits focusing on either cost effectiveness or production flexibility	Low volume plants can become orphans if not monitored carefully
Encourages customized development of production and management systems for products at different stages of their life style	
Product / Market Split	
Very responsive to market/customer needs and priorities	Duplication of resources across several facilities
Facilitates new product introduction	Product transfers become awkward
Permits specialization by market segment	Tendency to become unfocused as market shifts (high and low volume products produced in same plant)
Simplifies product cost estimation	Load imbalances develop as different markets grow at different rates
	Less emphasis on, and concentration of, technical skills in market-dominated environments
Process Split	
Concentrates technological expertise	Impedes radical changes in products or processes
Less duplication of equipment for producing common parts	Slows organization's response to totally new product/market requirements
Easier to balance loads among plants and keep utilization high	Longer cycle times and large pipeline inventories
Can develop customized process control systems	Higher cost of coordination
Encourages standardization	

Source: Hayes, Robert and Wheelwright, Steven. Restoring Our Competitive Edge (1984) Table 4-3.

- **Benefits and Tradeoffs of Focused Manufacturing**

According to Hill, "the choice of [a] focused manufacturing approach implies tradeoffs." "In arriving at this decision, companies must distinguish between the gains associated with size reduction and manageable units and those obtained from the approach adopted toward achieving focused manufacturing." "Smaller plants will bring advantages such as: the potential for improved communication; greater orientation toward a well-understood and agreed set of business objectives; simpler and more appropriate forms of control and managerial style; higher levels of employee participation and motivation; shorter process lead times; lower work-in-process inventory; reduced complexity of the production-control task; and a more accurate

assessment of financial performance. On the other hand, smaller plants run contrary to the concept of economies of scale and may well lead to the need to increase or duplicate certain processes or parts of the infrastructure such as procedures and specialist capabilities" (146).

- **Separate Plant vs. Plant-Within-a-Plant Focus**

As previously discussed, one important trade-off in focused manufacturing is that of plant size versus the economies of the organization scale (Hill 148; Pesch 34). Hill elaborates: "although the ideal would be plants individually focused to the needs of markets and arranged on the basis of alternatives explained before, this is often not practical because of the sizable investments of many companies, and also because of the lack of focus generated when businesses change" (148).

Even Skinner, as well as other authors, agree that in order to break down the present complexity, an implication of this concept seems to call for major investments in new plants, new equipment and new tooling. In this case, a more practical approach may be the "plant-within-a-plant" (PWP) notion in which the existing facilities are divided both organizationally and physically into several PWPs. Each of them can concentrate on its own particular manufacturing task, using its own workforce management approaches, production control, organizational structure, and so forth (Hayes and Wheelwright 103; Hill 148; Skinner 121). This reduces units to a more manageable task, incorporating the advantages of both focus and smaller size. In addition, this not only provides significant gains in sustaining, qualifying and meeting the order-winning criteria, it also decreases the likelihood of the agreed focus being undermined, which is known as focus regression (Hill 148-149).

II. Cluster Analysis

- **Concepts**

To market researchers, cluster analysis has been defined as an exploratory tool to divide markets into groups of customers showing similar buying behavior (Berry et al 363). In fact, clustering methods are the most appropriate interdependence statistical techniques for segmenting markets (Myers 68). Green and Tull define these methods as follows:

> The usual objective of cluster analysis is to separate objects into groups such that each object is more like other objects in its group than like objects outside the group. Cluster analysis is thus concerned ultimately with classification, and its techniques are part of the field of numerical taxonomy (qtd. in Myers 68).

Aldenderfer and Blashfield elaborate: "More specifically, a clustering method is a multivariate statistical procedure that starts with a data set containing information about a sample of entities and attempts to reorganize these entities into relatively homogenous groups" (qtd. in Myers 68).

However, the concept is more general and can be applied to other problems: a technique that classifies objects based on some measure of similarity between the objects. Kaufman and Rousseeuw interpreted cluster analysis as: "the art of finding groups in data," which is the title of their book (1). Their argument is basically the same as the ones mentioned above, except that the latter emphasizes the dissimilarity of objects joining different groups. Dissimilaries refer to nonnegative numbers used to indicate how remote two objects are (Kaufman and Rousseeuw 16, 20).

- **Cluster Analysis Decision-Making Process**

Various steps need to be followed when conducting a cluster analysis study: preparation of raw data, choice of distance metric, choice of clustering algorithm, and choice of criteria for best method. Under each step there are also different choices the analyst has to make, which may impact the results. Following is a description of these four steps and the different choices under each one.

Step 1: Preparation of Raw Data

There are various ways to handle the data:

a. **Normalize Variables.** This approach takes entire population and the largest value in a column is set to 1.00, thus the data ranges from 0 to 1. The purpose is to provide uniformity in the data when expressed in different units.
b. **Standardize Variables.** Variables are standardized by subtracting the means and dividing by the standard deviation before the distance matrix is calculated, which is a good idea if variables are in different units and it is desired to minimize the effect of scale differences. Many statistical packages like Minitab can do this.
c. **Use Raw Data.** This approach is suitable for variables using the same units.

Step 2: Choice of Distance Metric

The distance metric is a measure of similarity or dissimilarity between observations. It represents the p-space distance between two observations, where p represents the number of key variables. The selection of such distance depends on the properties of the data. Some choices include:

a. **Euclidean Distance**. Represents the standard mathematical measure of distance (square root of average of the sum of squared differences).

b. **Average-Weighted Distance.** This metric is a variation of the Euclidean distance in which the sum of squared differences is multiplied by the average of the two observations. The aim is to minimize the scaling effect of points is scale.

c. **Manhattan Distance.** This method calculates the sum of absolute differences, so that outliers receive less weight than they would if the Euclidean method were used.

d. **Weighted Euclidean Distance.** Each variable receives a weight according to its perceived importance.

e. **Pearson.** This method is used for standardizing and represents the square root of the sum of square distances divided by variances.

Step 3: Choice of Clustering Algorithm

Over the last 30 years, a wealth of algorithms and computer programs has been developed for cluster analysis. The reason for this diversity is that there exists no general definition of a cluster, and in fact there are several different kinds: spherical clusters, drawn-out clusters, linear clusters and so on. Moreover, different applications make use of different data types, such as continuous variables, discrete variables, similarities and dissimilarities. Therefore, different clustering methods are needed in order to adapt to the kind of application and the type of clusters preferred. Several algorithms are provided for transforming the data to perform cluster analysis and display the results graphically (Kaufman and Rousseeuw 3).

Analysts often experiment with more than one algorithm and then compare the results, an approach supported by Kaufman and Rousseeuw:

"It is permissible to try several algorithms on the same data, because cluster analysis is mostly used as a descriptive or exploratory tool, in contrast with statistical tests, which are carried out for inferential or confirmatory purposes. That is, we do not wish to prove a preconceived hypothesis, but to see what the data is trying to tell us" (37).

In some applications, changing the measurement units may lead to seeing a very different clustering structure, so to avoid this dependence, there is the option of standardizing the data by converting the original measurements to unitless variables. This approach does not really solve the problem however, because expressing a variable in smaller units will lead to a larger range for that variable, which will then have a larger effect on the resulting structure. This is better addressed by computing distances between the objects, in order to quantify their degree of dissimilarity. The most popular choice is the Euclidean distance, which generates a distance for each pair of objects i and j (Kaufman and Rousseeuw 5-6, 11). It is formulated as:
$Dij = \sum\sqrt{[(i1 - j1)^2 + (i2 - j2)^2]}$, where Dij = distance between two respondents, and i and j are coordinates (ratings) or attributes 1 and 2, across all attributes.

There are several different types of clustering methods. Everitt identifies five generic types, each having several specific algorithms (computer computation procedures) from which to select. However, in practice, only two major types are readily available in computer programs for most marketing researches; they are partitioning methods (also known as nodal methods) and hierarchical methods (also known as linkage methods). An interesting observation is that partitioning methods are the ones most widely used in market segmentation research today (qtd. in Myers 68-69).

3.1 Hierarchical Clustering

Hierarchical algorithms do not construct a single partition with k clusters, but they deal with all values of k in the same run. That is, the partition with k = 1 (all objects are together in the same cluster) is part of the output, as is the situation with k = n (each object forms a separate cluster with only a single element) (Kaufman and Rousseeuw 44).

The process starts by finding the single pair of respondents who have the shortest distance between them. Such dissimilarity coefficients may be computed using the Euclidean distance previously described. However, it is often difficult to spot a structure in a data set by merely looking at its dissimilarity matrix. For that reason, all hierarchical clustering programs also produce tree-like structures called dendograms (Kaufman and Rousseeuw 200, 202; Myers 115, 118).

There are two kinds of hierarchical techniques: the agglomerative and the divisive. They construct their hierarchy in the opposite direction, possibly yielding quite different results. For instance, agglomerative methods start with all objects apart, so one could say that that each object forms a small cluster of itself (that is, at step 0 we have n clusters). Then in each step, two clusters are merged until only one is left (Kaufman and Rousseuw 44).

One obvious advantage of hierarchical methods is that it provides dendograms, which are graphical representations of the order in which clusters are formed, also giving an idea of similarity between data points. On the other hand, there are some drawbacks, as expressed in the literature by Kaufman and Rousseuw: "First, graphical representation only makes use of the dissimilarity matrix and not of any measurements. Second, making dendograms is not as easy as it looks, because the algorithm often has to change the order of the objects to avoid crossing

lines" (205-206). Strengths and weaknesses of hierarchical clustering are summarized in the following table (Table 3) (Myers 128-129):

Table 3

Strengths and Weaknesses of Hierarchical Methods

Pros	*Cons*
"Because hierarchical clustering starts by joining cases and clusters that are known to be the most similar, it has a better chance of pointing the clustering effort toward the highest concentration of respondents, thus producing better clusters at the early stages of the process."	Clustering starts by producing a complete matrix of distances that results in large numbers of similarity measures that must be generated before the clustering can begin.
"Hierarchical clustering can be used as starting points for partition clustering."	"These plus other derived measures must be searched repeatedly as each step of clustering proceeds, and minimum distances must be determined over and over."
"Enables the decision-maker to visualize the linkage process, how they "build up" and what joins what at each stage."	Stages are irreversible.
"The decision-maker does not have to state in advance how many clusters are desired, as it can be decided as the dendogram builds."	"Dendograms for large numbers of respondents are awkward to handle physically and are especially difficult to interpret."

The use of hierarchical methods is optional, in which case the analyst decides on the linkage method. The linking method uses a distance metric and a joining rule to determine which observations or clusters will be linked together, and in what order. Several methods are available and depending on the characteristics of the data, some methods may be better choices than others. Thus, it is recommended to try different methods and compare the results. Some of the most commonly used linking methods include:

a. **Single Linkage.** Also known as "nearest neighbor," this linkage rule finds the minimum distance between an observation in one cluster and an observation in the cluster, which is a good choice when clusters are clearly separated.

b. **Complete Linkage.** Also known as "furthest neighbor," as opposed to the single linkage, this method finds the maximum distance between an observation in one cluster

and an observation in another cluster. This method ensures that all observations in a cluster are within a maximum distance and tends to produce clusters with similar diameters.

c. **Average Linkage.** This method computes the average distance between all possible pairs of objects in different clusters, and joins the observations with smallest average distance.

d. **Ward's Linkage.** With this method, the distance between two clusters is the sum of squared deviations from points to centroids. The objective is to minimize the within objects sum of squares.

3.2 Partitioning Methods

Whereas hierarchical clustering provides the analyst with a number of clusters, partition clustering, as Myers says, starts by requiring the decision-maker to state in advance how many clusters are to be formed. This is necessary because most of these algorithms are designed to produce only the most homogeneous clusters of some specified number, rather than to determine the number of clusters. What is needed is a computer program that does a preliminary sort through the data and indicates how many clusters exist (70). Hence, the data is classified into k groups: each group must contain at least one object and each object must belong to exactly one group ($k \leq n$) (Kaufman and Rousseeuw 38).

Since the number of groups, k, is fixed by the decision-maker, the process starts by initially dividing the sample into the desired number of cluster groups. Because there is not a clear idea of how homogeneous these groups are at this point, Myers suggests two generic ways of doing this (72-73):

- *Identify a single case (cluster seed) to serve as a temporary center of each cluster.* This is done in a two-step procedure. The first selects initial seeds for each of the number of clusters desired; the second calculates the distance from each of the remaining points to each of the seeds and then assigns each case to the nearest seed to form the initial clusters.
- *Sort the cases into groups on an a priori basis.* This could be done on the basis of some kind of previous cluster analysis of the same data (hierarchical clustering) or simply asking the computer to assign cases randomly into the required number of groups. Punj and Stewart suggest that a 2-stage cluster analysis methodology is preferred, that is, to apply a hierarchical clustering to identify good starting seeds prior to partition clustering (qtd. in Myers 73).

One important remark is that other authors such as Berry et al., who followed a two-step clustering method for demonstrating the application of cluster analysis using operation variables, share this last view. Several algorithms are available, but this research limited the selection to five partition methods: Agglomerative Hierarchy with k-Means Refinement Model, Plant Location Model, Covering Model, Fuzzy Set Method and Average-Weighted Distance Model.

Trying to understand the clustering models discussed throughout this thesis can be confusing when viewed separately. Therefore, Table 4 is constructed with the hope of clarifying and better illustrating the differences between these models and particularly, to show how they come up with different decisions to achieve their objectives.

Table 4

Summary of Clustering Methods Used

Clustering Method	**Outcome**	**Decision Process Summary**
Agglomerative Hierarchy w/k-Means Refinement	Assignment of objects to k groups, where k is chosen by decision-maker and cluster member variance is minimized.	1. Compute distance between all members. 2. Perform average linkage method until all objects are included into one all-inclusive cluster. 3. From dendogram, select a number of cluster groups. 4. Perform k-Means algorithm.
Plant Location Model	Assignment of objects to k groups, where k is chosen by decision-maker and objects of a group are chosen to minimize the distance between all members and the medoids of each cluster (group).	1. Compute distance between members. 2. Choose number of groups (k). 3. Solve binary integer optimization model that minimizes the distance between all members and the medoids of each cluster.
Covering Model	Assignment of objects to k groups, where k is an outcome of specifying a distance (D) from all objects to the nearest representative object.	1. Compute distance between all members. 2. Choose how close members must be by specifying the diameter of the cluster (D). 3. Solve binary integer optimization model that insures that all members are within a distance (D) or less from the representative object.
Fuzzy Set Method	Given the number of groups (k), the percent membership of each object to each group is computed.	1. Compute distance between all members. 2. Specify the number of groups (k). 3. Perform fuzzy set algorithm. 4. High percent membership indicates to which cluster an object should be assigned. 5. Use additional decision processes to determine to which group the objects with medium percent membership are assigned.
Average-Weighted Distance Model	Assignment of objects to k groups, where k is chosen by decision-maker and objects of a group are chosen to minimize the distance between all members and the medoids of each cluster of objects.	1. Compute average-weighted distance between all members to minimize scaling effect. 2. Choose the number of groups (k). 3. Solve binary integer optimization model that minimizes the distance between all members and the medoids of each cluster.

Step 4: Selection Criteria for Best Method

When evaluating the suitability of different methods, it is advisable to try different measures of performance for validation purposes. Included in this thesis are the average standard deviation and average sum of squares across the different variables for all solutions.

All the steps and the choices under each step are illustrated in Figure 1. It is worth to say that the steps aforementioned does not necessarily mean that a person interested in conducting a cluster analysis study should follow all of the choices under each step. This roadmap only suggests that there are many ways in which a specific problem can be analyzed and interpreted depending on the combination of choices selected. Therefore it is important for the analyst to understand these choices and to become familiar with the situations that seems to be more appropriate to use them.

However, because this research is only interested in examining the performance of the different partitioning methods, the choice selection is limited. The Euclidean distance was used to obtain the dissimilarities between products (in Chapter 3) or parts (in Chapter 4) for all the methods except for the Average-Weighted Distance Model that used the Average-Weighted Distance metric. Then, a two-stage clustering is conducted: hierarchical clustering using the average linkage method prior to evaluating each of the partitioning methods described earlier. The five partitioning methods were compared against the average standard deviation and the average sum of squares for all the solutions across all the variables.

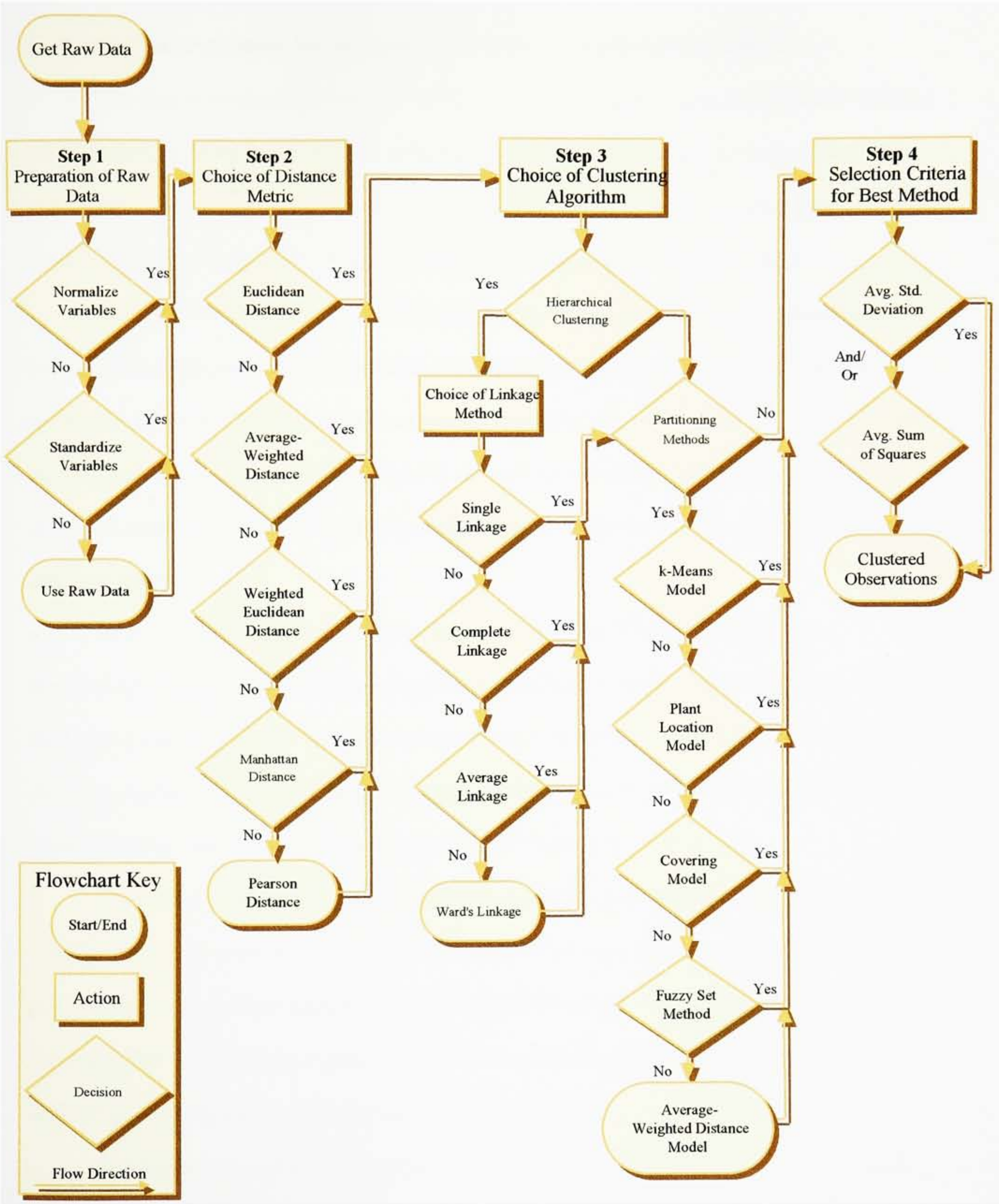


Figure 1. Roadmap to Cluster Analysis

- **Using Cluster Analysis for Segmentation Based on Operation Capabilities**

The idea of using cluster analysis to define market segments based on operation variables was first developed by Berry et al., as defined in their paper: "We propose a methodology that uses cluster analysis to categorize a set of products that are representative of a business's market, based on their similarity or dissimilarity across a set of operations variables" (366).

Their field of research indicates that the practice of segmenting markets has enabled businesses to implement targeted marketing strategies that yield important benefits such as products designed especially for different market segments. What is often not apparent to either marketing or operations managers is that it is difficult for a single operating unit to develop superior competitive capabilities serving market segments that require very different operations strategies (363).

Moreover, they believe that market segmentation is an effective technique for determining plant focus, a view that is supported by Skinner's original argument in developing the focused factory concept: "Focused manufacturing must be derived from an explicitly defined corporate strategy that has its roots in a corporate marketing plan, and that the choice of focus cannot be made independently by production people" (qtd. in Berry et al. 364).

In an effort to determine the best market-based focus, they adopted the manufacturing strategy framework reported by Hill. This framework provides one way of integrating marketing and operations (manufacturing) perspectives during the formulation of corporate strategy. In particular, Hill's use of order-winners and qualifiers provides a means of specifying the data required to segment markets for operations purposes (364).

It is important to point out that Hill's framework provides the basis for Berry's study in the sense that the latter extends the concept of order-winners for a market segment to include

additional key operational variables like process technology factors, volume factors and infrastructure. These order-winners, along with operation volumes and process information, are identified through discussion with general management and key marketing and operations executives (Berry et al. 365-366).

The goal of this clustering-based methodology is to break up operations into focused product groups in a two-step clustering approach. In the first step, a hierarchical clustering model is used to generate a dendogram that graphically illustrates the linking patterns between sample orders. Then, based on the results of the first step, a k-means clustering is used to identify the major operation-based segments found in a business's markets. In Chapter 3, the results obtained by Berry's study are analyzed and compared with other clustering models that differ in the decision approach to determine which products should be assigned to each focused factory.

III. Cellular Manufacturing

- **Background**

While there are many ways in which companies can arrange its products, according to Dhavale, the conventional view is that manufacturing systems are divided into two major types: process (or flow shops) and job shops. The former makes only one or a few products, but the demand is large enough to support the purchase of specialized machines. Job shops on the other hand, also known as functional-layout shops, group machines into departments based on machine functions (3).

This conventional view is often no longer adequate because there are many factors that affect success, such as global competition, rapid technological advancements, shorter product life cycles, increased customer focus and fast growing economies in third world countries.

Other authors, such as Wemmerlov and Hyer, have examined this phenomenum and found that the increased competition faced by U.S. manufacturers since the late 1970's has made them very receptive to ideas that promise improved competitiveness. Their paper that reports the finding of a survey study of 32 U.S. firms reveals that, whereas much attention has focused on setup reduction, pull systems and the application of statistical process control, other building blocks have not received the same attention. They refer to the group technology (GT) philosophy and in particular, its application to manufacturing systems called cellular manufacturing (Modie and Uzsoy, 5).

Wemmerlov defines cellular manufacturing as "an application of GT where a portion of a firm's manufacturing system has been converted to cells. A manufacturing cell is a cluster of dissimilar machines or processes located in close proximity and dedicated to the manufacture of a family of parts (a cell family). The parts are similar in their processing requirements (required operations, tolerances, machine tool capacities, etc)." Further, "the aim of cellular manufacturing is to reduce setup times (by using part-family tooling and sequencing) and flow times (by reducing setup and move times, wait time for moves and using small transfer batches) and therefore, to reduce inventories and market response times" (qtd. in Modie and Uzsoy, 5).

In cellular manufacturing, according to Dhavale, machines in cells are laid out so that parts have to follow only one sequence of machines, thus replacing the functional layout shops to improve manufacturing operations (3, 5).

- **Cell Formation Algorithms**

Roze and Wang summarize the cell formation steps as follows: "the design of a cellular manufacturing system usually begins with two fundamental tasks, part-family formation and

machine-cell formation. Part-family formation is to group together parts with similar geometric characteristics or processing requirements to take advantage of similarities for the design or manufacturing purpose. Machine-cell formation is to bring dissimilar machines together and dedicate them to the manufacture of one or more part families" (567).

Moodie and Uzsoy analyzed different studies that have shown no guarantee that a cellular manufacturing system will outperform a traditional system. Hence, the manner in which the manufacturing cells are established and parts are assigned to the cell is an important component of the ultimate success or failure of a cellular manufacturing system design (61).

The aforementioned authors illustrate some of the different approaches to the cell formation problem and concluded that overall, the cell formation problem in group technology has been addressed by a wide range of approaches and different techniques, including matrix manipulation, clustering techniques, integer programming, network flows, graph theory, expert systems, fuzzy mathematics and neural networks. However, while researchers have broadly developed algorithms capable of addressing different concerns, there is still a strong need for the development of a clear framework within which to evaluate the performance of these algorithms and to provide guidelines for which algorithm is appropriate in what circumstances.

- **Links between Cellular Manufacturing and Focused Factories**

There is a connection between cellular manufacturing and focused factories in the sense that, referring to Skinner's work, repetition and concentration in one area allows people to become effective and experienced in the task required for success (116).

In fact, according to Tatikonda and Wemmerlov, "group technology urges simplification and standardization of similar entities (parts, assemblies, process plants, etc.) in order to reduce

complexity and achieve economies of scale effects in batch manufacturing" (qtd. in Modie and Uzsoy, 26).

These authors have thus far considered cellular manufacturing and focused factories to be independent concepts. However, Dhavale considers grouping similar parts without regard to their end use to be a shortcoming. He prefers to think of a cell, or a collection of cells, designed not to make a family of similar parts but to make similar products (8-9). The rationale behind this theory is that the inherent inefficiencies of a job shop environment can be eliminated by evolving from "start-a-part, finish-a-part" operation. That is, once the product is started, the cells in a focused factory work only on that product until it is completed and shipped to a customer.

Dhavale defines cellular manufacturing and focused-factory systems as a group of cells that manufacture and perform assembly, inspection, and testing, and combine the flexibility of a job shop with the manufacturing efficiencies of an assembly line. "Focused factories are very conducive to worker empowerment because they operate as separate units independent of each other, and production management problems tend to be less complex. Cellular manufacturing needs a labor force with the ability to operate multiple machines; a trained labor force improves a cell's ability to handle imbalances and bottlenecks" (qtd. in Freedman, 63).

Further, Dhavale analyzes different factors that need to be considered to successfully implement cellular manufacturing: product mix (companies with an ever-changing mix of products also have used CM with great success), size of factories (the smaller size allows the workforce and equipment to remain adaptable enough to respond to the vagaries of the market), production volume (a CM system should be able to handle any capacity and changing lot sizes), and management and worker commitment (22-23).

With respect to production volume, Dhavale points out that "companies that face unstable demands for their products will experience unbalanced cells; that is, some cells will be idle while others experience back orders." He then examines some possible solutions such as reformulation of the cells to meet market demand, outsourcing the excess demand, redefining product lines so the demand within each of them is more stable, building excess capacity in the cell, and improving forecasting techniques to provide an adequate warning of changes (23).

It is obvious now that demand is another variable to be considered when grouping parts into cells. However, the literature has shown an extensive body of work that groups machines based on processing capabilities. Although embedded, it excludes other real world constraints that must be incorporated while designing cellular manufacturing systems.

Cell formation in practice must account for many technological, organizational and economic considerations as well. In the words of Rajagopalan and Batra: "It should be emphasized that for such an intensely complex and practical problem, any mathematical approach can only provide guidelines for making decisions. There are many intangibles in this situation, e.g., savings in the learning time of the operator, higher level of production control, the human factor, etc. which cannot readily be quantified and built into a mathematical model" (qtd. in Moodie and Uzsoy, 68-69).

The purpose of Chapter 4 is to analyze the performance of the different clustering algorithms discussed in Chapter 3 to form part cells based on dissimilarity coefficients. This can be viewed as an extension of previous research and presents an experimental study on part grouping for cellular manufacturing.

Chapter 3

Application of Clustering Methods for Focused Factories

This section provides a comparison of the different clustering models applied to approach focused manufacturing in a printed circuit board company. The data utilized is extracted from Berry's original study (367-372). Berry demonstrated his methodology using 3 different companies: two manufacturing and one from the service sector known as companies A, B, and C, respectively. This thesis shows the analysis on Company A.

- **Problem Statement**

The company under study serves various markets, including television, radio, video, appliance, and vending machine products. Using product type and life cycle criteria, the manufacturing organization was divided into four distinct units:

1. a pilot line for the development and introduction of new products,
2. a "catch-all" manufacturing unit,
3. a unit designed to handle circuit board requirements for the vending machine market, and
4. a Spares unit.

In the past, products remained in the manufacturing units to which they were originally assigned, despite changes in the volume levels and market requirements. Recently, management decided to reexamine the product groupings because of declining delivery performance and a shortage of skilled employees for the pilot line.

Management identified, through market analysis, three order-winners that characterized demand for their products: price, delivery speed and quality. Order volumes varied dramatically

within the sample. The projected average weekly production times in two years ranged from 1 hour per week for one spare part item to over 913 hours per week for a new product. The net result is that management had three factors to consider: order-winners, shifts in order-winners over time, and volume differences between a sample of products and their current manufacturing unit assignments. Table 5 shows the projected order-winners and volume measures for the representative sample of products and their current manufacturing unit assignments.

- ***Coding and Assigning Weights to the Key Variables***

The data for each product consists of four variables projected over the next two years: price, delivery speed, and quality order-winners, and production volume. Order-winner weights ranged from zero to 100, with a maximum of 100 points per order. The weights were based on management's judgements and were checked against available market data. Because of important volume differences between the OEM and spares business, the projected average weekly production was used to classify products as high or low volume products. If a product was considered to be a high volume item (mean weekly production $\geqslant$90 hours), then the volume variable was given a value of 200. Otherwise, this variable was set to 0. This rather heavy emphasis on volume is consistent with the process alternatives faced by Company A. For higher volume circuit boards, manufacturing can effectively use equipment with faster cycle times but longer (and more expensive) setups. For the lower volume items (such as spares), a slower, but more flexible, process having shorter setup times may prove better suited to meeting market needs.

When using cluster analysis in this context, it is important to recognize that the weightings are meant to capture differences in the order of magnitude between variables, not

exact numerical differences. Viewed in this light, the weightings used for Company A reflect management's feeling that volume is significantly more important for identifying operations-based market segments than are the other three variables.

Table 5

Current Products and Manufacturing Assignments, Company A

Recommended Manufacturing Unit	Prod #	Projected Weekly Prod (hrs)	Projected Order - Winners		
			Price	Delivery Speed	Quality
1 [Pilot Line]	1	913	80	20	0
	2	56	40	0	60
	3	8	30	50	0
	4	123	50	25	25
	5	178	100	0	0
	6	196	60	40	0
	7	200	20	0	0
Unit 1 Average:		239	54	19	12
Unit 1 Standard Deviation:		306	28	20	23
2 [Others]	8	15	0	25	0
	9	584	100	0	0
	10	34	50	50	0
	11	56	50	50	0
	12	279	100	0	0
	13	6	0	20	0
	14	522	80	0	20
Unit 2 Average:		214	54	21	3
Unit 2 Standard Deviation:		250	42	22	8
3 [Vending Machines]	15	77	60	0	40
	16	134	60	0	40
	17	13	30	0	30
	18	33	40	0	30
	19	29	40	0	20
	20	449	80	20	0
	21	94	50	0	0
Unit 3 Average:		118	51	3	23
Unit 3 Standard Deviation:		152	17	8	17
4 [Spares]	22	3	0	0	0
	23	16	0	0	0
	24	50	0	0	0
	25	4	0	0	0
	26	17	0	0	0
	27	1	0	0	0
	28	8	0	0	0
Unit 4 Average:		14	0	0	0
Unit 4 Standard Deviation:		17	0	0	0
Average Std Dev. Across All Four Units:		181	22	13	12

Source: Berry et al. "Factory Focus: Segmenting Markets from an Operations Perspective." Journal of Operations Management 10 3 (1991) Table 2.

1. Agglomerative Hierarchy with k-Means Refinement

a. Objective

To assign products to k focused factories, where k is chosen by the decision-maker (analyst) with the intention of minimizing the cluster member variance.

b. Procedure

First Step: Use an Agglomerative Hierarchical Clustering Model

In their study, Berry et al. used hierarchical clustering to group products according to their similarity across the key variables. The term “agglomerative”- as described before - refers to the fact that the model starts out with n observations, and continues linking observations until all observations have been assigned to a single group. According to Dillon and Goldstein, “hierarchical” refers to the notion that the allocation of an object to a cluster is irrevocable (qtd. in Berry et al. 382). A measure of similarity between observations and a linking method must be chosen, where the distance metric represents the p-space distance between two observations and p, the number of key variables. The methodology proposed in this study uses an average Euclidean Distance metric, defined below:

$$d_{ij} = \left\{ \frac{1}{p} \sum_{k=1}^{p} \left(x_{ik} - x_{jk} \right)^{2} \right\}^{1/2}$$

where Xik represents the value of the kth key variable for observation i.

This method works by computing the average distance between all possible pairs of objects in different clusters, and joining the observations/clusters with the smallest average distance. That is, the shortest distance between any two observations determines the first linkage

and then, distances are recomputed and the process continues until all the observations have been eventually linked into one cluster. The order of linkages can be graphically represented by a dendogram or tree diagram, which offers advantages such as giving a general idea of the similarity between products and clusters and of the number of "natural" clusters inherent in the data. Also, since distance is represented horizontally, drawing a vertical line through the dendogram can show how many clusters have been formed within a certain amount of distance. Specifically, the number of horizontal lines intersected by the vertical lines indicates the number of clusters present at that level (Berry et al. 382-383). See Figure 3 on page 30.

Figure 2 below shows Euclidean distances between the 28 products over the three order-winners.

Prods/ Cluster	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
1																											
2	106.771																										
3	104.163	39.370																									
4	19.685	102.408	102.042																								
5	14.142	108.628	108.858	30.619																							
6	14.142	106.771	101.242	15.411	28.284																						
7	31.623	104.881	103.199	23.184	40.000	28.284																					
8	107.732	38.161	19.526	103.833	112.500	104.672	101.273																				
9	14.142	108.628	108.858	30.619	0.000	28.284	40.000	112.500																			
10	102.225	39.370	10.000	101.550	106.066	100.250	104.163	27.951	106.066																		
11	102.225	39.370	10.000	101.550	106.066	100.250	104.163	27.951	106.066	0.000																	
12	14.142	108.628	108.858	30.619	0.000	28.284	40.000	112.500	0.000	106.066	106.066																
13	107.703	37.417	21.213	103.863	112.250	104.881	100.995	2.500	112.250	29.155	29.155	112.250															
14	14.142	103.923	106.536	19.685	14.142	24.495	31.623	108.886	14.142	104.642	104.642	14.142	108.628														
15	102.956	14.142	35.355	101.181	103.923	103.923	103.923	38.161	103.923	32.404	32.404	103.923	37.417	100.995													
16	24.495	100.995	106.066	15.411	28.284	28.284	28.284	107.034	28.284	105.119	105.119	28.284	106.771	14.142	100.000												
17	104.642	15.811	29.155	101.304	107.005	104.163	101.242	24.622	107.005	30.822	30.822	107.005	23.452	103.199	15.811	101.242											
18	103.562	15.000	29.580	100.933	105.475	103.562	101.612	27.951	105.475	29.580	29.580	105.475	26.926	102.103	11.180	100.623	5.000										
19	102.956	20.000	27.386	100.933	104.881	102.956	100.995	25.617	104.881	27.386	27.386	104.881	24.495	101.980	14.142	100.995	7.071	5.000									
20	0.000	106.771	104.163	19.685	14.142	14.142	31.623	107.732	14.142	102.225	102.225	14.142	107.703	14.142	102.956	24.495	104.642	103.562	102.956								
21	18.028	104.523	103.562	17.678	25.000	20.616	15.000	103.833	25.000	103.078	103.078	25.000	103.562	18.028	102.103	20.616	101.612	101.242	100.623	18.028							
22	108.167	36.056	29.155	104.583	111.803	106.301	100.499	12.500	111.803	35.355	35.355	111.803	10.000	108.167	36.056	106.301	21.213	25.000	22.361	108.167	103.078						
23	108.167	36.056	29.155	104.583	111.803	106.301	100.499	12.500	111.803	35.355	35.355	111.803	10.000	108.167	36.056	106.301	21.213	25.000	22.361	108.167	103.078	0.000					
24	108.167	36.056	29.155	104.583	111.803	106.301	100.499	12.500	111.803	35.355	35.355	111.803	10.000	108.167	36.056	106.301	21.213	25.000	22.361	108.167	103.078	0.000	0.000				
25	108.167	36.056	29.155	104.583	111.803	106.301	100.499	12.500	111.803	35.355	35.355	111.803	10.000	108.167	36.056	106.301	21.213	25.000	22.361	108.167	103.078	0.000	0.000	0.000			
26	108.167	36.056	29.155	104.583	111.803	106.301	100.499	12.500	111.803	35.355	35.355	111.803	10.000	108.167	36.056	106.301	21.213	25.000	22.361	108.167	103.078	0.000	0.000	0.000	0.000		
27	108.167	36.056	29.155	104.583	111.803	106.301	100.499	12.500	111.803	35.355	35.355	111.803	10.000	108.167	36.056	106.301	21.213	25.000	22.361	108.167	103.078	0.000	0.000	0.000	0.000	0.000	
28	108.167	36.056	29.155	104.583	111.803	106.301	100.499	12.500	111.803	35.355	35.355	111.803	10.000	108.167	36.056	106.301	21.213	25.000	22.361	108.167	103.078	0.000	0.000	0.000	0.000	0.000	0.000

Figure 2. Distance Matrix – Focused Factory Study

- ***Second Step: Use an Iterative-Based Clustering Technique***

A final solution is generated using an initial solution found in Step 1. Iterative-based methods start with a set number of clusters and use heuristics to reassign observations into clusters. So while a five-cluster hierarchical solution is found by "breaking apart" a four-cluster solution, an iterative-based model may rearrange all the objects in a given size cluster solution.

The specific type of iterative-based method used in this study is known as a k-Means approach. The algorithm chooses k observations as the initial estimates of cluster center, where k equals the expected number of clusters. Observations are assigned to minimize the Euclidean distance (or "error") between the observation and the mean vector for a cluster. As a result, the mean vector will change as observations are assigned to clusters, and reassignment of some observations will take place in an effort to minimize total errors among the observations (Berry et al. 385).

c. Results

Figure 3 shows the dendogram generated by the hierarchical clustering model (see Appendix A for the amalgamation steps performed by Minitab). A comparison of the product groupings shown in the dendogram with the current company data in Table 5 reveals that the clusters cleanly break into high and low volume groupings (products 22- 28 are low volume).

Using the dendogram as a guide, the k-Means model was run based on the four key variables mentioned above as grouping variables. With the aid of the Minitab program, three iterations of the k-Means model were performed, with the number of clusters set at two, three and four. A comparison of the solutions indicates that clusters membership was stable across all three solutions; that is, only splitting apart larger clusters formed new clusters. For a detailed

explanation of how to perform this method with minitab, consult Appendix B. It shows the k-Means cluster analysis for the Focused Factory study.

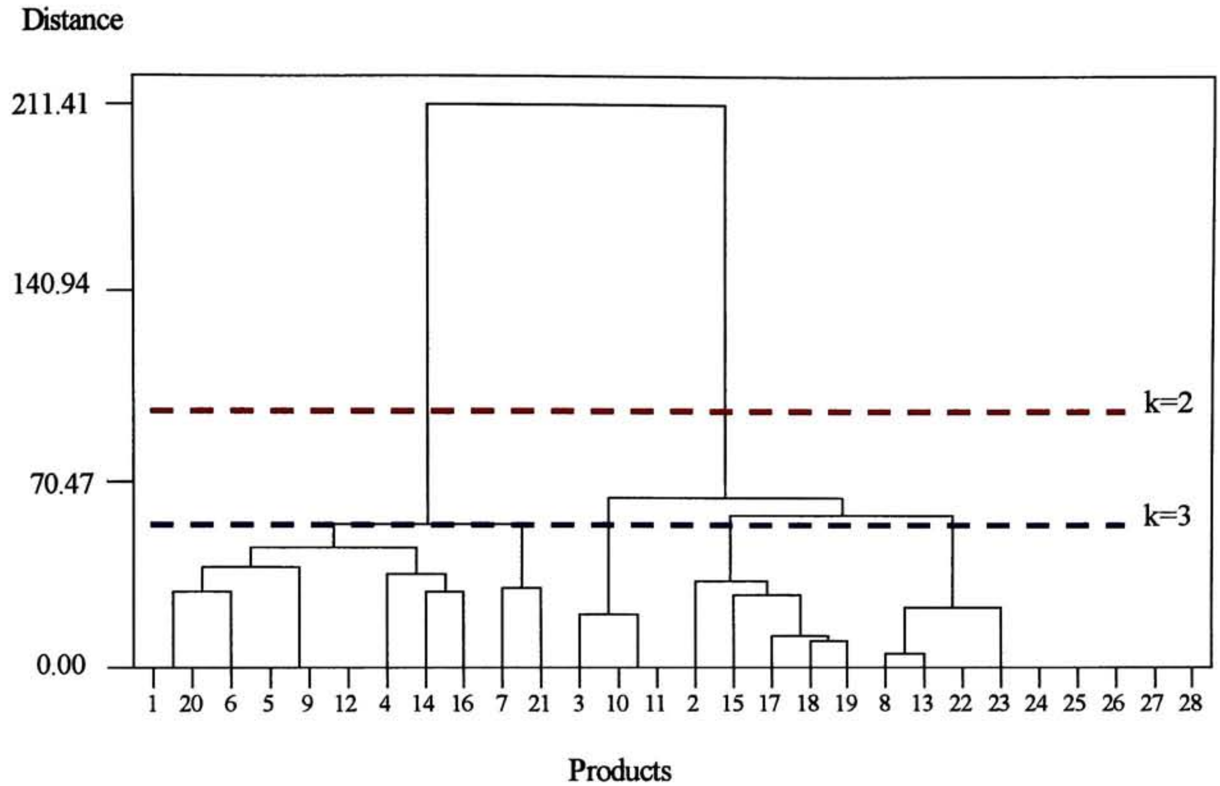


Figure 3. Dendogram for Company A, using Euclidean Distance and Average Linkage Method.

d. Manufacturing Implications

The two, three and four-cluster k-Means solution are shown in Table 7. The method was started with the same data set and enabled reproduction of some of Berry's results, although some disagreements were found related to the standard deviation across recommended units for the 3-cluster solution. Each of the clusters in the three-cluster solution represents a potential unit with a focused manufacturing task. Segment 1 includes high volume products sold to price sensitive customers. Segment 2 includes premium quality products sold to price sensitive customers who buy low volume products and segment 3 requires short delivery lead times to

customers who buy low volume OEM products and spares. A comparison of the order-winners and production volume data for the current manufacturing units in Table 6 and the recommended manufacturing units in Table 7 suggests that significant changes in the organization of manufacturing could lead to a major improvement in manufacturing focus (Berry et al. 368, 370):

- "It would permit management to assign highly skilled operators to Units 2 and 3 where their skills would be most appropriate due to short runs, frequent changeovers, and high quality requirements. (The recent increase in the need for highly skilled employees was caused by the major increase in volume experienced by maturing products that were still assigned to the 'pilot line')."
- "The requirement for low cost production in Unit 1 could be supported with lower skilled operators, and major efforts could be placed on reducing process cycle times in order to improve run time productivity."
- Focus on better service for customers requiring short delivery lead times. (Currently, short lead times are often obtained only by incurring extra changeover costs for the high volume, price sensitive work in the "pilot" and "other" production units).

Table 6

Agglomerative Hierarchy with k-Means Refinement

2 Clusters (K = 2)

Recommended Manufacturing Unit	Prod #	Projected Weekly Prod (hrs)	Projected Order - Winners Price	Delivery Speed	Quality
1	1	913	80	20	0
	4	123	50	25	25
	5	178	100	0	0
	6	196	60	40	0
	7	200	20	0	0
	9	584	100	0	0
	12	279	100	0	0
	14	522	80	0	20
	16	134	60	0	40
	20	449	80	20	0
	21	94	50	0	0
Unit 1 Average:		334	71	10	8
Unit 1 St Dev:		255	25	14	14
Unit 1 SS:		652792	6491	2023	1968
2	2	56	40	0	60
	3	8	30	50	0
	8	15	0	25	0
	10	34	50	50	0
	11	56	50	50	0
	13	6	0	20	0
	15	77	60	0	40
	17	13	30	0	30
	18	33	40	0	30
	19	29	40	0	20
	22	3	0	0	0
	23	16	0	0	0
	24	50	0	0	0
	25	4	0	0	0
	26	17	0	0	0
	27	1	0	0	0
	28	8	0	0	0
Unit 2 Average:		25	20	11	11
Unit 2 St Dev:		23	23	20	19
Unit 2 SS:		8241	8400	6288	5494
Across Units					
Avg Std Dev:		139	24	17	16
Avg SS:		330516	7445	4155	3731

3 Clusters (K = 3)

Recommended Manufacturing Unit	Prod #	Projected Weekly Prod (hrs)	Projected Order - Winners Price	Delivery Speed	Quality
1 (High volume, price sensitive)	1	913	80	20	0
	4	123	50	25	25
	5	178	100	0	0
	6	196	60	40	0
	7	200	20	0	0
	9	584	100	0	0
	12	279	100	0	0
	14	522	80	0	20
	16	134	60	0	40
	20	449	80	20	0
	21	94	50	0	0
Unit 1 Average:		334	71	10	8
Unit 1 St Dev:		255	25	14	14
Unit 1 SS:		652792	6491	2023	1968
2 (Quality products, price sensitive, low volume)	2	56	40	0	60
	15	77	60	0	40
	17	13	30	0	30
	18	33	40	0	30
	19	29	40	0	20
Unit 2 Average:		42	42	0	36
Unit 2 St Dev:		25	11	0	15
Unit 2 SS:		2511	480	0	920
3 (Short delivery lead times, low volume and spares)	3	8	30	50	0
	8	15	0	25	0
	10	34	50	50	0
	11	56	50	50	0
	13	6	0	20	0
	22	3	0	0	0
	23	16	0	0	0
	24	50	0	0	0
	25	4	0	0	0
	26	17	0	0	0
	27	1	0	0	0
	28	8	0	0	0
Unit 3 Average:		18	11	16	0
Unit 3 St Dev:		19	20	22	0
Unit 3 SS:		3792	4492	5356	0
Across Units					
Avg Std Dev:		100	19	12	10
Avg SS:		219698	3821	2460	963

4 Clusters (K = 4)

Recommended Manufacturing Unit	Prod #	Projected Weekly Prod (hrs)	Projected Order - Winners Price	Delivery Speed	Quality
1	1	913	80	20	0
	5	178	100	0	0
	6	196	60	40	0
	9	584	100	0	0
	12	279	100	0	0
	14	522	80	0	20
	20	449	80	20	0
Unit 1 Average:		446	86	11	3
Unit 1 Std Dev:		260	15	16	8
Unit 1 SS:		405131	1371	1486	343
2	2	56	40	0	60
	15	77	60	0	40
	17	13	30	0	30
	18	33	40	0	30
	19	29	40	0	20
Unit 2 Average:		42	42	0	36
Unit 2 Std Dev:		25	11	0	15
Unit 2 SS:		2511	480	0	920
3	3	8	30	50	0
	8	15	0	25	0
	10	34	50	50	0
	11	56	50	50	0
	13	6	0	20	0
	22	3	0	0	0
	23	16	0	0	0
	24	50	0	0	0
	25	4	0	0	0
	26	17	0	0	0
	27	1	0	0	0
	28	8	0	0	0
Unit 3 Average:		18	11	16	0
Unit 3 Std Dev:		19	20	22	0
Unit 3 SS:		3792	4492	5356	0
4	4	123	50	25	25
	7	200	20	0	0
	16	134	60	0	40
	21	94	50	0	0
Unit 4 Average:		138	45	6	16
Unit 4 Std Dev:		45	17	13	20
Unit 4 SS:		6021	900	469	1169
Across Units					
Avg Std Dev:		87	16	13	11
Avg SS:		104364	1811	1828	608

2. Plant Location Model

a. Objective

Assignment of products to k focused factories, where k is chosen by the decision-maker and product members of the focused factory are chosen to minimize the distance between all members and the medoids of each cluster of products.

b. Procedure

In order to obtain k clusters, this method selects k objects (which are called representative objects) in the data set. The corresponding clusters are then found by assigning each remaining object to the nearest representative object, which must be chosen so they are centrally located in the clusters they define. In others words, the average distance (or average dissimilarity) of the representative object to all the other objects of the same cluster is being minimized. For this reason, such an optimal representative object is called the medoid of its cluster, and the method of partitioning around medoids is called the k-medoid technique (Kaufman and Rousseeuw 40).

According to Kaufman and Rousseuw, this method is based on a location model (the k-medoid, median or centrotype model) with the following general formulation: "Given a finite number of users, whose demands for a given service are known and must be satisfied, and given a finite set of possible locations among which k must be chosen for the location of service centers, select the location in such a way as to minimize the total distance (or equivalently the average distance) traveled by the users." The location of a center is interpreted as the selection of an object as a representative object of a cluster, whereas the distance traveled by the user corresponds to the dissimilarity between an object and the representative object of the cluster to which it belongs (108).

The mathematical model is presented in Figure 4 below.

$$\text{minimize} \sum_{i=1}^{n}\sum_{j=1}^{n} d(i,j)z_{ij}$$

Subject to:

$$\sum_{i=1}^{n} z_{ij} = 1, \qquad j = 1,2,\ldots,n \qquad (9)$$

$$z_{ij} \le y_i, \qquad i,j = 1,2,\ldots,n \qquad (10)$$

$$\sum_{i=1}^{n} y_i = k, \qquad k = number.of.clusters \qquad (11)$$

$$y_i, z_{ij} \in \{0,1\}, \qquad i,j = 1,2,\ldots,n \qquad (12)$$

Figure 4. Plant Location Model Mathematical Formulation. Source: Kaufman, Leonard and Rousseeuw, Peter. Finding Groups in Data: An Introduction to Cluster Analysis. (1990): 109.

The main objective as formulated by the mathematical model is to minimize the Euclidean Distance between all members (the 28 products). Constraint 9 says that each object must belong to exactly one cluster. Constraint 10 mandates that a member cannot be assigned to an object unless it is the representative object. Constraint 11 specifies that there must be exactly 2, 3 or 4 representative objects (for this study). Finally, constraint 12 calls for integer binary variables, that is, values that can only be integers and are limited to 2 decisions: yes or no (0-1).

The first step is to compute the distance between the 28 products, that is, the dissimilarity matrix. This process was already done as a part of the Agglomerative Hierarchy Method (Figure 1). Second, a k number is chosen to specify the number of clusters or, in this case, focused factories desired. Third, a binary integer optimization model is solved using the program Lingo, to minimize the distance between all members and medoids of each cluster. The corresponding optimization model is shown in Appendix C.

c. Results

Since one usually does not know the number of clusters present in a data set and since most partitioning methods provide a fixed number of clusters, several values of k must be applied in order to find the most meaningful clustering (Kaufman and Rousseeuw 110). Using Berry's data as a guideline, three iterations have been performed setting k equals to two, three and four, which are shown in table 7 below.

Note that the bold face numbers under the product number column represent the medoid or representative of each segment (cluster solution).

Table 7

Plant Location Model Results

2 Clusters (K = 2)

Recommended Manufacturing Unit	Prod #	Projected Weekly Prod (hrs)	Projected Order - Winners: Price	Delivery Speed	Quality
1	1	913	80	20	0
	4	123	50	25	25
	5	178	100	0	0
	6	196	60	40	0
	7	200	20	0	0
	9	584	100	0	0
	12	279	100	0	0
	14	522	80	0	20
	16	134	60	0	40
	20	449	80	20	0
	21	94	50	0	0
Unit 1 Average:		334	71	10	8
Unit 1 St Dev:		255	25	14	14
Unit 1 SS:		652792	6491	2023	1968
2	2	56	40	0	60
	3	8	30	50	0
	8	15	0	25	0
	10	34	50	50	0
	11	56	50	50	0
	13	6	0	20	0
	15	77	60	0	40
	17	13	30	0	30
	18	33	40	0	30
	19	29	40	0	20
	22	3	0	0	0
	23	16	0	0	0
	24	50	0	0	0
	25	4	0	0	0
	26	17	0	0	0
	27	1	0	0	0
	28	8	0	0	0
Unit 2 Average:		25	20	11	11
Unit 2 St Dev:		23	23	20	19
Unit 2 SS:		8241	8400	6288	5494
Across Units					
Avg Std Dev:		139	24	17	16
Avg SS:		330516	7445	4155	3731

3 Clusters (K = 3)

Recommended Manufacturing Unit	Prod #	Projected Weekly Prod (hrs)	Projected Order - Winners: Price	Delivery Speed	Quality
1 (High volume, price sensitive)	1	913	80	20	0
	4	123	50	25	25
	5	178	100	0	0
	6	196	60	40	0
	7	200	20	0	0
	9	584	100	0	0
	12	279	100	0	0
	14	522	80	0	20
	16	134	60	0	40
	20	449	80	20	0
	21	94	50	0	0
Unit 1 Average:		334	71	10	8
Unit 1 St Dev:		255	25	14	14
Unit 1 SS:		652792	6491	2023	1968
2 (Quality products, short delivery price sensitive, low volume)	2	56	40	0	60
	10	34	50	50	0
	11	56	50	50	0
	15	77	60	0	40
	17	13	30	0	30
	18	33	40	0	30
	19	29	40	0	20
Unit 2 Average:		43	44	14	26
Unit 2 St Dev:		21	10	24	21
Unit 2 SS:		2770	571	3571	2771
3 (Short delivery lead times, low volume and spares)	3	8	30	50	0
	8	15	0	25	0
	13	6	0	20	0
	22	3	0	0	0
	23	16	0	0	0
	24	50	0	0	0
	25	4	0	0	0
	26	17	0	0	0
	27	1	0	0	0
	28	8	0	0	0
Unit 3 Average:		13	3	10	0
Unit 3 St Dev:		14	9	17	0
Unit 3 SS:		1822	810	2623	0
Across Units					
Avg Std Dev:		97	15	19	12
Avg SS:		219128	2624	2739	1580

4 Clusters (K = 4)

Recommended Manufacturing Unit	Prod #	Projected Weekly Prod (hrs)	Projected Order - Winners: Price	Delivery Speed	Quality
1	**1**	913	80	20	0
	4	123	50	25	25
	5	178	100	0	0
	6	196	60	40	0
	7	200	20	0	0
	9	584	100	0	0
	12	279	100	0	0
	14	522	80	0	20
	16	134	60	0	40
	20	449	80	20	0
	21	94	50	0	0
Unit 1 Average:		334	71	10	8
Unit 1 Std Dev:		255	25	14	14
Unit 1 SS:		652792	6491	2023	1968
2	3	8	30	50	0
	10	34	50	50	0
	11	56	50	50	0
Unit 2 Average:		33	43	50	0
Unit 2 Std Dev:		24	12	0	0
Unit 2 SS:		1155	267	0	0
3	2	56	40	0	60
	15	77	60	0	40
	17	13	30	0	30
	18	33	40	0	30
	19	29	40	0	20
Unit 3 Average:		42	42	0	36
Unit 3 Std Dev:		25	11	0	15
Unit 3 SS:		2511	480	0	920
4	8	15	0	25	0
	13	6	0	20	0
	22	3	0	0	0
	23	16	0	0	0
	24	50	0	0	0
	25	4	0	0	0
	26	17	0	0	0
	27	1	0	0	0
	28	8	0	0	0
Unit 4 Average:		13	0	5	0
Unit 4 Std Dev:		15	0	10	0
Unit 4 SS:		1796	0	800	0
Across Units					
Avg Std Dev:		80	12	6	7
Avg SS:		164563	1809	706	722

3. Covering Model

a. Objective

Assignment of products to k focused factories, where k is an outcome of a distance (D) from all the products to their representative object.

b. Procedure

This model is similar to the plant location model in the sense that both of them employ representative objects, but differ in the approach used to assign objects into clusters. That is, the number of clusters is not fixed. The decision-maker controls the distance D that separates all members of each cluster (the objective is to minimize the number of representative objects necessary to achieve this aim). As in the plant location model, once the representative objects have been selected, the clustering is obtained by assigning each object to its most similar selected representative object (Kaufman and Rousseuw 111). The statement of the mathematical formulation for this model is shown in Figure 5 below:

$$\text{minimize} \sum_{i=1}^{n} y_i \qquad (14)$$

subject to

$$\sum_{i=1}^{n} z_{ij} = 1, \qquad j = 1,2,\ldots,n \qquad (15)$$

$$z_{ij} \leq y_i, \qquad i, j = 1,2,\ldots,n \qquad (16)$$

$$\sum_{i=1}^{n} d(i,j) z_{ij} \leq D, \qquad j = 1,2,\ldots,n \qquad (17)$$

$$y_i, z_{ij} \in \{0,1\}, \qquad i, j = 1,2,\ldots,n \qquad (18)$$

Constraints (17) express that each object j must lie within a maximum dissimilarity D of its representative object.

Figure 5. Covering Model Mathematical Formulation. Source: Kaufman, Leonard and Rousseeuw, Peter. Finding Groups in Data: An Introduction to Cluster Analysis. (1990): 111.

The Objective is to minimize the number of representative objects for a given distance (D) (constraint 14). Constraint (15) ensures that each object should belong to exactly of 1 cluster, whereas constraint (16) assigns an object to the representative object . Then, the maximum dissimilarity (D) of its representative object within which objects must lie should be specified. Finally, the last constraint ensures the binary solution.

The solution process is as follows. First, compute the distance between all members (use the same dissimilarity matrix if already done). Secondly, specify the diameter of each cluster in order to choose how close members must be from each other. Finally, solve binary integer optimization model to insure that all members belong to a cluster of diameter (D) or less. The optimum result was obtained using Lingo, as shown in Appendix D.

c. Results

Different values of D were tried to find two, three and four cluster solutions, as shown in Tables 8 and 9. The outcomes were very similar to the previous models, except for one interesting result. That is, in the Covering Model, the same number of clusters was obtained by using different values of "D" (distance from all members to the representative object of each cluster). This creates some doubt about the robustness of the model if small changes do not result in different outcomes. Segments were formed by looking at the range of D's and which products remained uniform over the segments (see figure 6). This could be a drawback in the sense that a sensitivity analysis should be considered.

Table 8

Covering Model Results

2 Clusters (K = 2)

Recommended Manufacturing Unit	Prod #	Projected Weekly Prod (hrs)	Projected Order - Winners: Price	Delivery Speed	Quality
1	1	913	80	20	0
	4	123	50	25	25
	5	178	100	0	0
	6	196	60	40	0
	7	200	20	0	0
	9	584	100	0	0
	12	279	100	0	0
	14	522	80	0	20
	16	134	60	0	40
	20	449	80	20	0
	21	94	50	0	0
Unit 1 Average:		334	71	10	8
Unit 1 St Dev:		255	25	14	14
Unit 1 SS:		652792	6491	2023	1968
2	2	56	40	0	60
	3	8	30	50	0
	8	15	0	25	0
	10	34	50	50	0
	11	56	50	50	0
	13	6	0	20	0
	15	77	60	0	40
	17	13	30	0	30
	18	33	40	0	30
	19	29	40	0	20
	22	3	0	0	0
	23	16	0	0	0
	24	50	0	0	0
	25	4	0	0	0
	26	17	0	0	0
	27	1	0	0	0
	28	8	0	0	0
Unit 2 Average:		25	20	11	11
Unit 2 St Dev:		23	23	20	19
Unit 2 SS:		8241	8400	6288	5494
Across Units					
Avg Std Dev:		139	24	17	16
Avg SS:		330516	7445	4155	3731

3 Clusters (K = 3)

Recommended Manufacturing Unit	Prod #	Projected Weekly Prod (hrs)	Projected Order - Winners: Price	Delivery Speed	Quality
1	1	913	80	20	0
(High volume,	4	123	50	25	25
price sensitive)	5	178	100	0	0
	6	196	60	40	0
	7	200	20	0	0
	9	584	100	0	0
	12	279	100	0	0
	14	522	80	0	20
	16	134	60	0	40
	20	449	80	20	0
	21	94	50	0	0
Unit 1 Average:		334	71	10	8
Unit 1 St Dev:		255	25	14	14
Unit 1 SS:		652792	6491	2023	1968
2	2	56	40	0	60
(Quality products,	8	15	0	25	0
price sensitive,	13	6	0	20	0
low volume	15	77	60	0	40
and spares)	17	13	30	0	30
	18	33	40	0	30
	19	29	40	0	20
	22	3	0	0	0
	23	16	0	0	0
	24	50	0	0	0
	25	4	0	0	0
	26	17	0	0	0
	27	1	0	0	0
	28	8	0	0	0
Unit 2 Average:		23	15	3	13
Unit 2 St Dev:		23	22	8	20
Unit 2 SS:		6875	6150	880	5086
3	3	8	30	50	0
(Del speed, high	10	34	50	50	0
volume, price)	11	56	50	50	0
Unit 3 Average:		33	43	50	0
Unit 3 St Dev:		24	12	0	0
Unit 3 SS:		1155	267	0	0
Across Units					
Avg Std Dev:		101	20	7	11
Avg SS:		220274	4303	968	2351

4 Clusters (K = 4)

Recommended Manufacturing Unit	Prod #	Projected Weekly Prod (hrs)	Projected Order - Winners: Price	Delivery Speed	Quality
1	1	913	80	20	0
	4	123	50	25	25
	6	196	60	40	0
	7	200	20	0	0
	16	134	60	0	40
	20	449	80	20	0
	21	94	50	0	0
Unit 1 Average:		301	57	15	9
Unit 1 Std Dev:		294	21	16	16
Unit 1 SS:		520095	2543	1450	1621
2	3	8	30	50	0
	8	15	0	25	0
	10	34	50	50	0
	11	56	50	50	0
	13	6	0	20	0
Unit 2 Average:		24	26	39	0
Unit 2 Std Dev:		21	25	15	0
Unit 2 SS:		1785	2520	920	0
3	5	178	100	0	0
	9	584	100	0	0
	12	279	100	0	0
	14	522	80	0	20
Unit 3 Average:		391	95	0	5
Unit 3 Std Dev:		193	10	0	10
Unit 3 SS:		112323	300	0	300
4	2	56	40	0	60
	15	77	60	0	40
	17	13	30	0	30
	18	33	40	0	30
	19	29	40	0	20
	22	3	0	0	0
	23	16	0	0	0
	24	50	0	0	0
	25	4	0	0	0
	26	17	0	0	0
	27	1	0	0	0
	28	8	0	0	0
Unit 4 Average:		26	18	0	15
Unit 4 Std Dev:		24	23	0	21
Unit 4 SS:		6445	5625	0	4700
Across Units					
Avg Std Dev:		133	20	8	12
Avg SS:		160162	2747	593	1655

Table 9

Measure of Robustness

Recommended Manufacturing Unit	D (Distance)	Representative Objects	Products
2	30	19	2 3 8 10 11 13 15 17 18 19 22 23 24 25 26 27 28
		21	1 4 5 6 7 9 12 14 16 20 21
	29	16	1 4 5 6 7 9 12 14 16 20 21
		19	2 3 8 10 11 13 15 17 18 19 22 23 24 25 26 27 28
	28	19	2 3 8 10 1 13 15 17 18 19 22 23 24 25 26 27 28
		21	1 4 5 6 7 9 12 14 16 20 21
3	27	3	3 8 10 11
		18	2 13 15 17 18 19 22 23 24 25 26 27 28
		21	1 4 5 6 7 9 12 14 16 20 21
	26	3	3 10 11 13
		17	2 8 15 17 18 19 22 23 24 25 26 27 28
		21	1 4 5 6 7 9 12 14 16 20 21
	25	11	3 10 11
		17	2 8 13 15 17 18 19 22 23 24 25 26 27 28
		21	1 4 5 6 7 9 12 14 16 20 21
4	24	3	3 8 10 11 13
		19	2 15 17 18 19 22 23 24 25 26 27 28
		20	5 6 9 12 20
		21	1 4 7 14 16 21
	23	3	3 8 10 11 13
		9	5 9 12 14
		19	2 15 17 18 19 22 23 24 25 26 27 28
		21	1 4 6 7 16 20 21
	22	3	3 8 10 11 13
		5	5 9 12 14
		17	2 15 17 18 19 22 23 24 25 26 27 28
		21	1 4 6 7 16 20 21

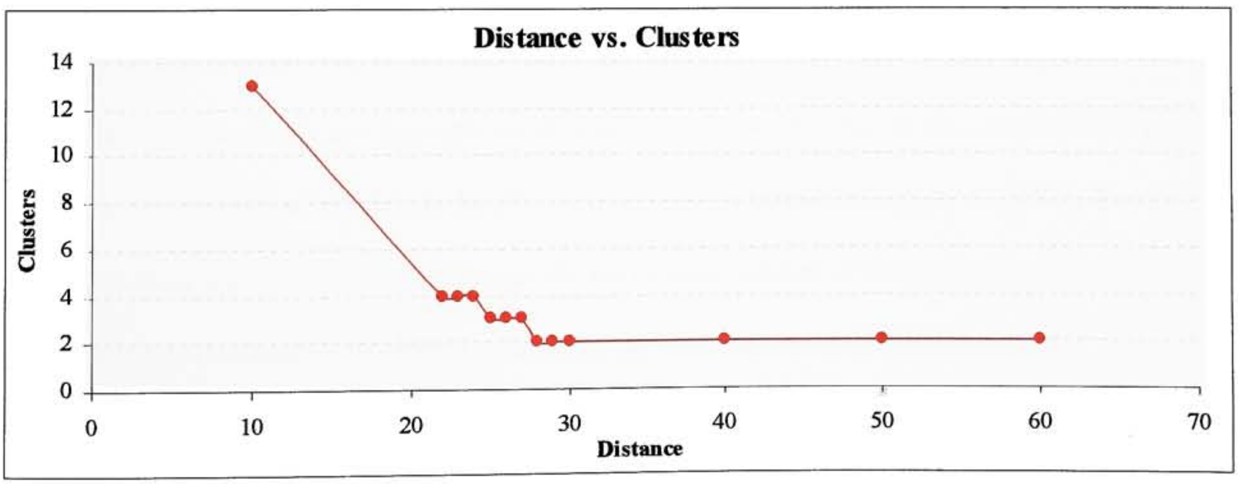


Figure 6. Relationship between different values of D and the different clusters formed

4. Fuzzy Set Method

a. Objective

Given the number of focused factories (k), compute the percent membership of each product to each focused factory.

b. Procedure

Fuzzy Clustering is a generalization of partitioning. In partitioning, each object of the data set is assigned to one and only one cluster. Therefore, partitioning methods are sometimes said to produce a hard clustering, because they make a clear-cut decision for each object. On the other hand, a fuzzy clustering method allows for some ambiguity in the data. Thus, this technique is much better to describe situations where each object may be spread out over various clusters. For instance, a fuzzy clustering method may indicate that object A belongs for the most part to one cluster, whereas object B may belong almost equally between several clusters. This degree of belonging is quantified by means of membership coefficients that range from 0 to 1, with the sum among all clusters equal to 1 (100%) (Kaufman and Rousseeuw 164).

The actual algorithm to calculate these membership coefficients is very different from other clustering methods and does not involve representative objects. In this research, the data, as well as the number of clusters to be obtained, is entered into the program FANNY. Fuzzy clustering does not allow for 1 cluster and the maximum number of clusters must be less than n/2, where n is the number of objects (Kaufman and Rousseeuw 44,166).

c. Results

For a given number of specified clusters, the Fuzzy Set Method yields a table of percent membership of each product to each cluster (see Table 11). The closest hard clustering assignment of product to focused factory is indicated by the greatest percent membership. For instance, Table 10 indicates that product 1 belongs 90.73% to cluster 1, 4.94% to cluster 2 and 4.33 % to cluster 3. Therefore, product 1 is assigned to cluster 1 because it shows the greatest percentage membership. Appendix E provides a sample output of the Fanny program, and Table 11 contains the hard clustering results for 2, 3 and 4 focused factories.

Table 10

Percent Membership for 3 Cluster Solution

FUZZY CLUSTERING

	1	2	3
001	.9073	.0494	.0433
002	.0851	.6646	.2503
003	.0889	.5654	.3457
004	.8009	.1056	.0935
005	.8726	.0681	.0593
006	.8008	.1055	.0937
007	.6947	.1582	.1471
008	.0597	.3207	.6196
009	.8725	.0681	.0593
010	.0930	.6202	.2868
011	.0930	.6203	.2867
012	.8726	.0681	.0593
013	.0507	.2763	.6730
014	.8804	.0639	.0557
015	.0699	.7300	.2002
016	.7835	.1154	.1011
017	.0443	.7306	.2251
018	.0388	.7966	.1646
019	.0418	.7593	.1989
020	.9073	.0494	.0433
021	.8279	.0905	.0817
022	.0032	.0166	.9802
023	.0032	.0166	.9802
024	.0032	.0166	.9802
025	.0032	.0166	.9802
026	.0032	.0166	.9802
027	.0032	.0166	.9802
028	.0032	.0166	.9802

Table 11

Fuzzy Set Method Results

2 Clusters (K = 2)

Recommended Manufacturing Unit	Prod #	Projected Weekly Prod (hrs)	Projected Order - Winners: Price	Delivery Speed	Quality
1	1	913	80	20	0
	4	123	50	25	25
	5	178	100	0	0
	6	196	60	40	0
	7	200	20	0	0
	9	584	100	0	0
	12	279	100	0	0
	14	522	80	0	20
	16	134	60	0	40
	20	449	80	20	0
	21	94	50	0	0
Unit 1 Average:		334	71	10	8
Unit 1 St Dev:		255	25	14	14
Unit 1 SS:		652792	6491	2023	1968
2	2	56	40	0	60
	3	8	30	50	0
	8	15	0	25	0
	10	34	50	50	0
	11	56	50	50	0
	13	6	0	20	0
	15	77	60	0	40
	17	13	30	0	30
	18	33	40	0	30
	19	29	40	0	20
	22	3	0	0	0
	23	16	0	0	0
	24	50	0	0	0
	25	4	0	0	0
	26	17	0	0	0
	27	1	0	0	0
	28	8	0	0	0
Unit 2 Average:		25	20	11	11
Unit 2 St Dev:		23	23	20	19
Unit 2 SS:		8241	8400	6288	5494
Across Units					
Avg Std Dev:		139	24	17	16
Avg SS:		330516	7445	4155	3731

3 Clusters (K = 3)

Recommended Manufacturing Unit	Prod #	Projected Weekly Prod (hrs)	Projected Order - Winners: Price	Delivery Speed	Quality
1 (High volume, price sensitive)	1	913	80	20	0
	4	123	50	25	25
	5	178	100	0	0
	6	196	60	40	0
	7	200	20	0	0
	9	584	100	0	0
	12	279	100	0	0
	14	522	80	0	20
	16	134	60	0	40
	20	449	80	20	0
	21	94	50	0	0
Unit 1 Average:		334	71	10	8
Unit 1 St Dev:		255	25	14	14
Unit 1 SS:		652792	6491	2023	1968
2 (Quality products, short delivery price sensitive, low volume)	2	56	40	0	60
	3	8	30	50	0
	10	34	50	50	0
	11	56	50	50	0
	15	77	60	0	40
	17	13	30	0	30
	18	33	40	0	30
	19	29	40	0	20
Unit 2 Average:		38	43	19	23
Unit 2 St Dev:		23	10	26	22
Unit 2 SS:		3816	750	4688	3350
3 (Low volume and spares)	8	15	0	25	0
	13	6	0	20	0
	22	3	0	0	0
	23	16	0	0	0
	24	50	0	0	0
	25	4	0	0	0
	26	17	0	0	0
	27	1	0	0	0
	28	8	0	0	0
Unit 3 Average:		13	0	5	0
Unit 3 St Dev:		15	0	10	0
Unit 3 SS:		1796	0	800	0
Across Units					
Avg Std Dev:		98	12	17	12
Avg SS:		219468	2414	2503	1773

4 Clusters (K = 4)

Recommended Manufacturing Unit	Prod #	Projected Weekly Prod (hrs)	Projected Order - Winners: Price	Delivery Speed	Quality
1	1	913	80	20	0
	4	123	50	25	25
	5	178	100	0	0
	6	196	60	40	0
	7	200	20	0	0
	9	584	100	0	0
	12	279	100	0	0
	14	522	80	0	20
	16	134	60	0	40
	20	449	80	20	0
	21	94	50	0	0
Unit 1 Average:		334	71	10	8
Unit 1 Std Dev:		255	25	14	14
Unit 1 SS:		652792	6491	2023	1968
2	3	8	30	50	0
	10	34	50	50	0
	11	56	50	50	0
Unit 2 Average:		33	43	50	0
Unit 2 Std Dev:		24	12	0	0
Unit 2 SS:		1155	267	0	0
3	2	56	40	0	60
	15	77	60	0	40
	17	13	30	0	30
	18	33	40	0	30
	19	29	40	0	20
Unit 3 Average:		42	42	0	36
Unit 3 Std Dev:		25	11	0	15
Unit 3 SS:		2511	480	0	920
4	8	15	0	25	0
	13	6	0	20	0
	22	3	0	0	0
	23	16	0	0	0
	24	50	0	0	0
	25	4	0	0	0
	26	17	0	0	0
	27	1	0	0	0
	28	8	0	0	0
Unit 4 Average:		13	0	5	0
Unit 4 Std Dev:		15	0	10	0
Unit 4 SS:		1796	0	800	0
Across Units					
Avg Std Dev:		80	12	6	7
Avg SS:		164563	1809	706	722

5. Average-Weighted Distance Model

a. Objective

Assignment of products to k focused factories, where k is chosen by the decision-maker and product members of the focused factory are chosen to minimize the distance between all members and the medoids of each cluster of products.

b. Procedure

This model is very similar to the plant location model in the sense that both of them use binary integer optimization to minimize the distance between all members and the centroids of each cluster. What is different, though, is the calculation of the Euclidean distance.

The Euclidean Distance interprets the separation of two points in space as the hypotenuse of a right triangle (see Figure 7).

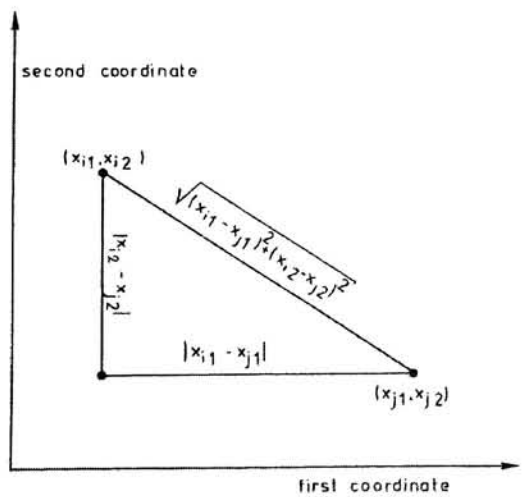


Figure 7. Illustration of the Euclidean Distance formula. (Kaufman and Rousseeuw, 1990) 12.

In reality, these coordinates are measurable variables such as time units, quantity, and even order-winners characteristics (e.g. price, delivery speed, quality, etc.). The problem is that

the Euclidean distance overlooks the scaling effect of points located in space. This idea is illustrated in Figure 8, where two variables are measured: delivery speed and price.

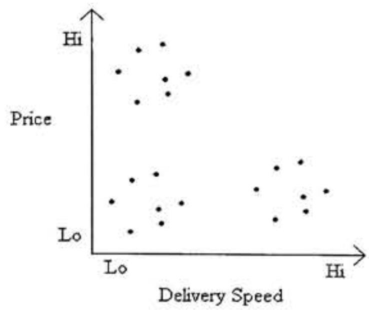


Figure 8. The concept of clusters

Note that each cluster is exactly the same, having the same size and the same variance within each cluster. However, their placement in relation to the variables against which they are measured is different. In an attempt to take the scaling effect into account, a new metric is proposed in this research to address this issue. The new metric modifies the Euclidean distance by multiplying the separation between each member across all variables by the average of these distances. It is defined as:

$$d_{ij} = \left\{ \frac{1}{p} \sum_{k=1}^{p} \left(\frac{x_{ik} + x_{jk}}{2} \right) \left(x_{ik} - x_{jk} \right)^{2} \right\}^{1/2}$$

The mathematical formulation is the same as the Plant Location model (refer to Figure 4). The distance matrix of the 28 products over the three order-winners is shown in figure 8.

c. Results

Three iterations were obtained setting k equals to two, three and four and the outcome is compiled in Table 12 following Figure 9. See Appendix F for the mathematical formulation and Lingo output.

Prods/ Cluster	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
1																											
2	1025.671																										
3	1020.906	208.567																									
4	129.301	1008.015	1005.889																								
5	100.000	1044.031	1046.542	225.347																							
6	100.000	1019.804	1005.609	71.807	200.000																						
7	214.476	1014.889	1008.092	108.541	309.839	154.919																					
8	1031.572	192.232	96.096	1008.751	1061.580	1014.312	1001.475																				
9	100.000	1044.031	1046.542	225.347	0.000	200.000	309.839	1061.580																			
10	1011.187	209.165	63.246	1003.899	1030.776	1001.249	1011.682	146.575	1030.776																		
11	1011.187	209.165	63.246	1003.899	1030.776	1001.249	1011.682	146.575	1030.776	0.000																	
12	100.000	1044.031	1046.542	225.347	0.000	200.000	309.839	1061.580	0.000	1030.776	1030.776																
13	1031.504	189.737	106.066	1008.820	1061.131	1014.889	1001.000	11.859	1061.131	153.297	153.297	1061.131															
14	44.721	1019.804	1025.183	129.301	100.000	126.491	214.476	1032.934	100.000	1015.505	1015.505	100.000	1032.473														
15	1007.968	100.000	183.712	1002.575	1019.804	1007.968	1011.929	192.232	1019.804	158.114	158.114	1019.804	189.737	1004.988													
16	126.491	1004.988	1016.735	71.807	200.000	126.491	154.919	1018.309	200.000	1012.423	1012.423	200.000	1017.841	100.000	1000.000												
17	1019.191	104.881	137.840	1003.058	1040.673	1010.693	1001.998	93.291	1040.673	151.658	151.658	1040.673	88.034	1017.349	104.881	1005.485											
18	1014.088	100.623	140.979	1001.624	1032.654	1008.154	1003.182	115.447	1032.654	141.863	141.863	1032.654	111.243	1012.238	76.649	1002.933	29.580										
19	1012.917	126.491	132.288	1001.608	1031.504	1006.976	1001.998	104.657	1031.504	133.229	133.229	1031.504	100.000	1011.929	89.443	1003.992	38.730	25.000									
20	0.000	1025.671	1020.906	129.301	100.000	100.000	214.476	1031.572	100.000	1011.187	1011.187	100.000	1031.504	44.721	1007.968	126.491	1019.191	1014.088	1012.917								
21	125.000	1013.965	1009.765	62.500	216.506	96.825	88.741	1008.751	216.506	1007.782	1007.782	216.506	1008.278	125.000	1004.677	96.825	1003.681	1002.247	1001.062	125.000							
22	1031.988	187.083	137.840	1009.718	1060.660	1017.349	1000.500	44.194	1060.660	176.777	176.777	1060.660	31.623	1031.988	187.083	1017.349	82.158	106.654	94.868	1031.988	1007.782						
23	1031.988	187.083	137.840	1009.718	1060.660	1017.349	1000.500	44.194	1060.660	176.777	176.777	1060.660	31.623	1031.988	187.083	1017.349	82.158	106.654	94.868	1031.988	1007.782	0.000					
24	1031.988	187.083	137.840	1009.718	1060.660	1017.349	1000.500	44.194	1060.660	176.777	176.777	1060.660	31.623	1031.988	187.083	1017.349	82.158	106.654	94.868	1031.988	1007.782	0.000	0.000				
25	1031.988	187.083	137.840	1009.718	1060.660	1017.349	1000.500	44.194	1060.660	176.777	176.777	1060.660	31.623	1031.988	187.083	1017.349	82.158	106.654	94.868	1031.988	1007.782	0.000	0.000	0.000			
26	1031.988	187.083	137.840	1009.718	1060.660	1017.349	1000.500	44.194	1060.660	176.777	176.777	1060.660	31.623	1031.988	187.083	1017.349	82.158	106.654	94.868	1031.988	1007.782	0.000	0.000	0.000	0.000		
27	1031.988	187.083	137.840	1009.718	1060.660	1017.349	1000.500	44.194	1060.660	176.777	176.777	1060.660	31.623	1031.988	187.083	1017.349	82.158	106.654	94.868	1031.988	1007.782	0.000	0.000	0.000	0.000	0.000	
28	1031.988	187.083	137.840	1009.718	1060.660	1017.349	1000.500	44.194	1060.660	176.777	176.777	1060.660	31.623	1031.988	187.083	1017.349	82.158	106.654	94.868	1031.988	1007.782	0.000	0.000	0.000	0.000	0.000	0.000

Figure 8. Distance Matrix for Average-Weighted Distance Model

Table 12

Average-Weighted Distance Model Results

2 Clusters (K = 2)

Recommended Manufacturing Unit	Prod #	Projected Weekly Prod (hrs)	Projected Order - Winners Price	Delivery Speed	Quality
1	1	913	80	20	0
	4	123	50	25	25
	5	178	100	0	0
	6	196	60	40	0
	7	200	20	0	0
	9	584	100	0	0
	12	279	100	0	0
	14	522	80	0	20
	16	134	60	0	40
	20	449	80	20	0
	21	94	50	0	0
Unit 1 Average:		334	71	10	8
Unit 1 St Dev:		255	25	14	14
Unit 1 SS:		652792	6491	2023	1968
2	2	56	40	0	60
	3	8	30	50	0
	8	15	0	25	0
	10	34	50	50	0
	11	56	50	50	0
	13	6	0	20	0
	15	77	60	0	40
	17	13	30	0	30
	18	33	40	0	30
	19	29	40	0	20
	22	3	0	0	0
	23	16	0	0	0
	24	50	0	0	0
	25	4	0	0	0
	26	17	0	0	0
	27	1	0	0	0
	28	8	0	0	0
Unit 2 Average:		25	20	11	11
Unit 2 St Dev:		23	23	20	19
Unit 2 SS:		8241	8400	6288	5494
Across Units					
Avg Std Dev:		139	24	17	16
Avg SS:		330516	7445	4155	3731

3 Clusters (K = 3)

Recommended Manufacturing Unit	Prod #	Projected Weekly Prod (hrs)	Projected Order - Winners Price	Delivery Speed	Quality
1 (High volume, price sensitive)	**1**	913	80	20	0
	4	123	50	25	25
	5	178	100	0	0
	6	196	60	40	0
	7	200	20	0	0
	9	584	100	0	0
	12	279	100	0	0
	14	522	80	0	20
	16	134	60	0	40
	20	449	80	20	0
	21	94	50	0	0
Unit 1 Average:		334	71	10	8
Unit 1 St Dev:		255	25	14	14
Unit 1 SS:		652792	6491	2023	1968
2 (Quality products, short delivery price sensitive, low volume)	2	56	40	0	60
	10	34	50	50	0
	11	56	50	50	0
	15	77	60	0	40
	17	13	30	0	30
	18	33	40	0	30
	19	29	40	0	20
Unit 2 Average:		43	44	14	26
Unit 2 St Dev:		21	10	24	21
Unit 2 SS:		2770	571	3571	2771
3 (Short delivery lead times, low volume and spares)	3	8	30	50	0
	8	15	0	25	0
	13	6	0	20	0
	22	3	0	0	0
	23	16	0	0	0
	24	50	0	0	0
	25	4	0	0	0
	26	17	0	0	0
	27	1	0	0	0
	28	8	0	0	0
Unit 3 Average:		13	3	10	0
Unit 3 St Dev:		14	9	17	0
Unit 3 SS:		1822	810	2623	0
Across Units					
Avg Std Dev:		97	15	19	12
Avg SS:		219128	2624	2739	1580

4 Clusters (K = 4)

Recommended Manufacturing Unit	Prod #	Projected Weekly Prod (hrs)	Projected Order - Winners Price	Delivery Speed	Quality
1	**4**	123	50	25	25
	6	196	60	40	0
	7	200	20	0	0
	16	134	60	0	40
	21	94	50	0	0
Unit 1 Average:		149	48	13	13
Unit 1 Std Dev:		47	16	19	19
Unit 1 SS:		8735	1080	1380	1380
2	1	913	80	20	0
	5	178	100	0	0
	9	584	100	0	0
	12	279	100	0	0
	14	522	80	0	20
	20	449	80	20	0
Unit 2 Average:		488	90	7	3
Unit 2 Std Dev:		258	11	10	8
Unit 2 SS:		332298	600	533	333
3	2	56	40	0	60
	10	34	50	50	0
	11	56	50	50	0
	15	77	60	0	40
	17	13	30	0	30
	18	33	40	0	30
	19	29	40	0	20
Unit 3 Average:		43	44	14	26
Unit 3 Std Dev:		21	10	24	21
Unit 3 SS:		2770	571	3571	2771
4	3	8	30	50	0
	8	15	0	25	0
	13	6	0	20	0
	22	3	0	0	0
	23	16	0	0	0
	24	50	0	0	0
	25	4	0	0	0
	26	17	0	0	0
	27	1	0	0	0
	28	8	0	0	0
Unit 4 Average:		13	3	10	0
Unit 4 Std Dev:		14	9	17	0
Unit 4 SS:		1822	810	2623	0
Across Units					
Avg Std Dev:		85	12	18	12
Avg SS:		86406	765	2027	1121

- **Discussion of Methods Chosen**

Overall, as noted by Berry et al., these results illustrate why a different segmentation basis is frequently required by operations in order to develop a consistent manufacturing task for individual operating units (372). The five methods, altogether, showed a similar cluster membership.

Table 13 shows a comparison of the different clustering models used in this thesis. Colors are used to differentiate clusters or focused factories and representative objects are in bold face. In addition, the manufacturing unit column indicates the results obtained from the dendogram in terms of each cluster size as suggested by the original dendogram partition.

Observations

In general, three segments have been identified from the k-Means Model. Segment 1 includes high volume products sold to price sensitive customers, segment 2 requires premium quality products sold to price sensitive customers who buy low volume products and segment 3 demands short delivery lead time products to customers who buy low volume products and spares. The remaining models generated similar results from which the following observations are made:

1. The two-cluster solution is the same for all the methods, but starts to differ as the number of groups become larger. Moreover, segment 1 remains the same for the three-cluster solution across all methods.
2. The Plant Location and the Average-Weighted Distance Model are the same except for the four-cluster solution, whereas the opposite occurs with Plant Location and Fuzzy Set Method, which have the same four-cluster solution.

3. In the Plant Location Model, segment 3 results include low volume products and spares but not much attention is paid to short delivery lead times. Segment 2 also includes premium quality products sold to price sensitive customers who buy low volume products along with products 10 and 11 which require short delivery lead times; this could explain why the standard deviation is higher when compared with segment 2 in the k-Means Model. Furthermore, cluster membership was not stable across all 3 solutions because in the four-cluster solution segment 2 was formed by grouping product 3 from segment 3 with products 10 and 11 from segment 2 in the three-cluster solution. This division generated 2 potential units with a focused manufacturing task: both of them are addressed to price sensitive customers who buy low volume products, but segment 2 requires short delivery lead times and segment 3 includes premium quality products.
4. In the Average-Weighted Distance Model, segment 1 from the three-cluster solution in the Plant Location Model is divided in 2 potential units for the four-cluster solution: medium to high volume products to somewhat price sensitive customers (segment 1) and high volume products sold to price sensitive customers (segment 2). Segments 3 and 4 in the Average-Weighted Distance Model are equal to segments 2 and 3 from the three-cluster solution in the same model.
5. In the Covering Model cluster membership is stable across all units. Segment 2 is a combination of segments 2 and 3 in the k-Means Model; it requires quality products sold to price sensitive customers who not only buy low volume products, but also spares. Segment 3 then is the same as segment 2 in the four cluster solution of the Plant Location Model: short delivery lead times for price sensitive customers who buy low volume products. This

category is extended in the four-cluster solution from this model because it includes products 8 and 13 from segment 2 in the three-cluster solution.

6. In the Fuzzy Set Method, segment 2 includes products 3, 10 and 11 which require short delivery lead times, leaving segment 3 as a low volume and spares unit, where delivery speed is not as important.

Table 13
Clustering Models Comparison – Focused Factory Study

Mfg. Unit	k-Means	Plant L.	Covering	Fuzzy	W. Aver
1	1	1	1	1	1
1	4	4	4	4	4
1	5	5	5	5	5
1	6	6	6	6	6
1	7	7	7	7	7
1	9	9	9	9	9
1	12	12	12	12	12
1	14	14	14	14	14
1	16	16	16	16	16
1	20	**20**	20	20	**20**
1	21	21	**21**	21	21
2	2	2	2	2	2
2	3	3	3	3	3
2	8	8	8	8	8
2	10	10	10	10	10
2	11	11	11	11	11
2	13	13	13	13	13
2	15	15	15	15	15
2	17	17	17	17	17
2	18	18	18	18	18
2	19	19	**19**	19	19
2	22	22	22	22	22
2	23	23	23	23	23
2	24	24	24	24	24
2	25	25	25	25	**25**
2	26	26	26	26	26
2	27	27	27	27	27
2	28	**28**	28	28	28

Mfg. Unit	k-Means	Plant L.	Covering	Fuzzy	W. Aver
1	1	1	1	1	**1**
1	4	4	4	4	4
1	5	5	5	5	5
1	6	6	6	6	6
1	7	7	7	7	7
1	9	9	9	9	9
1	12	12	12	12	12
1	14	14	14	14	14
1	16	16	16	16	16
1	20	**20**	20	20	20
1	21	21	**21**	21	21
2	2	2	2	2	2
2	15	10	8	3	10
2	17	11	13	10	11
3	18	15	15	11	15
3	19	17	**17**	15	17
3	3	**18**	18	17	**18**
3	8	19	19	18	19
3	10	3	22	19	3
3	11	8	23	8	8
3	13	13	24	13	13
3	22	22	25	22	22
3	23	23	26	23	**23**
3	24	24	27	24	24
3	25	25	28	25	25
3	26	26	3	26	26
3	27	**27**	10	27	27
3	28	28	**11**	28	28

Mfg. Unit	k-Means	Plant L.	Covering	Fuzzy	W. Aver
1	1	**1**	1	1	**4**
1	5	4	4	4	6
1	6	5	6	5	7
1	9	6	7	6	16
1	12	7	16	7	21
1	14	9	20	9	1
1	20	12	**21**	12	5
1	2	14	**3**	14	**9**
1	15	16	8	16	12
1	17	20	10	20	14
1	18	21	11	21	20
2	19	3	13	3	2
2	3	10	5	10	10
2	8	**11**	**9**	11	11
3	10	2	12	2	15
3	11	15	14	15	17
3	13	17	2	17	**18**
3	22	**18**	15	18	19
3	23	19	17	19	3
4	24	8	18	8	8
4	25	13	**19**	13	13
4	26	22	22	22	22
4	27	23	23	23	23
4	28	**24**	24	24	24
4	4	25	25	25	**25**
4	7	26	26	26	26
4	16	27	27	27	27
4	21	28	28	28	28

- **Statistical Comparison of Results**

Berry's analysis of results from the Agglomerative Hierarchy with k-Means Refinement Model suggests that the three-cluster solution provides a smaller standard deviation across all units when compared with the company's current manufacturing assignments. However, the validity of this statement is not clear because the four-cluster solution provides a lower average standard deviation across all units than the three-cluster solution.

Therefore, the best process by which to make statements and characterize the performance of these methods remains an open research question. Some possibilities, as well as their limitations, are explained next. The ANOVA, although it analyzes differences between groups, has the constraint that the sample size (e.g. number of products) must be constant. The standard deviation is a measure that ignores the effect on scale on the cluster centroid. Another measure could be the coefficient of variation. Despite these constraints, this research uses the average standard deviation as criteria for evaluation, along with the average sum of squares across all units for each projected order-winner and volume measure. Table 14 summarizes these two measures and ranks methods in ascending order from lowest average standard deviation and sum of squares to the highest.

There are many ways in which Table 14 can be visualized and interpreted. Two possibilities are vertical search (by columns) or horizontal search (by rows). The horizontal search consists of looking at the statistical measures independently and within a specific statistical measure, looking at the frequency each method shows across the different order-winners and volume measures. Make note of cluster size in which the frequency of a specific method is higher across all units. The vertical search looks at the order-winners and volume

measures independently, regardless of the statistical measure used. The aim is to find a method with the highest frequency per order-winner and volume measure.

Results from the horizontal search indicate that neither within the standard deviation nor the sum of squares, were any of the methods found twice in displaying the lowest standard deviation or sum of the squares for the three-cluster solution. Conversely, the four-cluster solution suggests that both the Plant Location Model and the Fuzzy Set Method display the lowest standard deviation across all dimensions, whereas as the k-Means Model was twice found to have the lowest sum of the squares. Both measures suggest four segments as potential units for manufacturing tasks.

From the vertical search, it can be inferred that the Plant Location Model has the lowest standard deviation for the weekly production measure in both the three and four solution. Similarly, the Covering Model has the lowest sum of squares for the delivery speed dimension, whereas the k-Means has the lowest sum of squares for the quality dimension. In any event, it should be emphasized that these are just possibilities that can be used as measures for effectiveness, it is not an exhaustive list.

Table 14

Average Standard Deviation and Sum of Squares across all Clustering Methods

Statistical Measure	Mfg Unit	Weekly Production	Price	Delivery Speed	Quality
Standard Deviation	2	All models are equal			
	3	1. Plant Location and Average-Weighted 2. Fuzzy Set 3. k-Means 4. Covering	1. Fuzzy Set 2. Plant Location and Average-Weighted 3. k-Means 4. Covering	1. Covering 2. k-Means 3. Fuzzy Set 4. Plant Location and Average-Weighted	1. k-Means 2. Covering 3. Plant Location, Average-Weighted and Fuzzy Set
	4	1. Plant Location and Fuzzy Set 2. Average-Weighted 3. k-Means 4. Covering	1. Plant Location, Average-Weighted and Fuzzy 2. k-Means 3. Covering	1. Plant Location and Fuzzy Set 2. Covering 3. k-Means 4. Average-Weighted	1. Plant Location and Fuzzy Set 2. k-Means 3. Covering and Average-Weighted
Sum of Squares	2	All models are equal			
	3	1. Plant Location and Average-Weighted Distance 2. Fuzzy Set 3. k-Means 4. Covering	1. Fuzzy Set 2. Plant Location and Average-Weighted 3. k-Means 4. Covering	1. Covering 2. k-Means 3. Fuzzy Set 4. Plant Location and Average-Weighted	1. k-Means 2. Plant Location and Average-Weighted Distance 3. Fuzzy Set 4. Covering
	4	1. k-Means 2. Covering 3. Plant Location, Average-Weighted and Fuzzy Set	1. Plant Location, Average-Weighted and Fuzzy Set 2. k-Means 3. Covering	1. Covering 2. Plant Location, Average-Weighted and Fuzzy Set 3. k-Means	1. k-Means 2. Plant Location, Average-Weighted and Fuzzy Set 3. Covering

Chapter 4

Case Study - Cellular Manufacturing

The following experimental report shows the technique used by Heragu in the book Facilities Design, Chapter 9. It illustrates a case in which other real-world constraints are incorporated in the design of a cellular manufacturing system. For example, a company may wish to arrange machines into cells and parts into families to minimize the number of trips made by the material-handling carriers between cells. Or, a company may be interested in forming machine cells for the most efficient machine utilization in each cell (315). However, the focus is on showing the performance of the clustering algorithms used in this thesis, some irrelevant information will be omitted.

The company under study- Mediquip Manufacturer Inc. – has eight machines belonging to [six] machine types. For several reasons, including recent drastic changes in product mix and volume, the current layout has led to such problems as high traffic congestion, lack of control in the system, low employee morale, and increased material-handling cost and time. The industrial engineering (IE) team at Mediquip strongly feels that regrouping the machines into cells (so that each is dedicated to the manufacture of a set of parts with similar processing characteristics) will alleviate many of the problems. The following information is available (320):

- Current layout drawn to scale (Figure 10);
- Routing information for 20 parts, including set-up times, processing times, batch size, and annual demand (Table 15);
- Material-handling devices used between machines and cost of moving a unit load through a unit distance on each (Table G.1 in Appendix G).

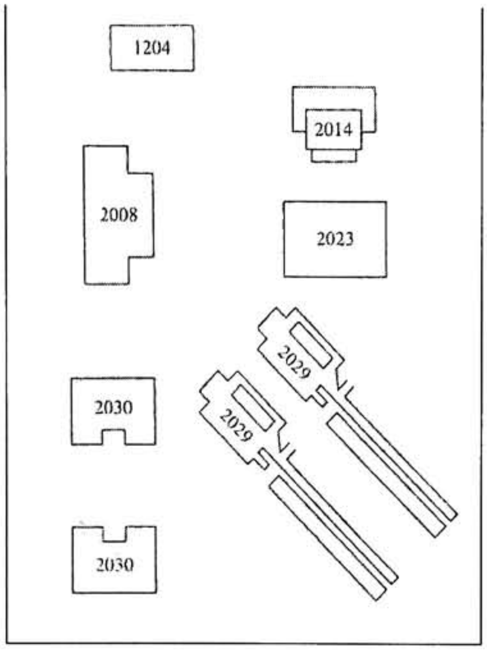


Figure 10. Current process layout for eight machines at Mediquip (Heragu, 1997) 320.

The production manager at Mediquip prefers to have no more than three machines in each cell because of a lack of skilled personnel. Further, if more than one unit of a machine type is available for processing, each batch of parts that require this machine type is split in equal proportion. The machine grouping and layout is to be determined using the GTLAYPC.EXE program (320, 322).

The first step is to determine whether there is sufficient capacity to process the projected part mix and volume. From the given demand and batch size, the number of batches of each part that are to be processed annually is determined by dividing the demand by the batch size (see fifth column in Table 16). Using the set-up and processing times per batch on Table 15, the time required on each machine type for processing each of the 20 part types is calculated by multiplying the number of batches by the sum of the set-up and processing times per batch (see the last six columns of Table 16) (322-323).

Table 15

Part information

Part Number	Part Name	Annual Demand (in Units)	Batch Size (in Units)	Machine Name	Set-up time (in Hours) Per Batch	Processing Time (in Hours) Per Batch
1	115110	220	10	01204	3.00	3.64
				02008	5.98	5.98
2	176022	500	10	01204	5.00	2.00
				02008	2.25	1.00
				02014	0.50	6.50
3	176023	500	10	01204	4.00	2.00
				02008	1.00	3.00
4	17007	500	10	02008	1.50	3.00
				02023	0.50	8.27
				02008	0.00	2.00
				02014	2.00	2.00
5	201052	220	10	02008	1.00	2.00
				02014	1.00	6.86
				02023	1.00	10.00
6	202102-501	120	10	02030	0.5	84.58
				02023	1.00	26.78
7	202103	700	10	02008	1.00	1.00
				02023	1.00	1.00
8	203009	900	10	02030	0.75	1.25
				02014	0.50	1.00
				02029	0.00	0.31
9	209001	200	10	02029	8.00	65.00
				02014	0.00	2.23
10	210001	250	10	02030	0.50	5.00
				02008	1.00	7.00
11	217002-2	400	10	02029	8.00	55.00
				02014	1.00	2.67
12	236235	300	10	01204	2.89	8.68
				02008	1.00	8.00
13	270071-501	250	10	02029	0.00	8.55
				02030	0.00	19.37
				02014	1.50	2.80
14	270080-501	600	10	02029	0.00	7.53
				02030	1.00	3.00
				02014	0.75	1.00
15	280011	450	10	02030	1.00	2.91
				02030	1.50	3.83
16	280024	500	10	01204	4.00	3.64
				02008	0.75	3.00
17	30130	400	10	02030	0.00	2.50
				02023	0.50	2.00
18	324008	300	10	02029	0.00	12.01
				02030	0.00	7.59
				02023	0.75	14.50
19	325003	10	10	02029	0.00	23.25
				02030	0.00	23.63
				02008	0.00	42.67
20	328008	250	10	02029	0.00	7.28
				02030	0.00	23.00
				02023	1.00	2.00

Source: Heragu, Sunderesh. Facilities Design (1997) Table 9.1

Table 16

Machine Capacity Calculations

Part Number	Machine Name	Set-up Time Per Batch	Processing Time Per Batch	Number of Batches	Time (in Hours) Required on Each Machine					
					01204	02008	2014	02023	02029	02030
1	01204	3.00	3.64	22	146					
	02008	5.98	5.98			198				
2	01204	5.00	2.00	50	350					
	02008	2.25	1.00			163				
	02014	0.50	6.50			350				
3	01204	4.00	2.00	50	300					
	02008	1.00	3.00			200				
4	02008	1.50	3.00	50		225				
	02023	0.50	8.27					439		
	02008	0.00	2.00			100				
	02014	2.00	2.00				200			
5	02008	1.00	2.00	22		66				
	02014	1.00	6.86				173			
	02023	1.00	10.00					242		
6	02030	0.5	84.58	12						1021
	02023	1.00	26.78					333		
7	02008	1.00	1.00	70		140				
	02023	1.00	1.00				140			
8	02030	0.75	1.25	90						180
	02014	0.50	1.00			135				
	02029	0.00	0.31						28	
9	02029	8.00	65.00	20					1460	
	02014	0.00	2.23				45			
10	02030	0.50	5.00	25						138
	02008	1.00	7.00			200				
11	02029	8.00	55.00	40					2520	
	02014	1.00	2.67				147			
12	01204	2.89	8.68	30	347					
	02008	1.00	8.00			270				
13	02029	0.00	8.55	25					214	
	02030	0.00	19.37							484
	02014	1.50	2.80				108			
14	02029	0.00	7.53	60					452	
	02030	1.00	3.00							240
	02014	0.75	1.00				105			
15	02030	1.00	2.91	45					176	
	02030	1.50	3.83							240
16	01204	4.00	3.64	50	382					
	02008	0.75	3.00			188				
17	02030	0.00	2.50	40						100
	02023	0.50	2.00					100		
18	02029	0.00	12.01	30					360	
	02030	0.00	7.59							228
	02023	0.75	14.50					458		
19	02029	0.00	23.25	1					23	
	02030	0.00	23.63							24
	02008	0.00	42.67			43				
20	02029	0.00	7.28	25				182		
	02030	0.00	23.00							575
	02023	1.00	2.00					75		
Machine hours required					1525	1791	1262	1786	5239	3405
Number of machines required					1.00	1.00	1.00	1.00	3.00	2.00

Source: Heragu, Sunderesh. Facilities Design (1997) Table 9.1

Summing up the values in each of the last six columns of Table 16 gives the hours required on each machine type (assuming that a set-up is incurred for each batch). Dividing these figures by the time available (assuming 40 hours per week of operation for 50 weeks at 90% efficiency or 1800 hours) provides an estimate of the number of machines of each type required. For the 90% assumption, it is found that three units of machine type 02029, two units of machine type 02030, and one unit of the other types is required (323).

Appendix G contains the input data file used to solve this problem as well as the output generated by GTLAYPC.EXE. For now on, the focus will be on the results obtained by Heragu's methodology and use of the information given to extend to the five clustering algorithms already explained in chapter 3.

- **Heragu's Methodology**

The information given in Tables 15 and 16 has been reproduced in Table 17 using a slightly different format. In order to simplify the handling of the data, part names are no longer being used, but rather part number. Notice in table 17.3 that three more columns are added to the original data (table 17.1), which correspond to the extra machines used (machines 5 and 6 need more than one unit for processing). Also, since each part that requires these machines is assumed to visit each of the units in equal proportion, columns 6 and 7 are identical to column 5, and column 9 is identical to column 8.

The Fractional Processing Matrix is also reproduced from the original results (Table 17.2). An error was found and corrected from the original source: part 3 uses machine 1 and 2, but the original source left a blank under machine 1 (consult Appendix G to see the original fractional processing matrix). Table 17.2 shows the fraction of each part type that visits each

unit of each machine type. A 0 (or blank) indicates that the part does not visit the machine, and a 1 indicates that 100 % of this part type is processed on the corresponding machine (324). Using this matrix (Table 17.2) and the original data (Table 17.1), a third table (Table 17.3) was constructed to show how processing and set-up times were divided between the extra machine units added.

Based on the output obtained from GTLAYPC.EXE (see Appendix G), Heragu's Results were compiled in Table 18, which indicates which parts should be included in each of the three cells.

To better understand these results, and analyze the proposed capacity for each recommended cell, three graphs were constructed, as shown in Figure 11. Table 19 displays new metrics based on the data from Heragu's results in terms of average annual demand, annual machine utilization, and annual set-up time per cell, respectively. This information is utilized in the clustering analysis described in the following sections.

Table 17

Data for Mediquip Manufacturer Inc.

Table 17.1 Original Data

Part #	Annual Dem Units	Set-up Time Per Batch (hrs) Machines 1	2	3	4	5	6	Process Time Per Batch (hrs) Machines 1	2	3	4	5	6
1	220	3.000	3.000					3.640	5.980				
2	500	5.000	2.250	0.500				2.000	1.000	6.500			
3	500	4.000	1.000					2.000	3.000				
4	500		1.500	2.000	0.500				5.000	2.000	8.270		
5	220		1.000	1.000	1.000				2.000	6.860	10.000		
6	120				1.000		0.500				26.780		84.580
7	700		1.000		1.000				1.000		1.000		
8	900			0.500		0.000	0.750			1.000		0.310	1.250
9	200			0.000		8.000				2.230		65.000	
10	250		1.000				0.500		7.000				5.000
11	400			1.000		8.000				2.670		55.000	
12	300	2.890	1.000					8.680	8.000				
13	250			1.500		0.000	0.000			2.800		8.550	19.370
14	600			0.750		0.000	1.000			1.000		7.530	3.000
15	450						2.500						6.740
16	500	4.000	0.750					3.640	3.000				
17	400				0.500		0.000				2.000		2.500
18	300				0.750	0.000	0.000				14.500	12.010	7.590
19	10		0.000			0.000	0.000		42.670			23.250	23.630
20	250				1.000	0.000	0.000				2.000	7.280	23.000

Table 17.2 Fractional Processing Matrix

Part #	Annual Dem Units	Part Fraction (Per Machine) Machines 1	2	3	4	5	6	7	8	9
1	220	1	1							
2	500	1	1	1						
3	500	1	1							
4	500		1	1	1					
5	220		1	1	1					
6	120				1				0.5	0.5
7	700		1		1					
8	900			1		0.33	0.33	0.33	0.5	0.5
9	200			1		0.33	0.33	0.33		
10	250		1						0.5	0.5
11	400			1		0.33	0.33	0.33		
12	300	1	1							
13	250			1		0.33	0.33	0.33	0.5	0.5
14	600			1		0.33	0.33	0.33	0.5	0.5
15	450								0.5	0.5
16	500	1	1							
17	400				1				0.5	0.5
18	300				1	0.33	0.33	0.33	0.5	0.5
19	10		1			0.33	0.33	0.33	0.5	0.5
20	250				1	0.33	0.33	0.33	0.5	0.5

Table 17.3 Modified Data

Part #	Annual Dem Units	Set-up Time Per Batch (hrs) Machines 1	2	3	4	5	6	7	8	9
1	220	3.000	3.000							
2	500	5.000	2.250	0.500						
3	500	4.000	1.000							
4	500		1.500	2.000	0.500					
5	220		1.000	1.000	1.000					
6	120				1.000				0.250	0.250
7	700		1.000		1.000					
8	900			0.500		0.000	0.000	0.000	0.375	0.375
9	200			0.000		2.640	2.640	2.640		
10	250		1.000						0.250	0.250
11	400			1.000		2.640	2.640	2.640		
12	300	2.890	1.000							
13	250			1.500		0.000	0.000	0.000	0.000	0.000
14	600			0.750		0.000	0.000	0.000	0.500	0.500
15	450								1.250	1.250
16	500	4.000	0.750							
17	400				0.500				0.000	0.000
18	300				0.750	0.000	0.000	0.000	0.000	0.000
19	10		0.000			0.000	0.000	0.000	0.000	0.000
20	250				1.000	0.000	0.000	0.000	0.000	0.000

Part #	Processing Time Per Batch (hrs) Machines 1	2	3	4	5	6	7	8	9
1	3.640	5.980							
2	2.000	1.000	6.500						
3	2.000	3.000							
4		5.000	2.000	8.270					
5		2.000	6.860	10.000					
6				26.780				42.290	42.290
7		1.000		1.000					
8			1.000		0.102	0.102	0.102	0.625	0.625
9			2.230		21.450	21.450	21.450		
10		7.000						2.500	2.500
11			2.670		18.150	18.150	18.150		
12	8.680	8.000							
13			2.800		2.822	2.822	2.822	9.685	9.685
14			1.000		2.485	2.485	2.485	1.500	1.500
15								3.370	3.370
16	3.640	3.000							
17				2.000				1.250	1.250
18				14.500	3.963	3.963	3.963	3.795	3.795
19		42.670			7.673	7.673	7.673	11.815	11.815
20				2.000	2.402	2.402	2.402	11.500	11.500

Table 18

Heragu's Results

Recommended Manufacturing Unit	Part #	Annual Demand Units	Set-up Time Per Batch									Process Time Per Batch								
			Machines									Machines								
			1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
1	1	220	3.000	3.000								3.640	5.980							
	2	500	5.000	2.250	0.500							2.000	1.000	6.500						
	3	500	4.000	1.000								2.000	3.000							
	4	500		1.500	2.000								5.000	2.000						
	5	220		1.000	1.000								2.000	6.860						
	7	700		1.000									1.000							
	8	900			0.500									1.000						
	9	200			0.000									2.230						
	10	250		1.000									7.000							
	11	400			1.000									2.670						
	12	300	2.890	1.000								8.680	8.000							
	13	250			1.500									2.800						
	14	600			0.750									1.000						
	16	500	4.000	0.750								3.640	3.000							
	19	10		0.000									42.670							
Unit 1 Average:		403.333	3.778	1.250	0.906							3.992	7.865	3.133						
Unit 1 St Dev:		228.619	0.864	0.833	0.626							2.746	12.472	2.291						
2	4	500				0.500									8.270					
	5	220				1.000									10.000					
	6	120				1.000				0.250	0.250				26.780				42.290	42.290
	7	700				1.000									1.000					
	8	900								0.375	0.375								0.625	0.625
	10	250								0.250	0.250								2.500	2.500
	13	250								0.000	0.000								9.685	9.685
	14	600								0.500	0.500								1.500	1.500
	15	450								1.250	1.250								3.370	3.370
	17	400				0.500				0.000	0.000				2.000				1.250	1.250
	18	300				0.750				0.000	0.000				14.500				3.795	3.795
	19	10								0.000	0.000								11.815	11.815
	20	250				1.000				0.000	0.000				2.000				11.500	11.500
Unit 2 Average:		380.769				0.821				0.263	0.263				9.221				8.833	8.833
Unit 2 St Dev:		246.322				0.238				0.393	0.393				9.216				12.517	12.517
3	8	900					0.000	0.000	0.000							0.102	0.102	0.102		
	9	200					2.640	2.640	2.640							21.450	21.450	21.450		
	11	400					2.640	2.640	2.640							18.150	18.150	18.150		
	13	250					0.000	0.000	0.000							2.822	2.822	2.822		
	14	600					0.000	0.000	0.000							2.485	2.485	2.485		
	18	300					0.000	0.000	0.000							3.963	3.963	3.963		
	19	10					0.000	0.000	0.000							7.673	7.673	7.673		
	20	250					0.000	0.000	0.000							2.402	2.402	2.402		
Unit 3 Average:		363.750					0.660	0.660	0.660							7.381	7.381	7.381		
Unit 3 St Dev:		274.275					1.222	1.222	1.222							8.002	8.002	8.002		
Avg Std Dev Across Recommended Units:		249.739	0.864	0.833	0.626	0.238	1.222	1.222	1.222	0.393	0.393	2.746	12.472	2.291	9.216	8.002	8.002	8.002	12.517	12.517

Table 19

Data Display - Heragu's Results

Average Annual Demand

Part #	Annual Demand (Units) Cell 1	Cell 2	Cell 3	Total Demand
1	220			220
2	500			500
3	500			500
4	250	250		500
5	110	110		220
6		120		120
7	350	350		700
8	300	300	300	900
9	100		100	200
10	250	250		500
11	200		200	400
12	300			300
13	83	83	84	250
14	200	200	200	600
15		450		450
16	500			500
17		400		400
18		150	150	300
19	3	3	4	10
20		125	125	250
Aver	258	215	145	
St Dev	155	133	90	
SS:	338501	212317	56126	
Across Units				
Aver Std Dev:		126		
Aver SS:		202315		

Machine Utilization

$$Machine\text{-}Utiliz(i) = \sum_{i=1}^{i=6} set\text{-}up(i) + proc.(i) * batchsiz.(j)$$

i = machine type (1 to 6)
j = product number (1 to 20)

Mach #	Utiliz (hr/yr) Cell 1	Cell 2	Cell 3
1	1525.18		
2	1791.23		
3	1261.82		
4		1786.36	
5			1728.87
6			1728.87
7			1728.87
8		1702.42	
9		1702.42	
Aver	1526.08	1730.40	1728.87
St Dev	264.71	48.46	0.00
SS:	140139	4697	0.00
Across Units			
Aver Std Dev:		104	
Aver SS:		48279	

Set-up Time (hr/yr)

$$Set - up\ (i) = \sum_{i=1}^{i=6} set - up\ (i) * batchsize\ (j)$$

i = machine type (1 to 6)
j = product number (1 to 20)

Mach #	Utiliz (hr/yr) Cell 1	Cell 2	Cell 3
1	802.70		
2	488.00		
3	314.50		
4		196.50	
5			158.40
6			158.40
7			158.40
8		129.25	
9		129.25	
Aver	535.07	151.67	158.40
St Dev	247.48	38.83	0.00
SS:	122493	3015	0.00
Across Units			
Aver Std Dev:		95	
Aver SS:		41836	

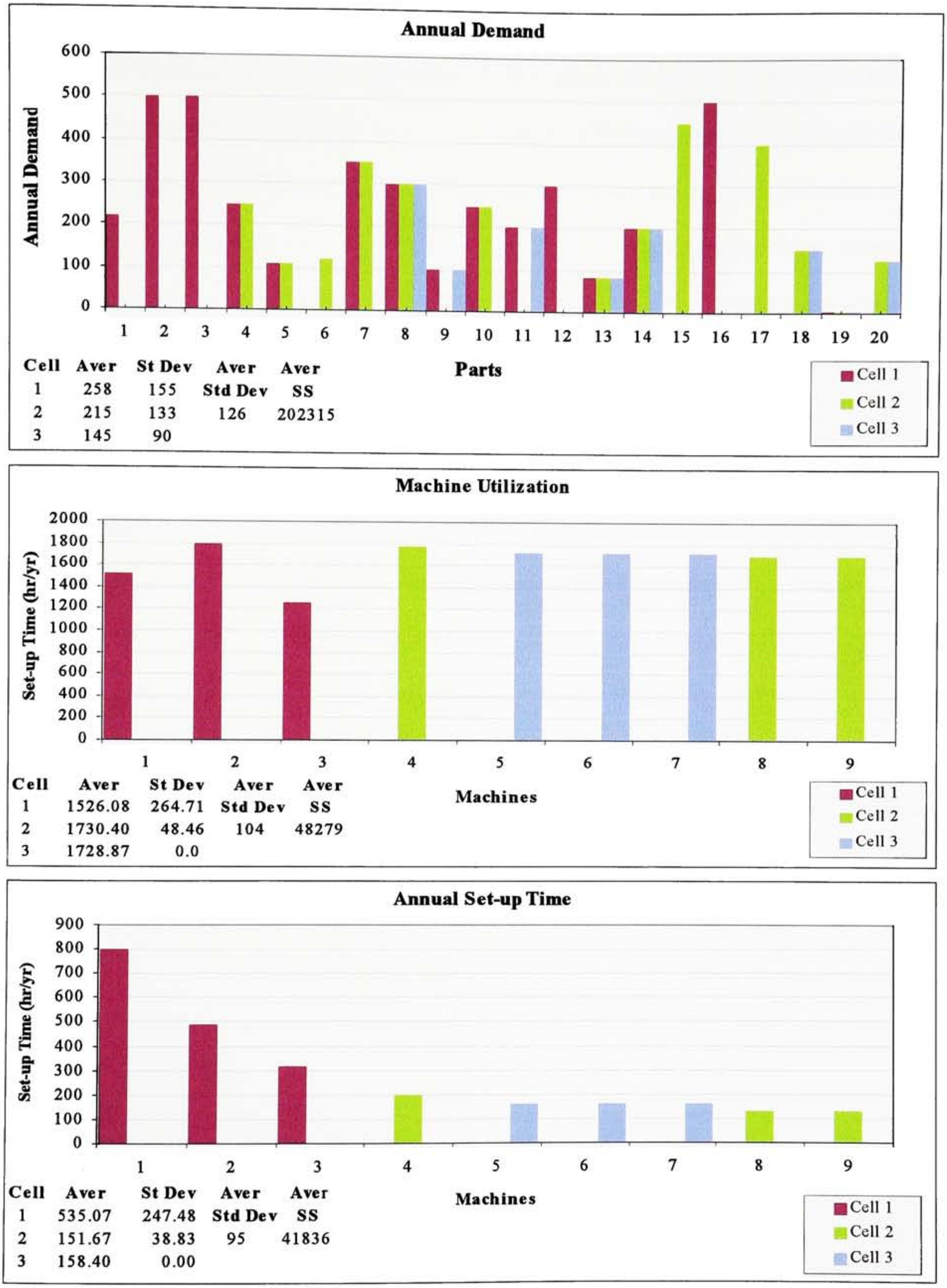


Figure 11. Graphs of Metrics from Heragu's Results

- **Clustering-Based Methodology**

Just as different clustering methods were used to group products into focused factory units, the following section illustrates the same approach applied to group parts into cells based on the data provided by Mediquip Manufacturer Inc. At first, the models were run using volume (annual demand), processing and set-up times, information found in Heragu's study. However, inconclusive results were obtained due to the large number of variables relative to the small sample size (number of parts). Hence, the dimensionality was reduced from 13 dimensions to 7. That is, only annual demand and set-up times of the 6 machines have been utilized.

The first model to present is the Agglomerative Hierarchy with k-Means Refinement. The initial step was to reduce the original data (Table 17.1) by eliminating the columns concerning processing times (Table 21). Since this data is given in different units (annual demand in units and set-up times in hours per batch), dividing it by the largest number of each column normalizes the data (Table 22), thereby assuring uniformity in scale. Further, based on the information of the normalized data provided in Table 22, the dissimilarity across the 7 dimensions is calculated using the Euclidean Distance and the Average Linkage Method (Figure 12).

Table 21

Original Data

Part #	Annual Demand Units	Set-up Time Per Batch (hrs)					
		Machines					
		1	2	3	4	5	6
1	220	3.000	3.000				
2	500	5.000	2.250	0.500			
3	500	4.000	1.000				
4	500		1.500	2.000	0.500		
5	220		1.000	1.000	1.000		
6	120				1.000		0.500
7	700		1.000		1.000		
8	900			0.500		0.000	0.750
9	200			0.000		8.000	
10	250		1.000				0.500
11	400			1.000		8.000	
12	300	2.890	1.000				
13	250			1.500		0.000	0.000
14	600			0.750		0.000	1.000
15	450						2.500
16	500	4.000	0.750				
17	400				0.500		0.000
18	300				0.750	0.000	0.000
19	10		0.000			0.000	0.000
20	250				1.000	0.000	0.000

Table 22

Normalized Data

Part #	Normalized Demand Units	Set-up Time Per Batch (hrs)					
		Machines					
		1	2	3	4	5	6
1	24.444	60.000	100.000				
2	55.556	100.000	75.000	25.000			
3	55.556	80.000	33.333				
4	55.556		50.000	100.000	50.000		
5	24.444		33.333	50.000	100.000		
6	13.333				100.000		20.000
7	77.778		33.333		100.000		
8	100.000			25.000		0.000	30.000
9	22.222			0.000		100.000	
10	27.778		33.333				20.000
11	44.444			50.000		100.000	
12	33.333	57.800	33.333				
13	27.778			75.000		0.000	0.000
14	66.667			37.500		0.000	40.000
15	50.000						100.000
16	55.556	80.000	25.000				
17	44.444				50.000		0.000
18	33.333				75.000	0.000	0.000
19	1.111		0.000			0.000	0.000
20	27.778				100.000	0.000	0.000

Part #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1																			
2	23.3541																		
3	28.8155	19.8606																	
4	52.8717	51.7549	52.3420																
5	54.1749	57.7297	53.2754	29.8704															
6	58.7044	63.7324	53.0415	49.5447	24.3033														
7	54.6272	57.1401	49.1264	43.5424	27.6314	28.4459													
8	54.5549	51.4092	39.6618	43.9160	51.1926	51.0408	43.3094												
9	58.0701	62.5198	51.5783	61.0750	58.0837	54.0886	58.7945	50.1061											
10	34.7554	43.9687	32.8886	44.6404	42.9470	40.2133	42.9285	31.7404	40.6061										
11	61.5282	61.3811	53.6318	50.1761	55.4348	58.3926	59.4285	45.6875	20.6807	45.1804									
12	25.4342	25.7343	11.8723	48.7156	47.6893	46.6782	46.7763	38.5846	45.6311	23.2124	49.3897								
13	52.4215	51.9569	44.5737	30.2291	40.9655	48.1569	52.4215	35.0837	47.2922	31.9287	39.4657	37.9997							
14	51.2542	50.0067	38.9887	38.9683	45.7479	45.7488	45.1044	13.9763	46.2625	25.1558	41.8331	34.9884	25.4070						
15	58.8619	61.2732	50.0599	59.7983	58.8754	50.3480	55.9116	33.8590	54.4736	33.8166	56.7335	45.8721	47.9863	27.4747					
16	31.6067	22.4404	3.1497	52.8137	53.3685	52.3828	49.2273	38.7765	50.9006	33.0390	52.9804	12.2830	43.7878	38.0877	49.3614				
17	48.5504	51.9250	38.0499	42.4659	30.4985	23.5065	25.9731	31.8735	43.0843	24.7527	46.2910	31.7926	34.6467	29.2772	42.3099	37.1261			
18	52.5140	56.5292	44.1263	44.1084	24.8283	14.2678	23.0261	40.6983	47.4318	31.9977	51.0580	37.9416	40.1441	37.3064	47.6637	43.3323	10.3403		
19	44.9515	52.3917	38.6843	50.6588	44.9691	38.8208	49.2644	40.1853	38.6295	17.8174	45.3207	28.0057	30.0859	32.3017	42.0716	37.7760	25.0079	30.8528	
20	58.0777	62.1307	51.1059	47.4667	22.7478	9.3246	22.7128	48.9038	53.4935	40.5518	57.0436	45.4860	47.2456	45.5421	54.1081	50.4219	19.9205	9.6796	39.1172

Figure 12. Dissimilarity Matrix – Cellular Manufacturing Study

1. Agglomerative Hierarchy with k-Means Refinement

Using Minitab, a dendogram generated by the hierarchical clustering was obtained for Mediquip Manufacturer Inc. (Figure 13). Although the program suggests standardizing the data with different units, this option was not considered because the data had been normalized before entering into the program. The dendogram indicates part groupings from two to three distinctive machine cells. See Appendix H for the Minitab output and amalgamation steps.

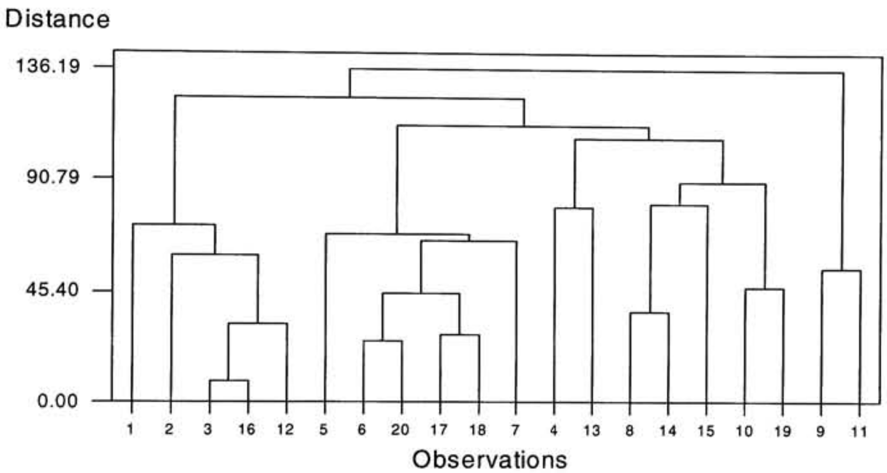


Figure 13. Dendogram for Mediquip Manufacturer Inc.

Using the dendogram as a guideline, the k-Means model was run using the seven dimensions mentioned above as grouping variables. Three iterations were performed with the number of clusters set at two, three, and four (Appendix I contains the Minitab output for the three-cluster solution, as well as the results for all three solutions). These results are shown on Table 22, and Figures 14 and 15 illustrate the annual demand and annual set-up times suggested by the model's results. The data used to generate these graphs is shown on Table J.1 in Appendix J. Further, the same analysis was done for the remaining clustering models and the results are indicated in the tables and figures below under each model section.

Table 22

k-Means Refinement – Cellular Manufacturing Study

2 Clusters (K = 2)

Recommended Manufacturing Unit	Part #	Annual Dem	Set-up Time Per Batch					
			Machines					
			1	2	3	4	5	6
1	1	220	3.000	3.000				
	2	500	5.000	2.250	0.500			
	3	500	4.000	1.000				
	4	500		1.500	2.000	0.500		
	8	900			0.500		0.000	0.750
	10	250		1.000				0.500
	11	400			1.000		8.000	
	12	300	2.890	1.000				
	13	250			1.500		0.000	0.000
	14	600			0.750		0.000	1.000
	16	500	4.000	0.750				
Unit 1 Average:		447.273	3.778	1.500	1.042	0.500	2.000	0.563
Unit 1 St Dev:		198.196	0.864	0.829	0.600	0.000	4.000	0.427
2	5	220		1.000	1.000	1.000		
	6	120				1.000		0.500
	7	700		1.000		1.000		
	9	200			0.000		8.000	
	15	450						2.500
	17	400				0.500		0.000
	18	300				0.750	0.000	0.000
	19	10		0.000			0.000	0.000
	20	250				1.000	0.000	0.000
Unit 2 Average:		294.444		0.667	0.500	0.875	2.000	0.500
Unit 2 St Dev:		202.368		0.577	0.707	0.209	4.000	1.000

Avg Std Dev Across
Recommended Units: 200.282 0.864 0.703 0.654 0.105 4.000 0.713

3 Clusters (K = 3)

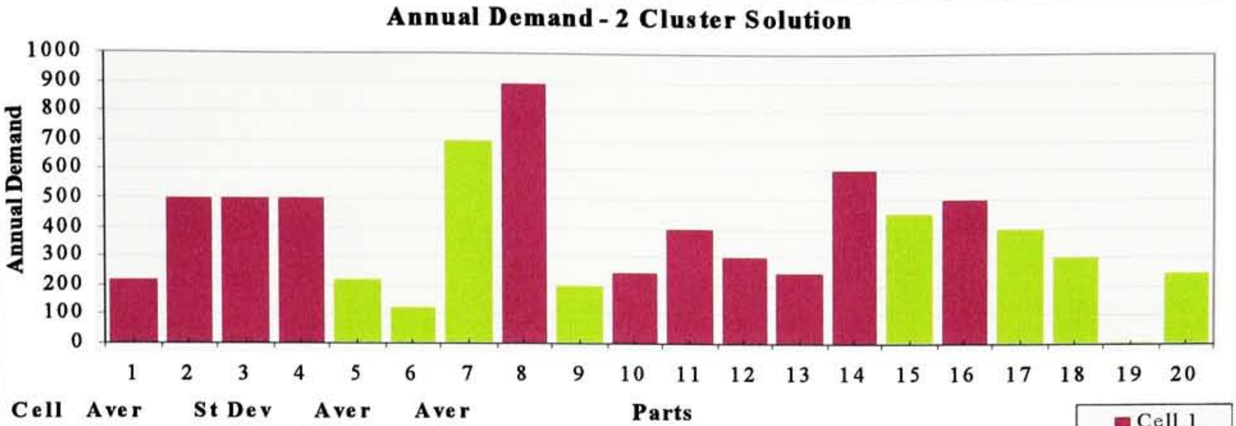
Recommended Manufacturing Unit	Part #	Annual Dem	Set-up Time Per Batch					
			Machines					
			1	2	3	4	5	6
1	1	220	3.000	3.000				
	2	500	5.000	2.250	0.500			
	3	500	4.000	1.000				
	12	300	2.890	1.000				
	16	500	4.000	0.750				
Unit 1 Average:		404.000	3.778	1.600	0.500			
Unit 1 St Dev:		134.462	0.864	0.978	0.000			
2	4	500		1.500	2.000	0.500		
	5	220		1.000	1.000	1.000		
Unit 2 Average:		360.000		1.250	1.500	0.750		
Unit 2 St Dev:		197.990		0.354	0.707	0.354		
3	6	120				1.000		0.500
	7	700		1.000		1.000		
	8	900			0.500		0.000	0.750
	9	200			0.000		8.000	
	10	250		1.000				0.500
	11	400			1.000		8.000	
	13	250			1.500		0.000	0.000
	14	600			0.750		0.000	1.000
	15	450						2.500
	17	400				0.500		0.000
	18	300				0.750	0.000	0.000
	19	10		0.000			0.000	0.000
	20	250				1.000	0.000	0.000
Unit 3 Average:		371.538		0.667	0.750	0.850	2.000	0.525
Unit 3 St Dev:		244.944		0.577	0.559	0.224	3.703	0.786

Avg Std Dev Across
Recommended Units: 192.465 0.864 0.636 0.422 0.289 3.703 0.786

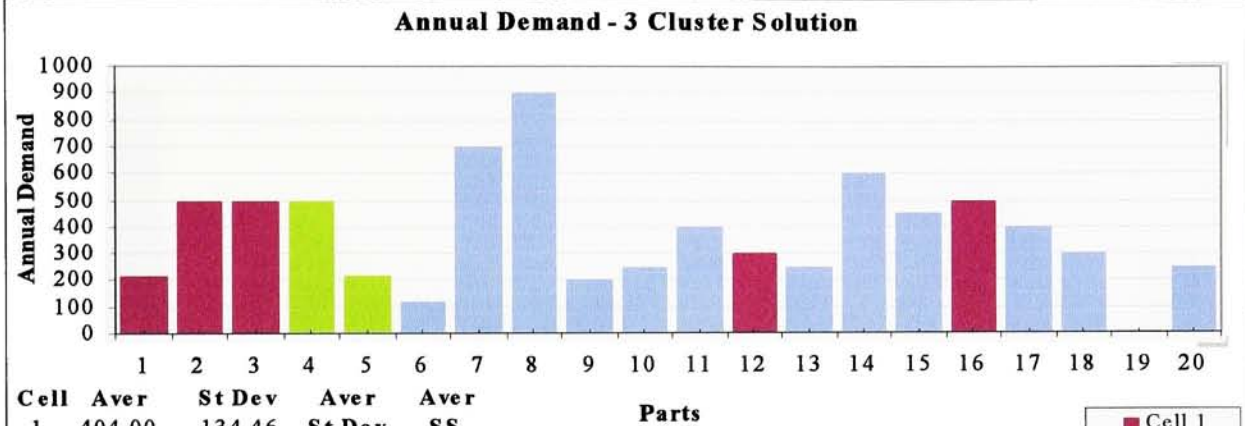
4 Clusters (K = 4)

Recommended Manufacturing Unit	Part #	Annual Dem	Set-up Time Per Batch					
			Machines					
			1	2	3	4	5	6
1	1	220	3.000	3.000				
Unit 1 Average:		220.000	3.000	3.000				
Unit 1 St Dev:		0.000	0.000	0.000				
2	2	500	5.000	2.250	0.500			
	3	500	4.000	1.000				
	12	300	2.890	1.000				
	16	500	4.000	0.750				
Unit 2 Average:		450.000	3.973	1.250	0.500			
Unit 2 St Dev:		100.000	0.862	0.677	0.000			
3	8	900			0.500		0.000	0.750
	9	200			0.000		8.000	
	10	250		1.000				0.500
	11	400			1.000		8.000	
	13	250			1.500		0.000	0.000
	14	600			0.750		0.000	1.000
	15	450						2.500
	19	10		0.000			0.000	0.000
Unit 3 Average:		382.500		0.500	0.750		2.667	0.792
Unit 3 St Dev:		274.421		0.707	0.559		4.131	0.928
4	4	500		1.500	2.000	0.500		
	5	220		1.000	1.000	1.000		
	6	120				1.000		0.500
	7	700		1.000		1.000		
	17	400				0.500		0.000
	18	300				0.750	0.000	0.000
	20	250				1.000	0.000	0.000
Unit 4 Average:		355.714		1.167	1.500	0.821	0.000	0.125
Unit 4 St Dev:		195.607		0.289	0.707	0.238	0.000	0.250

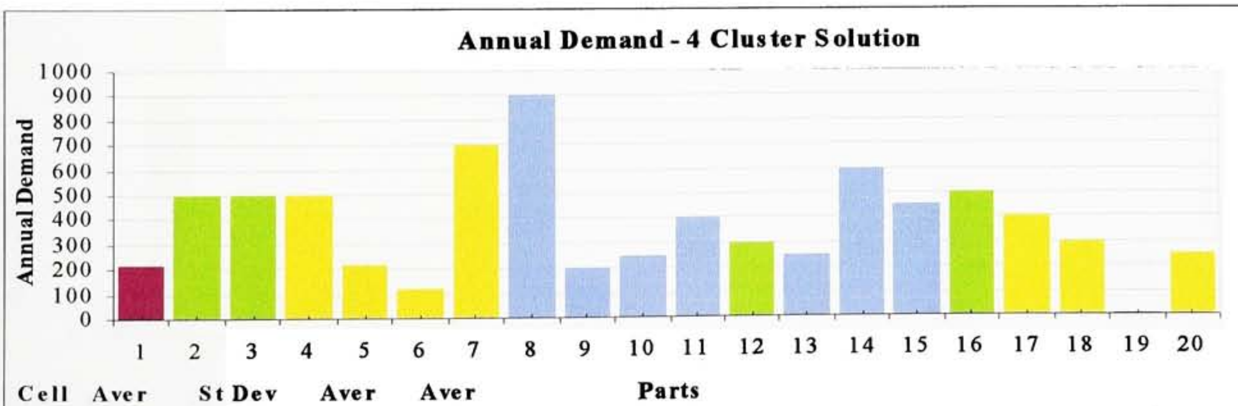
Avg Std Dev Across
Recommended Units: 142.507 0.431 0.418 0.422 0.238 2.066 0.589



Cell	Aver	St Dev	Aver St Dev	Aver SS
1	447.27	198.2		
2	294.44	202.37	200	392818

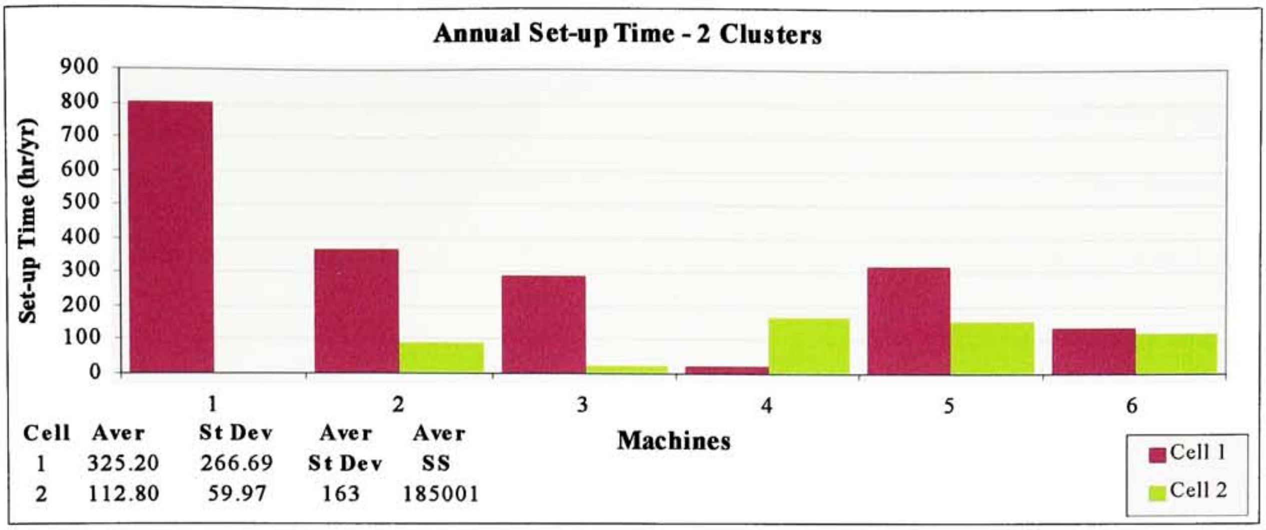


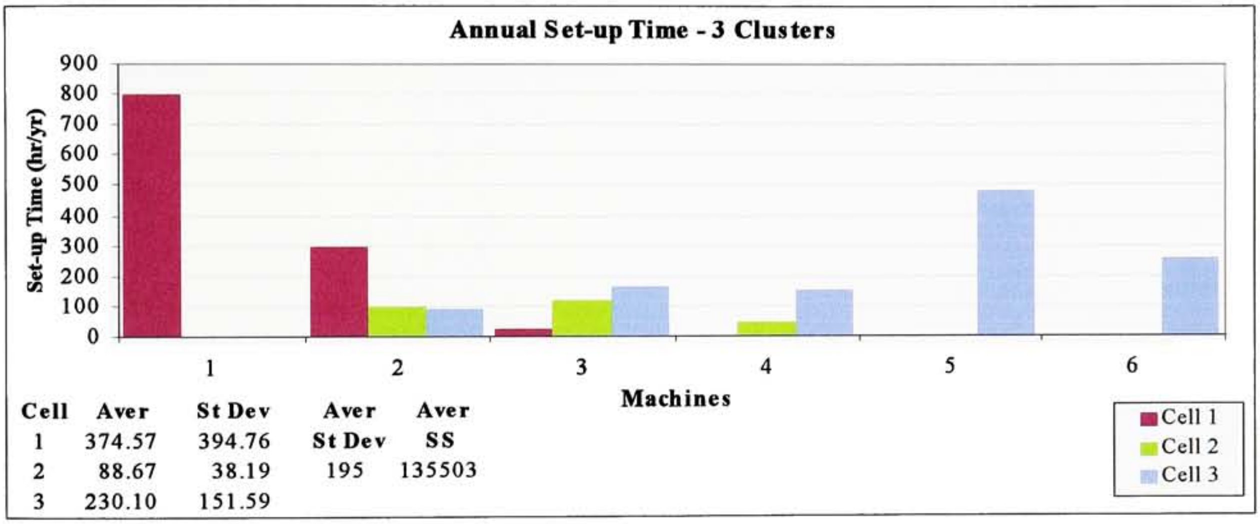
Cell	Aver	St Dev	Aver St Dev	Aver SS
1	404.00	134.46		
2	360.00	197.99	192	277163
3	371.54	244.94		



Cell	Aver	St Dev	Aver St Dev	Aver SS
1	220.00	0.00	143	196680
2	450.00	100.00		
3	382.50	274.42		
4	355.71	195.61		

Figure 14. Annual Demand for k-Means Model





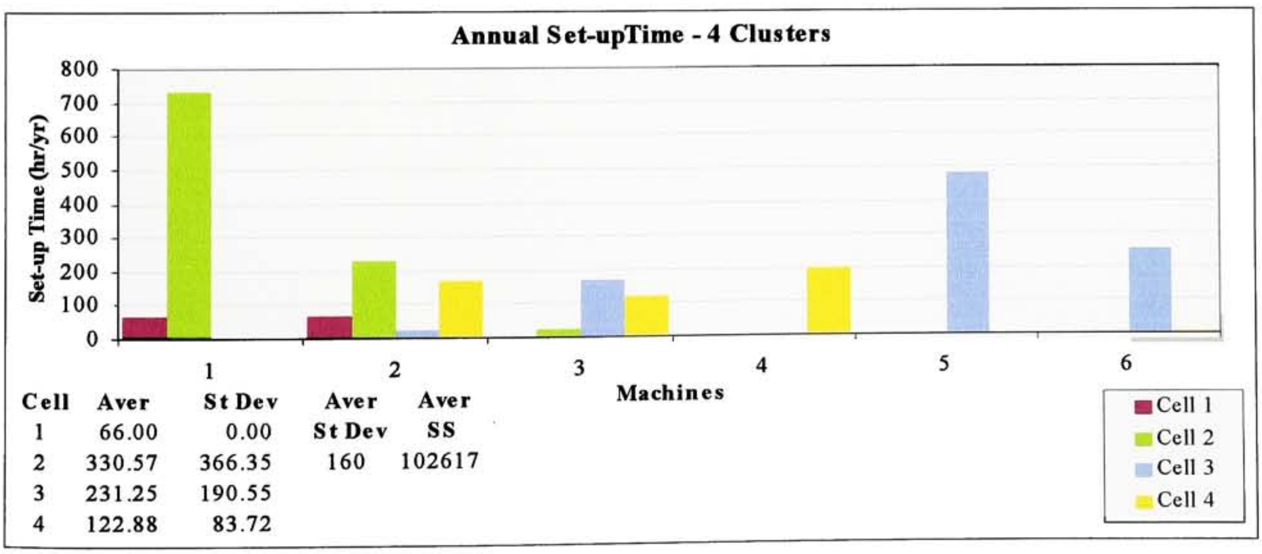


Figure 15. Annual Set-up Time for k-Means Model

2. Plant Location Model

The dissimilarities between parts, as illustrated in on Figure 11, were entered into Lingo following the Plant Location Model mathematical formulation (refer to Figure 4). The objective, as in the Focused Factory study, is to minimize the Euclidean Distance between all parts and the medoids of each cluster by assigning parts to k clusters or cells. In this case, three iterations were run setting the number of clusters to two, three and four. Results for all three solutions are summarized in Table 23 below, and Figures 16 and 17 illustrate the Annual Demand and Annual Set-up Time for each cell as suggested by the results from this model. Appendix J presents Table J.2, which displays the data used for calculating the metrics on these graphs.

Table 23

Plant Location Model – Cellular Manufacturing Model

2 Clusters (K = 2)

Recommended Manufacturing Unit	Part #	Annual Dem	Set-up Time Per Batch					
			Machines					
			1	2	3	4	5	6
1	1	220	3.000	3.000				
	2	500	5.000	2.250	0.500			
	3	500	4.000	1.000				
	12	300	2.890	1.000				
	16	500	4.000	0.750				
Unit 1 Average:		404.000	3.778	1.600	0.500			
Unit 1 St Dev:		134.462	0.864	0.978	0.000			
2	4	500		1.500	2.000	0.500		
	5	220		1.000	1.000	1.000		
	6	120				1.000		0.500
	7	700		1.000		1.000		
	8	900			0.500		0.000	0.750
	9	200			0.000		8.000	
	10	250		1.000				0.500
	11	400			1.000		8.000	
	13	250			1.500		0.000	0.000
	14	600			0.750		0.000	1.000
	15	450						2.500
	17	400				0.500		0.000
	18	300				0.750	0.000	0.000
	19	10		0.000			0.000	0.000
	20	250				1.000	0.000	0.000
Unit 2 Average:		370.000		0.900	0.964	0.821	2.000	0.525
Unit 2 St Dev:		232.901		0.548	0.652	0.238	3.703	0.786

Avg Std Dev Across
Recommended Units: 183.681 0.864 0.763 0.326 0.238 3.703 0.786

3 Clusters (K = 3)

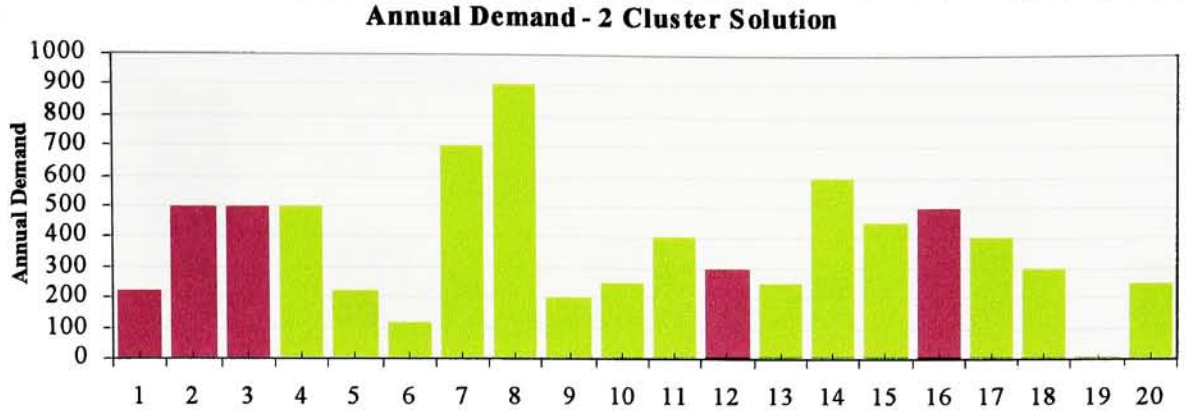
Recommended Manufacturing Unit	Part #	Annual Dem	Set-up Time Per Batch					
			Machines					
			1	2	3	4	5	6
1	1	220	3.000	3.000				
	2	500	5.000	2.250	0.500			
	3	500	4.000	1.000				
	12	300	2.890	1.000				
	16	500	4.000	0.750				
Unit 1 Average:		404.000	3.778	1.600	0.500			
Unit 1 St Dev:		134.462	0.864	0.978	0.000			
2	4	500		1.500	2.000	0.500		
	8	900			0.500		0.000	0.750
	9	200			0.000		8.000	
	10	250		1.000				0.500
	11	400			1.000		8.000	
	13	250			1.500		0.000	0.000
	14	600			0.750		0.000	1.000
	15	450						2.500
Unit 2 Average:		443.750		1.250	0.958	0.500	3.200	0.950
Unit 2 St Dev:		230.585		0.354	0.714	0.000	4.382	0.942
3	5	220		1.000	1.000	1.000		
	6	120				1.000		0.500
	7	700		1.000		1.000		
	17	400				0.500		0.000
	18	300				0.750	0.000	0.000
	19	10		0.000			0.000	0.000
	20	250				1.000	0.000	0.000
Unit 3 Average:		285.714		0.667	1.000	0.875	0.000	0.100
Unit 3 St Dev:		221.349		0.577	0.000	0.209	0.000	0.224

Avg Std Dev Across
Recommended Units: 195.465 0.864 0.636 0.238 0.105 2.191 0.583

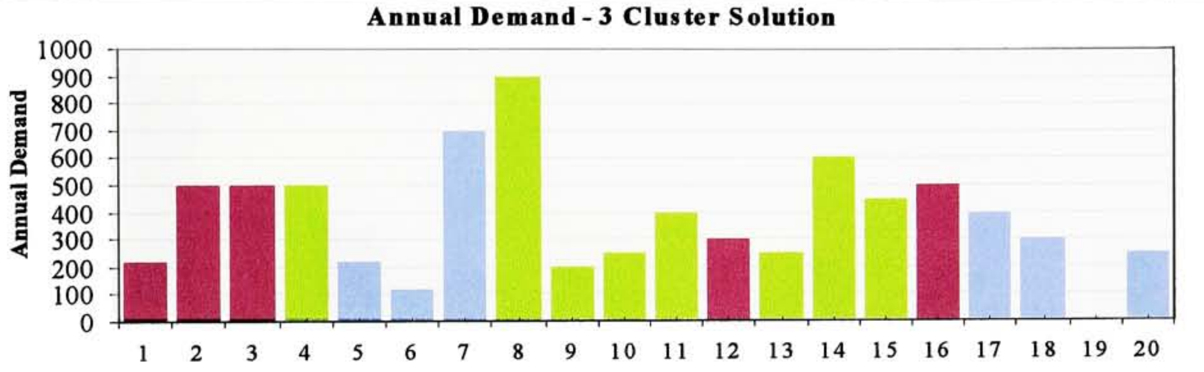
4 Clusters (K = 4)

Recommended Manufacturing Unit	Part #	Annual Dem	Set-up Time Per Batch					
			Machines					
			1	2	3	4	5	6
1	1	220	3.000	3.000				
	2	500	5.000	2.250	0.500			
	3	500	4.000	1.000				
	12	300	2.890	1.000				
	16	500	4.000	0.750				
Unit 1 Average:		404.000	3.778	1.600	0.500			
Unit 1 St Dev:		134.462	0.864	0.978				
2	9	200			0.000		8.000	
	11	400			1.000		8.000	
Unit 2 Average:		300.000			0.500		8.000	
Unit 2 St Dev:		141.421			0.707		0.000	
3	4	500		1.500	2.000	0.500		
	8	900			0.500		0.000	0.750
	10	250		1.000				0.500
	13	250			1.500		0.000	0.000
	14	600			0.750		0.000	1.000
	15	450						2.500
Unit 3 Average:		491.667		1.250	1.188	0.500	0.000	0.950
Unit 3 St Dev:		243.755		0.354	0.688	0.000	0.000	0.942
4	5	220		1.000	1.000	1.000		
	6	120				1.000		0.500
	7	700		1.000		1.000		
	17	400				0.500		0.000
	18	300				0.750	0.000	0.000
	19	10		0.000			0.000	0.000
	20	250				1.000	0.000	0.000
Unit 4 Average:		285.714		0.667	1.000	0.875	0.000	0.100
Unit 4 St Dev:		221.349		0.577	0.000	0.209	0.000	0.224

Avg Std Dev Across
Recommended Units: 185.247 0.864 0.636 0.465 0.105 0.000 0.583



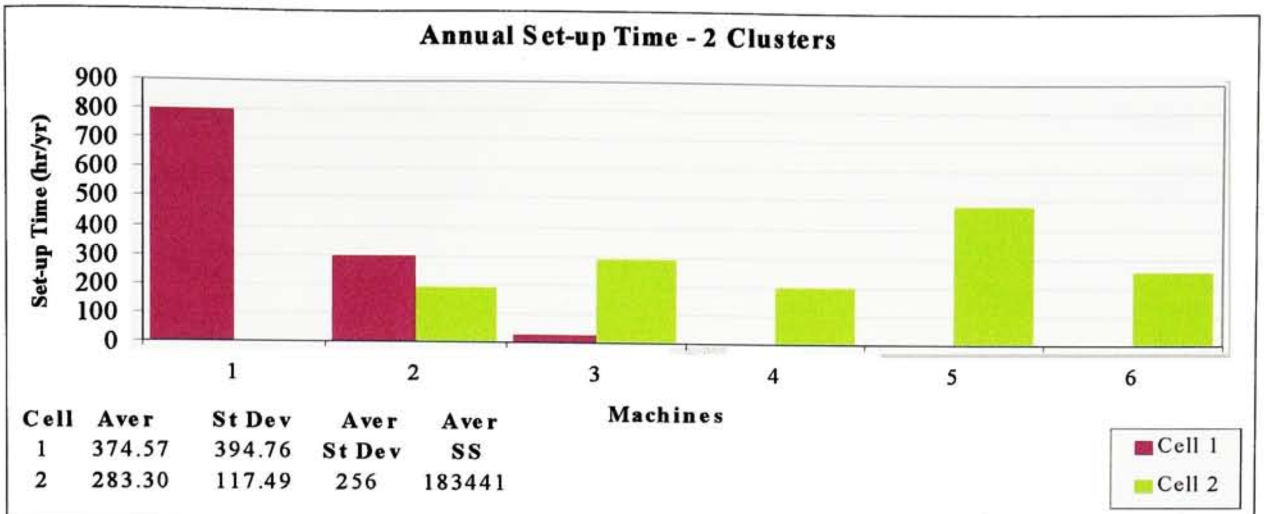
Cell	Aver	St Dev	Aver St Dev	Aver SS
1	404.00	134.46	184	415860
2	370.00	232.90		



Cell	Aver	St Dev	Aver St Dev	Aver SS
1	404.00	134.46	195	246160
2	443.75	230.59		
3	285.71	221.35		

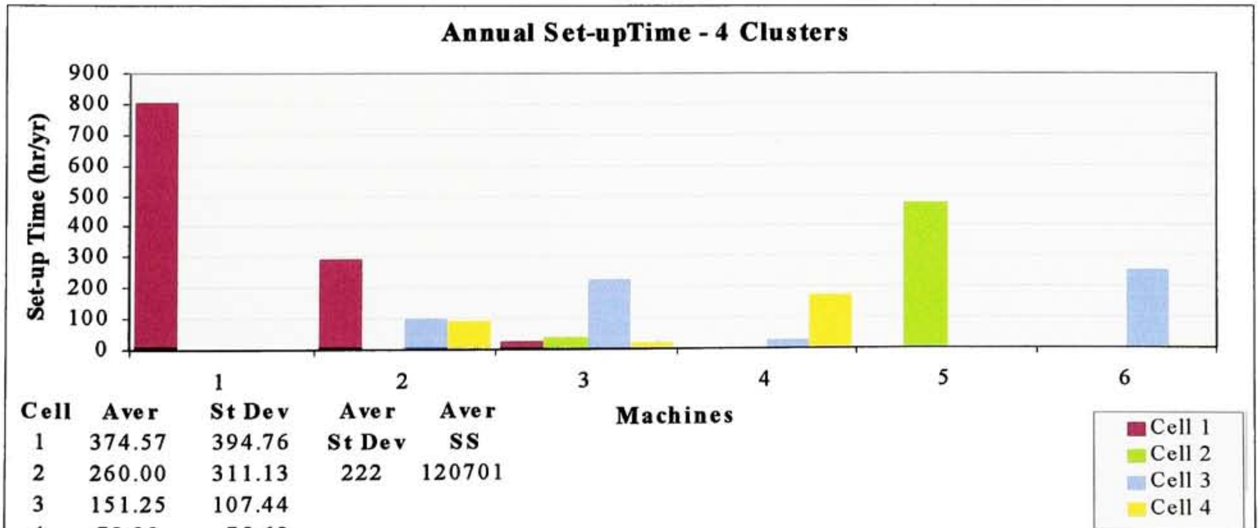
Cell	Aver	St Dev	Aver St Dev	Aver SS
1	404.00	134.46	185	170844
2	300.00	141.42		
3	491.67	243.76		
4	285.72	221.35		

Figure 16. Annual Demand for Plant Location Model



Cell	Aver	St Dev	Aver St Dev	Aver SS
1	374.57	394.76		
2	283.30	117.49	256	183441

Cell	Aver	St Dev	Aver St Dev	Aver SS
1	374.57	394.76		
2	161.00	102.39	191	125001
3	72.88	75.62		



Cell	Aver	St Dev	Aver St Dev	Aver SS
1	374.57	394.76		
2	260.00	311.13	222	120701
3	151.25	107.44		
4	72.88	75.62		

Figure 17. Annual Set-up Time for Plant Location Model

3. Covering Model

The model was run several times in search of the appropriate D (distance from all parts to the nearest representative object) value that would generate two, three and four clusters. The table below (Table 24) shows these results and compiles parts belonging to each cluster along with representative objects and the distances used.

Notice that D ranged from 30 to 45. Whereas 2 clusters were obtained with two values of D (44 and 45), only one three-cluster solution resulted when D was set to 43. More interesting is that for the four-cluster solution, D ranged from 30 to 42. In this case, it is more difficult to determine the appropriate D value and to obtain a more stable set of parts across the range.

Table 24
Measure of Robustness for Covering Model – Cellular Manufacturing Study

Manufacturing Cells	D (Distance)	Representative Objects	Parts
2	45	10	1 2 3 4 5 6 7 8 10 12 15 16 17 18 19 20
		11	9 11 13 14
	44	10	1 2 3 5 6 7 9 10 12 17 20
		14	4 8 11 13 14 15 16 18 19
3	43	10	1 3 5 6 7 9 10 17 20
		12	2 12 13 16 19
		14	4 8 11 14 15 19
4	42	5	4 5 6 7 18 20
		9	9 11
		12	1 2 3 10 12 13 16 17 19
		14	8 14 15
	41	6	5 6 7 10 17 20
		9	9 11
		12	1 2 3 12 13 16 19
		14	4 8 14 15 18
	40	9	9 11 19
		14	4 8 14 15 18
		16	1 2 3 10 16 17
		17	5 6 7 12 13 20
	39	1	1 2 3 10
		11	9 11
		14	4 8 14 15 18
		17	5 6 7 12 13 16 17 19 20
	38	5	4 5 6 7 18 20
		9	9 11
		10	1 3 8 10 14 15 17
		12	2 12 13 16 19

Table 24 / cont.

Manufacturing Cells	D (Distance)	Representative Objects	Parts
4	37	5	4 5 6 7 18 20
		10	1 3 8 13 15 17
		11	9 11
		12	2 10 12 14 16 19
	36	5	4 5 6 7 18 20
		10	1 3 8 13 15 17
		11	9 11
		12	2 10 12 14 16 19
	35	5	4 5 6 7 18 20
		9	9 11
		10	1 3 8 13 15 17
		12	2 10 12 14 16 19
	34	5	4 5 6 7 18 20
		10	3 8 10 13 14 15 17
		11	9 11
		12	1 2 12 16 19
	33	5	4 5 6 7 18 20
		9	9 11
		12	1 2 3 10 12 16 17 19
		14	8 13 14 15
	32	5	4 5 6 7 18 20
		11	9 11
		12	1 2 3 10 12 16 17 19
		14	8 13 14 15
	31	5	4 5 6 7 17 18 20
		11	9 11
		12	1 2 3 10 12 16 19
		14	8 13 14 15
	30	5	4 5 6 7 18 20
		9	9 11
		12	1 2 3 10 12 16 19
		14	8 13 14 15 17

The information from Table 24 was used to construct Table 25, which summarizes the more stable set of parts across the range for all three solutions. Then, Figures 18 and 19 show the Annual Demand and Annual Set-up Time for each cell, as suggested from the Covering Model Results. Appendix J presents in Table J.3 the data used to calculate these graphs.

Table 25

Covering Model Results – Cellular Manufacturing Study

2 Clusters (K = 2)

Recommended Manufacturing Unit	Part #	Annual Dem	Set-up Time Per Batch – Machines 1	2	3	4	5	6
1	1	220	3.000	3.000				
	2	500	5.000	2.250	0.500			
	3	500	4.000	1.000				
	5	220		1.000	1.000	1.000		
	6	120				1.000		0.500
	7	700		1.000		1.000		
	9	200			0.000		8.000	
	10	250		1.000				0.500
	12	300	2.890	1.000				
	17	400				0.500		0.000
	20	250				1.000	0.000	0.000
Unit 1 Average:		332.727	3.723	1.464	0.500	0.900	4.000	0.250
Unit 1 St Dev:		172.748	0.987	0.822	0.500	0.224	5.657	0.289
2	4	500		1.500	2.000	0.500		
	8	900			0.500		0.000	0.750
	11	400			1.000		8.000	
	13	250			1.500		0.000	0.000
	14	600			0.750		0.000	1.000
	15	450						2.500
	16	500	4.000	0.750				
	18	300				0.750	0.000	0.000
	19	10		0.000			0.000	0.000
Unit 2 Average:		434.444	4.000	0.750	1.150	0.625	1.333	0.708
Unit 2 St Dev:		246.582	0.000	0.750	0.602	0.177	3.266	0.980

Avg Std Dev Across
Recommended Units: 209.665 0.494 0.786 0.551 0.200 4.461 0.634

3 Clusters (K = 3)

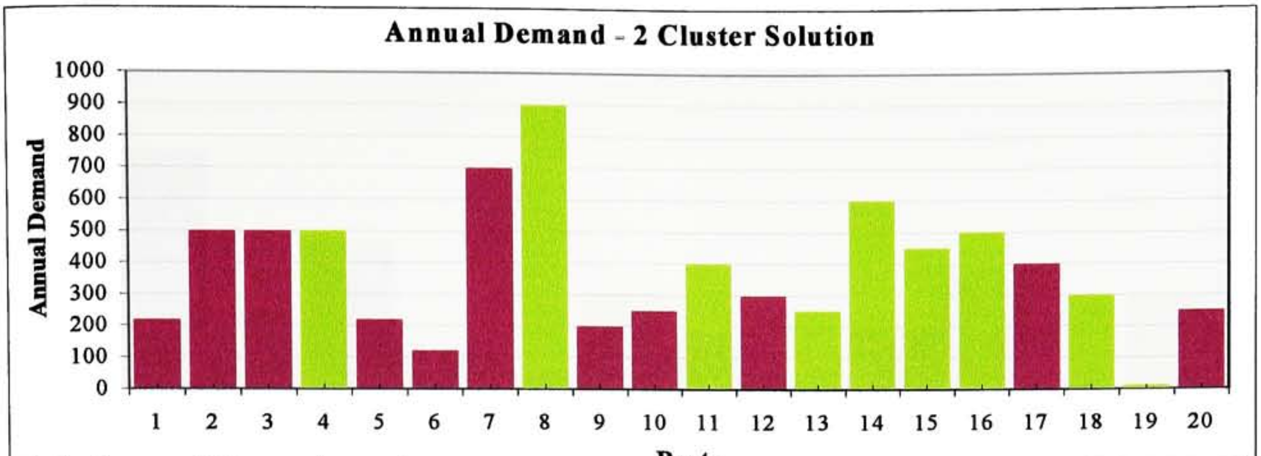
Recommended Manufacturing Unit	Part #	Annual Dem	Set-up Time Per Batch – Machines 1	2	3	4	5	6
1	1	220	3.000	3.000				
	3	500	4.000	1.000				
	5	220		1.000	1.000	1.000		
	6	120				1.000		0.500
	7	700		1.000		1.000		
	9	200			0.000		8.000	
	10	250		1.000				0.500
	17	400				0.500		0.000
	20	250				1.000	0.000	0.000
Unit 1 Average:		317.778	3.500	1.400	0.500	0.900	4.000	0.250
Unit 1 St Dev:		182.810	0.707	0.894	0.707	0.224	5.657	0.289
2	2	500	5.000	2.250	0.500			
	12	300	2.890	1.000				
	13	250			1.500		0.000	0.000
	16	500	4.000	0.750				
	19	10		0.000			0.000	0.000
Unit 2 Average:		312.000	3.963	1.000	1.000		0.000	0.000
Unit 2 St Dev:		203.642	1.055	0.935	0.707		0.000	0.000
3	4	500		1.500	2.000	0.500		
	8	900			0.500		0.000	0.750
	11	400			1.000		8.000	
	14	600			0.750		0.000	1.000
	15	450						2.500
	18	300				0.750	0.000	0.000
Unit 3 Average:		525.000		1.500	1.063	0.625	2.000	1.063
Unit 3 St Dev:		209.165		0.000	0.657	0.177	4.000	1.048

Avg Std Dev Across
Recommended Units: 198.539 0.881 0.610 0.691 0.200 3.219 0.446

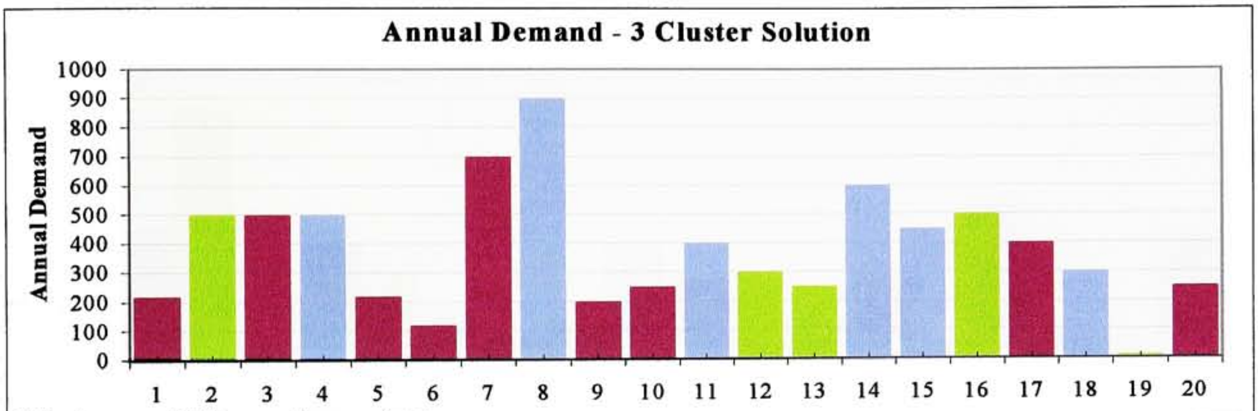
4 Clusters (K = 4)

Recommended Manufacturing Unit	Part #	Annual Dem	Set-up Time Per Batch – Machines 1	2	3	4	5	6
1	4	500		1.500	2.000	0.500		
	5	220		1.000	1.000	1.000		
	6	120				1.000		0.500
	7	700		1.000		1.000		
	18	300				0.750	0.000	0.000
	20	250				1.000	0.000	0.000
Unit 1 Average:		348.333		1.167	1.500	0.875	0.000	0.167
Unit 1 St Dev:		213.206		0.289	0.707	0.209	0.000	0.289
2	**9**	200			0.000		8.000	
	11	400			1.000		8.000	
Unit 2 Average:		300.000			0.500		8.000	
Unit 2 St Dev:		141.421			0.707		0.000	
3	1	220	3.000	3.000				
	2	500	5.000	2.250	0.500			
	3	500	4.000	1.000				
	10	250		1.000				0.500
	12	300	2.890	1.000				
	13	250			1.500		0.000	0.000
	16	500	4.000	0.750				
	17	400				0.500		0.000
	19	10		0.000			0.000	0.000
Unit 3 Average:		325.556	3.778	1.286	1.000	0.500	0.000	0.125
Unit 3 St Dev:		165.689	0.864	1.004	0.707	0.000	0.000	0.250
4	8	900			0.500		0.000	0.750
	14	600			0.750		0.000	1.000
	15	450						2.500
Unit 4 Average:		650.000			0.625		0.000	1.417
Unit 4 St Dev:		229.129			0.177		0.000	0.946

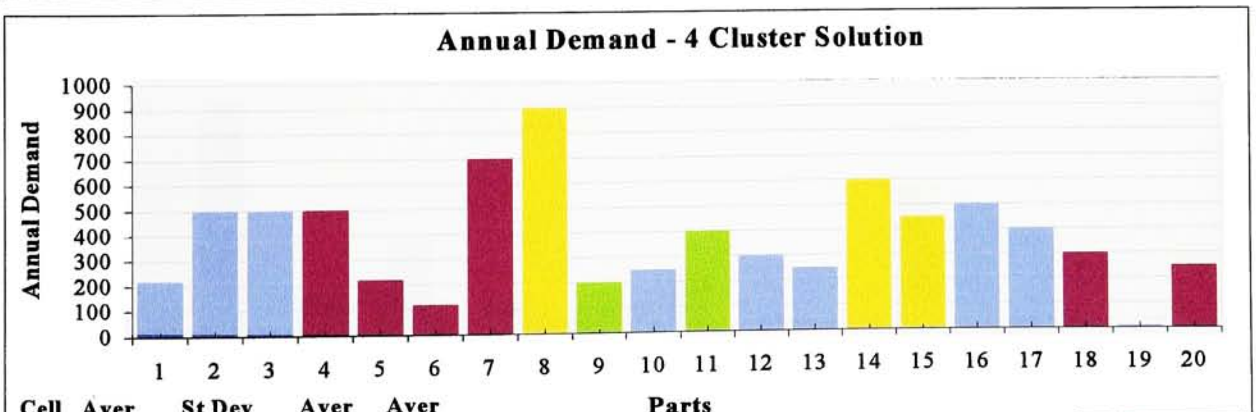
Avg Std Dev Across
Recommended Units: 187.361 0.864 0.647 0.575 0.105 0.000 0.495



Cell	Aver	St Dev	Aver St Dev	Aver SS
1	332.73	172.75	210	392420
2	434.44	246.58		

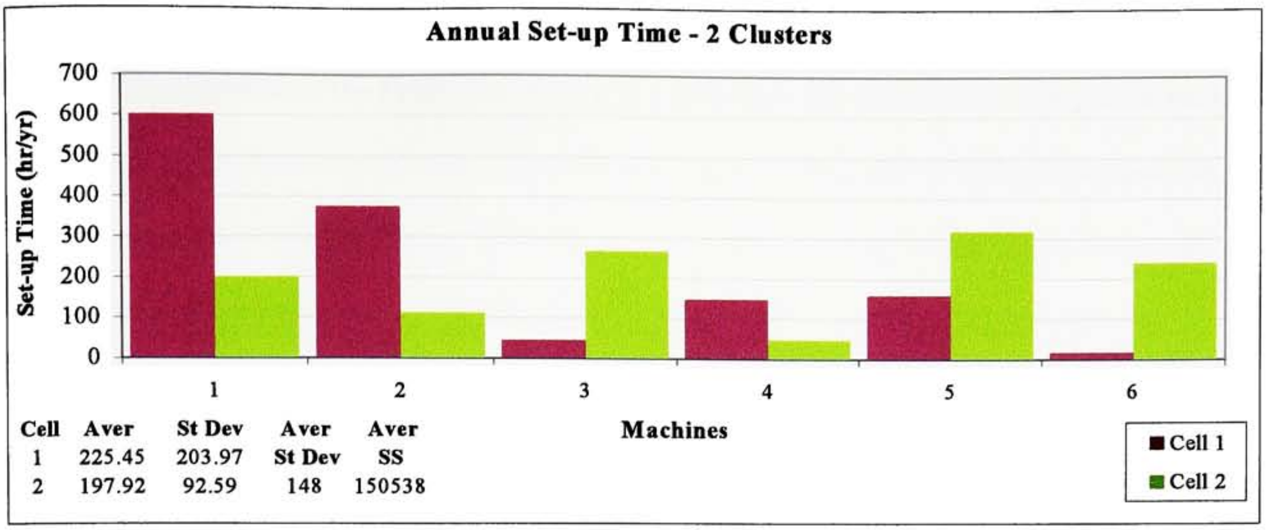


Cell	Aver	St Dev	Aver St Dev	Aver SS
1	317.78	182.81	199	217329
2	312.00	203.64		
3	525.00	209.17		

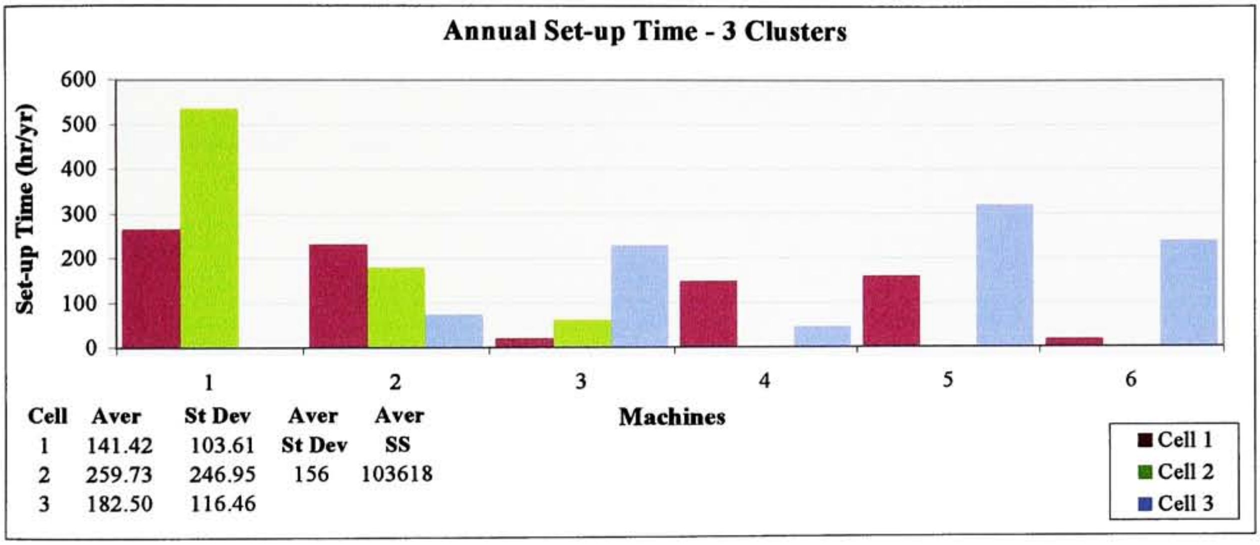


Cell	Aver	St Dev	Aver St Dev	Aver SS
1	348.33	213.21	187	142976
2	300.00	141.42		
3	325.26	165.69		
4	650.00	229.13		

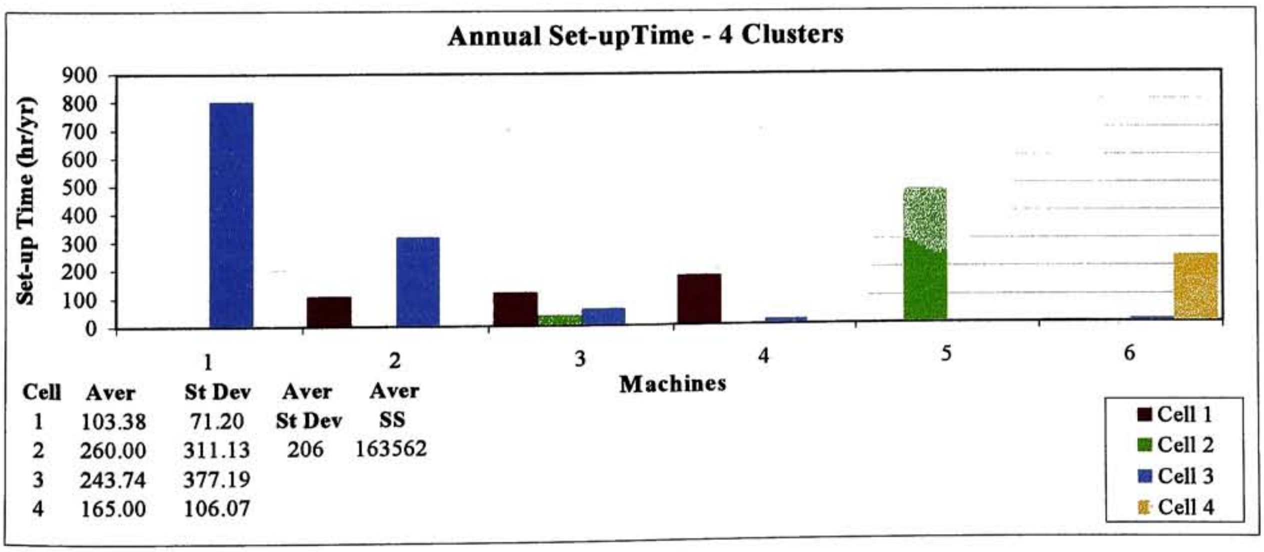
Figure 18. Annual Demand - Covering Model



Cell	Aver	St Dev	Aver St Dev	Aver SS
1	225.45	203.97		
2	197.92	92.59	148	150538



Cell	Aver	St Dev	Aver St Dev	Aver SS
1	141.42	103.61		
2	259.73	246.95	156	103618
3	182.50	116.46		



Cell	Aver	St Dev	Aver St Dev	Aver SS
1	103.38	71.20		
2	260.00	311.13	206	163562
3	243.74	377.19		
4	165.00	106.07		

Figure 19. Annual Set-up Time for Covering Model

4. Fuzzy Set Method

The objective of the Fuzzy Set Method is to determine the percent membership of each part to each manufacturing cell, after specifying the number of clusters or cells to be obtained. The data was entered into program FANNY with the number of clusters set to two, three and four. The program could clearly identify three clusters, but a four-cluster solution was not obtained. These results probably suggest too much fuzziness in the data as the reason for not indicating closest hard clustering assignment of parts to cells for the four-cluster solution.

Table 26 summarizes these results, which are illustrated in Figures 20 and 21 as Annual Demand and Annual Set-up Time for Fuzzy Set Method. The data used to calculate these graphs is shown in Table J.4 in Appendix J.

Table 26

Fuzzy Set Results – Cellular Manufacturing Study

2 Clusters (K = 2)

Recommended Manufacturing Unit	Part #	Annual Dem	Set-up Time Per Batch					
			Machines					
			1	2	3	4	5	6
1	1	220	3.00	3.00				
	2	500	5.00	2.25	0.50			
	3	500	4.00	1.00				
	8	900			0.50		0.00	0.75
	9	200			0.00		8.00	
	10	250		1.00				0.50
	11	400			1.00		8.00	
	12	300	2.89	1.00				
	13	250			1.50		0.00	0.00
	14	600			0.75		0.00	1.00
	15	450						2.50
	16	500	4.00	0.75				
	19	10		0.00			0.00	0.00
Unit 1 Average:		390.77	3.78	1.29	0.71		2.67	0.79
Unit 1 St Dev:		223.77	0.86	1.00	0.51		4.13	0.93
2	4	500		1.50	2.00	0.50		
	5	220		1.00	1.00	1.00		
	6	120				1.00		0.50
	7	700		1.00		1.00		
	17	400				0.50		0.00
	18	300				0.75	0.00	0.00
	20	250				1.00	0.00	0.00
Unit 2 Average:		355.71		1.17	1.50	0.82	0.00	0.13
Unit 2 St Dev:		195.61		0.29	0.71	0.24	0.00	0.25

Avg Std Dev Across Recommended Units:	209.69	0.86	0.65	0.61	0.24	2.07	0.59

3 Clusters (K = 3)

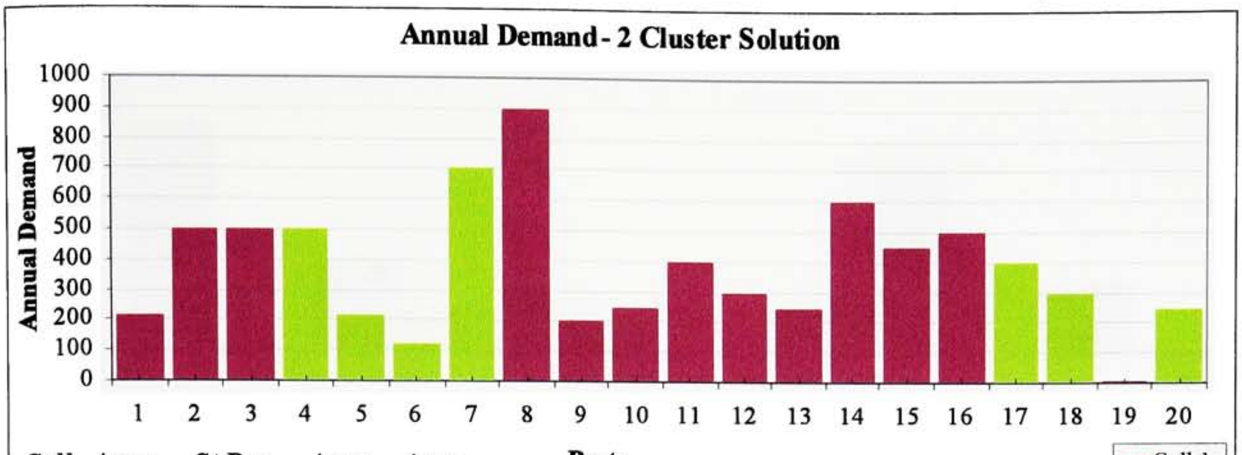
Recommended Manufacturing Unit	Part #	Annual Dem	Set-up Time Per Batch					
			Machines					
			1	2	3	4	5	6
1	1	220	3.00	3.00				
	2	500	5.00	2.25	0.50			
	3	500	4.00	1.00				
	12	300	2.89	1.00				
	16	500	4.00	0.75				
Unit 1 Average:		404.00	3.78	1.60	0.50			
Unit 1 St Dev:		134.46	0.86	0.98	0.00			
2	4	500		1.50	2.00	0.50		
	5	220		1.00	1.00	1.00		
	6	120				1.00		0.50
	7	700		1.00		1.00		
	17	400				0.50		0.00
	18	300				0.75	0.00	0.00
	20	250				1.00	0.00	0.00
Unit 2 Average:		355.71		1.17	1.50	0.82	0.00	0.13
Unit 2 St Dev:		195.61		0.29	0.71	0.24	0.00	0.25
3	8	900			0.50		0.00	0.75
	9	200			0.00		8.00	
	10	250		1.00				0.50
	11	400			1.00		8.00	
	13	250			1.50		0.00	0.00
	14	600			0.75		0.00	1.00
	15	450						2.50
	19	10		0.00			0.00	0.00
Unit 3 Average:		382.50		0.50	0.75		2.67	0.79
Unit 3 St Dev:		274.42		0.71	0.56		4.13	0.93

Avg Std Dev Across Recommended Units:	201.50	0.86	0.66	0.42	0.24	2.07	0.59

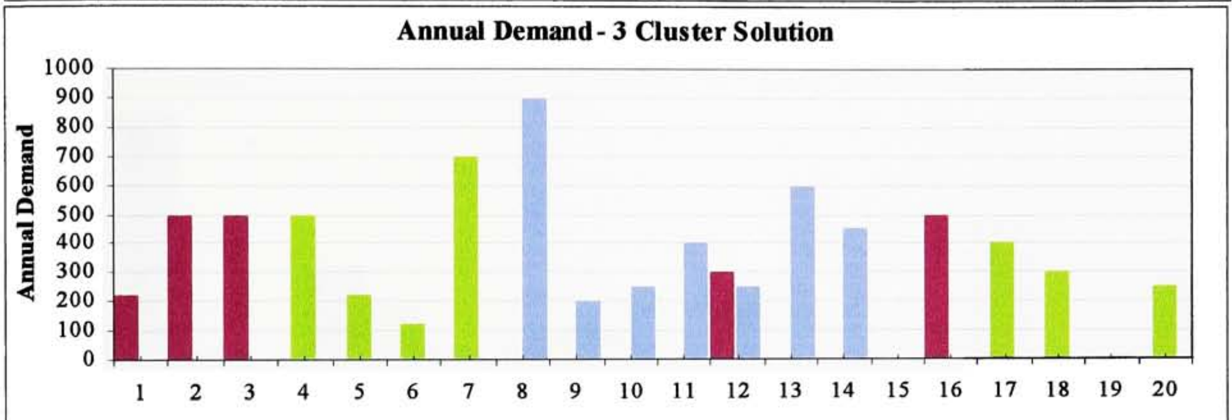
4 Clusters (K = 4)

Recommended Manufacturing Unit	Part #	Annual Dem	Set-up Time Per Batch					
			Machines					
			1	2	3	4	5	6
1	1	220	3.00	3.00				
	2	500	5.00	2.25	0.50			
	3	500	4.00	1.00				
	12	300	2.89	1.00				
	16	500	4.00	0.75				
Unit 1 Average:		404.00	3.78	1.60	0.50			
Unit 1 St Dev:		134.46	0.86	0.98	0.00			
2	4	500		1.50	2.00	0.50		
	8	900			0.50		0.00	0.75
	9	200			0.00		8.00	
	10	250		1.00				0.50
	11	400			1.00		8.00	
	13	250			1.50		0.00	0.00
	14	600			0.75		0.00	1.00
	15	450						2.50
	19	10		0.00			0.00	0.00
Unit 2 Average:		395.56		0.83	0.96	0.50	2.67	0.79
Unit 2 St Dev:		259.67		0.76	0.71	0.00	4.13	0.93
3	5	220		1.00	1.00	1.00		
	6	120				1.00		0.50
	7	700		1.00		1.00		
	17	400				0.50		0.00
	18	300				0.75	0.00	0.00
	20	250				1.00	0.00	0.00
Unit 3 Average:		331.67		1.00	1.00	0.88	0.00	0.13
Unit 3 St Dev:		202.62		0.00	0.00	0.21	0.00	0.25

Avg Std Dev Across Recommended Units:	198.92	0.86	0.58	0.24	0.10	2.07	0.59



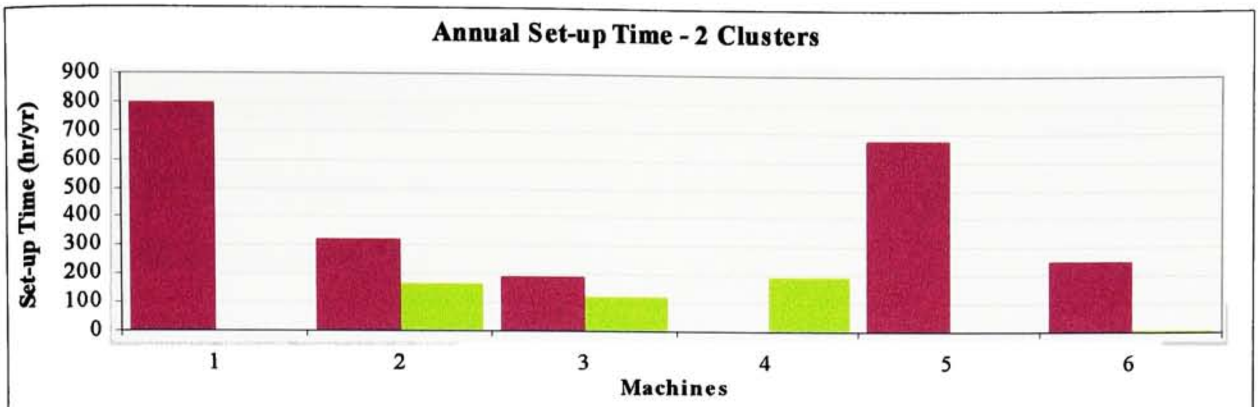
Cell	Aver	St Dev	Aver St Dev	Aver SS
1	390.77	223.77		
2	355.71	195.61	210	415232



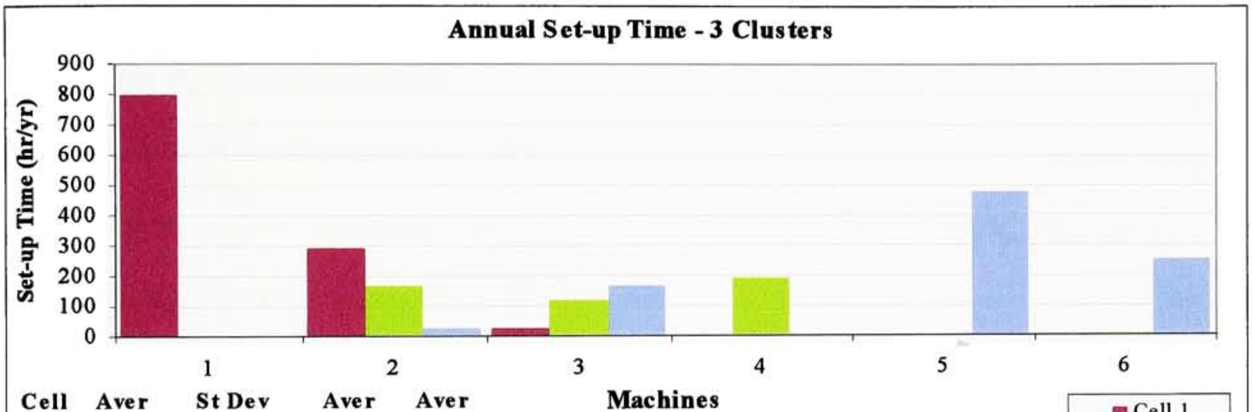
Cell	Aver	St Dev	Aver St Dev	Aver SS
1	404.00	134.46		
2	355.71	195.61	201	276347
3	382.50	274.42		

Cell	Aver	St Dev	Aver St Dev	Aver SS
1	404.00	134.46		
2	395.56	259.67	173	272342
3	331.67	202.62		
4				

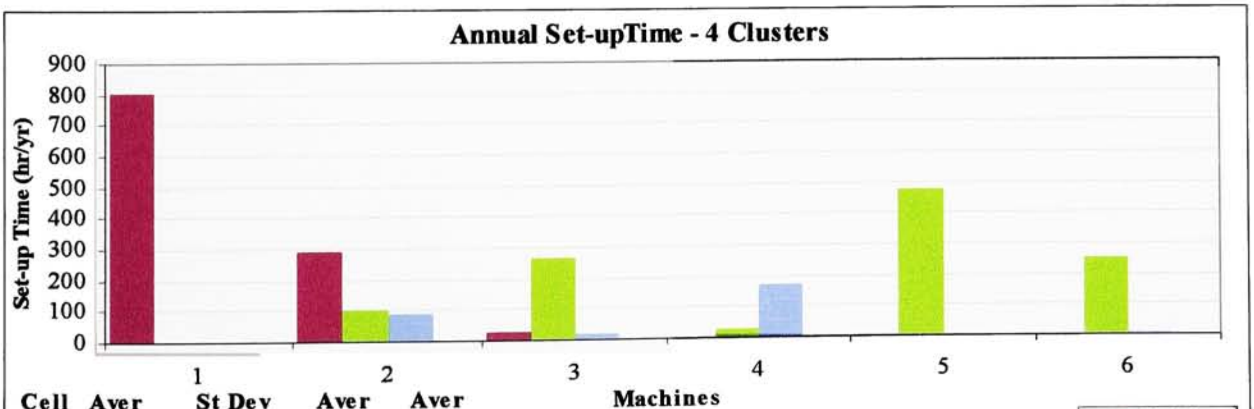
Figure 20. Annual Demand for Fuzzy Set Method



Cell	Aver	St Dev	Aver St Dev	Aver SS
1	449.74	273.52		
2	122.88	83.72	179	166179



Cell	Aver	St Dev	Aver St Dev	Aver SS
1	374.57	394.76		
2	98.30	90.98	225	151235
3	231.25	190.55		



Cell	Aver	St Dev	Aver St Dev	Aver SS
1	374.57	394.76		
2	225.00	175.51	215	152094
3	72.88	75.62		
4				

Figure 21. Annual Set-up Time for Fuzzy Set Method

5. Average-Weighted Distance Model

The purpose of this method is to assign parts into k manufacturing cells, where k is chosen by the decision-maker and part members of the manufacturing cells are chosen to minimize the distance between all members and the medoids of each cluster of parts. This method works by computing the average-weighted distance between all members, which considers the scaling effects of points in space. That is, this new distance, as opposed to the Euclidean Distance used on remaining clustering methods, takes into account the placement of parts in relation to the variables against which they are measured. Figure 22 below illustrates dissimilarity matrix for the 20 parts.

Further, three iterations were performed with Lingo, setting the value of k equal to two, three and four clusters. The results obtained are summarized in Table 27 below, followed by Figures 23 and 24 showing the Annual Demand and Annual Set-up Time for this model. Table J.5 in Appendix J presents the data used to generate these graphs.

Prods	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1																			
2	178.1015																		
3	227.7255	140.3322																	
4	241.0590	368.9996	344.3574																
5	359.9249	406.4116	349.9420	246.4868															
6	398.9866	428.3114	346.3943	341.3010	111.7078														
7	387.2391	402.6295	335.7128	323.3741	172.3297	173.9108													
8	373.6580	354.1736	253.3958	303.2438	360.6859	365.6583	288.7725												
9	397.8721	424.5939	341.7870	408.5182	392.9983	378.9845	409.3269	356.7706											
10	241.5978	301.8992	204.2933	295.2540	284.5523	273.2888	301.4120	227.4324	268.5335										
11	400.7676	420.9764	347.0676	341.9843	384.0194	395.4133	405.1313	329.4683	48.4929	291.5597									
12	206.6273	194.3149	89.3665	314.5455	307.3697	299.6193	317.6453	248.6184	300.4974	120.4120	312.2201								
13	296.1546	352.1541	271.9260	173.9613	282.3046	320.5411	350.7951	259.6007	267.4674	182.6222	280.0748	216.1181							
14	321.3004	328.4955	220.5287	248.7395	302.0329	305.2271	289.2853	120.1044	297.5595	135.3841	284.4219	180.8676	161.4744						
15	402.1921	417.5402	332.9838	401.1820	397.3713	363.8238	390.5702	270.9276	383.1940	245.4509	389.8640	299.1996	322.9644	205.1379					
16	244.4465	155.2935	17.0103	346.8094	350.3552	344.1796	336.1435	250.3598	343.1388	205.0002	344.8572	90.9710	269.0991	217.0333	330.6793				
17	312.6563	335.3938	221.4212	285.0245	200.8184	177.0640	197.8166	209.3256	287.5912	116.5267	298.8072	161.4089	201.2349	145.5541	283.8404	217.9401			
18	342.5159	368.7111	269.2327	298.4509	140.4018	98.5762	161.6634	274.7847	319.4564	182.9907	333.4321	215.8062	245.7694	215.4266	321.2724	266.3772	79.1595		
19	296.3966	338.6392	226.3070	318.2509	289.8214	268.6152	327.4063	271.4237	268.6473	68.4420	294.0533	137.8146	177.7675	170.7152	283.1156	222.9023	122.6328	180.7994	
20	397.9035	422.7293	339.4678	334.2690	107.7753	34.4109	146.6107	349.3995	378.1103	273.2133	391.4316	296.6508	318.6887	299.9485	381.5765	337.2076	167.9842	89.1472	269.9925

Figure 22. Dissimilarity Matrix for Average-Weighted Distance Model – Cellular Manufacturing Study

Table 27

Average-Weighted Distance Model Results – Cellular Manufacturing Study

2 Clusters (K = 2)

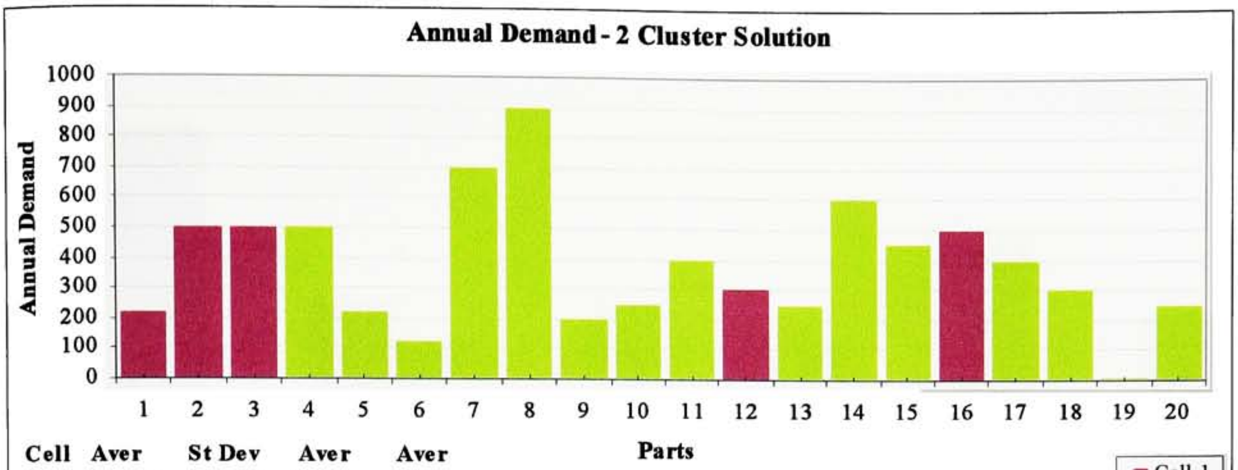
Recommended Manufacturing Unit	Part #	Annual Dem	Set-up Time Per Batch, Machines: 1	2	3	4	5	6
1	1	220	3.000	3.000				
	2	500	5.000	2.250	0.500			
	3	500	4.000	1.000				
	12	300	2.890	1.000				
	16	500	4.000	0.750				
Unit 1 Average:		404.000	3.778	1.600	0.500			
Unit 1 St Dev:		134.462	0.864	0.978	0.000			
2	4	500		1.500	2.000	0.500		
	5	220		1.000	1.000	1.000		
	6	120				1.000		0.500
	7	700		1.000		1.000		
	8	900			0.500		0.000	0.750
	9	200			0.000		8.000	
	10	250		1.000				0.500
	11	400			1.000		8.000	
	13	250			1.500		0.000	0.000
	14	600			0.750		0.000	1.000
	15	450						2.500
	17	400				0.500		0.000
	18	300				0.750	0.000	0.000
	19	10		0.000			0.000	0.000
	20	250				1.000	0.000	0.000
Unit 2 Average:		370.000		0.900	0.964	0.821	2.000	0.525
Unit 2 St Dev:		232.901		0.548	0.652	0.238	3.703	0.786
Avg Std Dev Across Recommended Units:		183.681	0.864	0.763	0.326	0.238	3.703	0.786

3 Clusters (K = 3)

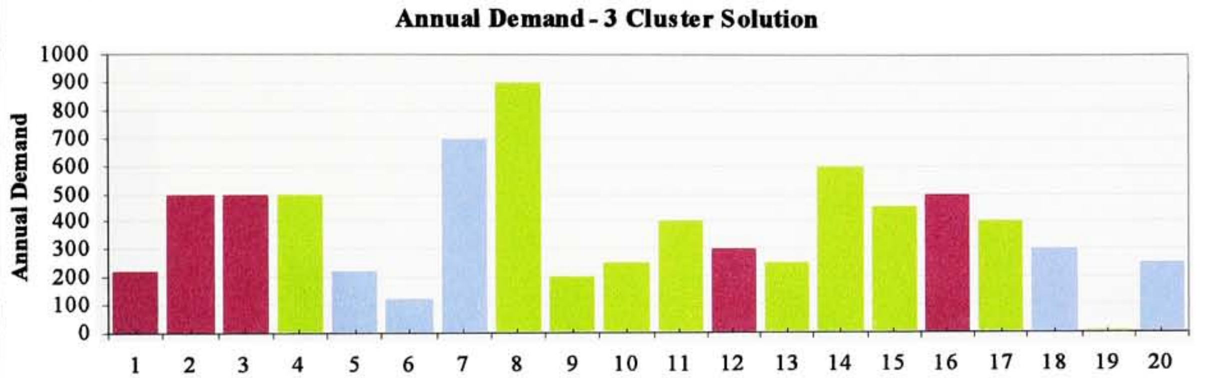
Recommended Manufacturing Unit	Part #	Annual Dem	Set-up Time Per Batch, Machines: 1	2	3	4	5	6
1	1	220	3.000	3.000				
	2	500	5.000	2.250	0.500			
	3	500	4.000	1.000				
	12	300	2.890	1.000				
	16	500	4.000	0.750				
Unit 1 Average:		404.000	3.778	1.600	0.500			
Unit 1 St Dev:		134.462	0.864	0.978	0.000			
2	4	500		1.500	2.000	0.500		
	8	900			0.500		0.000	0.750
	9	200			0.000		8.000	
	10	250		1.000				0.500
	11	400			1.000		8.000	
	13	250			1.500		0.000	0.000
	14	600			0.750		0.000	1.000
	15	450						2.500
	17	400				0.500		0.000
	19	10		0.000			0.000	0.000
Unit 2 Average:		396.000		0.833	0.958	0.500	2.667	0.679
Unit 2 St Dev:		244.822		0.764	0.714	0.000	4.131	0.898
3	5	220		1.000	1.000	1.000		
	6	120				1.000		0.500
	7	700		1.000		1.000		
	18	300				0.750	0.000	0.000
	20	250				1.000	0.000	0.000
Unit 3 Average:		318.000		1.000	1.000	0.950	0.000	0.167
Unit 3 St Dev:		223.428		0.000	0.000	0.112	0.000	0.289
Avg Std Dev Across Recommended Units:		200.904	0.864	0.581	0.238	0.056	2.066	0.593

4 Clusters (K = 4)

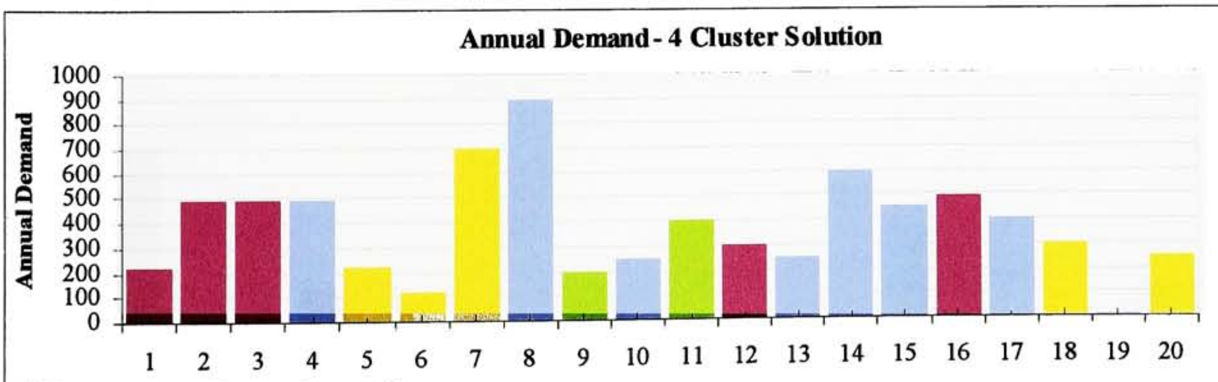
Recommended Manufacturing Unit	Part #	Annual Dem	Set-up Time Per Batch, Machines: 1	2	3	4	5	6
1	1	220	3.000	3.000				
	2	500	5.000	2.250	0.500			
	3	500	4.000	1.000				
	12	300	2.890	1.000				
	16	500	4.000	0.750				
Unit 1 Average:		404.000	3.778	1.600	0.500			
Unit 1 St Dev:		134.462	0.864	0.978				
2	9	200			0.000		8.000	
	11	400			1.000		8.000	
Unit 2 Average:		300.000			0.500		8.000	
Unit 2 St Dev:		141.421			0.707		0.000	
3	4	500		1.500	2.000	0.500		
	8	900			0.500		0.000	0.750
	10	250		1.000				0.500
	13	250			1.500		0.000	0.000
	14	600			0.750		0.000	1.000
	15	450						2.500
	17	400				0.500		0.000
	19	10		0.000			0.000	0.000
Unit 3 Average:		420.000		0.833	1.188	0.500	0.000	0.679
Unit 3 St Dev:		266.297		0.764	0.688	0.000	0.000	0.898
4	5	220		1.000	1.000	1.000		
	6	120				1.000		0.500
	7	700		1.000		1.000		
	18	300				0.750	0.000	0.000
	20	250				1.000	0.000	0.000
Unit 4 Average:		318.000		1.000	1.000	0.950	0.000	0.167
Unit 4 St Dev:		223.428		0.000	0.000	0.112	0.000	0.289
Avg Std Dev Across Recommended Units:		191.402	0.864	0.581	0.465	0.056	0.000	0.593



Cell	Aver	St Dev	Aver	Aver
1	404.00	134.46	St Dev	SS
2	370.00	232.90	184	415860

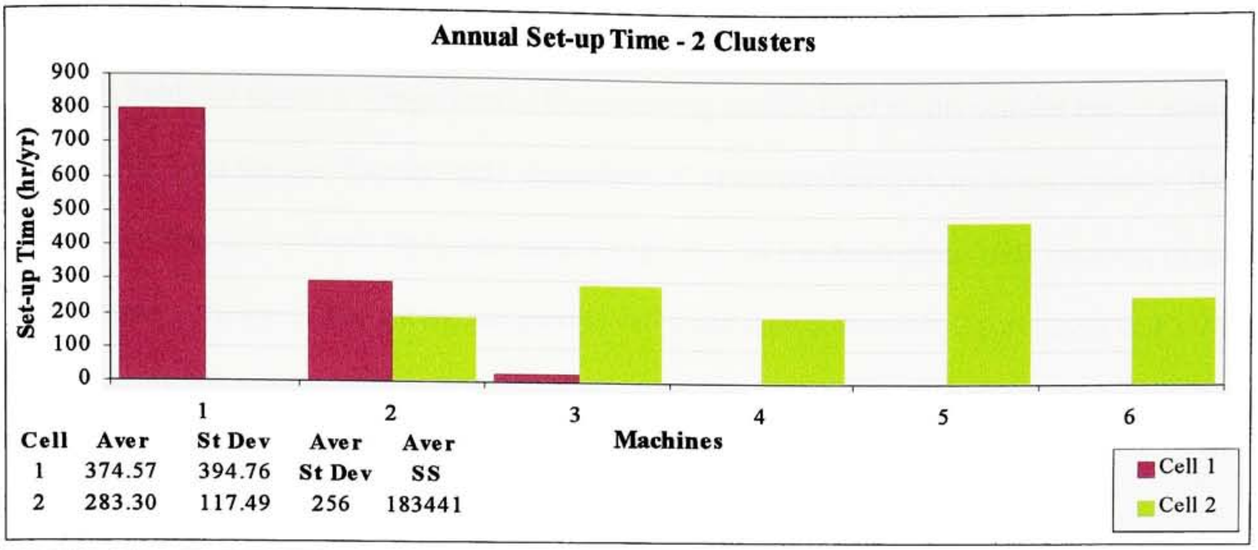


Cell	Aver	St Dev	Aver	Aver
1	404.00	134.46	St Dev	SS
2	396.00	244.82	201	270480
3	318.00	223.43		

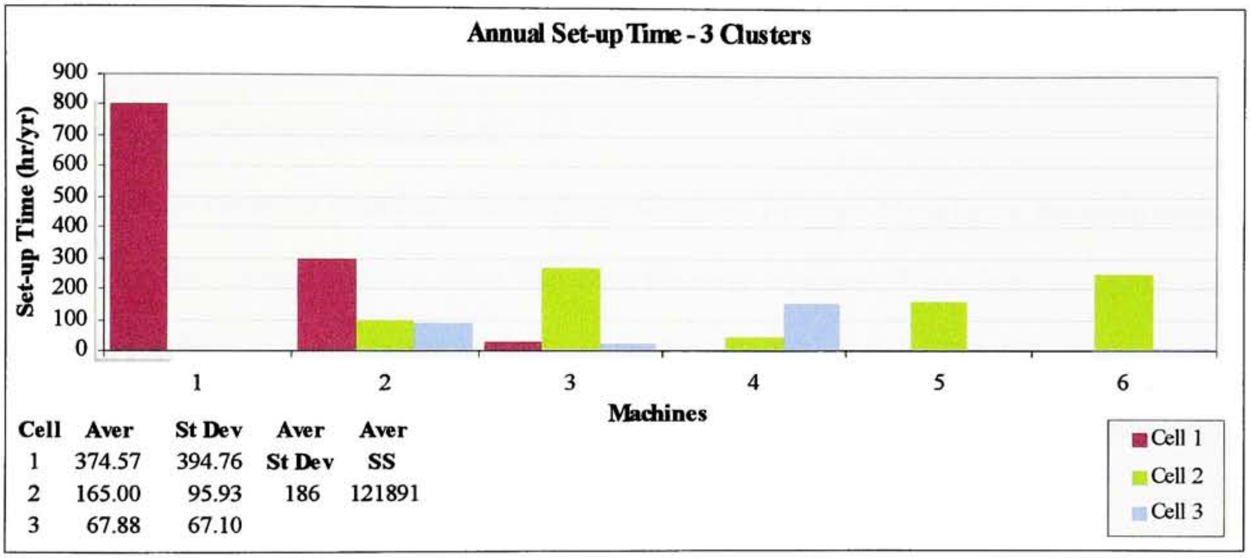


Cell	Aver	St Dev	Aver	Aver
1	404.00	134.46	St Dev	SS
2	300.00	141.42	191	197100
3	420.00	266.30		
4	318.00	223.43		

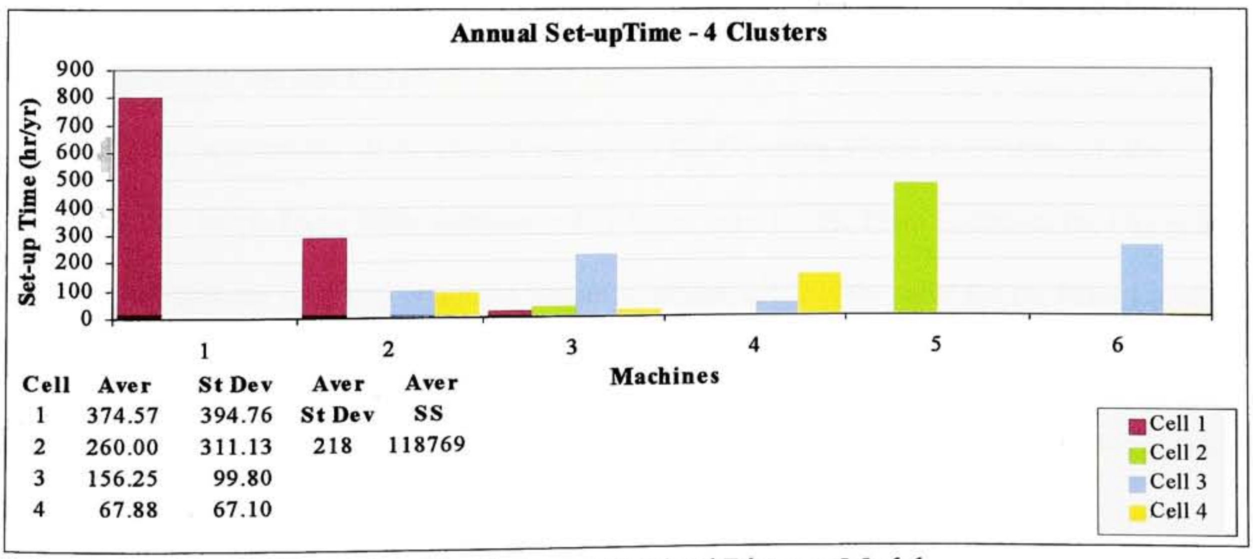
Figure 23. Annual Demand for Average-Weighted Distance Model



Cell	Aver	St Dev	Aver St Dev	Aver SS
1	374.57	394.76	256	183441
2	283.30	117.49		



Cell	Aver	St Dev	Aver St Dev	Aver SS
1	374.57	394.76	186	121891
2	165.00	95.93		
3	67.88	67.10		



Cell	Aver	St Dev	Aver St Dev	Aver SS
1	374.57	394.76	218	118769
2	260.00	311.13		
3	156.25	99.80		
4	67.88	67.10		

Figure 24. Annual Set-up Time for Average-Weighted Distance Model

- **Clustering Models Comparison**

Table 28 shows a comparison of the clustering models used in this cellular manufacturing study. As in the focused factory study, regardless of which products belong to each cluster, the manufacturing unit column represents results suggested by the dendogram. For instance, in the two cluster solution 18 parts were assigned to cell 1 and 2 parts to cell 2; 2 parts from cell 1 were assigned to cell 3 in the three cluster solution; further, this cell 3 is broken down to create cell 4.

- ***Observations***

Some observations are drawn below based on the graphs obtained by the various models in an effort to identify specific trends:

1. The Plant Location Model and the Average-Weighted Distance Model gave the same answer for the two-cluster solution. Also, they have the same representative objects, except for cell 3 in the three-cluster solution and cell 4 in the four-cluster solution. This leads the supposition that representative objects change as cells becomes larger. Cell 1 remains the same across all three solutions.
2. The four-cluster solution in the Plant Location Model has the same clustering partition as suggested by the dendogram.
3. Cell 1 is the same for all the models except for the Covering Model in the three-cluster solution. In the four cluster solution, cell 1 is the same for the Plant Location, the Fuzzy Set Method and the Average-Weighted Distance Model; cell 2 is the same for the Plant Location, Covering and Average-Weighted Distance Model.
4. The Plant Location and the Average-Weighted Distance Model are the only ones to show a stable membership across all units.

5. Since cluster membership is not stable across all solutions in the k-Means Model, as the number of segments increases, set-up time becomes a less important consideration for cell 2. That is, it changes from being a quick set-up cell for the two and three-cluster solution to set-up not being as important in the four-cluster solution. For the latter, cell 1 and 4 require quick set-ups. In spite of these changes, cell 2 remains a medium volume with somewhat uniform demand, which means that volume is a more important criteria for grouping in this model.
6. In the Plant Location and the Average-Weighted Distance Model it is observed that cell 1 has a high degree of difference in set-ups, but somewhat uniform demand. It is a medium volume cell in which there is a high degree of similarity between demand but set-up time is not as important. There is a tendency as the number of cells increases to group parts with low set-up times (cell 3 in the three cluster solution and cell 4 in the four cluster solution are quick set-up cells).
7. In the Covering Model, cluster membership changes rapidly and affects composition in cell 1, which moves from the set-up not being an important consideration to becoming a quick set-up cell in the four-cluster solution.
8. The Fuzzy Set Method becomes too fuzzy in the four-cluster solution, thus only 3 groups can be clearly identified. In this model, cell 1 shows a somewhat uniform demand across all three solutions, but set-up is not as important. Quick set-up cells are cell 2 in the two and three cluster solution and cell 3 in the four-cluster solution.

Table 28

Clustering Model Comparison – Cellular Manufacturing Study

Mfg. Unit	k-Means	Plant L.	Covering	Fuzzy	Avg W.
1	1	1	1	1	1
1	2	2	2	2	2
1	3	**3**	3	3	**3**
1	4	12	5	8	12
1	8	16	6	9	16
1	10	4	7	10	4
1	11	5	9	11	5
1	12	6	**10**	12	6
1	13	7	12	13	7
1	14	8	17	14	8
1	16	9	20	15	9
1	5	10	4	16	10
1	6	11	8	19	11
1	7	13	11	4	13
1	9	14	13	5	14
1	15	15	**14**	6	15
1	17	**17**	15	7	**17**
1	18	18	16	17	18
2	19	19	18	18	19
2	20	20	19	20	20

Mfg. Unit	k-Means	Plant L.	Covering	Fuzzy	Avg W.
1	1	1	1	1	1
1	2	2	3	2	2
1	3	**3**	5	3	**3**
1	12	12	6	12	12
1	16	16	7	16	16
2	4	4	9	4	4
2	5	8	**10**	5	8
3	6	9	17	6	9
3	7	10	20	7	10
3	8	11	2	17	11
3	9	13	**12**	18	13
3	10	**14**	13	20	**14**
3	11	15	16	8	15
3	13	5	19	9	17
3	14	6	4	10	19
3	15	7	8	11	5
3	17	17	11	13	6
3	18	**18**	**14**	14	7
3	19	19	15	15	18
3	20	20	18	19	**20**

Mfg. Unit	k-Means	Plant L.	Covering	Fuzzy	Avg W.
1	1	1	4	1	1
1	2	2	**5**	2	2
1	3	**3**	6	3	**3**
1	12	12	7	12	12
1	16	16	18	16	16
2	8	**9**	20	4	**9**
2	9	11	**9**	8	11
3	10	4	11	9	4
3	11	8	1	10	8
3	13	10	2	11	10
3	14	13	3	13	13
3	15	**14**	10	14	**14**
3	19	15	**12**	15	15
4	4	5	13	19	17
4	5	6	16	5	19
4	6	7	17	6	5
4	7	17	19	7	6
4	17	**18**	8	17	7
4	18	19	**14**	18	18
4	20	20	15	20	**20**

- **Statistical Comparison of results**

To measure the effectiveness of these methods, the average standard deviation and sum of squares are again used as statistical measures. The purpose of this analysis is to illustrate the significant differences between the different methods with respect to the annual demand and set-up times for the three-cluster solution.

Results from this examination are summarized in Table 29 below and indicate that the k-Means Model provides the lowest standard deviation in the four-cluster solution. Similarly, the Covering Model has the lowest sum of squares in the three-cluster solution. One interesting observation is that, overall, k-Means and Covering have the lowest standard deviation and sum of squares across the two dimensions (demand and set-up time). The Covering Model has the lowest standard deviation and sum of squares for the set-up time dimension and k-Means provides the lowest standard deviation for the annual demand.

Table 29

Average Standard Deviation and Sum of Squares across All Clustering Methods

Statistical Measure	Cell	Annual Demand (Units)	Set-up Time (hr/yr)
Standard Deviation	2	1. Plant Location and Average-Weighted Distance 2. k-Means 3. Covering and Fuzzy Set	1. Covering 2. k-Means 3. Fuzzy Set 4. Plant Location and Average-Weighted Distance
	3	1. k-Means 2. Plant Location 3. Covering 4. Fuzzy Set and Average-Weighted Distance	1. Covering 2. Average-Weighted Distance 3. Plant Location 4. k-Means 5. Fuzzy Set
	4	1. k-Means 2. Fuzzy Set 3. Plant Location 4. Covering 5. Average-Weighted Distance	1. k-Means 2. Covering 3. Fuzzy Set 4. Average-Weighted Distance 5. Plant Location
Sum of Squares	2	1. k-Means 2. Covering 3. Fuzzy Set 4. Average-Weighted Distance 5. Plant Location	1. Covering 2. Fuzzy Set 3. Plant Location and Average-Weighted Distance 4. k-Means
	3	1. Covering 2. Plant Location 3. Average-Weighted Distance 4. Fuzzy Set 5. k-Means	1. Covering 2. Average-Weighted Distance 3. Plant Location 4. k-Means 5. Fuzzy Set
	4	1. Covering 2. Plant Location 3. k-Means 4. Average-Weighted Distance 5. Fuzzy Set	1. k-Means 2. Average-Weighted Distance 3. Plant Location 4. Fuzzy Set 5. Covering

Chapter 5

Conclusions

This research has explored the use of multivariate clustering analysis to support the formation of focused factories and manufacturing cells. The objective of both systems is to concentrate manufacturing resources on a relatively narrow set of tasks.

In the case of focused factories, "narrowness" is traditionally defined by one easily identified similarity in manufacturing process, product line or manufacturing segment. In practice, multiple considerations may be more appropriate. Berry et al., for example, proposed the use of multiple market-based criteria, called order winning criteria, as a means of concentrating manufacturing operations on delivering the performance demanded by the market (such as low cost, high quality, or short delivery time). This approach enables one to consider how existing operations strengths can provide competitive advantage as they support different marketing initiatives. They employed Agglomerative Hierarchy with k-Means Refinement as a method of clustering products into focused factories. While many other multivariate clustering methods exist in the literature, the application of these methods to focused factory development is missing.

In the case of cellular manufacturing, existing methods rely solely on process similarity to define which products should be grouped together and manufactured in the same cell. Additional attributes, such as volume, setup time, quality and delivery lead times, have been ignored in the literature.

Motivated by the need to consider multiple criteria in the formation of focused factories and manufacturing cells, and the lack of analytical methods to support the decision maker, this

research has explored the performance of five clustering methods, each with differing clustering strategies:

1. The Agglomerative Hierarchy with k Means Refinement, which seeks to minimize the variance among the salient manufacturing attributes of products or parts, given the number of desired clusters.
2. The Plant Location Model, which seeks to minimize the difference between the salient manufacturing attributes among products or parts in a cluster relative to the centroid of the cluster, given the number of desired clusters.
3. The Covering Model, which seeks to determine cluster membership based on a maximum acceptable difference among salient manufacturing attributes and the cluster centroid. The number of clusters is an outcome in this case.
4. The Average-Weighted Distance Model, created by this research, seeks to minimize the *scale-adjusted* difference between the salient manufacturing attributes among products or parts in a cluster relative to the centroid of the cluster, given the number of desired clusters.
5. The Fuzzy Set Method, which seeks to determine the degree of similarity a product or part has to other products or parts given the number of desired clusters. Assignment of products or parts to clusters is made by choosing the cluster with the largest percent membership.

In practice, the choice of clustering methodology should be driven primarily by the choice of clustering strategy. That is, if minimal cluster variance is desired, the Agglomerative Hierarchy with k-Means Refinement would be a candidate method. However, if one wishes to

limit the maximum degree of dissimilarity among products in a cluster, the Covering Model would be a candidate. The choice of methodology is obviously case dependent. A secondary criterion for choosing a clustering method is the resultant performance as indicated by measures such as average cluster standard deviation or average sum-of-the-squares.

One of the benefits of using different algorithms is that the stability of the cluster memberships can be determined across the different units. Moreover, findings in this research reaffirm that cluster analysis can help management create new operating units, which are better focused than the old ones.

The use of these five methods has been illustrated using two cases drawn from the literature. Applying the methods to the focused factory case illustrates how different clustering strategies may produce different results. For example, in the case of two focused factories, all methods produced the same results, whereas in the four-focused factory case, only the Plant Location Model and the Fuzzy Set Method produced the same cluster solution. The differences are driven by the clustering strategy embedded in each method and the data preparation method, as discussed in a previous section. Using the secondary performance criteria of average cluster standard deviation, the plant location model and fuzzy set models also produced the best results in the four-factory case.

Applying the five methods to the cellular manufacturing case illustrates how attributes, other than process similarity, can be used to define manufacturing cells. In this case different cluster solutions were obtained throughout all the two, three and four cell options.

Since this research has only initiated the extension of multivariate clustering analysis to the definition of focused factories and manufacturing cells, there is significant opportunity for additional work.

First, the sensitivity of results to the choice of data preparation methods is not understood. For example, is the use of the Average-Weighted Distance metric proposed in this research actually preferable to the traditional Euclidean Distance metric? In order to explore this question, one might construct and test the following hypothesis: Ho: given a data set (X1, X2, X3,..., Xn) and a specified number of clusters C1, C2,..., Ck with centroids $|Y1| < |Y2| < \ldots < |Yn|$, then Var(Ci) given by the Average-Weighted Distance metric < Var(Ci) given by the Euclidean Distance metric; and HA: at least one Var(Ci) given by the Average-Weighted Distance metric is greater than or equal to the Var(Ci) given by the Euclidean Distance metric.

Second, since all cluster methods used in this research collapse a multivariate problem into a scalar problem, the influence of the scale of the constituent variables is not well understood. A carefully constructed sensitivity analysis, which varies the magnitude of the constituent variables, would provide insight into this effect.

Third, given that methods tend to provide different results, how can the overall performance be compared and contrasted taking into account the aggregate distance metric as well as the constituent variables? The average standard deviation and sum-of-the-squares across metrics utilized in this research are examples of comparison methods, but have proved inconclusive in the cases studied. One way to explore this issue further is to construct a multiple range test of the hypotheses Ho: Var(Ci) < Var(Cj) < ...< Var(Ck) and HA: that no such ordering exists.

Fourth, some sources of variability in results come from the choices the analyst has to make during the process. Because of these choices, the best method for all circumstances is not clear, providing opportunities for future research by examining the impact of choosing the different options.

Fifth, generation of candidate manufacturing cell or focused factory product options using clustering analysis may only be part of a larger decision-making process which involves other business considerations, such as facilities cost, risk, and labor availability. As such, additional decision analysis methods would be invoked to choose among an array of cluster options. One particularly suitable technique is the Analytical Hierarchy Process (AHP), which allows a decision-maker to decompose a given problem into elements in subsequent levels. An example using AHP is illustrated in Figure 25, in which the high level objective is to determine the best number of focused factories to construct. This decision depends on lower level considerations such as time, cost, risk, employee impact and return on investment. A three, four or five cluster solution, then, merely generates options to be further evaluated using the AHP by assigning weights and making pairwise comparisons at each level.

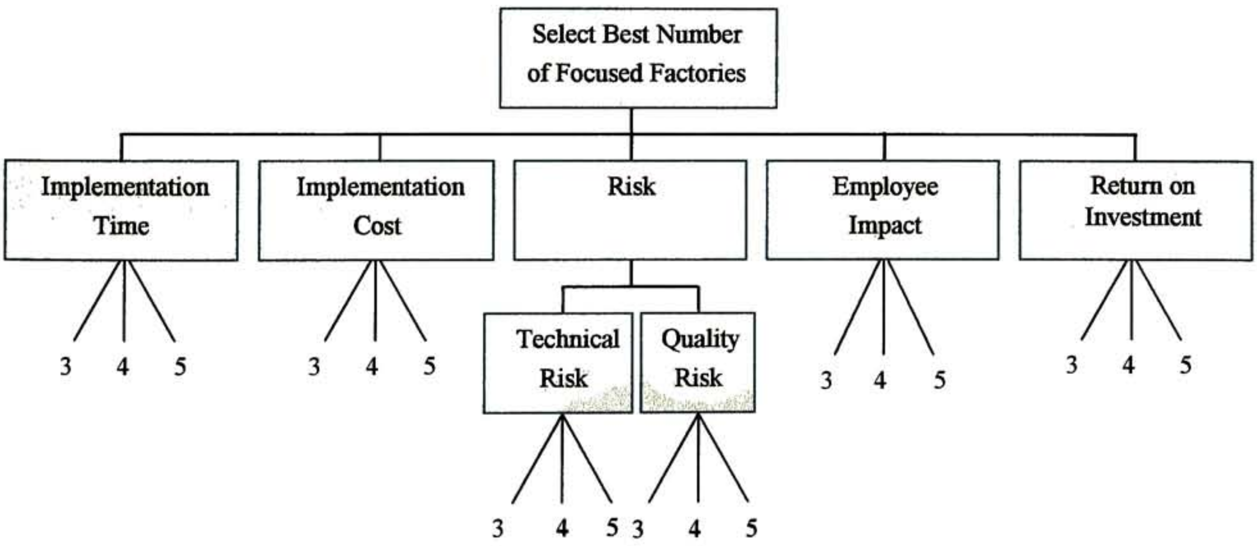


Figure 25. Analytic Hierarchy Process

Finally, there is an opportunity to create an integrated decision support tool to guide decision-makers through the choice of to attributes consider: data preparation method, choice of clustering method, comparison of results and selection of the best number of factories or cells given business criteria. For example, such a tool could be created as a Visual Basic Application (VBA) which provides a convenient interface for data entry, and display of results. This VBA could link to additional applications needed to perform the clustering results. For example, a link to LINGO would provide results for the Plant Location, Covering and Average-Weighted Distance methods and link to FANNY would provide the Fuzzy Set Method results.

While focused factories and manufacturing cells are commonplace in industry, there is a general absence of methodologies to assist decision-makers in determining the best number of units and in the identification of products best suited for each unit. This research has explored these issues, beyond the state of the literature, using several traditional clustering methods and one of our own creation. The analysis of data from two case studies has provided useful insight into the challenge of creating a suitable decision-making methodology. Numerous subsequent studies have been proposed to provide a better understanding of the intricacies of the analysis as a precursor to the development of a decision support tool.

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Appendix A

Hierarchical Cluster Analysis of Observations

- **Minitab Output**

Following is the output generated by Minitab with the aim to find group products:

Euclidean Distance, Average Linkage

Amalgamation Steps

Step	Number of clusters	Similarity level	Distance level	Clusters joined		New cluster	Number of obs. in new cluster
1	27	100.00	0.000	27	28	27	2
2	26	100.00	0.000	26	27	26	3
3	25	100.00	0.000	25	26	25	4
4	24	100.00	0.000	24	25	24	5
5	23	100.00	0.000	23	24	23	6
6	22	100.00	0.000	22	23	22	7
7	21	100.00	0.000	1	20	1	2
8	20	100.00	0.000	9	12	9	2
9	19	100.00	0.000	10	11	10	2
10	18	100.00	0.000	5	9	5	3
11	17	97.78	5.000	8	13	8	2
12	16	95.56	10.000	18	19	18	2
13	15	94.64	12.071	17	18	17	3
14	14	91.11	20.000	3	10	3	3
15	13	90.00	22.500	8	22	8	9
16	12	87.81	27.423	15	17	15	4
17	11	87.43	28.284	14	16	14	2
18	10	87.43	28.284	1	6	1	3
19	9	86.67	30.000	7	21	7	2
20	8	85.57	32.477	2	15	2	5
21	7	84.40	35.096	4	14	4	3
22	6	83.24	37.712	1	5	1	6
23	5	80.05	44.885	1	4	1	9
24	4	76.17	53.624	1	7	1	11
25	3	74.54	57.289	2	8	2	14
26	2	71.77	63.523	2	3	2	17
27	1	6.04	211.411	1	2	1	28

Final Partition

Number of clusters: 1

	Number of observations	Within cluster sum of squares	Average distance from centroid	Maximum distance from centroid
Cluster1	28	315195.536	104.122	136.196

When entering the data into Minitab, go to Stat>Multivariate>Cluster Observations and then enter the variables to be clustered (volume, price, delivery speed and quality). The distance metric to be used is the Euclidean distance and the linkage method is the average linkage (the distance between two clusters is the mean distance between an observation in one cluster and an observation in the other cluster). It is very important not to check the box for standardizing variables, because they were already weighted according to management criteria. Although this is a good idea when variables are in different units and the analyst wishes to minimize the effect of scale differences, the structure of the cluster can differ slightly if variables are standardized that don't need to be. Check the box to generate a dendogram.

As described in the help menu, these steps are displayed in the session window and at each step two clusters are joined. The following information is found in the table:

- ✓ The joined clusters
- ✓ The distance between them
- ✓ The corresponding similarity level (the percent of the minimum distance at that step relative to the maximum inter-observation distance in the data)
- ✓ The identification number of the new cluster (this number is always the smaller of the two numbers of the clusters joined)
- ✓ The number of observations in the new cluster
- ✓ The number of clusters

Amalgamation continues until there is just one cluster and finally, the information printed in the amalgamation table is displayed by the dendogram in the form of a tree diagram.

Appendix B

k-Means Cluster Analysis

k-Means clustering of observations using Minitab is generally used to classify observations into groups when these groups are initially unknown, in which case an initial partition column should be specified. However, since hierarchical clustering was already performed in a previous step, it would suffice to indicate partition by the desired number of clusters. The program is very easy to use and just a few steps need to be followed:

- ✓ Entering the data in the worksheet, go to Stat tab and choose Multivariate > Cluster K-Means
- ✓ Enter the variables by selecting from volume to quality
- ✓ Specify partition by the number of clusters desired (2, 3 or 4)
- ✓ Check Standardize variables
- ✓ Click Storage and in the cluster membership column, the name of a column created to store the final clusters

The following table is the result of the three-cluster solution iteration:

Number of clusters: 3

	Number of observations	Within cluster sum of squares	Average distance from centroid	Maximum distance from centroid
Cluster1	11	10481.818	29.176	52.369
Cluster2	5	1400.000	15.678	24.083
Cluster3	12	9847.917	25.360	51.702

Cluster Centroids

Variable	Cluster1	Cluster2	Cluster3	Grand centrd
Volume	200.0000	0.0000	0.0000	78.5714
Price	70.9091	42.0000	10.8333	40.0000
Speed	9.5455	0.0000	16.2500	10.7143
Quality	7.7273	36.0000	0.0000	9.4643

Distances Between Cluster Centroids

	Cluster1	Cluster2	Cluster3
Cluster1	0.0000	204.2699	209.0784
Cluster2	204.2699	0.0000	50.3133
Cluster3	209.0784	50.3133	0.0000

Minitab will display, in the first table, the number of observations in each cluster, the within cluster sum of squares, the average distance from observation to the cluster centroid and the maximum distance from observation to the cluster centroid. The centroid is the vector of variable means for the observations in that cluster and is used as a cluster midpoint. The centroids for the individual cluster are printed in the second table while the third table gives distances between cluster centroids.

- **Data Set and Solutions**

The table below shows the data set as input into Minitab's window session, as well as the product groupings when 2, 3 and 4 clusters are specified.

Product	Volume	Price	Speed	Quality	2-Cluster	3-Cluster	4-Cluster
1	200	80	20	0	1	1	1
2	0	40	0	60	2	2	2
3	0	30	50	0	2	3	3
4	200	50	25	25	1	1	4
5	200	100	0	0	1	1	1
6	200	60	40	0	1	1	1
7	200	20	0	0	1	1	4
8	0	0	25	0	2	3	3
9	200	100	0	0	1	1	1
10	0	50	50	0	2	3	3
11	0	50	50	0	2	3	3
12	200	100	0	0	1	1	1
13	0	0	20	0	2	3	3
14	200	80	0	20	1	1	1
15	0	60	0	40	2	2	2
16	200	60	0	40	1	1	4
17	0	30	0	30	2	2	2
18	0	40	0	30	2	2	2
19	0	40	0	20	2	2	2
20	200	80	20	0	1	1	1
21	200	50	0	0	1	1	4
22	0	0	0	0	2	3	3
23	0	0	0	0	2	3	3
24	0	0	0	0	2	3	3
25	0	0	0	0	2	3	3
26	0	0	0	0	2	3	3
27	0	0	0	0	2	3	3
28	0	0	0	0	2	3	3

Appendix C

Plant Location Model

- **Mathematical Model**

Notice that constraints (10) and (12) shown in the following mathematical model are presented in a different format than they were originally entered into Lingo (for illustration only, they are divided in 6 columns instead of 1). Using this model for the circuit board example, the resulting mathematical formulation is shown below:

MIN = 106.771*Zp1p2 + 104.163*Zp1p3 + 19.685*Zp1p4 + 14.142*Zp1p5 + 14.142*Zp1p6 + 31.623*Zp1p7 + 107.732*Zp1p8 + 14.142*Zp1p9 + 102.225*Zp1p10 + 102.225*Zp1p11 + 14.142*Zp1p12 + 107.703*Zp1p13 + 14.142*Zp1p14 + 102.956*Zp1p15 + 24.495*Zp1p16 + 104.642*Zp1p17 + 103.562*Zp1p18 + 102.956*Zp1p19 + 18.028*Zp1p21 + 108.167*Zp1p22 + 108.167*Zp1p23 + 108.167*Zp1p24 + 108.167*Zp1p25 + 108.167*Zp1p26 + 108.167*Zp1p27 + 108.167*Zp1p28 + 106.771*Zp2p1 + 39.370*Zp2p3 + 102.408*Zp2p4 + 108.628*Zp2p5 + 106.771*Zp2p6 + 104.881*Zp2p7 + 38.161*Zp2p8 + 108.628*Zp2p9 + 39.370*Zp2p10 + 39.370*Zp2p11 + 108.628*Zp2p12 + 37.417*Zp2p13 + 103.923*Zp2p14 + 14.142*Zp2p15 + 100.995*Zp2p16 + 15.811*Zp2p17 + 15.000*Zp2p18 + 20.000*Zp2p19 + 106.771*Zp2p20 + 104.523*Zp2p21 + 36.056*Zp2p22 + 36.056*Zp2p23 + 36.056*Zp2p24 + 36.056*Zp2p25 + 36.056*Zp2p26 + 36.056*Zp2p27 + 36.056*Zp2p28 + 104.163*Zp3p1 + 39.370*Zp3p2 + 102.042*Zp3p4 + 108.858*Zp3p5 + 101.242*Zp3p6 + 103.199*Zp3p7 + 19.526*Zp3p8 + 108.858*Zp3p9 + 10.000*Zp3p10 + 10.000*Zp3p11 + 108.858*Zp3p12 + 21.213*Zp3p13 + 106.536*Zp3p14 + 35.355*Zp3p15 + 106.066*Zp3p16 + 29.155*Zp3p17 + 29.580*Zp3p18 + 27.386*Zp3p19 + 104.163*Zp3p20 + 103.562*Zp3p21 + 29.155*Zp3p22 + 29.155*Zp3p23 + 29.155*Zp3p24 + 29.155*Zp3p25 + 29.155*Zp3p26 + 29.155*Zp3p27 + 29.155*Zp3p28 + 19.685*Zp4p1 + 102.408*Zp4p2 + 102.042*Zp4p3 + 30.619*Zp4p5 + 15.411*Zp4p6 + 23.184*Zp4p7 + 103.833*Zp4p8 + 30.619*Zp4p9 + 101.550*Zp4p10 + 101.550*Zp4p11 + 30.619*Zp4p12 + 103.863*Zp4p13 + 19.685*Zp4p14 + 101.181*Zp4p15 + 15.411*Zp4p16 + 101.304*Zp4p17 + 100.933*Zp4p18 + 100.933*Zp4p19 + 19.685*Zp4p20 + 17.678*Zp4p21 + 104.583*Zp4p22 + 104.583*Zp4p23 + 104.583*Zp4p24 + 104.583*Zp4p25 + 104.583*Zp4p26 + 104.583*Zp4p27 + 104.583*Zp4p28 + 14.142*Zp5p1 + 108.628*Zp5p2 + 108.858*Zp5p3 + 30.619*Zp5p4 + 28.284*Zp5p6 + 40.000*Zp5p7 + 112.500*Zp5p8 + 106.066*Zp5p10 + 106.066*Zp5p11 + 112.250*Zp5p13 + 14.142*Zp5p14 + 103.923*Zp5p15 + 28.284*Zp5p16 + 107.005*Zp5p17 + 105.475*Zp5p18 + 104.881*Zp5p19 + 14.142*Zp5p20 + 25.000*Zp5p21 + 111.803*Zp5p22 + 111.803*Zp5p23 + 111.803*Zp5p24 + 111.803*Zp5p25 + 111.803*Zp5p26 + 111.803*Zp5p27 + 111.803*Zp5p28 + 14.142*Zp6p1 + 106.771*Zp6p2 + 101.242*Zp6p3 + 15.411*Zp6p4 + 28.284*Zp6p5 + 28.284*Zp6p7 + 104.672*Zp6p8 + 28.284*Zp6p9 + 100.250*Zp6p10 + 100.250*Zp6p11 + 28.284*Zp6p12 + 104.881*Zp6p13 + 24.495*Zp6p14 + 103.923*Zp6p15 + 28.284*Zp6p16 + 104.163*Zp6p17 + 103.562*Zp6p18 + 102.956*Zp6p19 + 14.142*Zp6p20 + 20.616*Zp6p21 + 106.301*Zp6p22 + 106.301*Zp6p23 + 106.301*Zp6p24 + 106.301*Zp6p25 + 106.301*Zp6p26 + 106.301*Zp6p27 + 106.301*Zp6p28 + 31.623*Zp7p1 + 104.881*Zp7p2 + 103.199*Zp7p3 + 23.184*Zp7p4 + 40.000*Zp7p5 + 28.284*Zp7p6 + 101.273*Zp7p8 + 40.000*Zp7p9 + 104.163*Zp7p10 + 104.163*Zp7p11 + 40.000*Zp7p12 + 100.995*Zp7p13 + 31.623*Zp7p14 + 103.923*Zp7p15 + 28.284*Zp7p16 + 101.242*Zp7p17 + 101.612*Zp7p18 + 100.995*Zp7p19 + 31.623*Zp7p20 + 15.000*Zp7p21 + 100.499*Zp7p22 + 100.499*Zp7p23 + 100.499*Zp7p24 + 100.499*Zp7p25 + 100.499*Zp7p26 + 100.499*Zp7p27 + 100.499*Zp7p28 + 107.732*Zp8p1 + 38.161*Zp8p2 + 19.526*Zp8p3 + 103.833*Zp8p4 + 112.500*Zp8p5 + 104.672*Zp8p6 + 101.273*Zp8p7 + 112.500*Zp8p9 + 27.951*Zp8p10 + 27.951*Zp8p11 + 112.500*Zp8p12 + 2.500*Zp8p13 + 108.886*Zp8p14 + 38.161*Zp8p15 + 107.034*Zp8p16 + 24.622*Zp8p17 + 27.951*Zp8p18 + 25.617*Zp8p19 + 107.732*Zp8p20 + 103.833*Zp8p21 + 12.500*Zp8p22 + 12.500*Zp8p23 + 12.500*Zp8p24 + 12.500*Zp8p25 + 12.500*Zp8p26 + 12.500*Zp8p27 + 12.500*Zp8p28 + 14.142*Zp9p1 + 108.628*Zp9p2 + 108.858*Zp9p3 + 30.619*Zp9p4 + 28.284*Zp9p6 + 40.000*Zp9p7 + 112.500*Zp9p8 + 106.066*Zp9p10 + 106.066*Zp9p11 + 0.000*Zp9p12 + 112.250*Zp9p13 + 14.142*Zp9p14 + 103.923*Zp9p15 + 28.284*Zp9p16 + 107.005*Zp9p17 + 105.475*Zp9p18 + 104.881*Zp9p19 + 14.142*Zp9p20 + 25.000*Zp9p21 + 111.803*Zp9p22 + 111.803*Zp9p23 + 111.803*Zp9p24 + 111.803*Zp9p25 + 111.803*Zp9p26 + 111.803*Zp9p27 + 111.803*Zp9p28 + 102.225*Zp10p1 + 39.370*Zp10p2 + 10.000*Zp10p3 + 101.550*Zp10p4 + 106.066*Zp10p5 + 100.250*Zp10p6 + 104.163*Zp10p7 + 27.951*Zp10p8 + 106.066*Zp10p9 + 106.066*Zp10p12 + 29.155*Zp10p13 + 104.642*Zp10p14 + 32.404*Zp10p15 + 105.119*Zp10p16 + 30.822*Zp10p17 + 29.580*Zp10p18 + 27.386*Zp10p19 + 102.225*Zp10p20 + 103.078*Zp10p21 + 35.355*Zp10p22 + 35.355*Zp10p23 + 35.355*Zp10p24 + 35.355*Zp10p25 + 35.355*Zp10p26 + 35.355*Zp10p27 + 35.355*Zp10p28 + 102.225*Zp11p1 + 39.370*Zp11p2 + 10.000*Zp11p3 + 101.550*Zp11p4 + 106.066*Zp11p5 + 100.250*Zp11p6 + 104.163*Zp11p7 + 27.951*Zp11p8 + 106.066*Zp11p9 + 106.066*Zp11p12 + 29.155*Zp11p13 + 104.642*Zp11p14 + 32.404*Zp11p15 + 105.119*Zp11p16 + 30.822*Zp11p17 + 29.580*Zp11p18 + 27.386*Zp11p19 + 102.225*Zp11p20 + 103.078*Zp11p21 + 35.355*Zp11p22 + 35.355*Zp11p23 + 35.355*Zp11p24 + 35.355*Zp11p25 + 35.355*Zp11p26 + 35.355*Zp11p27 + 35.355*Zp11p28 + 14.142*Zp12p1 + 108.628*Zp12p2 + 108.858*Zp12p3 + 30.619*Zp12p4 + 28.284*Zp12p6 + 40.000*Zp12p7 + 112.500*Zp12p8 + 106.066*Zp12p10 + 106.066*Zp12p11 + 112.250*Zp12p13 + 14.142*Zp12p14 + 103.923*Zp12p15 + 28.284*Zp12p16 + 107.005*Zp12p17 + 105.475*Zp12p18 + 104.881*Zp12p19 + 14.142*Zp12p20 + 25.000*Zp12p21 + 111.803*Zp12p22 + 111.803*Zp12p23 + 111.803*Zp12p24 + 111.803*Zp12p25 + 111.803*Zp12p26 + 111.803*Zp12p27 + 111.803*Zp12p28

+ 107.703*Zp13p1 + 37.417*Zp13p2 + 21.213*Zp13p3 + 103.863*Zp13p4 + 112.250*Zp13p5 + 104.881*Zp13p6 + 100.995*Zp13p7 + 2.500*Zp13p8 + 112.250*Zp13p9 + 29.155*Zp13p10 + 29.155*Zp13p11 + 112.250*Zp13p12 + 108.628*Zp13p14 + 37.417*Zp13p15 + 106.771*Zp13p16 + 23.452*Zp13p17 + 26.926*Zp13p18 + 24.495*Zp13p19 + 107.703*Zp13p20 + 103.562*Zp13p21 + 10.000*Zp13p22 + 10.000*Zp13p23 + 10.000*Zp13p24 + 10.000*Zp13p25 + 10.000*Zp13p26 + 10.000*Zp13p27 + 10.000*Zp13p28 + 14.142*Zp14p1 + 103.923*Zp14p2 + 106.536*Zp14p3 + 19.685*Zp14p4 + 14.142*Zp14p5 + 24.495*Zp14p6 + 31.623*Zp14p7 + 108.886*Zp14p8 + 14.142*Zp14p9 + 104.642*Zp14p10 + 104.642*Zp14p11 + 14.142*Zp14p12 + 108.628*Zp14p13 + 100.995*Zp14p15 + 14.142*Zp14p16 + 103.199*Zp14p17 + 102.103*Zp14p18 + 101.980*Zp14p19 + 14.142*Zp14p20 + 18.028*Zp14p21 + 108.167*Zp14p22 + 108.167*Zp14p23 + 108.167*Zp14p24 + 108.167*Zp14p25 + 108.167*Zp14p26 + 108.167*Zp14p27 + 108.167*Zp14p28 + 102.956*Zp15p1 + 14.142*Zp15p2 + 35.355*Zp15p3 + 101.181*Zp15p4 + 103.923*Zp15p5 + 103.923*Zp15p6 + 103.923*Zp15p7 + 38.161*Zp15p8 + 103.923*Zp15p9 + 32.404*Zp15p10 + 32.404*Zp15p11 + 103.923*Zp15p12 + 37.417*Zp15p13 + 100.995*Zp15p14 + 100.000*Zp15p16 + 15.811*Zp15p17 + 11.180*Zp15p18 + 14.142*Zp15p19 + 102.956*Zp15p20 + 102.103*Zp15p21 + 36.056*Zp15p22 + 36.056*Zp15p23 + 36.056*Zp15p24 + 36.056*Zp15p25 + 36.056*Zp15p26 + 36.056*Zp15p27 + 36.056*Zp15p28 + 24.495*Zp16p1 + 100.995*Zp16p2 + 106.066*Zp16p3 + 15.411*Zp16p4 + 28.284*Zp16p5 + 28.284*Zp16p6 + 28.284*Zp16p7 + 107.034*Zp16p8 + 28.284*Zp16p9 + 105.119*Zp16p10 + 105.119*Zp16p11 + 28.284*Zp16p12 + 106.771*Zp16p13 + 14.142*Zp16p14 + 100.000*Zp16p15 + 101.242*Zp16p17 + 100.623*Zp16p18 + 100.995*Zp16p19 + 24.495*Zp16p20 + 20.616*Zp16p21 + 106.301*Zp16p22 + 106.301*Zp16p23 + 106.301*Zp16p24 + 106.301*Zp16p25 + 106.301*Zp16p26 + 106.301*Zp16p27 + 106.301*Zp16p28 + 104.642*Zp17p1 + 15.811*Zp17p2 + 29.155*Zp17p3 + 101.304*Zp17p4 + 107.005*Zp17p5 + 104.163*Zp17p6 + 101.242*Zp17p7 + 24.622*Zp17p8 + 107.005*Zp17p9 + 30.822*Zp17p10 + 30.822*Zp17p11 + 107.005*Zp17p12 + 23.452*Zp17p13 + 103.199*Zp17p14 + 15.811*Zp17p15 + 101.242*Zp17p16 + 5.000*Zp17p18 + 7.071*Zp17p19 + 104.642*Zp17p20 + 101.612*Zp17p21 + 21.213*Zp17p22 + 21.213*Zp17p23 + 21.213*Zp17p24 + 21.213*Zp17p25 + 21.213*Zp17p26 + 21.213*Zp17p27 + 21.213*Zp17p28 + 103.562*Zp18p1 + 15.000*Zp18p2 + 29.580*Zp18p3 + 100.933*Zp18p4 + 105.475*Zp18p5 + 103.562*Zp18p6 + 101.612*Zp18p7 + 27.951*Zp18p8 + 105.475*Zp18p9 + 29.580*Zp18p10 + 29.580*Zp18p11 + 105.475*Zp18p12 + 26.926*Zp18p13 + 102.103*Zp18p14 + 11.180*Zp18p15 + 100.623*Zp18p16 + 5.000*Zp18p17 + 5.000*Zp18p19 + 103.562*Zp18p20 + 101.242*Zp18p21 + 25.000*Zp18p22 + 25.000*Zp18p23 + 25.000*Zp18p24 + 25.000*Zp18p25 + 25.000*Zp18p26 + 25.000*Zp18p27 + 25.000*Zp18p28 + 102.956*Zp19p1 + 20.000*Zp19p2 + 27.386*Zp19p3 + 100.933*Zp19p4 + 104.881*Zp19p5 + 102.956*Zp19p6 + 100.995*Zp19p7 + 25.617*Zp19p8 + 104.881*Zp19p9 + 27.386*Zp19p10 + 27.386*Zp19p11 + 104.881*Zp19p12 + 24.495*Zp19p13 + 101.980*Zp19p14 + 14.142*Zp19p15 + 100.995*Zp19p16 + 7.071*Zp19p17 + 5.000*Zp19p18 + 102.956*Zp19p20 + 100.623*Zp19p21 + 22.361*Zp19p22 + 22.361*Zp19p23 + 22.361*Zp19p24 + 22.361*Zp19p25 + 22.361*Zp19p26 + 22.361*Zp19p27 + 22.361*Zp19p28 + 106.771*Zp20p2 + 104.163*Zp20p3 + 19.685*Zp20p4 + 14.142*Zp20p5 + 14.142*Zp20p6 + 31.623*Zp20p7 + 107.732*Zp20p8 + 14.142*Zp20p9 + 102.225*Zp20p10 + 102.225*Zp20p11 + 14.142*Zp20p12 + 107.703*Zp20p13 + 14.142*Zp20p14 + 102.956*Zp20p15 + 24.495*Zp20p16 + 104.642*Zp20p17 + 103.562*Zp20p18 + 102.956*Zp20p19 + 18.028*Zp20p21 + 108.167*Zp20p22 + 108.167*Zp20p23 + 108.167*Zp20p24 + 108.167*Zp20p25 + 108.167*Zp20p26 + 108.167*Zp20p27 + 108.167*Zp20p28 + 18.028*Zp21p1 + 104.523*Zp21p2 + 103.562*Zp21p3 + 17.678*Zp21p4 + 25.000*Zp21p5 + 20.616*Zp21p6 + 15.000*Zp21p7 + 103.833*Zp21p8 + 25.000*Zp21p9 + 103.078*Zp21p10 + 103.078*Zp21p11 + 25.000*Zp21p12 + 103.562*Zp21p13 + 18.028*Zp21p14 + 102.103*Zp21p15 + 20.616*Zp21p16 + 101.612*Zp21p17 + 101.242*Zp21p18 + 100.623*Zp21p19 + 18.028*Zp21p20 + 103.078*Zp21p22 + 103.078*Zp21p23 + 103.078*Zp21p24 + 103.078*Zp21p25 + 103.078*Zp21p26 + 103.078*Zp21p27 + 103.078*Zp21p28 + 108.167*Zp22p1 + 36.056*Zp22p2 + 29.155*Zp22p3 + 104.583*Zp22p4 + 111.803*Zp22p5 + 106.301*Zp22p6 + 100.499*Zp22p7 + 12.500*Zp22p8 + 111.803*Zp22p9 + 35.355*Zp22p10 + 35.355*Zp22p11 + 111.803*Zp22p12 + 10.000*Zp22p13 + 108.167*Zp22p14 + 36.056*Zp22p15 + 106.301*Zp22p16 + 21.213*Zp22p17 + 25.000*Zp22p18 + 22.361*Zp22p19 + 108.167*Zp22p20 + 103.078*Zp22p21 + 108.167*Zp23p1 + 36.056*Zp23p2 + 29.155*Zp23p3 + 104.583*Zp23p4 + 111.803*Zp23p5 + 106.301*Zp23p6 + 100.499*Zp23p7 + 12.500*Zp23p8 + 111.803*Zp23p9 + 35.355*Zp23p10 + 35.355*Zp23p11 + 111.803*Zp23p12 + 10.000*Zp23p13 + 108.167*Zp23p14 + 36.056*Zp23p15 + 106.301*Zp23p16 + 21.213*Zp23p17 + 25.000*Zp23p18 + 22.361*Zp23p19 + 108.167*Zp23p20 + 103.078*Zp23p21 + 108.167*Zp24p1 + 36.056*Zp24p2 + 29.155*Zp24p3 + 104.583*Zp24p4 + 111.803*Zp24p5 + 106.301*Zp24p6 + 100.499*Zp24p7 + 12.500*Zp24p8 + 111.803*Zp24p9 + 35.355*Zp24p10 + 35.355*Zp24p11 + 111.803*Zp24p12 + 10.000*Zp24p13 + 108.167*Zp24p14 + 36.056*Zp24p15 + 106.301*Zp24p16 + 21.213*Zp24p17 + 25.000*Zp24p18 + 22.361*Zp24p19 + 108.167*Zp24p20 + 103.078*Zp24p21 + 108.167*Zp25p1 + 36.056*Zp25p2 + 29.155*Zp25p3 + 104.583*Zp25p4 + 111.803*Zp25p5 + 106.301*Zp25p6 + 100.499*Zp25p7 + 12.500*Zp25p8 + 111.803*Zp25p9 + 35.355*Zp25p10 + 35.355*Zp25p11 + 111.803*Zp25p12 + 10.000*Zp25p13 + 108.167*Zp25p14 + 36.056*Zp25p15 + 106.301*Zp25p16 + 21.213*Zp25p17 + 25.000*Zp25p18 + 22.361*Zp25p19 + 108.167*Zp25p20 + 103.078*Zp25p21 + 108.167*Zp26p1 + 36.056*Zp26p2 + 29.155*Zp26p3 + 104.583*Zp26p4 + 111.803*Zp26p5 + 106.301*Zp26p6 + 100.499*Zp26p7 + 12.500*Zp26p8 + 111.803*Zp26p9 + 35.355*Zp26p10 + 35.355*Zp26p11 + 111.803*Zp26p12 + 10.000*Zp26p13 + 108.167*Zp26p14 + 36.056*Zp26p15 + 106.301*Zp26p16 + 21.213*Zp26p17 + 25.000*Zp26p18 + 22.361*Zp26p19 + 108.167*Zp26p20 + 103.078*Zp26p21 + 108.167*Zp27p1 + 36.056*Zp27p2 + 29.155*Zp27p3 + 104.583*Zp27p4 + 111.803*Zp27p5 + 106.301*Zp27p6 + 100.499*Zp27p7 + 12.500*Zp27p8 + 111.803*Zp27p9 + 35.355*Zp27p10 + 35.355*Zp27p11 + 111.803*Zp27p12 + 10.000*Zp27p13 + 108.167*Zp27p14 + 36.056*Zp27p15 + 106.301*Zp27p16 + 21.213*Zp27p17 + 25.000*Zp27p18 + 22.361*Zp27p19 + 108.167*Zp27p20 + 103.078*Zp27p21 + 108.167*Zp28p1 + 36.056*Zp28p2 + 29.155*Zp28p3 + 104.583*Zp28p4 + 111.803*Zp28p5 + 106.301*Zp28p6 + 100.499*Zp28p7 + 12.500*Zp28p8 + 111.803*Zp28p9 + 35.355*Zp28p10 + 35.355*Zp28p11 + 111.803*Zp28p12 + 10.000*Zp28p13 + 108.167*Zp28p14 + 36.056*Zp28p15 + 106.301*Zp28p16 + 21.213*Zp28p17 + 25.000*Zp28p18 + 22.361*Zp28p19 + 108.167*Zp28p20 + 103.078*Zp28p21;

Constraint (9)

Zp1p1 + Zp2p1 + Zp3p1 + Zp4p1 + Zp5p1 + Zp6p1 + Zp7p1 +Zp8p1 +Zp9p1 +Zp10p1 + Zp11p1 + Zp12p1 + Zp13p1 + Zp14p1 + Zp15p1 + Zp16p1 + Zp17p1 + Zp18p1 + Zp19p1 + Zp20p1 + Zp21p1 + Zp22p1 + Zp23p1 + Zp24p1 + Zp25p1 + Zp26p1 + Zp27p1 + Zp28p1 = 1;
Zp1p2 + Zp2p2 + Zp3p2 + Zp4p2 + Zp5p2 + Zp6p2 + Zp7p2 + Zp8p2 + Zp9p2 + Zp10p2 + Zp11p2 + Zp12p2 + Zp13p2 + Zp14p2 + Zp15p2 + Zp16p2 + Zp17p2 + Zp18p2 + Zp19p2 + Zp20p2 + Zp21p2 + Zp22p2 + Zp23p2 + Zp24p2 + Zp25p2 + Zp26p2 + Zp27p2 + Zp28p2 = 1;
Zp1p3 + Zp2p3 + Zp3p3 + Zp4p3 + Zp5p3 + Zp6p3 + Zp7p3 + Zp8p3 + Zp9p3 + Zp10p3 + Zp11p3 + Zp12p3 + Zp13p3 + Zp14p3 + Zp15p3 + Zp16p3 + Zp17p3 + Zp18p3 + Zp19p3 + Zp20p3 + Zp21p3 + Zp22p3 + Zp23p3 + Zp24p3 + Zp25p3 + Zp26p3 + Zp27p3 + Zp28p3 = 1;
Zp1p4 + Zp2p4 + Zp3p4 + Zp4p4 + Zp5p4 + Zp6p4 + Zp7p4 + Zp8p4 + Zp9p4 + Zp10p4 + Zp11p4 + Zp12p4 + Zp13p4 + Zp14p4 + Zp15p4 + Zp16p4 + Zp17p4 + Zp18p4 + Zp19p4 + Zp20p4 + Zp21p4 + Zp22p4 + Zp23p4 + Zp24p4 + Zp25p4 + Zp26p4 + Zp27p4 + Zp28p4 = 1;
Zp1p5 + Zp2p5 + Zp3p5 + Zp4p5 + Zp5p5 + Zp6p5 + Zp7p5 + Zp8p5 + Zp9p5 + Zp10p5 + Zp11p5 + Zp12p5 + Zp13p5 + Zp14p5 + Zp15p5 + Zp16p5 + Zp17p5 + Zp18p5 + Zp19p5 + Zp20p5 + Zp21p5 + Zp22p5 + Zp23p5 + Zp24p5 + Zp25p5 + Zp26p5 + Zp27p5 + Zp28p5 = 1;

Zp1p6 + Zp2p6 + Zp3p6 + Zp4p6 + Zp5p6 + Zp6p6 + Zp7p6 + Zp8p6 + Zp9p6 + Zp10p6 + Zp11p6 + Zp12p6 + Zp13p6 + Zp14p6 + Zp15p6 + Zp16p6 + Zp17p6 + Zp18p6 + Zp19p6 + Zp20p6 + Zp21p6 + Zp22p6 + Zp23p6 + Zp24p6 + Zp25p6 + Zp26p6 + Zp27p6 + Zp28p6 = 1;

Zp1p7 + Zp2p7 + Zp3p7 + Zp4p7 + Zp5p7 + Zp6p7 + Zp7p7 + Zp8p7 + Zp9p7 + Zp10p7 + Zp11p7 + Zp12p7 + Zp13p7 + Zp14p7 + Zp15p7 + Zp16p7 + Zp17p7 + Zp18p7 + Zp19p7 + Zp20p7 + Zp21p7 + Zp22p7 + Zp23p7 + Zp24p7 + Zp25p7 + Zp26p7 + Zp27p7 + Zp28p7 = 1;

Zp1p8 + Zp2p8 + Zp3p8 + Zp4p8 + Zp5p8 + Zp6p8 + Zp7p8 + Zp8p8 + Zp9p8 + Zp10p8 + Zp11p8 + Zp12p8 + Zp13p8 + Zp14p8 + Zp15p8 + Zp16p8 + Zp17p8 + Zp18p8 + Zp19p8 + Zp20p8 + Zp21p8 + Zp22p8 + Zp23p8 + Zp24p8 + Zp25p8 + Zp26p8 + Zp27p8 + Zp28p8 = 1;

Zp1p9 + Zp2p9 + Zp3p9 + Zp4p9 + Zp5p9 + Zp6p9 + Zp7p9 + Zp8p9 + Zp9p9 + Zp10p9 + Zp11p9 + Zp12p9 + Zp13p9 + Zp14p9 + Zp15p9 + Zp16p9 + Zp17p9 + Zp18p9 + Zp19p9 + Zp20p9 + Zp21p9 + Zp22p9 + Zp23p9 + Zp24p9 + Zp25p9 + Zp26p9 + Zp27p9 + Zp28p9 = 1;

Zp1p10 + Zp2p10 + Zp3p10 + Zp4p10 + Zp5p10 + Zp6p10 + Zp7p10 + Zp8p10 + Zp9p10 + Zp10p10 + Zp11p10 + Zp12p10 + Zp13p10 + Zp14p10 + Zp15p10 + Zp16p10 + Zp17p10 + Zp18p10 + Zp19p10 + Zp20p10 + Zp21p10 + Zp22p10 + Zp23p10 + Zp24p10 + Zp25p10 + Zp26p10 + Zp27p10 + Zp28p10 = 1;

Zp1p11 + Zp2p11 + Zp3p11 + Zp4p11 + Zp5p11 + Zp6p11 + Zp7p11 + Zp8p11 + Zp9p11 + Zp10p11 + Zp11p11 + Zp12p11 + Zp13p11 + Zp14p11 + Zp15p11 + Zp16p11 + Zp17p11 + Zp18p11 + Zp19p11 + Zp20p11 + Zp21p11 + Zp22p11 + Zp23p11 + Zp24p11 + Zp25p11 + Zp26p11 + Zp27p11 + Zp28p11 = 1;

Zp1p12 + Zp2p12 + Zp3p12 + Zp4p12 + Zp5p12 + Zp6p12 + Zp7p12 + Zp8p12 + Zp9p12 + Zp10p12 + Zp11p12 + Zp12p12 + Zp13p12 + Zp14p12 + Zp15p12 + Zp16p12 + Zp17p12 + Zp18p12 + Zp19p12 + Zp20p12 + Zp21p12 + Zp22p12 + Zp23p12 + Zp24p12 + Zp25p12 + Zp26p12 + Zp27p12 + Zp28p12 = 1;

Zp1p13 + Zp2p13 + Zp3p13 + Zp4p13 + Zp5p13 + Zp6p13 + Zp7p13 + Zp8p13 + Zp9p13 + Zp10p13 + Zp11p13 + Zp12p13 + Zp13p13 + Zp14p13 + Zp15p13 + Zp16p13 + Zp17p13 + Zp18p13 + Zp19p13 + Zp20p13 + Zp21p13 + Zp22p13 + Zp23p13 + Zp24p13 + Zp25p13 + Zp26p13 + Zp27p13 + Zp28p13 = 1;

Zp1p14 + Zp2p14 + Zp3p14 + Zp4p14 + Zp5p14 + Zp6p14 + Zp7p14 + Zp8p14 + Zp9p14 + Zp10p14 + Zp11p14 + Zp12p14 + Zp13p14 + Zp14p14 + Zp15p14 + Zp16p14 + Zp17p14 + Zp18p14 + Zp19p14 + Zp20p14 + Zp21p14 + Zp22p14 + Zp23p14 + Zp24p14 + Zp25p14 + Zp26p14 + Zp27p14 + Zp28p14 = 1;

Zp1p15 + Zp2p15 + Zp3p15 + Zp4p15 + Zp5p15 + Zp6p15 + Zp7p15 + Zp8p15 + Zp9p15 + Zp10p15 + Zp11p15 + Zp12p15 + Zp13p15 + Zp14p15 + Zp15p15 + Zp16p15 + Zp17p15 + Zp18p15 + Zp19p15 + Zp20p15 + Zp21p15 + Zp22p15 + Zp23p15 + Zp24p15 + Zp25p15 + Zp26p15 + Zp27p15 + Zp28p15 = 1;

Zp1p16 + Zp2p16 + Zp3p16 + Zp4p16 + Zp5p16 + Zp6p16 + Zp7p16 + Zp8p16 + Zp9p16 + Zp10p16 + Zp11p16 + Zp12p16 + Zp13p16 + Zp14p16 + Zp15p16 + Zp16p16 + Zp17p16 + Zp18p16 + Zp19p16 + Zp20p16 + Zp21p16 + Zp22p16 + Zp23p16 + Zp24p16 + Zp25p16 + Zp26p16 + Zp27p16 + Zp28p16 = 1;

Zp1p17 + Zp2p17+ Zp3p17 + Zp4p17 + Zp5p17 + Zp6p17 + Zp7p17 + Zp8p17 + Zp9p17 + Zp10p17 + Zp11p17 + Zp12p17 + Zp13p17 + Zp14p17 + Zp15p17 + Zp16p17 + Zp17p17 + Zp18p17 + Zp19p17 + Zp20p17 + Zp21p17 + Zp22p17 + Zp23p17 + Zp24p17 + Zp25p17 + Zp26p17 + Zp27p17 + Zp28p17 = 1;

Zp1p18 + Zp2p18 + Zp3p18 + Zp4p18 + Zp5p18 + Zp6p18 + Zp7p18 + Zp8p18 + Zp9p18 + Zp10p18 + Zp11p18 + Zp12p18 + Zp13p18 + Zp14p18 + Zp15p18 + Zp16p18 + Zp17p18 + Zp18p18 + Zp19p18 + Zp20p18 + Zp21p18 + Zp22p18 + Zp23p18 + Zp24p18 + Zp25p18 + Zp26p18 + Zp27p18 + Zp28p18 = 1;

Zp1p19 + Zp2p19 + Zp3p19 + Zp4p19 + Zp5p19 + Zp6p19 + Zp7p19 + Zp8p19 + Zp9p19 + Zp10p19 + Zp11p19 + Zp12p19 + Zp13p19 + Zp14p19 + Zp15p19 + Zp16p19 + Zp17p19 + Zp18p19 + Zp19p19 + Zp20p19 + Zp21p19 + Zp22p19 + Zp23p19 + Zp24p19 + Zp25p19 + Zp26p19 + Zp27p19 + Zp28p19 = 1;

Zp1p20 + Zp2p20 + Zp3p20 + Zp4p20 + Zp5p20 + Zp6p20 + Zp7p20 + Zp8p20 + Zp9p20 + Zp10p20 + Zp11p20 + Zp12p20 + Zp13p20 + Zp14p20 + Zp15p20 + Zp16p20 + Zp17p20 + Zp18p20 + Zp19p20 + Zp20p20 + Zp21p20 + Zp22p20 + Zp23p20 + Zp24p20 + Zp25p20 + Zp26p20 + Zp27p20 + Zp28p20 = 1;

Zp1p21 + Zp2p21 + Zp3p21 + Zp4p21 + Zp5p21 + Zp6p21 + Zp7p21 + Zp8p21 + Zp9p21 + Zp10p21 + Zp11p21 + Zp12p21 + Zp13p21 + Zp14p21 + Zp15p21 + Zp16p21 + Zp17p21 + Zp18p21 + Zp19p21 + Zp20p21 + Zp21p21 + Zp22p21 + Zp23p21 + Zp24p21 + Zp25p21 + Zp26p21 + Zp27p21 + Zp28p21 = 1;

Zp1p22 + Zp2p22 + Zp3p22 + Zp4p22 + Zp5p22 + Zp6p22 + Zp7p22 + Zp8p22 + Zp9p22 + Zp10p22 + Zp11p22 + Zp12p22 + Zp13p22 + Zp14p22 + Zp15p22 + Zp16p22 + Zp17p22 + Zp18p22 + Zp19p22 + Zp20p22 + Zp21p22 + Zp22p22 + Zp23p22 + Zp24p22 + Zp25p22 + Zp26p22 + Zp27p22 + Zp28p22 = 1;

Zp1p23 + Zp2p23 + Zp3p23 + Zp4p23 + Zp5p23 + Zp6p23 + Zp7p23 + Zp8p23 + Zp9p23 + Zp10p23 + Zp11p23 + Zp12p23 + Zp13p23 + Zp14p23 + Zp15p23 + Zp16p23 + Zp17p23 + Zp18p23 + Zp19p23 + Zp20p23 + Zp21p23 + Zp22p23 + Zp23p23 + Zp24p23 + Zp25p23 + Zp26p23 + Zp27p23 + Zp28p23 = 1;

Zp1p24 + Zp2p24 + Zp3p24 + Zp4p24 + Zp5p24 + Zp6p24 + Zp7p24 + Zp8p24 + Zp9p24 + Zp10p24 + Zp11p24 + Zp12p24 + Zp13p24 + Zp14p24 + Zp15p24 + Zp16p24 + Zp17p24 + Zp18p24 + Zp19p24 + Zp20p24 + Zp21p24 + Zp22p24 + Zp23p24 + Zp24p24 + Zp25p24 + Zp26p24 + Zp27p24 + Zp28p24 = 1;

Zp1p25 + Zp2p25 + Zp3p25 + Zp4p25 + Zp5p25 + Zp6p25 + Zp7p25 + Zp8p25 + Zp9p25 + Zp10p25 + Zp11p25 + Zp12p25 + Zp13p25 + Zp14p25 + Zp15p25 + Zp16p25 + Zp17p25 + Zp18p25 + Zp19p25 + Zp20p25 + Zp21p25 + Zp22p25 + Zp23p25 + Zp24p25 + Zp25p25 + Zp26p25 + Zp27p25 + Zp28p25 = 1;

Zp1p26 + Zp2p26 + Zp3p26 + Zp4p26 + Zp5p26 + Zp6p26 + Zp7p26 + Zp8p26 + Zp9p26 + Zp10p26 + Zp11p26 + Zp12p26 + Zp13p26 + Zp14p26 + Zp15p26 + Zp16p26 + Zp17p26 + Zp18p26 + Zp19p26 + Zp20p26 + Zp21p26 + Zp22p26 + Zp23p26 + Zp24p26 + Zp25p26 + Zp26p26 + Zp27p26 + Zp28p26 = 1;

Zp1p27 + Zp2p27 + Zp3p27 + Zp4p27 + Zp5p27 + Zp6p27 + Zp7p27 + Zp8p27 + Zp9p27 + Zp10p27 + Zp11p27 + Zp12p27 + Zp13p27 + Zp14p27 + Zp15p27 + Zp16p27 + Zp17p27 + Zp18p27 + Zp19p27 + Zp20p27 + Zp21p27 + Zp22p27 + Zp23p27 + Zp24p27 + Zp25p27 + Zp26p27 + Zp27p27 + Zp28p27 = 1;

Zp1p28 + Zp2p28 + Zp3p28 + Zp4p28 + Zp5p28 + Zp6p28 + Zp7p28 + Zp8p28 + Zp9p28 + Zp10p28 + Zp11p28 + Zp12p28 + Zp13p28 + Zp14p28 + Zp15p28 + Zp16p28 + Zp17p28 + Zp18p28 + Zp19p28 + Zp20p28 + Zp21p28 + Zp22p28 + Zp23p28 + Zp24p28 + Zp25p28 + Zp26p28 + Zp27p28 + Zp28p28 = 1;

Constraint (10)

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Zp1p3-y1< 0;
Zp1p4-y1< 0;
Zp1p5-y1< 0;
Zp1p6-y1< 0;
Zp1p7-y1< 0;
Zp1p8-y1< 0;
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Constraint (11)

y1+ y2 + y3 + y4 + y5 + y6 + y7 + y8 + y9 + y10 + y11 + y12 + y13 + y14 + y15 + y16 + y17 + y18 + y19 + y20 + y21 + y22 + y23 + y24 + y25 + y26 + y27 + y28 = 3;

Constraint (12)

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end
```

- **Lingo Output**

The representative objects have a value of one and are found at the very end of the output. Then, the products that are assigned to those representative objects also have a value of one.

```
Rows=     814 Vars=     812 No. integer vars=      28  ( all are linear)
 Nonzeros=    3113 Constraint nonz=  2380(  2380 are +- 1) Density=0.005
 Smallest and largest elements in abs value=     1.00000         112.500
 No. < : 784 No. =:  29 No. > :    0, Obj=MIN, GUBs <= 112
 Single cols=     0
```

```
Optimal solution found at step:        458
Objective value:                  311.5360
Branch count:                            0
```

Variable	Value	Reduced Cost
ZP1P2	0.0000000	91.77100
ZP1P3	0.0000000	75.00800
ZP1P4	0.0000000	0.5340576E-06
ZP1P5	0.0000000	-0.1983643E-06
ZP1P6	0.0000000	-0.1983643E-06
ZP1P7	0.0000000	0.8087158E-06
ZP1P8	0.0000000	95.23200
ZP1P9	0.0000000	0.0000000
ZP1P10	0.0000000	72.64500
ZP1P11	0.0000000	72.64500
ZP1P12	0.0000000	-0.1983643E-06
ZP1P13	0.0000000	97.70300
ZP1P14	0.0000000	-0.1983643E-06
ZP1P15	0.0000000	91.77600
ZP1P16	0.0000000	-0.8392334E-06
ZP1P17	0.0000000	99.64200
ZP1P18	0.0000000	103.5620
ZP1P19	0.0000000	97.95600
ZP1P21	0.0000000	0.1220703E-06
ZP1P22	0.0000000	108.1670
ZP1P23	0.0000000	108.1670
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ZP1P26	0.0000000	108.1670
ZP1P27	0.0000000	108.1670
ZP1P28	0.0000000	108.1670
ZP2P1	0.0000000	106.7710
ZP2P3	0.0000000	10.21500
ZP2P4	0.0000000	82.72300
ZP2P5	0.0000000	94.48600
ZP2P6	0.0000000	92.62900
ZP2P7	0.0000000	73.25800
ZP2P8	0.0000000	25.66100
ZP2P9	0.0000000	3.747003
ZP2P10	0.0000000	9.790000
ZP2P11	0.0000000	9.790000
ZP2P12	0.0000000	94.48600
ZP2P13	0.0000000	27.41700
ZP2P14	0.0000000	89.78100
ZP2P15	0.0000000	2.962000
ZP2P16	0.0000000	76.50000
ZP2P17	0.0000000	10.81100
ZP2P18	0.0000000	15.00000
ZP2P19	0.0000000	15.00000
ZP2P20	0.0000000	106.7710
ZP2P21	0.0000000	86.49500
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ZP2P28	0.0000000	36.05600
ZP3P1	0.0000000	104.1630
ZP3P2	0.0000000	24.37000
ZP3P4	0.0000000	82.35700
ZP3P5	0.0000000	94.71600
ZP3P6	0.0000000	87.10000
ZP3P7	0.0000000	71.57600
ZP3P8	0.0000000	7.026000
ZP3P9	0.0000000	3.977003
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ZP3P11	0.0000000	0.0000000
ZP3P12	0.0000000	94.71600
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ZP3P16	0.0000000	81.57100
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ZP3P28	0.0000000	29.15500
ZP4P1	0.0000000	19.68500
ZP4P2	0.0000000	87.40800
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ZP10P19	0.0000000	22.38600
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ZP23P26	0.0000000	0.0000000
ZP24P26	0.0000000	0.0000000
ZP25P26	0.0000000	0.0000000
ZP26P26	0.0000000	0.0000000
ZP27P26	1.000000	0.0000000
ZP28P26	0.0000000	0.0000000
ZP22P27	0.0000000	0.0000000
ZP23P27	0.0000000	0.0000000
ZP24P27	0.0000000	0.0000000
ZP25P27	0.0000000	0.0000000
ZP26P27	0.0000000	0.0000000
ZP27P27	1.000000	0.0000000
ZP28P27	0.0000000	0.0000000

ZP22P28	0.0000000	0.0000000
ZP23P28	0.0000000	0.0000000
ZP24P28	0.0000000	0.0000000
ZP25P28	0.0000000	0.0000000
ZP26P28	0.0000000	0.0000000
ZP27P28	1.000000	0.0000000
ZP28P28	0.0000000	0.0000000
Y1	0.0000000	-90.73900
Y2	0.0000000	-15.00000
Y3	0.0000000	-68.31500
Y4	0.0000000	-111.8200
Y5	0.0000000	-133.1650
Y6	0.0000000	-98.35200
Y7	0.0000000	-99.53200
Y8	0.0000000	-32.88700
Y9	0.0000000	-133.1650
Y10	0.0000000	-78.31500
Y11	0.0000000	-78.31500
Y12	0.0000000	-133.1650
Y13	0.0000000	-28.79200
Y14	0.0000000	-115.2340
Y15	0.0000000	-12.99600
Y16	0.0000000	-108.7050
Y17	0.0000000	-5.000000
Y18	1.000000	0.0000000
Y19	0.0000000	-11.15700
Y20	1.000000	-90.73900
Y21	0.0000000	-120.4180
Y22	0.0000000	0.0000000
Y23	0.0000000	0.0000000
Y24	0.0000000	0.0000000
Y25	0.0000000	0.0000000
Y26	0.0000000	0.0000000
Y27	1.000000	0.0000000
Y28	0.0000000	0.0000000

Appendix D

Covering Model

- **Mathematical Model**

As in the Plant Location Model and later in the Average-Weighted Distance Model, constraints (16) and (18) were originally entered in one column, but for space considerations and illustration purposes, a modified version in which these constraints are split in 6 columns is presented next:

MIN = y1 + y2 + y3 + y4 + y5 + y6 + y7 + y8 + y9 + y10 + y11 + y12 + y13 + y14 + y15 + y16 + y17 + y18 + y19 + y20 + y21 + y22 + y23 + y24 + y25 + y26 + y27 + y28;

Constraint (15)

Zp1p1 + Zp2p1 + Zp3p1 + Zp4p1 + Zp5p1 + Zp6p1 + Zp7p1 +Zp8p1 +Zp9p1 +Zp10p1 + Zp11p1 + Zp12p1 + Zp13p1 + Zp14p1 + Zp15p1 + Zp16p1 + Zp17p1 + Zp18p1 + Zp19p1 + Zp20p1 + Zp21p1 + Zp22p1 + Zp23p1 + Zp24p1 + Zp25p1 + Zp26p1 + Zp27p1 + Zp28p1 = 1;
Zp1p2 + Zp2p2 + Zp3p2 + Zp4p2 + Zp5p2 + Zp6p2 + Zp7p2 + Zp8p2 + Zp9p2 + Zp10p2 + Zp11p2 + Zp12p2 + Zp13p2 + Zp14p2 + Zp15p2 + Zp16p2 + Zp17p2 + Zp18p2 + Zp19p2 + Zp20p2 + Zp21p2 + Zp22p2 + Zp23p2 + Zp24p2 + Zp25p2 + Zp26p2 + Zp27p2 + Zp28p2 = 1;
Zp1p3 + Zp2p3 + Zp3p3 + Zp4p3 + Zp5p3 + Zp6p3 + Zp7p3 + Zp8p3 + Zp9p3 + Zp10p3 + Zp11p3 + Zp12p3 + Zp13p3 + Zp14p3 + Zp15p3 + Zp16p3 + Zp17p3 + Zp18p3 + Zp19p3 + Zp20p3 + Zp21p3 + Zp22p3 + Zp23p3 + Zp24p3 + Zp25p3 + Zp26p3 + Zp27p3 + Zp28p3 = 1;
Zp1p4 + Zp2p4 + Zp3p4 + Zp4p4 + Zp5p4 + Zp6p4 + Zp7p4 + Zp8p4 + Zp9p4 + Zp10p4 + Zp11p4 + Zp12p4 + Zp13p4 + Zp14p4 + Zp15p4 + Zp16p4 + Zp17p4 + Zp18p4 + Zp19p4 + Zp20p4 + Zp21p4 + Zp22p4 + Zp23p4 + Zp24p4 + Zp25p4 + Zp26p4 + Zp27p4 + Zp28p4 = 1;
Zp1p5 + Zp2p5 + Zp3p5 + Zp4p5 + Zp5p5 + Zp6p5 + Zp7p5 + Zp8p5 + Zp9p5 + Zp10p5 + Zp11p5 + Zp12p5 + Zp13p5 + Zp14p5 + Zp15p5 + Zp16p5 + Zp17p5 + Zp18p5 + Zp19p5 + Zp20p5 + Zp21p5 + Zp22p5 + Zp23p5 + Zp24p5 + Zp25p5 + Zp26p5 + Zp27p5 + Zp28p5 = 1;
Zp1p6 + Zp2p6 + Zp3p6 + Zp4p6 + Zp5p6 + Zp6p6 + Zp7p6 + Zp8p6 + Zp9p6 + Zp10p6 + Zp11p6 + Zp12p6 + Zp13p6 + Zp14p6 + Zp15p6 + Zp16p6 + Zp17p6 + Zp18p6 + Zp19p6 + Zp20p6 + Zp21p6 + Zp22p6 + Zp23p6 + Zp24p6 + Zp25p6 + Zp26p6 + Zp27p6 + Zp28p6 = 1;
Zp1p7 + Zp2p7 + Zp3p7 + Zp4p7 + Zp5p7 + Zp6p7 + Zp7p7 + Zp8p7 + Zp9p7 + Zp10p7 + Zp11p7 + Zp12p7 + Zp13p7 + Zp14p7 + Zp15p7 + Zp16p7 + Zp17p7 + Zp18p7 + Zp19p7 + Zp20p7 + Zp21p7 + Zp22p7 + Zp23p7 + Zp24p7 + Zp25p7 + Zp26p7 + Zp27p7 + Zp28p7 = 1;
Zp1p8 + Zp2p8 + Zp3p8 + Zp4p8 + Zp5p8 + Zp6p8 + Zp7p8 + Zp8p8 + Zp9p8 + Zp10p8 + Zp11p8 + Zp12p8 + Zp13p8 + Zp14p8 + Zp15p8 + Zp16p8 + Zp17p8 + Zp18p8 + Zp19p8 + Zp20p8 + Zp21p8 + Zp22p8 + Zp23p8 + Zp24p8 + Zp25p8 + Zp26p8 + Zp27p8 + Zp28p8 = 1;
Zp1p9 + Zp2p9 + Zp3p9 + Zp4p9 + Zp5p9 + Zp6p9 + Zp7p9 + Zp8p9 + Zp9p9 + Zp10p9 + Zp11p9 + Zp12p9 + Zp13p9 + Zp14p9 + Zp15p9 + Zp16p9 + Zp17p9 + Zp18p9 + Zp19p9 + Zp20p9 + Zp21p9 + Zp22p9 + Zp23p9 + Zp24p9 + Zp25p9 + Zp26p9 + Zp27p9 + Zp28p9 = 1;
Zp1p10 + Zp2p10 + Zp3p10 + Zp4p10 + Zp5p10 + Zp6p10 + Zp7p10 + Zp8p10 + Zp9p10 + Zp10p10 + Zp11p10 + Zp12p10 + Zp13p10 + Zp14p10 + Zp15p10 + Zp16p10 + Zp17p10 + Zp18p10 + Zp19p10 + Zp20p10 + Zp21p10 + Zp22p10 + Zp23p10 + Zp24p10 + Zp25p10 + Zp26p10 + Zp27p10 + Zp28p10 = 1;
Zp1p11 + Zp2p11 + Zp3p11 + Zp4p11 + Zp5p11 + Zp6p11 + Zp7p11 + Zp8p11 + Zp9p11 + Zp10p11 + Zp11p11 + Zp12p11 + Zp13p11 + Zp14p11 + Zp15p11 + Zp16p11 + Zp17p11 + Zp18p11 + Zp19p11 + Zp20p11 + Zp21p11 + Zp22p11 + Zp23p11 + Zp24p11 + Zp25p11 + Zp26p11 + Zp27p11 + Zp28p11 = 1;
Zp1p12 + Zp2p12 + Zp3p12 + Zp4p12 + Zp5p12 + Zp6p12 + Zp7p12 + Zp8p12 + Zp9p12 + Zp10p12 + Zp11p12 + Zp12p12 + Zp13p12 + Zp14p12 + Zp15p12 + Zp16p12 + Zp17p12 + Zp18p12 + Zp19p12 + Zp20p12 + Zp21p12 + Zp22p12 + Zp23p12 + Zp24p12 + Zp25p12 + Zp26p12 + Zp27p12 + Zp28p12 = 1;
Zp1p13 + Zp2p13 + Zp3p13 + Zp4p13 + Zp5p13 + Zp6p13 + Zp7p13 + Zp8p13 + Zp9p13 + Zp10p13 + Zp11p13 + Zp12p13 + Zp13p13 + Zp14p13 + Zp15p13 + Zp16p13 + Zp17p13 + Zp18p13 + Zp19p13 + Zp20p13 + Zp21p13 + Zp22p13 + Zp23p13 + Zp24p13 + Zp25p13 + Zp26p13 + Zp27p13 + Zp28p13 = 1;
Zp1p14 + Zp2p14 + Zp3p14 + Zp4p14 + Zp5p14 + Zp6p14 + Zp7p14 + Zp8p14 + Zp9p14 + Zp10p14 + Zp11p14 + Zp12p14 + Zp13p14 + Zp14p14 + Zp15p14 + Zp16p14 + Zp17p14 + Zp18p14 + Zp19p14 + Zp20p14 + Zp21p14 + Zp22p14 + Zp23p14 + Zp24p14 + Zp25p14 + Zp26p14 + Zp27p14 + Zp28p14 = 1;
Zp1p15 + Zp2p15 + Zp3p15 + Zp4p15 + Zp5p15 + Zp6p15 + Zp7p15 + Zp8p15 + Zp9p15 + Zp10p15 + Zp11p15 + Zp12p15 + Zp13p15 + Zp14p15 + Zp15p15 + Zp16p15 + Zp17p15 + Zp18p15 + Zp19p15 + Zp20p15 + Zp21p15 + Zp22p15 + Zp23p15 + Zp24p15 + Zp25p15 + Zp26p15 + Zp27p15 + Zp28p15 = 1;
Zp1p16 + Zp2p16 + Zp3p16 + Zp4p16 + Zp5p16 + Zp6p16 + Zp7p16 + Zp8p16 + Zp9p16 + Zp10p16 + Zp11p16 + Zp12p16 + Zp13p16 + Zp14p16 + Zp15p16 + Zp16p16 + Zp17p16 + Zp18p16 + Zp19p16 + Zp20p16 + Zp21p16 + Zp22p16 + Zp23p16 + Zp24p16 + Zp25p16 + Zp26p16 + Zp27p16 + Zp28p16 = 1;

Zp1p17 + Zp2p17+ Zp3p17 + Zp4p17 + Zp5p17 + Zp6p17 + Zp7p17 + Zp8p17 + Zp9p17 + Zp10p17 + Zp11p17 + Zp12p17 + Zp13p17 + Zp14p17 + Zp15p17 + Zp16p17 + Zp17p17 + Zp18p17 + Zp19p17 + Zp20p17 + Zp21p17 + Zp22p17 + Zp23p17 + Zp24p17 + Zp25p17 + Zp26p17 + Zp27p17 + Zp28p17 = 1;
Zp1p18 + Zp2p18 + Zp3p18 + Zp4p18 + Zp5p18 + Zp6p18 + Zp7p18 + Zp8p18 + Zp9p18 + Zp10p18 + Zp11p18 + Zp12p18 + Zp13p18 + Zp14p18 + Zp15p18 + Zp16p18 + Zp17p18 + Zp18p18 + Zp19p18 + Zp20p18 + Zp21p18 + Zp22p18 + Zp23p18 + Zp24p18 + Zp25p18 + Zp26p18 + Zp27p18 + Zp28p18 = 1;
Zp1p19 + Zp2p19 + Zp3p19 + Zp4p19 + Zp5p19 + Zp6p19 + Zp7p19 + Zp8p19 + Zp9p19 + Zp10p19 + Zp11p19 + Zp12p19 + Zp13p19 + Zp14p19 + Zp15p19 + Zp16p19 + Zp17p19 + Zp18p19 + Zp19p19 + Zp20p19 + Zp21p19 + Zp22p19 + Zp23p19 + Zp24p19 + Zp25p19 + Zp26p19 + Zp27p19 + Zp28p19 = 1;
Zp1p20 + Zp2p20 + Zp3p20 + Zp4p20 + Zp5p20 + Zp6p20 + Zp7p20 + Zp8p20 + Zp9p20 + Zp10p20 + Zp11p20 + Zp12p20 + Zp13p20 + Zp14p20 + Zp15p20 + Zp16p20 + Zp17p20 + Zp18p20 + Zp19p20 + Zp20p20 + Zp21p20 + Zp22p20 + Zp23p20 + Zp24p20 + Zp25p20 + Zp26p20 + Zp27p20 + Zp28p20 = 1;
Zp1p21 + Zp2p21 + Zp3p21 + Zp4p21 + Zp5p21 + Zp6p21 + Zp7p21 + Zp8p21 + Zp9p21 + Zp10p21 + Zp11p21 + Zp12p21 + Zp13p21 + Zp14p21 + Zp15p21 + Zp16p21 + Zp17p21 + Zp18p21 + Zp19p21 + Zp20p21 + Zp21p21 + Zp22p21 + Zp23p21 + Zp24p21 + Zp25p21 + Zp26p21 + Zp27p21 + Zp28p21 = 1;
Zp1p22 + Zp2p22 + Zp3p22 + Zp4p22 + Zp5p22 + Zp6p22 + Zp7p22 + Zp8p22 + Zp9p22 + Zp10p22 + Zp11p22 + Zp12p22 + Zp13p22 + Zp14p22 + Zp15p22 + Zp16p22 + Zp17p22 + Zp18p22 + Zp19p22 + Zp20p22 + Zp21p22 + Zp22p22 + Zp23p22 + Zp24p22 + Zp25p22 + Zp26p22 + Zp27p22 + Zp28p22 = 1;
Zp1p23 + Zp2p23 + Zp3p23 + Zp4p23 + Zp5p23 + Zp6p23 + Zp7p23 + Zp8p23 + Zp9p23 + Zp10p23 + Zp11p23 + Zp12p23 + Zp13p23 + Zp14p23 + Zp15p23 + Zp16p23 + Zp17p23 + Zp18p23 + Zp19p23 + Zp20p23 + Zp21p23 + Zp22p23 + Zp23p23 + Zp24p23 + Zp25p23 + Zp26p23 + Zp27p23 + Zp28p23 = 1;
Zp1p24 + Zp2p24 + Zp3p24 + Zp4p24 + Zp5p24 + Zp6p24 + Zp7p24 + Zp8p24 + Zp9p24 + Zp10p24 + Zp11p24 + Zp12p24 + Zp13p24 + Zp14p24 + Zp15p24 + Zp16p24 + Zp17p24 + Zp18p24 + Zp19p24 + Zp20p24 + Zp21p24 + Zp22p24 + Zp23p24 + Zp24p24 + Zp25p24 + Zp26p24 + Zp27p24 + Zp28p24 = 1;
Zp1p25 + Zp2p25 + Zp3p25 + Zp4p25 + Zp5p25 + Zp6p25 + Zp7p25 + Zp8p25 + Zp9p25 + Zp10p25 + Zp11p25 + Zp12p25 + Zp13p25 + Zp14p25 + Zp15p25 + Zp16p25 + Zp17p25 + Zp18p25 + Zp19p25 + Zp20p25 + Zp21p25 + Zp22p25 + Zp23p25 + Zp24p25 + Zp25p25 + Zp26p25 + Zp27p25 + Zp28p25 = 1;
Zp1p26 + Zp2p26 + Zp3p26 + Zp4p26 + Zp5p26 + Zp6p26 + Zp7p26 + Zp8p26 + Zp9p26 + Zp10p26 + Zp11p26 + Zp12p26 + Zp13p26 + Zp14p26 + Zp15p26 + Zp16p26 + Zp17p26 + Zp18p26 + Zp19p26 + Zp20p26 + Zp21p26 + Zp22p26 + Zp23p26 + Zp24p26 + Zp25p26 + Zp26p26 + Zp27p26 + Zp28p26 = 1;
Zp1p27 + Zp2p27 + Zp3p27 + Zp4p27 + Zp5p27 + Zp6p27 + Zp7p27 + Zp8p27 + Zp9p27 + Zp10p27 + Zp11p27 + Zp12p27 + Zp13p27 + Zp14p27 + Zp15p27 + Zp16p27 + Zp17p27 + Zp18p27 + Zp19p27 + Zp20p27 + Zp21p27 + Zp22p27 + Zp23p27 + Zp24p27 + Zp25p27 + Zp26p27 + Zp27p27 + Zp28p27 = 1;
Zp1p28 + Zp2p28 + Zp3p28 + Zp4p28 + Zp5p28 + Zp6p28 + Zp7p28 + Zp8p28 + Zp9p28 + Zp10p28 + Zp11p28 + Zp12p28 + Zp13p28 + Zp14p28 + Zp15p28 + Zp16p28 + Zp17p28 + Zp18p28 + Zp19p28 + Zp20p28 + Zp21p28 + Zp22p28 + Zp23p28 + Zp24p28 + Zp25p28 + Zp26p28 + Zp27p28 + Zp28p28 = 1;

Constraint (16)

Zp1p1-y1< 0;
Zp1p2-y1< 0;
Zp1p3-y1< 0;
Zp1p4-y1< 0;
Zp1p5-y1< 0;
Zp1p6-y1< 0;
Zp1p7-y1< 0;
Zp1p8-y1< 0;
Zp1p9-y1< 0;
Zp1p10-y1< 0;
Zp1p11-y1< 0;
Zp1p12-y1< 0;
Zp1p13-y1< 0;
Zp1p14-y1< 0;
Zp1p15-y1< 0;
Zp1p16-y1< 0;
Zp1p17-y1< 0;
Zp1p18-y1< 0;
Zp1p19-y1< 0;
Zp1p20-y1< 0;
Zp1p21-y1< 0;
Zp1p22-y1< 0;
Zp1p23-y1< 0;
Zp1p24-y1< 0;
Zp1p25-y1< 0;
Zp1p26-y1< 0;
Zp1p27-y1< 0;
Zp1p28-y1< 0;
Zp2p1-y2< 0;
Zp2p2-y2< 0;
Zp2p3-y2< 0;
Zp2p4-y2< 0;
Zp2p5-y2< 0;
Zp2p6-y2< 0;
Zp2p7-y2< 0;
Zp2p8-y2< 0;
Zp2p9-y2< 0;
Zp2p10-y2< 0;
Zp2p11-y2< 0;
Zp2p12-y2< 0;
Zp2p13-y2< 0;
Zp2p14-y2< 0;
Zp2p15-y2< 0;
Zp2p16-y2< 0;
Zp2p17-y2< 0;
Zp2p18-y2< 0;
Zp2p19-y2< 0;
Zp2p20-y2< 0;
Zp2p21-y2< 0;
Zp2p22-y2< 0;
Zp2p23-y2< 0;
Zp2p24-y2< 0;
Zp2p25-y2< 0;
Zp2p26-y2< 0;
Zp2p27-y2< 0;
Zp2p28-y2< 0;
Zp3p1-y3< 0;
Zp3p2-y3< 0;
Zp3p3-y3< 0;
Zp3p4-y3< 0;
Zp3p5-y3< 0;
Zp3p6-y3< 0;
Zp3p7-y3< 0;
Zp3p8-y3< 0;
Zp3p9-y3< 0;
Zp3p10-y3< 0;
Zp3p11-y3< 0;
Zp3p12-y3< 0;
Zp3p13-y3< 0;
Zp3p14-y3< 0;
Zp3p15-y3< 0;
Zp3p16-y3< 0;
Zp3p17-y3< 0;
Zp3p18-y3< 0;
Zp3p19-y3< 0;
Zp3p20-y3< 0;
Zp3p21-y3< 0;
Zp3p22-y3< 0;
Zp3p23-y3< 0;
Zp3p24-y3< 0;
Zp3p25-y3< 0;
Zp3p26-y3< 0;
Zp3p27-y3< 0;
Zp3p28-y3< 0;
Zp4p1-y4< 0;
Zp4p2-y4< 0;
Zp4p3-y4< 0;
Zp4p4-y4< 0;
Zp4p5-y4< 0;
Zp4p6-y4< 0;
Zp4p7-y4< 0;
Zp4p8-y4< 0;
Zp4p9-y4< 0;
Zp4p10-y4< 0;
Zp4p11-y4< 0;
Zp4p12-y4< 0;
Zp4p13-y4< 0;
Zp4p14-y4< 0;
Zp4p15-y4< 0;
Zp4p16-y4< 0;
Zp4p17-y4< 0;
Zp4p18-y4< 0;
Zp4p19-y4< 0;
Zp4p20-y4< 0;
Zp4p21-y4< 0;
Zp4p22-y4< 0;
Zp4p23-y4< 0;
Zp4p24-y4< 0;
Zp4p25-y4< 0;
Zp4p26-y4< 0;
Zp4p27-y4< 0;
Zp4p28-y4< 0;
Zp5p1-y5< 0;
Zp5p2-y5< 0;
Zp5p3-y5< 0;
Zp5p4-y5< 0;
Zp5p5-y5< 0;
Zp5p6-y5< 0;
Zp5p7-y5< 0;
Zp5p8-y5< 0;
Zp5p9-y5< 0;
Zp5p10-y5< 0;
Zp5p11-y5< 0;
Zp5p12-y5< 0;
Zp5p13-y5< 0;
Zp5p14-y5< 0;
Zp5p15-y5< 0;
Zp5p16-y5< 0;
Zp5p17-y5< 0;
Zp5p18-y5< 0;
Zp5p19-y5< 0;
Zp5p20-y5< 0;

Zp5p21-y5< 0;
Zp5p22-y5< 0;
Zp5p23-y5< 0;
Zp5p24-y5< 0;
Zp5p25-y5< 0;
Zp5p26-y5< 0;
Zp5p27-y5< 0;
Zp5p28-y5< 0;
Zp6p1-y6< 0;
Zp6p2-y6< 0;
Zp6p3-y6< 0;
Zp6p4-y6< 0;
Zp6p5-y6< 0;
Zp6p6-y6< 0;
Zp6p7-y6< 0;
Zp6p8-y6< 0;
Zp6p9-y6< 0;
Zp6p10-y6< 0;
Zp6p11-y6< 0;
Zp6p12-y6< 0;
Zp6p13-y6< 0;
Zp6p14-y6< 0;
Zp6p15-y6< 0;
Zp6p16-y6< 0;
Zp6p17-y6< 0;
Zp6p18-y6< 0;
Zp6p19-y6< 0;
Zp6p20-y6< 0;
Zp6p21-y6< 0;
Zp6p22-y6< 0;
Zp6p23-y6< 0;
Zp6p24-y6< 0;
Zp6p25-y6< 0;
Zp6p26-y6< 0;
Zp6p27-y6< 0;
Zp6p28-y6< 0;
Zp7p1-y7< 0;
Zp7p2-y7< 0;
Zp7p3-y7< 0;
Zp7p4-y7< 0;
Zp7p5-y7< 0;
Zp7p6-y7< 0;
Zp7p7-y7< 0;
Zp7p8-y7< 0;
Zp7p9-y7< 0;
Zp7p10-y7< 0;
Zp7p11-y7< 0;
Zp7p12-y7< 0;
Zp7p13-y7< 0;
Zp7p14-y7< 0;
Zp7p15-y7< 0;
Zp7p16-y7< 0;
Zp7p17-y7< 0;
Zp7p18-y7< 0;
Zp7p19-y7< 0;
Zp7p20-y7< 0;
Zp7p21-y7< 0;
Zp7p22-y7< 0;
Zp7p23-y7< 0;
Zp7p24-y7< 0;
Zp7p25-y7< 0;
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Constraint (17)

106.771 *Zp2p1 + 104.163 *Zp3p1 + 19.685 *Zp4p1 + 14.142 *Zp5p1 + 14.142 *Zp6p1 + 31.623 *Zp7p1 + 107.732 *Zp8p1 + 14.142 *Zp9p1 + 102.225 *Zp10p1 + 102.225 *Zp11p1 + 14.142 *Zp12p1 + 107.703 *Zp13p1 + 14.142 *Zp14p1 + 102.956 *Zp15p1 + 24.495 *Zp16p1 + 104.642 *Zp17p1 + 103.562 *Zp18p1 + 102.956 *Zp19p1 + 18.028 *Zp21p1 + 108.167 *Zp22p1 + 108.167 *Zp23p1 + 108.167 *Zp24p1 + 108.167 *Zp25p1 + 108.167 *Zp26p1 + 108.167 *Zp27p1 + 108.167 *Zp28p1 - D < 0;

106.771 *Zp1p2 + 39.370 *Zp3p2 + 102.408 *Zp4p2 + 108.628 *Zp5p2 + 106.771 *Zp6p2 + 104.881 *Zp7p2 + 38.161 *Zp8p2 + 108.628 *Zp9p2 + 39.370 *Zp10p2 + 39.370 *Zp11p2 + 108.628 *Zp12p2 + 37.417 *Zp13p2 + 103.923 *Zp14p2 + 14.142 *Zp15p2 + 100.995 *Zp16p2 + 15.811 *Zp17p2 + 15.000 *Zp18p2 + 20.000 *Zp19p2 + 106.771 *Zp20p2 + 104.523 *Zp21p2 + 36.056 *Zp22p2 + 36.056 *Zp23p2 + 36.056 *Zp24p2 + 36.056 *Zp25p2 + 36.056 *Zp26p2 + 36.056 *Zp27p2 + 36.056 *Zp28p2 - D < 0;

104.163 *Zp1p3 + 39.370 *Zp2p3 + 102.042 *Zp4p3 + 108.858 *Zp5p3 + 101.242 *Zp6p3 + 103.199 *Zp7p3 + 19.526 *Zp8p3 + 108.858 *Zp9p3 + 10.000 *Zp10p3 + 10.000 *Zp11p3 + 108.858 *Zp12p3 + 21.213 *Zp13p3 + 106.536 *Zp14p3 + 35.355 *Zp15p3 + 106.066 *Zp16p3 + 29.155 *Zp17p3 + 29.580 *Zp18p3 + 27.386 *Zp19p3 + 104.163 *Zp20p3 + 103.562 *Zp21p3 + 29.155 *Zp22p3 + 29.155 *Zp23p3 + 29.155 *Zp24p3 + 29.155 *Zp25p3 + 29.155 *Zp26p3 + 29.155 *Zp27p3 + 29.155 *Zp28p3 - D < 0;

19.685 *Zp1p4 + 102.408 *Zp2p4 + 102.042 *Zp3p4 + 30.619 *Zp5p4 + 15.411 *Zp6p4 + 23.184 *Zp7p4 + 103.833 *Zp8p4 + 30.619 *Zp9p4 + 101.550 *Zp10p4 + 101.550 *Zp11p4 + 30.619 *Zp12p4 + 103.863 *Zp13p4 + 19.685 *Zp14p4 + 101.181 *Zp15p4 + 15.411 *Zp16p4 + 101.304 *Zp17p4 + 100.933 *Zp18p4 + 100.933 *Zp19p4 + 19.685 *Zp20p4 + 17.678 *Zp21p4 + 104.583 *Zp22p4 + 104.583 *Zp23p4 + 104.583 *Zp24p4 + 104.583 *Zp25p4 + 104.583 *Zp26p4 + 104.583 *Zp27p4 + 104.583 *Zp28p4 - D < 0;

14.142 *Zp1p5 + 108.628 *Zp2p5 + 108.858 *Zp3p5 + 30.619 *Zp4p5 + 28.284 *Zp6p5 + 40.000 *Zp7p5 + 112.500 *Zp8p5 + 106.066 *Zp10p5 + 106.066 *Zp11p5 + 112.250 *Zp13p5 + 14.142 *Zp14p5 + 103.923 *Zp15p5 + 28.284 *Zp16p5 + 107.005 *Zp17p5 + 105.475 *Zp18p5 + 104.881 *Zp19p5 + 14.142 *Zp20p5 + 25.000 *Zp21p5 + 111.803 *Zp22p5 + 111.803 *Zp23p5 + 111.803 *Zp24p5 + 111.803 *Zp25p5 + 111.803 *Zp26p5 + 111.803 *Zp27p5 + 111.803 *Zp28p5 - D < 0;

14.142 *Zp1p6 + 106.771 *Zp2p6 + 101.242 *Zp3p6 + 15.411 *Zp4p6 + 28.284 *Zp5p6 + 28.284 *Zp7p6 + 104.672 *Zp8p6 + 28.284 *Zp9p6 + 100.250 *Zp10p6 + 100.250 *Zp11p6 + 28.284 *Zp12p6 + 104.881 *Zp13p6 + 24.495 *Zp14p6 + 103.923 *Zp15p6 + 28.284 *Zp16p6 + 104.163 *Zp17p6 + 103.562 *Zp18p6 + 102.956 *Zp19p6 + 14.142 *Zp20p6 + 20.616 *Zp21p6 + 106.301 *Zp22p6 + 106.301 *Zp23p6 + 106.301 *Zp24p6 + 106.301 *Zp25p6 + 106.301 *Zp26p6 + 106.301 *Zp27p6 + 106.301 *Zp28p6 - D < 0;
31.623 *Zp1p7 + 104.881 *Zp2p7 + 103.199 *Zp3p7 + 23.184 *Zp4p7 + 40.000 *Zp5p7 + 28.284 *Zp6p7 + 101.273 *Zp8p7 + 40.000 *Zp9p7 + 104.163 *Zp10p7 + 104.163 *Zp11p7 + 40.000 *Zp12p7 + 100.995 *Zp13p7 + 31.623 *Zp14p7 + 103.923 *Zp15p7 + 28.284 *Zp16p7 + 101.242 *Zp17p7 + 101.612 *Zp18p7 + 100.995 *Zp19p7 + 31.623 *Zp20p7 + 15.000 *Zp21p7 + 100.499 *Zp22p7 + 100.499 *Zp23p7 + 100.499 *Zp24p7 + 100.499 *Zp25p7 + 100.499 *Zp26p7 + 100.499 *Zp27p7 + 100.499 *Zp28p7 - D < 0;

107.732 *Zp1p8 + 38.161 *Zp2p8 + 19.526 *Zp3p8 + 103.833 *Zp4p8 + 112.500 *Zp5p8 + 104.672 *Zp6p8 + 101.273 *Zp7p8 + 112.500 *Zp9p8 + 27.951 *Zp10p8 + 27.951 *Zp11p8 + 112.500 *Zp12p8 + 2.500 *Zp13p8 + 108.886 *Zp14p8 + 38.161 *Zp15p8 + 107.034 *Zp16p8 + 24.622 *Zp17p8 + 27.951 *Zp18p8 + 25.617 *Zp19p8 + 107.732 *Zp20p8 + 103.833 *Zp21p8 + 12.500 *Zp22p8 + 12.500 *Zp23p8 + 12.500 *Zp24p8 + 12.500 *Zp25p8 + 12.500 *Zp26p8 + 12.500 *Zp27p8 + 12.500 *Zp28p8 - D < 0;

14.142 *Zp1p9 + 108.628 *Zp2p9 + 108.858 *Zp3p9 + 30.619 *Zp4p9 + 28.284 *Zp6p9 + 40.000 *Zp7p9 + 112.500 *Zp8p9 + 106.066 *Zp10p9 + 106.066 *Zp11p9 + 112.250 *Zp13p9 + 14.142 *Zp14p9 + 103.923 *Zp15p9 + 28.284 *Zp16p9 + 107.005 *Zp17p9 + 105.475 *Zp18p9 + 104.881 *Zp19p9 + 14.142 *Zp20p9 + 25.000 *Zp21p9 + 111.803 *Zp22p9 + 111.803 *Zp23p9 + 111.803 *Zp24p9 + 111.803 *Zp25p9 + 111.803 *Zp26p9 + 111.803 *Zp27p9 + 111.803 *Zp28p9 - D < 0;

102.225 *Zp1p10 + 39.370 *Zp2p10 + 10.000 *Zp3p10 + 101.550 *Zp4p10 + 106.066 *Zp5p10 + 100.250 *Zp6p10 + 104.163 *Zp7p10 + 27.951 *Zp8p10 + 106.066 *Zp9p10 + 106.066 *Zp12p10 + 29.155 *Zp13p10 + 104.642 *Zp14p10 + 32.404 *Zp15p10 + 105.119 *Zp16p10 + 30.822 *Zp17p10 + 29.580 *Zp18p10 + 27.386 *Zp19p10 + 102.225 *Zp20p10 + 103.078 *Zp21p10 + 35.355 *Zp22p10 + 35.355 *Zp23p10 + 35.355 *Zp24p10 + 35.355 *Zp25p10 + 35.355 *Zp26p10 + 35.355 *Zp27p10 + 35.355 *Zp28p10 - D < 0;

102.225 *Zp1p11 + 39.370 *Zp2p11 + 10.000 *Zp3p11 + 101.550 *Zp4p11 + 106.066 *Zp5p11 + 100.250 *Zp6p11 + 104.163 *Zp7p11 + 27.951 *Zp8p11 + 106.066 *Zp9p11 + 106.066 *Zp12p11 + 29.155 *Zp13p11 + 104.642 *Zp14p11 + 32.404 *Zp15p11 + 105.119 *Zp16p11 + 30.822 *Zp17p11 + 29.580 *Zp18p11 + 27.386 *Zp19p11 + 102.225 *Zp20p11 + 103.078 *Zp21p11 + 35.355 *Zp22p11 + 35.355 *Zp23p11 + 35.355 *Zp24p11 + 35.355 *Zp25p11 + 35.355 *Zp26p11 + 35.355 *Zp27p11 + 35.355 *Zp28p11 - D < 0;

14.142 *Zp1p12 + 108.628 *Zp2p12 + 108.858 *Zp3p12 + 30.619 *Zp4p12 + 28.284 *Zp6p12 + 40.000 *Zp7p12 + 112.500 *Zp8p12 + 106.066 *Zp10p12 + 106.066 *Zp11p12 + 112.250 *Zp13p12 + 14.142 *Zp14p12 + 103.923 *Zp15p12 + 28.284 *Zp16p12 + 107.005 *Zp17p12 +

105.475 *Zp18p12 + 104.881 *Zp19p12 + 14.142 *Zp20p12 + 25.000 *Zp21p12 + 111.803 *Zp22p12 + 111.803 *Zp23p12 + 111.803 *Zp24p12 + 111.803 *Zp25p12 + 111.803 *Zp26p12 + 111.803 *Zp27p12 + 111.803 *Zp28p12 - D < 0;

107.703 *Zp1p13 + 37.417 *Zp2p13 + 21.213 *Zp3p13 + 103.863 *Zp4p13 + 112.250 *Zp5p13 + 104.881 *Zp6p13 + 100.995 *Zp7p13 + 2.500 *Zp8p13 + 112.250 *Zp9p13 + 29.155 *Zp10p13 + 29.155 *Zp11p13 + 112.250 *Zp12p13 + 108.628 *Zp14p13 + 37.417 *Zp15p13 + 106.771 *Zp16p13 + 23.452 *Zp17p13 + 26.926 *Zp18p13 + 24.495 *Zp19p13 + 107.703 *Zp20p13 + 103.562 *Zp21p13 + 10.000 *Zp22p13 + 10.000 *Zp23p13 + 10.000 *Zp24p13 + 10.000 *Zp25p13 + 10.000 *Zp26p13 + 10.000 *Zp27p13 + 10.000 *Zp28p13 - D < 0;
14.142 *Zp1p14 + 103.923 *Zp2p14 + 106.536 *Zp3p14 + 19.685 *Zp4p14 + 14.142 *Zp5p14 + 24.495 *Zp6p14 + 31.623 *Zp7p14 + 108.886 *Zp8p14 + 14.142 *Zp9p14 + 104.642 *Zp10p14 + 104.642 *Zp11p14 + 14.142 *Zp12p14 + 108.628 *Zp13p14 + 100.995 *Zp15p14 + 14.142 *Zp16p14 + 103.199 *Zp17p14 + 102.103 *Zp18p14 + 101.980 *Zp19p14 + 14.142 *Zp20p14 + 18.028 *Zp21p14 + 108.167 *Zp22p14 + 108.167 *Zp23p14 + 108.167 *Zp24p14 + 108.167 *Zp25p14 + 108.167 *Zp26p14 + 108.167 *Zp27p14 + 108.167 *Zp28p14 - D < 0;

102.956 *Zp1p15 + 14.142 *Zp2p15 + 35.355 *Zp3p15 + 101.181 *Zp4p15 + 103.923 *Zp5p15 + 103.923 *Zp6p15 + 103.923 *Zp7p15 + 38.161 *Zp8p15 + 103.923 *Zp9p15 + 32.404 *Zp10p15 + 32.404 *Zp11p15 + 103.923 *Zp12p15 + 37.417 *Zp13p15 + 100.995 *Zp14p15 + 100.000 *Zp16p15 + 15.811 *Zp17p15 + 11.180 *Zp18p15 + 14.142 *Zp19p15 + 102.956 *Zp20p15 + 102.103 *Zp21p15 + 36.056 *Zp22p15 + 36.056 *Zp23p15 + 36.056 *Zp24p15 + 36.056 *Zp25p15 + 36.056 *Zp26p15 + 36.056 *Zp27p15 + 36.056 *Zp28p15 - D < 0;

24.495 *Zp1p16 + 100.995 *Zp2p16 + 106.066 *Zp3p16 + 15.411 *Zp4p16 + 28.284 *Zp5p16 + 28.284 *Zp6p16 + 28.284 *Zp7p16 + 107.034 *Zp8p16 + 28.284 *Zp9p16 + 105.119 *Zp10p16 + 105.119 *Zp11p16 + 28.284 *Zp12p16 + 106.771 *Zp13p16 + 14.142 *Zp14p16 + 100.000 *Zp15p16 + 101.242 *Zp17p16 + 100.623 *Zp18p16 + 100.995 *Zp19p16 + 24.495 *Zp20p16 + 20.616 *Zp21p16 + 106.301 *Zp22p16 + 106.301 *Zp23p16 + 106.301 *Zp24p16 + 106.301 *Zp25p16 + 106.301 *Zp26p16 + 106.301 *Zp27p16 + 106.301 *Zp28p16 - D < 0;

104.642 *Zp1p17 + 15.811 *Zp2p17 + 29.155 *Zp3p17 + 101.304 *Zp4p17 + 107.005 *Zp5p17 + 104.163 *Zp6p17 + 101.242 *Zp7p17 + 24.622 *Zp8p17 + 107.005 *Zp9p17 + 30.822 *Zp10p17 + 30.822 *Zp11p17 + 107.005 *Zp12p17 + 23.452 *Zp13p17 + 103.199 *Zp14p17 + 15.811 *Zp15p17 + 101.242 *Zp16p17 + 5.000 *Zp18p17 + 7.071 *Zp19p17 + 104.642 *Zp20p17 + 101.612 *Zp21p17 + 21.213 *Zp22p17 + 21.213 *Zp23p17 + 21.213 *Zp24p17 + 21.213 *Zp25p17 + 21.213 *Zp26p17 + 21.213 *Zp27p17 + 21.213 *Zp28p17 - D < 0;

103.562 *Zp1p18 + 15.000 *Zp2p18 + 29.580 *Zp3p18 + 100.933 *Zp4p18 + 105.475 *Zp5p18 + 103.562 *Zp6p18 + 101.612 *Zp7p18 + 27.951 *Zp8p18 + 105.475 *Zp9p18 + 29.580 *Zp10p18 + 29.580 *Zp11p18 + 105.475 *Zp12p18 + 26.926 *Zp13p18 + 102.103 *Zp14p18 + 11.180 *Zp15p18 + 100.623 *Zp16p18 + 5.000 *Zp17p18 + 5.000 *Zp19p18 + 103.562 *Zp20p18 + 101.242 *Zp21p18 + 25.000 *Zp22p18 + 25.000 *Zp23p18 + 25.000 *Zp24p18 + 25.000 *Zp25p18 + 25.000 *Zp26p18 + 25.000 *Zp27p18 + 25.000 *Zp28p18 - D < 0;

102.956 *Zp1p19 + 20.000 *Zp2p19 + 27.386 *Zp3p19 + 100.933 *Zp4p19 + 104.881 *Zp5p19 + 102.956 *Zp6p19 + 100.995 *Zp7p19 + 25.617 *Zp8p19 + 104.881 *Zp9p19 + 27.386 *Zp10p19 + 27.386 *Zp11p19 + 104.881 *Zp12p19 + 24.495 *Zp13p19 + 101.980 *Zp14p19 + 14.142 *Zp15p19 + 100.995 *Zp16p19 + 7.071 *Zp17p19 + 5.000 *Zp18p19 + 102.956 *Zp20p19 + 100.623 *Zp21p19 + 22.361 *Zp22p19 + 22.361 *Zp23p19 + 22.361 *Zp24p19 + 22.361 *Zp25p19 + 22.361 *Zp26p19 + 22.361 *Zp27p19 + 22.361 *Zp28p19 - D < 0;

106.771 *Zp2p20 + 104.163 *Zp3p20 + 19.685 *Zp4p20 + 14.142 *Zp5p20 + 14.142 *Zp6p20 + 31.623 *Zp7p20 + 107.732 *Zp8p20 + 14.142 *Zp9p20 + 102.225 *Zp10p20 + 102.225 *Zp11p20 + 14.142 *Zp12p20 + 107.703 *Zp13p20 + 14.142 *Zp14p20 + 102.956 *Zp15p20 + 24.495 *Zp16p20 + 104.642 *Zp17p20 + 103.562 *Zp18p20 + 102.956 *Zp19p20 + 18.028 *Zp21p20 + 108.167 *Zp22p20 + 108.167 *Zp23p20 + 108.167 *Zp24p20 + 108.167 *Zp25p20 + 108.167 *Zp26p20 + 108.167 *Zp27p20 + 108.167 *Zp28p20 - D < 0;

18.028 *Zp1p21 + 104.523 *Zp2p21 + 103.562 *Zp3p21 + 17.678 *Zp4p21 + 25.000 *Zp5p21 + 20.616 *Zp6p21 + 15.000 *Zp7p21 + 103.833 *Zp8p21 + 25.000 *Zp9p21 + 103.078 *Zp10p21 + 103.078 *Zp11p21 + 25.000 *Zp12p21 + 103.562 *Zp13p21 + 18.028 *Zp14p21 + 102.103 *Zp15p21 + 20.616 *Zp16p21 + 101.612 *Zp17p21 + 101.242 *Zp18p21 + 100.623 *Zp19p21 + 18.028 *Zp20p21 + 103.078 *Zp22p21 + 103.078 *Zp23p21 + 103.078 *Zp24p21 + 103.078 *Zp25p21 + 103.078 *Zp26p21 + 103.078 *Zp27p21 + 103.078 *Zp28p21 - D < 0;

108.167 *Zp1p22 + 36.056 *Zp2p22 + 29.155 *Zp3p22 + 104.583 *Zp4p22 + 111.803 *Zp5p22 + 106.301 *Zp6p22 + 100.499 *Zp7p22 + 12.500 *Zp8p22 + 111.803 *Zp9p22 + 35.355 *Zp10p22 + 35.355 *Zp11p22 + 111.803 *Zp12p22 + 10.000 *Zp13p22 + 108.167 *Zp14p22 + 36.056 *Zp15p22 + 106.301 *Zp16p22 + 21.213 *Zp17p22 + 25.000 *Zp18p22 + 22.361 *Zp19p22 + 108.167 *Zp20p22 + 103.078 *Zp21p22 - D < 0;

108.167 *Zp1p23 + 36.056 *Zp2p23 + 29.155 *Zp3p23 + 104.583 *Zp4p23 + 111.803 *Zp5p23 + 106.301 *Zp6p23 + 100.499 *Zp7p23 + 12.500 *Zp8p23 + 111.803 *Zp9p23 + 35.355 *Zp10p23 + 35.355 *Zp11p23 + 111.803 *Zp12p23 + 10.000 *Zp13p23 + 108.167 *Zp14p23 + 36.056 *Zp15p23 + 106.301 *Zp16p23 + 21.213 *Zp17p23 + 25.000 *Zp18p23 + 22.361 *Zp19p23 + 108.167 *Zp20p23 + 103.078 *Zp21p23 - D < 0;

108.167 *Zp1p24 + 36.056 *Zp2p24 + 29.155 *Zp3p24 + 104.583 *Zp4p24 + 111.803 *Zp5p24 + 106.301 *Zp6p24 + 100.499 *Zp7p24 + 12.500 *Zp8p24 + 111.803 *Zp9p24 + 35.355 *Zp10p24 + 35.355 *Zp11p24 + 111.803 *Zp12p24 + 10.000 *Zp13p24 + 108.167 *Zp14p24 + 36.056 *Zp15p24 + 106.301 *Zp16p24 + 21.213 *Zp17p24 + 25.000 *Zp18p24 + 22.361 *Zp19p24 + 108.167 *Zp20p24 + 103.078 *Zp21p24 - D < 0;

108.167 *Zp1p25 + 36.056 *Zp2p25 + 29.155 *Zp3p25 + 104.583 *Zp4p25 + 111.803 *Zp5p25 + 106.301 *Zp6p25 + 100.499 *Zp7p25 + 12.500 *Zp8p25 + 111.803 *Zp9p25 + 35.355 *Zp10p25 + 35.355 *Zp11p25 + 111.803 *Zp12p25 + 10.000 *Zp13p25 + 108.167 *Zp14p25 + 36.056 *Zp15p25 + 106.301 *Zp16p25 + 21.213 *Zp17p25 + 25.000 *Zp18p25 + 22.361 *Zp19p25 + 108.167 *Zp20p25 + 103.078 *Zp21p25 - D < 0;

108.167 *Zp1p26 + 36.056 *Zp2p26 + 29.155 *Zp3p26 + 104.583 *Zp4p26 + 111.803 *Zp5p26 + 106.301 *Zp6p26 + 100.499 *Zp7p26 + 12.500 *Zp8p26 + 111.803 *Zp9p26 + 35.355 *Zp10p26 + 35.355 *Zp11p26 + 111.803 *Zp12p26 + 10.000 *Zp13p26 + 108.167 *Zp14p26 +

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36.056 *Zp15p26 + 106.301 *Zp16p26 + 21.213 *Zp17p26 + 25.000 *Zp18p26 + 22.361 *Zp19p26 + 108.167 *Zp20p26 + 103.078 *Zp21p26 -
D < 0;

108.167 *Zp1p27 + 36.056 *Zp2p27 + 29.155 *Zp3p27 + 104.583 *Zp4p27 + 111.803 *Zp5p27 + 106.301 *Zp6p27 + 100.499 *Zp7p27 +
12.500 *Zp8p27 + 111.803 *Zp9p27 + 35.355 *Zp10p27 + 35.355 *Zp11p27 + 111.803 *Zp12p27 + 10.000 *Zp13p27 + 108.167 *Zp14p27 +
36.056 *Zp15p27 + 106.301 *Zp16p27 + 21.213 *Zp17p27 + 25.000 *Zp18p27 + 22.361 *Zp19p27 + 108.167 *Zp20p27 + 103.078 *Zp21p27 -
D < 0;

108.167 *Zp1p28 + 36.056 *Zp2p28 + 29.155 *Zp3p28 + 104.583 *Zp4p28 + 111.803 *Zp5p28 + 106.301 *Zp6p28 + 100.499 *Zp7p28 +
12.500 *Zp8p28 + 111.803 *Zp9p28 + 35.355 *Zp10p28 + 35.355 *Zp11p28 + 111.803 *Zp12p28 + 10.000 *Zp13p28 + 108.167 *Zp14p28 +
36.056 *Zp15p28 + 106.301 *Zp16p28 + 21.213 *Zp17p28 + 25.000 *Zp18p28 + 22.361 *Zp19p28 + 108.167 *Zp20p28 + 103.078 *Zp21p28 -
D < 0;


D = 25;
```

Constraint (18)

```
@bin (y1);
@bin (y2);
@bin (y3);
@bin (y4);
@bin (y5);
@bin (y6);
@bin (y7);
@bin (y8);
@bin (y9);
@bin (y10);
@bin (y11);
@bin (y12);
@bin (y13);
@bin (y14);
@bin (y15);
@bin (y16);
@bin (y17);
@bin (y18);
@bin (y19);
@bin (y20);
@bin (y21);
@bin (y22);
@bin (y23);
@bin (y24);
@bin (y25);
@bin (y26);
@bin (y27);
@bin (y28);
@BIN(Zp1p1);
@BIN(Zp2p1);
@BIN(Zp3p1);
@BIN(Zp4p1);
@BIN(Zp5p1);
@BIN(Zp6p1);
@BIN(Zp7p1);
@BIN(Zp8p1);
@BIN(Zp9p1);
@BIN(Zp10p1);
@BIN(Zp11p1);
@BIN(Zp12p1);
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@BIN(Zp17p1);
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@BIN(Zp19p1);
@BIN(Zp20p1);
@BIN(Zp21p1);
@BIN(Zp22p1);
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@BIN(Zp26p1);
@BIN(Zp27p1);
@BIN(Zp28p1);
@BIN(Zp1p2);
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@BIN(Zp3p2);
@BIN(Zp4p2);
@BIN(Zp5p2);
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@BIN(Zp8p2);
@BIN(Zp9p2);
@BIN(Zp10p2);
@BIN(Zp11p2);
@BIN(Zp12p2);
@BIN(Zp13p2);
@BIN(Zp14p2);
@BIN(Zp15p2);
@BIN(Zp16p2);
@BIN(Zp17p2);
@BIN(Zp18p2);
@BIN(Zp19p2);
@BIN(Zp20p2);
@BIN(Zp21p2);
@BIN(Zp22p2);
@BIN(Zp23p2);
@BIN(Zp24p2);
@BIN(Zp25p2);
@BIN(Zp26p2);
@BIN(Zp27p2);
@BIN(Zp28p2);
@BIN(Zp1p3);
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end
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- **Lingo Output**

```
Rows=     841 Vars=     812 No. integer vars=     812  ( all are linear)
 Nonzeros=   3140 Constraint nonz=  3056(  2352 are +- 1) Density=0.005
 Smallest and largest elements in abs value=    1.00000        112.500
 No. < : 784 No. =:   56 No. > :    0, Obj=MIN, GUBs <= 179
 Single cols=     0

 Optimal solution found at step:         251
 Objective value:                   3.000000
 Branch count:                             0

                       Variable           Value        Reduced Cost
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                             Y2       0.0000000            1.000000
                             Y3       0.0000000            1.000000
                             Y4       0.0000000            1.000000
                             Y5       0.0000000            1.000000
                             Y6       0.0000000            1.000000
                             Y7       0.0000000            1.000000
                             Y8       0.0000000            1.000000
                             Y9       0.0000000            1.000000
                            Y10       0.0000000            1.000000
                            Y11        1.000000            1.000000
                            Y12       0.0000000            1.000000
                            Y13       0.0000000            1.000000
                            Y14       0.0000000            1.000000
                            Y15       0.0000000            1.000000
                            Y16       0.0000000            1.000000
                            Y17        1.000000            1.000000
                            Y18       0.0000000            1.000000
                            Y19       0.0000000            1.000000
                            Y20       0.0000000            1.000000
                            Y21        1.000000            1.000000
                            Y22       0.0000000            1.000000
                            Y23       0.0000000            1.000000
                            Y24       0.0000000            1.000000
                            Y25       0.0000000            1.000000
                            Y26       0.0000000            1.000000
                            Y27       0.0000000            1.000000
                            Y28       0.0000000            1.000000
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                          ZP2P1       0.0000000           0.0000000
                          ZP3P1       0.0000000           0.0000000
                          ZP4P1       0.0000000           0.0000000
                          ZP5P1       0.0000000           0.0000000
                          ZP6P1       0.0000000           0.0000000
                          ZP7P1       0.0000000           0.0000000
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ZP12P19	0.0000000	0.0000000
ZP13P19	0.0000000	0.0000000
ZP14P19	0.0000000	0.0000000
ZP15P19	0.0000000	0.0000000
ZP16P19	0.0000000	0.0000000
ZP17P19	1.000000	0.0000000
ZP18P19	0.0000000	0.0000000
ZP19P19	0.0000000	0.0000000
ZP20P19	0.0000000	0.0000000
ZP21P19	0.0000000	0.0000000
ZP22P19	0.0000000	0.0000000
ZP23P19	0.0000000	0.0000000
ZP24P19	0.0000000	0.0000000
ZP25P19	0.0000000	0.0000000
ZP26P19	0.0000000	0.0000000
ZP27P19	0.0000000	0.0000000
ZP28P19	0.0000000	0.0000000
ZP1P20	0.0000000	0.0000000
ZP2P20	0.0000000	0.0000000
ZP3P20	0.0000000	0.0000000
ZP4P20	0.0000000	0.0000000
ZP5P20	0.0000000	0.0000000
ZP6P20	0.0000000	0.0000000
ZP7P20	0.0000000	0.0000000
ZP8P20	0.0000000	0.0000000
ZP9P20	0.0000000	0.0000000
ZP10P20	0.0000000	0.0000000
ZP11P20	0.0000000	0.0000000
ZP12P20	0.0000000	0.0000000
ZP13P20	0.0000000	0.0000000
ZP14P20	0.0000000	0.0000000
ZP15P20	0.0000000	0.0000000
ZP16P20	0.0000000	0.0000000
ZP17P20	0.0000000	0.0000000
ZP18P20	0.0000000	0.0000000
ZP19P20	0.0000000	0.0000000
ZP20P20	0.0000000	0.0000000
ZP21P20	1.000000	0.0000000
ZP22P20	0.0000000	0.0000000
ZP23P20	0.0000000	0.0000000
ZP24P20	0.0000000	0.0000000
ZP25P20	0.0000000	0.0000000
ZP26P20	0.0000000	0.0000000
ZP27P20	0.0000000	0.0000000
ZP28P20	0.0000000	0.0000000
ZP1P21	0.0000000	0.0000000
ZP2P21	0.0000000	0.0000000
ZP3P21	0.0000000	0.0000000
ZP4P21	0.0000000	0.0000000
ZP5P21	0.0000000	0.0000000
ZP6P21	0.0000000	0.0000000
ZP7P21	0.0000000	0.0000000
ZP8P21	0.0000000	0.0000000
ZP9P21	0.0000000	0.0000000
ZP10P21	0.0000000	0.0000000
ZP11P21	0.0000000	0.0000000
ZP12P21	0.0000000	0.0000000
ZP13P21	0.0000000	0.0000000
ZP14P21	0.0000000	0.0000000
ZP15P21	0.0000000	0.0000000
ZP16P21	0.0000000	0.0000000

ZP17P21	0.0000000	0.0000000
ZP18P21	0.0000000	0.0000000
ZP19P21	0.0000000	0.0000000
ZP20P21	0.0000000	0.0000000
ZP21P21	1.000000	0.0000000
ZP22P21	0.0000000	0.0000000
ZP23P21	0.0000000	0.0000000
ZP24P21	0.0000000	0.0000000
ZP25P21	0.0000000	0.0000000
ZP26P21	0.0000000	0.0000000
ZP27P21	0.0000000	0.0000000
ZP28P21	0.0000000	0.0000000
ZP1P22	0.0000000	0.0000000
ZP2P22	0.0000000	0.0000000
ZP3P22	0.0000000	0.0000000
ZP4P22	0.0000000	0.0000000
ZP5P22	0.0000000	0.0000000
ZP6P22	0.0000000	0.0000000
ZP7P22	0.0000000	0.0000000
ZP8P22	0.0000000	0.0000000
ZP9P22	0.0000000	0.0000000
ZP10P22	0.0000000	0.0000000
ZP11P22	0.0000000	0.0000000
ZP12P22	0.0000000	0.0000000
ZP13P22	0.0000000	0.0000000
ZP14P22	0.0000000	0.0000000
ZP15P22	0.0000000	0.0000000
ZP16P22	0.0000000	0.0000000
ZP17P22	1.000000	0.0000000
ZP18P22	0.0000000	0.0000000
ZP19P22	0.0000000	0.0000000
ZP20P22	0.0000000	0.0000000
ZP21P22	0.0000000	0.0000000
ZP22P22	0.0000000	0.0000000
ZP23P22	0.0000000	0.0000000
ZP24P22	0.0000000	0.0000000
ZP25P22	0.0000000	0.0000000
ZP26P22	0.0000000	0.0000000
ZP27P22	0.0000000	0.0000000
ZP28P22	0.0000000	0.0000000
ZP1P23	0.0000000	0.0000000
ZP2P23	0.0000000	0.0000000
ZP3P23	0.0000000	0.0000000
ZP4P23	0.0000000	0.0000000
ZP5P23	0.0000000	0.0000000
ZP6P23	0.0000000	0.0000000
ZP7P23	0.0000000	0.0000000
ZP8P23	0.0000000	0.0000000
ZP9P23	0.0000000	0.0000000
ZP10P23	0.0000000	0.0000000
ZP11P23	0.0000000	0.0000000
ZP12P23	0.0000000	0.0000000
ZP13P23	0.0000000	0.0000000
ZP14P23	0.0000000	0.0000000
ZP15P23	0.0000000	0.0000000
ZP16P23	0.0000000	0.0000000
ZP17P23	1.000000	0.0000000
ZP18P23	0.0000000	0.0000000
ZP19P23	0.0000000	0.0000000
ZP20P23	0.0000000	0.0000000
ZP21P23	0.0000000	0.0000000
ZP22P23	0.0000000	0.0000000
ZP23P23	0.0000000	0.0000000
ZP24P23	0.0000000	0.0000000
ZP25P23	0.0000000	0.0000000
ZP26P23	0.0000000	0.0000000
ZP27P23	0.0000000	0.0000000
ZP28P23	0.0000000	0.0000000
ZP1P24	0.0000000	0.0000000
ZP2P24	0.0000000	0.0000000
ZP3P24	0.0000000	0.0000000

ZP4P24	0.0000000	0.0000000
ZP5P24	0.0000000	0.0000000
ZP6P24	0.0000000	0.0000000
ZP7P24	0.0000000	0.0000000
ZP8P24	0.0000000	0.0000000
ZP9P24	0.0000000	0.0000000
ZP10P24	0.0000000	0.0000000
ZP11P24	0.0000000	0.0000000
ZP12P24	0.0000000	0.0000000
ZP13P24	0.0000000	0.0000000
ZP14P24	0.0000000	0.0000000
ZP15P24	0.0000000	0.0000000
ZP16P24	0.0000000	0.0000000
ZP17P24	1.000000	0.0000000
ZP18P24	0.0000000	0.0000000
ZP19P24	0.0000000	0.0000000
ZP20P24	0.0000000	0.0000000
ZP21P24	0.0000000	0.0000000
ZP22P24	0.0000000	0.0000000
ZP23P24	0.0000000	0.0000000
ZP24P24	0.0000000	0.0000000
ZP25P24	0.0000000	0.0000000
ZP26P24	0.0000000	0.0000000
ZP27P24	0.0000000	0.0000000
ZP28P24	0.0000000	0.0000000
ZP1P25	0.0000000	0.0000000
ZP2P25	0.0000000	0.0000000
ZP3P25	0.0000000	0.0000000
ZP4P25	0.0000000	0.0000000
ZP5P25	0.0000000	0.0000000
ZP6P25	0.0000000	0.0000000
ZP7P25	0.0000000	0.0000000
ZP8P25	0.0000000	0.0000000
ZP9P25	0.0000000	0.0000000
ZP10P25	0.0000000	0.0000000
ZP11P25	0.0000000	0.0000000
ZP12P25	0.0000000	0.0000000
ZP13P25	0.0000000	0.0000000
ZP14P25	0.0000000	0.0000000
ZP15P25	0.0000000	0.0000000
ZP16P25	0.0000000	0.0000000
ZP17P25	1.000000	0.0000000
ZP18P25	0.0000000	0.0000000
ZP19P25	0.0000000	0.0000000
ZP20P25	0.0000000	0.0000000
ZP21P25	0.0000000	0.0000000
ZP22P25	0.0000000	0.0000000
ZP23P25	0.0000000	0.0000000
ZP24P25	0.0000000	0.0000000
ZP25P25	0.0000000	0.0000000
ZP26P25	0.0000000	0.0000000
ZP27P25	0.0000000	0.0000000
ZP28P25	0.0000000	0.0000000
ZP1P26	0.0000000	0.0000000
ZP2P26	0.0000000	0.0000000
ZP3P26	0.0000000	0.0000000
ZP4P26	0.0000000	0.0000000
ZP5P26	0.0000000	0.0000000
ZP6P26	0.0000000	0.0000000
ZP7P26	0.0000000	0.0000000
ZP8P26	0.0000000	0.0000000
ZP9P26	0.0000000	0.0000000
ZP10P26	0.0000000	0.0000000
ZP11P26	0.0000000	0.0000000
ZP12P26	0.0000000	0.0000000
ZP13P26	0.0000000	0.0000000
ZP14P26	0.0000000	0.0000000
ZP15P26	0.0000000	0.0000000
ZP16P26	0.0000000	0.0000000
ZP17P26	1.000000	0.0000000
ZP18P26	0.0000000	0.0000000

ZP19P26	0.0000000	0.0000000
ZP20P26	0.0000000	0.0000000
ZP21P26	0.0000000	0.0000000
ZP22P26	0.0000000	0.0000000
ZP23P26	0.0000000	0.0000000
ZP24P26	0.0000000	0.0000000
ZP25P26	0.0000000	0.0000000
ZP26P26	0.0000000	0.0000000
ZP27P26	0.0000000	0.0000000
ZP28P26	0.0000000	0.0000000
ZP1P27	0.0000000	0.0000000
ZP2P27	0.0000000	0.0000000
ZP3P27	0.0000000	0.0000000
ZP4P27	0.0000000	0.0000000
ZP5P27	0.0000000	0.0000000
ZP6P27	0.0000000	0.0000000
ZP7P27	0.0000000	0.0000000
ZP8P27	0.0000000	0.0000000
ZP9P27	0.0000000	0.0000000
ZP10P27	0.0000000	0.0000000
ZP11P27	0.0000000	0.0000000
ZP12P27	0.0000000	0.0000000
ZP13P27	0.0000000	0.0000000
ZP14P27	0.0000000	0.0000000
ZP15P27	0.0000000	0.0000000
ZP16P27	0.0000000	0.0000000
ZP17P27	1.000000	0.0000000
ZP18P27	0.0000000	0.0000000
ZP19P27	0.0000000	0.0000000
ZP20P27	0.0000000	0.0000000
ZP21P27	0.0000000	0.0000000
ZP22P27	0.0000000	0.0000000
ZP23P27	0.0000000	0.0000000
ZP24P27	0.0000000	0.0000000
ZP25P27	0.0000000	0.0000000
ZP26P27	0.0000000	0.0000000
ZP27P27	0.0000000	0.0000000
ZP28P27	0.0000000	0.0000000
ZP1P28	0.0000000	0.0000000
ZP2P28	0.0000000	0.0000000
ZP3P28	0.0000000	0.0000000
ZP4P28	0.0000000	0.0000000
ZP5P28	0.0000000	0.0000000
ZP6P28	0.0000000	0.0000000
ZP7P28	0.0000000	0.0000000
ZP8P28	0.0000000	0.0000000
ZP9P28	0.0000000	0.0000000
ZP10P28	0.0000000	0.0000000
ZP11P28	0.0000000	0.0000000
ZP12P28	0.0000000	0.0000000
ZP13P28	0.0000000	0.0000000
ZP14P28	0.0000000	0.0000000
ZP15P28	0.0000000	0.0000000
ZP16P28	0.0000000	0.0000000
ZP17P28	1.000000	0.0000000
ZP18P28	0.0000000	0.0000000
ZP19P28	0.0000000	0.0000000
ZP20P28	0.0000000	0.0000000
ZP21P28	0.0000000	0.0000000
ZP22P28	0.0000000	0.0000000
ZP23P28	0.0000000	0.0000000
ZP24P28	0.0000000	0.0000000
ZP25P28	0.0000000	0.0000000
ZP26P28	0.0000000	0.0000000
ZP27P28	0.0000000	0.0000000
ZP28P28	0.0000000	0.0000000
D	25.00000	0.0000000

Appendix E

Fuzzy Set Method

- **Program Description and Interpretation of Results**

Following is the output generated by FANNY, which provides in the first part some general information on the type of data (measurements or dissimilarities) and the chosen options. The program can handle up to 100 objects divided into 2 to 10 clusters and using between 1 to 80 variables. When prompted for this study, dissimilarities are chosen and the location of this data was specified (in this case, fuzzy.dat). Then, 28 objects were to be clustered into 2, 3 and 4 groups (results for k=3 are the ones shown below under Fanny Output heading and specified under number of clusters 3).

The algorithm does not use any representative object, but rather attempts to minimize the objective function, which contains the dissimilarities and the membership coefficients that it is trying to find. This iterative process stops when the objective function converges, so the output shows that the algorithm needed 6 iterations and that the final value of the objective function is 150.3682).

Next, the actual memberships are printed and divided in three columns because k = 3. This table illustrates the percentages in which each object is a member of each cluster. It is important to point out that some clusters are more fuzzy than others. For instance, when an object has equal membership in all clusters it is known as complete fuzziness. However, there are cases in which an object has a membership of 1 in some cluster, so that the clustering is said to be entirely hard. Hence, the Dunn's partition coefficient is computed to measure how hard a fuzzy clustering is. Notice that each object (product) is identified by a three-character label (computer default), but specifying a label name is optional. The closest hard clustering

summarizes the fuzzy clustering by assigning each object to the cluster in which it has the largest membership.

Silhouettes are graphical displays that provide many types of information. In addition to illustrating the members and size of each cluster, it also shows how well members are within each cluster. For instance, product 3 has a low s(I) value of 0.05, indicating that this product is not well-clustered. Moreover, the second column (NEIG) shows that the neighbor of product 3 is cluster 2, so product 3 is intermediate between its own cluster (cluster 2) and cluster 3 (notice that product 3 is listed at the bottom because the s(I) ranks objects in decreasing order).

Below the plot some summary values are given. For each cluster the average silhouette width is listed (this is the average of the s(I) values), which shows that cluster 3 is more pronounced than the other clusters. Finally, the average of the entire data set is listed. When compared with different values of k, the best average s(I) is called the silhouette coefficient (SC) and leads the analyst to assume the value of k that represents a good choice for the data.

- **FANNY Output**

This program performs fuzzy clustering on a data set
of measurements or dissimilarities.
More information can be found in chapter 4 of:

L. Kaufman and P.J. Rousseeuw (1990),
Finding Groups in Data: An Introduction to Cluster Analysis,
Wiley, New York.

TITLE: Fuzzy Set Method using distance matrix

DATA SPECIFICATIONS AND CHOSEN OPTIONS

THERE ARE 28 OBJECTS
LABELS OF OBJECTS ARE NOT READ
INPUT OF DISSIMILARITIES
LARGE OUTPUT IS WANTED
GRAPHICAL OUTPUT IS WANTED (SILHOUETTES)
CLUSTERINGS ARE CARRIED OUT IN 2 TO 4 CLUSTERS
THE DISSIMILARITIES WILL BE READ IN FREE FORMAT

YOUR DATA RESIDE ON FILE: h:fuzzy.dat

DISSIMILARITY MATRIX

001
002 106.77
003 104.16 39.37
004 19.68 102.41 102.04
005 14.14 108.63 108.86 30.62
006 14.14 106.77 101.24 15.41 28.28
007 31.62 104.88 103.20 23.18 40.00 28.28
008 107.73 38.16 19.53 103.83 112.50 104.67 101.27
009 14.14 108.63 108.86 30.62 .00 28.28 40.00 112.50
010 102.22 39.37 10.00 101.55 106.07 100.25 104.16 27.95 106.07
011 102.22 39.37 10.00 101.55 106.07 100.25 104.16 27.95 106.07 .00
012 14.14 108.63 108.86 30.62 .00 28.28 40.00 112.50 .00 106.07 106.07
013 107.70 37.42 21.21 103.86 112.25 104.88 101.00 2.50 112.25 29.16 29.16 112.25
014 14.14 103.92 106.54 19.68 14.14 24.50 31.62 108.89 14.14 104.64 104.64 14.14 108.63
015 102.96 14.14 35.35 101.18 103.92 103.92 103.92 38.16 103.92 32.40 32.40 103.92 37.42 101.00
016 24.50 101.00 106.07 15.41 28.28 28.28 28.28 107.03 28.28 105.12 105.12 28.28 106.77 14.14 100.00
017 104.64 15.81 29.16 101.30 107.00 104.16 101.24 24.62 107.00 30.82 30.82 107.00 23.45 103.20 15.81 101.24
018 103.56 15.00 29.58 100.93 105.47 103.56 101.61 27.95 105.47 29.58 29.58 105.47 26.93 102.10 11.18 100.62
5.00
019 102.96 20.00 27.39 100.93 104.88 102.96 101.00 25.62 104.88 27.39 27.39 104.88 24.50 101.98 14.14 101.00
7.07 5.00
020 .00 106.77 104.16 19.68 14.14 14.14 31.62 107.73 14.14 102.22 102.22 14.14 107.70 14.14 102.96 24.50
104.64 103.56 102.96
021 18.03 104.52 103.56 17.68 25.00 20.62 15.00 103.83 25.00 103.08 103.08 25.00 103.56 18.03 102.10 20.62
101.61 101.24 100.62 18.03
022 108.17 36.06 29.16 104.58 111.80 106.30 100.50 12.50 111.80 35.35 35.35 111.80 10.00 108.17 36.06 106.30
21.21 25.00 22.36 108.17 103.08
023 108.17 36.06 29.16 104.58 111.80 106.30 100.50 12.50 111.80 35.35 35.35 111.80 10.00 108.17 36.06 106.30
21.21 25.00 22.36 108.17 103.08 .00
024 108.17 36.06 29.16 104.58 111.80 106.30 100.50 12.50 111.80 35.35 35.35 111.80 10.00 108.17 36.06 106.30
21.21 25.00 22.36 108.17 103.08 .00 .00
025 108.17 36.06 29.16 104.58 111.80 106.30 100.50 12.50 111.80 35.35 35.35 111.80 10.00 108.17 36.06 106.30
21.21 25.00 22.36 108.17 103.08 .00 .00 .00
026 108.17 36.06 29.16 104.58 111.80 106.30 100.50 12.50 111.80 35.35 35.35 111.80 10.00 108.17 36.06 106.30
21.21 25.00 22.36 108.17 103.08 .00 .00 .00 .00
027 108.17 36.06 29.16 104.58 111.80 106.30 100.50 12.50 111.80 35.35 35.35 111.80 10.00 108.17 36.06 106.30
21.21 25.00 22.36 108.17 103.08 .00 .00 .00 .00 .00
028 108.17 36.06 29.16 104.58 111.80 106.30 100.50 12.50 111.80 35.35 35.35 111.80 10.00 108.17 36.06 106.30
21.21 25.00 22.36 108.17 103.08 .00 .00 .00 .00 .00 .00

NUMBER OF CLUSTERS: 3

ITERATION	OBJECTIVE FUNCTION
1	165.9812
2	150.5632
3	150.3745
4	150.3685
5	150.3682
6	150.3682

FUZZY CLUSTERING

	1	2	3
001	.9073	.0494	.0433
002	.0851	.6646	.2503
003	.0889	.5654	.3457
004	.8009	.1056	.0935
005	.8726	.0681	.0593
006	.8008	.1055	.0937
007	.6947	.1582	.1471
008	.0597	.3207	.6196
009	.8725	.0681	.0593
010	.0930	.6202	.2868
011	.0930	.6203	.2867
012	.8726	.0681	.0593
013	.0507	.2763	.6730

```
014 .8804 .0639 .0557
015 .0699 .7300 .2002
016 .7835 .1154 .1011
017 .0443 .7306 .2251
018 .0388 .7966 .1646
019 .0418 .7593 .1989
020 .9073 .0494 .0433
021 .8279 .0905 .0817
022 .0032 .0166 .9802
023 .0032 .0166 .9802
024 .0032 .0166 .9802
025 .0032 .0166 .9802
026 .0032 .0166 .9802
027 .0032 .0166 .9802
028 .0032 .0166 .9802
```

PARTITION COEFFICIENT OF DUNN = .72
ITS NORMALIZED VERSION = .57

CLOSEST HARD CLUSTERING

CLUSTER NUMBER	SIZE	OBJECTS
1	11	001 004 005 006 007 009 012 014 016 020 021
2	8	002 003 010 011 015 017 018 019
3	9	008 013 022 023 024 025 026 027 028

CLUSTERING VECTOR

```
1 2 2 1 1 1 1 3 1 2 2 1 3 1 2 1 2 2 2 1
1 3 3 3 3 3 3 3
```

SILHOUETTES

```
            0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
            . . . . . . . . . . . . . . . . . . . . . . . . .
            0 0 0 1 1 2 2 2 3 3 4 4 4 5 5 6 6 6 7 7 8 8 8 9 9 0
            0 4 8 2 6 0 4 8 2 6 0 4 8 2 6 0 4 8 2 6 0 4 8 2 6 0

CLU NEIG S(I) I+++++++++++++++++++++++++++++++++++++++++++++++++++
               +                         +
 1   2   .84  001+******************************************     +
 1   2   .84  020+******************************************     +
 1   2    83  014+*****************************************     +
 1   2   .82  005+****************************************     +
 1   2   .82  009+****************************************     +
 1   2   .82  012+****************************************     +
 1   2   .80  021+****************************************     +
 1   2   .78  004+***************************************     +
 1   2   .78  006+**************************************     +
 1   2   .77  016+**************************************     +
 1   3   .69  007+**********************************       +
               +                         +
 2   3   .39  015+*******************           +
 2   3   .30  018+***************             +
 2   3   .28  010+**************             +
 2   3   .28  011+**************             +
 2   3   .28  002+**************             +
 2   3   .20  019+**********               +
 2   3   .12  017+******                 +
 2   3   .05  003+**                    +
               +                         +
 3   2   .91  022+*********************************************    +
 3   2   .91  023+*********************************************    +
 3   2    91  024+*********************************************    +
 3   2   .91  025+*********************************************    +
```

```
3      2   .91  026+**********************************************    +
3      2   .91  027+**********************************************    +
3      2   .91  028+**********************************************    +
3      2   .68  013+***********************************          +
3      2   .61  008+*******************************            +
                +                              +
                ++++++++++++++++++++++++++++++++++++++++++++++++++++

                0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
                . . . . . . . . . . . . . . . . . . . . . . . . . .
                0 0 0 1 1 2 2 2 3 3 4 4 4 5 5 6 6 6 7 7 8 8 8 9 9 0
                0 4 8 2 6 0 4 8 2 6 0 4 8 2 6 0 4 8 2 6 0 4 8 2 6 0
```

CLUSTER 1 HAS AVERAGE SILHOUETTE WIDTH .80
CLUSTER 2 HAS AVERAGE SILHOUETTE WIDTH .24
CLUSTER 3 HAS AVERAGE SILHOUETTE WIDTH .85

FOR THE ENTIRE DATA SET, THE AVERAGE SILHOUETTE WIDTH IS .65

The output is on file : h:fuzzy.fan

Appendix F

Average-Weighted Distance Model

- **Mathematical Model**

MIN = 1025.671 *Zp1p2 + 1020.906 *Zp1p3 + 129.301 *Zp1p4 + 100.000 *Zp1p5 + 100.000 *Zp1p6 + 214.476 *Zp1p7 + 1031.572 *Zp1p8 +
100.000 *Zp1p9 + 1011.187 *Zp1p10 + 1011.187 *Zp1p11 + 100.000 *Zp1p12 + 1031.504 *Zp1p13 + 44.721 *Zp1p14 + 1007.968 *Zp1p15 +
126.491 *Zp1p16 + 1019.191 *Zp1p17 + 1014.088 *Zp1p18 + 1012.917 *Zp1p19 + 125.000 *Zp1p21 + 1031.988 *Zp1p22 + 1031.988
*Zp1p23 + 1031.988 *Zp1p24 + 1031.988 *Zp1p25 + 1031.988 *Zp1p26 + 1031.988 *Zp1p27 + 1031.988 *Zp1p28 + 1025.671 *Zp2p1 +
208.567 *Zp2p3 + 1008.015 *Zp2p4 + 1044.031 *Zp2p5 + 1019.804 *Zp2p6 + 1014.889 *Zp2p7 + 192.232 *Zp2p8 + 1044.031 *Zp2p9 +
209.165 *Zp2p10 + 209.165 *Zp2p11 + 1044.031 *Zp2p12 + 189.737 *Zp2p13 + 1019.804 *Zp2p14 + 100.000 *Zp2p15 + 1004.988 *Zp2p16 +
104.881 *Zp2p17 + 100.623 *Zp2p18 + 126.491 *Zp2p19 + 1025.671 *Zp2p20 + 1013.965 *Zp2p21 + 187.083 *Zp2p22 + 187.083 *Zp2p23 +
187.083 *Zp2p24 + 187.083 *Zp2p25 + 187.083 *Zp2p26 + 187.083 *Zp2p27 + 187.083 *Zp2p28 + 1020.906 *Zp3p1 + 208.567 *Zp3p2 +
1005.889 *Zp3p4 + 1046.542 *Zp3p5 + 1005.609 *Zp3p6 + 1008.092 *Zp3p7 + 96.096 *Zp3p8 + 1046.542 *Zp3p9 + 63.246 *Zp3p10 + 63.246
*Zp3p11 + 1046.542 *Zp3p12 + 106.066 *Zp3p13 + 1025.183 *Zp3p14 + 183.712 *Zp3p15 + 1016.735 *Zp3p16 + 137.840 *Zp3p17 + 140.979
*Zp3p18 + 132.288 *Zp3p19 + 1020.906 *Zp3p20 + 1009.765 *Zp3p21 + 137.840 *Zp3p22 + 137.840 *Zp3p23 + 137.840 *Zp3p24 + 137.840
*Zp3p25 + 137.840 *Zp3p26 + 137.840 *Zp3p27 + 137.840 *Zp3p28 + 129.301 *Zp4p1 + 1008.015 *Zp4p2 + 1005.889 *Zp4p3 + 225.347
*Zp4p5 + 71.807 *Zp4p6 + 108.541 *Zp4p7 + 1008.751 *Zp4p8 + 225.347 *Zp4p9 + 1003.899 *Zp4p10 + 1003.899 *Zp4p11 + 225.347
*Zp4p12 + 1008.820 *Zp4p13 + 129.301 *Zp4p14 + 1002.575 *Zp4p15 + 71.807 *Zp4p16 + 1003.058 *Zp4p17 + 1001.624 *Zp4p18 +
1001.608 *Zp4p19 + 129.301 *Zp4p20 + 62.500 *Zp4p21 + 1009.718 *Zp4p22 + 1009.718 *Zp4p23 + 1009.718 *Zp4p24 + 1009.718 *Zp4p25
+ 1009.718 *Zp4p26 + 1009.718 *Zp4p27 + 1009.718 *Zp4p28 + 100.000 *Zp5p1 + 1044.031 *Zp5p2 + 1046.542 *Zp5p3 + 225.347 *Zp5p4 +
200.000 *Zp5p6 + 309.839 *Zp5p7 + 1061.580 *Zp5p8 + 1030.776 *Zp5p10 + 1030.776 *Zp5p11 + 1061.131 *Zp5p13 + 100.000 *Zp5p14 +
1019.804 *Zp5p15 + 200.000 *Zp5p16 + 1040.673 *Zp5p17 + 1032.654 *Zp5p18 + 1031.504 *Zp5p19 + 100.000 *Zp5p20 + 216.506 *Zp5p21
+ 1060.660 *Zp5p22 + 1060.660 *Zp5p23 + 1060.660 *Zp5p24 + 1060.660 *Zp5p25 + 1060.660 *Zp5p26 + 1060.660 *Zp5p27 + 1060.660
*Zp5p28 + 100.000 *Zp6p1 + 1019.804 *Zp6p2 + 1005.609 *Zp6p3 + 71.807 *Zp6p4 + 200.000 *Zp6p5 + 154.919 *Zp6p7 + 1014.312 *Zp6p8
+ 200.000 *Zp6p9 + 1001.249 *Zp6p10 + 1001.249 *Zp6p11 + 200.000 *Zp6p12 + 1014.889 *Zp6p13 + 126.491 *Zp6p14 + 1007.968 *Zp6p15
+ 126.491 *Zp6p16 + 1010.693 *Zp6p17 + 1008.154 *Zp6p18 + 1006.976 *Zp6p19 + 100.000 *Zp6p20 + 96.825 *Zp6p21 + 1017.349 *Zp6p22
+ 1017.349 *Zp6p23 + 1017.349 *Zp6p24 + 1017.349 *Zp6p25 + 1017.349 *Zp6p26 + 1017.349 *Zp6p27 + 1017.349 *Zp6p28 + 214.476
*Zp7p1 + 1014.889 *Zp7p2 + 1008.092 *Zp7p3 + 108.541 *Zp7p4 + 309.839 *Zp7p5 + 154.919 *Zp7p6 + 1001.475 *Zp7p8 + 309.839 *Zp7p9
+ 1011.682 *Zp7p10 + 1011.682 *Zp7p11 + 309.839 *Zp7p12 + 1001.000 *Zp7p13 + 214.476 *Zp7p14 + 1011.929 *Zp7p15 + 154.919
*Zp7p16 + 1001.998 *Zp7p17 + 1003.182 *Zp7p18 + 1001.998 *Zp7p19 + 214.476 *Zp7p20 + 88.741 *Zp7p21 + 1000.500 *Zp7p22 +
1000.500 *Zp7p23 + 1000.500 *Zp7p24 + 1000.500 *Zp7p25 + 1000.500 *Zp7p26 + 1000.500 *Zp7p27 + 1000.500 *Zp7p28 + 1031.572
*Zp8p1 + 192.232 *Zp8p2 + 96.096 *Zp8p3 + 1008.751 *Zp8p4 + 1061.580 *Zp8p5 + 1014.312 *Zp8p6 + 1001.475 *Zp8p7 + 1061.580
*Zp8p9 + 146.575 *Zp8p10 + 146.575 *Zp8p11 + 1061.580 *Zp8p12 + 11.859 *Zp8p13 + 1032.934 *Zp8p14 + 192.232 *Zp8p15 + 1018.309
*Zp8p16 + 93.291 *Zp8p17 + 115.447 *Zp8p18 + 104.657 *Zp8p19 + 1031.572 *Zp8p20 + 1008.751 *Zp8p21 + 44.194 *Zp8p22 + 44.194
*Zp8p23 + 44.194 *Zp8p24 + 44.194 *Zp8p25 + 44.194 *Zp8p26 + 44.194 *Zp8p27 + 44.194 *Zp8p28 + 100.000 *Zp9p1 + 1044.031 *Zp9p2
+ 1046.542 *Zp9p3 + 225.347 *Zp9p4 + 200.000 *Zp9p6 + 309.839 *Zp9p7 + 1061.580 *Zp9p8 + 1030.776 *Zp9p10 + 1030.776 *Zp9p11 +
1061.131 *Zp9p13 + 100.000 *Zp9p14 + 1019.804 *Zp9p15 + 200.000 *Zp9p16 + 1040.673 *Zp9p17 + 1032.654 *Zp9p18 + 1031.504
*Zp9p19 + 100.000 *Zp9p20 + 216.506 *Zp9p21 + 1060.660 *Zp9p22 + 1060.660 *Zp9p23 + 1060.660 *Zp9p24 + 1060.660 *Zp9p25 +
1060.660 *Zp9p26 + 1060.660 *Zp9p27 + 1060.660 *Zp9p28 + 1011.187 *Zp10p1 + 209.165 *Zp10p2 + 63.246 *Zp10p3 + 1003.899 *Zp10p4
+ 1030.776 *Zp10p5 + 1001.249 *Zp10p6 + 1011.682 *Zp10p7 + 146.575 *Zp10p8 + 1030.776 *Zp10p9 + 1030.776 *Zp10p12 + 153.297
*Zp10p13 + 1015.505 *Zp10p14 + 158.114 *Zp10p15 + 1012.423 *Zp10p16 + 151.658 *Zp10p17 + 141.863 *Zp10p18 + 133.229 *Zp10p19 +
1011.187 *Zp10p20 + 1007.782 *Zp10p21 + 176.777 *Zp10p22 + 176.777 *Zp10p23 + 176.777 *Zp10p24 + 176.777 *Zp10p25 + 176.777
*Zp10p26 + 176.777 *Zp10p27 + 176.777 *Zp10p28 + 1011.187 *Zp11p1 + 209.165 *Zp11p2 + 63.246 *Zp11p3 + 1003.899 *Zp11p4 +
1030.776 *Zp11p5 + 1001.249 *Zp11p6 + 1011.682 *Zp11p7 + 146.575 *Zp11p8 + 1030.776 *Zp11p9 + 1030.776 *Zp11p12 + 153.297
*Zp11p13 + 1015.505 *Zp11p14 + 158.114 *Zp11p15 + 1012.423 *Zp11p16 + 151.658 *Zp11p17 + 141.863 *Zp11p18 + 133.229 *Zp11p19 +
1011.187 *Zp11p20 + 1007.782 *Zp11p21 + 176.777 *Zp11p22 + 176.777 *Zp11p23 + 176.777 *Zp11p24 + 176.777 *Zp11p25 + 176.777
*Zp11p26 + 176.777 *Zp11p27 + 176.777 *Zp11p28 + 100.000 *Zp12p1 + 1044.031 *Zp12p2 + 1046.542 *Zp12p3 + 225.347 *Zp12p4 +
200.000 *Zp12p6 + 309.839 *Zp12p7 + 1061.580 *Zp12p8 + 1030.776 *Zp12p10 + 1030.776 *Zp12p11 + 1061.131 *Zp12p13 + 100.000
*Zp12p14 + 1019.804 *Zp12p15 + 200.000 *Zp12p16 + 1040.673 *Zp12p17 + 1032.654 *Zp12p18 + 1031.504 *Zp12p19 + 100.000 *Zp12p20
+ 216.506 *Zp12p21 + 1060.660 *Zp12p22 + 1060.660 *Zp12p23 + 1060.660 *Zp12p24 + 1060.660 *Zp12p25 + 1060.660 *Zp12p26 +
1060.660 *Zp12p27 + 1060.660 *Zp12p28 + 1031.504 *Zp13p1 + 189.737 *Zp13p2 + 106.066 *Zp13p3 + 1008.820 *Zp13p4 + 1061.131
*Zp13p5 + 1014.889 *Zp13p6 + 1001.000 *Zp13p7 + 11.859 *Zp13p8 + 1061.131 *Zp13p9 + 153.297 *Zp13p10 + 153.297 *Zp13p11 +
1061.131 *Zp13p12 + 1032.473 *Zp13p14 + 189.737 *Zp13p15 + 1017.841 *Zp13p16 + 88.034 *Zp13p17 + 111.243 *Zp13p18 + 100.000
*Zp13p19 + 1031.504 *Zp13p20 + 1008.278 *Zp13p21 + 31.623 *Zp13p22 + 31.623 *Zp13p23 + 31.623 *Zp13p24 + 31.623 *Zp13p25 +
31.623 *Zp13p26 + 31.623 *Zp13p27 + 31.623 *Zp13p28 + 44.721 *Zp14p1 + 1019.804 *Zp14p2 + 1025.183 *Zp14p3 + 129.301 *Zp14p4 +
100.000 *Zp14p5 + 126.491 *Zp14p6 + 214.476 *Zp14p7 + 1032.934 *Zp14p8 + 100.000 *Zp14p9 + 1015.505 *Zp14p10 + 1015.505
*Zp14p11 + 100.000 *Zp14p12 + 1032.473 *Zp14p13 + 1004.988 *Zp14p15 + 100.000 *Zp14p16 + 1017.349 *Zp14p17 + 1012.238 *Zp14p18
+ 1011.929 *Zp14p19 + 44.721 *Zp14p20 + 125.000 *Zp14p21 + 1031.988 *Zp14p22 + 1031.988 *Zp14p23 + 1031.988 *Zp14p24 + 1031.988
*Zp14p25 + 1031.988 *Zp14p26 + 1031.988 *Zp14p27 + 1031.988 *Zp14p28 + 1007.968 *Zp15p1 + 100.000 *Zp15p2 + 183.712 *Zp15p3 +
1002.575 *Zp15p4 + 1019.804 *Zp15p5 + 1007.968 *Zp15p6 + 1011.929 *Zp15p7 + 192.232 *Zp15p8 + 1019.804 *Zp15p9 + 158.114
*Zp15p10 + 158.114 *Zp15p11 + 1019.804 *Zp15p12 + 189.737 *Zp15p13 + 1004.988 *Zp15p14 + 1000.000 *Zp15p16 + 104.881 *Zp15p17 +
76.649 *Zp15p18 + 89.443 *Zp15p19 + 1007.968 *Zp15p20 + 1004.677 *Zp15p21 + 187.083 *Zp15p22 + 187.083 *Zp15p23 + 187.083
*Zp15p24 + 187.083 *Zp15p25 + 187.083 *Zp15p26 + 187.083 *Zp15p27 + 187.083 *Zp15p28 + 126.491 *Zp16p1 + 1004.988 *Zp16p2 +

1016.735 *Zp16p3 + 71.807 *Zp16p4 + 200.000 *Zp16p5 + 126.491 *Zp16p6 + 154.919 *Zp16p7 + 1018.309 *Zp16p8 + 200.000 *Zp16p9 + 1012.423 *Zp16p10 + 1012.423 *Zp16p11 + 200.000 *Zp16p12 + 1017.841 *Zp16p13 + 100.000 *Zp16p14 + 1000.000 *Zp16p15 + 1005.485 *Zp16p17 + 1002.933 *Zp16p18 + 1003.992 *Zp16p19 + 126.491 *Zp16p20 + 96.825 *Zp16p21 + 1017.349 *Zp16p22 + 1017.349 *Zp16p23 + 1017.349 *Zp16p24 + 1017.349 *Zp16p25 + 1017.349 *Zp16p26 + 1017.349 *Zp16p27 + 1017.349 *Zp16p28 + 1019.191 *Zp17p1 + 104.881 *Zp17p2 + 137.840 *Zp17p3 + 1003.058 *Zp17p4 + 1040.673 *Zp17p5 + 1010.693 *Zp17p6 + 1001.998 *Zp17p7 + 93.291 *Zp17p8 + 1040.673 *Zp17p9 + 151.658 *Zp17p10 + 151.658 *Zp17p11 + 1040.673 *Zp17p12 + 88.034 *Zp17p13 + 1017.349 *Zp17p14 + 104.881 *Zp17p15 + 1005.485 *Zp17p16 + 29.580 *Zp17p18 + 38.730 *Zp17p19 + 1019.191 *Zp17p20 + 1003.681 *Zp17p21 + 82.158 *Zp17p22 + 82.158 *Zp17p23 + 82.158 *Zp17p24 + 82.158 *Zp17p25 + 82.158 *Zp17p26 + 82.158 *Zp17p27 + 82.158 *Zp17p28 + 1014.088 *Zp18p1 + 100.623 *Zp18p2 + 140.979 *Zp18p3 + 1001.624 *Zp18p4 + 1032.654 *Zp18p5 + 1008.154 *Zp18p6 + 1003.182 *Zp18p7 + 115.447 *Zp18p8 + 1032.654 *Zp18p9 + 141.863 *Zp18p10 + 141.863 *Zp18p11 + 1032.654 *Zp18p12 + 111.243 *Zp18p13 + 1012.238 *Zp18p14 + 76.649 *Zp18p15 + 1002.933 *Zp18p16 + 29.580 *Zp18p17 + 25.000 *Zp18p19 + 1014.088 *Zp18p20 + 1002.247 *Zp18p21 + 106.654 *Zp18p22 + 106.654 *Zp18p23 + 106.654 *Zp18p24 + 106.654 *Zp18p25 + 106.654 *Zp18p26 + 106.654 *Zp18p27 + 106.654 *Zp18p28 + 1012.917 *Zp19p1 + 126.491 *Zp19p2 + 132.288 *Zp19p3 + 1001.608 *Zp19p4 + 1031.504 *Zp19p5 + 1006.976 *Zp19p6 + 1001.998 *Zp19p7 + 104.657 *Zp19p8 + 1031.504 *Zp19p9 + 133.229 *Zp19p10 + 133.229 *Zp19p11 + 1031.504 *Zp19p12 + 100.000 *Zp19p13 + 1011.929 *Zp19p14 + 89.443 *Zp19p15 + 1003.992 *Zp19p16 + 38.730 *Zp19p17 + 25.000 *Zp19p18 + 1012.917 *Zp19p20 + 1001.062 *Zp19p21 + 94.868 *Zp19p22 + 94.868 *Zp19p23 + 94.868 *Zp19p24 + 94.868 *Zp19p25 + 94.868 *Zp19p26 + 94.868 *Zp19p27 + 94.868 *Zp19p28 + 1025.671 *Zp20p2 + 1020.906 *Zp20p3 + 129.301 *Zp20p4 + 100.000 *Zp20p5 + 100.000 *Zp20p6 + 214.476 *Zp20p7 + 1031.572 *Zp20p8 + 100.000 *Zp20p9 + 1011.187 *Zp20p10 + 1011.187 *Zp20p11 + 100.000 *Zp20p12 + 1031.504 *Zp20p13 + 44.721 *Zp20p14 + 1007.968 *Zp20p15 + 126.491 *Zp20p16 + 1019.191 *Zp20p17 + 1014.088 *Zp20p18 + 1012.917 *Zp20p19 + 125.000 *Zp20p21 + 1031.988 *Zp20p22 + 1031.988 *Zp20p23 + 1031.988 *Zp20p24 + 1031.988 *Zp20p25 + 1031.988 *Zp20p26 + 1031.988 *Zp20p27 + 1031.988 *Zp20p28 + 125.000 *Zp21p1 + 1013.965 *Zp21p2 + 1009.765 *Zp21p3 + 62.500 *Zp21p4 + 216.506 *Zp21p5 + 96.825 *Zp21p6 + 88.741 *Zp21p7 + 1008.751 *Zp21p8 + 216.506 *Zp21p9 + 1007.782 *Zp21p10 + 1007.782 *Zp21p11 + 216.506 *Zp21p12 + 1008.278 *Zp21p13 + 125.000 *Zp21p14 + 1004.677 *Zp21p15 + 96.825 *Zp21p16 + 1003.681 *Zp21p17 + 1002.247 *Zp21p18 + 1001.062 *Zp21p19 + 125.000 *Zp21p20 + 1007.782 *Zp21p22 + 1007.782 *Zp21p23 + 1007.782 *Zp21p24 + 1007.782 *Zp21p25 + 1007.782 *Zp21p26 + 1007.782 *Zp21p27 + 1007.782 *Zp21p28 + 1031.988 *Zp22p1 + 187.083 *Zp22p2 + 137.840 *Zp22p3 + 1009.718 *Zp22p4 + 1060.660 *Zp22p5 + 1017.349 *Zp22p6 + 1000.500 *Zp22p7 + 44.194 *Zp22p8 + 1060.660 *Zp22p9 + 176.777 *Zp22p10 + 176.777 *Zp22p11 + 1060.660 *Zp22p12 + 31.623 *Zp22p13 + 1031.988 *Zp22p14 + 187.083 *Zp22p15 + 1017.349 *Zp22p16 + 82.158 *Zp22p17 + 106.654 *Zp22p18 + 94.868 *Zp22p19 + 1031.988 *Zp22p20 + 1007.782 *Zp22p21 + 1031.988 *Zp23p1 + 187.083 *Zp23p2 + 137.840 *Zp23p3 + 1009.718 *Zp23p4 + 1060.660 *Zp23p5 + 1017.349 *Zp23p6 + 1000.500 *Zp23p7 + 44.194 *Zp23p8 + 1060.660 *Zp23p9 + 176.777 *Zp23p10 + 176.777 *Zp23p11 + 1060.660 *Zp23p12 + 31.623 *Zp23p13 + 1031.988 *Zp23p14 + 187.083 *Zp23p15 + 1017.349 *Zp23p16 + 82.158 *Zp23p17 + 106.654 *Zp23p18 + 94.868 *Zp23p19 + 1031.988 *Zp23p20 + 1007.782 *Zp23p21 + 1031.988 *Zp24p1 + 187.083 *Zp24p2 + 137.840 *Zp24p3 + 1009.718 *Zp24p4 + 1060.660 *Zp24p5 + 1017.349 *Zp24p6 + 1000.500 *Zp24p7 + 44.194 *Zp24p8 + 1060.660 *Zp24p9 + 176.777 *Zp24p10 + 176.777 *Zp24p11 + 1060.660 *Zp24p12 + 31.623 *Zp24p13 + 1031.988 *Zp24p14 + 187.083 *Zp24p15 + 1017.349 *Zp24p16 + 82.158 *Zp24p17 + 106.654 *Zp24p18 + 94.868 *Zp24p19 + 1031.988 *Zp24p20 + 1007.782 *Zp24p21 + 1031.988 *Zp25p1 + 187.083 *Zp25p2 + 137.840 *Zp25p3 + 1009.718 *Zp25p4 + 1060.660 *Zp25p5 + 1017.349 *Zp25p6 + 1000.500 *Zp25p7 + 44.194 *Zp25p8 + 1060.660 *Zp25p9 + 176.777 *Zp25p10 + 176.777 *Zp25p11 + 1060.660 *Zp25p12 + 31.623 *Zp25p13 + 1031.988 *Zp25p14 + 187.083 *Zp25p15 + 1017.349 *Zp25p16 + 82.158 *Zp25p17 + 106.654 *Zp25p18 + 94.868 *Zp25p19 + 1031.988 *Zp25p20 + 1007.782 *Zp25p21 + 1031.988 *Zp26p1 + 187.083 *Zp26p2 + 137.840 *Zp26p3 + 1009.718 *Zp26p4 + 1060.660 *Zp26p5 + 1017.349 *Zp26p6 + 1000.500 *Zp26p7 + 44.194 *Zp26p8 + 1060.660 *Zp26p9 + 176.777 *Zp26p10 + 176.777 *Zp26p11 + 1060.660 *Zp26p12 + 31.623 *Zp26p13 + 1031.988 *Zp26p14 + 187.083 *Zp26p15 + 1017.349 *Zp26p16 + 82.158 *Zp26p17 + 106.654 *Zp26p18 + 94.868 *Zp26p19 + 1031.988 *Zp26p20 + 1007.782 *Zp26p21 + 1031.988 *Zp27p1 + 187.083 *Zp27p2 + 137.840 *Zp27p3 + 1009.718 *Zp27p4 + 1060.660 *Zp27p5 + 1017.349 *Zp27p6 + 1000.500 *Zp27p7 + 44.194 *Zp27p8 + 1060.660 *Zp27p9 + 176.777 *Zp27p10 + 176.777 *Zp27p11 + 1060.660 *Zp27p12 + 31.623 *Zp27p13 + 1031.988 *Zp27p14 + 187.083 *Zp27p15 + 1017.349 *Zp27p16 + 82.158 *Zp27p17 + 106.654 *Zp27p18 + 94.868 *Zp27p19 + 1031.988 *Zp27p20 + 1007.782 *Zp27p21 + 1031.988 *Zp28p1 + 187.083 *Zp28p2 + 137.840 *Zp28p3 + 1009.718 *Zp28p4 + 1060.660 *Zp28p5 + 1017.349 *Zp28p6 + 1000.500 *Zp28p7 + 44.194 *Zp28p8 + 1060.660 *Zp28p9 + 176.777 *Zp28p10 + 176.777 *Zp28p11 + 1060.660 *Zp28p12 + 31.623 *Zp28p13 + 1031.988 *Zp28p14 + 187.083 *Zp28p15 + 1017.349 *Zp28p16 + 82.158 *Zp28p17 + 106.654 *Zp28p18 + 94.868 *Zp28p19 + 1031.988 *Zp28p20 + 1007.782 *Zp28p21;

Constraint (9)

Zp1p1 + Zp2p1 + Zp3p1 + Zp4p1 + Zp5p1 + Zp6p1 + Zp7p1 +Zp8p1 +Zp9p1 +Zp10p1 + Zp11p1 + Zp12p1 + Zp13p1 + Zp14p1 + Zp15p1 + Zp16p1 + Zp17p1 + Zp18p1 + Zp19p1 + Zp20p1 + Zp21p1 + Zp22p1 + Zp23p1 + Zp24p1 + Zp25p1 + Zp26p1 + Zp27p1 + Zp28p1 = 1;
Zp1p2 + Zp2p2 + Zp3p2 + Zp4p2 + Zp5p2 + Zp6p2 + Zp7p2 + Zp8p2 + Zp9p2 + Zp10p2 + Zp11p2 + Zp12p2 + Zp13p2 + Zp14p2 + Zp15p2 + Zp16p2 + Zp17p2 + Zp18p2 + Zp19p2 + Zp20p2 + Zp21p2 + Zp22p2 + Zp23p2 + Zp24p2 + Zp25p2 + Zp26p2 + Zp27p2 + Zp28p2 = 1;
Zp1p3 + Zp2p3 + Zp3p3 + Zp4p3 + Zp5p3 + Zp6p3 + Zp7p3 + Zp8p3 + Zp9p3 + Zp10p3 + Zp11p3 + Zp12p3 + Zp13p3 + Zp14p3 + Zp15p3 + Zp16p3 + Zp17p3 + Zp18p3 + Zp19p3 + Zp20p3 + Zp21p3 + Zp22p3 + Zp23p3 + Zp24p3 + Zp25p3 + Zp26p3 + Zp27p3 + Zp28p3 = 1;
Zp1p4 + Zp2p4 + Zp3p4 + Zp4p4 + Zp5p4 + Zp6p4 + Zp7p4 + Zp8p4 + Zp9p4 + Zp10p4 + Zp11p4 + Zp12p4 + Zp13p4 + Zp14p4 + Zp15p4 + Zp16p4 + Zp17p4 + Zp18p4 + Zp19p4 + Zp20p4 + Zp21p4 + Zp22p4 + Zp23p4 + Zp24p4 + Zp25p4 + Zp26p4 + Zp27p4 + Zp28p4 = 1;
Zp1p5 + Zp2p5 + Zp3p5 + Zp4p5 + Zp5p5 + Zp6p5 + Zp7p5 + Zp8p5 + Zp9p5 + Zp10p5 + Zp11p5 + Zp12p5 + Zp13p5 + Zp14p5 + Zp15p5 + Zp16p5 + Zp17p5 + Zp18p5 + Zp19p5 + Zp20p5 + Zp21p5 + Zp22p5 + Zp23p5 + Zp24p5 + Zp25p5 + Zp26p5 + Zp27p5 + Zp28p5 = 1;
Zp1p6 + Zp2p6 + Zp3p6 + Zp4p6 + Zp5p6 + Zp6p6 + Zp7p6 + Zp8p6 + Zp9p6 + Zp10p6 + Zp11p6 + Zp12p6 + Zp13p6 + Zp14p6 + Zp15p6 + Zp16p6 + Zp17p6 + Zp18p6 + Zp19p6 + Zp20p6 + Zp21p6 + Zp22p6 + Zp23p6 + Zp24p6 + Zp25p6 + Zp26p6 + Zp27p6 + Zp28p6 = 1;
Zp1p7 + Zp2p7 + Zp3p7 + Zp4p7 + Zp5p7 + Zp6p7 + Zp7p7 + Zp8p7 + Zp9p7 + Zp10p7 + Zp11p7 + Zp12p7 + Zp13p7 + Zp14p7 + Zp15p7 + Zp16p7 + Zp17p7 + Zp18p7 + Zp19p7 + Zp20p7 + Zp21p7 + Zp22p7 + Zp23p7 + Zp24p7 + Zp25p7 + Zp26p7 + Zp27p7 + Zp28p7 = 1;
Zp1p8 + Zp2p8 + Zp3p8 + Zp4p8 + Zp5p8 + Zp6p8 + Zp7p8 + Zp8p8 + Zp9p8 + Zp10p8 + Zp11p8 + Zp12p8 + Zp13p8 + Zp14p8 + Zp15p8 + Zp16p8 + Zp17p8 + Zp18p8 + Zp19p8 + Zp20p8 + Zp21p8 + Zp22p8 + Zp23p8 + Zp24p8 + Zp25p8 + Zp26p8 + Zp27p8 + Zp28p8 = 1;
Zp1p9 + Zp2p9 + Zp3p9 + Zp4p9 + Zp5p9 + Zp6p9 + Zp7p9 + Zp8p9 + Zp9p9 + Zp10p9 + Zp11p9 + Zp12p9 + Zp13p9 + Zp14p9 + Zp15p9 + Zp16p9 + Zp17p9 + Zp18p9 + Zp19p9 + Zp20p9 + Zp21p9 + Zp22p9 + Zp23p9 + Zp24p9 + Zp25p9 + Zp26p9 + Zp27p9 + Zp28p9 = 1;

Zp1p10 + Zp2p10 + Zp3p10 + Zp4p10 + Zp5p10 + Zp6p10 + Zp7p10 + Zp8p10 + Zp9p10 + Zp10p10 + Zp11p10 + Zp12p10 + Zp13p10 + Zp14p10 + Zp15p10 + Zp16p10 + Zp17p10 + Zp18p10 + Zp19p10 + Zp20p10 + Zp21p10 + Zp22p10 + Zp23p10 + Zp24p10 + Zp25p10 + Zp26p10 + Zp27p10 + Zp28p10 = 1;
Zp1p11 + Zp2p11 + Zp3p11 + Zp4p11 + Zp5p11 + Zp6p11 + Zp7p11 + Zp8p11 + Zp9p11 + Zp10p11 + Zp11p11 + Zp12p11 + Zp13p11 + Zp14p11 + Zp15p11 + Zp16p11 + Zp17p11 + Zp18p11 + Zp19p11 + Zp20p11 + Zp21p11 + Zp22p11 + Zp23p11 + Zp24p11 + Zp25p11 + Zp26p11 + Zp27p11 + Zp28p11 = 1;
Zp1p12 + Zp2p12 + Zp3p12 + Zp4p12 + Zp5p12 + Zp6p12 + Zp7p12 + Zp8p12 + Zp9p12 + Zp10p12 + Zp11p12 + Zp12p12 + Zp13p12 + Zp14p12 + Zp15p12 + Zp16p12 + Zp17p12 + Zp18p12 + Zp19p12 + Zp20p12 + Zp21p12 + Zp22p12 + Zp23p12 + Zp24p12 + Zp25p12 + Zp26p12 + Zp27p12 + Zp28p12 = 1;
Zp1p13 + Zp2p13 + Zp3p13 + Zp4p13 + Zp5p13 + Zp6p13 + Zp7p13 + Zp8p13 + Zp9p13 + Zp10p13 + Zp11p13 + Zp12p13 + Zp13p13 + Zp14p13 + Zp15p13 + Zp16p13 + Zp17p13 + Zp18p13 + Zp19p13 + Zp20p13 + Zp21p13 + Zp22p13 + Zp23p13 + Zp24p13 + Zp25p13 + Zp26p13 + Zp27p13 + Zp28p13 = 1;
Zp1p14 + Zp2p14 + Zp3p14 + Zp4p14 + Zp5p14 + Zp6p14 + Zp7p14 + Zp8p14 + Zp9p14 + Zp10p14 + Zp11p14 + Zp12p14 + Zp13p14 + Zp14p14 + Zp15p14 + Zp16p14 + Zp17p14 + Zp18p14 + Zp19p14 + Zp20p14 + Zp21p14 + Zp22p14 + Zp23p14 + Zp24p14 + Zp25p14 + Zp26p14 + Zp27p14 + Zp28p14 = 1;
Zp1p15 + Zp2p15 + Zp3p15 + Zp4p15 + Zp5p15 + Zp6p15 + Zp7p15 + Zp8p15 + Zp9p15 + Zp10p15 + Zp11p15 + Zp12p15 + Zp13p15 + Zp14p15 + Zp15p15 + Zp16p15 + Zp17p15 + Zp18p15 + Zp19p15 + Zp20p15 + Zp21p15 + Zp22p15 + Zp23p15 + Zp24p15 + Zp25p15 + Zp26p15 + Zp27p15 + Zp28p15 = 1;
Zp1p16 + Zp2p16 + Zp3p16 + Zp4p16 + Zp5p16 + Zp6p16 + Zp7p16 + Zp8p16 + Zp9p16 + Zp10p16 + Zp11p16 + Zp12p16 + Zp13p16 + Zp14p16 + Zp15p16 + Zp16p16 + Zp17p16 + Zp18p16 + Zp19p16 + Zp20p16 + Zp21p16 + Zp22p16 + Zp23p16 + Zp24p16 + Zp25p16 + Zp26p16 + Zp27p16 + Zp28p16 = 1;
Zp1p17 + Zp2p17+ Zp3p17 + Zp4p17 + Zp5p17 + Zp6p17 + Zp7p17 + Zp8p17 + Zp9p17 + Zp10p17 + Zp11p17 + Zp12p17 + Zp13p17 + Zp14p17 + Zp15p17 + Zp16p17 + Zp17p17 + Zp18p17 + Zp19p17 + Zp20p17 + Zp21p17 + Zp22p17 + Zp23p17 + Zp24p17 + Zp25p17 + Zp26p17 + Zp27p17 + Zp28p17 = 1;
Zp1p18 + Zp2p18 + Zp3p18 + Zp4p18 + Zp5p18 + Zp6p18 + Zp7p18 + Zp8p18 + Zp9p18 + Zp10p18 + Zp11p18 + Zp12p18 + Zp13p18 + Zp14p18 + Zp15p18 + Zp16p18 + Zp17p18 + Zp18p18 + Zp19p18 + Zp20p18 + Zp21p18 + Zp22p18 + Zp23p18 + Zp24p18 + Zp25p18 + Zp26p18 + Zp27p18 + Zp28p18 = 1;
Zp1p19 + Zp2p19 + Zp3p19 + Zp4p19 + Zp5p19 + Zp6p19 + Zp7p19 + Zp8p19 + Zp9p19 + Zp10p19 + Zp11p19 + Zp12p19 + Zp13p19 + Zp14p19 + Zp15p19 + Zp16p19 + Zp17p19 + Zp18p19 + Zp19p19 + Zp20p19 + Zp21p19 + Zp22p19 + Zp23p19 + Zp24p19 + Zp25p19 + Zp26p19 + Zp27p19 + Zp28p19 = 1;
Zp1p20 + Zp2p20 + Zp3p20 + Zp4p20 + Zp5p20 + Zp6p20 + Zp7p20 + Zp8p20 + Zp9p20 + Zp10p20 + Zp11p20 + Zp12p20 + Zp13p20 + Zp14p20 + Zp15p20 + Zp16p20 + Zp17p20 + Zp18p20 + Zp19p20 + Zp20p20 + Zp21p20 + Zp22p20 + Zp23p20 + Zp24p20 + Zp25p20 + Zp26p20 + Zp27p20 + Zp28p20 = 1;
Zp1p21 + Zp2p21 + Zp3p21 + Zp4p21 + Zp5p21 + Zp6p21 + Zp7p21 + Zp8p21 + Zp9p21 + Zp10p21 + Zp11p21 + Zp12p21 + Zp13p21 + Zp14p21 + Zp15p21 + Zp16p21 + Zp17p21 + Zp18p21 + Zp19p21 + Zp20p21 + Zp21p21 + Zp22p21 + Zp23p21 + Zp24p21 + Zp25p21 + Zp26p21 + Zp27p21 + Zp28p21 = 1;
Zp1p22 + Zp2p22 + Zp3p22 + Zp4p22 + Zp5p22 + Zp6p22 + Zp7p22 + Zp8p22 + Zp9p22 + Zp10p22 + Zp11p22 + Zp12p22 + Zp13p22 + Zp14p22 + Zp15p22 + Zp16p22 + Zp17p22 + Zp18p22 + Zp19p22 + Zp20p22 + Zp21p22 + Zp22p22 + Zp23p22 + Zp24p22 + Zp25p22 + Zp26p22 + Zp27p22 + Zp28p22 = 1;
Zp1p23 + Zp2p23 + Zp3p23 + Zp4p23 + Zp5p23 + Zp6p23 + Zp7p23 + Zp8p23 + Zp9p23 + Zp10p23 + Zp11p23 + Zp12p23 + Zp13p23 + Zp14p23 + Zp15p23 + Zp16p23 + Zp17p23 + Zp18p23 + Zp19p23 + Zp20p23 + Zp21p23 + Zp22p23 + Zp23p23 + Zp24p23 + Zp25p23 + Zp26p23 + Zp27p23 + Zp28p23 = 1;
Zp1p24 + Zp2p24 + Zp3p24 + Zp4p24 + Zp5p24 + Zp6p24 + Zp7p24 + Zp8p24 + Zp9p24 + Zp10p24 + Zp11p24 + Zp12p24 + Zp13p24 + Zp14p24 + Zp15p24 + Zp16p24 + Zp17p24 + Zp18p24 + Zp19p24 + Zp20p24 + Zp21p24 + Zp22p24 + Zp23p24 + Zp24p24 + Zp25p24 + Zp26p24 + Zp27p24 + Zp28p24 = 1;
Zp1p25 + Zp2p25 + Zp3p25 + Zp4p25 + Zp5p25 + Zp6p25 + Zp7p25 + Zp8p25 + Zp9p25 + Zp10p25 + Zp11p25 + Zp12p25 + Zp13p25 + Zp14p25 + Zp15p25 + Zp16p25 + Zp17p25 + Zp18p25 + Zp19p25 + Zp20p25 + Zp21p25 + Zp22p25 + Zp23p25 + Zp24p25 + Zp25p25 + Zp26p25 + Zp27p25 + Zp28p25 = 1;
Zp1p26 + Zp2p26 + Zp3p26 + Zp4p26 + Zp5p26 + Zp6p26 + Zp7p26 + Zp8p26 + Zp9p26 + Zp10p26 + Zp11p26 + Zp12p26 + Zp13p26 + Zp14p26 + Zp15p26 + Zp16p26 + Zp17p26 + Zp18p26 + Zp19p26 + Zp20p26 + Zp21p26 + Zp22p26 + Zp23p26 + Zp24p26 + Zp25p26 + Zp26p26 + Zp27p26 + Zp28p26 = 1;
Zp1p27 + Zp2p27 + Zp3p27 + Zp4p27 + Zp5p27 + Zp6p27 + Zp7p27 + Zp8p27 + Zp9p27 + Zp10p27 + Zp11p27 + Zp12p27 + Zp13p27 + Zp14p27 + Zp15p27 + Zp16p27 + Zp17p27 + Zp18p27 + Zp19p27 + Zp20p27 + Zp21p27 + Zp22p27 + Zp23p27 + Zp24p27 + Zp25p27 + Zp26p27 + Zp27p27 + Zp28p27 = 1;
Zp1p28 + Zp2p28 + Zp3p28 + Zp4p28 + Zp5p28 + Zp6p28 + Zp7p28 + Zp8p28 + Zp9p28 + Zp10p28 + Zp11p28 + Zp12p28 + Zp13p28 + Zp14p28 + Zp15p28 + Zp16p28 + Zp17p28 + Zp18p28 + Zp19p28 + Zp20p28 + Zp21p28 + Zp22p28 + Zp23p28 + Zp24p28 + Zp25p28 + Zp26p28 + Zp27p28 + Zp28p28 = 1;

Constraint (10)

Zp1p1-y1< 0;
Zp1p2-y1< 0;
Zp1p3-y1< 0;
Zp1p4-y1< 0;
Zp1p5-y1< 0;
Zp1p6-y1< 0;
Zp1p7-y1< 0;
Zp1p8-y1< 0;
Zp1p9-y1< 0;
Zp1p10-y1< 0;
Zp1p11-y1< 0;
Zp1p12-y1< 0;
Zp1p13-y1< 0;
Zp1p14-y1< 0;
Zp1p15-y1< 0;
Zp1p16-y1< 0;
Zp1p17-y1< 0;
Zp1p18-y1< 0;
Zp1p19-y1< 0;
Zp1p20-y1< 0;
Zp1p21-y1< 0;
Zp1p22-y1< 0;
Zp1p23-y1< 0;
Zp1p24-y1< 0;
Zp1p25-y1< 0;
Zp1p26-y1< 0;
Zp1p27-y1< 0;
Zp1p28-y1< 0;
Zp2p1-y2< 0;
Zp2p2-y2< 0;
Zp2p3-y2< 0;
Zp2p4-y2< 0;
Zp2p5-y2< 0;
Zp2p6-y2< 0;
Zp2p7-y2< 0;
Zp2p8-y2< 0;
Zp2p9-y2< 0;
Zp2p10-y2< 0;
Zp2p11-y2< 0;
Zp2p12-y2< 0;
Zp2p13-y2< 0;
Zp2p14-y2< 0;
Zp2p15-y2< 0;
Zp2p16-y2< 0;
Zp2p17-y2< 0;
Zp2p18-y2< 0;
Zp2p19-y2< 0;
Zp2p20-y2< 0;

Zp2p21-y2< 0;
Zp2p22-y2< 0;
Zp2p23-y2< 0;
Zp2p24-y2< 0;
Zp2p25-y2< 0;
Zp2p26-y2< 0;
Zp2p27-y2< 0;
Zp2p28-y2< 0;
Zp3p1-y3< 0;
Zp3p2-y3< 0;
Zp3p3-y3< 0;
Zp3p4-y3< 0;
Zp3p5-y3< 0;
Zp3p6-y3< 0;
Zp3p7-y3< 0;
Zp3p8-y3< 0;
Zp3p9-y3< 0;
Zp3p10-y3< 0;
Zp3p11-y3< 0;
Zp3p12-y3< 0;
Zp3p13-y3< 0;
Zp3p14-y3< 0;
Zp3p15-y3< 0;
Zp3p16-y3< 0;
Zp3p17-y3< 0;
Zp3p18-y3< 0;
Zp3p19-y3< 0;
Zp3p20-y3< 0;
Zp3p21-y3< 0;
Zp3p22-y3< 0;
Zp3p23-y3< 0;
Zp3p24-y3< 0;
Zp3p25-y3< 0;
Zp3p26-y3< 0;
Zp3p27-y3< 0;
Zp3p28-y3< 0;
Zp4p1-y4< 0;
Zp4p2-y4< 0;
Zp4p3-y4< 0;
Zp4p4-y4< 0;
Zp4p5-y4< 0;
Zp4p6-y4< 0;
Zp4p7-y4< 0;
Zp4p8-y4< 0;
Zp4p9-y4< 0;
Zp4p10-y4< 0;
Zp4p11-y4< 0;
Zp4p12-y4< 0;
Zp4p13-y4< 0;
Zp4p14-y4< 0;
Zp4p15-y4< 0;
Zp4p16-y4< 0;
Zp4p17-y4< 0;
Zp4p18-y4< 0;
Zp4p19-y4< 0;
Zp4p20-y4< 0;
Zp4p21-y4< 0;
Zp4p22-y4< 0;
Zp4p23-y4< 0;
Zp4p24-y4< 0;
Zp4p25-y4< 0;
Zp4p26-y4< 0;
Zp4p27-y4< 0;
Zp4p28-y4< 0;
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Zp5p2-y5< 0;
Zp5p3-y5< 0;
Zp5p4-y5< 0;
Zp5p5-y5< 0;
Zp5p6-y5< 0;
Zp5p7-y5< 0;
Zp5p8-y5< 0;
Zp5p9-y5< 0;
Zp5p10-y5< 0;
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Zp5p12-y5< 0;
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Zp5p16-y5< 0;
Zp5p17-y5< 0;
Zp5p18-y5< 0;
Zp5p19-y5< 0;
Zp5p20-y5< 0;
Zp5p21-y5< 0;
Zp5p22-y5< 0;
Zp5p23-y5< 0;
Zp5p24-y5< 0;
Zp5p25-y5< 0;
Zp5p26-y5< 0;
Zp5p27-y5< 0;
Zp5p28-y5< 0;
Zp6p1-y6< 0;
Zp6p2-y6< 0;
Zp6p3-y6< 0;
Zp6p4-y6< 0;
Zp6p5-y6< 0;
Zp6p6-y6< 0;
Zp6p7-y6< 0;
Zp6p8-y6< 0;
Zp6p9-y6< 0;
Zp6p10-y6< 0;
Zp6p11-y6< 0;
Zp6p12-y6< 0;
Zp6p13-y6< 0;
Zp6p14-y6< 0;
Zp6p15-y6< 0;
Zp6p16-y6< 0;
Zp6p17-y6< 0;
Zp6p18-y6< 0;
Zp6p19-y6< 0;
Zp6p20-y6< 0;
Zp6p21-y6< 0;
Zp6p22-y6< 0;
Zp6p23-y6< 0;
Zp6p24-y6< 0;
Zp6p25-y6< 0;
Zp6p26-y6< 0;
Zp6p27-y6< 0;
Zp6p28-y6< 0;
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Zp7p2-y7< 0;
Zp7p3-y7< 0;
Zp7p4-y7< 0;
Zp7p5-y7< 0;
Zp7p6-y7< 0;
Zp7p7-y7< 0;
Zp7p8-y7< 0;
Zp7p9-y7< 0;
Zp7p10-y7< 0;
Zp7p11-y7< 0;
Zp7p12-y7< 0;
Zp7p13-y7< 0;
Zp7p14-y7< 0;
Zp7p15-y7< 0;
Zp7p16-y7< 0;
Zp7p17-y7< 0;
Zp7p18-y7< 0;
Zp7p19-y7< 0;
Zp7p20-y7< 0;
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Zp7p28-y7< 0;
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Zp8p9-y8< 0;
Zp8p10-y8< 0;
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Zp9p14-y9< 0;
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Zp9p16-y9< 0;
Zp9p17-y9< 0;
Zp9p18-y9< 0;
Zp9p19-y9< 0;
Zp9p20-y9< 0;
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Zp9p25-y9< 0;
Zp9p26-y9< 0;
Zp9p27-y9< 0;
Zp9p28-y9< 0;
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Zp10p4-y10< 0;
Zp10p5-y10< 0;
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Zp10p8-y10< 0;
Zp10p9-y10< 0;
Zp10p10-y10< 0;
Zp10p11-y10< 0;
Zp10p12-y10< 0;
Zp10p13-y10< 0;
Zp10p14-y10< 0;
Zp10p15-y10< 0;
Zp10p16-y10< 0;
Zp10p17-y10< 0;
Zp10p18-y10< 0;
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Zp11p3-y11< 0;
Zp11p4-y11< 0;
Zp11p5-y11< 0;
Zp11p6-y11< 0;
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Zp11p8-y11< 0;
Zp11p9-y11< 0;
Zp11p10-y11< 0;
Zp11p11-y11< 0;
Zp11p12-y11< 0;
Zp11p13-y11< 0;
Zp11p14-y11< 0;
Zp11p15-y11< 0;
Zp11p16-y11< 0;
Zp11p17-y11< 0;
Zp11p18-y11< 0;
Zp11p19-y11< 0;
Zp11p20-y11< 0;
Zp11p21-y11< 0;
Zp11p22-y11< 0;
Zp11p23-y11< 0;
Zp11p24-y11< 0;
Zp11p25-y11< 0;
Zp11p26-y11< 0;
Zp11p27-y11< 0;
Zp11p28-y11< 0;
Zp12p1-y12< 0;
Zp12p2-y12< 0;
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Constraint (11)

y1+ y2 + y3 + y4 + y5 + y6 + y7 + y8 + y9 + y10 + y11 + y12 + y13 + y14 + y15 + y16 + y17 + y18 + y19 + y20 + y21 + y22+ y23 + y24 + y25 + y26 + y27 + y28 = 3;

Constraint (12)

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end
```

- **Lingo Output**

```
Rows=     814 Vars=     812 No. integer vars=     784  ( all are linear)
Nonzeros=    3113 Constraint nonz=  2380(  2380 are +- 1) Density=0.005
Smallest and largest elements in abs value=    1.00000          1061.58
No. < : 784 No. =:  29 No. > .   0, Obj=MIN, GUBs <= 112
Single cols=    0

Optimal solution found at step:        315
```

Objective value: 1769.224
Branch count: 0

Variable	Value	Reduced Cost
ZP1P2	0.0000000	1025.671
ZP1P3	0.0000000	1020.906
ZP1P4	1.000000	129.3010
ZP1P5	1.000000	100.0000
ZP1P6	1.000000	100.0000
ZP1P7	1.000000	214.4760
ZP1P8	0.0000000	1031.572
ZP1P9	1.000000	100.0000
ZP1P10	0.0000000	1011.187
ZP1P11	0.0000000	1011.187
ZP1P12	1.000000	100.0000
ZP1P13	0.0000000	1031.504
ZP1P14	1.000000	44.72100
ZP1P15	0.0000000	1007.968
ZP1P16	1.000000	126.4910
ZP1P17	0.0000000	1019.191
ZP1P18	0.0000000	1014.088
ZP1P19	0.0000000	1012.917
ZP1P21	1.000000	125.0000
ZP1P22	0.0000000	1031.988
ZP1P23	0.0000000	1031.988
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ZP1P27	0.0000000	1031.988
ZP1P28	0.0000000	1031.988
ZP2P1	0.0000000	1025.671
ZP2P3	0.0000000	208.5670
ZP2P4	0.0000000	1008.015
ZP2P5	0.0000000	1044.031
ZP2P6	0.0000000	1019.804
ZP2P7	0.0000000	1014.889
ZP2P8	0.0000000	192.2320
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ZP2P18	0.0000000	100.6230
ZP2P19	0.0000000	126.4910
ZP2P20	0.0000000	1025.671
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ZP2P27	0.0000000	187.0830
ZP2P28	0.0000000	187.0830
ZP3P1	0.0000000	1020.906
ZP3P2	0.0000000	208.5670
ZP3P4	0.0000000	1005.889
ZP3P5	0.0000000	1046.542
ZP3P6	0.0000000	1005.609
ZP3P7	0.0000000	1008.092
ZP3P8	0.0000000	96.09600
ZP3P9	0.0000000	1046.542
ZP3P10	0.0000000	63.24600
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ZP3P13	0.0000000	106.0660
ZP3P14	0.0000000	1025.183

ZP3P15	0.0000000	183.7120
ZP3P16	0.0000000	1016.735
ZP3P17	0.0000000	137.8400
ZP3P18	0.0000000	140.9790
ZP3P19	0.0000000	132.2880
ZP3P20	0.0000000	1020.906
ZP3P21	0.0000000	1009.765
ZP3P22	0.0000000	137.8400
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ZP3P27	0.0000000	137.8400
ZP3P28	0.0000000	137.8400
ZP4P1	0.0000000	129.3010
ZP4P2	0.0000000	1008.015
ZP4P3	0.0000000	1005.889
ZP4P5	0.0000000	225.3470
ZP4P6	0.0000000	71.80700
ZP4P7	0.0000000	108.5410
ZP4P8	0.0000000	1008.751
ZP4P9	0.0000000	225.3470
ZP4P10	0.0000000	1003.899
ZP4P11	0.0000000	1003.899
ZP4P12	0.0000000	225.3470
ZP4P13	0.0000000	1008.820
ZP4P14	0.0000000	129.3010
ZP4P15	0.0000000	1002.575
ZP4P16	0.0000000	71.80700
ZP4P17	0.0000000	1003.058
ZP4P18	0.0000000	1001.624
ZP4P19	0.0000000	1001.608
ZP4P20	0.0000000	129.3010
ZP4P21	0.0000000	62.50000
ZP4P22	0.0000000	1009.718
ZP4P23	0.0000000	1009.718
ZP4P24	0.0000000	1009.718
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ZP4P26	0.0000000	1009.718
ZP4P27	0.0000000	1009.718
ZP4P28	0.0000000	1009.718
ZP5P1	0.0000000	100.0000
ZP5P2	0.0000000	1044.031
ZP5P3	0.0000000	1046.542
ZP5P4	0.0000000	225.3470
ZP5P6	0.0000000	200.0000
ZP5P7	0.0000000	309.8390
ZP5P8	0.0000000	1061.580
ZP5P10	0.0000000	1030.776
ZP5P11	0.0000000	1030.776
ZP5P13	0.0000000	1061.131
ZP5P14	0.0000000	100.0000
ZP5P15	0.0000000	1019.804
ZP5P16	0.0000000	200.0000
ZP5P17	0.0000000	1040.673
ZP5P18	0.0000000	1032.654
ZP5P19	0.0000000	1031.504
ZP5P20	0.0000000	100.0000
ZP5P21	0.0000000	216.5060
ZP5P22	0.0000000	1060.660
ZP5P23	0.0000000	1060.660
ZP5P24	0.0000000	1060.660
ZP5P25	0.0000000	1060.660
ZP5P26	0.0000000	1060.660
ZP5P27	0.0000000	1060.660
ZP5P28	0.0000000	1060.660
ZP6P1	0.0000000	100.0000
ZP6P2	0.0000000	1019.804
ZP6P3	0.0000000	1005.609
ZP6P4	0.0000000	71.80700
ZP6P5	0.0000000	200.0000

ZP6P7	0.0000000	154.9190
ZP6P8	0.0000000	1014.312
ZP6P9	0.0000000	200.0000
ZP6P10	0.0000000	1001.249
ZP6P11	0.0000000	1001.249
ZP6P12	0.0000000	200.0000
ZP6P13	0.0000000	1014.889
ZP6P14	0.0000000	126.4910
ZP6P15	0.0000000	1007.968
ZP6P16	0.0000000	126.4910
ZP6P17	0.0000000	1010.693
ZP6P18	0.0000000	1008.154
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ZP6P20	0.0000000	100.0000
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ZP7P12	0.0000000	309.8390
ZP7P13	0.0000000	1001.000
ZP7P14	0.0000000	214.4760
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ZP7P17	0.0000000	1001.998
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ZP7P19	0.0000000	1001.998
ZP7P20	0.0000000	214.4760
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ZP7P23	0.0000000	1000.500
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ZP8P9	0.0000000	1061.580
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ZP8P12	0.0000000	1061.580
ZP8P13	0.0000000	11.85900
ZP8P14	0.0000000	1032.934
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ZP8P21	0.0000000	1008.751
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ZP9P4	0.0000000	225.3470
ZP9P6	0.0000000	200.0000
ZP9P7	0.0000000	309.8390
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ZP9P10	0.0000000	1030.776
ZP9P11	0.0000000	1030.776
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ZP9P14	0.0000000	100.0000
ZP9P15	0.0000000	1019.804
ZP9P16	0.0000000	200.0000
ZP9P17	0.0000000	1040.673
ZP9P18	0.0000000	1032.654
ZP9P19	0.0000000	1031.504
ZP9P20	0.0000000	100.0000
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ZP9P22	0.0000000	1060.660
ZP9P23	0.0000000	1060.660
ZP9P24	0.0000000	1060.660
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ZP10P1	0.0000000	1011.187
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ZP10P4	0.0000000	1003.899
ZP10P5	0.0000000	1030.776
ZP10P6	0.0000000	1001.249
ZP10P7	0.0000000	1011.682
ZP10P8	0.0000000	146.5750
ZP10P9	0.0000000	1030.776
ZP10P12	0.0000000	1030.776
ZP10P13	0.0000000	153.2970
ZP10P14	0.0000000	1015.505
ZP10P15	0.0000000	158.1140
ZP10P16	0.0000000	1012.423
ZP10P17	0.0000000	151.6580
ZP10P18	0.0000000	141.8630
ZP10P19	0.0000000	133.2290
ZP10P20	0.0000000	1011.187
ZP10P21	0.0000000	1007.782
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ZP10P23	0.0000000	176.7770
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ZP10P27	0.0000000	176.7770
ZP10P28	0.0000000	176.7770
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ZP11P6	0.0000000	1001.249
ZP11P7	0.0000000	1011.682
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ZP11P9	0.0000000	1030.776
ZP11P12	0.0000000	1030.776
ZP11P13	0.0000000	153.2970
ZP11P14	0.0000000	1015.505
ZP11P15	0.0000000	158.1140
ZP11P16	0.0000000	1012.423
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ZP12P2	0.0000000	1044.031
ZP12P3	0.0000000	1046.542
ZP12P4	0.0000000	225.3470
ZP12P6	0.0000000	200.0000
ZP12P7	0.0000000	309.8390
ZP12P8	0.0000000	1061.580
ZP12P10	0.0000000	1030.776
ZP12P11	0.0000000	1030.776
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ZP12P14	0.0000000	100.0000
ZP12P15	0.0000000	1019.804
ZP12P16	0.0000000	200.0000
ZP12P17	0.0000000	1040.673
ZP12P18	0.0000000	1032.654
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ZP12P22	0.0000000	1060.660
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ZP13P1	0.0000000	1031.504
ZP13P2	0.0000000	189.7370
ZP13P3	0.0000000	106.0660
ZP13P4	0.0000000	1008.820
ZP13P5	0.0000000	1061.131
ZP13P6	0.0000000	1014.889
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ZP13P10	0.0000000	153.2970
ZP13P11	0.0000000	153.2970
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ZP13P16	0.0000000	1017.841
ZP13P17	0.0000000	88.03400
ZP13P18	0.0000000	111.2430
ZP13P19	0.0000000	100.0000
ZP13P20	0.0000000	1031.504
ZP13P21	0.0000000	1008.278
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ZP13P23	0.0000000	31.62300
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ZP13P26	0.0000000	31.62300
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ZP14P1	0.0000000	44.72100
ZP14P2	0.0000000	1019.804
ZP14P3	0.0000000	1025.183
ZP14P4	0.0000000	129.3010
ZP14P5	0.0000000	100.0000
ZP14P6	0.0000000	126.4910
ZP14P7	0.0000000	214.4760
ZP14P8	0.0000000	1032.934

ZP14P9	0.0000000	100.0000
ZP14P10	0.0000000	1015.505
ZP14P11	0.0000000	1015.505
ZP14P12	0.0000000	100.0000
ZP14P13	0.0000000	1032.473
ZP14P15	0.0000000	1004.988
ZP14P16	0.0000000	100.0000
ZP14P17	0.0000000	1017.349
ZP14P18	0.0000000	1012.238
ZP14P19	0.0000000	1011.929
ZP14P20	0.0000000	44.72100
ZP14P21	0.0000000	125.0000
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ZP15P1	0.0000000	1007.968
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ZP15P3	0.0000000	183.7120
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ZP15P5	0.0000000	1019.804
ZP15P6	0.0000000	1007.968
ZP15P7	0.0000000	1011.929
ZP15P8	0.0000000	192.2320
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ZP15P17	0.0000000	104.8810
ZP15P18	0.0000000	76.64900
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ZP15P20	0.0000000	1007.968
ZP15P21	0.0000000	1004.677
ZP15P22	0.0000000	187.0830
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ZP16P1	0.0000000	126.4910
ZP16P2	0.0000000	1004.988
ZP16P3	0.0000000	1016.735
ZP16P4	0.0000000	71.80700
ZP16P5	0.0000000	200.0000
ZP16P6	0.0000000	126.4910
ZP16P7	0.0000000	154.9190
ZP16P8	0.0000000	1018.309
ZP16P9	0.0000000	200.0000
ZP16P10	0.0000000	1012.423
ZP16P11	0.0000000	1012.423
ZP16P12	0.0000000	200.0000
ZP16P13	0.0000000	1017.841
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ZP16P15	0.0000000	1000.000
ZP16P17	0.0000000	1005.485
ZP16P18	0.0000000	1002.933
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ZP16P27	0.0000000	1017.349
ZP16P28	0.0000000	1017.349
ZP17P1	0.0000000	1019.191
ZP17P2	0.0000000	104.8810
ZP17P3	0.0000000	137.8400
ZP17P4	0.0000000	1003.058
ZP17P5	0.0000000	1040.673
ZP17P6	0.0000000	1010.693
ZP17P7	0.0000000	1001.998
ZP17P8	0.0000000	93.29100
ZP17P9	0.0000000	1040.673
ZP17P10	0.0000000	151.6580
ZP17P11	0.0000000	151.6580
ZP17P12	0.0000000	1040.673
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ZP17P14	0.0000000	1017.349
ZP17P15	0.0000000	104.8810
ZP17P16	0.0000000	1005.485
ZP17P18	0.0000000	29.58000
ZP17P19	0.0000000	38.73000
ZP17P20	0.0000000	1019.191
ZP17P21	0.0000000	1003.681
ZP17P22	0.0000000	82.15800
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ZP17P27	0.0000000	82.15800
ZP17P28	0.0000000	82.15800
ZP18P1	0.0000000	1014.088
ZP18P2	1.000000	100.6230
ZP18P3	0.0000000	140.9790
ZP18P4	0.0000000	1001.624
ZP18P5	0.0000000	1032.654
ZP18P6	0.0000000	1008.154
ZP18P7	0.0000000	1003.182
ZP18P8	0.0000000	115.4470
ZP18P9	0.0000000	1032.654
ZP18P10	1.000000	141.8630
ZP18P11	1.000000	141.8630
ZP18P12	0.0000000	1032.654
ZP18P13	0.0000000	111.2430
ZP18P14	0.0000000	1012.238
ZP18P15	1.000000	76.64900
ZP18P16	0.0000000	1002.933
ZP18P17	1.000000	29.58000
ZP18P19	1.000000	25.00000
ZP18P20	0.0000000	1014.088
ZP18P21	0.0000000	1002.247
ZP18P22	0.0000000	106.6540
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ZP18P26	0.0000000	106.6540
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ZP19P2	0.0000000	126.4910
ZP19P3	0.0000000	132.2880
ZP19P4	0.0000000	1001.608
ZP19P5	0.0000000	1031.504
ZP19P6	0.0000000	1006.976
ZP19P7	0.0000000	1001.998
ZP19P8	0.0000000	104.6570
ZP19P9	0.0000000	1031.504
ZP19P10	0.0000000	133.2290
ZP19P11	0.0000000	133.2290
ZP19P12	0.0000000	1031.504
ZP19P13	0.0000000	100.0000
ZP19P14	0.0000000	1011.929
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ZP19P17	0.0000000	38.73000
ZP19P18	0.0000000	25.00000
ZP19P20	0.0000000	1012.917
ZP19P21	0.0000000	1001.062
ZP19P22	0.0000000	94.86800
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ZP19P26	0.0000000	94.86800
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ZP19P28	0.0000000	94.86800
ZP20P2	0.0000000	1025.671
ZP20P3	0.0000000	1020.906
ZP20P4	0.0000000	129.3010
ZP20P5	0.0000000	100.0000
ZP20P6	0.0000000	100.0000
ZP20P7	0.0000000	214.4760
ZP20P8	0.0000000	1031.572
ZP20P9	0.0000000	100.0000
ZP20P10	0.0000000	1011.187
ZP20P11	0.0000000	1011.187
ZP20P12	0.0000000	100.0000
ZP20P13	0.0000000	1031.504
ZP20P14	0.0000000	44.72100
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ZP20P16	0.0000000	126.4910
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ZP20P18	0.0000000	1014.088
ZP20P19	0.0000000	1012.917
ZP20P21	0.0000000	125.0000
ZP20P22	0.0000000	1031.988
ZP20P23	0.0000000	1031.988
ZP20P24	0.0000000	1031.988
ZP20P25	0.0000000	1031.988
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ZP20P27	0.0000000	1031.988
ZP20P28	0.0000000	1031.988
ZP21P1	0.0000000	125.0000
ZP21P2	0.0000000	1013.965
ZP21P3	0.0000000	1009.765
ZP21P4	0.0000000	62.50000
ZP21P5	0.0000000	216.5060
ZP21P6	0.0000000	96.82500
ZP21P7	0.0000000	88.74100
ZP21P8	0.0000000	1008.751
ZP21P9	0.0000000	216.5060
ZP21P10	0.0000000	1007.782
ZP21P11	0.0000000	1007.782
ZP21P12	0.0000000	216.5060
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ZP21P20	0.0000000	125.0000
ZP21P22	0.0000000	1007.782
ZP21P23	0.0000000	1007.782
ZP21P24	0.0000000	1007.782
ZP21P25	0.0000000	1007.782
ZP21P26	0.0000000	1007.782
ZP21P27	0.0000000	1007.782
ZP21P28	0.0000000	1007.782
ZP22P1	0.0000000	1031.988
ZP22P2	0.0000000	187.0830
ZP22P3	0.0000000	137.8400
ZP22P4	0.0000000	1009.718
ZP22P5	0.0000000	1060.660
ZP22P6	0.0000000	1017.349

ZP22P7	0.0000000	1000.500
ZP22P8	0.0000000	44.19400
ZP22P9	0.0000000	1060.660
ZP22P10	0.0000000	176.7770
ZP22P11	0.0000000	176.7770
ZP22P12	0.0000000	1060.660
ZP22P13	0.0000000	31.62300
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ZP22P15	0.0000000	187.0830
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ZP22P17	0.0000000	82.15800
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ZP22P19	0.0000000	94.86800
ZP22P20	0.0000000	1031.988
ZP22P21	0.0000000	1007.782
ZP23P1	0.0000000	1031.988
ZP23P2	0.0000000	187.0830
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ZP23P4	0.0000000	1009.718
ZP23P5	0.0000000	1060.660
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ZP23P8	1.000000	44.19400
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ZP23P10	0.0000000	176.7770
ZP23P11	0.0000000	176.7770
ZP23P12	0.0000000	1060.660
ZP23P13	1.000000	31.62300
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ZP23P15	0.0000000	187.0830
ZP23P16	0.0000000	1017.349
ZP23P17	0.0000000	82.15800
ZP23P18	0.0000000	106.6540
ZP23P19	0.0000000	94.86800
ZP23P20	0.0000000	1031.988
ZP23P21	0.0000000	1007.782
ZP24P1	0.0000000	1031.988
ZP24P2	0.0000000	187.0830
ZP24P3	0.0000000	137.8400
ZP24P4	0.0000000	1009.718
ZP24P5	0.0000000	1060.660
ZP24P6	0.0000000	1017.349
ZP24P7	0.0000000	1000.500
ZP24P8	0.0000000	44.19400
ZP24P9	0.0000000	1060.660
ZP24P10	0.0000000	176.7770
ZP24P11	0.0000000	176.7770
ZP24P12	0.0000000	1060.660
ZP24P13	0.0000000	31.62300
ZP24P14	0.0000000	1031.988
ZP24P15	0.0000000	187.0830
ZP24P16	0.0000000	1017.349
ZP24P17	0.0000000	82.15800
ZP24P18	0.0000000	106.6540
ZP24P19	0.0000000	94.86800
ZP24P20	0.0000000	1031.988
ZP24P21	0.0000000	1007.782
ZP25P1	0.0000000	1031.988
ZP25P2	0.0000000	187.0830
ZP25P3	0.0000000	137.8400
ZP25P4	0.0000000	1009.718
ZP25P5	0.0000000	1060.660
ZP25P6	0.0000000	1017.349
ZP25P7	0.0000000	1000.500
ZP25P8	0.0000000	44.19400
ZP25P9	0.0000000	1060.660
ZP25P10	0.0000000	176.7770
ZP25P11	0.0000000	176.7770
ZP25P12	0.0000000	1060.660
ZP25P13	0.0000000	31.62300
ZP25P14	0.0000000	1031.988

ZP25P15	0.0000000	187.0830
ZP25P16	0.0000000	1017.349
ZP25P17	0.0000000	82.15800
ZP25P18	0.0000000	106.6540
ZP25P19	0.0000000	94.86800
ZP25P20	0.0000000	1031.988
ZP25P21	0.0000000	1007.782
ZP26P1	0.0000000	1031.988
ZP26P2	0.0000000	187.0830
ZP26P3	0.0000000	137.8400
ZP26P4	0.0000000	1009.718
ZP26P5	0.0000000	1060.660
ZP26P6	0.0000000	1017.349
ZP26P7	0.0000000	1000.500
ZP26P8	0.0000000	44.19400
ZP26P9	0.0000000	1060.660
ZP26P10	0.0000000	176.7770
ZP26P11	0.0000000	176.7770
ZP26P12	0.0000000	1060.660
ZP26P13	0.0000000	31.62300
ZP26P14	0.0000000	1031.988
ZP26P15	0.0000000	187.0830
ZP26P16	0.0000000	1017.349
ZP26P17	0.0000000	82.15800
ZP26P18	0.0000000	106.6540
ZP26P19	0.0000000	94.86800
ZP26P20	0.0000000	1031.988
ZP26P21	0.0000000	1007.782
ZP27P1	0.0000000	1031.988
ZP27P2	0.0000000	187.0830
ZP27P3	0.0000000	137.8400
ZP27P4	0.0000000	1009.718
ZP27P5	0.0000000	1060.660
ZP27P6	0.0000000	1017.349
ZP27P7	0.0000000	1000.500
ZP27P8	0.0000000	44.19400
ZP27P9	0.0000000	1060.660
ZP27P10	0.0000000	176.7770
ZP27P11	0.0000000	176.7770
ZP27P12	0.0000000	1060.660
ZP27P13	0.0000000	31.62300
ZP27P14	0.0000000	1031.988
ZP27P15	0.0000000	187.0830
ZP27P16	0.0000000	1017.349
ZP27P17	0.0000000	82.15800
ZP27P18	0.0000000	106.6540
ZP27P19	0.0000000	94.86800
ZP27P20	0.0000000	1031.988
ZP27P21	0.0000000	1007.782
ZP28P1	0.0000000	1031.988
ZP28P2	0.0000000	187.0830
ZP28P3	0.0000000	137.8400
ZP28P4	0.0000000	1009.718
ZP28P5	0.0000000	1060.660
ZP28P6	0.0000000	1017.349
ZP28P7	0.0000000	1000.500
ZP28P8	0.0000000	44.19400
ZP28P9	0.0000000	1060.660
ZP28P10	0.0000000	176.7770
ZP28P11	0.0000000	176.7770
ZP28P12	0.0000000	1060.660
ZP28P13	0.0000000	31.62300
ZP28P14	0.0000000	1031.988
ZP28P15	0.0000000	187.0830
ZP28P16	0.0000000	1017.349
ZP28P17	0.0000000	82.15800
ZP28P18	0.0000000	106.6540
ZP28P19	0.0000000	94.86800
ZP28P20	0.0000000	1031.988
ZP28P21	0.0000000	1007.782
ZP1P1	1.000000	0.0000000

ZP20P1	0.0000000	0.0000000
ZP2P2	0.0000000	0.0000000
ZP3P3	0.0000000	0.0000000
ZP4P4	0.0000000	0.0000000
ZP5P5	0.0000000	0.0000000
ZP9P5	0.0000000	0.0000000
ZP12P5	0.0000000	0.0000000
ZP6P6	0.0000000	0.0000000
ZP7P7	0.0000000	0.0000000
ZP8P8	0.0000000	0.0000000
ZP5P9	0.0000000	0.0000000
ZP9P9	0.0000000	0.0000000
ZP12P9	0.0000000	0.0000000
ZP10P10	0.0000000	0.0000000
ZP11P10	0.0000000	0.0000000
ZP10P11	0.0000000	0.0000000
ZP11P11	0.0000000	0.0000000
ZP5P12	0.0000000	0.0000000
ZP9P12	0.0000000	0.0000000
ZP12P12	0.0000000	0.0000000
ZP13P13	0.0000000	0.0000000
ZP14P14	0.0000000	0.0000000
ZP15P15	0.0000000	0.0000000
ZP16P16	0.0000000	0.0000000
ZP17P17	0.0000000	0.0000000
ZP18P18	1.000000	0.0000000
ZP19P19	0.0000000	0.0000000
ZP1P20	1.000000	0.0000000
ZP20P20	0.0000000	0.0000000
ZP21P21	0.0000000	0.0000000
ZP22P22	0.0000000	0.0000000
ZP23P22	1.000000	0.0000000
ZP24P22	0.0000000	0.0000000
ZP25P22	0.0000000	0.0000000
ZP26P22	0.0000000	0.0000000
ZP27P22	0.0000000	0.0000000
ZP28P22	0.0000000	0.0000000
ZP22P23	0.0000000	0.0000000
ZP23P23	1.000000	0.0000000
ZP24P23	0.0000000	0.0000000
ZP25P23	0.0000000	0.0000000
ZP26P23	0.0000000	0.0000000
ZP27P23	0.0000000	0.0000000
ZP28P23	0.0000000	0.0000000
ZP22P24	0.0000000	0.0000000
ZP23P24	1.000000	0.0000000
ZP24P24	0.0000000	0.0000000
ZP25P24	0.0000000	0.0000000
ZP26P24	0.0000000	0.0000000
ZP27P24	0.0000000	0.0000000
ZP28P24	0.0000000	0.0000000
ZP22P25	0.0000000	0.0000000
ZP23P25	1.000000	0.0000000
ZP24P25	0.0000000	0.0000000
ZP25P25	0.0000000	0.0000000
ZP26P25	0.0000000	0.0000000
ZP27P25	0.0000000	0.0000000
ZP28P25	0.0000000	0.0000000
ZP22P26	0.0000000	0.0000000
ZP23P26	1.000000	0.0000000
ZP24P26	0.0000000	0.0000000
ZP25P26	0.0000000	0.0000000
ZP26P26	0.0000000	0.0000000
ZP27P26	0.0000000	0.0000000
ZP28P26	0.0000000	0.0000000
ZP22P27	0.0000000	0.0000000
ZP23P27	1.000000	0.0000000
ZP24P27	0.0000000	0.0000000
ZP25P27	0.0000000	0.0000000
ZP26P27	0.0000000	0.0000000
ZP27P27	0.0000000	0.0000000

ZP28P27	0.0000000	0.0000000
ZP22P28	0.0000000	0.0000000
ZP23P28	1.000000	0.0000000
ZP24P28	0.0000000	0.0000000
ZP25P28	0.0000000	0.0000000
ZP26P28	0.0000000	0.0000000
ZP27P28	0.0000000	0.0000000
ZP28P28	0.0000000	0.0000000
Y1	1.000000	0.0000000
Y2	0.0000000	0.0000000
Y3	0.0000000	0.0000000
Y4	0.0000000	0.0000000
Y5	0.0000000	0.0000000
Y6	0.0000000	0.0000000
Y7	0.0000000	0.0000000
Y8	0.0000000	0.0000000
Y9	0.0000000	0.0000000
Y10	0.0000000	0.0000000
Y11	0.0000000	0.0000000
Y12	0.0000000	0.0000000
Y13	0.0000000	0.0000000
Y14	0.0000000	0.0000000
Y15	0.0000000	0.0000000
Y16	0.0000000	0.0000000
Y17	0.0000000	0.0000000
Y18	1.000000	0.0000000
Y19	0.0000000	0.0000000
Y20	0.0000000	0.0000000
Y21	0.0000000	0.0000000
Y22	0.0000000	0.0000000
Y23	1.000000	0.0000000
Y24	0.0000000	0.0000000
Y25	0.0000000	0.0000000
Y26	0.0000000	0.0000000
Y27	0.0000000	0.0000000
Y28	0.0000000	0.0000000

Appendix G

Heragu's Methodology

- **Input Data**

The input data file constructed by Heragu is shown in figure G.1 below and illustrates that 3 additional machines are needed: 2 for machine type 02029 and 1 for machine type 02030, which correspond to the columns 6, 7 and 9, respectively. Moreover, because each batch of parts that require these machine types are assumed to visit each of the units in equal proportion, note that column 6 and 7 are identical to column 5 and column 9 is identical to 8. Columns 1,2,3,4,5 and 8 correspond to machines 01204, 02008, 02014, 02023, 02029 and 02030, respectively. Notice that the data file is no longer using part names, but part numbers, as shown in the part-machine processing indicator matrix (figure G.2).

Number of machines	9	
Number of parts	20	
Number of dummy machines	1	
Machine names	1	01204
	2	02008
	3	02014
	4	02023
	5	02029A
	6	02029B
	7	02029C
	8	02030A
	9	02030B
Part names	1	115110
	2	176022
	3	176023
	4	177007
	5	201052
	6	202102-501
	7	202103
	8	203009
	9	209001
	10	210001
	11	217002-2
	12	236235
	13	270071-501
	14	270080-501
	15	280011
	16	280024
	17	30130
	18	324008
	19	325003
	20	328008

Figure G.1 Input Data File for Mediquip Manufacturer Inc. (Heragu, 1991) 326.

Part–Machine Processing Indicator Matrix

	1	2	3	4	5	6	7	8	9	2		
1	1	2									220	10
2	1	2	3								500	10
3	1	2									500	10
4		1	4	2						3	300	10
5		1	2	3							220	10
6				2				1	1		120	10
7		1		2							700	10
8			2		3	3	3	1	1		900	10
9			2		1	1	1				200	10
10		2						1	1		250	10
11			2		1	1	1				400	10
12	[illegible]	2									300	10
13			3		1	1	1	2	2		250	10
14			3		1	1	1	2	2		600	10
15								1	1		450	10
16	1	2									500	10
17				2				1	1		400	10
18				3	1	1	1	2	2		300	10
19		2			1	1	1	2	2		10	10
20				3	1	1	1	2	2		250	10

Figure G.2. Part-Machine Processing Indicator Matrix. (Heragu, 1991) 327.

Figure G.2 shows when parts visit each machine for the first, second, third and fourth operations. Normally, when a part visits the same machine for multiple, nonconsecutive operations, the corresponding entry in the part-machine incidence matrix has to be modified as a vector. In this example, an alternative approach suggested by Heragu and Kakuturi is used (323). Therefore, because part 4 visits machine 2 for the first and third operations, a dummy column is added to indicate that the third operation is done by machine 2. According to Heragu, this dummy column is only used to calculate the flow within machines. This approach is preferred because not only does it allow the capture of nonconsecutive operation information, but does so with minimal changes to the matrix and memory requirements. It is noted also that although part 15 requires two operations on machine 8, it is treated as a single operation because the part stays on the same machine between the two operations. Hence, there isn't material movement that could affect the flow matrix (323-324).

The first matrix in figure G.3 shows the fractional processing matrix used to illustrate the fraction of each part type that visits each unit of each machine type. It was pointed out earlier

that part type 3 visits machine type 1 and 2 also, but the original matrix shown here omitted the visit to machine type 1. This error was corrected when the matrix was reproduced on Table 18.2.

The second matrix in Figure G.3 (called the material-handling cost matrix) shows the cost of handling a unit load through a unit distance between each machine pair. This matrix is based on relative cost obtained from Table G.1 below.

Fractional Processing Matrix

	1	2	3	4	5	6	7	8	9	2
1	1	1								
2	1	1	1							
3		1								
4		1	1	1						1
5		1	1	1						
6				1				0.5	0.5	
7		1		1						
8			1		0.33	0.33	0.33	0.5	0.5	
9			1		0.33	0.33	0.33			
10		1						0.5	0.5	
11			1		0.33	0.33	0.33			
12	1	1								
13			1		0.33	0.33	0.33	0.5	0.5	
14			1		0.33	0.33	0.33	0.5	0.5	
15								0.5	0.5	
16	1	1								
17				1				0.5	0.5	
18				1	0.33	0.33	0.33	0.5	0.5	
19		1			0.33	0.33	0.33	0.5	0.5	
20				1	0.33	0.33	0.33	0.5	0.5	

Material-Handling Cost Matrix

	1	2	3	4	5	6	7	8	9
1	0	2	2	2	2	2	2	2	2
2	1	0	1	1	1	1	1	1	1
3	3	3	0	1	3	3	3	3	3
4	1	1	3	0	2	2	2	3	3
5	1	1	1	1	0	1	1	1	1
6	1	1	1	1	1	0	1	1	1
7	1	1	1	1	1	1	0	1	1
8	2	2	2	2	2	2	2	0	2
9	2	2	2	2	2	2	2	2	0

Figure G.3 Fractional Processing Matrix and Material-Handling Cost Matrix (Heragu, 1997) 328.

Table G.1

Material-Handling Devices Used*

From Machine (s)	To Machine (s)	Device Used
1204	All others	Truck
2008	All others	Manually by cart
2014	All others	AGV
2023	1204, 2008	Manually by cart
2023	2029	Truck
2023	2014, 2030	AGV
2029	All others	Manually by cart
2030	All others	Truck

*Although the exact costs of handling a unit load through a unit distance on each type of material-handling device is difficult to obtain, it is known that the AGV is 3 times as expensive (per load per unit distance) as transporting manually via carts and that transporting via trucks is twice as expensive as transporting manually via carts, for this example.

Source: Heragu, Sunderesh. Facilities Design (1997) Table 9.2.

Finally, Figure 9.4 shows the material relationship indicator matrix, which tells whether or not there are special adjacency restrictions for machine pairs. For instance, a 1 in this matrix indicates that the corresponding pair must be placed in the same cell, whereas a 0 indicates no special adjacency relationship and the machine pair placement is to be determined by the layout algorithm. Notice that there is no special adjacency relationship in this problem. It is also assumed that the maximum number of machines allowed in a cell would be three.

Material Relationship Matrix

	1	2	3	4	5	6	7	8	9
1	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0

Maximum machines in any cell	3
Machine lengths	20 20 15 25 45 45 45 20 20
Machine widths	10 25 10 20 10 10 10 15 15

Figure G.4 Material Relationship Matrix (Heragu, 1997) 329.

- **Output File**

The GTLAYPC.EXE program goes beyond the scope of this thesis in the sense that not only does it identify machine cells (and the corresponding part families) but also their layout and the layout of machines within each cell. It is based on the three-stage approach, starting with machines being grouped into cells and the corresponding parts into part families. Then, in the second stage the layout of machines within the cells is determined and in the third stage the layout of cells themselves is determined.

Since the layout of cells is not the focus in this research, only a brief explanation of the algorithm used in the first stage presented here.

When grouping machines into cells (stage 1), some constraints must be considered. Machine adjacency requirements must be met, the number of machines in each cell cannot exceed a user-specified value and the number of cells cannot exceed a user-specified value. Additional features, including the identification of bottleneck machines, must be incorporated. As explained by Heragu, the algorithm identifies the pair of machines that have the greatest flow and places them together. It then adds the remaining machines one by one to the two machines already placed so that the machine added has the greatest interaction with the machines being placed together until a cell of the required size is formed. This procedure is repeated for the remaining parts and machines (329-330).

The output generated by the GTLAYPC.EXE program is presented in Figure G.5 in the following pages. In addition, Figure G.6 shows the layout constructed from this output. According to Heragu, although some parts do visit multiple cells, the overall intercellular movement is minimized (334).

```
 1. GROUP TECHNOLOGY PROGRAM WILL START NOW
FINAL MACHINE CELLS
 MACHINES IN CELL 1
       MACH. NO.      MACH NAME
           1            01204
           2            02008
           3            02014
 MACHINES IN CELL 2
       MACH. NO.      MACH NAME
           4            02023
           9            02030B
           8            02030A
 MACHINES IN CELL 3
       MACH. NO.      MACH NAME
           5            02029A
           6            02029B
           7            02029C
FINAL PART FAMILIES
 PARTS THAT GO INTO CELL    1ARE
       PART. NO.      PART NAME
           1            115110
           2            176022
           3            176023
          12            236235
          16            280024
           4            177007
           5            201052
           7            202103
          10            210001
          19            325003
           8            203009
           9            209001
          11            217002-3
          13            270071-501
          14            270080-501
 PARTS THAT GO INTO CELL    2ARE
       PART. NO.      PART NAME
           4            177007
           5            201052
           6            202102-501
           7            202103
          17            30130
          18            324008
          20            328008
           8            203009
          10            210001
          13            270071-501
          14            270080-501
          15            280011
          19            325003
```

Figure G.5 Output File Generated by GTLAYPC.EXE. (Heragu, 1997) 331.
(Continued next page)

```
PARTS THAT GO INTO CELL    3ARE
      PART. NO.      PART NAME
           8           203009
           9           210001
          11           217002-3
          13           270071-501
          14           270080-501
          18           324008
          19           325003
          20           328008
GROUPING OF MACHINES AND PARTS IN MATRIX FORM
      1  2  3  4  9  8  5  6  7

 1    1  2  0  0  0  0  0  0  0
 2    1  2  3  0  0  0  0  0  0
 3    1  2  0  0  0  0  0  0  0
12    1  2  0  0  0  0  0  0  0
16    1  2  0  0  0  0  0  0  0
 4    0  1  4  2  0  0  0  0  0
 5    0  1  2  3  0  0  0  0  0
 7    0  1  0  2  0  0  0  0  0
10    0  2  0  0  1  1  0  0  0
19    0  2  0  0  2  2  1  1  1
 8    0  0  2  0  1  1  3  3  3
 9    0  0  2  0  0  0  1  1  1
11    0  0  2  0  0  0  1  1  1
13    0  0  3  0  2  2  1  1  1
14    0  0  3  0  2  2  1  1  1
 6    0  0  0  2  1  1  0  0  0
17    0  0  0  2  1  1  0  0  0
18    0  0  0  3  2  2  1  1  1
20    0  0  0  3  2  2  1  1  1
15    0  0  0  0  1  1  0  0  0
GROUP TECHNOLOGY PART OVER
LAYOUT OF MACHINES WITHIN CELLS WILL START NOW

DETAILS AND LAYOUT OF CELL 1

   LAYOUT SCALE 1:  3.5000
   TOTAL CELL LENGTH=35.00
   TOTAL CELL WIDTH=35.00
   OPTIMUM VALUE OF FLOW TIMES DISTANCE
   FOR THE CELL IS  12620.000
POSI  MACH  MACH   LENGTH    WIDTH     LAYOUT
TION  NO.   NAME
 1     2   2008 0    20.00     25.00   2 2 2 2 2 2 3 3 3 3
 2     3   014 02    15.00     10.00   2 2 2 2 2 2 3 3 3 3
 3     1   01204     20.00     10.00   2 2 2 2 2 2 3 3 3 3
                                       2 2 2 2 2 2 0 0 0 0
```

Figure G.5. (Continued next page)

```
                                            2 2 2 2 2 2 0 0 0 0
                                            2 2 2 2 2 2 0 0 0 0
                                            2 2 2 2 2 2 0 0 0 0
                                            1 1 1 1 1 1 0 0 0 0
                                            1 1 1 1 1 1 0 0 0 0
                                            1 1 1 1 1 1 0 0 0 0
 DETAILS AND LAYOUT OF CELL 2

    LAYOUT SCALE 1:  4.5000
    TOTAL CELL LENGTH=45.00
    TOTAL CELL WIDTH=35.00
    OPTIMUM VALUE OF FLOW TIMES DISTANCE
    FOR THE CELL IS  7222.500
POSI   MACH   MACH   LENGTH    WIDTH      LAYOUT
TION   NO.    NAME
  1    4   23  020     25.00      20.00   4 4 4 4 4 4 9 9 9 9
  2    9   0B          20.00      15.00   4 4 4 4 4 4 9 9 9 9
  3    8   0A  020     20.00      15.00   4 4 4 4 4 4 9 9 9 9
                                          4 4 4 4 4 4 0 0 0 0
                                          8 8 8 8 0 0 0 0 0 0
                                          8 8 8 8 0 0 0 0 0 0
                                          8 8 8 8 0 0 0 0 0 0
 DETAILS AND LAYOUT OF CELL 3

    LAYOUT SCALE 1: 9.0000
    TOTAL CELL LENGTH=90.00
    TOTAL CELL WIDTH=20.00
    OPTIMUM VALUE OF FLOW TIMES DISTANCE
    FOR THE CELL IS  0.000
POSI   MACH   MACH   LENGTH    WIDTH      LAYOUT
TION   NO.    NAME
  1     5  9A  020     45.00      10.00   5 5 5 5 5 6 6 6 6 6
  2     6  9B  020     45.00      10.00   7 7 7 7 7 0 0 0 0 0
  3     7  9C  020     45.00      10.00
  LAYOUT OF MACHINES WITHIN THE CELLS OVER
  LAYOUT OF FINALCELLS WILL START NOW

  DETAILS AND FINAL LAYOUT OF ALL CELLS

    FINAL LAYOUT SCALE 1:  9.0000
    TOTAL FINAL LAYOUT LENGTH=90.00
    TOTAL FINAL LAYOUT WIDTH=55.00
    OPTIMUM VALUE OF FLOW TIMES DISTANCE
    FOR THE FINAL LAYOUT IS58511.55
POSITION  CELL. NO.  LENGTH    WIDTH      FINAL LAYOUT
   1          2        35.00     35.00    2 2 2 2 2 1 1 1 1 0
   2          1        45.00     35.00    2 2 2 2 2 1 1 1 1 0
   3          3        90.00     20.00    2 2 2 2 2 1 1 1 1 0
                                          2 2 2 2 2 1 1 1 1 0
                                          3 3 3 3 3 3 3 3 3 3
                                          3 3 3 3 3 3 3 3 3 3
```

Figure G.5. (Continued)

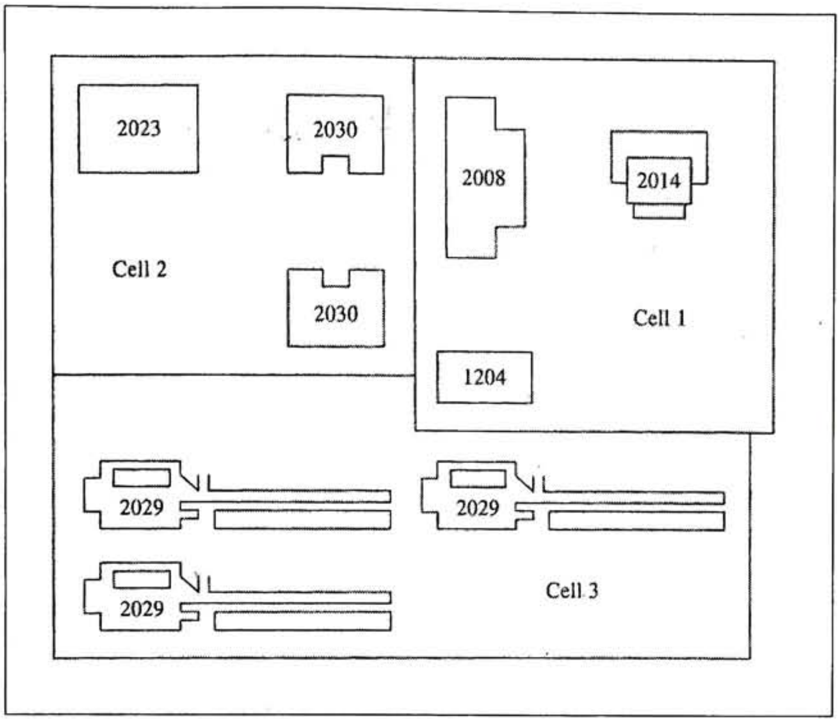

Figure G.6 Layout Developed using output shown in Figure G.5. (Heragu, 1997) 334.

Appendix H

Hierarchical Cluster Analysis of Observations – Cellular Manufacturing Study

- **Data Display**

Row	Part No.	Demand	Mach1	Mach2	Mach3	Mach4	Mach5	Mach6
1	1	24.44	60.00	100.00	0.00	0.00	0.00	0.00
2	2	55.56	100.00	75.00	25.00	0.00	0.00	0.00
3	3	55.56	80.00	33.33	0.00	0.00	0.00	0.00
4	4	55.56	0.00	50.00	100.00	50.00	0.00	0.00
5	5	24.44	0.00	33.33	50.00	100.00	0.00	0.00
6	6	13.33	0.00	0.00	0.00	100.00	0.00	20.00
7	7	77.78	0.00	33.33	0.00	100.00	0.00	0.00
8	8	100.00	0.00	0.00	25.00	0.00	0.00	30.00
9	9	22.22	0.00	0.00	0.00	0.00	100.00	0.00
10	10	27.78	0.00	33.33	0.00	0.00	0.00	20.00
11	11	44.44	0.00	0.00	50.00	0.00	100.00	0.00
12	12	33.33	57.80	33.33	0.00	0.00	0.00	0.00
13	13	27.78	0.00	0.00	75.00	0.00	0.00	0.00
14	14	66.67	0.00	0.00	37.50	0.00	0.00	40.00
15	15	50.00	0.00	0.00	0.00	0.00	0.00	100.00
16	16	55.56	80.00	25.00	0.00	0.00	0.00	0.00
17	17	44.44	0.00	0.00	0.00	50.00	0.00	0.00
18	18	33.33	0.00	0.00	0.00	75.00	0.00	0.00
19	19	1.11	0.00	0.00	0.00	0.00	0.00	0.00
20	20	27.78	0.00	0.00	0.00	100.00	0.00	0.00

- **Results**

Euclidean Distance, Average Linkage

Amalgamation Steps

Step	Number of clusters	Similarity level	Distance level	Clusters joined		New cluster	Number of obs. in new cluster
1	19	95.06	8.333	3	16	3	2
2	18	85.37	24.671	6	20	6	2
3	17	83.78	27.358	17	18	17	2
4	16	81.05	31.955	3	12	3	3
5	15	78.07	36.978	8	14	8	2
6	14	73.57	44.564	6	17	6	4
7	13	72.04	47.140	10	19	10	2
8	12	67.55	54.716	9	11	9	2
9	11	64.42	60.002	2	3	2	4
10	10	60.71	66.248	6	7	6	5

11	9	59.20	68.794	5	6	5	6
12	8	57.16	72.236	1	2	1	5
13	7	52.57	79.979	4	13	4	2
14	6	51.88	81.137	8	15	8	3
15	5	46.32	90.516	8	10	8	5
16	4	35.91	108.072	4	8	4	7
17	3	32.87	113.198	4	5	4	13
18	2	25.95	124.857	1	4	1	18
19	1	19.23	136.188	1	9	1	20

Final Partition

Number of clusters: 1

	Number of observations	Within cluster sum of squares	Average distance from centroid	Maximum distance from centroid
Cluster1	20	128450.648	77.882	104.160

Appendix I

k-means Cluster Analysis – Cellular Manufacturing Study

- **Number of clusters: 3**

Number of	Within cluster	Average distance	Maximum distance	observations
sum of squares	from centroid	from centroid		
Cluster1	5	6837.192	35.492	53.506
Cluster2	2	3122.873	39.515	39.515
Cluster3	13	69282.326	70.398	98.952

Cluster Centroids

Variable	Cluster1	Cluster2	Cluster3	Grand centroid
Demand	44.8890	40.0000	41.2820	42.0556
Mach1	75.5600	0.0000	0.0000	18.8900
Mach2	53.3332	41.6665	5.1282	20.8332
Mach3	5.0000	75.0000	14.4231	18.1250
Mach4	0.0000	75.0000	32.6923	28.7500
Mach5	0.0000	0.0000	15.3846	10.0000
Mach6	0.0000	0.0000	16.1538	10.5000

Distances Between Cluster Centroids

	Cluster1	Cluster2	Cluster3
Cluster1	0.0000	128.0403	98.4950
Cluster2	128.0403	0.0000	85.4039
Cluster3	98.4950	85.4039	0.0000

- **Data Display**

Part No.	Demand	Mach1	Mach2	Mach3	Mach4	Mach5	Mach6	2-Cluster	3-Cluster	4-Cluster
1	24.444	60	100	0	0	0	0	2	1	1
2	55.556	100	75	25	0	0	0	2	1	2
3	55.556	80	33.333	0	0	0	0	2	1	2
4	55.556	0	50	100	50	0	0	2	2	4
5	24.444	0	33.333	50	100	0	0	1	2	4
6	13.333	0	0	0	100	0	20	1	3	4
7	77.778	0	33.333	0	100	0	0	1	3	4
8	100	0	0	25	0	0	30	2	3	3
9	22.222	0	0	0	0	100	0	1	3	3
10	27.778	0	33.333	0	0	0	20	2	3	3
11	44.444	0	0	50	0	100	0	2	3	3
12	33.333	57.8	33.333	0	0	0	0	2	1	2
13	27.778	0	0	75	0	0	0	2	3	3
14	66.667	0	0	37.5	0	0	40	2	3	3
15	50	0	0	0	0	0	100	1	3	3
16	55.556	80	25	0	0	0	0	2	1	2
17	44.444	0	0	0	50	0	0	1	3	4
18	33.333	0	0	0	75	0	0	1	3	4
19	1.111	0	0	0	0	0	0	1	3	3
20	27.778	0	0	0	100	0	0	1	3	4

Appendix J

Clustering Models Data Display Results

Table J.1

k-Means Model Annual Demand Set-up Time

Part #	Annual Demand (Units)								
	2 Clusters		3 Clusters			4 Clusters			
	Cell 1	Cell 2	Cell 1	Cell 2	Cell 3	Cell 1	Cell 2	Cell 3	Cell 4
1	220		220			220			
2	500		500				500		
3	500		500				500		
4	500			500					500
5		220		220					220
6		120			120				120
7		700			700				700
8	900				900			900	
9		200			200			200	
10	250				250			250	
11	400				400			400	
12	300		300				300		
13	250				250			250	
14	600				600			600	
15		450			450			450	
16	500		500				500		
17		400			400				400
18		300			300				300
19		10			10			10	
20		250			250				250
Aver:	447	294	404	360	372	220	450	383	356
St Dev:	198	202	134	198	245	0	100	274	196
SS:	392818	327622	72320	39200	719969	0	30000	527150	229571
Across Units									
Aver Std Dev:	200		192			143			
Aver SS:	360220		277163			196680			

Mach #	Utilization (hr/yr)								
	2 Clusters		3 Clusters			4 Clusters			
	Cell 1	Cell 2	Cell 1	Cell 2	Cell 3	Cell 1	Cell 2	Cell 3	Cell 4
1	802.70		802.70			66.00	736.70		
2	371.00	92.00	296.00	97.00	95.00	66.00	230.00	25.00	167.00
3	292.50	22.00	25.00	122.00	167.50		25.00	167.50	122.00
4	25.00	171.50		47.00	149.50				196.50
5	320.00	160.00			480.00			480.00	0.00
6	140.00	118.50			258.50			252.50	6.00
Aver :	325.20	112.80	374.57	88.67	230.10	66.00	330.57	231.25	122.88
St Dev:	266.69	59.97	394.76	38.19	151.59	0.00	366.35	190.55	83.72
SS:	355619	14383	311668	2917	91924	0	268429	108931	33107
Across Units									
Aver Std Dev:	163		195			160			
Aver SS:	185001		135503			102617			

Table J.2

Plant Location Model Annual Demand and Set-up Time

Part #	Annual Demand (Units)								
	2 Clusters		3 Clusters			4 Clusters			
	Cell 1	Cell 2	Cell 1	Cell 2	Cell 3	Cell 1	Cell 2	Cell 3	Cell 4
1	220		220			220			
2	500		500			500			
3	500		500			500			
4		500		500				500	
5		220			220				220
6		120			120				120
7		700			700				700
8		900		900				900	
9		200		200			200		
10		250		250				250	
11		400		400			400		
12	300		300			300			
13		250		250				250	
14		600		600				600	
15		450		450				450	
16	500		500			500			
17		400			400				400
18		300			300				300
19		10			10				10
20		250			250				250
Aver:	404	370	404	444	286	404	300	492	286
St Dev:	134	233	134	231	221	134	141	244	221
SS:	72320	759400	72320	372188	293971	72320	20000	297083	293971
Across Units									
Aver Std Dev:	184		195			185			
Aver SS:	415860		246160			170844			

Mach #	Utilization (hr/yr)								
	2 Clusters		3 Clusters			4 Clusters			
	Cell 1	Cell 2	Cell 1	Cell 2	Cell 3	Cell 1	Cell 2	Cell 3	Cell 4
1	802.70		802.70			802.70			
2	296.00	192.00	296.00	100.00	92.00	296.00		100.00	92.00
3	25.00	289.50	25.00	267.50	22.00	25.00	40.00	227.50	22.00
4		196.50		25.00	171.50			25.00	171.50
5		480.00		160.00	0.00		480.00	0.00	0.00
6		258.50		252.50	6.00			252.50	6.00
Aver :	374.57	283.30	374.57	161.00	72.88	374.57	260.00	151.25	72.88
St Dev:	394.76	117.49	394.76	102.39	75.62	394.76	311.13	107.44	75.62
SS:	311668	55214	311668	41933	21402	311668	96800	52933	21402
Across Units									
Aver Std Dev:	256		191			222			
Aver SS:	183441		125001			120701			

Table J.3

Covering Model Annual Demand and Set-up Time

Part #	Annual Demand (Units) 2 Clusters Cell 1	2 Clusters Cell 2	3 Clusters Cell 1	3 Clusters Cell 2	3 Clusters Cell 3	4 Clusters Cell 1	4 Clusters Cell 2	4 Clusters Cell 3	4 Clusters Cell 4
1	220		220					220	
2	500			500				500	
3	500		500					500	
4		500			500	500			
5	220		220			220			
6	120		120			120			
7	700		700			700			
8		900			900				900
9	200		200				200		
10	250		250					250	
11		400			400		400		
12	300			300				300	
13		250		250				250	
14		600			600				600
15		450			450				450
16		500		500				500	
17	400		400					400	
18		300			300	300			
19		10		10				10	
20	250		250			250			
Aver:	333	426	318	312	525	348	300	326	650
St Dev:	173	247	183	204	209	213	141	166	229
SS:	298418	486422	267356	165880	218750	227283	20000	219622	105000
Across Units									
Aver Std Dev:	210		199			187			
Aver SS:	392420		217329			142976			

Mach #	Utilization (hr/yr) 2 Clusters Cell 1	2 Clusters Cell 2	3 Clusters Cell 1	3 Clusters Cell 2	3 Clusters Cell 3	4 Clusters Cell 1	4 Clusters Cell 2	4 Clusters Cell 3	4 Clusters Cell 4
1	602.70	200.00	266.00	536.70				802.70	
2	375.50	112.50	233.00	180.00	75.00	109.00		321.00	
3	47.00	267.50	22.00	62.50	230.00	122.00	40.00	62.50	90.00
4	149.00	47.50	149.00		47.50	176.50		20.00	
5	160.00	320.00	160.00	0.00	320.00	0.00	480.00	0.00	0.00
6	18.50	240.00	18.50	0.00	240.00	6.00		12.50	240.00
Aver :	225.45	197.92	141.42	259.73	182.50	103.38	260.00	243.74	165.00
St Dev:	203.97	92.59	103.61	246.95	116.46	71.20	311.13	337.19	106.07
SS:	249634	51443	53680	202923	54250	23757	96800	504293	29400
Across Units									
Aver Std Dev:	148		156			206			
Aver SS:	150538		103618			163562			

Table J.4

Fuzzy Set Method Annual Demand and Set-up Time

Part #	Annual Demand (Units)							
	2 Clusters		3 Clusters			4 Clusters		
	Cell 1	Cell 2	Cell 1	Cell 2	Cell 3	Cell 1	Cell 2	Cell 3
1	220		220			220		
2	500		500			500		
3	500		500			500		
4		500		500			500	
5		220		220				220
6		120		120				120
7		700		700				700
8	900				900		900	
9	200				200		200	
10	250				250		250	
11	400				400		400	
12	300		300		250	300		
13	250				600		250	
14	600				450		600	
15	450						450	
16	500		500			500		
17		400		400				400
18		300		300				300
19	10				10		10	
20		250		250				250
Aver:	391	356	404	356	383	404	396	332
St Dev:	224	196	134	196	274	134	260	203
SS:	600892	229571	72320	229571	527150	72320	539422	205283
Across Units								
Aver Std Dev:	210		201			199		
Aver SS:	415232		276347			272342		

Mach #	Utilization (hr/yr)							
	2 Clusters		3 Clusters			4 Clusters		
	Cell 1	Cell 2	Cell 1	Cell 2	Cell 3	Cell 1	Cell 2	Cell 3
1	802.70		802.70			802.70		
2	321.00	167.00	296.00	167.00	25.00	296.00	100.00	92.00
3	192.50	122.00	25.00	122.00	167.50	25.00	267.50	22.00
4		196.50		196.50			25.00	171.50
5	680.00	0.00		0.00	480.00		480.00	0.00
6	252.50	6.00		6.00	252.50		252.50	6.00
Aver :	449.74	122.88	374.57	98.30	231.25	374.57	225.00	72.88
St Dev:	273.52	83.72	394.76	90.98	190.55	394.76	175.51	75.62
SS:	299250	33107	311668	33107	108931	311668	123213	21402
Across Units								
Aver Std Dev:	179		225			215		
Aver SS:	166179		151235			152094		

Table J.5

Average-Weighted Distance Model Annual Demand and Set-up Time

Part #	Annual Demand (Units)								
	2 Clusters		3 Clusters			4 Clusters			
	Cell 1	Cell 2	Cell 1	Cell 2	Cell 3	Cell 1	Cell 2	Cell 3	Cell 4
1	220		220			220			
2	500		500			500			
3	500		500			500			
4		500		500				500	
5		220			220				220
6		120			120				120
7		700			700				700
8		900		900				900	
9		200		200			200		
10		250		250				250	
11		400		400			400		
12	300		300			300			
13		250		250				250	
14		600		600				600	
15		450		450				450	
16	500		500			500			
17		400		400				400	
18		300			300				300
19		10		10				10	
20		250			250				250
Aver:	404	370	404	396	318	404	300	420	318
St Dev:	134	233	134	245	223	134	141	266	223
SS:	72320	759400	72320	539440	199680	72320	20000	496400	199680
Across Units									
Aver Std Dev:	184		201			191			
Aver SS:	415860		270480			197100			

Mach #	Utilization (hr/yr)								
	2 Clusters		3 Clusters			4 Clusters			
	Cell 1	Cell 2	Cell 1	Cell 2	Cell 3	Cell 1	Cell 2	Cell 3	Cell 4
1	802.70		802.70			802.70			
2	296.00	192.00	296.00	100.00	92.00	296.00		100.00	92.00
3	25.00	289.50	25.00	267.50	22.00	25.00	40.00	227.50	22.00
4		196.50		45.00	151.50			45.00	151.50
5		480.00		160.00	0.00		480.00	0.00	0.00
6		258.50		252.50	6.00			252.50	6.00
Aver :	374.57	283.30	374.57	165.00	67.88	374.57	260.00	156.25	67.88
St Dev:	394.76	117.49	394.76	95.93	67.10	394.76	311.13	99.80	67.10
SS:	311668	55214	311668	36813	17194	311668	96800	49413	17194
Across Units									
Aver Std Dev:	256		186			218			
Aver SS:	183441		121891			118769			