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Optimization of inpatient hemodialysis scheduling considering efficiency and treatment delays to minimize length of stay

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OPTIMIZATION OF INPATIENT HEMODIALYSIS SCHEDULING CONSIDERING EFFICIENCY AND TREATMENT DELAYS TO MINIMIZE LENGTH OF STAY

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ABSTRACT

Inpatient dialysis units face an uncertain daily demand of hemodialysis procedures for end-stage renal disease (ESRD) patients hospitalized for health conditions that may or may not be directly related to their renal disease. While hospitalized, these patients must receive hemodialysis in addition to any medical services needed for their primary diagnosis. As a result, when demand for inpatient dialysis is high, treatments and procedures required by these inpatients may be delayed increasing their length of stays (LOS). This research presents an optimization approach for daily scheduling of inpatient hemodialysis to maximize the efficiency of the dialysis unit while minimizing delays of other scheduled procedures that could extend the LOS of the inpatients. The optimization approach takes into account the dialysis protocols prescribed by a treating nephrologist for each dialysis patient, the variable duration of the dialysis treatments, the limited capacity of the dialysis equipment and personnel, as well as the isolation requirements used to mitigate the spread of healthcare-associated infections (HAI). In addition, a variant of the optimization approach is developed that considers uncertainty associated with rescheduling procedures that are delayed and the expected impact on LOS. An experimental performance evaluation illustrates the capability and effectiveness of the proposed scheduling methodologies. The results of this research indicate that the optimization-based scheduling approaches developed in this study could be used on a daily basis by an inpatient dialysis unit to create efficient dialysis schedules.

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1. INTRODUCTION

End-stage renal disease (ESRD) is a chronic condition due to kidney failure that requires intensive and expensive dialysis treatments and often transplantation. Since the 1980s, the prevalence and incidence of ESRD patients have steadily increased. In 2009, the prevalent ESRD population in the United States reached a total number of 571,000 patients, with an incidence of 116,000 new patients (USRDS, 2011). Over 65% of the ESRD patients required hemodialysis and they represent 92% of all new patients diagnosed with ESRD.

The growth of the ESRD population in the U.S. is alarming, from both a medical and financial perspective. From the financial perspective, the cost burden of hemodialysis to Medicare, which provides near-universal coverage for this population regardless of age (CMS, 2012; Dor, Pauly, Eichleay & Held, 2007). The United States Renal Data System (USRDS) reported that ESRD expenditures reached $29 billion dollars in 2009, representing about 6% of the total Medicare costs (USRDS, 2011). Moreover, over 85% of the ESRD expenditures were spent in hemodialysis patients. The USRDS also registered that hemodialysis patients are hospitalized an average of 1.90 times per year, with a mean yearly length of stay (LOS) per patient of 11.90 days. Thus, reducing the LOS of the ESRD inpatients can have a significant economic and social impact for both patients and providers.

One reason for the relatively high hospitalization rate and LOS of ESRD patients is that they are prone to develop additional medical conditions (DOPPS, 2010), and thus they require treatment for both ESRD and their other conditions. Furthermore, ESRD patients admitted to a hospital for problems not necessarily associated with their renal condition tend to have longer

LOS than patients with similar problems that do not have ESRD (Cowen, Huang, Lebow, Devivo & Hawkins, 1995; Forrest, Nagao, Iqbal & Kakar, 2005; Forrest, 2004). The longer LOS is due, in part, to the need of ESRD patients to receive dialysis while hospitalized. Forrest (2004) claims that treatments for these inpatients are often missed or delayed due to scheduling conflicts with their planned dialysis procedure. In a follow-up study, Forrest et al. (2005) provide evidence that if ESRD inpatients undergo all their procedures when planned, their LOS could be very close to the LOS of non-ESRD patients.

This research aims to minimize the LOS of ESRD inpatients by scheduling the hemodialysis procedures in such a way that inpatients undergo dialysis and their other procedures as close as possible to their scheduled appointment time. Two optimization approaches are developed to assist with the daily scheduling of an inpatient hemodialysis unit that aims to maximize efficiency of the unit, while minimizing delays of other scheduled procedures that could extend the LOS of the inpatients. The optimization approaches take into account the dialysis protocols prescribed by a treating nephrologist, the variable duration of the dialysis treatments, the limited capacity of the dialysis equipment and personnel in the inpatient hemodialysis unit, as well as the isolation requirements used to control the spread of healthcare-associated infections (HAI). (A detailed description of inpatient hemodialysis units and their scheduling requirements is presented in chapter 4.) The initial optimization approach focuses on minimizing the overall time delays (tardiness) of rescheduled procedures in order to mitigate its impact over the LOS of inpatients. In addition, a variant of the optimization approach accounts the uncertainty surrounding whether or not procedures could be rescheduled without impacting the LOS of the inpatients. This variant also considers the medical needs and time-based

criticality of the inpatients for undergoing dialysis by a given due time during the day. As a result, the optimization approaches could be used on a daily basis by an inpatient dialysis unit to create efficient schedules.

The remainder of this thesis is organized as follows. Chapter 2 describes the problem statement and states the objectives of this research. Chapter 3 provides a relevant literature review. Chapter 4 describes the inpatient dialysis unit and the scheduling challenges the unit faces. Chapter 5 presents the initial optimization approach for the inpatient hemodialysis scheduling problem. Chapter 6 introduces the variant of the optimization approach of this research. Finally, chapter 7 presents the conclusions and future recommendations.

2. PROBLEM STATEMENT

The prevalence of ESRD patients and the resulting expenditures for this population are an important concern in the U.S. healthcare system. To cope with the significant occurrence of ESRD patients and complications regarding their care, hospitals and dialysis units need to strategically plan their capacity and the quality of patient care provided (Holland, 1994; Knauf & Aronson, 2009). The options to manage these problems may include increasing the capacity of dialysis units by adding dialysis stations, building new dialysis centers or extending current operating hours. However, these options are very expensive. As an alternative, we focus on improving the scheduling of inpatient dialysis procedures in a manner that efficiently utilizes the existing resources, while mitigating delays in other procedures that these patients may have in other units.

The hospital care of ESRD patients is very complex because these patients are frequently admitted to a hospital by a condition not related to their renal disease. Comorbidities not only affect the patients' health, but also the medical services that hospitals and dialysis units personnel must provide to ESRD patients. Moreover, several studies have shown that the number of hospital days per year for ESRD patients with comorbidities is higher than for those patients with the same diagnoses but no renal conditions (Cowen et al., 1995; Forrest et al., 2005; Forrest, 2004). Longer stays are caused, in part, by the need of ESRD patients to receive dialysis in addition to other procedures required while hospitalized. Since the dialysis demand of ESRD inpatients varies from day to day, when demand is high, scheduling dialysis for all inpatients while accommodating other scheduled procedures becomes difficult, potentially resulting in extended LOS. Forrest et al. (2005) claims that the LOS for ESRD patients can be minimized if

all procedures are performed as planned. Based on that last claim, this study specifically addresses the following question:

> *On a daily basis, when should the ESRD inpatients undergo dialysis and procedures in other hospital units so that delays that could extend their length of stays (LOS) are minimized, while considering dialysis scheduling priorities?*

Currently, we are not aware of any supporting tools for the hemodialysis inpatient scheduling problem that consider all its challenges, including uncertain daily demand, limited capacity of machines and personnel, different prescribed dialysis duration per inpatient, non-dialysis scheduled procedures, as well as isolations protocols to prevent the propagation of HAI. Therefore, the goal of this study is to develop scheduling methodologies to help inpatient dialysis units to schedule hemodialysis procedures that maximize the unit's efficiency, while minimizing the time delays (tardiness) of the non-dialysis procedures and the number of inpatients affected by such delays, thus attempting to minimize the LOS of ESRD inpatients. In an effort to provide efficient inpatient hemodialysis schedules, optimization methods are investigated. The key objectives for this research are:

- *Develop optimization approaches to schedule hemodialysis inpatients*: The inpatient hemodialysis scheduling problem is modeled as an assignment problem that focuses on assigning when (time of the day) and where (dialysis station) hemodialysis inpatients should undergo dialysis throughout a working day. The methods to be developed for solving this problem should be capable of identifying the most cost-efficient assignment of inpatients, such that the use of dialysis stations is maximized while the delays in other scheduled procedures are minimized. The targeted information to be provided by the

optimization methods to the dialysis unit administrators (decision-makers) includes a dialysis schedule indicating when and where the inpatients are to undergo dialysis on the day of interest, as well as which non-dialysis procedures need to be rescheduled with their corresponding delay times.

- *Conduct an experimental performance evaluation*: Sets of experiments will be performed to test the capability and limitations of the proposed optimization approaches. In particular, the first set of experiments will be targeted to analyze performance of the initial approach under problem instances considering different resource utilization rates (i.e., the relation between the inpatient demand and the capacity of the dialysis unit during regular hours) and different scheduling priorities or settings. The scheduling settings include prioritizing dialysis procedures, prioritizing the on-time delivery of non-dialysis procedures, and a balanced approach priority. In addition, the performance of the two optimization methods will be compared to evaluate the worth of the additional considerations of the second method and the schedules produced.

The development of these scheduling optimization approaches could enable the dialysis units to manage their resources more efficiently, to minimize treatment delays, and to avoid extending LOS of ESRD inpatients due to treatment scheduling complications. By using the proposed optimization approaches, it is expected that the dialysis unit decision-makers can determine dialysis schedules that consider the tradeoffs of using fixed and mobile dialysis stations, as well as the efficiency of the unit and the on-time delivery of other scheduled procedures. Moreover, decision-makers would be allowed to manage the preference of appointment time through the day, i.e., when it is better to schedule dialysis procedures based on

the goals of the dialysis unit. By scheduling efficiently (in terms of station utilization and operation hours) and minimizing delays that could extend the LOS of ESRD patients (making it closer to that of non-ESRD patients), the scheduling methods have the potential to positively impact the overall U.S. healthcare costs associated with hemodialysis for ESRD patients.

3. LITERATURE REVIEW

This chapter summarizes the current body of knowledge that is relevant to the research conducted for this thesis, including a general background of the ESRD treatment modalities, ESRD comorbidities and LOS, HAI, as well as related optimization and scheduling approaches used in healthcare settings. The potential contribution that this research can provide to the current literature is discussed at the end of the chapter.

3.1 ESRD and Treatments Modalities Background

The function of the kidneys is to filter wastes and excess fluids from the blood, and expelled them through the urine. Chronic kidney disease (CKD) represents the degradation over time of the kidneys' function, and it becomes ESRD when the kidneys are no longer capable to remove enough waste from the human body (MedlinePlus, 2012b). ESRD is considered the complete or almost complete failure of the kidney functions, requiring special and costly treatments like dialysis and often kidney transplantation (MedlinePlus, 2012b).

Dialysis is the treatment that artificially performs the function of the kidneys removing the wastes, salt and extra fluids from the blood. There are two different modalities of dialysis, hemodialysis and peritoneal dialysis. Hemodialysis uses a machine to accomplish the dialysis treatment, and it can be performed in hospitals, special dialysis centers, or at home (NKF, 2012a). For hemodialysis, doctors have to create an access or entrance into the blood vessels of the patient in order to be able to connect the patient to the dialysis machine and a special filter called dialyzer or artificial kidney. There are three types of access: fistula or arteriovenous fistula (AV fistula), graft or AV graft, and catheter or plastic tube. The first two type of access are

permanent access and require minor surgery to connect an artery and a vein, but fistulas are safer and last longer (MedlinePlus, 2012a). The latter, catheter access, is a temporary access that is inserted into a large vein, usually in the neck, chest or leg area; however, this method is more likely to get infected or have clots (AKF, 2012a; MedlinePlus, 2012a).

Before performing hemodialysis, the patients are connected to the machine with two needles, one for the extraction and the other for the insertion of the blood. During the procedure, the blood comes out of the patient through a tube and passes over the dialyzer or artificial kidney to remove the waste and extra fluid of the blood. The dialyzer filters and cleans small ounces of blood at a time with a washing fluid or cleansing solution called dialysate; then, the treated blood goes back to the patient through another tube (AKF, 2012a; MedlinePlus, 2012a; NKF, 2012a). The duration of the procedure is determined by a nephrologist and depends on the health conditions of the patient. In general, an ESRD patient requires 9 to 12 hours of dialysis per week, regularly divided in 3 or 4 equal time sessions (DOPPS, 2010; USRDS, 2011). Previous studies suggest that getting the right amount of dialysis improves the overall health of patients, enabling them to avoid hospitalizations and live a better and longer life (NKF, 2012a).

On the other hand, in peritoneal dialysis the inside lining of the abdomen of the person acts as a natural filter or cleaner (AKF, 2012c; NKF, 2012b; WebMD, 2012). To use this dialysis method, a catheter or plastic tube must be placed in the belly or peritoneal cavity of the patient by surgery. To perform peritoneal dialysis, the patient adds the dialysate into his abdomen through the catheter. After a prescribed amount of time, usually 4 to 6 hours, the dialysate draws the waters, chemicals and extra fluids from the blood, and it is drained from the patients'

abdomen (WebMD, 2012). During the day, in a 24 hours period, a patient has to do 3 to 5 exchanges, which is the process to drain and re-fill his peritoneal cavity with the dialysate (NKF, 2012b; WebMD, 2012). This dialysis method is very practical because the blood cleaning process can take place while sleeping, or during any other regular daily activity a person could perform (AKF, 2012c; WebMD, 2012).

The other alternative to treat ESRD is kidney transplantation. Kidney transplant is the surgery procedure to replace the damaged or diseased kidney of the patient, with a healthy kidney from a donor. The kidneys can be donated by a living or a deceased (i.e., cadaver) donor (AKF, 2012b; MedlinePlus, 2012c). A living donor may be a related person, such as parent or sibling, or an unrelated person, such as a friend, a spouse, a coworker, or someone willing to donate his kidney to the person in need. A deceased donor is someone who recently died and was willing to donate his organs after his death. In order to perform the transplant, the patient must perform matching tests to make sure that the healthy kidney is compatible with the diseased person, and to make sure that the patient is healthy enough to proceed with the operation (AKF, 2012b; MedlinePlus, 2012c).

The USRDS (2011) reports Medicare costs per person per year of more than $70,000 overall, ranging from $30,000 for transplant patients to $82,000 for patients undergoing hemodialysis therapy. Knauf and Aronson (2009) claim that even though kidney transplantation is more cost-effective than dialysis and is the preferred modality of treatment, it is limited by the number of organ donations. Likewise, even though peritoneal dialysis is cheaper than hemodialysis, the latter remains the most used and common treatment for the ESRD population

in United States. Some studies suggests that the United States has one of the highest expenditures per ESRD patient, yet its health outcomes of dialysis in ESRD care are relatively poor compared to other countries (Dor et al., 2007; Foley & Hakim, 2009). According to Zenios and Fuloria (2000), this phenomenon may be due to the reimbursement rate, which is 50% lower in the United States than in Europe, and to the enrollment of older and sicker patients in the United States.

3.2 ESRD Comorbidities and LOS

Patients with ESRD have a high prevalence of other health conditions or comorbidities and present low levels of physical fitness and function (Forrest, 2004), which may cause them to be hospitalized. When hospitalized, the presence of comorbidities impacts the inpatients' LOS. The 2010 Annual Report of the Dialysis Outcomes and Practice Patterns Study (DOPPS), prepared by Arbor Research Collaborative for Health, indicates that the most common comorbidities among ESRD patients are diabetes with a prevalence of over 60% and cardiovascular diseases, led by hypertension, congestive heart failure (CHF), and coronary artery disease (CAD) with a prevalence of over 85%, 47%, and 45%, respectively (DOPPS, 2010). The same report indicates that diabetes is also one of the major causes of ESRD, representing about 54% of all the causes of ESRD, and it is associated with the presence of cardiovascular disease.

Hence, depending on the medical condition or primary diagnosis of the patient during the admission, other hospital units may schedule a set of procedures and tests, such as surgeries, x-rays, computerized tomographies, among others, which may or not have higher priority over the inpatients' dialysis procedures. Consequently, procedures could be delayed or rescheduled,

impacting the LOS of the inpatients. According to USRD, on average hemodialysis inpatients who have diabetes are hospitalized 13.70 hospital days per year in 2.11 admissions, and hemodialysis inpatients who have hypertension are hospitalized 10.60 days per year in 1.74 admissions (USRDS, 2011).

Cowen et al. (1995) evaluate the functional outcomes and LOS of ESRD patients admitted to a rehabilitation unit and compare them to the outcomes of other patients with the same diagnoses treated at the same facility; this study reports that the ESRD patients experienced a slightly longer LOS than other patients. Moreover, the difficult task of scheduling dialysis procedures and other medical services often results in patients missing medical appointments. For example, Forrest (2004) reports that in 2001 in an Albany medical center, ESRD patients missed 27% of their rehabilitation therapy (i.e., occupational and physical therapy) sessions due to the scheduling conflicts between the times to undergo dialysis and other treatments. The average LOS was considerably longer in the dialysis group, with 16.03 days per year compared to 10.63 days per year in the non-dialysis group. However, the study could not determine if the longer LOS was due to the renal condition of the ESRD patients or the fact that they missed their therapy sessions. In a follow-up study, Forrest et al. (2005) claim, based on a statistical analysis on patient records in 2003 and 2004 from the same Albany medical center, that guaranteeing the rehabilitation therapy times by changing the medical procedure schedules results in a reduction of missed appointments, and that the LOS of ESRD patients dropped from 16.03 to 12.10 days per year, while the LOS of non-dialysis patients remained about the same.

Moreover, the LOS and costs of ESRD patients can also be affected by the care provider (i.e., internists or nephrologists). For example, Kshirsagar et al. (2000) claim that, despite the belief that non-specialists may use fewer resources than specialists, the LOS and costs for hemodialysis patients under the care of nephrologists was significantly shorter than for those under the care of internists.

Several studies evaluate the impact of comorbidities in the ESRD population. Knauf and Aronson (2009) consider that comorbidities among renal patients have become the norm instead of the exception. Moreover, Knauf and Aronson (2009) claim that the Medicare expenditures are greater the older the patient and can be as twice as high for those ESRD patients with both diabetes and CHF than without these comorbidities. Prichard (2000) analyzes the outcomes associated with the major comorbidities (i.e., diabetes and cardiovascular diseases: ischemic heart disease, hypertension, CAD) in patients with ESRD, and he claims that in order to improve the outcomes of these patients, not only the renal replacement therapy itself has to be improved, but also a better understanding and management of these coexisting diagnoses must be achieved. Miskulin et al. (2009) identify which comorbid conditions are associated with survival, and which comorbidities are most prognostic in comparison to the information provided by routinely measured laboratory and clinical parameters. Miskulin et al. (2009) conclude that comorbidities explain the variance for survival better than case-mix factors (i.e., age or physical impairments). In another study, Power et al. (2009) claim that ESRD patients commonly present to the emergency department with cardiovascular diseases, diabetic emergencies, and other dialysis-related complications (e.g., vascular access problems, infections, among others).

3.3 Healthcare-Associated Infections

The CDC (2010) defines HAI as infections that patients acquire during the course of receiving healthcare treatment for other conditions. Burke (2003) claims that HAI are by far the most common complications affecting hospitalized patients. In addition, Sherman et al. (2006) suggest that HAI rates have become a significant concern in the U.S. healthcare community. Klevens et al. (2007) report that in 2002 the estimated number of HAI in U.S. hospitals was about 1.70 million, with the estimated casualties associated to HAI being around 99,000 patients. In another study, Scott II (2009) analyzes the costs of HAI in the U.S. healthcare system in 2007 accounting for an overall annual direct cost of over $30 billion dollars.

To mitigate the propagation of HAI and reduce these outcomes, many international healthcare organizations have defined isolation protocols and guidelines to follow in the hospital care of patients. For example, the World Health Organization (WHO) provides a general background on HAI and defines general infection control practices to prevent HAI (WHO, 2003). The Center for Disease Control and Prevention (CDC) claims that the proper implementation of these guidelines reduces the occurrence of HAI (CDC, 2011). Nonetheless, despite this success, the implementation of isolation protocols still has challenges that affect the ability of hospitals and inpatient dialysis units to manage different operational aspects of the daily care of patients.

Moreover, ESRD patients are considered a group at high risk of acquiring HAI due to the intensive use of catheters, ventilators, and the transfusions necessary during hemodialysis procedures (CDC, 2010). Therefore, to prevent the spread of HAI, dialysis units must implement

isolation protocols considering the patients' conditions and isolation needs during the dialysis scheduling process.

3.4 Scheduling and Optimization Approaches

Power et al. (2009) outline the principles of dialysis and relevant medical considerations to treat ESRD patients to support healthcare physicians responsible of the delivery of care. Moreover, Holland (1994) details the sequence of steps that must be followed before a patient undergoes hemodialysis. In the setting of an outpatient dialysis clinic, Holland (1994) compares the effects of scheduled appointment times and the number of patients scheduled per appointment time-slot on the utilization of dialysis machines, the length of service hours, and the capacity of the dialysis unit. Holland (1994) claims that spreading the appointments throughout the day enables the dialysis unit to serve more patients with shorter operating hours while obtaining higher utilization rates of the dialysis machines than when patients are scheduled to arrive in large batches at the same time. In an outpatient setting, dialysis treatments are typically planned well in advance and specific appointments can be scheduled; whereas, in an inpatient setting, dialysis units face an uncertain demand of the patients requiring dialysis.

In addition, operations research models have been applied to scheduling problems in healthcare. An extensive literature review on patient scheduling can be found (Cardoen, Demeulemeester & Beliën, 2010; Cayirli & Veral, 2003; Gupta & Denton, 2008). Gupta and Denton (2008) highlight that the appointment scheduling problems can often be formulated either as cost or penalty minimization problems or as profit maximization problems and often consider the use of time-slots or periods to allocate the resources. Moreover, Gupta and Denton

(2008) claim that there are many optimization models focused in manufacturing, transportation and logistics areas that cannot be easily transformed to fit the healthcare environment, urging for the need of developing more healthcare-specific optimization approaches. Cayirli and Veral (2003) categorize patient scheduling in two broad groups: static and dynamic. In static scheduling, the most common appointment system in health care, all decisions are taken prior to the beginning of the clinic session; whereas, in dynamic scheduling future arrivals are reviewed continuously throughout the current day, based on the current state of the system. This classification is important since the optimization approach can vary significantly depending on the different scheduling type.

Additional studies address the uncertainty in patient scheduling. For example, Zhang et al. (2009) describe a mixed-integer program (MIP) for allocating operating room (OR) capacity to medical specialties and minimize inpatients' LOS; in addition, their study assesses the robustness of the provided optimal solutions with a simulation model that captures the uncertainty of the system. Lamiri et al. (2008) present a stochastic model and a MIP integrated with a Monte Carlo simulation for OR planning for two classes of patients and uncertain demand to minimize the OR costs.

Moreover, scheduling problems have been studied in other areas such as manufacturing. Li and Ierapetritou (2008) present a literature review on production scheduling techniques under uncertainties and identify the main challenges in this area. In addition, Bassett, Pekny and Reklaitis (1997) use Monte Carlo simulation to evaluate stochastic processing times and determine due dates which met certain reliability for the production planning problem.

Balasubramanian and Grossmann (2003) propose MIPs for different scheduling problems using fuzzy set theory to model the uncertainty in the processing time, with specific applications into the flow shop scheduling problem and the new product development scheduling problem. Seo, Klein and Jang (2005) study the single machine scheduling problem with stochastic processing times and propose 4 non-linear integer models to minimize the expected number of tardy jobs.

Simulation-based methods have been popular to schedule patients and evaluate appointments policies and uncertainties. Jeang (1990) uses discrete simulation modeling to describe the inpatient admission system in a hospital, aiming to maximize the number of inpatients scheduled and the occupancy and to minimize the LOS of the inpatients. Ogulata et al. (2009) assess the efficiency of a slack capacity scheduling approach and propose appropriate parameter values for the scheduling policy, such as when patients should have their appointments in order to reduce delays in treatments. In addition, Vermeulen et al. (2009) analyze how to schedule patients with various levels of urgency and preferences to time-slots for treatment. Vermeulen et al. (2009) evaluate the tradeoffs of scheduling performance and fulfillment of patient preferences, allowing the hospital departments remain in control of the scheduling.

In addition, Mageshwari and Kanaga (2012) identify three broad categories for patient scheduling when using agent-based simulation models: distributed, dynamic and coordinated patient scheduling. The distributed patient scheduling considers that hospitals are decentralized structures comprised of autonomous wards and ancillary units, in which patients demand medical services to the hospital units. Paulussen et al. (2003) propose a multi-agent simulation based approach to model the distributed nature of the hospitals, where patients competed over the

limited resources. The dynamic approach considers the active changes in the hospital, such as stochastic duration of the procedure times, delay in patients' arrival, among others. Paulussen et al. (2004a, 2004b) consider the uncertainty related to the medical needs of the patients at the beginning of the treatments, and the variable duration of those treatments. These follow-up studies introduced a novel multi-agent simulation based model to schedule patients under variable pathways and stochastic process times. In addition, they implemented their system and benchmarked different coordination mechanisms including the current practice in hospitals. Finally, the coordinated patient scheduling takes the distributed nature of the hospital and allows coordination and interaction between the agents. This approach intends reducing the response time of the distributed system, and simplifies the scheduling problem. Decker and Li (1998) propose a new mutli-agent model that considers the agents providing higher utilization rates for the resources while decreasing patients' hospital days.

In manufacturing settings, job flow-shop scheduling models have considered the earliness-tardiness problem that a particular job could experience in a manufacturing floor. Ronconi and Birgin (2012) present and compare the performance of several MIPs for the job flow-shop scheduling problem, whose objective is to minimize the earliness and tardiness of the system. Hooker (2005) presents a hybrid optimization approach for minimizing late tasks, combining integer linear programming with constraint programming. In his work, Hooker (2005) discusses two different objectives, minimizing the number of late tasks and minimizing the total tardiness of the system.

Finally, as discussed in section 3.3, HAI represents a significant concern in the healthcare community and, to mitigate their spread, many international healthcare organizations have defined isolation protocols and guidelines to follow in the hospital care of patients. Cignarale et al. (2012) studies patient scheduling considering isolation requirements. Their study presents an integer program (IP) formulation to assign single and double-rooms in a hospital unit, while implementing all necessary isolations required for controlling HAI.

3.5 Discussion

While reviewing the literature relevant to the scheduling optimization in healthcare settings, certain opportunities have been identified. One opportunity is that even though there is a considerable amount of research done in appointment and patient scheduling, none has directly addressed the scheduling of hemodialysis for inpatients. As previously indicated, in inpatient settings the hospital units (e.g., inpatient dialysis units) face an uncertain demand of the inpatients requiring medical services, which can make scheduling of such services difficult, and if not handled properly, it can result in procedures delays and potential extended LOS for the patients. In addition, inpatient dialysis units manage different type of resources (dialysis stations) that can be used to provide the hemodialysis services. Furthermore, a limited number of research studies have considered isolation protocols as a factor for patient scheduling. In addition, many studies account the uncertainty of the demand or the processing times in a manufacturing setting, but a limited research has applied those concepts to healthcare settings and most of them implement simulation-based methods but no optimization or mathematical programming-based methods.

Scheduling optimization approaches are developed to address the lack of scheduling tools for the inpatient hemodialysis scheduling problem and expand the current patient scheduling literature, which includes OR scheduling, general procedure appointment scheduling, patient admissions and bed assignment. The proposed optimization approaches (presented in chapters 5 and 6) consider the medical conditions of the inpatients (isolation needs), the limited capacity of the dialysis unit, the other scheduled treatments, as well as the uncertainty surrounding whether or not rescheduling non-dialysis procedures significantly impact the LOS of the inpatients.

4. INPATIENT DIALYSIS UNITS

Inpatient dialysis units are specialized centers that provide hemodialysis services to ESRD patients. These units face an uncertain daily demand of inpatient dialysis that complicates scheduling, and thus, when demand for inpatient dialysis is high, required treatments and procedures may be delayed potentially causing an increased LOS.

Each day a nephrologist prescribes the dialysis protocol to follow for each inpatient that needs to undergo dialysis. This protocol describes the duration and frequency of the dialysis procedure that the patient must undergo while hospitalized. The dialysis procedure time may be accurately estimated per patient, but can vary among patients. Typically, an ESRD patient requires 9 to 12 hours of dialysis per week, evenly distributed in 3 to 4 sessions (DOPPS, 2010; USRDS, 2011). It is very important to ensure the continuity of treatment for the dialysis procedure, i.e., once started, the dialysis procedure must not be interrupted. Moreover, the dialysis protocol may include the preferred time for initiating dialysis for each inpatient.

In general, inpatient dialysis units have a limited number of fixed (stationary) dialysis stations, distributed in various configurations in rooms or blocks of stations, and have limited personnel (e.g., doctors, nurses, and technicians) to manage the dialysis treatments. The dialysis units also typically have a set of mobile stations that are used to treat inpatients in their hospital rooms. Mobile stations are often used to treat inpatients with special conditions such as infectious diseases or patients who cannot be moved to the dialysis unit. However, mobile stations require dedicated personnel and additional medical resources. Furthermore, dialysis units also consider the transportation time of moving inpatients to and from the dialysis unit. In

general, patients that undergo dialysis in a fixed station need to be transported to the unit, whereas patients that use mobile dialysis stations receive care in their rooms.

In addition, the scheduling of dialysis units must consider the needs of ESRD patients with unusual medical complications or special conditions, such as infectious diseases (e.g., the flu (influenza), tuberculosis or MRSA). Focusing on the infectious diseases, hospital dialysis units must implement isolation requirements to mitigate the spread of HAI, such as avoiding assigning patients to a multiple-units block, if at least one of the patients poses a risk of infection to the others. Furthermore, patients with conditions that can be spread by airborne pathogens must not be co-located with non-infected patients to avoid exposure to illness. Therefore, to minimize the propagation of HAI and avoid cross-contamination among patients, the dialysis unit must ensure that at any given period, each block of stations only treats patients with at most one type of condition requiring isolation.

Once the protocols have been defined, unit administrators schedule when and where inpatients should receive dialysis, taking into consideration other scheduled procedures these inpatients must undergo elsewhere in the hospital. Considering scheduled procedures in other hospital units is very important since patients who do not undergo them as planned may extend their LOS, thus reducing the hospital's capacity to accommodate new inpatients. Additionally, uncertainty associated with when procedures can be rescheduled could further impact the LOS of the inpatients.

Finally, dialysis units can divide their working day into time-slots or working periods. The granularity of these time-slots (i.e. their duration) is important because decision-makers must know when they are enabled to schedule the start of dialysis, and it allows them to manage the schedules of procedures in other hospital units more efficiently. In particular, it is considered that the beginning of each dialysis procedure coincides with the beginning of any time-slot, and that the duration of a dialysis and other scheduled procedures can be expressed in multiples of the time-slot duration.

To exemplify the nature of the task of scheduling inpatient hemodialysis, consider the following scenario. An inpatient unit is comprised of 6 nurses and 6 blocks of stations, 3 of which have 2 fixed dialysis units, and 3 are single-unit (i.e., mobile station) blocks. There are 20 ESRD inpatients requiring dialysis, 8 of which present with various isolation needs. The length of the dialysis varies per inpatient. In addition, some inpatients must initiate dialysis by a prescribed time before the end of the day, and others must undergo dialysis in a mobile station. Note that a nurse must be present the whole length of the dialysis treatment for inpatients requiring mobile stations. Finally, all inpatients have one additional non-dialysis procedure scheduled during the day. In particular, in this example decision-makers would have over 1,000 decisions to make for the assignment of inpatients to come up with a schedule for the dialysis unit while considering the dialysis requirements per inpatient, the other scheduled procedures, as well as the isolation needs of each inpatient. Notice that currently, the task of scheduling inpatient hemodialysis is often made based on the experience of the personnel in charge. Consequently, it is practically impossible that a person is enabled to consider all possible combinations to configure an optimal schedule that satisfies the aforementioned considerations

while maximizing the efficiency of the unit to reduce cost, and minimizing the delays of scheduled procedures in other hospital units. This example (including specific details associated with the condition and scheduling requirement of each inpatient) will be used in the next chapters to demonstrate the optimization approached developed in this research.

The following chapters introduce the optimization methods for daily scheduling of inpatients hemodialysis during time-slots throughout the day while accounting for scheduled procedures in other hospital units to attempt to minimize delays that could impact the LOS of the inpatients.

5. INPATIENT HEMODIALYSIS SCHEDULING: OPTIMIZING EFFICIENCY AND TARDINESS TRADEOFF

This chapter introduces an optimization method which aims to maximize the efficiency of the dialysis unit while minimizing a combination of (a) the time delays (tardiness) of other scheduled procedures the inpatients must undergo elsewhere in the hospital and (b) the number of inpatients that require rescheduling of their non-dialysis treatments in order to mitigate its impact over the LOS of inpatients. The optimization method presented in this chapter aims to determine in which time-slot and in which dialysis station the inpatients are scheduled to undergo dialysis. The implementation of a penalty system is used to allow the dialysis unit decision-makers to control the following aspects (or factors):

- The efficiency (in terms of station utilization and operating hours) of the dialysis unit;
- Using single-unit (mobile station) blocks and multiple-units (fixed station) blocks;
- The number of patients affected by delays and the lengths of the delays; and
- The preference of appointment times through the day.

With the proposed optimization approach we try to minimize the impact on the LOS by minimizing the overall delays of non-dialysis scheduled procedures. In chapter 6, a variant of this optimization model is presented which takes into account the uncertainty associated with rescheduling procedures that are delayed and the expected impact on LOS. Notice that the optimization approaches considered in this research for the hemodialysis scheduling problem focuses on the static scheduling defined by Cayirli and Veral (2003), in which it is assumed that the scheduling decisions are made prior to the operating day. Furthermore, the penalty system

used to determine when and where patients must undergo dialysis is supported in previous work (Jeang, 1990; Ogulata et al., 2009). Moreover, the concepts of minimizing tardiness presented by Hooker (2005) and by Ronconi and Birgin (2012) are considered to address the delays of non-dialysis procedures when required, as well as the concepts for isolation requirements presented by Cignarale et al. (2012) are considered for the implementation of the isolation protocols to mitigate the propagation of HAI.

In particular, the optimization method presented in this chapter for the hemodialysis scheduling problem can be used to address the following questions:

- When and in which dialysis station are the inpatients undergoing dialysis in a given day?
- Which non-dialysis procedures should be rescheduled in order to provide hemodialysis to all inpatients in a given day?
- What is the minimum time delay that these non-dialysis procedures would experience?

This chapter is organized as follows. Section 5.1 states the assumptions considered in the first method of the optimization approach. Section 5.2 presents the notation and optimization model for this first method. Section 5.3 presents an illustrative experimental example and the results of applying the optimization approach. Section 5.4 describes a computationally efficient algorithm engineered for solving the experimental instances. Finally, section 5.5 presents an experimental performance evaluation run for the optimization approach.

5.1 Assumptions Considered in the Optimization Method

The hemodialysis scheduling problem can be classified as an assignment problem, in which inpatients are assigned to time-slots and hemodialysis stations. This problem considers the limited capacity of the dialysis unit, the dialysis protocols to follow for each inpatient, their other scheduled procedures in other hospital units, as well as the isolation protocols to facilitate the control of HAI.

For the proposed optimization approach, it is assumed that the following information is known for the decision-makers in the inpatient unit when deciding the dialysis schedules:

- The set of inpatients requiring dialysis.
- The number of the operating hours and medical resources (i.e., dialysis stations and personnel) available for the given day.
- Inpatients' current appointment times for non-dialysis treatments.
- The clinical conditions and isolation needs of each inpatient.
- The number of equal-length time-slots or working periods available to provide dialyses in the inpatient unit.
- The number of fixed (stationary) and mobile dialysis stations.
- The number of blocks of stationary dialysis stations in which the unit has been arranged, and the number of stationary stations per block.

Additionally, the optimization approach is designed considering the following assumptions:

- The blocks of dialysis station are used to facilitate the implementation of isolation requirements by ensuring that only inpatients with the same isolation needs can be treated during the same time-slot or period.
- Inpatients can have none or multiple non-dialysis procedures scheduled during the day.
- Dialysis treatments must occur during the current day and must not be interrupted.
- Dialysis treatments can have a due time within the current day (i.e., a medical required time to have dialysis.)
- The length of dialysis treatments includes time to transport the inpatient to the dialysis unit (or to carry over the mobile station to the room of the inpatient) and set-up and disinfection time of the stations.
- Non-dialysis procedures may be delayed to accommodate the dialysis treatment (unless otherwise specified).
- Decision-makers can specify which other scheduled procedures must not be delayed.
- The duration and starting time of the procedures are considered deterministic (fixed and known.)
- The magnitude of the weights used to penalize when dialysis is scheduled are defined by the decision-makers.

5.2 Optimization Method Notation and Mathematical Model

The optimization method can be described as follows in terms of the objective function and constraints:

- *Minimize Weighted Penalties* = *f* { (Type of resources used for performing dialysis),

  (Tardiness of delayed non-dialysis procedures),

  (Number of delayed non-dialysis procedures),

  (Dialysis appointment time)}

- *Subject to constraints for*:
  - Dialysis requirements (uncertain daily demand, uninterrupted service, service due time)
  - Capacity (dialysis machines, personnel, operating hours)
  - Isolation requirements
  - Non-dialysis procedures appointments
  - Non-overlapping procedures
  - Mobile station requirements

The notation used in this optimization method is as follows:

- **<u>SETS:</u>**

P : Set of inpatients $p \in P$

C : Set of medical conditions or isolation needs $c \in C$

B : Set of blocks of stations $b \in B$

T : Set of time-slots or working periods $t \in T$

G_p : Set of non-dialysis treatments $g \in G_p$ for inpatient $p \in P$

- **PARAMETERS:**

MB : Number of single-unit (mobile station) blocks

$DTime$: Number of operating periods per day available to schedule dialysis

$$s_{p.g,t} \begin{cases} 1, & \text{If inpatient } p \in P \text{ has non– dialysis procedure } g \in G_p \text{ scheduled to start at period } t \in T \\ 0, & \text{Otherwise} \end{cases}$$

$d_{p,g}$: Expected length (duration or number of periods) of the non-dialysis procedure $g \in G_p$ scheduled for inpatient $p \in P$ that started at period corresponding to $s_{p,g,t}$

$$A_{p,g} \begin{cases} 1, & \text{if non– dialysis procedure } g \in G_p \text{ for inpatient } p \in P \text{ cannot be delayed} \\ 0, & \text{Otherwise} \end{cases}$$

β_b : Penalty incurred for using a station from type of block $b \in B$

α_t : Penalty incurred for scheduling inpatients to period $t \in T$

δ : Penalty incurred for delaying/pushing a non-dialysis treatment

τ : Penalty incurred for the tardiness of pushing a non-dialysis treatment

k_b : Number of stations available in block $b \in B$

l_p : Expected length (duration) of the dialysis procedure for inpatient $p \in P$ (includes set-up and cleaning time)

M : Large constant

n_t : Total personnel time available to manage the dialysis procedure per period $t \in T$

r_b : Personnel time required to manage a patient's dialysis procedure with a station of block $b \in B$

i_p : Isolation need $c \in C$ of inpatient $p \in P$

w_p : Last period of the day by when inpatient $p \in P$ can start undergoing dialysis

$$m_p \begin{cases} 1, & \text{if inpatient } p \in P \text{ must undergo dialysis in a mobile station} \\ 0, & \text{Otherwise} \end{cases}$$

- **<u>VARIABLES:</u>**

$$X_{p,b,t} \begin{cases} 1, & \text{If inpatient } p \in P \text{ is treated at block } b \in B \text{ at period } t \in T: t \leq Dtime \\ 0, & \text{Otherwise} \end{cases}$$

$$Y_{c,b,t} \begin{cases} 1, & \text{If at least one inpatient } p \in P \text{ with isolation need } c \in C \text{ is treated at} \\ & \text{block } b \in B \text{ during period } t \in T: t \leq Dtime \\ 0, & \text{Otherwise} \end{cases}$$

$$U_{p,b,t} \begin{cases} 1, & \text{If inpatient } p \in P \text{ is assigned to undergo dialysis in block } b \text{ during period} \\ & t-1, \text{and also requires dialysis in period } t \in T: t \leq Dtime \\ 0, & \text{Otherwise} \end{cases}$$

$$Z_{p,g,t} \begin{cases} 1, & \text{If inpatient } p \in P \text{ starts undergoing his non– dialysis procedure } g \in G_p \text{ at} \\ & \text{period } t \in T \\ 0, & \text{Otherwise} \end{cases}$$

$$D_{p,g,t} \begin{cases} 1, & \text{If inpatient } p \in P \text{ undergoes his non– dialysis procedure } g \in G_p \text{ during} \\ & \text{period } t \in T \\ 0, & \text{Otherwise} \end{cases}$$

$Push_{p,g}$: Flags when treatment $g \in G_p$ is pushed/rescheduled for inpatient $p \in P$

Minimize:

$$\sum_{p \in P} \sum_{b \in B} \sum_{t \in T: t \leq DTime} X_{p,b,t} * \beta_b + \sum_{p \in P} \sum_{g \in G_p} \tau * \left(\sum_{t \in T} t * Z_{p,g,t} - \sum_{t' \in T} t' * s_{p,g,t'} \right) + \sum_{p \in P} \sum_{g \in G_p} \delta * Push_{p,g} + \sum_{p \in P} \sum_{b \in B} \sum_{t \in T: t \leq DTime} X_{p,b,t} * \alpha_t \qquad (5.0)$$

Subject to:

Dialysis length:

$$\sum_{b \in B} \sum_{t \in T} X_{p,b,t} = l_p \quad , \quad \forall\, p \in P \tag{5.1}$$

Dialysis continuity:

$$X_{p,b,t-1} + X_{p,b,t} \geq 2 * \; U_{p,b,t} \quad , \; \forall\, p \in P,\; \forall\, b \in B,\; \forall\, t \in T{:}\, t > 1 \; and \; t \leq DTime \tag{5.2}$$

$$\sum_{t \in T: t > 1} \sum_{b \in B} U_{p,b,t} = \; \left(l_p - 1\right) \;\; , \; \forall\, p \in P \tag{5.3}$$

Dialysis due time:

$$\sum_{b \in B} t * X_{p,b,t} \leq \left(w_p + l_p - 1\right) \quad , \quad \forall\, p \in P,\; \forall\, t \in T{:}\, t \leq DTime \tag{5.4}$$

Capacity:

$$\sum_{p \in P} X_{p,b,t} \leq k_b \quad , \quad \forall\, b \in B,\; \forall\, t \in T{:}\, t \leq DTime \tag{5.5}$$

Personnel:

$$\sum_{p \in P} \sum_{b \in B} X_{p,b,t} * r_b \leq n_t \quad , \quad \forall\, t \in T : t \leq DTime \tag{5.6}$$

Isolation requirements:

$$X_{p,b,t} \leq Y_{i_p,b,t} \quad , \qquad \forall\, p \in P,\; \forall\, b \in B,\; \forall\, t \in T{:}\, t \leq DTime \tag{5.7}$$

$$\sum_{c \in C} Y_{c,b,t} \leq \; 1 \;\; , \; \forall\, b \in B,\; \forall\, t \in T{:}\, t \leq DTime \tag{5.8}$$

$$\sum_{c \in C} Y_{c,b,t} \leq \sum_{p \in P} X_{p,b,t} \quad , \; \forall\, b \in B,\; \forall\, t \in T{:}\, t \leq DTime \tag{5.9}$$

Non– dialysis procedures:

$$\sum_{t' \in T} t' * s_{p,g,t'} \leq \sum_{t \in T} t * Z_{p,g,t} \quad , \qquad \forall\, p \in P,\; \forall\, g \in G_p \tag{5.10}$$

$$\sum_{t \in T} Z_{p,g,t} \leq 1, \qquad \forall\, p \in P,\ \ \forall\, g \in G_p \tag{5.11}$$

$$\sum_{p \in P} \sum_{g \in G_p} \sum_{t \in T: t > \|T\| - d_{p,g} - 1} Z_{p,g,t} = 0; \tag{5.12}$$

$$\left(\sum_{t \in T} t * Z_{p,g,t} - \sum_{t' \in T} t' * s_{p,g,t'} \right) \leq M * Push_{p,g} \ \ , \ \ \forall\, p \in P,\ \ \forall\, g \in G_p \tag{5.13}$$

$$\sum_{t \in T} D_{p,g,t} = d_{p,g} \ \ , \ \ \forall\, p \in P,\ \ \forall\, g \in G_p \tag{5.14}$$

$$d_{p,g} * Z_{p,g,t} \leq \sum_{h=0}^{d_{p,g}-1} D_{p,g,t+h}, \ \ \forall\, p \in P,\ \ \forall\, g \in G_p,\ \ \forall\, t \in T: t \leq \|T\| - d_{p,g} - 1 \tag{5.15}$$

Overlaping procedures:

$$X_{p,b,t} \leq \left(1 - D_{p,g,t}\right) \ , \ \ \forall\, p \in P,\ \ \forall\, b \in B,\ \ \forall\, t \in T,\ \ \forall\, g \in G_p \tag{5.16}$$

$$\sum_{g \in G_p} D_{p,g,t} \leq 1 \ \ , \ \ \forall\, p \in P, \quad \forall\, t \in T \tag{5.17}$$

Non– dialysis procedure fixed schedule:

$$\sum_{t' \in T} t' * s_{p,g,t'} = \sum_{t \in T} t * Z_{p,g,t} \quad , \quad \forall\, p \in P,\ \ \forall\, g \in G_p: A_{p,g} = 1 \tag{5.18}$$

Mobile station requirement:

$$\sum_{b \in B: \boldsymbol{b} > \|B\| - MB} \sum_{t \in T} X_{p,b,t} = l_p \quad , \quad \forall\, p \in P: m_p = 1 \tag{5.19}$$

The objective function (5.0) minimizes total penalties associated with assigning dialysis procedures to different time-slots or work periods, such that the dialysis unit maximizes the efficiency of using its stations while minimizing of the number and length (tardiness) of delays of other scheduled procedures that inpatients must undergo elsewhere in the hospital. The objective function (5.0) can simultaneously penalize when during the day dialysis is scheduled. This penalty system allows the decision-makers to control the tradeoffs between using single-unit (mobile station) blocks and using multiple-units (fixed station) blocks. For example, given that mobile stations require dedicated nursing time to provide patient care, and that in fixed stations the nursing time could be shared with other patients in the unit, the decision-makers may favor the use of fixed stations. In addition, the decision-makers can control the tradeoffs between the number of inpatients that may be affected by delays in the schedule of their non-dialysis treatments and the length of the delays. For example, it may be better to reschedule and delay 1 inpatient for 4 hours than to reschedule and delay 4 inpatients for 1 hour each, or vice-versa. Furthermore, the decision-makers can manage the preference of appointments time through the day. For example, the dialysis units may prefer to schedule dialysis procedures as early in the day as possible to minimize the potential need of overtime, thus penalty may be applied for dialysis scheduled in later time-slots.

In order to ensure that the scheduling of dialysis treatments is adequate and meets the planning objectives, the set of constraints (5.1 - 5.19) must be met. Constraint (5.1) guarantees that all inpatients expected to have dialysis are treated during the considered day. Along with constraint (5.1), constraints (5.2) and (5.3) ensure that if a dialysis procedure occurs over multiple periods, it happens uninterruptedly in the same station. Particularly, constraint (5.2)

allows scheduling dialysis over a consecutive number of periods while constraint (5.3) determines the number of consecutive periods required to ensure the continuity of treatment based on the length of the dialysis procedure. For example, an inpatient requiring 3 periods of dialysis needs 2 consecutive periods to avoid interruptions in his procedure. Constraint (5.4) establishes the last working period of the day by when inpatients must be scheduled to undergo dialysis. Constraint (5.5) ensures that during a working period the capacity of each block of stations is not exceeded. Similarly, constraint (5.6) guarantees that the total personnel time available per period is not exceeded.

Constraints (5.7), (5.8) and (5.9) help control the implementation of isolation protocols to prevent the propagation of infectious diseases. Constraint (5.7) ensures that an inpatient can be assigned to a block of stations only if inpatients with the same isolation needs are receiving care in the same block at a given period. Constraint (5.8) guarantees that at any given period, each block of stations cares for inpatients with at most one type of isolation needs. Therefore, constraint (5.8) prevents cross contamination among inpatients who may have different conditions or infectious diseases. Constraint (5.9) ensures that if there are no inpatients scheduled to undergo dialysis at a given dialysis block in a period, then there should not be any medical condition assigned to that block at that period.

Constraints (5.10 - 5.17) manage the non-dialysis procedures an inpatient may have and their rescheduling (if needed). Constraint (5.10) ensures that the non-dialysis procedures start as close as possible to their appointment time. Constraint (5.11) ensures that each scheduled non-dialysis procedure occurs at most once during the day. Constraint (5.12) forbids starting non-

dialysis procedures in the periods where the continuity of treatment is not guaranteed (e.g., if the length of the procedure is 5 and there are 20 working periods, then the procedure cannot be scheduled to start after period 16 as it would not allow the procedure to be completed during the time horizon). Constraint (5.13) detects or "flags" when a non-dialysis treatment has been pushed or delayed. Constraint (5.14) ensures that non-dialysis procedures are performed according to their expected duration. Constraint (5.15) guarantees the continuity of the non-dialysis treatment over a consecutive number of periods. Constraint (5.16) prevents scheduling dialysis procedures in periods dedicated for non-dialysis procedures, while constraint (5.17) prevents overlapping between the non-dialysis procedures. Constraint (5.18) specifies that the non-dialysis procedures that cannot be rescheduled must respect their appointment time, i.e. must start as scheduled. Finally, Constraint (5.19) ensures that inpatients that must undergo dialysis in single-unit blocks undergo dialysis in single-units (mobile station) blocks.

5.2.1 Modification to personnel constraint

Decision-makers may find it easier to indicate the number of personnel available and the number of stations each personnel can serve at a time. (e.g., a nurse could serve the 2 dialysis machines in a multiple-units (fixed stations) block; whereas the nurse could only serve 1 single-unit (mobile station) bock at a time). Therefore, to implement such considerations, parameters n_t and r_b must be redefined as follows:

n_t : Number of personnel available to manage dialysis procedures per period $t \in T$

r_b : Number of dialysis stations per type of block $b \in B$ a personnel can manage at the same time

In addition, constraint (5.6) must be changed with constraint (5.20), which essentially prevents exceeding the available personnel capacity per working period:

Personnel:

$$\sum_{p \in P} \sum_{b \in B} X_{p,b,t} / r_b \le n_t \quad , \quad \forall\, t \in T : t \le DTime \tag{5.20}$$

For the illustrative example and the experimental performance evaluation, the original model with constraint (5.6) described in section 5.2 is considered.

5.3 Illustrative Example

This section discusses the results of applying the aforementioned optimization approach to an illustrative example. The implementation was performed in GUROBI through an AMPL interface. Refer to Appendix C for data and output file.

Consider the problem discussed in chapter 4, corresponding to a problem instance for which a dialysis unit is comprised of 6 nurses, 6 blocks of stations, 3 of which have 2 fixed dialysis units, and 3 are single-unit (i.e., mobile station) blocks. The hospital works for 24 hours, but the dialysis unit operates for 16 hours (10 regular hours and 6 overtime hours) and divides its working time in time-slots or periods of 2 hours each. It is assumed that the nursing time required to manage the dialysis procedure of an inpatient in a fixed station is 0.50 hours, whereas an inpatient assigned to a mobile station requires a nurse to be present during the whole procedure. Equivalently, this assumption can be understood as if a nurse can manage the dialysis treatments occurring at a given block of stations, which means that a nurse can serve 2 inpatients

at the same time in a fixed station block and only 1 inpatient in a mobile unit block. There are 20 ESRD inpatients requiring dialysis; 12 inpatients do not have any isolation needs while 8 inpatients have isolation needs as detailed in Table 5.1 (each number represents a particular medical condition requiring isolation with 0 corresponding to no isolation requirement). In addition, the expected dialysis duration (in periods), the due times (in periods) and the mobile station requirement for each inpatient are also presented in Table 5.1. Figure 5.1 presents the appointed times for the non-dialysis procedures that have been scheduled for the set of 20 inpatients, as well as their isolation needs. Figure 5.1 also indicates the non-dialysis procedures that cannot be rescheduled.

Table 5.1: Dialysis length, isolation needs, due times, and requirements for mobile stations

Inpatient	**Dialysis length** (l_p)	**Isolation needs** (i_p)	**Due time** (w_p)	**Mobile station req.** (m_p)
1	2	0 (None)	None	0 (No)
2	3	0 (None)	3	0 (No)
3	1	0 (None)	1	0 (No)
4	3	0 (None)	None	0 (No)
5	2	0 (None)	None	0 (No)
6	2	0 (None)	None	0 (No)
7	2	0 (None)	None	0 (No)
8	2	0 (None)	None	0 (No)
9	2	0 (None)	None	0 (No)
10	2	0 (None)	None	0 (No)
11	2	0 (None)	None	0 (No)
12	2	0 (None)	None	0 (No)
13	2	1	None	0 (No)
14	2	1	1	0 (No)
15	2	2	None	1 (Yes)
16	2	2	None	1 (Yes)
17	2	3	None	0 (No)
18	1	3	None	0 (No)
19	1	3	None	0 (No)
20	3	4	None	1 (Yes)

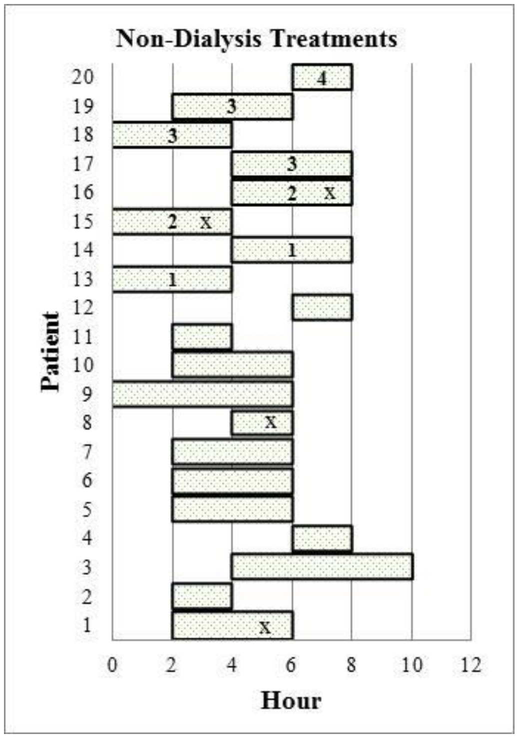

Figure 5.1: Non-dialysis procedures appointments and isolation needs
('x' marks procedures that cannot be delayed)

For this example, it is assumed that undergoing dialysis with a mobile station is twice as expensive as with a fixed station due to the need of dedicated personnel time. Moreover, in order to avoid the likelihood of overtime, the penalty of having dialysis is assumed to increase during the day (refer to Table 5.2 for penalty per period). In addition, the optimization model is solved considering different set of priority weights, which define three different scheduling settings related to the possible objectives of the decision-makers: the baseline (dialysis priority) setting, the non-dialysis priority setting, and the balanced approach setting. The baseline setting, based on our perception of how dialysis units currently schedule, consists of prioritizing the dialysis treatment, where the dialysis unit schedules procedures without any concern for the treatments in other hospital units. In the other extreme, the non-dialysis priority setting focuses on non-dialysis

procedures, enforcing the need to avoid any delay in such procedures. Finally, in the balanced approach, dialysis procedures are scheduled seeking an equal penalty balance of efficiency of the dialysis unit and delays of non-dialysis procedures. To model the different settings, the magnitudes of the scheduling penalties were defined relatively to each other. For the dialysis priority approach, penalties for delaying non-dialysis treatments received very low values in comparison to using fixed and mobile stations (efficiency of the unit), whereas in the non-dialysis priority approach these penalties were large. For the balanced approach, the penalties were assumed to be equal in magnitude. Table 5.3 presents the penalties for each setting. Note that a more in-depth assessment of these settings is discussed in section 5.5.

Table 5.2: Penalties associated with appointment time

Period	Penalty (α_t)
1	0.1
2	0.2
3	0.3
4	0.4
5	0.5
6	6.0
7	6.1
8	6.2

Table 5.3: Penalties associated with using dialysis stations and delaying other scheduled procedures for the 3 scheduling alternatives

Setting	Fixed Station (β_b^F)	Mobile Station (β_b^M)	Rescheduling (δ)	Tardiness (τ)
Dialysis Priority	1	2	0.001	0.001
Non-dialysis Priority	1	2	15	15
Balanced Approach	1	2	1	1

The results of applying the optimization approach to the example problem under the three settings are summarized in Table 5.4 and Figures 5.2 and 5.3. As expected, the solution of the dialysis priority setting experienced a higher number of non-dialysis treatment delays. A total of 4 inpatients had their non-dialysis procedure rescheduled, with an overall time delay or tardiness of 16 hours. For the non-dialysis priority setting, there were no delays in the other hospital units. However, the dialysis unit encountered higher costs resulting from scheduling 2 additional hours of overtime and lower overall machine utilization in regular service hours. Finally, under the balanced approach setting, only 1 inpatient had his non-dialysis procedure rescheduled with a total time delay of 2 hours. Moreover, working overtime was not required to provide dialysis services. Results from Table 5.4 show how, with the implemented penalty system, the unit's decision-makers can control the tradeoffs between different scheduling settings, and thus create efficient dialysis schedules. Further experimentation is performed in section 5.5 to provide a more profound assessment of the capability of the optimization approach and the tradeoffs of the scheduling settings.

Table 5.4: Summary table for the optimal schedules for the 3 scheduling alternatives

Category	Dialysis Priority	Non-Dialysis Priority	Balanced Approach
Number of non-dialysis procedures delayed	4	0	1
Total tardiness (Hrs.)	16	0	2
Fixed stations overall utilization	96.67%	87.10%	90.00%
Mobile stations overall utilization	73.33%	86.67%	86.67%
Dialysis unit Operating hours	10	12	10

Figure 5.2 shows the inpatient treatment schedules for the three described settings, highlighting the treatments delayed, and illustrates that procedures that could not be delayed remained scheduled as planned. Figure 5.3 presents the dialysis treatment scheduled for the dialysis unit, describing the sequence of inpatients that use each machine during the day.

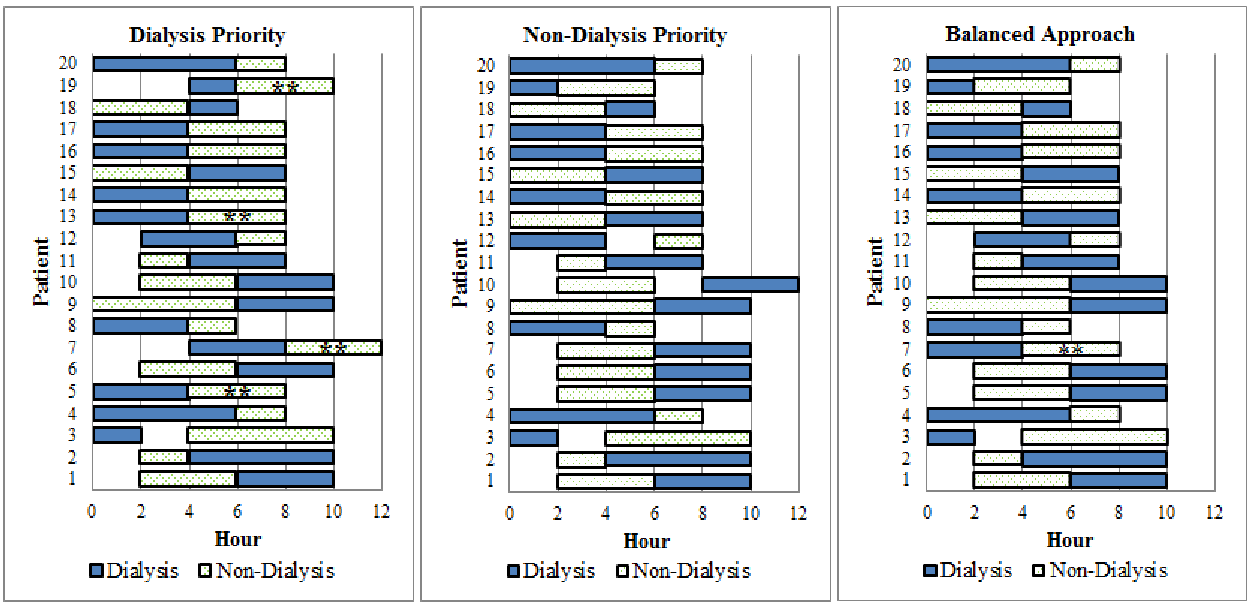

Figure 5.2: Inpatient treatment schedules for the 3 scheduling alternatives
(** represents the rescheduled treatments)

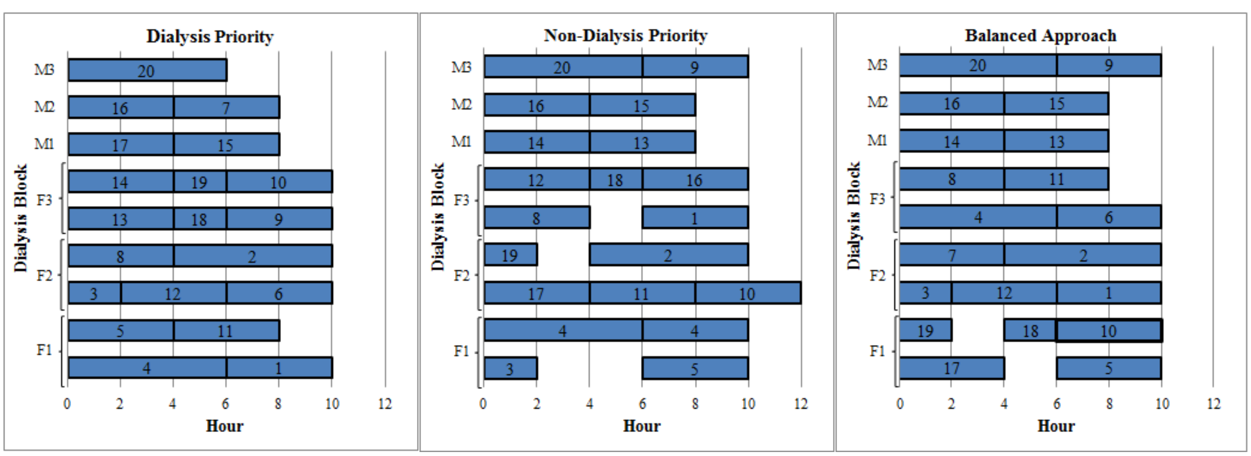

Figure 5.3: Dialysis unit schedules for the 3 alternatives (inpatient number indicated on bar)

5.4 Computationally Efficient Algorithm for Solving the Optimization Method

As described in chapter 4, it is very important to determine the granularity, i.e., the length of the working periods of the operating hours, in which the inpatient hemodialysis scheduling problem must be solved. Specifically, a granularity of 15-minutes periods is considered to provide efficient solutions, since it allows decision-makers to manage the starting and ending time of procedures more efficiently. As the granularity is smaller (i.e., as the number of working periods increases), the number of decision variables increases extending the solution time considerably. To generate solutions in a useful time, this section discusses how to interact with the solver by solving each optimization problem through a sequence of phases. A phase correspond to the steps taken to solve the optimization problem (e.g., solving a problem in two phases means solving the problem to provide an initial guess (phase A) and solve it again with a greater level of detail (phase B)). In particular, a fixed budgeted amount of time, the optimality gap tolerance and different sequences of phases are considered.

To efficiently solve any instance of the proposed optimization problem with the solver for small granularity, we considered the following alternatives:

- *Solve problem in 1 phase*: Solve the problem directly with 15-mins periods.
- *Solve problem in 2 phases*: Provide an initial solution from phase A with 2-hrs periods, and then solve the phase B with 15-mins periods.
- *Solve problem in 3 phases*: Add an intermediate step to have phase A with 2-hrs periods, phase B with 1-hr periods and finally phase C with 15-mins periods.

Moreover, this study evaluated 5 random instances with the general characteristics described in Table 5.5 with a budgeted time of 20 minutes.

Table 5.5: General characteristics of experimental instances to design solving algorithm

Parameter	Value
Expected utilization	80%
Dialysis hours of operation	16 (10 of regular time)
Dialysis unit regular time capacity (hrs.)	120 (4 2-fixed dialysis stations blocks; 4 1-mobile dialysis station blocks)
Nurses available per working period	8
Dialysis length (15-mins.)	3%-UNIF(8, 11); 32%-UNIF(11, 13); 61%-UNIF(13, 17); 4%-UNIF(17, 33)
Number of different conditions (isolation needs) per instance	10%-1, 30%-2, 28%-3, 25%-4, 7%-5
Inpatients with isolation needs	UNIF(20%, 40%)
Inpatients with due time to undergo dialysis	10%
Inpatients with mobile station requirement	10% if $i_p \neq 0$; 5% if $i_p = 0$
Non-dialysis procedures assignable per instance	10%-0; 60%-1; 25%-2; 5%-3
Non-dialysis procedures length (hrs.)	UNIF(2, 6)
Non-dialysis procedures that cannot be rescheduled	5%

Solving the problem directly with 15-mins periods did not provide any feasible answer in the 20 minutes limit, which suggests at least one additional phase must be included in order to provide an initial feasible solution to the desired granularity of the problem.

To solve the problem in 2 phases (2-hrs – 15-mins), two alternatives were evaluated:

- *Alternative 1*: Assign 1/3 of time (6.67 minutes) and a 1% optimality gap tolerance to phase A with 2-hrs periods and the rest of the time (13.33 minutes) and 10% optimality gap tolerance to phase B with 15-mins periods.
- *Alternative 2*: Assign 2 minutes and 1% optimality gap tolerance to phase A and 18 minutes and 10% optimality gap tolerance to phase B.

As shown in Tables 5.6 and 5.7, providing an initial feasible solution is beneficial. Solving phase A with 2-hrs periods can yield an optimal solution in less than 2 minutes, thus it enables allocating the rest of the time to the phase B with 15-mins periods. Nonetheless, for this particular study, the comparison between Tables 5.6 and 5.7 indicates that providing more time to the final phase was better only for the last instance.

Table 5.6: Results for the first alternative of 2 phases (2-hrs – 15-mins)

Phase A – 2-hrs Instances (1% Tolerance)							
Instance	**Initial Gap**	**Initial Objective**	**Initial Bound**	**GAP**	**Objective**	**Bound**	**Time (min.)**
1	—	—	—	0.00%	103.700	103.700	1.22
2	—	—	—	0.00%	86.500	86.500	0.73
3	—	—	—	0.00%	82.400	82.400	0.32
4	—	—	—	0.00%	85.800	85.800	0.42
5	—	—	—	0.00%	109.200	109.200	1.40
Phase B – 15-mins Instances (10% Tolerance)							
Instance	**Initial Gap**	**Initial Objective**	**Initial Bound**	**GAP**	**Objective**	**Bound**	**Time (min.)**
1	28.10%	89.375	64.250	21.19%	81.525	64.250	13.33
2	22.30%	73.325	56.988	14.72%	67.025	57.160	13.33
3	25.30%	71.575	53.500	22.40%	69.090	53.585	13.33
4	20.30%	71.825	57.213	15.98%	68.138	57.247	13.33
5	24.10%	84.238	63.975	20.52%	80.525	63.398	13.33

Table 5.7: Results for the second alternative of 2 phases (2-hrs – 15-mins)

Phase A – 2-hrs Instances (1% Tolerance)							
Instance	**Initial Gap**	**Initial Objective**	**Initial Bound**	**GAP**	**Objective**	**Bound**	**Time (min.)**
1	—	—	—	0.00%	103.700	103.700	0.87
2	—	—	—	0.00%	86.500	86.500	0.51
3	—	—	—	0.00%	82.400	82.400	0.29
4	—	—	—	0.00%	85.800	85.800	0.33
5	—	—	—	0.00%	109.200	109.200	1.03
Phase B – 15-mins Instances (10% Tolerance)							
Instance	**Initial Gap**	**Initial Objective**	**Initial Bound**	**GAP**	**Objective**	**Bound**	**Time (min.)**
1	28.10%	89.375	64.250	21.19%	81.525	64.250	18.00
2	22.30%	73.325	56.988	14.76%	67.025	57.135	18.00
3	25.30%	71.575	53.500	22.40%	69.050	53.585	18.00
4	20.30%	71.825	57.213	15.98%	68.138	57.247	18.00
5	24.10%	84.238	63.975	14.04%	74.425	63.975	18.00

Following the 2 phases trial, an experiment was run with 3 phases (2-hrs – 1-hr – 15-mins) under the two following alternatives:

- *Alternative 1*: Assign 2 minutes and 1% optimality gap tolerance to phase A with 2-hrs periods, 6.67 minutes and 5% optimality gap tolerance to phase B with 1-hr periods, and finally, 11.33 minutes and 10% optimality gap tolerance to phase C with 15-mins periods.
- *Alternative 2*: Assign 2 minutes to phase A but assign a considerable amount of time to the next phases to assess if a better solution can be achieved when the second phase is run for a longer time (larger than the budgeted time of 20 minutes). Specifically, we assign 60 minutes to phase B and 30 minutes to phase C. Each phase was assigned a 1% optimality gap tolerance.

The results from Tables 5.8 and 5.9 indicate that having an additional phase (with 1-hr periods) provides similar or better solutions in the same amount of time. Better solutions can be obtained because by stepping down in the sequence of phases, the solutions are rectified and improved by eliminating the extra slack that is needed to solve the problem in the phase A (e.g., an inpatient may require only 2.5 hours of dialysis (10 15-mins periods), which would correspond to 4 hours (2 2-hrs periods) in the phase and only 3 hours (3 1-hr periods) in phase C). Moreover, notice that running the solver for a longer time may not necessarily provide a better solution. This is, in general, because the previous phases provide optimal, or near optimal solutions, thus only the bound may increase. In Figures 5.4 to 5.8, it can actually be noticed that for both phases B and C the best possible solution is reached in less than 600 seconds.

Table 5.8: Results for the first alternative of 3 phases (2-hrs – 1-hr – 15-mins)

Phase A – 2-hrs Instances (1% Tolerance)							
Instance	**Initial Gap**	**Initial Objective**	**Initial Bound**	**GAP**	**Objective**	**Bound**	**Time (min.)**
1	—	—	—	0.00%	103.700	103.700	1.57
2	—	—	—	0.00%	86.500	86.500	0.58
3	—	—	—	0.00%	82.400	82.400	0.39
4	—	—	—	0.00%	85.800	85.800	0.56
5	—	—	—	0.00%	109.200	109.200	1.11
Phase B – 1-hr Instances (5% Tolerance)							
Instance	**Initial Gap**	**Initial Objective**	**Initial Bound**	**GAP**	**Objective**	**Bound**	**Time (min.)**
1	22.80%	94.100	72.650	14.56%	91.700	78.350	6.67
2	19.20%	81.600	65.900	10.35%	74.400	66.700	6.67
3	24.40%	75.700	57.250	12.96%	71.350	62.100	6.67
4	19.50%	80.500	64.800	12.95%	77.250	67.250	6.67
5	24.40%	93.050	70.350	6.46%	77.350	72.350	6.67

Continuation of Table 5.8: Results for the second alternative of 3 phases

Phase A – 2-hrs Instances (10% Tolerance)							
Instance	**Initial Gap**	**Initial Objective**	**Initial Bound**	**GAP**	**Objective**	**Bound**	**Time (min.)**
1	23.60%	84.050	64.250	20.26%	80.575	64.250	11.33
2	14.90%	66.950	56.988	12.40%	65.137	57.037	11.33
3	21.20%	67.913	53.500	18.68%	65.875	53.500	11.33
4	16.80%	68.788	57.213	13.87%	66.438	57.225	11.33
5	11.40%	72.238	63.945	9.88%	70.988	63.975	2.55

Table 5.9: Results for the second alternative of 3 phases (2-hrs – 1-hr – 15-mins)

Phase A – 2-hrs Instances (1% Tolerance)							
Instance	**Initial Gap**	**Initial Objective**	**Initial Bound**	**GAP**	**Objective**	**Bound**	**Time (min.)**
1	—	—	—	0.00%	103.700	103.700	1.57
2	—	—	—	0.92%	86.500	85.700	1.65
3	—	—	—	0.00%	82.400	82.400	0.39
4	—	—	—	0.00%	85.800	85.800	0.56
5	—	—	—	0.00%	109.200	109.200	1.11
Phase B – 1-hr Instances (1% Tolerance)							
Instance	**Initial Gap**	**Initial Objective**	**Initial Bound**	**GAP**	**Objective**	**Bound**	**Time (min.)**
1	22.80%	94.100	72.650	9.48%	89.750	81.250	60.00
2	18.70%	81.100	65.900	5.04%	74.400	70.650	60.00
3	23.70%	75.000	57.250	1.75%	71.650	70.000	60.00
4	19.50%	80.500	64.800	11.78%	77.250	68.150	60.00
5	24.40%	93.050	70.350	4.46%	77.350	73.900	60.00
Phase C – 15-min Instances (1% Tolerance)							
Instance	**Initial Gap**	**Initial Objective**	**Initial Bound**	**GAP**	**Objective**	**Bound**	**Time (min.)**
1	23.30%	83.800	64.250	19.50%	79.863	64.289	30.00
2	15.10%	67.088	56.988	14.51%	66.838	57.140	30.00
3	21.10%	67.813	53.500	19.20%	66.313	53.582	30.00
4	17.00%	68.913	57.213	14.06%	66.600	57.234	30.00
5	11.40%	72.238	63.945	7.62%	69.250	63.975	60.00

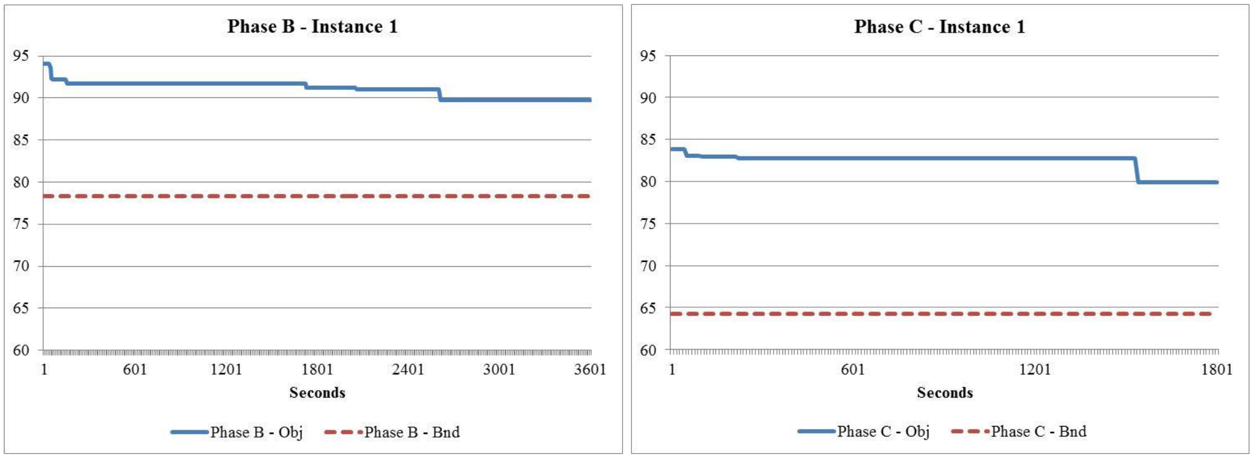

Figure 5.4: ‘Long Run’ objective vs. bound for phases B (left) and C (right) of instance 1

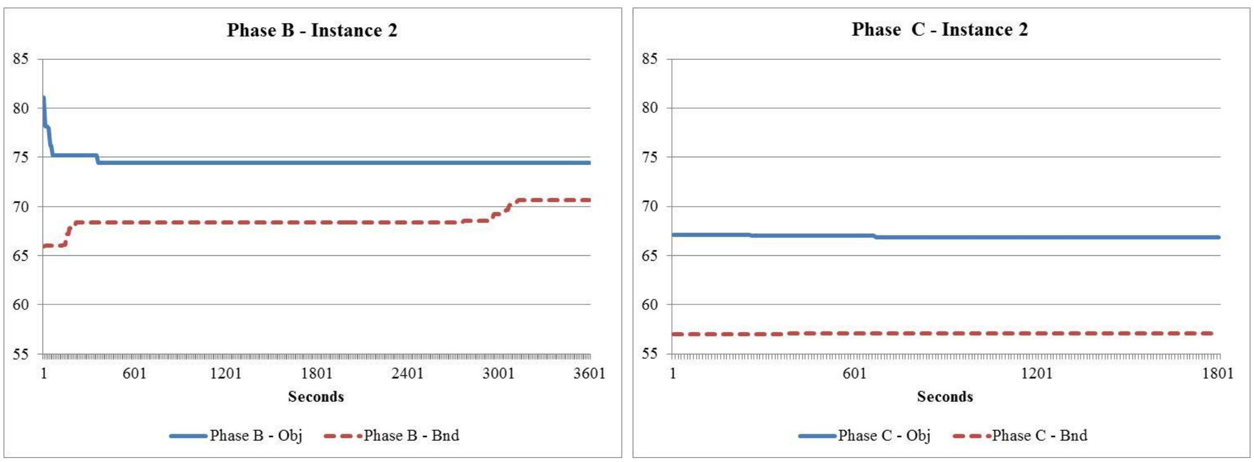

Figure 5.5: ‘Long Run’ objective vs. bound for phases B (left) and C (right) of instance 2

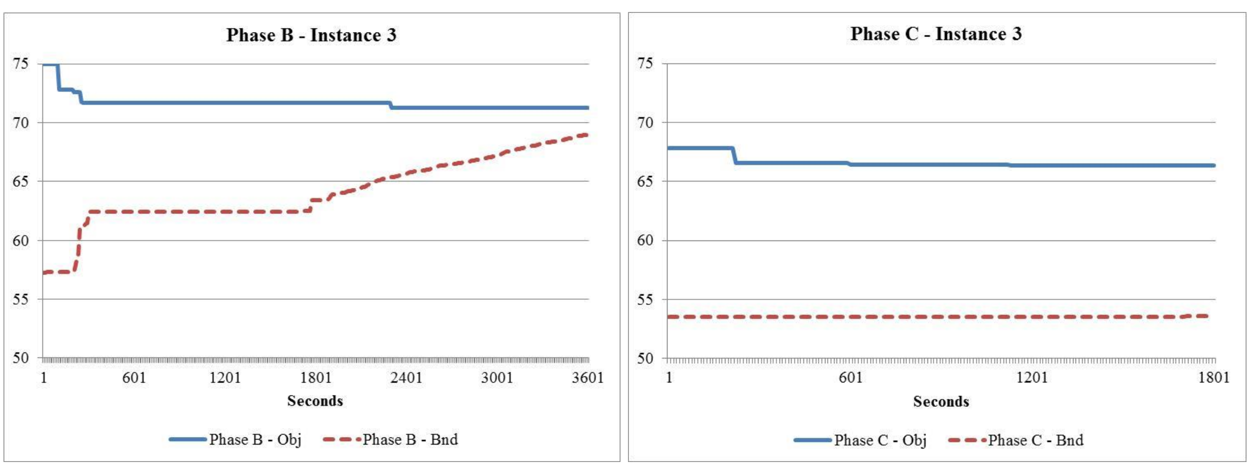

Figure 5.6: ‘Long Run’ objective vs. bound for phases B (left) and C (right) of instance 3

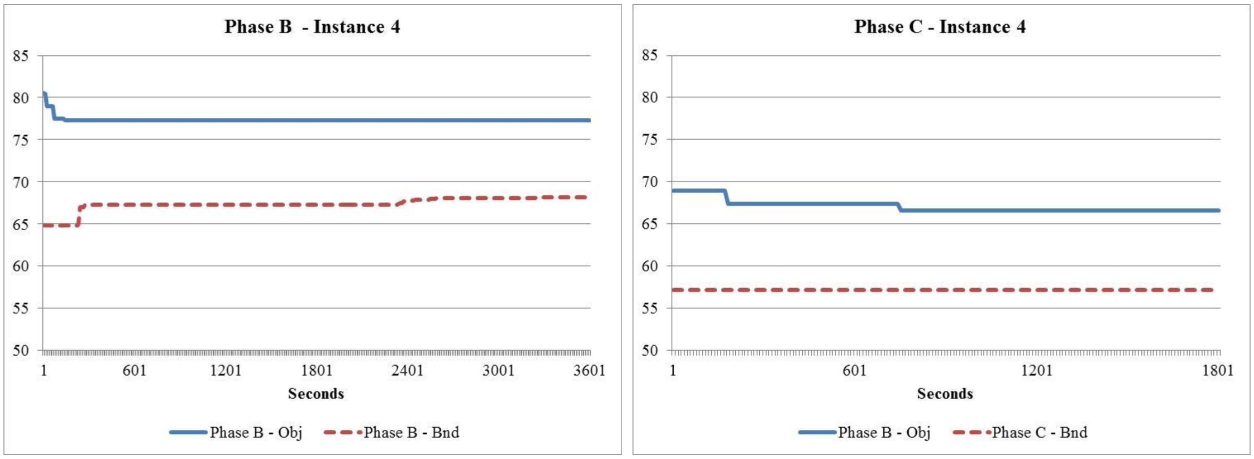

Figure 5.7: 'Long Run' objective vs. bound for phases B (left) and C (right) of instance 4

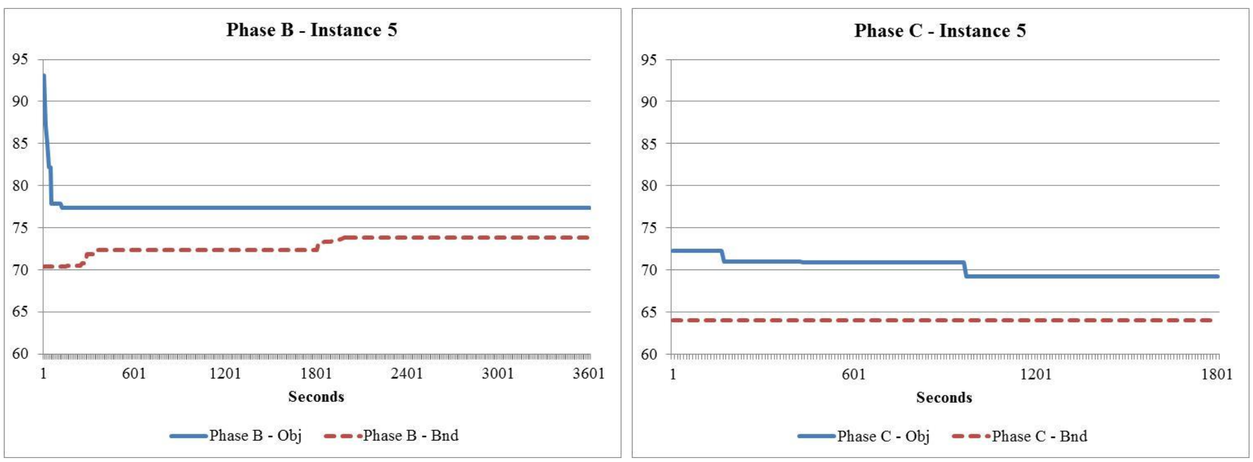

Figure 5.8: 'Long Run' objective vs. bound for phases B (left) and C (right) of instance 5

Given the previous results, for the experimental performance evaluation of this thesis we adopt the use of a 3-phases solving algorithm, allocating the time for the different phases in increasing amounts, e.g. 17% to phase A, 33% to phase B and, finally, 50% to phase C. In addition, the selected optimality gap tolerances for these tests suggest that it should be fixed to 1% for each phase, given that a better solution may be reached below the specified time limit. As a result, the selected solving algorithm for future experimentation is as follows:

- Solve phase A (2-hrs periods) with an allocated time of 5 minutes and 1% optimality gap tolerance.
- Use the solution from phase A as the initial basis, and solve phase B (1-hr periods) with an allocated time of 10 minutes and 1% optimality gap tolerance.
- Use the solution from Phase B as the initial basis, and solve phase C (15-mins periods) with a time limit of 15 minutes and 1% optimality gap tolerance.

Note that for practical and real life data, the solving algorithm could budget less time (e.g., 15 to 20 minutes total) to provide optimal, or near optimal solutions.

5.5 Experimentation

This section presents an experimental performance evaluation to investigate the capability and performance of the initial optimization approach. In terms of performance and efficiency, a good schedule allows:

- Administering dialysis to all the inpatients within the available time of the day (hours of operation) without breaking the prescribed protocols per inpatient and respecting the isolation needs of the inpatients.
- Having a high utilization rate of dialysis stations avoiding overtime hours.
- Performing non-dialysis procedures as close as possible to their original planned schedules.

In particular, a first set of experiments is designed to test how the first optimization method performs under different resource utilization rates of the dialysis unit across different

scheduling settings. In addition, given that the relative values for the penalties in the first set of experiments are arbitrarily chosen, a second set of experiments performs a sensitivity analysis of the weights given in the penalty system to assess the tradeoffs in the dialysis schedules and the resulting performance measures.

5.5.1 Definition of performance measures

Before discussing the experimental performance evaluation, this section defines the performance measures used to compare the different scenarios:

- *Number of other procedures delayed*: As its name states, this metric accounts the number of non-dialysis procedures that are pushed or delayed to accommodate the dialysis treatments.
- *Total tardiness of non-dialysis procedures (Hrs.)*: Total time delays that are registered for rescheduling or pushing non-dialysis procedures.
- *Fixed stations overall utilization*: Total number of busy time-slots or working periods using fixed stations over the total number of working periods in which the given stations were available for use. In other words, it measures the gaps or open spaces in which the fixed stations were not used during the hours of operation. For example, consider there are 2 fixed stations and 15 working periods (10 of regular hours and 5 of overtime hours). Station 1 is used continuously from period 1 to period 12, whereas Station 2 is used from period 1 to period 8 and from period 12 to period 14. The total utilization would be:

$$Fixed\ station\ overall\ utilization = \left(\frac{12 + (8 + 3)}{12 + (10 + 4)}\right) = 88.46\%$$

Note that the utilization of the second machine is comprised of 2 parts, the "regular time utilization" which is always over the regular working periods, and the "overtime

utilization" which considers the 'gaps' in which the machine was idle while the dialysis unit was still operating during the day.

- *Mobile stations overall utilization*: Similar to the fixed stations utilization, it consists of the total number of busy working periods using mobile stations over the total number of working periods in which the mobile stations were available for use.
- *Dialysis unit operating hours or completion time (Hrs.)*: Total time the dialysis unit needs to accommodate the dialysis demand of the current day.
- *Number of dialysis scheduled in overtime*: Number of inpatients that are scheduled to undergo dialysis after regular hours (overtime).

5.5.2 Experiments: Expected utilization across different scheduling priority settings

The first set of experiments focuses on two main factors a) the complexity of the scheduling problem in terms of the resource utilization rate (or expected utilization) of dialysis unit (i.e., the relation between the dialysis demand of inpatients and the capacity of the dialysis unit); and b) the priority given to the efficiency of the unit and the on-time delivery of other scheduled procedures. Factor b) can be considered an experimental block since the different levels of factor a) are evaluated under all levels of factor b).

The levels of the factors a) and b) are:

- *Factor A*: Complexity of the scheduling problem:
    1) 65% of expected utilization of the unit during regular operating hours.
    2) 80% of expected utilization of the unit during regular operating hours.
    3) 95% of expected utilization of the unit during regular operating hours.

- *Factor B*: Priority given to the efficiency of the dialysis unit and the on-time delivery of other scheduled procedures:
    1) Dialysis priority setting: penalties for delaying non-dialysis procedures were significantly low with regards to the penalties associated with assignment of dialysis inpatients (efficiency of the unit).
    2) Non-dialysis priority setting: penalties for delaying non-dialysis procedures were significantly high with regards to the penalties associated with the inpatients assignment (efficiency of the unit).
    3) Balanced approach: penalties of the system were equal in magnitude.

To evaluate the initial optimization method, 30 instances of each level of factor A were randomly generated with the general characteristics described in Table 5.10 and analyzed under the 3 priority settings (levels of factor B). Table 5.11 shows the general penalties associated with the appointment time for dialysis and the usage of the dialysis machines. Table 5.12 indicates the relative penalty weights associated with using dialysis stations and delaying of other scheduled procedures for the three scheduling alternatives.

Table 5.10: General characteristics for minimum tardiness experimental instances

Parameter	**Value**
Dialysis hours of operation	16 (10 of regular time)
Dialysis unit regular time capacity (hrs.)	120 (4 2-fixed dialysis stations blocks; 4 1-mobile dialysis station blocks)
Nurses available per working period	8
Dialysis length (15-mins.)	1%-2; 36%-3; 62%-4; 1%-UNIF(18, 32)
Number of different conditions (isolation needs) per instance	10%-1; 30%-2; 28%-3; 25%-4; 7%-5
Inpatients with isolation needs	UNIF(20%, 40%)
Inpatients with due time to undergo dialysis	15%
Inpatients with mobile station requirement	10% if $i_p \neq 0$; 5% if $i_p = 0$
Non-dialysis procedures assignable per instance	10%-0; 55%-1; 30%-2; 5%-3
Non-dialysis procedures length (hrs.)	UNIF(2, 6)
Non-dialysis procedures that cannot be rescheduled	5%

Table 5.11: Penalties associated with appointment time

Period	**Penalty (α_t)**
1	0.1
2	0.2
3	0.3
4	0.4
5	0.5
6	6.0
7	6.1
8	6.2

Table 5.12: Penalties associated with using dialysis stations and delaying other scheduled procedures for the 3 alternatives (magnitudes corresponding to Phase A (2hrs))

Setting	Fixed station (β_b^F)	Mobile station (β_b^M)	Rescheduling (δ)	Tardiness (τ)
Dialysis Priority	1	2	0.001	0.001
Non-dialysis Priority	1	2	30	30
Balanced Approach	1	2	1	1

5.5.2.1 Cases with 65% of expected utilization across different scheduling settings

This experimentation considers problem instances with an average demand of 65% of the capacity of the dialysis unit during regular operating hours (level 1 of factor A), in which the sets of inpatients are scheduled considering the 3 different scheduling priority settings (levels of factor B).

The results from the schedules of the 30 simulated instances of an inpatient dialysis unit with an average of 65% demand are summarized in Table 5.13. For detailed results per instance refer to Appendix D. As expected, on average the schedules provided under the dialysis priority setting experienced a higher number of non-dialysis procedures delayed, while the non-dialysis priority setting provided on average the fewest delays. In addition, higher costs are expected to be incurred in the non-dialysis priority approach since the dialysis unit required a considerable amount of overtime to serve all inpatients.

Table 5.13: 65% expected utilization across scheduling settings summarized results

Category	Dialysis Priority		Non-Dialysis Priority		Balanced Approach	
	Avg.	Std. Dev.	Avg.	Std. Dev.	Avg.	Std. Dev.
Number of non-dialysis procedures delayed	6.90	2.34	2.13	1.48	4.30	2.05
Total tardiness (Hrs.)	21.13	12.60	4.27	4.62	8.98	6.47
Fixed stations overall utilization	81.68%	5.03%	78.96%	11.67%	80.78%	5.74%
Mobile stations overall utilization	31.67%	13.02%	24.73%	11.67%	31.67%	11.28%
Dialysis unit operating hours	10.12	0.95	14.11	1.77	10.63	1.49
Number of dialysis scheduled in overtime	0.07	0.25	3.00	1.46	0.80	1.06

5.5.2.2 Cases with 80% of expected utilization across different scheduling settings

This experimentation considers problem instances with an average demand of 80% of the capacity of the dialysis unit during regular operating hours (level 2 of factor A), in which the sets of inpatients are scheduled under the 3 different scheduling priority settings (levels of factor B).

Table 5.14 summarizes the results from the schedules of the 30 simulated instances of an inpatient dialysis unit with an 80% expected resource utilization rate. For detailed results per instance refer to Appendix D. Just as in the previous case (65% resource utilization rate), the balanced approach seems to provide the most efficient schedules for the hemodialysis inpatients.

Table 5.14: 80% expected utilization across scheduling settings summarized results

Category	Dialysis Priority		Non-Dialysis Priority		Balanced Approach	
	Avg.	**Std. Dev.**	**Avg.**	**Std. Dev.**	**Avg.**	**Std. Dev.**
Number of non-dialysis procedures delayed	6.79	2.22	2.43	1.48	4.79	2.53
Total tardiness (Hrs.)	19.02	10.57	5.60	4.65	10.90	6.60
Fixed stations overall utilization	89.68%	3.05%	87.85%	14.27%	88.70%	3.03%
Mobile stations overall utilization	60.31%	10.53%	47.08%	14.27%	61.41%	10.95%
Dialysis unit operating hours	10.42	0.74	13.89	1.90	10.73	0.76
Number of dialysis scheduled in overtime	0.50	1.07	3.29	2.24	0.82	0.98

5.5.2.3 Cases with 95% of expected utilization across different scheduling settings

This experimentation considers problem instances with an average demand of 95% of the capacity of the dialysis unit during regular operating hours (level 3 of factor A), in which the sets of inpatients are scheduled under the 3 different scheduling priority settings (levels of factor B).

Table 5.15 summarizes the results from the schedules of the 30 simulated instances of an inpatient dialysis unit with 95% expected resource utilization rate For detailed results per instance refer to Appendix D. The results here follow the same pattern described for the previous two cases.

Table 5.15: 95% expected utilization across scheduling settings summarized results

Category	Dialysis Priority		Non-Dialysis Priority		Balanced Approach	
	Avg.	**Std. Dev.**	**Avg.**	**Std. Dev.**	**Avg.**	**Std. Dev.**
Number of non-dialysis procedures delayed	6.57	3.21	2.46	1.95	5.07	2.64
Total tardiness (Hrs.)	18.48	11.02	5.35	6.36	10.93	7.33
Fixed stations overall utilization	94.66%	2.42%	91.93%	13.02%	94.50%	2.11%
Mobile stations overall utilization	82.36%	6.62%	69.31%	13.02%	80.75%	7.03%
Dialysis unit operating hours	11.82	0.48	14.46	1.32	12.12	1.10
Number of dialysis scheduled in overtime	3.00	1.52	4.57	1.75	3.32	1.31

Figures 5.9 to 5.11 illustrate the performance measures for this set of experiments:

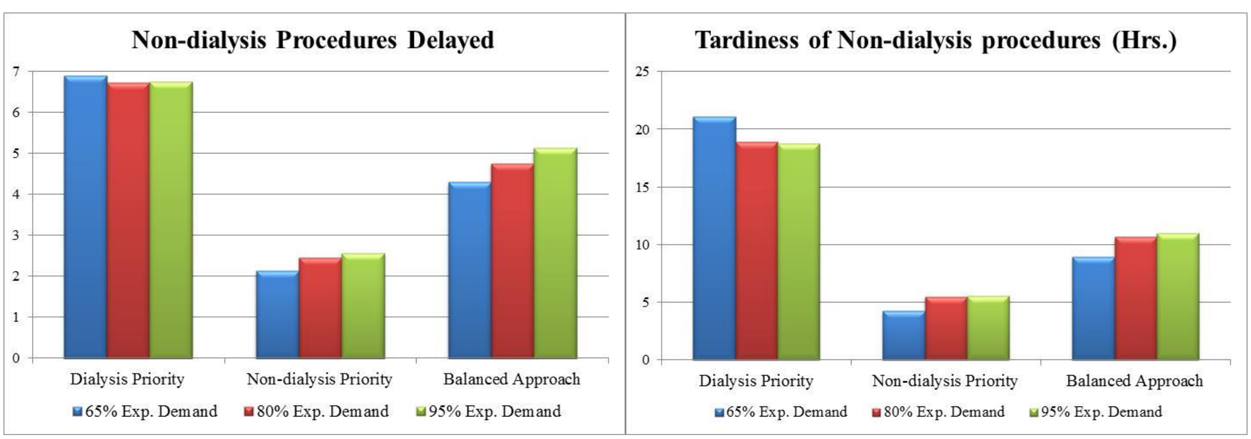

Figure 5.9: Average number of procedures delayed (left) and tardiness (right) for studied cases

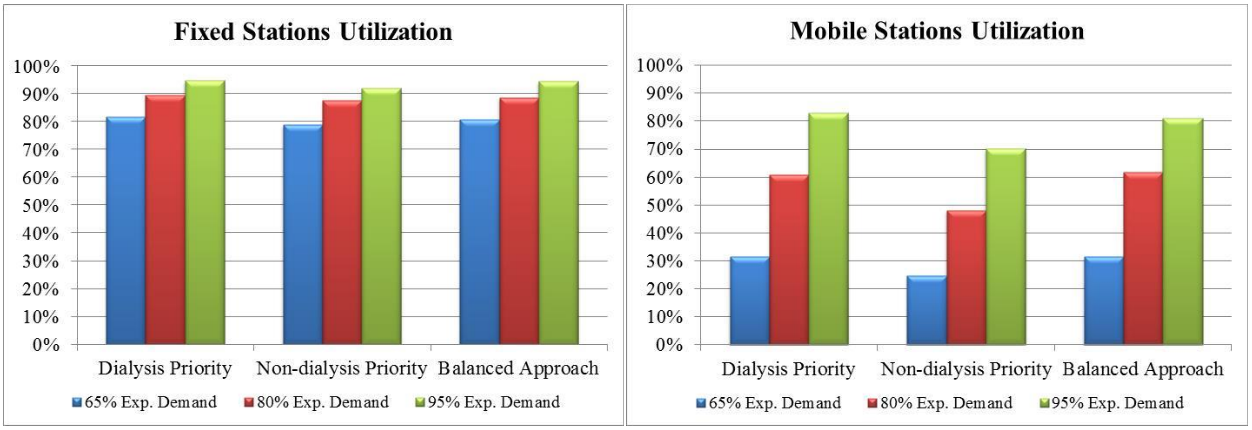

Figure 5.10: Utilization of fixed stations (left) and mobile stations (right) for studied cases

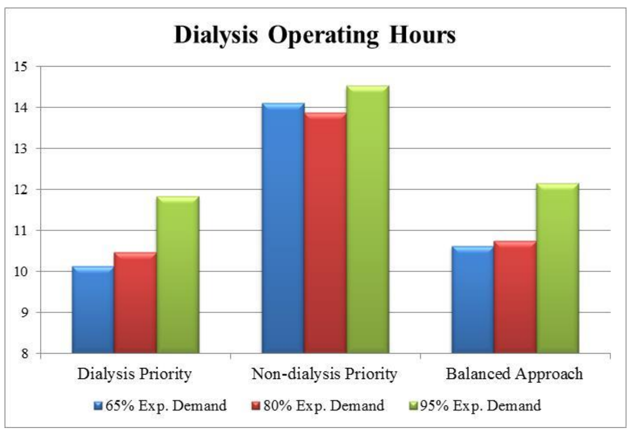

Figure 5.11: Dialysis operating hours for studied cases

5.5.2.4 Discussion of results

The optimization approach in this research provides an optimal, or near optimal solution to the inpatient hemodialysis scheduling problem regardless of the expected resource utilization rates experienced by the dialysis unit or the scheduling setting considered. As appreciated in Figures 5.9 to 5.11, scheduling under the dialysis priority setting resulted on average in a higher number of procedures delayed and a more efficient use of the dialysis unit, i.e., less time required to accommodate all inpatients and higher utilizations of the dialysis machines. On the other hand,

scheduling under the non-dialysis priority setting presented on average fewer delays but lower utilization of the dialysis machines since the dialysis unit required more hours to serve all dialysis demand. Nonetheless, notice that even when scheduling under the dialysis priority setting, the method still tries to accommodate as many treatments as possible, and that when scheduling under the non-dialysis priority setting, it still tries to be as efficient as possible.

Moreover, the results from Tables 5.13 to 5.15 show that the balanced approach setting is practically a perfect mix of the two other alternatives since it tries to accomplish the benefits of both at the same time, and thus it provides the most efficient schedules. Specifically, the efficiency of the unit (in terms of utilization and operating hours) was very close to the efficiency achieved under the dialysis priority setting while the number of non-dialysis procedures rescheduled was in between the two other settings. Note that there are cases, such as Instance 2 in Tables D.1 to D.3 in Appendix D or instance 6 from Tables D.4 to D.6 in Appendix D, where the relative value of the penalties (the scheduling priority setting) does not have a significant impact on the resulting schedule. This is mainly due to the characteristics of the inpatients requesting medical services. Consequently, the results suggest that scheduling under the balanced approach setting provides a better schedule only when there are complicated choices to make other than the ones that can be taken under the extreme scheduling settings (dialysis priority and non-dialysis priority.)

On the other hand, the results suggest that there are no statistically significant differences in applying the optimization approach for different expected resource utilizations. Notice that if the dialysis demand is considerably low, then the schedules provided under the three different

alternatives would be similar. Nonetheless, the results show that the hours required to serve all hemodialysis inpatients and the number of inpatients treated in overtime increased with the expected demand. Likewise, the utilization of mobile stations increased with higher expected demand.

In conclusion, this set of experiments suggests that the proposed optimization method consistently provides optimal schedules related to the interests or alternative adopted by the decision-makers. However, considering the set of priority weights, scheduling under the balanced approach setting is recommended because it provides the best tradeoffs of the efficiency of the unit (in terms of station utilization and operating hours) and the on-time delivery of non-dialysis scheduled procedures.

5.5.3 Experiments: Tradeoffs of weights of penalties over resulting schedule

Since the relative penalty values used in the first set of experiments are arbitrary chosen, a second set of experiments is carried out to evaluate the sensitivity of the priority weights over the schedules and resulting performance measures. An initial experiment focuses on the penalty weights assigned to the priority of the efficiency (in terms of utilization and operating hours) of the unit and the on-time delivery of other scheduled procedures. Specifically, starting from the weights assigned in the balanced approach setting, the penalties for using dialysis stations were fixed (blocked) and the penalties for delaying procedures and the tardiness associated such delays were increased. Similarly, the penalties for delaying procedures were blocked, and the usage penalties of dialysis machines were increased. A second experiment focuses on the priority given to rescheduling and time delays when rescheduling of non-dialysis procedures is

imminent, i.e., priority given to the number of delays and the time of the delays. In particular, with the penalties for using dialysis stations blocked, the penalties for delaying procedures and tardiness were weighed under different proportions.

Table 5.16 displays the different configurations for the settings of experiment A, and Table 5.17 shows the different settings for experiment B. Note that using mobile stations is twice the cost of using fixed stations due to the dedicated personnel time required to manage the dialysis procedure. For the general appointment penalties, refer to Table 5.11.

Table 5.16: Penalties associated with experiment A: Priority of efficiency and on-time delivery of other scheduled procedures (magnitudes corresponding to Phase A (2hrs))

Setting (efficiency, delays/tardiness)	**Fixed station (β_b^F)**	**Mobile station (β_b^M)**	**Rescheduling (δ)**	**Tardiness (τ)**
(1,4)	1	2	4	4
(1,3)	1	2	3	3
(1,2)	1	2	2	2
(1,1) (Balanced Approach)	1	2	1	1
(2,1)	2	4	1	1
(3,1)	3	6	1	1
(4,1)	4	8	1	1

Table 5.17: Penalties associated with experiment B: Priority given to number of rescheduled procedures and time delays (magnitudes corresponding to Phase A (2hrs))

Setting (delays, tardiness)	**Fixed station (β_b^F)**	**Mobile station (β_b^M)**	**Rescheduling (δ)**	**Tardiness (τ)**
(2, 0)	1	2	2	0
(1.5, 0.5)	1	2	1.5	0.5
(1, 1) (Balanced Approach)	1	2	1	1
(0.5, 1.5)	1	2	0.5	1.5
(0, 2)	1	2	0	2

As stated in section 5.5.2.4, in the set of randomly generated instances there were some cases that resulted in very little, if any, conflict between the scheduling settings focused on efficiency of the dialysis unit (dialysis priority) and on-time delivery of other procedures (non-dialysis priority). This means that, for these cases, the selection of the penalty values does not have a significant impact on the resulting schedules, and therefore the resulting schedule would be the same regardless of what parameters are chosen, i.e., there is no correlation between the values of the penalties and the resulting schedules. Moreover, the expected resource utilization rate did not seem to impact significantly the results or the capability of the optimization method. Consequently, to evaluate the tradeoffs of how the penalties impact the schedule and resulting performance measures, only 10 instances were selected from the previous experiment with an 80% of expected station utilization rate where the relative magnitude of the penalties had an impact in the resulting schedule. These instances are 1-5, 7, 10-12, and 15. For further details regarding the results obtained in such instances refer to Tables D.4 to D.6 in Appendix D.

5.5.3.1 Efficiency of dialysis unit and on-time delivery of other scheduled procedures tradeoffs evaluations

This experiment assesses the tradeoffs of the penalty values given to the efficiency of the unit and the on-time delivery of other scheduled procedures or efficiency-delays/tardiness (ET) experimental pairs.

The (1, 1) efficiency-delays/tardiness pair or balanced approach results for the 10 selected instances are displayed again in Table 5.18. The results of the remaining settings or levels to evaluate the tradeoffs of efficiency and on-time delivery of other scheduled procedures are summarized in Table 5.19. For detailed results per instance refer to Appendix E.

Table 5.18: Summarized results for (1, 1) efficiency-delays/tardiness pair (balanced approach)

Category	**Balanced Approach**	
	Avg.	**Std. Dev.**
Number of non-dialysis procedures delayed	4.77	2.45
Total tardiness (Hrs.)	10.68	6.43
Fixed stations overall utilization	88.76%	2.97%
Mobile stations overall utilization	61.84%	11.20%
Dialysis unit Operating Hours	10.75	0.74
Number of dialysis scheduled in overtime	0.90	1.03

Table 5.19: Efficiency-delays/tardiness tradeoffs summarized results

Category	(1, 4)		(1, 3)		(1, 2)	
	Avg.	Std. Dev.	Avg.	Std. Dev.	Avg.	Std. Dev.
Number of non-dialysis procedures delayed	3.80	1.87	4.30	1.89	4.90	2.02
Total tardiness (Hrs.)	8.20	4.96	9.50	4.60	10.80	5.65
Fixed stations overall utilization	90.39%	3.80%	90.99%	3.44%	90.68%	3.95%
Mobile stations overall utilization	53.37%	14.74%	56.69%	11.58%	58.92%	11.63%
Dialysis unit operating hours	12.73	1.30	11.63	0.77	11.30	0.82
Number of dialysis scheduled in overtime	2.70	1.42	2.30	1.64	1.80	1.62

Continuation of Table 5.19: Efficiency-delays/tardiness tradeoffs summarized results

Category	(2, 1)		(3, 1)		(4, 1)	
	Avg.	Std. Dev.	Avg.	Std. Dev.	Avg.	Std. Dev.
Number of non-dialysis procedures delayed	5.50	1.96	5.30	2.00	5.20	2.49
Total tardiness (Hrs.)	12.08	5.97	11.90	5.31	12.38	6.16
Fixed stations overall utilization	90.39%	2.80%	93.77%	3.37%	95.83%	4.02%
Mobile stations overall utilization	59.67%	9.73%	49.18%	12.16%	42.00%	13.73%
Dialysis unit operating hours	11.45	0.95	11.80	0.63	12.00	1.21
Number of dialysis scheduled in overtime	1.40	1.43	2.90	1.37	3.50	1.84

The results from Tables 5.18 and 5.19 or the pattern described in their graphic representations in Figures 5.12 and 5.13 suggest that there is an evident tradeoff of the penalty values given to the efficiency of the unit and the treatments delayed and their tardiness. The number of non-dialysis procedures delayed, as well as their tardiness increase when the focus is toward efficiency of the dialysis unit, i.e., the higher we penalize using dialysis stations, the higher the number and time delays experienced. In addition, as the penalty weights depart from those corresponding to the balanced approach (1, 1), the operating hours the dialysis unit required of to serve the dialysis demand increases.

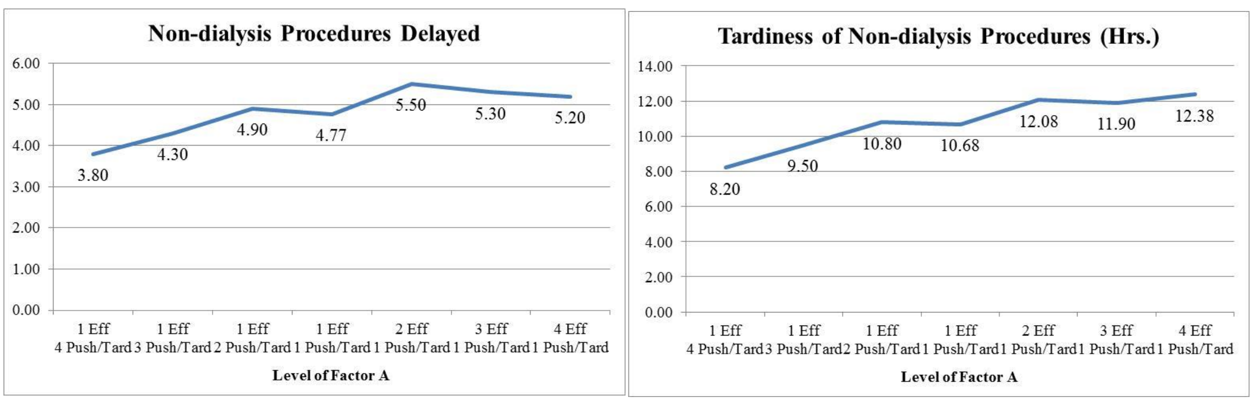

Figure 5.12: Average number of non-dialysis procedures pushed/delayed (left) and tardiness (right) for ET pairs

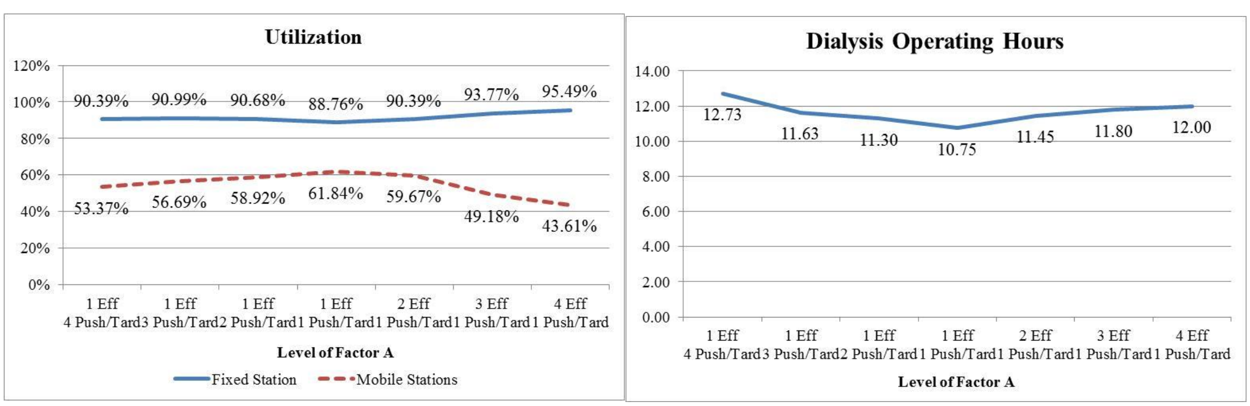

Figure 5.13: Average dialysis stations utilization (left) and operating hours required by the dialysis unit (right) for ET pairs

5.5.3.2 Efficiency for rescheduling and time delays tradeoffs evaluations

This experiment studies the tradeoffs between the number of non-dialysis procedures delayed and the time delays for the cases when rescheduling other procedures is inevitable. This experiment considers rescheduling-tardiness (RT) experimental pairs.

Summarized results for the 10 selected instances under the different efficiency-delays/tardiness pairs are shown in Tables 5.20. For detailed results per instance refer to Appendix E. Refer to Table 5.18 for results of the (1, 1) efficiency-delays/tardiness pair or balanced approach case.

Table 5.20: Rescheduling/tardiness tradeoffs summarized results

Category	**Push 2 - 0 Tardiness**		**Push 0.5 - 1.5 Tardiness**	
	Avg.	**Std. Dev.**	**Avg.**	**Std. Dev.**
Number of other treatments delayed	5.00	1.56	5.10	1.97
Total tardiness (Hrs.)	55.10	22.28	21.00	7.19
Fixed stations overall utilization	89.64%	3.44%	90.30%	3.01%
Mobile stations overall utilization	64.50%	11.41%	63.50%	11.80%
Dialysis unit operating hours	10.13	1.90	10.75	0.72
Number of dialysis scheduled in overtime	0.80	1.03	0.90	0.99

Continuation of Table 5.20: Rescheduling/tardiness tradeoffs summarized results

Category	Push 1.5 - 0.5 Tardiness		Push 0 - 2 Tardiness	
	Avg.	**Std. Dev.**	**Avg.**	**Std. Dev.**
Number of other treatments delayed	5.30	2.31	5.10	2.42
Total tardiness (Hrs.)	11.20	4.73	8.68	3.14
Fixed stations overall utilization	91.12%	4.19%	90.34%	4.61%
Mobile stations overall utilization	59.41%	13.14%	60.66%	13.51%
Dialysis unit operating hours	11.35	0.82	11.25	0.79
Number of dialysis scheduled in overtime	1.80	1.69	1.90	1.60

Figures 5.14 and 5.15 illustrate the average values of the performance measures for each level of factor B.

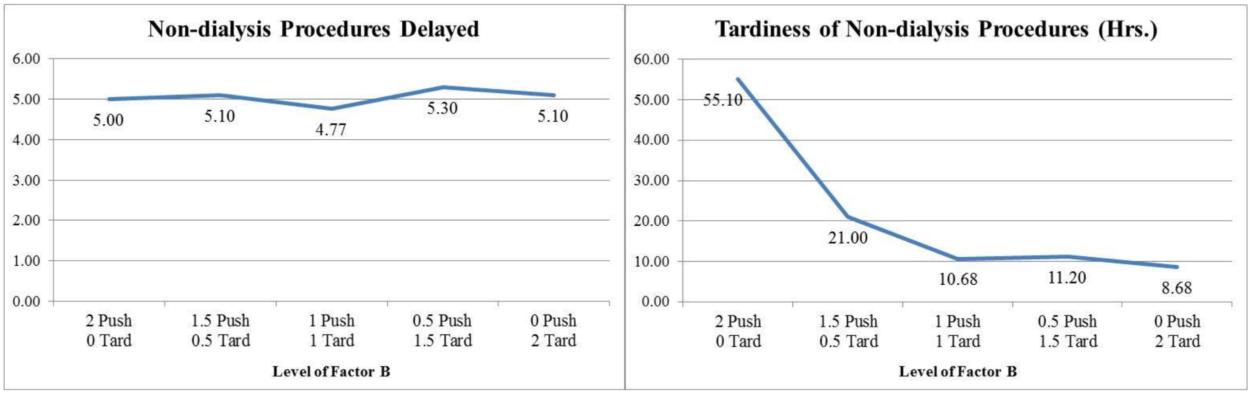

Figure 5.14: Average number of non-dialysis procedures delayed (left) and tardiness (right) for RT pairs

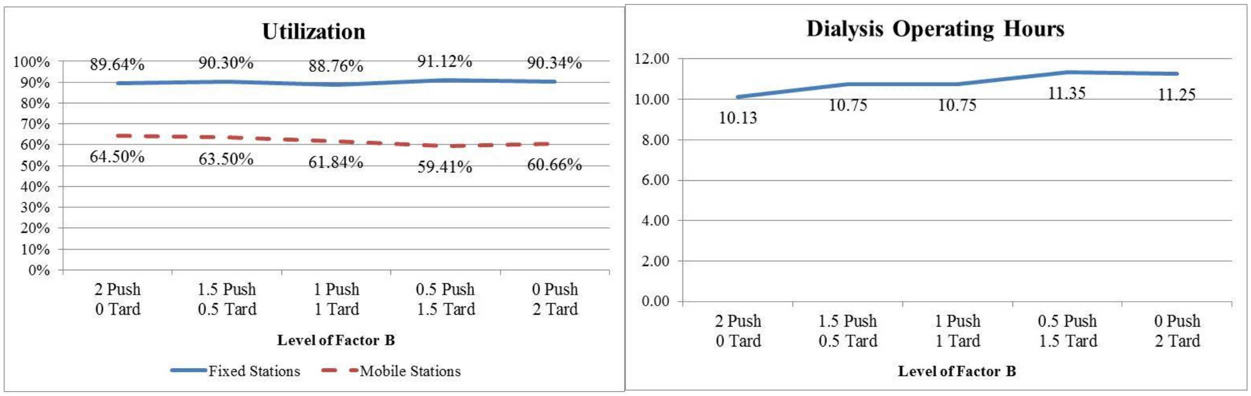

Figure 5.15: Average dialysis stations utilization (left) and operating hours required by the dialysis unit (right) for RT pairs

5.5.3.3 Discussion of results

For experiment A with efficiency-delays/tardiness (ET) experimental pairs, there is an obvious tradeoff of the penalties given to the efficiency of the dialysis unit and the number of rescheduled treatments and their corresponding time delays. The more we penalize the efficiency of the unit (using dialysis stations), the more non-dialysis procedures are delayed. There is also a tendency for the operating hours required to serve the dialysis demand to increase as we get farther from the balanced approach setting. Although this behavior is expected when the focus is toward on-time delivery of non-dialysis procedures, it is not that evident for when the focus is on efficiency of the dialysis unit. Nonetheless, this finding can be explained because the use of mobile dialysis stations is highly penalized in comparison to the other penalties of the objective function; recall that using a mobile station is twice as expensive as with a fixed station (Tables 5.16 and 5.17).

The decision-makers can then decide what is more important for the dialysis unit based on the conditions of the inpatient demand of the day and their other scheduled procedures. A fewer number of non-dialysis procedures delayed and lower time delays can be achieved by working additional hours in the dialysis unit. In other words, a potentially lower LOS can be achieved with a higher cost to the dialysis unit.

For experiment B with rescheduling-tardiness (RT) experimental pairs, the weights of the penalties do not seem to affect the number of non-dialysis procedures delayed. On the other hand, and as expected, the less we penalize the tardiness of non-dialysis procedures, the more likely that time delays increases. Moreover, there is a slight increasing tendency for the operating hours of the dialysis unit when the focus is toward tardiness. Notice that for certain sets of inpatients (e.g., instance 1; refer to Tables E.8 and E.11 in Appendix E), prioritizing over tardiness provides schedules with fewer rescheduled treatments, lower time delays and the same hours of operations at the dialysis unit.

In conclusion, the second set of experiments also suggests that the balanced approach setting provides the best tradeoff between efficiency and treatments delays.

5.5.4 Summary of experimental performance evaluation

The experimental performance evaluation shows that the proposed optimization model can serve as a decision-support tool to help an inpatient hemodialysis unit maximize its efficiency while minimizing any delays of scheduled non-dialysis procedures that ESRD inpatients may require in other hospital units.

In both set of experiments, the proposed optimization method is shown to be effective in identifying optimal, or near optimal schedules for hemodialysis inpatients that result, considering decision-makers priorities, in the maximum efficiency of the unit, as well as the minimum number of treatments delayed and minimum time delays of non-dialysis scheduled procedures. It is noteworthy that scheduling under the balanced approach setting provides the best tradeoffs between efficiency of the dialysis unit and on-time delivery of non-dialysis procedures. Thus, the experimental performance evaluation shows that the optimization method can consistently provide optimal schedules for the hemodialysis scheduling problem to potentially minimize the LOS of the inpatients.

Moreover, the experimental performance evaluation provided an in-depth assessment of the tradeoffs of the weights of the penalties and their impact over the schedules and resulting performance measures. In general, a potentially lower LOS can be achieved at a higher cost to the dialysis unit. As a result, the decision-makers have the option to ponder the weights of the penalties based on the needs of the inpatients and the strategy of the dialysis unit.

6. INPATIENT HEMODIALYSIS SCHEDULING: OPTMIZING EFFICIENCY AND LOS UNDER RESCHEDULING UNCERTAINTY

In this chapter, we extend the optimization method discussed in chapter 5 which focuses on minimizing the overall time delays (tardiness) of the rescheduled procedures such that procedures are administered as close as planned to mitigate any impact over the LOS of the inpatients. The method in chapter 5 proved to consistently provide optimal or near optimal dialysis schedules based on the priorities of the dialysis decision-makers, yet it does not consider what happens with the schedules when non-dialysis procedures can only afford a certain delay, after which they must be rescheduled during the next day. Doctors and nurses can provide a permissible timeframe in which procedures in other hospital units, if rescheduled, could still be administered during the inpatient's stay without significantly impacting his LOS. In addition, in the dialysis protocols, nephrologists may provide a due time by when inpatients must undergo their dialysis or otherwise affect their health.

As a result, to consider more realistic problem instances, a variant of the optimization approach is developed to account for any uncertainty surrounding whether or not procedures could be rescheduled without forcing patients to be hospitalized for an extra day. Moreover, this variant also considers the medical need and time-based criticality of the inpatients for undergoing dialysis by a given due time during the day.

In the next sections, the notation and mathematical model for the variant of the optimization approach are presented. In addition, the two optimization methods are compared to assess if considering the additional details provide better and more practical schedules to mitigate the LOS of the inpatients.

6.1 Assumptions considered in the Optimization Method

The second optimization model follows the same set of general assumptions described in section 5.1 but additionally considers the following:

- Non-dialysis scheduled procedures have a timeframe during which they can be rescheduled without forcing patients to be hospitalized for an extra day.
- There are non-dialysis scheduled procedures that would not affect the LOS of the inpatients, even if pushed to the next day (e.g., a nutrition class that must be received any time before being released from the hospital.)
- There are non-dialysis scheduled procedures that cannot be rescheduled without impacting the LOS (e.g., a surgery for the inpatient's primary diagnosis.)
- The closer an inpatient is scheduled to undergo dialysis to their prescribed starting due time, the higher the need or criticality of his dialysis procedure.

6.2 Rescheduling Uncertainty of Other Scheduled Procedures

This variant optimization method considers the uncertainty surrounding whether or not other procedures could be rescheduled without impacting the LOS of the inpatients. For illustration purposes, consider the next parameters:

S : Scheduled start time of the non-dialysis procedure

s : Scheduled start working period of the non-dialysis procedure

$\hat{u}$: Last time of the day by when the non-dialysis procedure could be rescheduled without impacting LOS (else, it is rescheduled to the next day)

u : Last period of the day by when the procedure could be rescheduled without impacting LOS (else, it is rescheduled to the next day)

T : End of day

Λ_q : Probability for rescheduling the non-dialysis procedure at time q, such that $q \leq \hat{u}$

λ_t : Probability for rescheduling the non-dialysis procedure at period t, such that $t \leq u$

Ω_q : Penalty incurred for time delay associated with rescheduling at time q, for $q \leq \hat{u}$

ω_t : Penalty incurred for time delay associated with rescheduling at period t, for $t \leq u$

θ : Penalty incurred for rescheduling the non-dialysis procedure at time q, such that $q > \hat{u}$ (or at period t, such that $t \leq u$)

Φ_q : Expected penalty for rescheduling the non-dialysis procedure to time q

φ_t : Expected penalty for rescheduling the non-dialysis procedure to period t

Note that θ considers the rescheduling costs of the procedure by itself in addition to any medical costs or implications associated with such replacement. In addition, the expected penalty Φ_q can be defined as follows:

$$\Phi_q = \begin{cases} 0, & q \leq S \\ (\Lambda_q * \Omega_q) + \left((1 - \Lambda_q) * \theta\right), & S < q \leq \hat{u} \\ \theta, & q > \hat{u} \end{cases}$$

Now consider the case illustrated in Figure 6.1, which represents an example with a linear decreasing probability function and a linear increasing penalty function for rescheduling without impacting LOS. Note that the functions describing the probabilities (Λ_q) and penalties (Ω_q) associated with the likelihood of rescheduling the procedure between S and $\hat{u}$ could follow any probability distribution. Figure 6.1 displays the curves of the continuous distribution for Λ_q and Ω_q. Note that the values for Λ_q and Ω_q can be selected directly from the curves, as shown in Table 6.1.

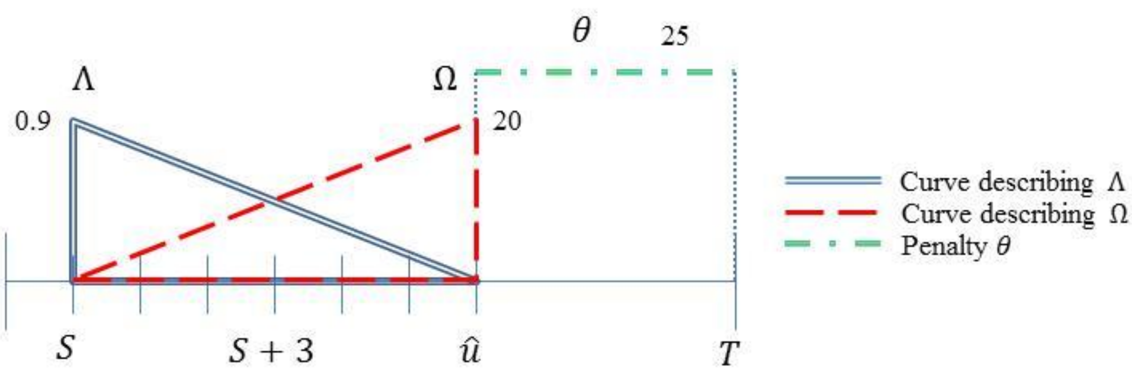

Figure 6.1: Representation of uncertainty surrounding the likelihood of a procedure to be rescheduled within the day of interest so that it does not significantly impact LOS

However, since this research considers discrete periods, the calculation of λ_t and ω_t must be discretized; see Figure 6.2. The discretizing of the parameters is achieved by calculating the average value of the parameter for each period. For this illustrative example, the discretization consists of averaging the times conforming the desired period (e.g., for period $t = 2$, time $q = 1$ and $q = 2$). Specifically, the next mathematical expression can be used:

$$\lambda_t = \Lambda_{t-1} + \Lambda_t \qquad ; \qquad \omega_t = \Omega_{t-1} + \Omega_t$$

$$\varphi_t = \begin{cases} 0, & t \leq s \\ (\lambda_t * \omega_t) + \big((1-\lambda_t) * \theta\big), & s < t \leq u \\ \theta, & t > u \end{cases}$$

As a result, the expected penalty φ_t can be calculated as demonstrated above (using λ_t and ω_t), or it can be calculated directly integrating the previous transformations:

$$\varphi_t = \begin{cases} 0, & t \leq s \\ \big((\Lambda_{t-1} + \Lambda_t) * (\Omega_{t-1} + \Omega_t)\big) + \Big(\big(1 - (\Omega_{t-1} + \Omega_t)\big) * \theta\Big), & s < t \leq u \\ \theta, & t > u \end{cases}$$

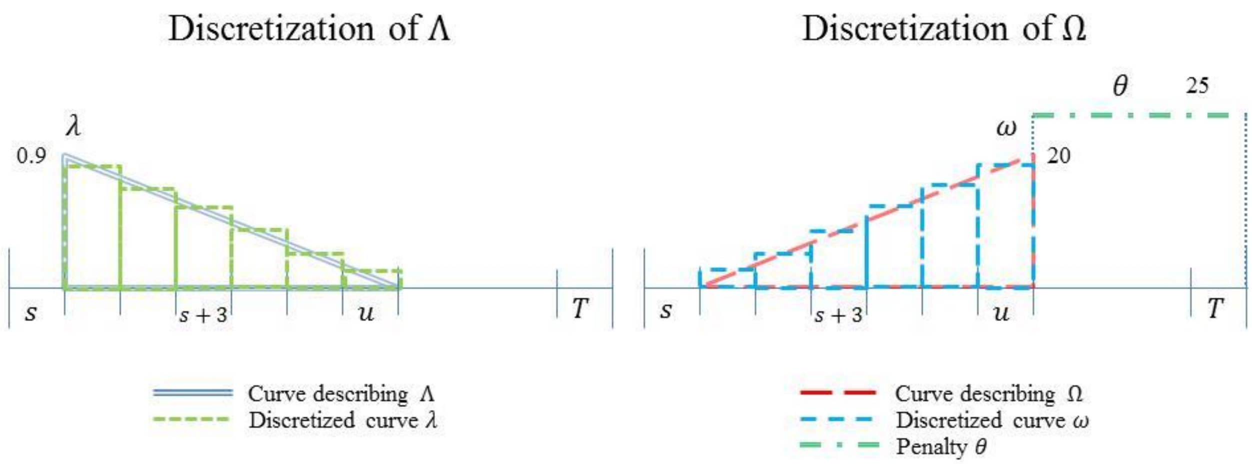

Figure 6.2: Discretization of probabilities (left chart) and penalties (right chart) associated with rescheduling at time t without impacting LOS

After discretizing the curves, the final values for both continuous and discrete parameters are shown in Table 6.1:

Table 6.1: Parameters for continuous and discretized distributions

Continuous Distribution Parameters				Discretized Distribution Parameters			
Time (q)	**Probability** (Λ_q)	**Penalty** (Ω_q)	**Exp. Penalty** (Φ_q)	**Period** (t)	**Probability** (λ_t)	**Penalty** (ω_t)	**Exp. Penalty** (φ_t)
S	0.900	0.000	0.000	s	—	—	—
$S+1$	0.750	3.333	8.750	$s+1$	0.825	1.667	5.750
$S+2$	0.600	6.667	14.000	$s+2$	0.675	5.000	11.500
$S+3$	0.450	10.000	18.250	$s+3$	0.525	8.333	16.250
$S+4$	0.300	13.333	21.500	$s+4$	0.375	11.667	20.000
$S+5$	0.150	16.667	23.750	$s+5$	0.225	15.000	22.750
$\hat{u}$	0.000	20.000	25.000	u	0.075	18.333	24.500
$\hat{u}$ to T	0.000	25.000	25.000	u to T	0.000	25.000	25.000

6.3 Optimization Method Notation and Mathematical Model

The variant optimization approach introduced in this chapter can be described as follows in terms of the objective function and constraints:

- *Minimize Weighted Penalties* = *f* { (Type of resources used for performing dialysis),

  (Delays of non-dialysis procedures),

  (Dialysis appointment time),

  (Time-based criticality for undergoing dialysis)}

- *Subject to constraints for*:
  - Dialysis requirements (uncertain daily demand, uninterrupted service, service due time)
  - Capacity (dialysis machines, personnel, operating hours)
  - Isolation requirements
  - Non-dialysis procedures appointments

- Non-overlapping procedures
- Mobile station requirements

This variant optimization approach also utilizes cost (or penalties) to determine in which time-slot and in which dialysis station the inpatients are scheduled to undergo dialysis. However, besides controlling the tradeoffs between using single-unit (mobile station) blocks and using multiple-units (fixed station) blocks, and manage the preference of appointments time throughout the day, the penalty system allows to control the non-dialysis scheduled procedures that could be rescheduled without affecting the LOS of the inpatients. Furthermore, the decision-makers can manage the time-based criticality of undergoing dialysis for inpatients.

The notation used in this optimization model is described next:

- **SETS:**

P : Set of inpatients $p \in P$

C : Set of medical conditions or isolation needs $c \in C$

B : Set of blocks of stations $b \in B$

T : Set of time-slots or working periods $t \in T$

G_p : Set of non-dialysis treatments $g \in G_p$ for inpatient $p \in P$

- **PARAMETERS:**

MB : Number of single-unit (mobile station) blocks

$DTime$: Number of operating periods per day available to schedule dialysis

$$s_{p.g,t} \begin{cases} 1, & \text{If inpatient } p \in P \text{ has non– dialysis procedure } g \in G_p \text{ scheduled to start at period } t \in T \\ 0, & \text{Otherwise} \end{cases}$$

$d_{p,g}$: Expected length (duration or number of periods) of the non-dialysis procedure$g \in G_p$ scheduled for inpatient $p \in P$ that started at period corresponding to $s_{p,g,t}$

$u_{p,g}$: Last period of the day by when non-dialysis treatment $g \in G_p$ for inpatient $p \in P$ could be rescheduled without significantly impacting LOS

$\lambda_{p,g,t}$: Probability for rescheduling non-dialysis treatment $g \in G_p$ for inpatient $p \in P$ at period $t \in T$ until $u_{p,g}$

$\omega_{p,g,t}$: Penalty incurred for the tardiness of delaying non-dialysis treatment $g \in G_p$ for inpatient $p \in P$ to period $t \in T : t \leq u_{p,g}$

$\theta_{p,g}$: Penalty incurred for rescheduling non-dialysis treatment $g \in G_p$ for inpatient $p \in P$ farther than $u_{p,g}$ (i.e., to the next day)

$\varphi_{p,g,t}$: Expected penalty for rescheduling non-dialysis treatment $g \in G_p$ for patient $p \in P$ to period$t \in T$; It is defined by:

$$\varphi_{p,g,t} = \begin{cases} 0, & t \leq \sum_{t' \in T} t' * s_{p,g,t'} \\ \left(\lambda_{p,g,t} * \omega_{p,g,t}\right) + \left(\left(1 - \lambda_{p,g,t}\right) * \theta_{p,g}\right), & \sum_{t' \in T} t' * s_{p,g,t'} < t \leq u_{p,g} \\ \theta_{p,g}, & t > u_{p,g} \end{cases}$$

$A_{p,g}$ $\begin{cases} 1, & \text{if non– dialysis procedure } g \in G_p \text{ for inpatient } p \in P \text{ cannot be delayed} \\ 0, & \text{Otherwise} \end{cases}$

β_b : Penalty incurred for using a given station from type of block $b \in B$

α_t : Penalty incurred for scheduling inpatients to period $t \in T$

k_b : Number of stations available in block $b \in B$

l_p : Expected length (duration) of the dialysis procedure for inpatient $p \in P$ (includes set-up and cleaning time)

M : Large constant

n_t : Total personnel time available to manage the dialysis procedure per period $t \in T$

r_b : Personnel time required to manage a patient dialysis procedure with a station of block $b \in B$

i_p : Isolation need $c \in C$ of the inpatient $p \in P$

w_p : Last period of the day by when inpatient $p \in P$ can start undergoing dialysis

$\gamma_{p,t}$: Penalty incurred for starting dialysis for inpatient $p \in P$ at period $t \in T: t \leq w_p$

m_p $\begin{cases} 1, & \text{if inpatient } p \in P \text{ must undergo dialysis in a mobile station} \\ 0, & \text{Otherwise} \end{cases}$

- **VARIABLES:**

$$SX_{p,b,t} \begin{cases} 1, & \text{if inpatient } p \in P \text{ starts undergoing dialysis in block } b \in B \text{ at period } t \in T: t \leq Dtime \\ 0, & \text{Otherwise} \end{cases}$$

$$X_{p,b,t} \begin{cases} 1, & \text{If inpatient } p \in P \text{ is treated at block } b \in B \text{ at period } t \in T: t \leq Dtime \\ 0, & \text{Otherwise} \end{cases}$$

$$Y_{c,b,t} \begin{cases} 1, & \text{If at least one inpatient } p \in P \text{ with isolation need } c \in C \text{ is treated at block } b \in B \text{ during period } t \in T: t \leq Dtime \\ 0, & \text{Otherwise} \end{cases}$$

$$Z_{p,g,t} \begin{cases} 1, & \text{If inpatient } p \in P \text{ starts undergoing his non–dialysis procedure } g \in G_p \text{ at period } t \in T \\ 0, & \text{Otherwise} \end{cases}$$

$$D_{p,g,t} \begin{cases} 1, & \text{If inpatient } p \in P \text{ undergoes his non–dialysis procedure } g \in G_p \text{ during period } t \in T \\ 0, & \text{Otherwise} \end{cases}$$

$Push_{p,g}$: Flags when treatment $g \in G_p$ is pushed/rescheduled for inpatient $p \in P$

Minimize:

$$\sum_{p \in P} \sum_{b \in B} \sum_{t \in T: t \leq DTime} X_{p,b,t} * \beta_b + \sum_{p \in P} \sum_{g \in G_p} \sum_{t \in T} \varphi_{p,g,t} * Z_{p,g,t} + \sum_{p \in P} \sum_{b \in B} \sum_{t \in T: t \leq DTime} X_{p,b,t} * \alpha_t + \sum_{p \in P} \sum_{b \in B} \sum_{t \in T: t \leq w_p} SX_{p,b,t} * \gamma_{p,t} \quad (6.0)$$

Subject to:

Dialysis length:

$$\sum_{b \in B} \sum_{t \in T} X_{p,b,t} = l_p \quad , \quad \forall\, p \in P \quad (6.1)$$

Dialysis Starting Time and continuity:

$$\sum_{t\in T:t\leq DTime}\sum_{b\in B} SX_{p,b,t} = 1, \qquad \forall\, p \in P \tag{6.2}$$

$$l_p * SX_{p,b,t} \leq \sum_{h=0}^{l_p-1} X_{p,b,t+h}, \quad \forall\, p \in P,\ \ \forall\, b \in B,\ \ \forall\, t \in T: t \leq DTime - l_p - 1, \tag{6.3}$$

$$\sum_{t\in T:t>DTime-l_p+1 \text{ and } t\leq Dtime}\ \sum_{b\in B} SX_{p,b,t} = 0, \qquad \forall\, p \in P \tag{6.4}$$

Dialysis due time:

$$\sum_{b\in B} t * X_{p,j,t} \leq w_p + l_p - 1 \quad , \quad \forall\, p \in P,\ \ \forall\, t \in T: t \leq DTime \tag{6.5}$$

Capacity:

$$\sum_{i\in P} X_{p,b,t} \leq k_b \quad , \quad \forall\, b \in B,\ \ \forall\, t \in T: t \leq DTime \tag{6.6}$$

Personnel:

$$\sum_{p\in P}\sum_{b\in B} X_{p,b,t} * r_b \leq n_t \quad , \quad \forall\, t \in T : t \leq DTime \tag{6.7}$$

Isolation requirements:

$$X_{p,b,t} \leq Y_{i_p,b,t} \quad , \qquad \forall\, p \in P,\ \ \forall\, b \in B,\ \ \forall\, t \in T: t \leq DTime \tag{6.8}$$

$$\sum_{c\in C} Y_{c,b,t} \leq\ 1 \quad , \quad \forall\, b \in B,\ \ \forall\, t \in T: t \leq DTime \tag{6.9}$$

$$\sum_{c\in C} Y_{c,b,t} \leq \sum_{p\in P} X_{p,b,t} \quad , \quad \forall\, b \in B,\ \ \forall\, t \in T: t \leq DTime \tag{6.10}$$

Non– dialysis procedures:

$$\sum_{t'\in T} t' * s_{p,g,t'} \leq \sum_{t\in T} t * Z_{p,g,t} \quad , \qquad \forall\, p \in P,\ \ \forall\, g \in G_p \tag{6.11}$$

$$\sum_{t\in T} Z_{p,g,t} \leq 1, \qquad \forall\, p \in P,\ \ \forall\, g \in G_p \tag{6.12}$$

$$\sum_{p\in P}\sum_{g\in G_p}\sum_{t\in T: t>\|T\|-d_{p,g}-1} Z_{p,g,t} = 0; \tag{6.13}$$

$$\left(\sum_{t\in T} t * Z_{p,g,t} - \sum_{t'\in T} t' * s_{p,g,t'}\right) \leq M * Push_{p,g} \ , \ \forall p \in P, \ \forall g \in G_p \tag{6.14}$$

$$\sum_{t\in T} D_{p,g,t} = d_{p,g} \ , \ \forall p \in P, \ \forall g \in G_p \tag{6.15}$$

$$d_{p,g} * Z_{p,g,t} \leq \sum_{h=0}^{d_{p,g}-1} D_{p,g,t+h}, \ \forall p \in P, \ \forall g \in G_p, \ \forall t \in T: t \leq \|T\| - d_{p,g} - 1 \tag{6.16}$$

Overlaping procedures:

$$X_{p,b,t} \leq \left(1 - D_{p,g,t}\right) \ , \ \forall p \in P, \ \forall b \in B, \ \forall t \in T, \ \forall g \in G_p \tag{6.17}$$

$$\sum_{g\in G_p} D_{p,g,t} \leq 1 \ , \ \forall p \in P, \ \ \forall t \in T \tag{6.18}$$

Non– dialysis procedure fixed schedule:

$$\sum_{t'\in T} t' * s_{p,o,t'} = \sum_{t\in T} t * Z_{p,o,t} \quad , \quad \forall p \in P, \ \forall o \in O[p]: K_{p,o} = 1 \tag{6.19}$$

Mobile station requirement:

$$\sum_{b\in B: \boldsymbol{b}>\|B\|-MB}\sum_{t\in T} X_{p,b,t} = l_p \quad , \quad \forall p \in P: m_p = 1 \tag{6.20}$$

The objective function (6.0) minimizes total penalties associated with assigning dialysis procedures to different time-slots or work periods, such that the dialysis unit maximizes the efficiency of using its stations while minimizing the number of other procedures that are delayed beyond the permissible timeframe to avoid a significant impact over the LOS of the inpatients. In addition, the objective function (6.0) can simultaneously penalize when during the day dialysis is

scheduled, and it allows the decision-makers to manage the time-based criticality of undergoing dialysis for inpatients. For example, if two inpatients can be scheduled at a given appointment time, the inpatient with the higher criticality would be scheduled over the other one.

In order to ensure that the scheduling of dialysis treatments is adequate and meets the planning objectives, the set of constraints (6.1-6.20) must be met. Constraint (6.1) guarantees that all inpatients expected to have dialysis are treated during the considered day. Along with constraint (6.1), constraint (6.2) defines the starting period of the dialysis procedure and ensures that if a dialysis procedure occurs over multiple periods, it happens uninterruptedly in the same station. Constraint (6.3) ensures that each required dialysis occurs at most once during the day. Constraint (6.4) prevents starting dialysis in the periods where the continuity of treatment is not guaranteed (e.g., if the length of the procedure is 5 and there are 16 working periods, then the procedure cannot be scheduled to start after period 11 as it would not allow the procedure to be completed). Constraint (6.5) establishes the last working period of the day by when inpatients must be scheduled to undergo dialysis. Constraint (6.6) ensures that during a working period the capacity of each block of stations is not exceeded. Similarly, constraint (6.7) guarantees that the total personnel time available per period is not exceeded.

Constraints (6.8), (6.9) and (6.10) help control the implementation of isolation requirement to prevent the propagation of infectious diseases. Constraint (6.8) ensures that an inpatient can be assigned to a block of stations only if inpatients with the same isolation needs are receiving care in the same block at a given period. Constraint (6.9) guarantees that at any given period, each block of stations cares for inpatients with at most one type of isolation needs.

Constraint (6.9) prevents cross contamination among inpatients with different conditions or infectious diseases. Constraint (6.10) ensures that if there are no inpatients scheduled to undergo dialysis at a given dialysis block in a period, then there should not be any medical condition assigned to that block at that time period.

Constraints (6.11-6.18) manage the non-dialysis procedures an inpatient may have and their rescheduling (if needed). Constraint (6.11) ensures that the non-dialysis procedures start as close as possible to their appointment time. Constraint (6.12) ensures that each scheduled non-dialysis procedure occurs at most once during the day. Constraint (6.13) prevents starting non-dialysis procedures in the periods where the continuity of treatment is not guaranteed (e.g., if the length of the procedure is 5 and there are 20 working periods, then the procedure cannot be scheduled to start after period 16 as it would not allow the procedure to be completed during the day). Constraint (6.14) detects or "flags" when a non-dialysis treatment has been delayed. Constraint (6.15) ensures that non-dialysis procedures are performed according to their expected duration. Constraint (6.16) guarantees the continuity of the non-dialysis treatment over a consecutive number of periods. Constraint (6.17) prevents scheduling dialysis procedures in periods dedicated for non-dialysis procedures, while constraint (6.18) prevents overlapping between the non-dialysis procedures. Constraint (6.19) specifies that the non-dialysis procedures that cannot be rescheduled must respect their appointment time, i.e. must start as scheduled. Finally, Constraint (6.20) ensures that inpatients that must undergo dialysis in single-unit blocks undergo dialysis in single-units (mobile station) blocks.

6.4 Illustrative Example: Dialysis Scheduling under Rescheduling Uncertainty

This section discusses the results of applying the proposed second method of the optimization approach to an illustrative example. The proposed optimization model was implemented in GUROBI through an AMPL interface. Refer to Appendix F for data file and output file.

Consider a problem instance for the same dialysis unit described in section 5.3, which is comprised of 6 nurses, 6 blocks of stations (3 of 2 fixed dialysis units, 3 of single mobile units) and works 16 hours (10 regular hours and 6 overtime hours) out of the 24 operating hours of the hospital. For this instance, there are 19 ESRD inpatients requiring dialysis; 14 inpatients do not have any isolation needs while 5 inpatients have given isolation needs as detailed in Table 6.2 (each number represents a particular medical condition requiring isolation with 0 corresponding to no isolation requirement). In addition, the expected dialysis duration (in periods), the due times (in periods), the mobile station requirement for each inpatient, as well as the cost penalty associated with rescheduling non-dialysis procedures impacting LOS of the given inpatient are also presented in Table 6.2 (a magnitude of 35 if it represents a considerable impact on LOS, 1 if not). Note that the magnitudes of this penalty are arbitrarily chosen. Later, in section 6.6, an experimental performance evaluation is done to provide a more in-depth assessment of the magnitudes of the penalties and assess if considering uncertainty enables to provide more efficient schedules to avoid extending the LOS of inpatients.

Table 6.2: Dialysis length, isolation needs, due times, and requirements for mobile stations, and penalty for rescheduling after $u_{p,g}$ (i.e., potentially impacting LOS)

Inpatient	**Dialysis length** (l_p)	**Isolation needs** (i_p)	**Due time** (w_p)	**Mobile station req.** (m_p)	**Rescheduling impacting LOS** ($\theta_{p,g}$)
1	2	0 (None)	3	0 (No)	35
2	2	0 (None)	4	0 (No)	35
3	2	0 (None)	None	0 (No)	35
4	2	0 (None)	None	0 (No)	35
5	2	0 (None)	None	0 (No)	35
6	2	0 (None)	None	0 (No)	35
7	2	0 (None)	None	0 (No)	1
8	2	0 (None)	None	1 (Yes)	35
9	2	0 (None)	2	0 (No)	35
10	2	0 (None)	None	0 (No)	35
11	1	0 (None)	None	0 (No)	35
12	2	0 (None)	None	0 (No)	35
13	2	0 (None)	None	0 (No)	1
14	2	0 (None)	4	0 (No)	35
15	2	1	2	0 (No)	1
16	2	1	None	0 (No)	35
17	2	2	None	1 (Yes)	35
18	2	2	None	0 (No)	35
19	2	2	None	0 (No)	35

Figure 6.3 presents the appointment time for the non-dialysis procedures scheduled for the set of 19 inpatients, as well as their isolation needs. Moreover, Figure 6.3 indicates the non-dialysis procedures that cannot be rescheduled, as well as the last period by when the non-dialysis procedures could be delayed to so that it does not result in an additional day of hospitalization for the patient. Table 6.3 indicates the probabilities and penalties associated with rescheduling without impacting LOS. Note that these probabilities and penalties are randomly assigned. In addition, it is assumed that undergoing dialysis with a mobile station is twice as

expensive as with a fixed station due to the need of dedicated personnel time. Moreover, in order to avoid the likelihood of overtime, the penalty of having dialysis is assumed to increase during the day.

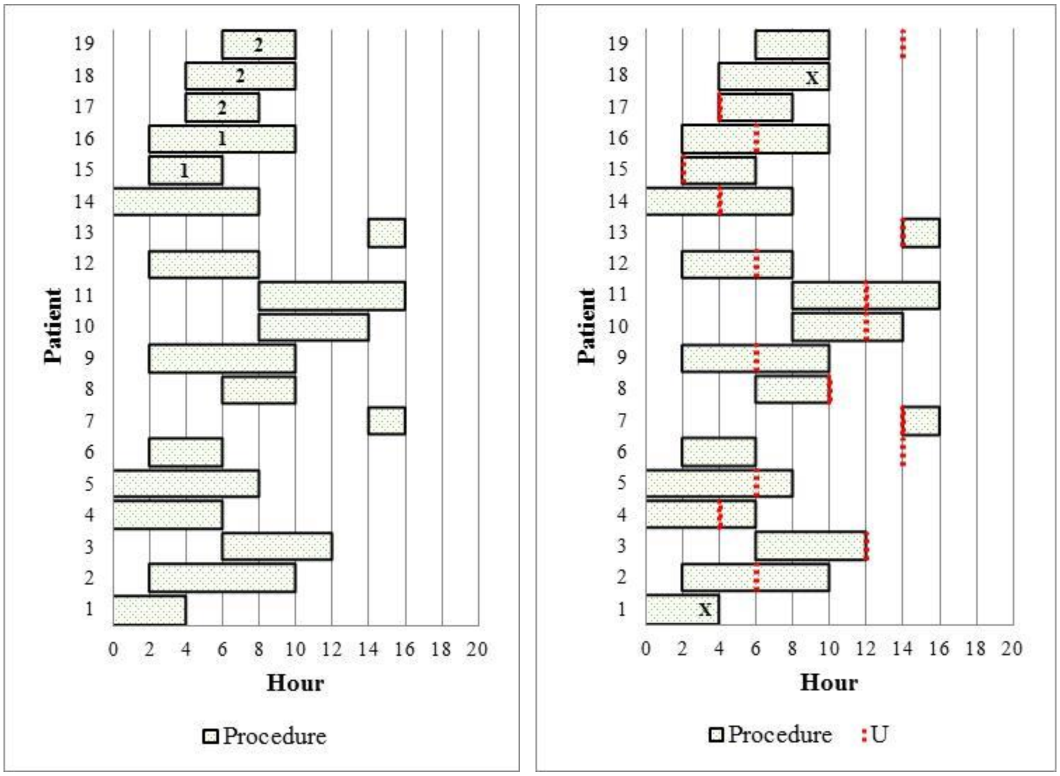

Figure 6.3: Non-dialysis procedures appointments and isolation needs (left) and last period by which rescheduling does not affect LOS (right) (‘x’ marks procedures that cannot be delayed)

Table 6.3: Inpatients' probabilities and penalties for rescheduling at period t w/o impacting LOS

Inpatient	Period	Prob. of rescheduling w/o impacting LOS ($\lambda_{p,g,t}$)	Rescheduling w/o impacting LOS ($\omega_{p,g,t}$)
2	3	0.0536	29.17
3	5	0.6020	8.50
	6	0.1561	24.68
4	2	0.0776	26.75
5	2	0.4491	10.24
	3	0.0345	30.54
6	3	0.6632	4.25
	4	0.4947	9.49
	5	0.3263	14.72
	6	0.1579	19.95
	7	0.0187	29.91
8	5	0.2257	21.00
9	3	0.0264	33.14
10	6	0.3694	18.87
11	6	0.0478	31.38
12	3	0.2430	16.23
14	2	0.1317	26.76
16	3	0.0091	33.76
19	5	0.6181	6.12
	6	0.2225	15.03
	7	0.0027	33.28

The results of the example are summarized in Table 6.4 and the proposed schedule is displayed in Figure 6.4. A total of 4 non-dialysis procedures were rescheduled, but only 2 would result in an extended LOS. As for the remaining 2, they presented a total tardiness of 4 hours. Note that this level of detail cannot be provided with the optimization method described in chapter 5. It is a slight but interesting difference since now the decision-makers can know exactly which procedures can really be administered during the given day, and which ones would be pushed to the next day.

Table 6.4: Summary table for the optimal schedule of example 6.4

Category	Value
Number of other procedures delayed w/o impact on LOS	2
Number of other procedures delayed impacting LOS	2
Tardiness of procedures w/o impact on LOS (Hrs.)	4
Fixed Stations Overall Utilization	91.18%
Mobile Stations Overall Utilization	40.00%
Dialysis Unit Completion Time (Hrs.)	14
Number of dialysis scheduled in overtime	3

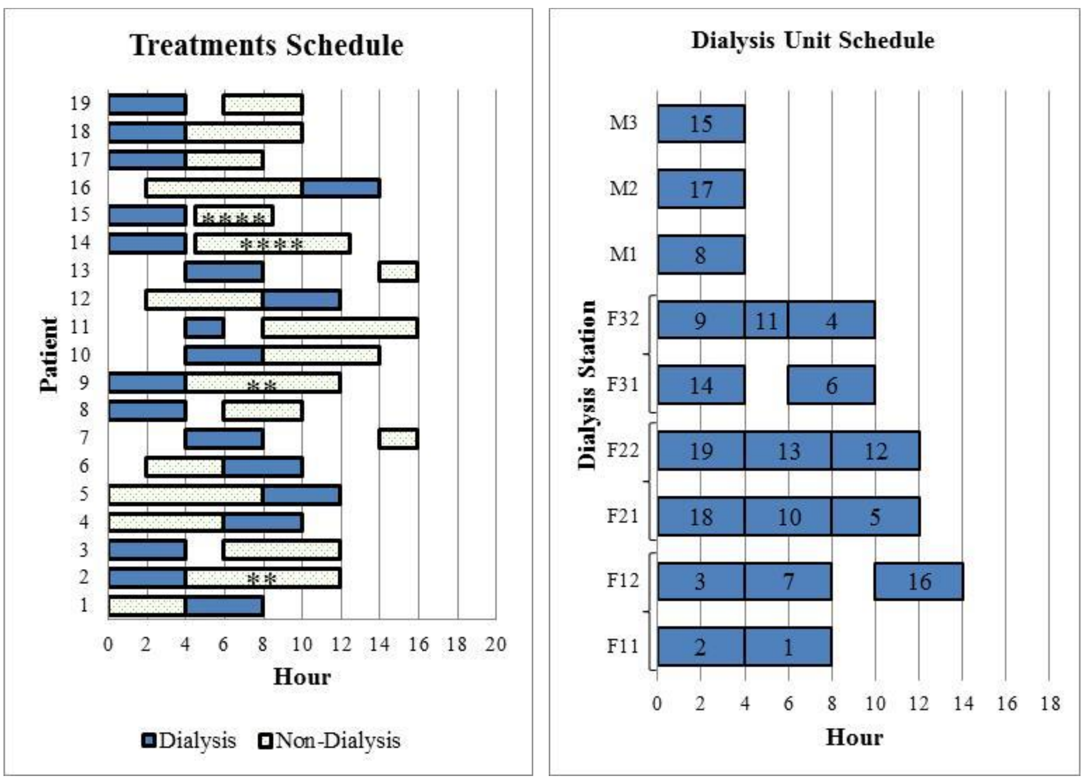

Figure 6.4: Inpatient treatments schedule (left) and dialysis unit schedule (right)

(For left chart, ** represents the rescheduled treatments without impact on LOS, **** represents those that impacts LOS; for right chart, inpatient number indicated on bar)

6.5 Computationally Efficient Algorithm for Solving the Optimization Method

As discussed in section 5.4, a solving algorithm needs to be determined to efficiently solve any instance of the proposed optimization problem with the solver for small granularity. In particular, this research considers a granularity of 15-minutes periods.

After attempting to directly solve 5 random instances using 1-phase, we did not obtain any feasible answer after the budgeted time of 20 minutes. Thus, it is assumed that the solving algorithm for this method would behave similarly to the one discussed in section 5.4. As a result, the selected solving algorithm for applying the variant optimization approach is as follows:

- Solve phase A (2-hrs periods) with an allocated time of 5 minutes and 1% optimality gap tolerance.
- Use the solution from phase A as the initial basis, and solve phase B (1-hr periods) with an allocated time of 10 minutes and 1% optimality gap tolerance.
- Use the solution from Phase B as the initial basis, and solve phase C (15-mins periods) with a time limit of 15 minutes and 1% optimality gap tolerance.

Recall that for practical and real life data, the solving algorithm could budget less time (e.g., 15 to 20 minutes total) to provide optimal, or near optimal solutions.

6.6 Experimentation

Considering the variant optimization approach incorporates the uncertainty surrounding whether or not procedures could be rescheduled without impacting the LOS of the inpatients, a set of experiments is designed to assess if providing the additional information regarding other

scheduled procedures enables a better and more accurate decision making with regards to which procedures should be rescheduled and by how much time.

To make a fair comparison between the two proposed methods, the objective function of the initial method (presented in chapter 5) must be slightly modified. The changes are required because the initial method aims to minimize the number and minimum length of time delays (tardiness) of other scheduled procedures separately, whereas this variant approach considers them combined. In particular, if a procedure is rescheduled, then it is penalized by $\theta_{p,g}$ instead of δ and τ. Moreover, an additional term must be added to this modified formulation to replicate the effect of rescheduling the procedures as early as possible. Finally, the time-based criticality penalty term is included as well. The resulting objective function is:

Minimize:

$$\sum_{p\in P}\sum_{b\in B}\sum_{t\in T:t\leq DTime} X_{p,b,t} * \beta_b + \sum_{p\in P}\sum_{g\in G_p} Push_{p,g} * \theta_{p,g} + \sum_{p\in P}\sum_{g\in G_p}\sum_{t\in T} Z_{p,g,t} * t * 0.001 + \sum_{p\in P}\sum_{b\in B}\sum_{t\in T:t\leq DTime} X_{p,b,t} * \alpha_t + \sum_{p\in P}\sum_{b\in B}\sum_{t\in T:t\leq w_p} SX_{p,b,t} * \gamma_{p,t} \tag{6.21}$$

The constraints (6.1-6.20) remain unchanged.

This experimental performance evaluation focuses on one single main factor, the magnitude of penalizing for rescheduling impacting the LOS of the inpatients. The levels of the factor are:

Factor A: Penalty for rescheduling non-dialysis procedures:

1) Constant $\theta_{p,g}$ among other scheduled procedures, with magnitude higher than all penalties associated with the likelihood of rescheduling without impacting the LOS ($\omega_{p,g,t}$).
2) Different $\theta_{p,g}$ among other scheduled procedures.

The comparison of the two optimization approaches under these two levels enables to understand if any differences detected between the methods are due to the magnitude of the penalties or if they are due to the general considerations of the variant approach.

To perform this set of experiments, another set of 30 instances for each level was randomly generated with the general characteristics presented in Table 6.5. For this set of experiments, the expected resource utilization rate for the dialysis unit is selected to be 80%, because our previous sets of experiments (in chapter 5) suggest that the utilization rate of the unit is not a significant factor relative to the performance measures of interest. Moreover, the assignment of the new penalties for rescheduling uncertainty does not fall directly into any of the scheduling priority settings described previously, because it is patient dependent. In particular, when non-dialysis procedures have high rescheduling penalties, the scheduling setting can be considered to behave similarly to the non-dialysis priority setting, whereas when the rescheduling penalties are low, the scheduling setting falls into the balanced approach setting.

Table 6.5: Characteristics of experimental instances

Parameter	**Value**
Expected utilization	80%
Dialysis hours of operation	16 (10 of regular time)
Dialysis unit capacity (hrs.)	120 (4 2-fixed dialysis stations blocks; 4 1-mobile dialysis station blocks)
Nurses available per working period	8
Dialysis length (15-mins.)	1%-2; 36%-3; 62%-4; 1%-UNIF(18, 32)
Inpatients with due time to undergo dialysis	15%
Inpatients with mobile station requirement	10% if $i_p \neq 0$; 5% if $i_p = 0$
Number of different conditions (isolation needs) per instance	10%-1; 30%-2; 28%-3; 25%-4; 7%-5
Inpatients with isolation needs	UNIF(20%, 40%)
Non-dialysis procedures assignable per instance	10%-0; 55%-1; 30%-2; 5%-3
Non-dialysis procedures length (hrs.)	UNIF(2,6)
Non-dialysis procedures that cannot be rescheduled	5%
Last period by when procedures could be rescheduled without impacting LOS ($u_{p,g}$)	UNIF($s_{p.g,t}$, $DTime - d_{p.g} + 1$)
Highest probability of linearly decreasing probability distribution for rescheduling without impacting LOS ($\lambda_{p,g,t}$)	UNIF(75%, 100%)
Highest penalty of linearly increasing probability distribution for rescheduling without impacting LOS ($\omega_{p,g,t}$)	UNIF(20, 40)
Penalty associated with rescheduling procedures impacting LOS ($\theta_{p,g}$)	If $A_{p,g} = 1$ then 70% - 1, 30% - 48 ; Else ($\omega_{p,g,t}$)*1.20
Penalty for time-based criticality of undergoing dialysis before due time ($\gamma_{p,t}$)	t*0.05

For completeness, Table 6.6 shows the general penalties associated with the appointment time for dialysis and the use of the dialysis machines considered for this experiment.

Table 6.6: Penalties associated with appointment time and usage of dialysis machines

Period	**Penalty (α_t)**
1	0.1
2	0.2
3	0.3
4	0.4
5	0.5
6	6.0
7	6.1
8	6.2

Block	**Penalty (β_b)**
Multiple Units (Fixed machines)	1
Single Units (Mobile machines)	2

6.6.1. Cases with constant $\theta_{p,g}$ across non-dialysis procedures

This section considers problem instances where the penalty for rescheduling non-dialysis procedures beyond the permissible timeframe or rescheduled to the next day (i.e., impacting LOS) is equal among procedures.

The results from schedules of the 30 simulated instances of an inpatient dialysis unit a constant $\theta_{p,g}$ across other scheduled procedures are summarized in Table 6.7 (refer to Appendix G for detailed results per instance). Results suggest that considering uncertainty results in a slight increase the number of non-dialysis procedures performed as planned and/or without impacting LOS.

Table 6.7: Summarized comparison between optimization methods for cases with constant $\theta_{p,g}$ across non-dialysis procedures

Category	**Uncertain Parameters**		**Deterministic Parameters**	
	Avg.	**Std. Dev.**	**Avg.**	**Std. Dev.**
Number of other procedures delayed w/o impact on LOS	1.07	0.94	0.71	0.81
Number of other procedures delayed impacting LOS	1.82	1.16	2.00	1.31
Tardiness of procedures w/o impact on LOS (Hrs.)	1.63	1.70	1.38	2.38
Fixed Stations Overall Utilization	89.20%	5.38%	90.18%	4.05%
Mobile Stations Overall Utilization	43.80%	12.13%	42.14%	13.35%
Dialysis Unit Completion Time (Hrs.)	14.54	1.30	14.38	1.26
Number of dialysis scheduled in overtime	3.50	1.67	3.61	1.77

A graphic representation of the performance measures is presented in Figures 6.5 and 6.6.

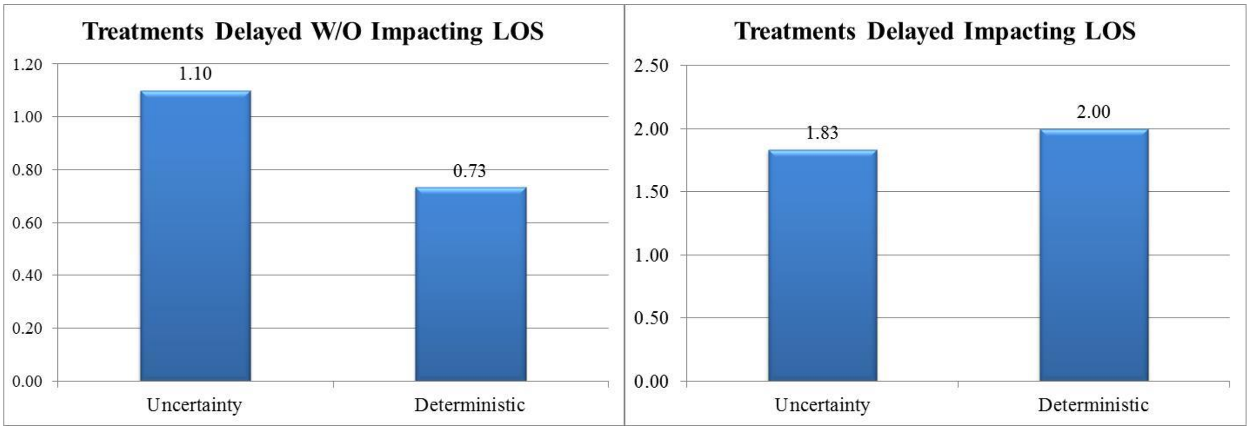

Figure 6.5: Average number of non-dialysis procedures delayed without impacting LOS (left) and impacting LOS (right) for cases with constant $\theta_{p,g}$ across non-dialysis procedures

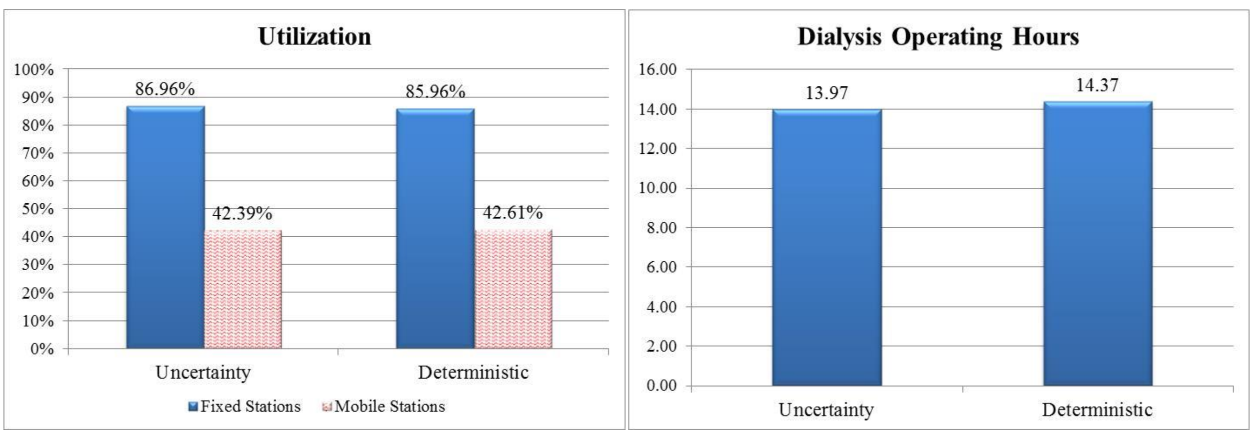

Figure 6.6: Average dialysis stations utilization (left) and operating hours required by the dialysis unit (right) for cases with constant $\theta_{p,g}$ across non-dialysis procedures

6.6.2. Cases with variable $\theta_{p,g}$ across non-dialysis procedures

This section considers problem instances where the penalty for rescheduling non-dialysis procedures beyond the permissible timeframe or scheduled to the next day (i.e., impacting LOS) is equal among procedures.

Results for the 30 generated instances of an inpatient dialysis unit with a variable $\theta_{p,g}$ across other scheduled procedures are summarized in Table 6.8 (refer to Appendix G for detailed results per instance). Following the same pattern described in the previous case (constant $\theta_{p,g}$ across non-dialysis procedures), the results suggest that the extra effort of considering uncertainty can increase the number of non-dialysis procedures performed without impacting LOS.

Table 6.8: Summarized comparison between optimization methods for cases with variable $\theta_{p,g}$ across non-dialysis procedures

Category	Uncertain Parameters		Deterministic Parameters	
	Avg.	**Std. Dev.**	**Avg.**	**Std. Dev.**
Number of other procedures delayed w/o impact on LOS	1.07	1.02	0.75	0.93
Number of other procedures delayed impacting LOS	1.89	1.37	2.04	1.50
Tardiness of procedures w/o impact on LOS (Hrs.)	1.38	1.66	1.10	1.53
Fixed Stations Overall Utilization	86.63%	15.97%	85.83%	17.43%
Mobile Stations Overall Utilization	42.39%	13.18%	42.26%	12.92%
Dialysis Unit Completion Time (Hrs.)	13.99	2.87	14.39	1.27
Number of dialysis scheduled in overtime	3.68	1.72	3.75	1.71

A graphic representation of the performance measures is presented in Figures 6.7 and 6.8.

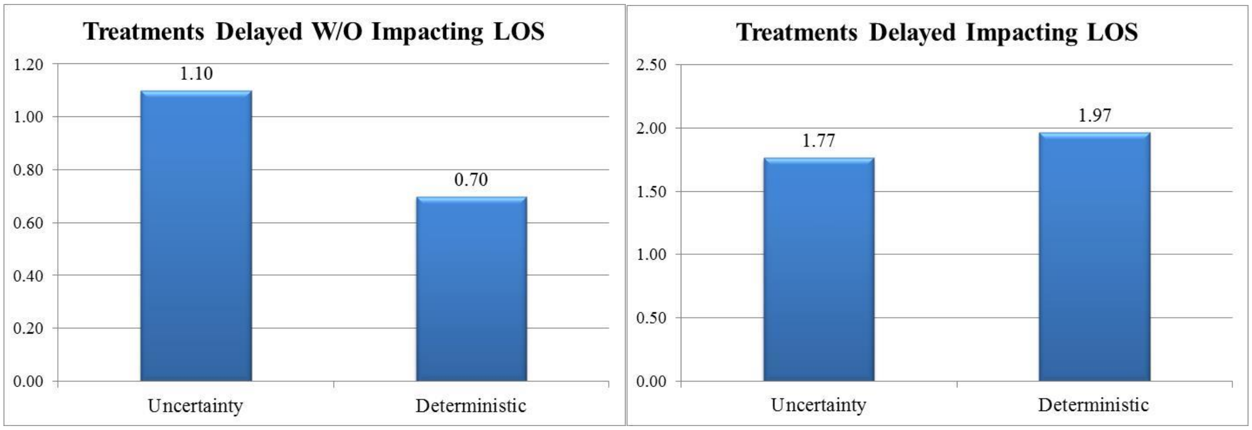

Figure 6.7: Average number of non-dialysis procedures delayed without impacting LOS (left) and impacting LOS (right) for cases with variable $\theta_{p,g}$ across non-dialysis procedures

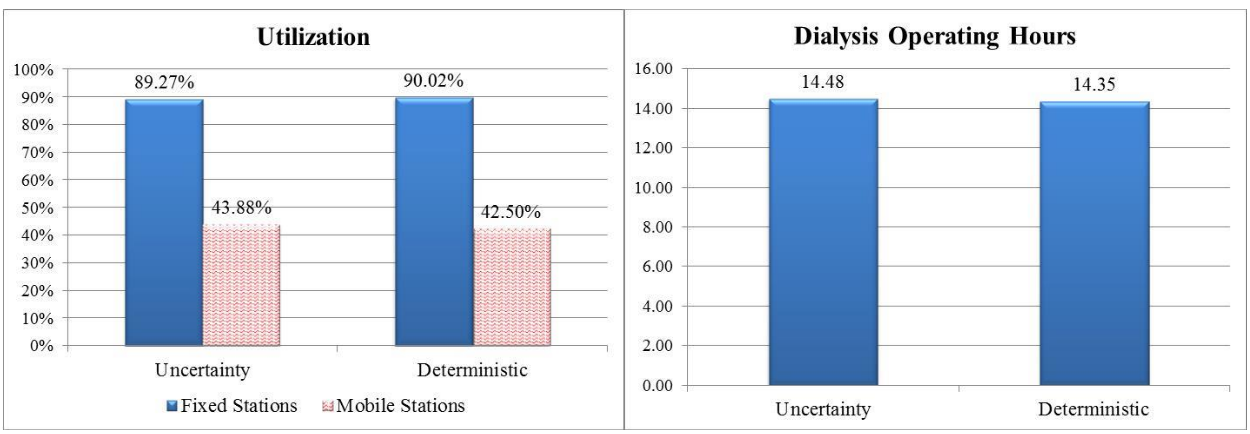

Figure 6.8: Average dialysis stations utilization (left) and operating hours required by the dialysis unit (right) for cases with variable $\theta_{p,g}$ across non-dialysis procedures

6.6.3 Discussion of results & summary of experimental performance evaluation

The second optimization method is developed to consider the uncertainty surrounding whether or not rescheduling other scheduled procedures would impact the LOS of inpatients. In general, for the conditions evaluated, the results suggest that considering uncertainty reduces the

number of non-dialysis procedures delayed that do not have a significant impact on the LOS. In other words, when considering uncertainty, the number of non-dialysis procedures that are rescheduled without impacting the LOS increases while the number of non-dialysis procedures impacting LOS decreases. Although the results may not seem significant, considering the average increment of 0.40 non-dialysis procedures that does not impact LOS per day represents over 140 procedures or patients that are not affected by extended LOS.

Moreover, there is no significant difference between assigning a constant or variable penalty ($\theta_{p,g}$) across other scheduled procedures when rescheduling impacts LOS. Tables 6.7 and 6.8 indicate an almost identical behavior regardless of the penalty assignment.

It is noteworthy that the initial optimization approach is a subset of this variant approach in which the penalties follow an uniform distribution with $t * s_{p,g,t} = u_{p,g}$. In other words, if the rescheduling uncertainty information of non-dialysis procedures cannot be obtained, then this variant optimization approach reduces to the initial method presented in chapter 5.

In conclusion, the experimental performance evaluation suggests that the extra effort of considering uncertainty may provide better schedules and performance measures, but that the initial optimization method can consistently and robustly provide optimal or near optimal solutions (schedules) that maximizes the efficiency of the unit while minimizing non-dialysis procedures that could result in extending the LOS of inpatients.

7. CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE RESEARCH

The goal of this research was to address the inpatient hemodialysis scheduling problem to maximize the efficiency of the unit (in terms of utilization and operating hours), while minimizing delays of other scheduled procedures that could result in extending the LOS of the inpatients. In general, the hemodialysis scheduling problem considers an uncertain demand of inpatients requiring dialysis services, the dialysis protocols prescribed by a treating nephrologist, the variable duration of the dialysis treatments, the limited capacity of the dialysis equipment and personnel in the inpatient hemodialysis units, as well as the isolation requirements used to mitigate the spread of healthcare-associated infections (HAI). The result of this effort is the development of two optimization methods to address the gap identified in the literature, which is the lack of optimization approaches that account the hemodialysis scheduling problem. The conclusions drawn for each one of the methods are presented in the next section.

7.1 Conclusions

The scheduling methods presented in this research are designed to provide good schedules for dialysis units that administer hemodialysis to inpatients on a daily basis. The scheduling approaches are designed to take into consideration the dialysis protocols per inpatient and the scheduled appointments that the inpatients may have in other hospital units during the day. Furthermore, the schedules are designed to assign dialysis appointment times to patients in a way the efficiently utilizes the equipment and personnel of the dialysis unit while ensuring the medical needs and restrictions of the inpatients. By accommodating other scheduled

appointments, delays in treatments and extended lengths of stay can be avoided, thus potentially benefitting both the care provided to the patient and the expenses incurred by the hospital.

An initial optimization method focuses on minimizing the minimum time delays of rescheduled procedures in order to mitigate its impact over the LOS of inpatients. This method was tested under different scheduling settings and under different levels of complexity (defined by the expected resource utilization rate of the unit) to verify its capability and performance. The experimental performance evaluation suggests that the method can consistently provide optimal, or near optimal solutions to the scheduling problem with efficient schedules regardless of the complexity of the problem instance. Also, this evaluation shows that the penalty system implemented in the method allows the decision-makers to establish their own scheduling priorities (i.e., ponder the weights of the penalties) to accommodate the inpatients. In particular, the tradeoffs on how the penalties impacted the schedules and resulting performance measures were assessed, with results suggesting that, in general, a potentially lower LOS can be achieved to the extent of higher dialysis unit costs. In any case, scheduling under the balanced approach setting is recommended since it efficiently maximizes the efficiency of the unit while minimizing delays in other scheduled procedures that could result in extending the LOS of inpatients.

A variant optimization method considers more realistic instances and accounts the uncertainty surrounding whether or not procedures could be rescheduled without impacting the LOS of the inpatients, as well as the time-based criticality of the inpatients for undergoing dialysis by a given due time during the day. A set of experiments was performed to assess if providing the additional information regarding other scheduled procedures enables to take better

and more accurate decisions with regards to which non-dialysis procedures should be rescheduled and by how much time. After evaluating under constant and variable penalties for rescheduling over the permissible timeframe to avoid impacting LOS, the results suggest that considering uncertainty can provide better schedules and increase the number of procedures that do not impact LOS.

Moreover, the variant approach can be reduced to the initial optimization method when considering that the penalties follow an uniform distribution with $t * s_{p,g,t} = u_{p,g}$. As a result, the initial optimization method can consistently and robustly provide optimal or near optimal solutions that maximizes the efficiency of the unit while minimizing procedures that could result in extending the LOS of inpatients.

Providing solutions to the hemodialysis scheduling problem for the desired granularity of 15-minutes periods involves large solution times when solving the problem directly due to the number of decision variables and constraints. Thus to efficiently solve any instance of the proposed optimization problem with the solver for the desired granularity in a useful and practical time, we adopted a 3-phases solving algorithm, allocating a budgeted time for the different phases in increasing amounts, where a phase correspond to the steps taken to solve the optimization problem. Specifically, we assigned 5 minutes to phase A (2-hrs periods), 10 minutes to phase B (1-hr periods), and 15 minutes to phase C (15-mins periods) with 1% optimality gap tolerance for each phase.

In conclusion, the use of a decision-support tool based on the proposed optimization methods can help an inpatient hemodialysis unit maximize the efficiency of the dialysis unit, while minimizing any delays of scheduled non-dialysis treatments that ESRD inpatients need to undergo in other hospital units. As a result, implementing these optimization approaches enables the possibility of minimizing the LOS of ESRD patients, making their LOS closer to that of non-ESRD patients, and enabling hospital units to accommodate new inpatients. Finally, by scheduling efficiently (in terms of utilization and operating hours) and minimizing delays that could extend the LOS of ESRD patients, the scheduling methods have the potential have the potential to minimize the overall U.S. expenditures associated with hemodialysis for ESRD patients.

7.2 Recommendations for Future Research

The optimization approach presented in this research expands the current literature related to inpatient scheduling. The following list future research recommendations:

- Consider daily rolling schedules with weekly time-horizon for dialysis and other required procedures in other hospital units, i.e., every day schedule for the next 7 days all procedures that inpatients need to undergo in the dialysis unit and other hospital units.
- Consider a centralized scheduling unit to schedule all procedures at once, since the proposed methods do not consider how/when other units schedule their procedures.
- Consider assigning a starting and ending time for procedures instead of using time-slots (periods).
- Incorporate natural (stochastic) uncertainties such as procedures start times and procedure lengths.

- Implement optimization approaches into commercially available software (e.g., Excel) to aid dialysis units with the scheduling of hemodialysis inpatients.

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APPENDIX

Appendix A: First Optimization Method Model – AMPL Implementation

The AMPL implementation of the initial optimization method introduced in chapter 5 is presented in this appendix.

```
#Model File - Scheduling Inpatients for Dialysis Treatment
#SETS:
set P;                          #Set of inpatients
set C;                          #Set of medical conditions (infectious diseases)
set B;                          #Set of blocks of stations
set T;                          #Set of time-slots or working periods
set G{p in P} default {};       #Set of non-dialysis procedures for inpatient p


#PARAMETERS:
param MB;                       #Number of single-unit (mobile station) blocks
param DTime;                    #Number of operating periods per day available to schedule dialysis
param hperiod, default 2;       #AUX. parameter for hours per 'time period' (Granularity)
param s{p in P,G[p],T}, default 0; #If inpatient p has non-dialysis procedure g scheduled to start at period t
param d{p in P,G[p]}, default 0;   #Expected length (in hrs) of non-dialysis procedure g for inpatient p
param beta{B}, default hperiod/2;  #Penalty incurred for using a station from block b
param alpha{T};                 #Penalty incurred for scheduling inpatients to period t (appointment time)
param delta;                    #Penalty incurred for pushing a non-dialysis procedure
param tau;                      #Penalty incurred for the tardiness of pushing a non-dialysis procedure
param k{B};                     #Number of stations per block b
param l{P}, default 4/hperiod;  #Expected length (in hrs) of the dialysis procedure for inpatient p
param M = 100;                  #Large number
param n{T};                     #Total personnel time available to manage the dialysis procedure at time t
param r{B};                     #Personnel time required to manage the dialysis procedure in a station of block
                                b
param i{P}, default 0;          #Isolation need c of inpatient p
param w{P}, default DTime;      #Last period of the day by when inpatient p can start undergoing dialysis
param m{P}, default 0;          #If inpatient p must undergo dialysis in a mobile block (station)
param A{p in P, g in G[p]}, default 0;       #If non-dialysis procedure g for inpatient p cannot be delayed
param laux{P};                  #AUX. Length of dialysis procedures to provide initial solution subsequent
                                phases


#VARIABLES:
var X{P,B,t in T:t<=DTime} binary;        #Indicates if inpatient p is treated at block b during work period t
var Y{C,B,t in T:t<=DTime} binary;        #Indicates if at least one inpatient with condition c is treated at block b
                                          during period t
var U{P,B,t in T: t > 1 and t<=DTime} binary;     #Indicates if inpatient p is assigned to undergo dialysis in
                                                  block b in time t-1, and also requires treatment in t
var Z{p in P,G[p],T} binary;              #Indicates if inpatient p starts undergoing his non-dialysis procedure g
                                          at period t
```

```
var D{p in P,G[p],T} binary;            #Indicates if inpatient p receives his non-dialysis procedure during
                                        period t
var Push{p in P,G[p]} integer >=0;      #Indicates number of non-dialysis treatments rescheduled (pushed) to
                                        start later


#FORMULATION:
minimize Penalties:
sum{p in P, b in B, t in T:t<=DTime} X[p,b,t]*beta[b] +
sum{p in P, g in G[p]:p>0} tau*(sum{t in T}t*Z[p,g,t] - sum{tt in T}tt*s[p,g,tt]) +
sum{p in P,g in G[p]:p>0} delta*Push[p,g] +
sum{p in P, b in B, t in T:t<=DTime} X[p,b,t]*alpha[t];

#*****Dialysis Length*****#
s.t. Service{p in P}: sum{b in B, t in T:t<=DTime} X[p,b,t] = l[p];

#*****Dialysis Continuity*****#
s.t. ServiceCont {p in P, b in B, t in T: t > 1 and t<=DTime}: X[p,b,t-1] + X[p,b,t] >= 2*U[p,b,t];
s.t. ServiceCont2 {p in P}: sum{b in B, t in T: t > 1 and t<=DTime} U[p,b,t] = (l[p] - 1);

#*****Dialysis Due Time*****#
s.t. ServiceTime{p in P, t in T:t<=DTime}: sum{b in B} t*X[p,b,t] <= w[p] + l[p] - 1;

#*****Dialysis Unit Capacity*****#
s.t. Capacity{b in B, t in T:t<=DTime}: sum{p in P} X[p,b,t] <= k[b];

#*****Personnel Capacity*****#
s.t. Nurse{t in T:t<=DTime}: sum{p in P, b in B} (X[p,b,t]*r[b]) <= n[t];
###s.t. Nurse{t in T:t<=DTime}: sum{p in P, b in B} (X[p,b,t]/r[b]) <= n[t]; ### Modified personnel constraint

#*****Isolation Needs (Requirements)*****#
s.t. Condition{p in P, b in B, t in T:t<=DTime}: X[p,b,t] <= Y[i[p],b,t];
s.t. Condition2{b in B, t in T:t<=DTime}: sum{c in C} Y[c,b,t] <= 1;
s.t. Condition3{b in B, t in T:t<=DTime}: sum{c in C} Y[c,b,t] <= sum{p in P} X[p,b,t];

#*****Non-Dialysis Procedures*****#
s.t. OTStart{p in P, g in G[p]:p>0}: sum{tt in T} tt*s[p,g,tt] <= sum{t in T} t*Z[p,g,t];
s.t. OTStart2{p in P, g in G[p]:p>0}: sum{t in T} Z[p,g,t] <= 1;
s.t. OTStart3: sum{p in P, g in G[p], t in T: p>0 and t > card(T)-d[p,g] + 1} Z[p,g,t]=0;
s.t. OTPush{p in P, g in G[p]:p>0}: (sum{t in T} t*Z[p,g,t] - sum{tt in T} tt*s[p,g,tt]) <= M*Push[p,g];
s.t. OTDuration{p in P, g in G[p]:p>0}: sum{t in T} D[p,g,t] = d[p,g];
s.t. OTCont{p in P, g in G[p], t in T: p>0 and t <= card(T)-d[p,g] + 1}: d[p,g]*Z[p,g,t] <= sum{h in 0..(d[p,g]-1)}
D[p,g,t + h];

#*****Overlapping Procedures****#
s.t. OTTime{p in P, b in B, t in T, g in G[p]:p>0 and t<=DTime}: X[p,b,t] <= (1-D[p,g,t]);
s.t. NonD{p in P, t in T}: sum{g in G[p]: p>0} D[p,g,t] <=1;

#*****Non-dialysis Procedures Flag*****#
s.t. OTFlag{p in P, g in G[p]: A[p,g]=1}: sum{t in T} t*Z[p,g,t] = sum{tt in T} tt*s[p,g,tt];

#*****Mobile Station Requirement*****#
s.t. Mobilestation{p in P: m[p] = 1}: sum{b in B, t in T: b>card(B)-MB and t<=DTime} X[p,b,t]=l[p];
```

Appendix B: Variant Optimization Method Model – AMPL Implementation

The AMPL implementation of the variant optimization method discussed in chapter 6 is presented in this appendix.

```
#Model File - Scheduling Inpatients for Dialysis Treatment
#SETS:
set P;                              #Set of inpatients
set C;                              #Set of medical conditions (infectious diseases)
set B;                              #Set of blocks of stations
set T;                              #Set of time-slots or working periods
set G{p in P} default {};           #Set of non-dialysis procedures for inpatient p

#PARAMETERS:
param MB;                           #Number of single-unit (mobile station) blocks
param DTime;                        #Number of operating periods per day available to schedule dialysis
param hperiod, default 2;           #AUX. parameter for hours per 'time period' (Granularity)
param s{p in P,G[p],T}, default 0;  #If inpatient p has non-dialysis procedure g scheduled to start at period t
param d{p in P,G[p]}, default 0;    #Expected length (in hrs) of non-dialysis procedure g for inpatient p
param alpha{T};                     #Penalty incurred for scheduling inpatients to period t (appointment time)
param beta{B}, default hperiod/2;   #Penalty incurred for using a station from block b

param u{p in P, g in G[p]}, default sum{t in T}t*s[p,g,t];   #Last period of the day by which non-dialysis
                                                             procedure g for inpatient p could be rescheduled
                                                             without impacting LOS
param lambda{p in P, G[p], T}, default 1;   #Probability of starting the non-dialysis treatment g in period s<t<u
                                            (without impacting LOS)
param omega{p in P, G[p], T}, default 0;    #Penalty incurred for the tardiness of pushing a non-dialysis treatment
                                            in s<t<u (without impacting LOS)
param theta{p in P, G[p]};                  #Penalty incurred for rescheduling after u (impacting LOS)

param phi{p in P, g in G[p], t in T}, default (lambda[p,g,t]*omega[p,g,t])+((1-lambda[p,g,t])*theta[p,g]);
#Expected penalty for rescheduling non-dialysis procedure g for inpatient p to period t
#Remaining Phi assignment in Data file: let {p in P, g in G[p], t in T:t>u[p,g]} phi[p,g,t]:= theta[p,g];

param k{B};                         #Number of stations per block b
param l{P}, default 4/hperiod;      #Length (in hrs) of the dialysis procedure for inpatient p
param M = 100;                      #Large number
param n{T};                         #Total personnel time available to manage the dialysis procedure at time t
param r{B};                         #Personnel time required to manage the dialysis procedure in a station of block
                                    b
param i{P}, default 0;              #Isolation need c of inpatient p
param w{P}, default DTime;          #Last period of the day by when inpatient p can start undergoing dialysis
param m{P}, default 0;              #If inpatient p must undergo dialysis in a mobile block (station)
param A{p in P, g in G[p]}, default 0;      #If non-dialysis procedure g for inpatient p cannot be delayed
param gamma{p in P,t in T:t<=w[p]}, default 0;   #Penalty incurred for starting dialysis for inpatient p at period t
param laux{P};                      #AUX. Length of dialysis procedures to provide initial solution subsequent
                                    phases

#VARIABLES:
var SX{P, B, t in T:t<=DTime} binary;       #Indicates start time of dialysis at block b at work period t
var X{P,B,t in T:t<=DTime} binary;          #Indicates if inpatient p is treated at block b during work period t
```

```
var Y{C,B,t in T:t<=DTime} binary;          #Indicates if at least one inpatient with condition c is treated at block b during period t
var Z{p in P,G[p],T} binary;                #Indicates if inpatient p starts undergoing his non-dialysis procedure g at period t
var D{p in P,G[p],T} binary;                #Indicates if inpatient p undergoes his non-dialysis procedure during period t
var Push{p in P,G[p]} integer >=0;          #Indicates number of non-dialysis treatments rescheduled (pushed) to start later

#FORMULATION:
minimize Penalties:
sum{p in P, b in B, t in T:t<=DTime} X[p,b,t]*beta[b] +
sum{p in P, g in G[p],t in T:p>0} phi[p,g,t]*Z[p,g,t] +
sum{p in P, b in B, t in T:t<=DTime} X[p,b,t]*alpha[t] +
sum{p in P, b in B, t in T: t<=w[p]} SX[p,b,t]*gamma[p,t];

#*****Dialysis Length*****#
s.t. Service{p in P}: sum{b in B, t in T:t<=DTime} X[p,b,t] = l[p];

#*****Dialysis Start Time and Continuity*****#
s.t. StartDialysis{p in P}: sum{t in T,b in B:t<=DTime} SX[p,b,t] = 1;
s.t. ContDialysis{p in P, b in B, t in T: t <=DTime-l[p]+1}:l[p]*SX[p,b,t] <= sum{h in 0..(l[p]-1)}X[p,b,t + h];
s.t. NonStart{p in P}: sum{b in B, t in T: t> DTime-l[p]+1 and t<=DTime} SX[p,b,t]=0;

#*****Dialysis Start Due Time*****#
s.t. ServiceTime{p in P, t in T:t<=DTime}: sum{b in B} t*X[p,b,t] <= w[p] + l[p] - 1;

#*****Dialysis Unit Capacity*****#
s.t. Capacity{b in B, t in T:t<=DTime}: sum{p in P} X[p,b,t] <= k[b];

#*****Personnel Capacity*****#
s.t. Nurse{t in T:t<=DTime}: sum{p in P, b in B} (X[p,b,t]*r[b]) <= n[t];

#*****Isolation Needs (Requirements)*****#
s.t. Condition{p in P, b in B, t in T:t<=DTime}: X[p,b,t] <= Y[i[p],b,t];
s.t. Condition2{b in B, t in T:t<=DTime}: sum{c in C} Y[c,b,t] <= 1;
s.t. Condition3{b in B, t in T:t<=DTime}: sum{c in C} Y[c,b,t] <= sum{p in P} X[p,b,t];

#*****Non-Dialysis Procedures*****#
s.t. OTStart{p in P, g in G[p]:p>0}: sum{tt in T} tt*s[p,g,tt] <= sum{t in T} t*Z[p,g,t];
s.t. OTStart2{p in P, g in G[p]:p>0}: sum{t in T} Z[p,g,t] <= 1;
s.t. OTStart3: sum{p in P, g in G[p], t in T: p>0 and t > card(T)-d[p,g] + 1} Z[p,g,t]=0;
s.t. OTPush{p in P, g in G[p]:p>0}: (sum{t in T} t*Z[p,g,t] - sum{tt in T} tt*s[p,g,tt]) <= M*Push[p,g];
s.t. OTDuration{p in P, g in G[p]:p>0}: sum{t in T} D[p,g,t] = d[p,g];
s.t. OTCont{p in P, g in G[p], t in T: p>0 and t <= card(T)-d[p,g] + 1}: d[p,g]*Z[p,g,t] <= sum{h in 0..(d[p,g]-1)}
D[p,g,t + h];

#*****Overlapping Procedures****#
s.t. OTTime{p in P, b in B, t in T, g in G[p]:p>0 and t<=DTime}: X[p,b,t] <= (1-D[p,g,t]);
s.t. NonD{p in P, t in T}: sum{g in G[p]: p>0} D[p,g,t] <=1;

#*****Non-dialysis Procedures Flag*****#
s.t. OTFlag{p in P, g in G[p]: A[p,g]=1}: sum{t in T} t*Z[p,g,t] = sum{tt in T} tt*s[p,g,tt];

#*****Mobile Station Requirement*****#
s.t. Mobilestation{p in P: m[p] = 1}: sum{b in B, t in T: b>card(B)-MB and t<=DTime} X[p,b,t]=l[p];
```

Appendix C: Data file and Output file for Example of Section 5.3

The data file used for the example of section 5.3 for the AMPL implementation of the model presented in Appendix A is as follows:

```
##**Data File – Example 5.3 **##

#SETS:
set P:= 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20;
set C:= 0 1 2 3 4;
set B:= 1 2 3 4 5 6;
set T:= 1 2 3 4 5 6 7 8 9 10 11 12;
set G[1]:=1;
set G[2]:=1;
set G[3]:=1;
set G[4]:=1;
set G[5]:=1;
set G[6]:=1;
set G[7]:=1;
set G[8]:=1;
set G[9]:=1;
set G[10]:=1;
set G[11]:=1;
set G[12]:=1;
set G[13]:=1;
set G[14]:=1;
set G[15]:=1;
set G[16]:=1;
set G[17]:=1;
set G[18]:=1;
set G[19]:=1;
set G[20]:=1;


#PARAMETERS:
param hperiod:=2;

param MB:=3;

param DTime:=8;

param s:=
1  1 2   1
2  1 2   1
3  1 3   1
4  1 4   1
5  1 2   1
6  1 2   1
7  1 2   1
8  1 3   1
9  1 1   1
```

```
10 1 2   1
11 1 2   1
12 1 4   1
13 1 1   1
14 1 3   1
15 1 1   1
16 1 3   1
17 1 3   1
18 1 1   1
19 1 2   1
20 1 4   1
;

param d:=
1  1  2
2  1  1
3  1  3
4  1  1
5  1  2
6  1  2
7  1  2
8  1  1
9  1  3
10 1  2
11 1  1
12 1  1
13 1  2
14 1  2
15 1  2
16 1  2
17 1  2
18 1  2
19 1  2
20 1  1
;

param k:=
1 2
2 2
3 2
4 1
5 1
6 1
;


param r:=
4 1
5 1
6 1
;

param beta:=
4 2
5 2
6 2
```

```
;

param alpha:=
 1  0.1
 2  0.2
 3  0.3
 4  0.4
 5  0.5
 6  6.1
 7  6.2
 8  6.3
 9  6.4
10  6.5
;


param l:=
2  3
3  1
4  3
18 1
19 1
20 3
;

param i:=
13 1
14 1
15 2
16 2
17 3
18 3
19 3
20 4
;

param w:=
2  3
3  1
14 1
;

param m:=
15 1
16 1
20 1
;

param A :=
1  1   1
8  1   1
15 1   1
16 1   1
;
```

On the other hand, after solving the problem, there are significant results that are meaningful to display. The next example of an output file presents the objective function value, the number and ID of procedures that are pushed, the new proposed starting time for those procedures, the hemodialysis schedules, the unit utilization, and, if any, the inpatients treated in overtime:

```
############## Output file – Example 5.3 ##############

Penalties = 66.5

###Treatments Delayed =  1

Push :=
7 1   1
;

###Tardiness(hr) =  2

:        s   Z    :=
1  1 2    1   1
2  1 2    1   1
3  1 3    1   1
4  1 4    1   1
5  1 2    1   1
6  1 2    1   1
7  1 2    1   0
7  1 3    0   1
8  1 3    1   1
9  1 1    1   1
10 1 2    1   1
11 1 2    1   1
12 1 4    1   1
13 1 1    1   1
14 1 3    1   1
15 1 1    1   1
16 1 3    1   1
17 1 3    1   1
18 1 1    1   1
19 1 2    1   1
20 1 4    1   1
;

X [*,1,*]
:   1   2   3   4   5    :=
2    0   0   1   1   1
11   0   0   1   1   0
17   1   1   0   0   0
```

```
19   1   0   0   0   0


 [*,2,*]
:   1   2   3   4   5    :=
1    0   0   0   1   1
3    1   0   0   0   0
4    1   1   1   0   0
10   0   0   0   1   1
12   0   1   1   0   0

 [*,3,*]
:   1   2   3   4   5    :=
5    0   0   0   1   1
6    0   0   0   1   1
7    1   1   0   0   0
8    1   1   0   0   0
18   0   0   1   0   0

 [*,4,*]
:   1   2   3   4    :=
15   0   0   1   1
16   1   1   0   0

 [*,5,*]
:   1   2   3   4    :=
13   0   0   1   1
14   1   1   0   0

 [*,6,*]
:   1   2   3   4   5    :=
9    0   0   0   1   1
20   1   1   1   0   0
;

###Fixed Reg. Time Utilization = 0.9

###Mobile Reg. Time Utilization = 0.8666666666666667

###Overall Utilization = 0.8888888888888888

###Inpatients/Block/Periods in Overtime =
X[p,b,t] :=
;
```

Appendix D: Detailed Results of Set of Experiments of Section 5.5.2

This appendix shows the detailed results of the set of experiments run in section 5.5.2. This set of experiments tested the optimization approach under different expected utilization rates of the dialysis unit across different scheduling priority settings:

Table D.1: Detailed results for 65% resource utilization rate under dialysis priority setting

Instance	Procedures Delayed	Tardiness (Hrs.)	Completion Time (Hrs.)	Inpatients in Overtime	Fixed	Mobile
1	9	40.25	10.00	0	81.25%	47.50%
2	6	14.00	13.75	1	72.84%	45.00%
3	5	8.25	9.00	0	76.25%	35.00%
4	6	14.75	9.00	0	81.25%	35.00%
5	7	13.75	9.00	0	78.75%	27.50%
6	2	16.75	10.00	0	80.00%	7.50%
7	10	19.75	10.00	0	75.00%	30.00%
8	6	6.75	10.00	0	82.50%	52.50%
9	8	27.75	10.00	0	87.50%	10.00%
10	13	52.75	9.00	0	82.50%	30.00%
11	8	16.50	10.00	0	99.69%	0.00%
12	6	9.00	10.00	0	82.50%	37.50%
13	8	33.50	10.00	0	83.75%	35.00%
14	3	3.75	10.00	0	85.00%	30.00%
15	8	21.75	10.00	0	81.25%	30.00%
16	10	48.50	10.00	0	76.25%	42.50%
17	8	29.50	10.00	0	81.25%	30.00%
18	8	15.00	10.00	0	84.06%	40.00%
19	7	37.25	10.00	0	87.50%	25.00%
20	8	20.00	10.00	0	83.75%	20.00%
21	6	31.75	10.00	0	75.00%	37.50%
22	4	6.75	10.00	0	82.50%	20.00%
23	5	26.50	10.00	0	82.50%	37.50%
24	4	11.75	10.00	0	85.31%	27.50%
25	5	14.75	12.75	1	84.59%	35.00%
26	7	13.50	10.00	0	78.75%	17.50%
27	6	15.00	11.00	0	75.31%	25.00%
28	11	36.50	10.00	0	81.88%	62.50%
29	7	13.50	10.00	0	81.88%	40.00%
30	6	14.50	10.00	0	80.00%	37.50%

Table D.2: Detailed results for 65% resource utilization rate under non-dialysis priority setting

Instance	Procedures Delayed	Tardiness (Hrs.)	Completion Time (Hrs.)	Inpatients in Overtime	Fixed	Mobile
1	1	1.25	16.00	4	80.44%	27.50%
2	5	7.75	14.00	2	75.58%	36.88%
3	2	2.00	16.00	3	75.14%	25.00%
4	1	0.50	15.00	2	82.29%	17.50%
5	1	1.50	16.00	2	68.60%	37.50%
6	2	5.75	11.00	1	79.01%	7.50%
7	2	2.75	12.00	1	64.63%	47.50%
8	0	0.00	11.75	2	83.95%	45.51%
9	5	8.50	16.00	3	77.78%	10.00%
10	3	8.50	16.00	5	77.56%	19.51%
11	3	4.00	14.25	4	92.20%	0.00%
12	3	5.25	12.00	2	89.16%	17.50%
13	1	0.25	16.00	4	83.04%	25.00%
14	0	0.00	12.50	2	78.08%	37.50%
15	2	2.25	12.00	2	80.00%	27.50%
16	2	2.50	14.00	7	69.91%	40.00%
17	2	4.75	15.00	5	79.35%	10.00%
18	1	0.50	13.25	4	80.95%	26.35%
19	3	5.50	12.25	2	87.69%	17.50%
20	1	3.00	15.00	3	71.85%	20.00%
21	2	4.75	16.00	1	69.77%	37.50%
22	0	0.00	14.00	3	78.11%	19.16%
23	3	23.00	12.00	2	84.59%	27.50%
24	1	2.50	16.00	2	88.11%	15.22%
25	1	0.50	16.00	4	85.32%	17.50%
26	2	10.25	13.00	3	72.62%	17.50%
27	2	4.25	12.25	4	73.49%	23.67%
28	6	9.50	16.00	6	85.64%	17.50%
29	4	4.25	16.00	3	73.30%	40.48%
30	3	2.50	12.00	2	80.72%	30.00%

Table D.3: Detailed results for 65% resource utilization rate under balanced approach setting

Instance	Procedures Delayed	Tardiness (Hrs.)	Completion Time (Hrs.)	Inpatients in Overtime	Fixed	Mobile
1	4	7.25	11.00	1	80.25%	47.50%
2	5	7.75	14.00	2	75.58%	35.00%
3	4	4.25	9.00	0	76.25%	35.00%
4	4	7.75	9.00	0	81.25%	35.00%
5	3	4.00	10.00	0	68.75%	47.50%
6	2	5.75	11.00	1	79.01%	7.50%
7	3	6.25	8.00	0	66.25%	47.50%
8	1	0.50	11.00	1	85.19%	45.00%
9	8	12.00	10.00	0	82.50%	20.00%
10	5	9.50	10.75	3	81.23%	30.00%
11	7	15.25	10.00	0	92.19%	15.00%
12	4	7.00	11.00	1	85.19%	30.00%
13	2	1.50	11.00	3	80.72%	35.00%
14	2	2.25	10.00	0	81.25%	37.50%
15	4	3.75	10.00	0	82.50%	27.50%
16	7	9.25	11.00	1	75.31%	42.50%
17	6	10.75	11.00	2	89.02%	10.00%
18	5	11.25	10.00	0	84.06%	40.00%
19	4	9.50	10.00	0	81.25%	37.50%
20	4	6.50	10.00	0	83.75%	20.00%
21	2	14.00	10.00	0	75.00%	37.50%
22	3	5.25	10.00	0	82.50%	20.00%
23	5	24.75	10.00	0	82.50%	37.50%
24	3	2.75	10.00	0	86.56%	25.00%
25	3	3.00	12.75	2	88.36%	25.00%
26	4	12.25	10.00	0	75.00%	25.00%
27	4	7.50	11.00	2	78.05%	17.50%
28	11	24.50	11.25	2	88.15%	45.00%
29	6	8.25	10.00	0	80.63%	42.50%
30	4	25.00	16.00	3	75.28%	30.00%

Table D.4: Detailed results for 80% resource utilization rate under dialysis priority setting

Instance	Procedures Delayed	Tardiness (Hrs.)	Completion Time (Hrs.)	Inpatients in Overtime	Fixed	Mobile
1	7	39.75	10.00	0	95.00%	50.00%
2	9	36.75	12.00	1	90.24%	66.25%
3	8	37.00	10.00	0	90.00%	65.00%
4	8	19.75	12.00	2	90.77%	77.50%
5	12	32.25	10.00	0	86.25%	65.00%
6	4	6.25	10.00	0	87.50%	57.50%
7	5	22.25	10.00	0	95.00%	57.50%
8	11	29.50	11.00	1	87.96%	72.50%
9	7	15.25	12.00	2	94.88%	60.00%
10	6	13.50	10.00	0	86.25%	62.50%
11	6	13.50	11.00	1	90.12%	75.00%
12	8	29.50	11.00	1	88.89%	65.00%
13	9	10.00	10.00	0	93.75%	37.50%
14	8	29.75	10.75	1	89.16%	47.50%
15	5	10.25	10.00	0	87.50%	57.50%
16	6	3.25	10.00	0	87.50%	77.50%
17	8	25.75	10.00	0	92.50%	55.00%
18	7	20.25	10.00	0	88.75%	50.00%
19	9	22.75	10.00	0	88.44%	77.50%
20	1	3.50	10.00	0	92.50%	55.00%
21	4	7.00	10.00	0	91.56%	62.50%
22	7	11.00	10.00	0	88.13%	52.50%
23	5	12.75	12.00	5	91.38%	77.50%
24	6	10.25	10.00	0	90.00%	55.00%
25	6	8.25	10.00	0	91.88%	50.00%
26	5	24.50	10.00	0	86.25%	55.00%
27	7	17.00	10.00	0	82.50%	50.00%
28	6	21.00	10.00	0	86.25%	55.00%
29	6	11.00	10.00	0	88.75%	62.50%
30	6	24.25	12.00	2	89.16%	77.50%

Table D.5: Detailed results for 80% resource utilization rate under non-dialysis priority setting

Instance	Procedures Delayed	Tardiness (Hrs.)	Completion Time (Hrs.)	Inpatients in Overtime	Fixed	Mobile
1	4	18.25	11.25	3	92.49%	47.50%
2	3	6.00	14.00	7	93.51%	35.00%
3	4	6.25	14.00	1	86.36%	55.00%
4	4	10.25	16.00	4	84.14%	80.49%
5	5	8.50	16.00	5	81.77%	46.67%
6	1	0.50	10.50	1	86.96%	57.50%
7	1	2.50	13.75	3	93.98%	42.50%
8	3	6.25	16.00	9	88.26%	23.81%
9	2	3.25	15.00	2	91.76%	55.00%
10	2	3.75	12.00	2	81.93%	65.00%
11	1	6.00	14.00	2	88.10%	72.50%
12	5	9.25	15.00	5	87.78%	43.18%
13	5	7.50	16.00	2	84.27%	37.50%
14	3	6.50	16.00	3	87.50%	35.00%
15	1	1.50	11.00	1	86.42%	57.50%
16	2	0.75	16.00	4	87.33%	50.00%
17	4	17.25	14.00	3	84.09%	55.00%
18	3	11.25	13.75	2	88.12%	37.50%
19	3	8.50	12.25	5	96.82%	45.00%
20	0	0.00	14.75	1	92.04%	45.00%
21	1	2.00	10.00	0	87.81%	70.00%
22	2	4.50	14.00	3	91.83%	25.00%
23	0	0.00	16.00	9	94.03%	49.08%
24	2	2.50	16.00	4	82.35%	25.00%
25	3	4.00	12.00	3	92.08%	37.50%
26	1	5.50	13.00	1	84.34%	52.50%
27	2	2.00	11.75	4	86.80%	30.00%
28	1	2.25	15.00	3	76.92%	42.50%
29	3	5.00	15.00	3	85.56%	47.50%
30	3	3.00	12.50	3	88.62%	75.61%

Table D.6: Detailed results for 80% resource utilization rate under balanced approach setting

Instance	Procedures Delayed	Tardiness (Hrs.)	Completion Time (Hrs.)	Inpatients in Overtime	Fixed	Mobile
1	6	24.75	11.00	1	93.83%	50.00%
2	7	11.25	11.50	3	89.76%	65.00%
3	6	12.25	12.00	1	90.24%	60.00%
4	7	15.75	11.00	2	86.73%	90.24%
5	9	20.00	11.00	1	85.19%	65.00%
6	1	0.50	10.50	1	86.96%	57.50%
7	3	8.50	12.00	1	91.46%	60.00%
8	11	23.00	11.00	1	90.43%	67.50%
9	4	12.00	10.00	0	91.25%	74.38%
10	3	12.75	10.00	0	86.25%	62.50%
11	2	6.75	11.00	1	87.65%	80.00%
12	8	18.75	11.00	1	83.95%	75.00%
13	7	8.00	10.00	0	93.75%	37.50%
14	3	4.50	11.00	1	88.89%	47.50%
15	1	1.50	10.50	1	86.96%	57.50%
16	5	1.75	12.00	1	90.24%	67.50%
17	5	18.25	11.00	1	91.36%	55.00%
18	5	16.25	10.00	0	86.25%	55.00%
19	6	12.75	12.00	2	91.27%	65.00%
20	1	3.50	10.00	0	92.50%	55.00%
21	1	1.50	10.00	0	91.56%	62.50%
22	5	6.75	10.00	0	88.13%	52.50%
23	6	13.75	12.00	4	89.53%	77.50%
24	5	8.25	10.00	0	88.75%	57.50%
25	5	6.25	10.00	0	89.38%	55.00%
26	2	12.75	10.00	0	85.00%	57.50%
27	6	8.25	10.00	0	82.50%	50.00%
28	4	15.00	10.00	0	83.75%	60.00%
29	5	9.00	11.00	1	91.36%	55.00%
30	4	6.25	11.00	3	87.80%	80.49%

Table D.7: Detailed results for 95% resource utilization rate under dialysis priority setting

Instance	Procedures Delayed	Tardiness (Hrs.)	Completion Time (Hrs.)	Inpatients in Overtime	Fixed	Mobile
1	6	29.75	12.00	4	97.67%	80.00%
2	5	18.75	12.00	2	90.48%	85.00%
3	4	3.75	12.00	1	90.24%	90.00%
4	5	20.50	11.00	1	92.59%	67.50%
5	4	6.50	12.00	3	95.24%	82.50%
6	8	8.50	11.00	1	95.06%	82.50%
7	3	6.00	11.00	4	94.05%	85.00%
8	6	16.00	12.00	2	96.39%	85.00%
9	6	17.75	12.00	6	95.40%	80.00%
10	8	31.75	12.00	2	89.29%	82.50%
11	4	20.50	11.00	3	97.59%	75.00%
12	5	12.75	13.00	1	95.00%	77.50%
13	11	45.25	12.00	4	94.19%	80.00%
14	3	10.75	12.00	5	94.80%	70.00%
15	11	35.50	12.00	4	97.67%	85.00%
16	6	25.00	12.00	2	92.47%	95.00%
17	6	8.50	12.00	2	97.59%	85.00%
18	11	27.75	12.00	6	96.67%	85.00%
19	14	23.50	12.00	5	93.66%	85.63%
20	5	8.25	12.00	3	97.62%	85.00%
21	10	25.25	12.00	5	96.76%	85.19%
22	12	26.50	12.00	2	96.41%	87.50%
23	3	5.25	12.00	3	95.35%	72.50%
24	2	2.75	11.00	1	95.06%	87.50%
25	7	19.50	12.00	3	93.59%	70.00%
26	2	6.25	12.00	2	90.24%	90.24%
27	9	32.00	12.00	3	94.31%	90.00%
28	8	23.00	11.00	4	95.21%	80.00%
29	6	23.25	12.00	3	92.77%	90.24%
30	13	23.25	12.00	5	97.73%	95.12%

Table D.8: Detailed results for 95% resource utilization rate under non-dialysis priority setting

Instance	Procedures Delayed	Tardiness (Hrs.)	Completion Time (Hrs.)	Inpatients in Overtime	Fixed	Mobile
1	5	23.25	12.00	4	97.70%	77.50%
2	4	5.25	15.00	3	88.76%	73.33%
3	2	1.50	12.00	2	89.02%	90.24%
4	1	1.00	14.00	3	91.36%	50.00%
5	0	0.00	14.25	5	85.96%	74.42%
6	3	3.25	12.50	4	90.17%	76.19%
7	1	2.50	13.00	5	91.95%	80.49%
8	1	2.25	14.00	4	94.32%	72.09%
9	3	3.50	16.00	5	90.57%	77.50%
10	4	6.00	16.00	5	91.49%	55.00%
11	1	11.25	15.00	4	95.19%	55.00%
12	1	1.75	14.00	2	94.74%	72.50%
13	2	3.00	16.00	6	91.62%	64.92%
14	0	0.00	16.00	3	92.39%	62.50%
15	4	18.00	14.00	6	91.64%	78.13%
16	2	2.00	15.00	4	93.46%	72.50%
17	2	4.00	15.00	4	94.44%	75.00%
18	4	7.50	15.25	7	90.77%	81.87%
19	6	8.25	16.00	9	91.55%	45.00%
20	0	0.00	15.00	7	96.88%	56.44%
21	8	23.25	13.00	6	90.74%	82.72%
22	4	6.25	16.00	7	89.04%	45.00%
23	2	3.50	12.00	4	95.29%	74.53%
24	1	0.50	14.00	1	94.05%	82.50%
25	2	4.75	15.00	5	93.35%	42.50%
26	0	0.00	15.00	3	88.00%	82.93%
27	4	5.25	16.00	5	86.28%	72.50%
28	2	2.00	14.00	5	93.42%	67.50%
29	1	2.00	15.00	4	94.51%	70.00%
30	7	14.25	16.00	7	89.38%	95.12%

Table D.9: Detailed results for 95% resource utilization rate under balanced approach setting

Instance	Procedures Delayed	Tardiness (Hrs.)	Completion Time (Hrs.)	Inpatients in Overtime	Fixed	Mobile
1	5	23.00	12.00	4	95.35%	85.00%
2	6	13.50	12.00	2	92.77%	82.50%
3	4	4.75	12.00	2	92.86%	80.00%
4	4	11.00	12.00	2	97.59%	52.50%
5	2	2.75	12.00	3	92.77%	85.71%
6	4	6.75	12.00	3	92.26%	82.50%
7	2	3.50	11.00	4	95.18%	82.93%
8	4	7.75	12.00	3	95.24%	85.00%
9	4	3.75	16.00	5	92.31%	77.50%
10	7	8.00	12.00	1	91.46%	82.50%
11	4	19.50	12.00	2	95.24%	77.50%
12	2	2.25	12.50	1	94.55%	80.00%
13	7	19.25	12.00	3	93.98%	83.33%
14	3	4.75	12.00	2	97.59%	72.50%
15	7	24.25	12.00	4	93.10%	87.50%
16	4	5.25	11.75	3	95.22%	87.50%
17	5	7.00	11.00	3	97.56%	85.37%
18	7	14.75	12.00	7	97.73%	83.33%
19	13	19.50	12.00	5	92.79%	91.07%
20	4	7.75	12.00	4	97.65%	82.50%
21	9	22.75	12.00	4	96.80%	82.72%
22	10	23.75	12.00	5	96.59%	76.25%
23	3	5.25	12.00	3	95.35%	72.50%
24	2	1.75	11.00	3	95.18%	82.50%
25	6	15.00	11.50	3	91.32%	80.00%
26	2	5.25	11.00	4	92.77%	82.93%
27	7	11.00	15.50	4	91.16%	82.50%
28	5	12.25	12.00	4	93.68%	75.00%
29	2	2.00	12.00	4	97.70%	72.50%
30	10	22.00	13.00	7	93.80%	97.67%

Appendix E: Detailed Results of Set of Experiments of Section 5.5.3

This appendix shows the detailed results of the set of experiments run in section 5.5.3. This set of experiments performed a sensitivity analysis of the weights given in the penalty system to assess the tradeoffs in the dialysis schedules and the resulting performance measures:

Table E.1: Detailed results for Case A – (1, 4) ordered pair
(Efficiency, On-time delivery of other procedures)

Instance	Procedures Delayed	Tardiness (Hr)	Completion Time	Inpatients in Overtime	Fixed	Mobile
1	4	18.25	11.25	3	92.49%	47.50%
2	4	8.00	12.00	5	92.88%	45.00%
3	4	6.25	14.00	3	89.77%	47.50%
4	5	10.75	13.00	4	88.92%	72.50%
5	6	13.50	12.00	4	87.32%	51.22%
7	1	2.50	14.00	3	93.71%	42.50%
10	2	3.25	15.00	2	93.16%	52.50%
11	2	3.75	12.00	1	81.71%	67.50%
12	3	9.50	11.00	1	90.00%	77.50%
15	7	6.25	13.00	1	93.98%	30.00%

Table E.2: Detailed results for Case A – (1, 3) ordered pair
(Efficiency, On-time delivery of other procedures)

Instance	Procedures Delayed	Tardiness (Hr)	Completion Time	Inpatients in Overtime	Fixed	Mobile
1	4	18.25	11.25	3	92.49%	47.50%
2	5	8.75	12.00	5	94.22%	47.50%
3	5	10.25	12.00	2	90.48%	55.00%
4	5	12.75	12.00	4	90.72%	72.50%
5	6	13.25	12.00	4	87.06%	51.85%
7	2	3.25	12.00	2	94.61%	50.00%
10	3	5.50	12.75	1	93.96%	62.50%
11	2	3.75	11.25	1	83.69%	65.00%
12	3	9.50	11.00	1	90.12%	75.00%
15	8	9.75	10.00	0	92.50%	40.00%

Table E.3: Detailed results for Case A – (1, 2) ordered pair

(Efficiency, On-time delivery of other procedures)

Instance	Procedures Delayed	Tardiness (Hr)	Completion Time	Inpatients in Overtime	Fixed	Mobile
1	5	23.25	11.00	2	91.46%	52.50%
2	5	8.75	12.00	5	93.95%	47.50%
3	6	12.25	12.00	1	90.24%	60.00%
4	5	10.50	12.00	4	90.43%	73.13%
5	8	15.00	11.00	2	86.42%	61.73%
7	2	3.25	12.00	2	93.41%	52.50%
10	4	12.00	10.00	0	95.00%	66.88%
11	3	4.00	12.00	1	82.01%	62.50%
12	3	9.25	11.00	1	90.12%	75.00%
15	8	9.75	10.00	0	93.75%	37.50%

Table E.4: Detailed results for Case A – (1, 1) ordered pair (Balanced Approach)

(Efficiency, On-time delivery of other procedures)

Instance	Procedures Delayed	Tardiness (Hr)	Completion Time	Inpatients in Overtime	Fixed	Mobile
1	6	24.75	11.00	1	93.83%	50.00%
2	7	11.25	11.50	3	89.76%	65.00%
3	6	12.25	12.00	1	90.24%	60.00%
4	7	15.75	11.00	2	86.73%	90.24%
5	9	20.00	11.00	1	85.19%	65.00%
7	3	8.50	12.00	1	91.46%	60.00%
10	4	12.00	10.00	0	91.25%	74.38%
11	3	12.75	10.00	0	86.25%	62.50%
12	2	6.75	11.00	1	87.65%	80.00%
15	7	8.00	10.00	0	93.75%	37.50%

Table E.5: Detailed results for Case A – (2, 1) ordered pair

(Efficiency, On-time delivery of other procedures)

Instance	Procedures Delayed	Tardiness (Hr)	Completion Time	Inpatients in Overtime	Fixed	Mobile
1	6	25.75	10.75	1	92.88%	52.50%
2	5	8.25	12.00	5	94.22%	47.50%
3	7	13.75	12.00	1	90.24%	60.00%
4	7	15.75	12.00	2	89.46%	70.00%
5	8	16.00	11.00	2	86.42%	61.73%
7	3	7.50	12.00	1	93.90%	55.00%
10	3	5.50	12.75	1	91.54%	67.50%
11	4	8.25	10.00	0	86.25%	62.50%
12	4	10.25	12.00	1	89.02%	75.00%
15	8	9.75	10.00	0	90.00%	45.00%

Table E.6: Detailed results for Case A – (3, 1) ordered pair

(Efficiency, On-time delivery of other procedures)

Instance	Procedures Delayed	Tardiness (Hrs.)	Completion Time (Hrs.)	Inpatients in Overtime	Fixed	Mobile
1	5	23.25	11.00	4	96.72%	37.50%
2	5	8.75	12.00	5	96.02%	40.00%
3	6	13.25	12.00	2	91.57%	55.00%
4	6	17.25	12.00	4	94.48%	65.00%
5	9	13.25	11.00	3	89.02%	54.32%
7	3	11.75	11.00	3	97.62%	42.50%
10	3	5.50	13.00	4	95.39%	50.00%
11	4	6.00	12.00	1	87.80%	55.00%
12	4	10.25	12.00	2	92.77%	65.00%
15	8	9.75	12.00	1	96.34%	27.50%

Table E.7: Detailed results for Case A – (4, 1) ordered pair

(Efficiency, On-time delivery of other procedures)

Instance	Procedures Delayed	Tardiness (Hrs.)	Completion Time (Hrs.)	Inpatients in Overtime	Fixed	Mobile
1	5	23.25	11.00	4	96.72%	37.50%
2	6	9.00	12.00	7	98.90%	27.50%
3	6	14.25	12.00	2	95.24%	45.00%
4	6	16.25	12.00	6	96.88%	55.00%
5	9	20.00	11.00	2	89.02%	55.00%
7	2	7.50	15.00	4	100.00%	25.00%
10	3	5.50	12.75	3	98.83%	45.00%
11	3	5.00	11.25	1	88.62%	55.00%
12	3	9.50	12.00	3	95.29%	55.00%
15	9	13.50	11.00	3	98.80%	20.00%

Table E.8: Detailed results for Case B – (2, 0) for (Push, Tardiness) ordered pair

Instance	Procedures Delayed	Tardiness (Hrs.)	Completion Time (Hrs.)	Inpatients in Overtime	Fixed	Mobile
1	7	87.75	10.00	0	90.00%	60.00%
2	6	72.75	5.25	2	88.02%	67.50%
3	5	57.25	10.00	0	88.75%	67.50%
4	5	25.00	12.00	3	91.96%	75.00%
5	7	80.00	12.00	1	89.02%	55.00%
7	3	37.25	10.00	0	91.25%	65.00%
10	5	70.25	11.00	1	92.28%	70.00%
11	3	32.75	10.00	0	81.25%	72.50%
12	3	32.50	11.00	1	90.12%	75.00%
15	6	55.50	10.00	0	93.75%	37.50%

Table E.9: Detailed results for Case B – (1.5, 0.5) for (Push, Tardiness) ordered pair

Instance	Procedures Delayed	Tardiness (Hrs.)	Completion Time (Hrs.)	Inpatients in Overtime	Fixed	Mobile
1	6	27.75	11.00	1	90.12%	57.50%
2	7	28.25	11.50	2	89.63%	67.50%
3	5	30.50	10.00	0	90.00%	65.00%
4	5	21.00	12.00	3	91.96%	75.00%
5	9	24.25	10.00	0	86.25%	65.00%
7	3	21.25	11.00	1	95.06%	55.00%
10	4	12.00	11.00	1	92.28%	70.00%
11	3	13.75	10.00	0	86.25%	62.50%
12	3	9.50	11.00	1	87.65%	80.00%
15	6	21.75	10.00	0	93.75%	37.50%

Table E.10: Detailed results for Case B – (0.5, 1.5) for (Push, Tardiness) ordered pair

Instance	Procedures Delayed	Tardiness (Hrs.)	Completion Time (Hrs.)	Inpatients in Overtime	Fixed	Mobile
1	5	22.25	11.00	4	92.97%	50.00%
2	5	9.50	12.00	5	93.06%	50.00%
3	6	12.25	12.00	1	90.24%	60.00%
4	6	12.75	12.00	3	92.88%	72.50%
5	10	12.50	12.00	1	83.75%	69.14%
7	3	7.50	12.00	2	97.59%	45.00%
10	4	12.00	10.00	0	92.19%	72.50%
11	3	4.00	11.50	1	84.66%	62.50%
12	3	9.50	11.00	1	90.12%	75.00%
15	8	9.75	10.00	0	93.75%	37.50%

Table E.11: Detailed results for Case B – (0, 2) for (Push, Tardiness) ordered pair

Instance	Procedures Delayed	Tardiness (Hrs.)	Completion Time (Hrs.)	Inpatients in Overtime	Fixed	Mobile
1	4	7.50	11.00	2	92.50%	55.00%
2	5	8.50	12.00	5	93.06%	50.00%
3	7	11.50	12.00	1	86.59%	67.50%
4	6	12.00	12.00	4	88.39%	80.49%
5	9	12.25	11.00	2	85.19%	64.20%
7	2	3.25	11.50	2	97.01%	45.00%
10	5	8.75	10.00	0	95.00%	66.88%
11	2	3.75	12.00	1	82.93%	65.00%
12	3	9.50	11.00	2	89.02%	75.00%
15	8	9.75	10.00	0	93.75%	37.50%

Appendix F: Data file and Output file for Example of Section 6.4

The data file used for the example of section 6.4 for the AMPL implementation of the model presented in Appendix B is as follows:

```
##**Data File – Example 6.3 **##

#SETS:
set P:= 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19;
set C:= 0 1 2 3;
set B:= 1 2 3 4 5 6;
set T:= 1 2 3 4 5 6 7 8 9 10 11 12;
set G[1] := 1;
set G[2] := 1;
set  G[3] := 1;
set G[4] := 1;
set G[5] := 1;
set G[6] := 1;
set G[7] := 1;
set G[8] := 1;
set G[9] := 1;
set G[10] := 1;
set G[11] := 1;
set G[12] := 1;
set G[13] := 1;
set G[14] := 1;
set G[15] := 1;
set G[16] := 1;
set G[17] := 1;
set G[18] := 1;
set G[19] := 1;

##PARAMETERS##
param hperiod:=2;

param MB:=3;

param DTime:=8;

param s:=
1  1 1   1
2  1 2   1
3  1 4   1
4  1 1   1
5  1 1   1
6  1 2   1
7  1 8   1
8  1 4   1
9  1 2   1
10 1 5   1
11 1 5   1
```

```
12 1 2   1
13 1 8   1
14 1 1   1
15 1 2   1
16 1 2   1
17 1 3   1
18 1 3   1
19 1 4   1
;

param d:=
1  1   2
2  1   4
3  1   3
4  1   3
5  1   4
6  1   2
7  1   1
8  1   2
9  1   4
10 1   3
11 1   4
12 1   3
13 1   1
14 1   4
15 1   2
16 1   4
17 1   2
18 1   3
19 1   2
;

param u:=
1  1   5
2  1   3
3  1   6
4  1   2
5  1   3
6  1   7
7  1   8
8  1   5
9  1   3
10 1   6
11 1   6
12 1   3
13 1   8
14 1   2
15 1   2
16 1   3
17 1   3
18 1   5
19 1   7
;
```

```
param lambda:=
1  1   2   0.6433
1  1   3   0.3922
1  1   4   0.1412
1  1   5   0.0017
2  1   3   0.0536
3  1   5   0.602
3  1   6   0.1561
4  1   2   0.0776
5  1   2   0.4491
5  1   3   0.0345
6  1   3   0.6632
6  1   4   0.4947
6  1   5   0.3263
6  1   6   0.1579
6  1   7   0.0187
8  1   5   0.2257
9  1   3   0.0264
10 1   6   0.3694
11 1   6   0.0478
12 1   3   0.243
14 1   2   0.1317
16 1   3   0.0091
18 1   4   0.395
18 1   5   0.016
19 1   5   0.6181
19 1   6   0.2225
19 1   7   0.0027
;

param omega:=
1  1   2   7.22
1  1   3   14.02
1  1   4   20.82
1  1   5   33.8
2  1   3   29.17
3  1   5   8.5
3  1   6   24.68
4  1   2   26.75
5  1   2   10.24
5  1   3   30.54
6  1   3   4.25
6  1   4   9.49
6  1   5   14.72
6  1   6   19.95
6  1   7   29.91
8  1   5   21
9  1   3   33.14
10 1   6   18.87
11 1   6   31.38
12 1   3   16.23
14 1   2   26.76
16 1   3   33.76
18 1   4   11.53
18 1   5   31.88
19 1   5   6.12
```

```
19 1   6   15.03
19 1   7   33.28
;

param theta :=
1  1   35
2  1   35
3  1   35
4  1   35
5  1   35
6  1   35
7  1    1
8  1   35
9  1   35
10 1   35
11 1   35
12 1   35
13 1    1
14 1   35
15 1    1
16 1   35
17 1   35
18 1   35
19 1   35
;

param k:=
1  2
2  2
3  2
4  1
5  1
6  1
;

param r:=
4  1
5  1
6  1
;

param alpha:=
 1  0.1
 2  0.2
 3  0.3
 4  0.4
 5  0.5
 6  6.1
 7  6.2
 8  6.3
 9  6.4
10  6.5
11  6.6
12  6.7
;
```

```
param beta:=
4  2
5  2
6  2
;

param l:=
2   2
5   2
7   2
10  2
11  1
15  2
18  2
;

param
i [*] :=
15  1
16  1
17  3
18  3
19  3
;

param w:=
 1  3
 2  4
 3  8
 4  8
 5  8
 6  8
 7  8
 8  8
 9  2
10  8
11  8
12  8
13  8
14  4
15  2
16  8
17  8
18  8
19  8
;

param gamma :=
1  1   0.5
1  2   1
1  3   1.5
2  1   0.5
2  2   1
2  3   1.5
2  4   2
9  1   0.5
```

```
9  2   1
14 1   0.5
14 2   1
14 3   1.5
14 4   2
15 1   0.5
15 2   1
;

param  :=
8    1
17  1
;

param A :=
1   1 1
18 1 1;


let {p in P, g in G[p], t in T:t>u[p,g]} phi[p,g,t]:= theta[p,g];
```

The next example of an output file presents the objective function value, the number and ID of procedures that are pushed and distinguish between those that would impact LOS (scheduled after its corresponding u) and those that would not (scheduled before u). Moreover, it displays the tardiness of the rescheduled procedures, the new proposed starting time for those procedures, the hemodialysis schedules, the unit utilization, and, if any, the inpatients treated in overtime:

```
############## Output file – Example 6.4 ##############

Penalties = 185.238

###Treatments Delayed =  4

Push :=
2  1   1
9  1   1
14 1   1
15 1   1
;

###Treatments Delayed Before u =  2
```

```
###Treatments Delayed After u =  2

###Tardiness before u(hr) =  4


:         s   Z    :=
1  1 1    1   1
2  1 2    1   0
2  1 3    0   1
3  1 4    1   1
4  1 1    1   1
5  1 1    1   1
6  1 2    1   1
7  1 8    1   1
8  1 4    1   1
9  1 2    1   0
9  1 3    0   1
10 1 5    1   1
11 1 5    1   1
12 1 2    1   1
13 1 8    1   1
14 1 1    1   0
14 1 9    0   1
15 1 2    1   0
15 1 7    0   1
16 1 2    1   1
17 1 3    1   1
18 1 3    1   1
19 1 4    1   1
;

X [*,1,*]
:   1   2   3   4   6   7    :=
1    0   0   1   1   0   0
2    1   1   0   0   0   0
3    1   1   0   0   0   0
7    0   0   1   1   0   0
16   0   0   0   0   1   1

 [*,2,*]
:   1   2   3   4   5   6    :=
5    0   0   0   0   1   1
10   0   0   1   1   0   0
12   0   0   0   0   1   1
13   0   0   1   1   0   0
18   1   1   0   0   0   0
19   1   1   0   0   0   0

 [*,3,*]
:   1   2   3   4   5    :=
4    0   0   0   1   1
6    0   0   0   1   1
9    1   1   0   0   0
11   0   0   1   0   0
14   1   1   0   0   0
```

```
 [*,4,*]
:   1   2    :=
8    1   1


 [*,5,*]
:   1   2    :=
17   1   1

 [*,6,*]
:   1   2    :=
15   1   1
;
```

###Fixed Reg. Time Utilization = 0.9117647058823529

###Mobile Reg. Time Utilization = 0.4

###Overall Utilization = 0.7551020408163265

###Inpatients/Block/Periods in Overtime =

```
X[p,b,t] :=
5  2 6   1
12 2 6   1
16 1 6   1
16 1 7   1
;
```

Appendix G: Detailed Results of Set of Experiments of Section 6.6

This appendix shows the detailed results of the set of experiments run in section 6.6. This set of experiments assessed the differences between the two proposed optimization methods considering the magnitude of the penalties for rescheduling non-dialysis procedures to the next day (impacting LOS):

Table G.1: Detailed results for Case A1 – scheduling under uncertainty

Instance	**Procedures Delayed Before u**	**Procedures Delayed After u**	**Completion Time (Hrs.)**	**Inpatients in Overtime**	**Fixed**	**Mobile**
1	1	3	14.00	2	92.75%	47.50%
2	1	1	12.50	7	91.53%	39.02%
3	3	2	16.00	3	88.64%	47.50%
4	3	1	13.00	4	93.98%	32.50%
5	0	1	14.75	2	89.14%	57.50%
6	0	1	15.75	4	79.11%	67.50%
7	0	5	16.00	9	90.19%	48.19%
8	2	3	13.00	4	95.16%	25.00%
9	1	2	15.00	5	88.77%	42.50%
10	0	0	14.00	1	92.86%	42.50%
11	2	1	16.00	4	83.67%	35.00%
12	3	1	15.00	3	77.89%	44.44%
13	1	3	0.95	4	97.11%	29.38%
14	1	3	15.00	5	87.75%	41.72%
15	1	1	16.00	3	9.53%	40.00%
16	0	1	16.00	6	89.49%	59.38%
17	0	6	15.25	3	88.73%	45.00%
18	1	3	12.50	2	95.27%	50.00%
19	0	0	13.00	3	93.50%	50.00%
20	2	2	12.25	2	95.24%	47.50%
21	2	1	12.50	3	90.32%	45.00%
22	1	1	15.00	4	93.09%	20.00%
23	2	2	14.00	3	90.23%	50.30%
24	2	2	16.00	4	76.81%	35.00%
25	1	1	13.25	1	84.08%	72.50%
26	0	1	16.00	5	91.10%	20.00%
27	0	3	14.00	4	92.13%	19.39%
28	0	2	15.00	3	87.67%	32.50%
29	3	0	12.25	2	91.29%	37.50%
30	0	2	15.00	5	91.82%	47.50%

Table G.2: Detailed results for Case A1 – scheduling without uncertainty

Instance	Procedures Delayed Before u	Procedures Delayed After u	Completion Time (Hrs.)	Inpatients in Overtime	Fixed	Mobile
1	1	3	14.00	2	92.75%	47.50%
2	1	1	12.50	7	0.00%	37.27%
3	3	2	15.25	3	89.40%	47.50%
4	2	2	13.00	4	93.98%	32.50%
5	0	1	14.75	2	89.14%	57.50%
6	0	1	15.75	4	79.11%	67.50%
7	0	5	16.00	8	91.01%	46.63%
8	0	5	13.00	4	91.12%	35.00%
9	0	3	15.00	5	89.01%	42.50%
10	0	0	13.50	1	93.41%	42.50%
11	1	1	16.00	5	83.09%	25.00%
12	3	1	15.00	3	87.06%	44.44%
13	0	1	14.50	6	83.65%	40.86%
14	1	3	15.00	4	92.57%	32.50%
15	1	1	15.50	3	84.78%	40.00%
16	0	2	16.00	6	88.30%	59.38%
17	0	6	15.25	3	87.11%	47.50%
18	1	3	12.50	2	95.27%	50.00%
19	0	0	13.00	3	93.77%	50.00%
20	2	2	12.25	2	96.43%	45.00%
21	2	1	12.50	3	90.32%	45.00%
22	1	1	15.00	4	81.25%	27.50%
23	0	3	14.00	4	89.83%	47.90%
24	1	3	16.00	5	80.80%	25.00%
25	1	1	13.25	1	84.08%	72.50%
26	0	1	16.00	5	91.82%	20.00%
27	0	3	13.50	4	92.66%	19.39%
28	0	1	15.00	2	91.59%	35.00%
29	1	1	13.00	3	83.63%	47.50%
30	0	2	15.00	5	91.82%	47.50%

Table G.3: Detailed results for Case A2 – scheduling under uncertainty

Instance	Procedures Delayed Before u	Procedures Delayed After u	Completion Time (Hrs.)	Inpatients in Overtime	Fixed	Mobile
1	1	3	14.00	2	92.75%	47.50%
2	1	1	13.00	5	95.77%	29.81%
3	3	2	16.00	3	88.64%	47.50%
4	3	1	13.00	4	91.07%	40.00%
5	0	1	14.75	2	89.14%	57.50%
6	0	1	15.75	4	79.95%	67.50%
7	1	4	15.50	9	90.43%	49.08%
8	2	3	13.00	3	93.60%	32.50%
9	1	2	15.00	5	87.94%	45.00%
10	0	0	13.50	1	93.41%	42.50%
11	2	1	16.00	4	82.21%	35.00%
12	2	2	15.00	3	86.05%	43.90%
13	1	1	14.25	5	84.51%	47.31%
14	2	3	16.00	4	88.52%	37.50%
15	1	1	15.50	3	84.10%	40.00%
16	0	2	16.00	6	89.25%	59.38%
17	1	5	15.25	3	87.11%	47.50%
18	2	2	12.50	2	95.27%	50.00%
19	0	0	13.00	3	93.77%	50.00%
20	2	2	12.25	2	96.43%	45.00%
21	1	2	12.50	2	90.59%	45.00%
22	1	1	15.00	4	89.89%	27.50%
23	0	3	14.00	4	89.83%	47.62%
24	2	2	16.00	4	72.82%	45.00%
25	1	1	16.00	1	83.33%	72.50%
26	0	1	16.00	5	94.82%	20.00%
27	0	3	13.50	3	94.80%	19.39%
28	0	1	15.00	2	91.59%	35.00%
29	3	0	12.25	2	91.29%	37.50%
30	0	2	15.00	4	89.24%	52.50%

Table G.4: Detailed results for Case A2 – scheduling without uncertainty

Instance	Procedures Delayed Before u	Procedures Delayed After u	Completion Time (Hrs.)	Inpatients in Overtime	Fixed	Mobile
1	2	2	14.00	2	92.75%	47.50%
2	1	1	12.50	6	93.71%	37.27%
3	2	2	16.00	3	85.23%	55.00%
4	1	3	13.00	4	92.05%	35.00%
5	0	1	14.75	2	89.14%	57.50%
6	0	1	15.75	4	79.95%	67.50%
7	2	3	15.50	9	91.49%	45.51%
8	2	3	13.00	3	93.60%	32.50%
9	0	3	15.00	5	89.01%	42.50%
10	0	0	13.50	1	93.41%	42.50%
11	0	2	16.00	5	88.24%	40.00%
12	2	2	15.00	3	86.05%	44.44%
13	0	1	15.25	6	91.25%	23.66%
14	1	3	15.00	4	90.40%	35.00%
15	1	1	15.50	3	90.25%	32.50%
16	0	1	15.25	6	90.22%	59.38%
17	0	6	15.25	3	87.61%	47.50%
18	1	3	12.50	2	95.27%	50.00%
19	0	0	13.00	3	93.50%	50.00%
20	2	2	12.25	2	96.43%	45.00%
21	1	2	12.50	2	90.59%	45.00%
22	1	1	14.50	4	93.58%	20.00%
23	0	3	14.00	4	89.83%	47.90%
24	0	4	16.00	4	80.80%	25.00%
25	1	1	13.25	1	84.08%	72.50%
26	0	1	16.00	5	90.19%	25.00%
27	0	3	13.50	3	94.80%	19.39%
28	0	1	15.00	2	91.59%	35.00%
29	1	1	13.00	3	83.48%	47.50%
30	0	2	14.75	5	92.06%	47.50%