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A Dynamic inventory optimization method applied to printer fleet management

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A DYNAMIC INVENTORY OPTIMIZATION METHOD APPLIED TO PRINTER FLEET MANAGEMENT

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Abstract

Current optimization methods for inventory management of toner cartridges for printer fleets typically focus on aggregate cartridge demand. However, with the development of printer technology, toner consumption algorithms are being developed which can accurately quantify the amount of toner that has been consumed over time, based on print job characteristics. This research introduces a dynamic inventory optimization approach for a fleet of printers over a rolling time horizon. Given, the consumption algorithm for the printer system, the cumulative toner consumed per cartridge per printer can be tracked. A forecasting method is developed which utilizes this toner consumption data for individual printers to forecast toner cartridge replacement times. Taking into account the uncertainty related to demand, demand forecast and lead time, an optimization model has been developed to determine the order placement times and order quantities to minimize the total cost subject to a specified service level. An experimental performance evaluation has been conducted on the parameters of the dynamic inventory management algorithm. Based on the results of this evaluation, the implementation of this dynamic inventory optimization methodology could have a positive impact on printer fleet management.

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1 Introduction

A key aspect in managing inventory is dealing with the uncertainties in demand and supply lead time. The major challenges in managing inventory systems of toner cartridges for a fleet of printers are when to order and what quantity to order to achieve a desired service level and minimize the total cost associated with the ordering policy. The traditional optimization methods for toner cartridge inventory systems are based on the aggregate cartridge demand. The uniqueness of this problem when compared with the inventory replenishment systems already existing in literature is that there exists variability in the aggregate demand due to the variability in independent demands for the cartridges per printer. However, with the development of printer technology, toner consumption algorithms are being developed which can accurately quantify the amount of toner that has been consumed over time, based on print job characteristics. Currently, some companies like Xerox (Lobiondo et al., 1994) use tracking mechanisms to record the aggregate consumption of toner cartridges and the policy is to order whenever the inventory goes below a threshold value. Even with this level of real time or near real time consumption data on hand, a forecasting model and an optimization model are required to give the optimal inventory management strategy for a given time horizon.

Effectively managing the inventory systems of printer cartridges is significant in organizations housing a large number of printers. For example, in the University of Pennsylvania Medical Center there are 10,000 printers for 43000 employees with about $7 billion spent on printing. In such a case managing the cartridge supplies efficiently is very necessary as eliminating additional inventory costs can help save a lot of money on a large scale (Syzmanski, 2008).

This research introduces a dynamic inventory optimization approach for a fleet of printers over a rolling time horizon. The idea is to first track the toner consumption per cartridge per printer using a consumption algorithm. This toner consumption data for individual printers is then used to forecast toner cartridge replacement times. Taking into account the uncertainty related to demand, demand forecast and lead time, an optimization model is developed to determine the order placement times and order quantities to minimize the total cost subject to a specified inventory level.

Before placing an order, the trade-off among the critical costs associated with the ordering policy, which includes the holding, ordering and penalty costs have to be evaluated. The holding or carrying costs represent the cost of having the inventory on hand, such as storage and investment costs; the ordering costs are mainly the costs involved in processing the orders as well as the communication costs; and the penalty or shortage costs are the costs incurred due to absence of on-hand inventory (Emmet, 2005).

The target is to find a balance between the cost of the ordering policy and the cost of providing the required service level desired by the customer. A high level of inventory would provide a higher service level, but at a higher cost and vice versa. The ideal scenario is to achieve low cost with high service. In practice the toner cartridge inventory system is associated with uncertainties due to the demand, demand forecast and the lead time. The uncertainty in the independent demands of products is due to the random and unpredictable consumer behavior.

The focus of the research is to develop an algorithm to dynamically determine the replenishment strategy over a specified time horizon, for the inventory of toner cartridges for a printer fleet system. Figure 1.1 gives the general outline of the problem system. The

printer fleet consists of a set of printers with different usage characteristics. The utilization of each printer in the group is variable and user dependent. The consumption of a printer's toner cartridge is dependent on the print job characteristics such as the size of the job, the number of pixels in the image, the positional relationship of the pixels and the colors specified, which leads to variable replacement times for the cartridges of each printer in the fleet. In the system, every time a printer runs out of toner it is replaced with a cartridge from the inventory. The toner consumption can be monitored using a toner consumption algorithm which calculates toner consumed as a function of the print job's characteristics. Using this data from the consumption algorithm, the approximate replacement time of the cartridge in use and the subsequent replacement times are predicted using a demand forecast calculation methodology. This demand forecast for each printer and the existing levels of inventory are sent as inputs to an optimization model to calculate the number of cartridges to be ordered and when to place this order while attaining a minimal cost at a required service level associated with this order placement.

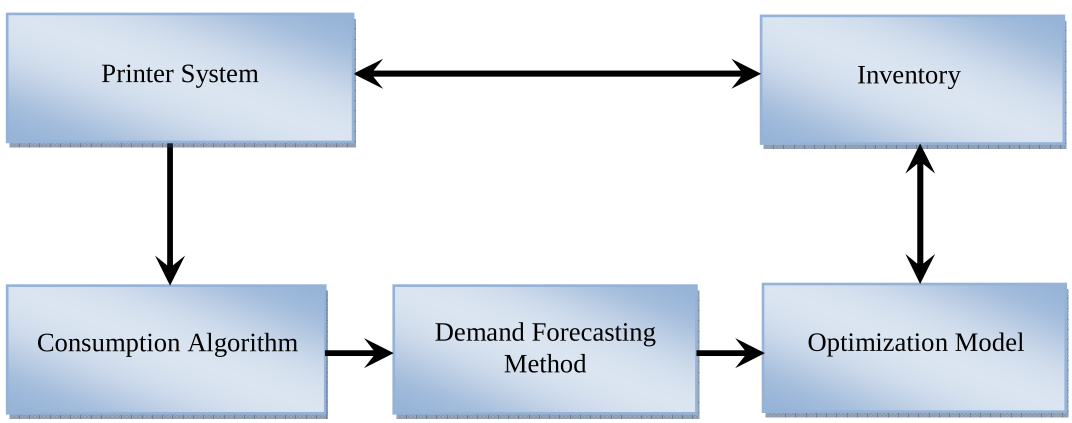

Figure 1.1: Overview of the printer fleet inventory management system

The remainder of the thesis is organized as follows. In Chapter 2, the problem detail and the end goal of the current research are described. Chapter 3 gives an overview of all the existing literature reviewed to aid in developing the idea and methodology for the current research. Followed by this in Chapter 4 is the description of the scope of this research and methodology to develop the proposed algorithm. The implementation methodology for the developed algorithm is described in Chapter 5. Chapter 6 presents the experimental evaluation of the developed inventory management algorithm, to test for its robustness in various scenarios. Finally, further steps are discussed in Chapter 7. All the models developed using computer software, sample outputs from these and the user guide to the algorithm are illustrated in the Appendix.

2 Problem Statement

In stochastic inventory systems having uncertainty in demand, demand forecast and the supply lead time, formulating an optimal ordering strategy over a time horizon which minimizes the cost and maximizes the service levels is still an area of research interest. Periodically reviewing these ordering strategies over a rolling time horizon basis is essential to dynamically update the replenishment strategy. This checks for the need to order based on the expected future demand and the time taken to provide this inventory by the supplier, to avoid additional shortage or surplus costs, loss of customer good will and loss of reputation to the business.

The focus of this research is to manage inventory levels in an organization housing a fleet of printers, where each printer has its own individual requirement for toner cartridges over a time period due to the variability in the demand for print jobs in each printer. The variability in demand is due to the different usage patterns for each printer over a time period. Here, though the toner cartridges may be interchangeable between printers, there still exists an uncertainty in forecasting the aggregate demand for the cartridges of all the printers in the fleet. Given forecasts for individual demand units and lead time uncertainty, the objective is to calculate a minimal total inventory cost ordering strategy subject to a required service level, over a specified time horizon.

The goal of this research is to design and develop a method to minimize the total inventory cost of an inventory system subject to a specified service level over a finite time horizon by determining the optimal times of order placement and order quantities given forecasts for individual demand units under uncertainty associated with demand, the

demand forecast, and order lead time. To accomplish this goal, the objectives of this research are to:

- Develop a method, specifically for the printer application that takes inputs in terms of current inventory level and demand forecasts based on an extrapolation of the consumption algorithm data to give solutions for the order time and order quantity over a time horizon;
- Construct a simulation model to represent the printer system - Simulation facilitates dynamic generation of print jobs to replicate the real-world printing scenario. The simulation component of the model can be replaced with the real printer systems once the model is tested for its robustness;
- Select and implement a toner consumption algorithm – The toner consumption algorithm is used to calculate the toner consumed per print job per cartridge. The consumption algorithm selected can be updated as newer and better algorithms are developed;
- Calculate the forecasted demands – The demand forecast is calculated by extrapolating the toner consumption data of the cartridge in use from the toner consumption algorithm;
- Develop an optimization model which takes the demand forecasts and the current inventory level as inputs to calculate an optimal ordering strategy which minimizes the costs subject to a specified service level over a time horizon;
- Conduct experimental performance evaluation to test the capabilities and computational efficiencies of the method; and

- Draw inferences about the effectiveness of the proposed inventory management system.

With the development of this method for discrete time, dynamic, stochastic inventory systems, purchasing managers of companies with a fleet of printers will have a tool to aid in determining an optimal ordering policy with minimum cost and required customer service levels.

3 Literature Review

"Every management mistake ends up in inventory."

- Michel C. Bergerac (Ballou, 2007)

Efficient and cost effective management of stochastic inventory systems has been a challenge for many years. Designing optimization methods to dynamically determine a replenishment policy for a stochastic inventory system over a time horizon, still remains an area of interest. This section summarizes the existing methods and indentifies the gap between the methods described in literature and the current practices.

3.1 Inventory Systems

The basic challenge of managing an inventory system is to calculate a well defined minimal cost ordering policy. In order to determine the optimal order quantity and order placement time, inventory models suggested in literature include the EOQ models, Single and Multi-period stochastic inventory systems, Continuous and Periodic review systems (Nahmias, 1997). The Economic Order Quantity (EOQ) model is a deterministic demand model where the demand rate is known and is a constant value over a period of time and there is no lead time. Figure 3.1 shows the inventory levels for the EOQ model.

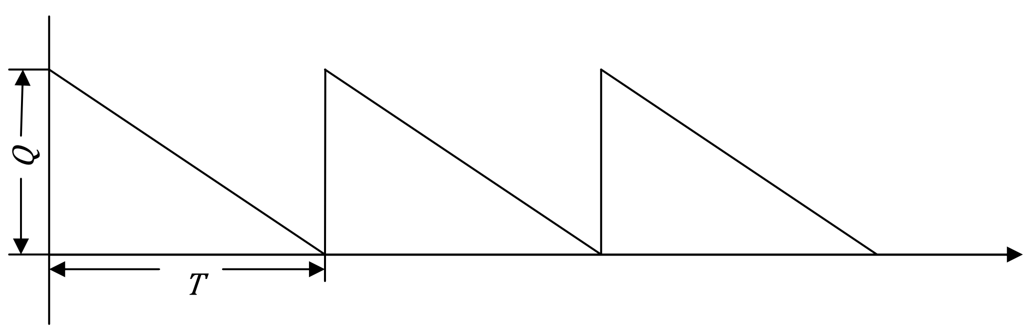

Figure 3.1: Inventory levels for the EOQ model

Stochastic inventory models consider the sources of uncertainty which include variability in customer needs and demands, variability in predicting these demands and variability in service levels. The uncertainty in demand leads to the uncertainty in predicting it which makes demand forecasting for stochastic demands an important task. The general practice is to predict the future probable demand distribution from related previous experience or data. The inventory control models subject to uncertainty are of two types- Continuous Review models and Periodic Review models. For example the Lot Size-Reorder Point (Q, R) model is a continuous review inventory system. Here as the level of on-hand inventory reaches R, an order is placed for Q units that will arrive in τ units of time. Figure 3.2 shows the inventory over time for a basic (Q, R) model.

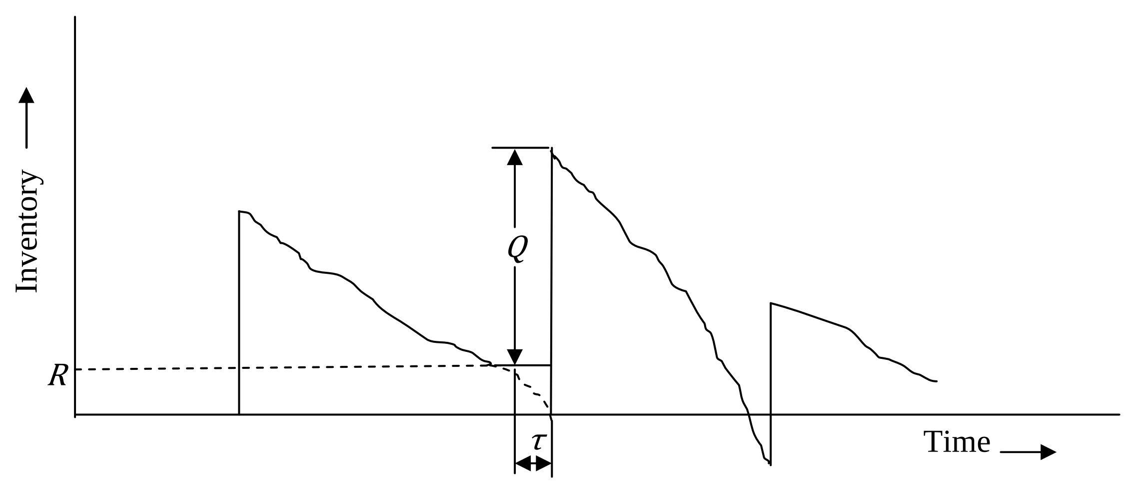

Figure 3.2: Change in inventory over time for a (Q, R) system

In a Periodic review model as the inventory levels are known only at discrete points of time, an Order-Up-to-Level (s, S) policy is a classic example of the same. According to this policy, whenever the on-hand inventory is less than or equal to s, an order for the difference between the inventory and S is placed (Nahmias, 1997). Figure 3.3 shows the inventory over time for an (s, S) model.

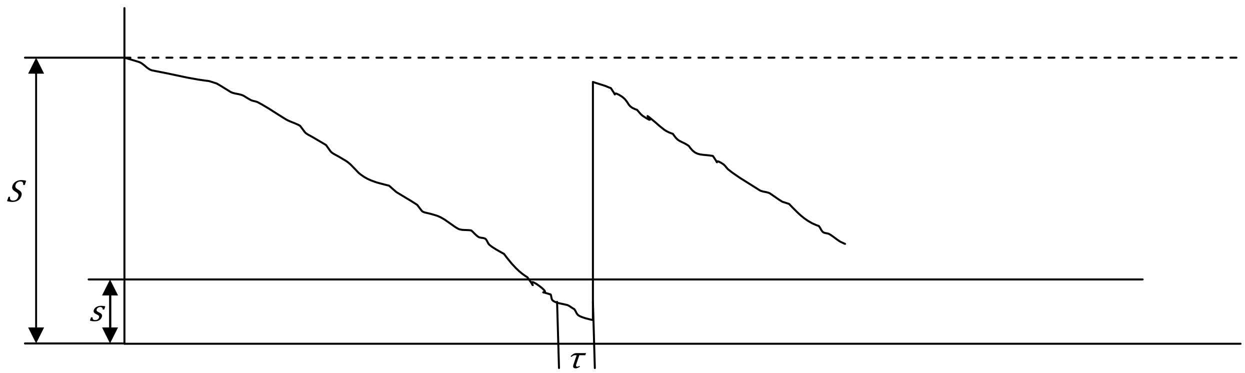

Figure 3.3: Inventory over time for an (s,S) model

This policy has been further developed for a single item periodic review system such that the buyer decides when the quantity has to be ordered to receive it on a particular day (Chiang, 2008). For systems with stochastic demand rates, lead times and service levels, considerable research has been done to calculate an optimum replenishment policy with demand arrivals in the form of a Poisson process (Berman et al., 2003). In particular, inventory systems with uncertainty have been investigated for various combinations of the distributions of demand and lead time including normal demand and gamma lead times (Burgin, 1972), normal demand and exponential lead time (Das, 1976) and Poisson demand and exponential lead time (Carlson, 1982). Further, systems having constant demand and variable lead time with backorders are considered in Ben-Daya, (1994) and Ouyang et al. (2001). Tarim et al. (2004) calculate the minimum total expected cost to maintain a multi-period, single item stochastic inventory while considering minimum service levels.

For inventory models with a demand distribution and uncertainty in lead time, a mixed inventory backorder and lost sales problem is studied by considering lead time and order quantity as decision variables in determining an optimal inventory model using

service level as a constraint in the optimization process instead of using stock-out cost which is difficult to quantify (Chu et al., 2005). The model assumes a particular probability distribution for the demand during lead time, continuous review of inventory and the optimal order quantity and re-order point are calculated by minimizing the total cost. The cost components considered are ordering cost, holding cost and lead time crash cost. The resulting objective function is:

$$\text{Minimize } TotalCost(Q, L) = A\frac{D}{Q} + h\left[\frac{Q}{2}k\sigma\sqrt{L} + (1-\beta)B(r)\right] + \frac{D}{Q}C(L)$$

where, A is fixed ordering cost per order, h is inventory holding cost per item per year, L is length of lead time, $C(L)$ is the lead time crashing cost, k is the safety factor, r is the reorder point, β is the fraction of demand during the stock-out period that will be backordered, α is the proportion of demands that are not met from the stock, i.e. service level is (1- α) and $B(r)$ is the expected demand shortage at the end of the cycle. Solving this objective function is subject to the lead time constraint that the proportion of expected demand shortage at the end of cycle should be less than accepted level of backorders

$$\frac{B(r)}{Q} \leq \alpha.$$

By solving these analytically using a developed algorithm, optimal solutions of lead time and order quantity at a reduced total cost have been calculated. The model is extended to a free-distribution of lead time while considering a normal distribution for the demand.

This model can be further extended to a single-product, discrete-time, non-stationary, inventory replenishment problem with both supply and demand uncertainty, capacity limits on replenishment quantities, and service level requirements, where a simulation-based optimization approach is used to test a heuristic methodology developed to calculate

the quantity and timing of the order placement (Bollapragada et al., 2005). Nevison et al., (1984) propose an optimal solution for inventory systems with uncertainty in lead time but deterministic demands by considering interdependence of the probabilistic lead times. The method developed in this research considers uncertainty due to both demand and lead time to calculate the optimal ordering strategy over a given time period.

3.2 Inventory Monitoring Systems

The rate at which toner is used in a printer is dependent on various factors such as model of the printer, environmental variables, average print job length, color of print job, density of the print, and sensitiveness of the toner level sensor (Kendall, 2008). A method for predicting when a toner cartridge of a printer should be replaced was developed by Frankel et al. (2006). The model uses replaceable toner cartridges for four different colors in the printer. Various processes and algorithms are provided that calculate the end of life of the cartridge in use and also alerts the user if the cartridge has to be changed in between or to the end of a job. Hopper et al. (2003) provides a method, system and program for monitoring the depletion of a toner cartridge in a monitored system. The toner consumption is monitored based on the factors such as contrast and boldness of the image. Filbrich et al. (2007) calculated the amount of toner used based on pixel count and location. The method includes determining the pixel count for a page and the plurality of proximity factors and thereby providing an estimate of toner consumed per page. Kendall (2008) developed a method for estimating the quantity of toner remaining in a toner cartridge. A cost effective toner consumption algorithm which is adjusted based on actual printer toner usage rate has been developed and the toner used is defined as:

Toner used = (Number indicating wear incurred)(Wear constant) + (Number indicating the number and toner of the pixels printed)*(Pixel Constant)*(Adaptive Term).*

3.3 Simulation and Supply Chain Management

In the early 1990s, supply chain management was defined as a process of integrating/utilizing suppliers, manufacturers, warehouses, and retailers, so that goods are produced and delivered at the right quantities, and at the right time, while minimizing costs as well as satisfying customer requirements (Chang et al., 2001). He heightens the benefits of using discrete event simulation to evaluate the performance of a supply chain. Simulation helps to understand the overall supply chain processes and characteristics by graphics/animation, captures system dynamics: using probability distribution and can minimize the risk of changes in planning process. In a system where demand uncertainty exists, the problem of determining the safety stock level to use to meet a desired level of customer satisfaction can be addressed using a simulation based optimization approach (Jung et al., 2004). Takashi et al. (1998) proposes a concept called virtual printer which is a simulation of the actual printing system and printer. Simulation is used for running experiments and data generation as it makes the testing of the model cost-efficient and gives and insight into the functioning of the model (Grabau et al., 1997). This gives an idea to implement simulation techniques to represent the real-world scenario of the printer fleet inventory system.

3.4 Demand Forecasting

Demand forecasting is the process of predicting the future demand for a product. Forecasting can be classified in several ways (Nahmias, 1997). One classification is on the basis of time horizon as: Short term, Intermediate term and Long term forecasting. Short

term forecasts are usually measured in days or weeks and used mostly in inventory management. The demand forecasting associated with the current research model is a good example of short term forecasting methods. Intermediate forecasting techniques generally measure in weeks or months and are used for sales and resource requirement forecasting. Long term production and management decisions require Long term forecasting methods. Another classification of forecasting methods based on the data analysis techniques is subjective methods and objective methods. Subjective forecasting techniques are based on human judgment. Objective forecasting methods are based on data analysis. These are further classified as causal models and time series methods of forecasting. Causal models use data from factors related to the data series under consideration, i.e. forecast for a trend is a function of the variables ($X_1, X_2 ... X_n$) related to Y. A special case of causal models are the econometric models in which there exists a linear relationship between Y and ($X_1, X_2 ... X_n$).

$$Y = \alpha_0 + \alpha_1 X_1 + \alpha_2 X_2 + \cdots + \alpha_n X_n$$

The other types of causal methods based on a trend of the independent variable's data are the regression analysis, Holt's method and the Box Jenkins method (Nahmias, 1997).

The time series methods are used to calculate a forecast based on the historic data drawn at discrete intervals of time of the variable being predicted. In this method of forecasting, the data collected from previous observations is classified based on the patterns associated with it. The various patterns that can be observed in the data are:

- A *Trend* in which it exhibits a steady pattern of growth or decline
- A *Seasonal* pattern in which the data pattern repeats at fixed intervals
- A *Cyclic* variation which is similar to the seasonal pattern except that the length of each cycle may be different

- A pure *Random* series where no recognizable pattern can be associated with the data available

One particular type of demand forecasting is predicting the intermittent demand which is characterized by a frequent mix of zero values with non-zero demand values. Viswanathan et al. (2008) has investigated various demand forecasting techniques such as top down and bottom up approach methods to estimate the aggregate data series when the sub-aggregate time series components are intermittent. This led to a patenting a method to calculate the forecast of intermittent demand over a lead time using statistical and sample reuse techniques.

3.5 Periodic Review Policies

"One inventory cost-control practice that qualifies as the best of the best is the periodic review of inventory position" (IOMA, 2006). Periodic review of the inventory position is necessary to dynamically update the replenishment strategy for the inventory and also to minimize the errors due to a static forecast horizon. Rolling-Horizon method and Rolling Schedule methods are two such periodic review algorithms for stochastic inventory systems. Bollapragada et al. (2005) illustrates with examples how the solution to a static planning problem may be implemented in a rolling-horizon method. In a stochastic system a rolling-horizon framework for periodic review helps update the system parameters dynamically. Girlich (1989) suggests adopting a rolling schedule method for reviewing inventory systems involving uncertainty, which involves updating the inventory policy for every time period.

3.6 Current Practices

Reliable Technologies Inc. (http://www.reliabletechnologiesinc.com/), a printer services management company, indicates that the key to sustaining the savings obtained from managing a printer fleet is to adjust the inventory policies on a continuous basis. Companies like Xerox Corporation (Lobiondo et al., 1994) are using reprographic machines integrated with a computer to keep track of the consumables supplies of a group of printers with the help of recording mechanisms. The reprographic machines calculate the aggregate demand for consumable supplies in all the printers in the group and place an order once this quantity goes below a specified threshold value. In comparison to these methods, the solution to the research algorithm not only gives an ordering policy designed on the basis on individual printer cartridge demands but also minimizes the cost associated with the order and maximizes the service levels required.

4 Methodology

The purpose of the research is to develop an algorithm that can be periodically reviewed to dynamically replenish the inventory of the toner cartridges at a minimum cost and desired service level. This section presents the scope and methodology of the research and the means adopted to develop the dynamic inventory optimization system.

The primary objective of the research is to develop a system optimization method to dynamically determine the replenishment strategy for inventory over a specified time horizon for a stochastic inventory system for printer cartridges to minimize the total cost of inventory and meet a desired service level. In particular, the research will focus on:

- Calculating an accurate forecast for the demand (replenishment of a printer cartridge) while considering the uncertainties due to demand, demand forecasts and supplier lead time; and
- Using this forecasted demand in an optimization model to determine order quantities and dates of order placement over a given time horizon to minimize the total cost over the time horizon given a specified service level requirement.

The uncertainty in demand for toner in each printer is due to the variations in size and characteristics of the print jobs. The uncertainty in demand forecasting is due to the forecast errors resulting from system downtime, scheduling errors and lag in trends. The variability in supplier lead time is due to factors such as transportation time, time associated with information processing, manufacturing or supply-chain issues. The toner cartridges are assumed to be interchangeable among printers and are only black in color. This limitation is made in order to simplify the printer system to variability in consumption

of a single color cartridge. The consumption algorithm can be extended to multiple color cartridges in the future.

This section discusses in detail the methods that are used to design and develop the above discussed algorithm. The algorithm addresses the objective of the replenishment policy- minimum cost at a specified service level. The structure of the system is modular and each section of the model is represented as a part by itself. So, the various components of the system can be substituted and updated as new and better systems are available.

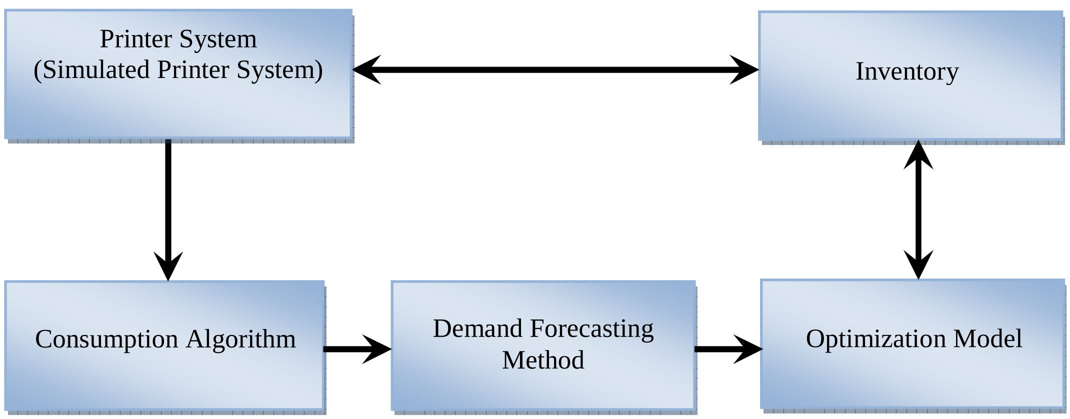

Figure 4.1: Overview of the printer fleet inventory system

Figure 4.1 describes the overview of the inventory system under consideration. The system consists of five components, the printer system, consumption algorithm, demand forecasting method, optimization method and an inventory system. The detailed methodology for developing this system is illustrated with an example in this chapter.

Example: *A printer fleet consisting of one hundred printers is considered. Each printer in the group has its own set of print jobs arriving randomly. The print jobs are considered to be of three types – word document, picture file, and graphics file. The maximum toner in each printer's toner cartridge is considered to be 75 grams/cartridge. The holding and penalty costs are assumed to be $2 and $5 per unit per day and the ordering cost is $10 per order. An*

initial inventory of 1 unit is assumed to be present at the start of the system run. The service level required is 90%. The current date is assumed to be day 6 of the time horizon of 30 days.

4.1 Printer System

The printer system consists of a group of printers where each printer has an individual demand of print jobs. In the physical system each printer would monitor the print jobs it produces and be monitored by a toner consumption algorithm tool for each toner cartridge. To enable experimentation and testing of the inventory management system under many system configurations, a simulation model is created to represent the physical printer fleet. The simulation model is developed using a discrete event simulation software. The arrivals of print jobs are modeled as stochastic processes. The model currently simulates three kinds of print jobs including text documents, picture documents, and graphics documents. The printing process time and the quantity of toner user per job is calculated based on the printer type and print job type. The simulation model results in the simulated *actual* amount of toner consumed by each job. The resulting randomly generated print job characteristics are then utilized by the consumption algorithm to *estimate* the amount of toner consumed. Table 4.1 shows the 5 random inter-arrival distributions between print jobs. A distribution is randomly assigned to a printer. Shown in Figures 4.1 to 4.5 are some potential sample shapes that might be observed using these distributions.

Table 4.1: Random arrival distributions of print jobs for the printers

Arrival Distribution (time in seconds)
NORMAL(UNIFORM(300, 700), UNIFORM(20,80))
EXPONENTIAL(UNIFORM(100, 800))
GAMMA(UNIFORM(50, 100), UNIFORM(1, 5))
LOGNORMAL(UNIFORM(200, 800), UNIFORM(30, 90))
ERLANG(UNIFORM(10, 30), UNIF(2, 5))

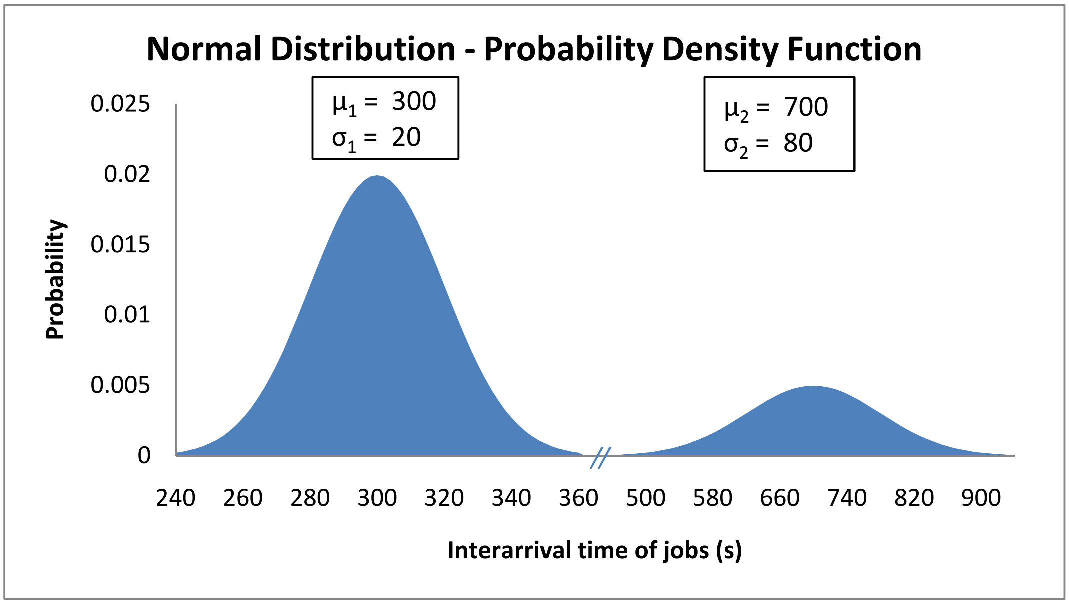

Figure 4.1: Normal distributions with means of 700, 300 and std. dev. of 80, 20

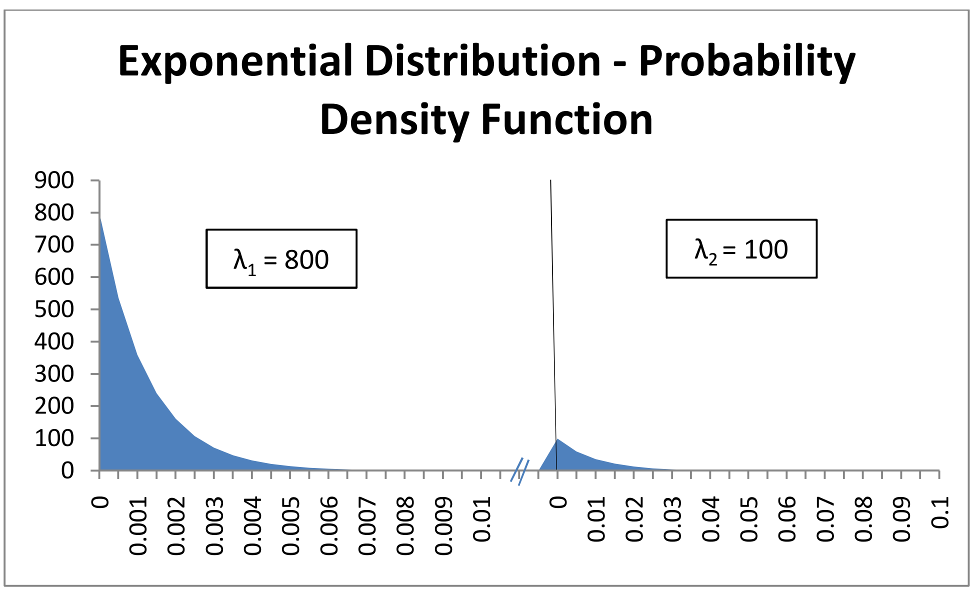

Figure 4.2: Exponential distributions with means of 800 and 100

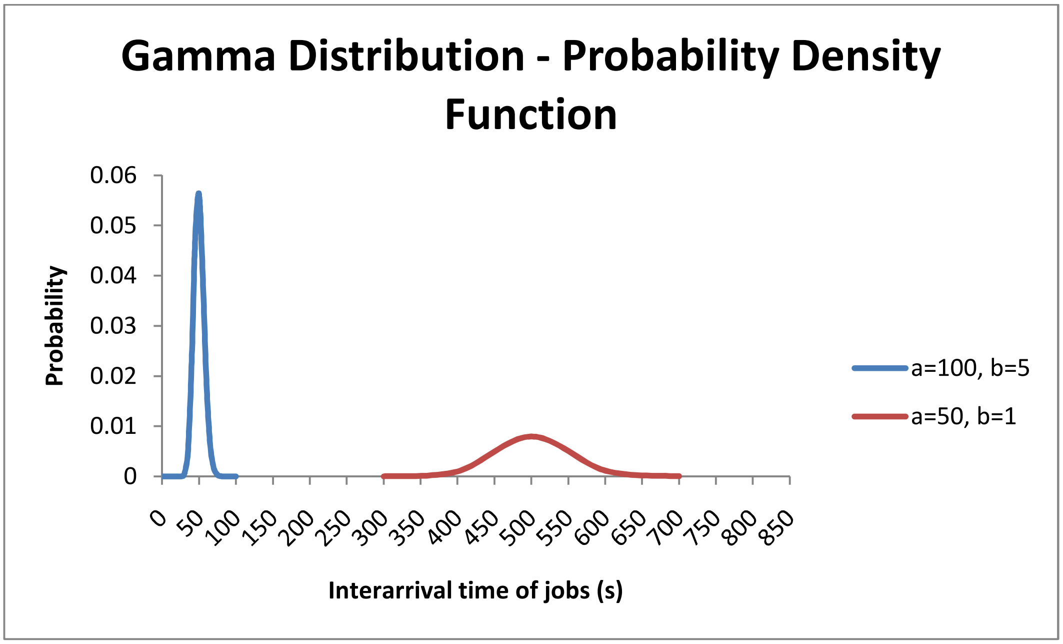

Figure 4.3: Gamma distributions

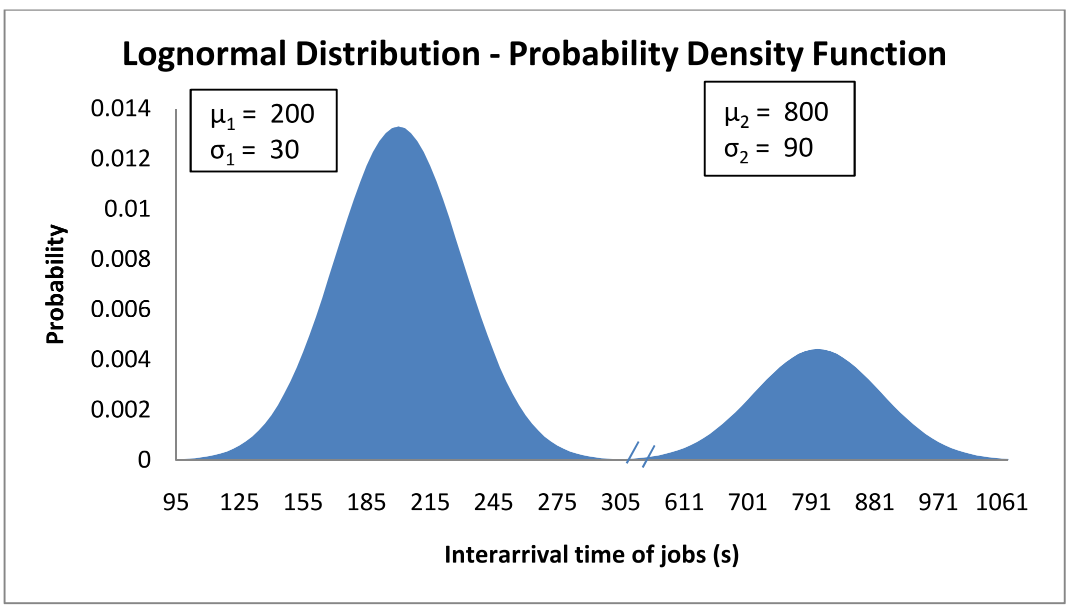

Figure 4.4: Lognormal distribution with means of 200, 800 and std. dev. of 30, 90

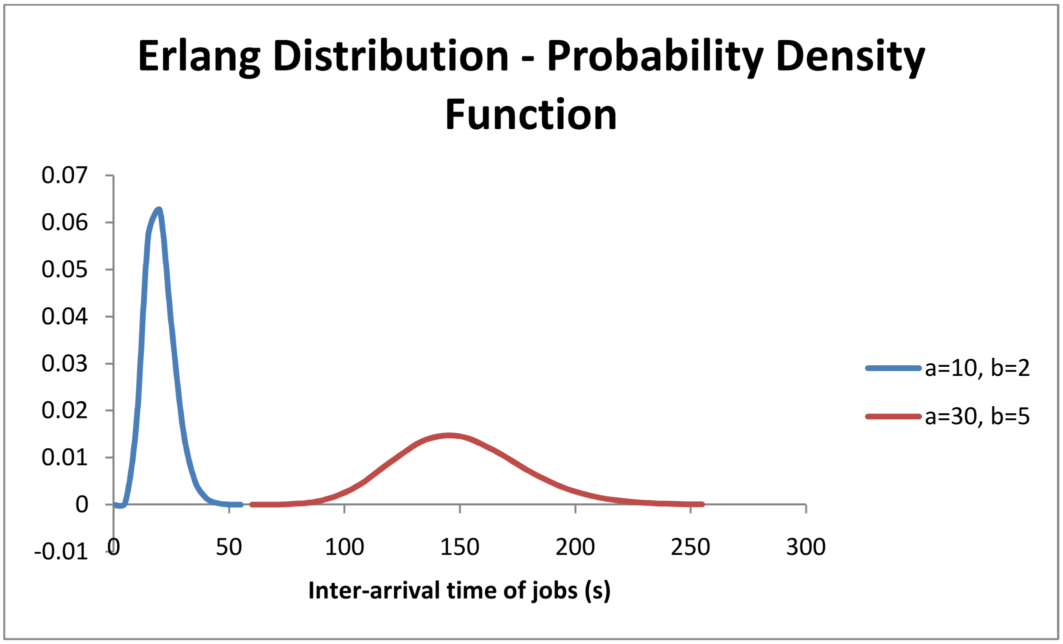

Figure 4.5: Erlang distributions with a=10, 30 and b=2, 5

These arriving print jobs are then separated based on the type of user and his printing characteristics. Three types of users are considered namely – office user, mid-end user, and graphics artist. Each of these users has their own printing preferences which are considered as listed below in Table 4.2:

Table 4.2: Printing preferences of users

Type of User	Type of print job preference (%)		
	Text file	Graphics file	Picture file
Office user	60	30	10
Mid-end user	30	50	20
Graphics artist	15	40	45

After deciding on the type of the print job, the job length and toner coverage per page per print job type are calculated. The length of the job is assumed to be a lognormal

distribution of a mean of 10, 15, and 20 pages and a standard deviation of 2 pages for the text file, pictures file, and the graphics file respectively. The coverage of the print jobs are considered to be a normal distribution with an average coverage of 5%, 20% and 40% and a standard deviation of 1%, 2%, and 3% for text file, pictures file, and graphics file respectively and the lowest coverage job is assumed to be 0.005. Figure 4.6 gives the logic flow in the simulated printer system model.

Table 4.3: Notations used in Figure 4.6

Notation	Description
XX	Printer index
ConAused	Toner consumed per print job calculated according to the consumption algorithm
Actual	Actual toner consumed per print job
TonerUsed	Cumulative actual toner consumed over time
CAUsed	Cumulative toner consumed calculated suing the consumption algorithm
TotToner	Maximum amount of toner in a cartridge

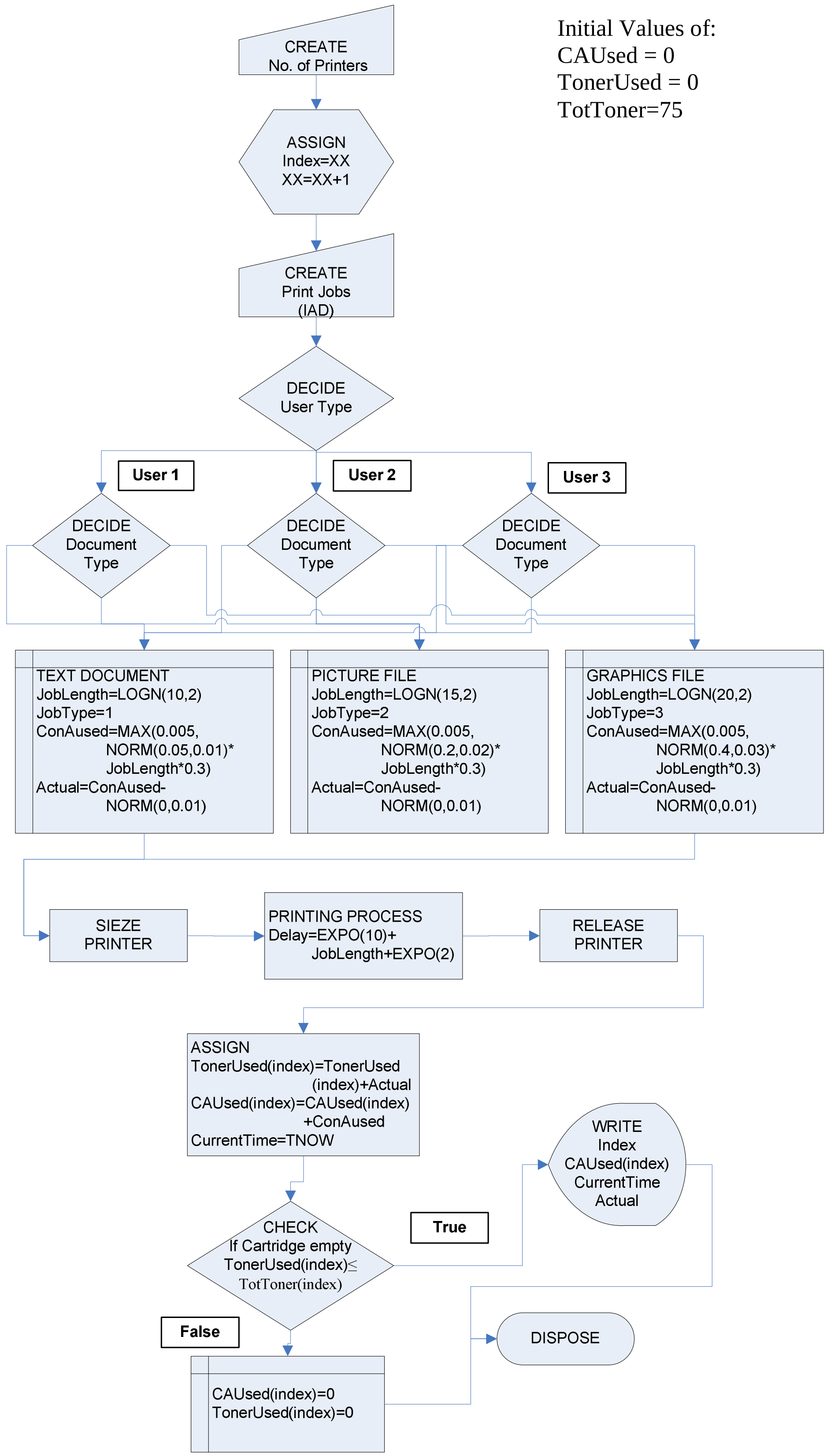

Figure 4.6: Logic flow in the simulated printer system model

4.2 Toner Consumption Algorithm

Having decided the print job properties, the toner consumed per print job is calculated using the toner consumption algorithm. The toner consumption algorithm is a toner level tracking mechanism that predicts the amount of toner consumed. The consumption algorithm estimates the cumulative amount of toner consumed per cartridge as print jobs are processed. The rate at which toner is used in a printer is dependent on various factors such as model of the printer, environmental variables, average print job length, color of print job, density of the print, and sensitiveness of the toner level sensor (Kendall, 2008). The consumption algorithm currently used in this research has the form:

$$Toner\ Consumed = TP \cdot CV \cdot PP \tag{4.1}$$

where TP is the toner (grams) consumed per full page coverage, CV is the proportion of page coverage for the print job, and PP is the number of print job pages. The simulated actual amount of toner consumed is calculated as:

$$Actual = Estimated\ \pm\ error \tag{4.2}$$

where, *Actual* is the actual amount of toner consumed per print job, *Estimated* is the predicted amount of toner consumed per print job and *error* is the deviation in the consumption algorithm forecast. In the model the error in the consumption algorithm data is assumed to be normally distributed with a mean of zero and a standard deviation of 1%.

Figures 4.7 and 4.8 illustrate a plot of the toner consumed from the cartridges of printer 1 and printer 3 respectively.

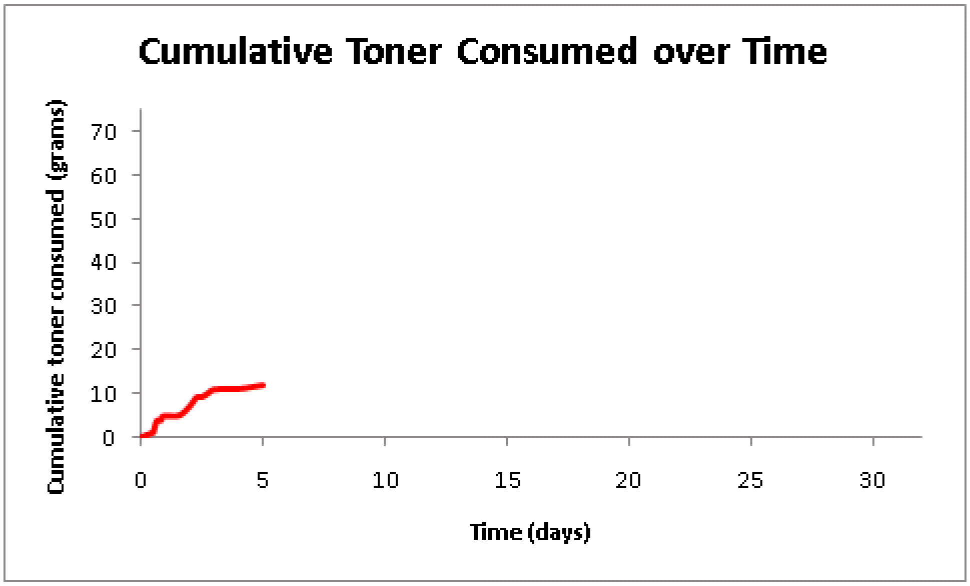

Figure 4.7: Cumulative toner consumed over time for Printer 1

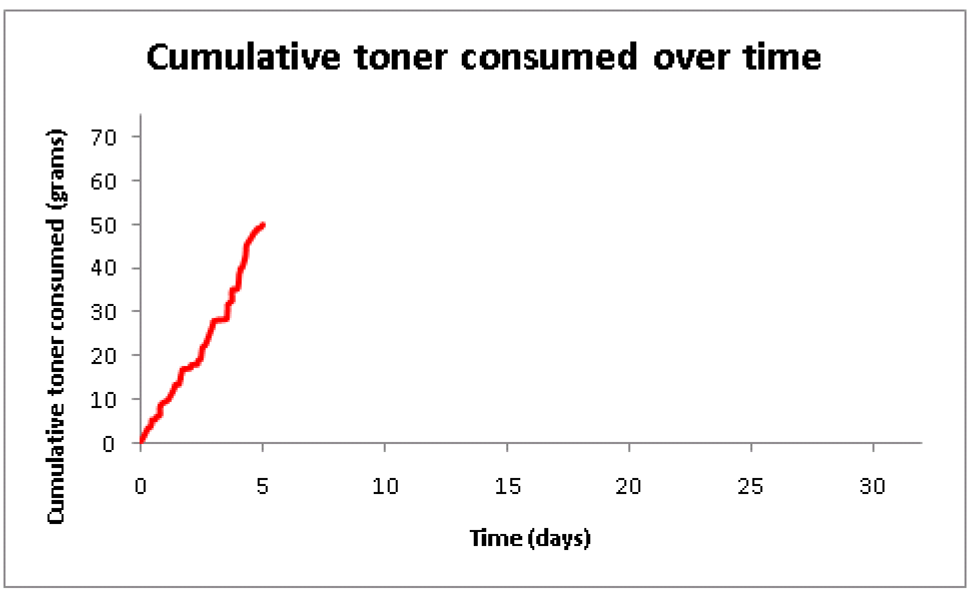

Figure 4.8: Cumulative toner consumed over time for Printer 3

Static and dynamic analysis of the system is performed on all the components of the system. For the static analysis, the model is executed only once for the given time period on the current day and the ordering strategy suggested using the forecasted demands is applied to the actual demands. In the dynamic analysis, the model is executed everyday or at user specified intervals to keep revising the ordering strategy using the updated demand forecast and lead time. Table 4.4 shows the cumulative toner consumed calculated from the toner consumption algorithm for a sample of 10 printers out of 100 on day 5 and Table 4.5 shows the cumulative toner consumed for cartridges in use up to day 6.

Table 4.4: Cumulative toner consumed from cartridge in use

Printer number	Cumulative toner consumed up to day 5 (gms)
1	12.03
2	15.54
3	50.07
4	12.68
5	42.56
6	20.91
7	53.63
8	14.90
9	12.44
10	18.29

Table 4.5 shows the cumulative toner consumed for cartridges in use on day 6.

Printer number	Cumulative toner consumed up to day 6 (gms)
1	18.67
2	18.61
3	62.97
4	16.73
5	49.26
6	27.35
7	61.65
8	20.90
9	14.46
10	23.93

Since the methodology of creating the system follows a modular approach, the toner consumption algorithm component can be replaced and updated as new and better consumption algorithms are developed. The toner consumption algorithm provides cartridge toner consumption data to the demand forecasting method which extrapolates the current and historic consumption data.

4.3 Demand Forecasting Methods

The demand forecasting technique utilizes toner consumption data to predict the replacement date for the cartridge in use and the subsequent replacement times for cartridges of a printer. The forecasting method stores previous cartridge replacement information over a time horizon. This method is periodically reviewed to account for the changes in demand. A linear regression forecasting method is used in this research to predict the replacement times for the current and subsequent toner cartridges of each printer, as there is a relationship between the consumption data and the time period.

The confidence intervals of the forecasted times are used to calculate the probable demand at given time period. The forecast limits become wider and wider as the forecast time horizon becomes larger and larger (Wei, 1994).

A simple straight line model is considered for the demand forecast. It is represented by (Bedworth et al., 1987):

$$y(n) = a + b\,x(n) \tag{4.3}$$

where *y(n)* represents the forecasted time to consume x amount of toner. The forecast equation can be written as:

$$Y = Xp \tag{4.4}$$

Where Y is the prediction vector, X is the relationship matrix and p is the estimated parameter vector. The p matrix is then calculated as:

$$p = (X^T X)^{-1} X^T Y \tag{4.5}$$

Equation (4.5) gives the parameters of the forecast line. Now, the range of the confidence intervals on the forecast is calculated as follows:

$$MSE = \frac{\sum_{i=1}^{n} e_i^2}{n-p} \tag{4.6}$$

where MSE is the mean square error, e_i is the forecast error and $(n\text{-}p)$ are the degrees of freedom.

$$S^2\left(Y_{obs}(n) - \hat{Y}\right) = MSE\ (1 + x_n^t (X^t X)^{-1} x_n) \tag{4.7}$$

Here x_n is the vector representing the maximum amount of toner in the cartridge.

$$Y_{obs}(n) - t_{\left(\frac{1-\delta}{2}, n-p\right)} S^2\left(Y_{obs}(n) - \hat{Y}\right) \le E(Y(n))$$
$$\le Y_{obs}(n) + t_{\left(\frac{1-\delta}{2}, n-p\right)} S^2\left(Y_{obs}(n) - \hat{Y}\right) \tag{4.8}$$

Equation (4.8) gives the range of the confidence interval on the forecasted demand value.

Current date, number of printers, time horizon and the toner consumption data are the input parameters to the demand forecasting model. From the toner consumption data, the previous cartridge replacements for the printer till current date and the start date of the current cartridge in use are recorded. A linear regression line is fitted to this consumption data for each printer's cartridge in use to forecast the current replacement time and its subsequent replacement times. The confidence interval for each forecast is calculated over the specified time horizon. Here a 99% confidence interval is assumed for the distribution. Tables 4.6 and 4.7 show the forecasted replacement times and their range of confidence interval of the cartridge in use for the 10 printers on day 5 and day 6.

Table 4.6: Forecasted replacement times and their range of confidence interval on day 5

Printer number	Cumulative toner consumed up to day 5 (gms)	Expected cartridge replacement time (day)	Minimum value of Y_n (day)	Maximum value of Y_n (day)
1	12.03	27.19	24.53	29.86
2	15.54	21.97	19.62	24.32
3	50.07	7.50	7.38	7.63
4	12.68	26.96	24.48	29.43
5	42.56	8.95	8.74	9.15
6	20.91	19.29	18.63	19.96
7	53.63	7.19	7.12	7.27
8	14.90	22.62	20.90	24.35
9	12.44	30.41	29.24	31.59
10	18.29	20.49	19.60	21.37

Table 4.7: Forecasted replacement times and their range of confidence interval on day 6

Printer number	Cumulative toner consumed upto day 6 (gms)	Expected cartridge replacement time (day)	Minimum value of Y_n (day)	Maximum value of Y_n (day)
1	18.67	26.07	24.78	27.36
2	18.61	24.40	22.35	26.44
3	62.97	7.18	7.07	7.28
4	16.73	24.95	23.07	26.82
5	49.26	8.99	8.85	9.13
6	27.35	17.66	17.07	18.25
7	61.65	7.14	7.08	7.19
8	20.90	20.90	19.75	22.05
9	14.46	29.47	28.54	30.40
10	23.93	19.60	18.97	20.23

A plot of the toner consumed over time up to day 5 and the forecasted replacement time for the cartridge in use based on this data for Printer 1 and Printer 3 are shown in Figure 4.9 and 4.10. This is the static case of the model.

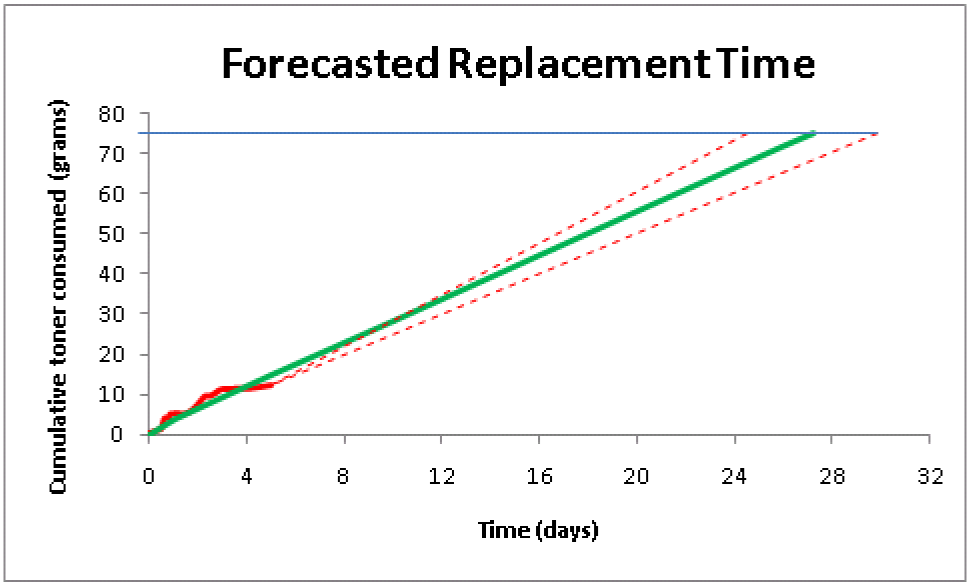

Figure 4.9: Forecasted replacement time of cartridge of Printer 1 on day 5

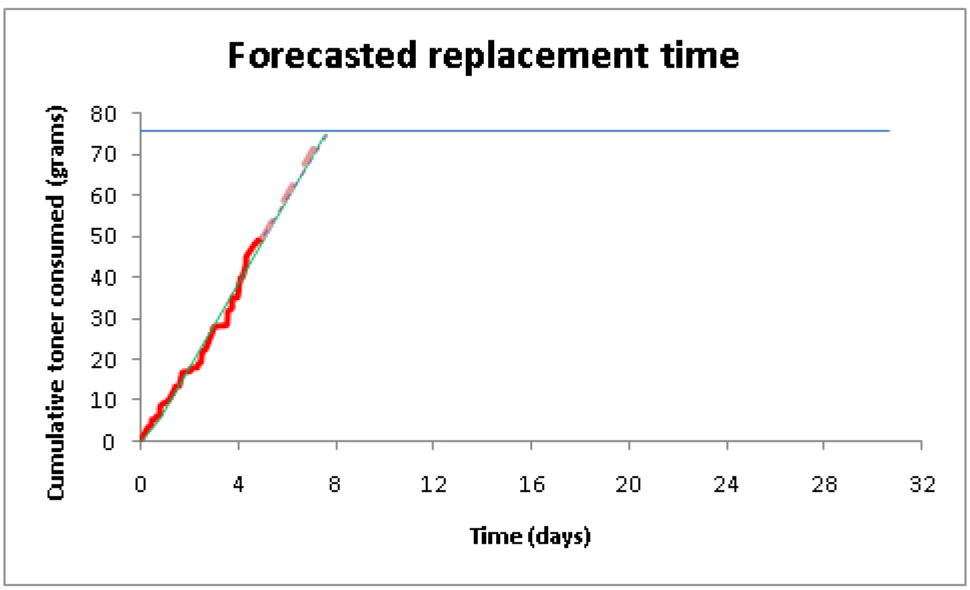

Figure 4.10: Forecasted replacement time of cartridge of Printer 3 on day 5

The forecast is reviewed continuously until the end of the time horizon. Figures 4.11 and 4.12 show the dynamic case for an updated forecast for the cartridges of Printer 1 and 3 based on consumption data upto day 6.

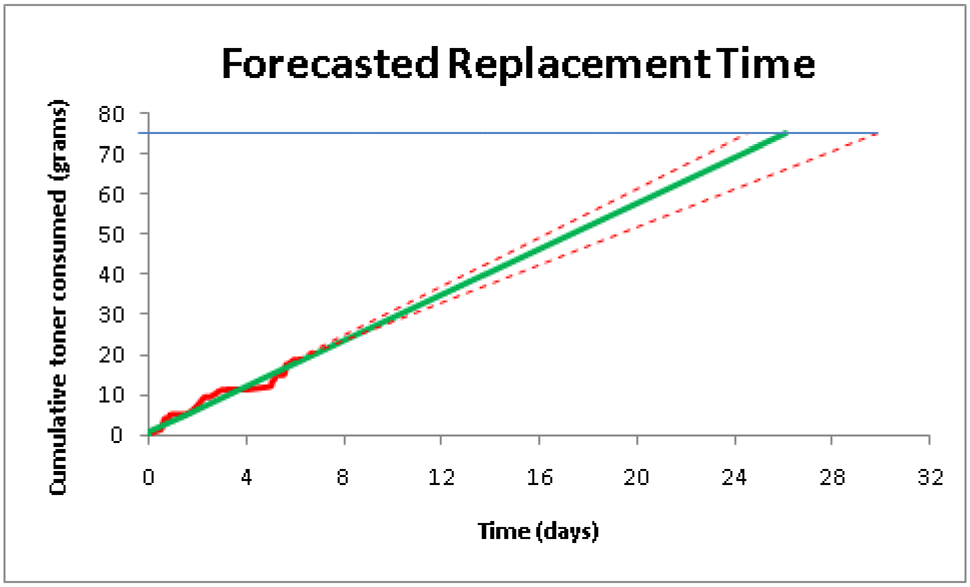

Figure 4.11: Forecasted replacement time of cartridge of Printer 1 on day 6

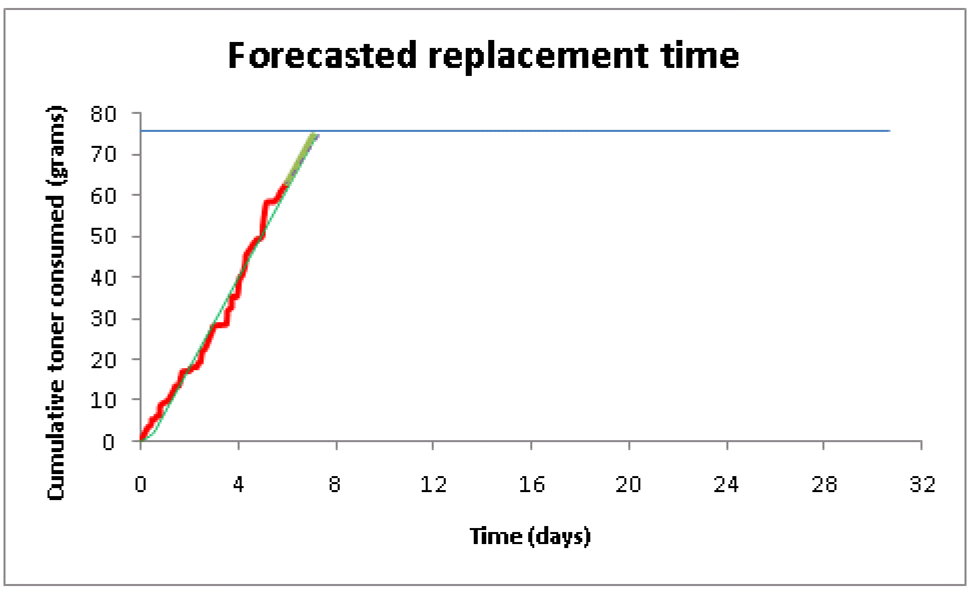

Figure 4.12: Forecasted replacement time of cartridge of Printer 3 on day 6

Probable demands for toner cartridges for each printer over the time horizon are calculated based on the forecasted demand and its confidence interval range. The probable demands are calculated by assuming the occurrence of the demand for a cartridge to be

normally distributed with the mean to be the forecasted replacement date and the distribution is truncated at a 99% confidence level.

The aggregate of this probable demand of cartridges over the time horizon for all the printers is the output from the demand forecast model. This output serves as an input to the optimization model. Figure 4.13 is the algorithm to calculate the demand forecast over a time horizon.

Inputs: Current day to start execution (*sd*), length of Time horizon (*T*), consumption history upto current day for all printers.
Forecast Calculation:

```
for (i = 1 to total number of printers)
{
        Record the start date of the current cartridge in use (= cday)
        if (cday == sd)
                Calculate forecast based on previous cartridge's life;
        else
        {
                Calculate X^{-1}, X^tY, P, PX, error, X_n^t X^{-1} X_n, Standard Deviation to fit a Linear
                Regression line;
                Check the goodness of fit of the line using R^2 test;
                Calculate the expected forecast day (Y_n) and the range of its confidence
                interval (llimit, ulimit);
                Calculate the probable demands on all the days within the confidence
                interval range;
                while (Yn < T)
                {
                        Calculate the subsequent replacement time, their Confidence intervals
                        and probable demand distribution;
                }
        }
}
Calculate aggregate expected demand on each day for all the printers (PD);
Print PD;
```

Figure: 4.13: Algorithm to calculate the demand forecast over a time horizon

4.4 Optimization Model

Given the forecasted replacement times the objective of the optimization model is to calculate the minimal total cost ordering policy over the time horizon for the desired service level. The components of the optimization model are:

- Objective function – minimum cost at a specified service level
- Decision variables – when to place an order and how much to order
- Constraints – balance existing inventory with the forecasted demand for cartridges at the specified service level

While considering the uncertainties in demand, demand forecast and the lead time, the solution of the optimization model is designed to determine how much to order and when to place these orders over a specified time horizon.

The developed model is a mixed integer programming model solved using a linear and integer programming software. The mathematical program for the optimization method is shown below. The notation used in the mathematical model is shown in Table 4.8.

Table 4.8: Notations used in equations

Notation	Description
	Inputs
d_i	Demand for cartridges on day $i+1$
$I0$	Inventory on hand at the beginning of the time horizon
N	Number of orders placed
M	Large constant number (big-M)
h	Holding cost per unit per day
pen	Penalty cost per unit per day
k	Ordering cost per order
P_j	Probability distribution for receiving an order j days after placing an order
nP	Number of printers
$gamma$	Required service level
$delta$	Deviation from service level
$ndpo_j$	Number of days prior to the current period the previous order XP_j was placed
no	Number of orders placed prior to the current period that have not arrived yet
Al	Average length of the lead time
T	Length of time horizon
	Decision Variables
X_i	Number of cartridges ordered on day $i+1$
R_{ij}	Expected number of cartridges received on j days after placing an order on day i
ER_i	Expected cumulative number of cartridges received during day $i+1$
Y_i	Indicator variable for order placement on day $i+1$ 0 if is $X_i \leq 0$ (no order placed on day i) 1 if $X_i > 0$ (order placed on day i)
I_i	Inventory on hand at the beginning of day i
$PP_{j,i}$	Expected number of cartridges received on day i from previously placed orders j
XP_j	Order quantities placed in previous periods that will arrive in the current period
Service Level	Service level over the time horizon
	Calculation Variables
holdi,	Inventory holding or penalty time
HP_i	Inventory shortage time
HM_i	Inventory holding time
$penalty_i$	Inventory shortage time
$PenM_i$	Inventory holding time

$PenP_i$	inventory backorder time
z_i	Variable used to calculate the proportion of holding time
zz_i	Variable used to calculate the proportion of penalty time

$$Minimize\ k * Y_i + h * HP_i + pen * PenP_i + 100000 * delta \tag{4.9}$$

Subject to constraints:

$$I_0 = I0 \tag{4.10}$$

$$PP_{order,j} = 0 \quad \forall order = 1,..,no \ \ \forall j\ in\ al, \dots, (T-1) \tag{4.11}$$

$$if(ndpo_{order} == 1)\ \ PP_{order,j} = P_{j+1} \quad \forall order = 1,..,no \ \ \forall j\ in\ 1,..,alr \tag{4.12}$$

$$if(ndpo_{order} == 2) PP_{order,j} = \frac{P_{j+1}}{P_2 + P_3} \quad \forall order = 1,..,no \ \ \forall j\ in\ 1,..,al \tag{4.13}$$

$$if(ndpo_{order} == 3) PP_{order,1} = 1 \quad \forall order = 1,..,no \tag{4.14}$$

$$R_{0,j} == \sum_{order=1}^{no} \left(XP_{order} * PP_{o,j}\right) + X_0 * P_j \qquad \forall j\ in\ 0, \dots, (T-1); \forall j \geq 0 \tag{4.15}$$

$$R_{i,j} = X_i * P_{j-1} \qquad \forall i\ in\ 1, \dots, (T-1)\ \forall j\ in\ 0, \dots, (T-1) \forall j \geq i \tag{4.16}$$

$$R_{i,j} = 0 \qquad \forall i\ in\ 0, \dots, (T-1)\ \forall j\ in\ 0, \dots, (T-1) \quad \forall j < i \tag{4.17}$$

$$ER_j = \sum_{i=1}^{t} R_{i,j} \qquad \forall j = 1, \dots, T \tag{4.18}$$

$$ER_j + I_{j-1} - d_j = I_j \qquad \forall j = 1, \dots, T \tag{4.19}$$

$$hold_i = I_{i+1} - ER_i \qquad \forall i\ in\ 0, \dots, (T-1) \tag{4.20}$$

$$hold_i = HP_i - HM_i \qquad \forall i\ in\ 0, \dots, (T-1) \tag{4.21}$$

$$penalty_i = -I_{i+1} \qquad \forall i\ in\ 0, \dots, (T-1) \tag{4.22}$$

$$penalty_i = PenP_i - PenM_i \qquad \forall i\ in\ 0, \dots, (T-1) \tag{4.23}$$

$$HP_i \leq M * (1 - z_i) \qquad \forall i\ in\ 0, \dots, (T-1) \tag{4.24}$$

$$HM_i \leq M * z_i \qquad \forall i\ in\ 0, \dots, (T-1) \tag{4.25}$$

$$PenP_i \leq M * (1 - zz_i) \qquad \forall i \; in \; 0, \dots, (T-1) \tag{4.26}$$

$$PenM_i \leq M * zz_i \qquad \forall i \; in \; 0, \dots, (T-1) \tag{4.27}$$

$$X_i \leq M * Y_i \qquad \forall i = 1, \dots, T \tag{4.28}$$

$$X_i \geq Y_i \qquad \forall i = 1, \dots, T \tag{4.29}$$

$$N = \sum_{i=1}^{T} Y_i \tag{4.30}$$

$$\sum_{i=0}^{j} PenP_i \leq (1 - gamma + delta) * nP * (j+1) \qquad \forall \, j \; in \; 0, \dots, (T-1) \tag{4.31}$$

$$ServiceLevel = 1 - \left(\frac{\sum_{i=0}^{(T-1)} PenP_i}{nP * T} \right) \tag{4.32}$$

The mathematical program developed utilizes forecasts based on the extrapolation of the consumption history for each printer. This model is developed for discrete time increments (the time unit is one day in the base example).

This general formulation can be modified to model the cases of:

(a) Deterministic demand forecasts and lead time where backorders are allowed;

(b) Stochastic demand forecasts and deterministic lead time where backorders are allowed; and

(c) Stochastic demand and stochastic lead time where backorders are allowed.

The objective function (4.9) calculates the minimum total inventory cost consisting of holding cost, penalty cost and the ordering cost. Constraints (4.11) to (4.14) are used to calculate the probability of receiving the orders placed in the previous periods in the current periods. The assumption here is that a maximum of only 3 orders can be placed in the previous time period. Constraints (4.15) to (4.19) calculate the quantity to be ordered

based on the probability of receiving it on any day. Constraint (4.32) establishes the service level is met by requiring the number of printer hours available to be atleast the service level times the total number of printer hours over the time horizon. Total printer hours refers to the number of printers times the length of time horizon. In this model, the service level is calculated based on the total number of printer hours available instead of number of printers available as it is more desirable to have the penalty considered as a measure of printer downtime versus considering it as a measure of the number of printers that are inoperable within the time window.

The solution from the optimization algorithm is then reviewed every day on a rolling time horizon basis to check for the errors in forecasting the demand for cartridges. This accounts for the dynamic nature of the model. The cost of the revised ordering strategy is observed to be lower than the static solution cost. A likely explanation of the observed difference in ordering strategies is due to the reduction in error associated with the forecast as the toner consumption history data gets updated. This results in more accurate and less spread out probable demands for all the printers. In the given example, each of the printer numbers 3, 5 and 7 have about 10 grams of toner consumed within one day which influences the demand forecast calculation and thereby results in a shortened range of the probable demand distribution. In order to incorporate this variability in demand and to attain the minimum required service level, a revised strategy is suggested by the solution from the optimization model. This shows that dynamically updating the model helps reduce additional inventory costs due to the variabilities.

The mathematical program discussed above is developed for system configurations over discrete time. Extension of this formulation for continuous time is intended. The

mathematical formulation developed for system configurations over continuous time is given in Appendix-E.

4.5 Inventory

Inventory consists of toner cartridges on-hand and the replenishment order of toner cartridges as obtained from the optimization model solution. Whenever a printer's cartridge has to be replaced, a cartridge is taken from the inventory, if available. This decision is based on the inventory position, which is a measure of the current inventory level, equal to the sum of on-hand inventory and inventory on order minus backorders (Nahmias, 1997). The inventory position is reviewed on the basis of a rolling time horizon based on the status of the continually monitored toner consumption.

5 Implementation

This section discusses the design and implementation of the model using the softwares. This is explained on the basis of the example described in chapter 4.

5.1 Simulation Model and Toner Consumption Algorithm

The simulation model is developed using a discrete event simulation software with a modular structure. The model is developed using ARENA simulation software. The purpose of the simulation model is to represent the real-world printer scenario and thereby generating print jobs per printer. Figure 5.1 shows the simulated printer system developed in ARENA. The current simulation model incorporates the printing process and the consumption algorithm. In the figure below, sections 5.1 a, b and, c correspond to the printing system and 5.1d section corresponds to the consumption algorithm calculations.

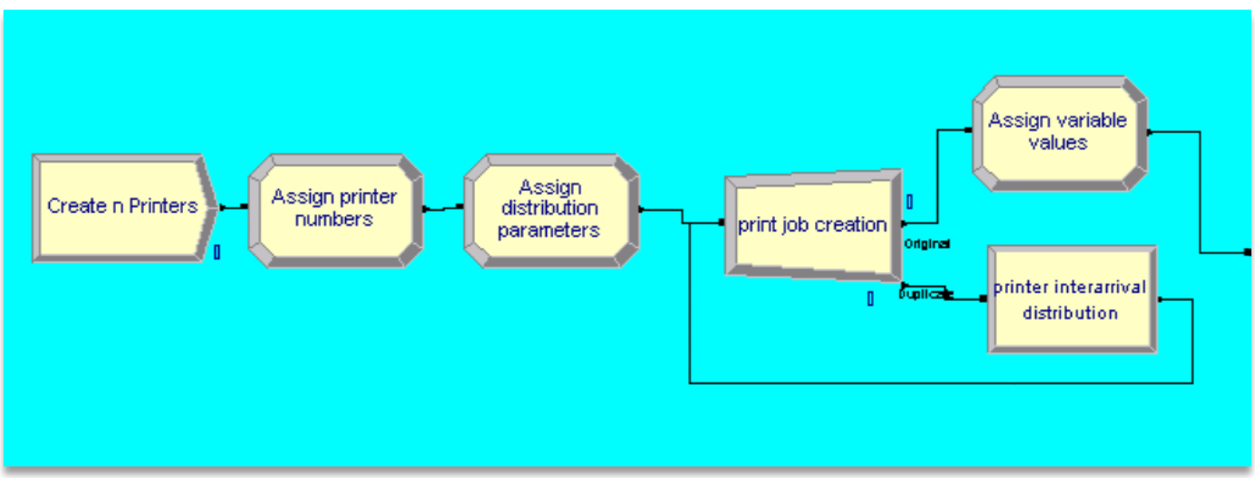

Figure 5.1a: Creation of printers, print jobs and assigning inter-arrival distributions

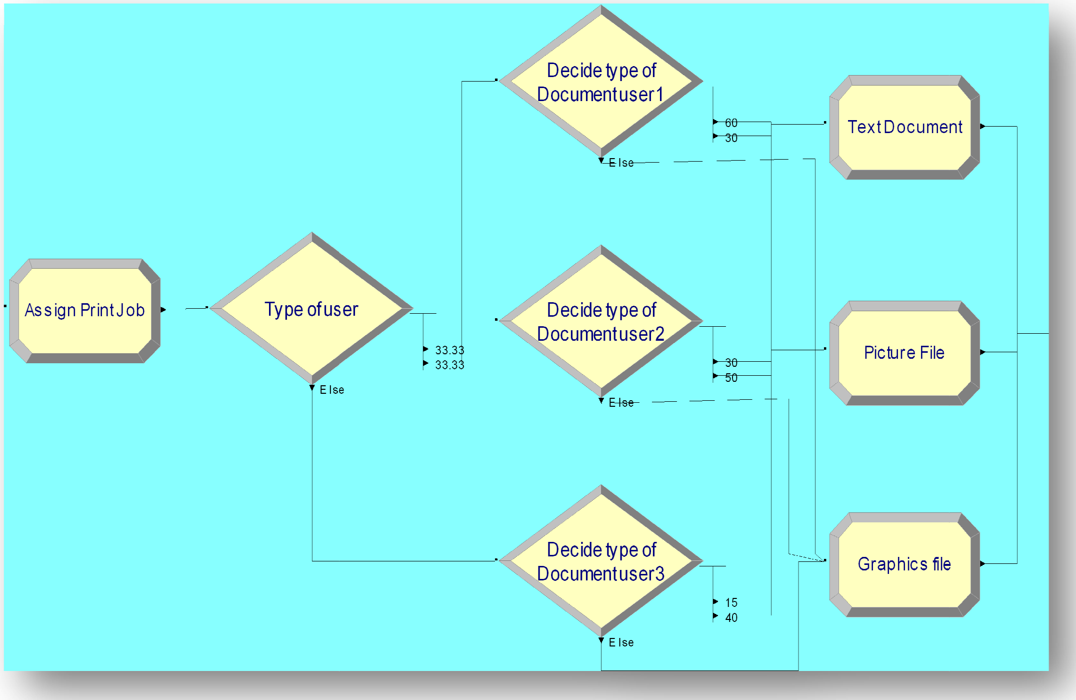

Figure 5.1 b: Assigning print job characteristics based on user type

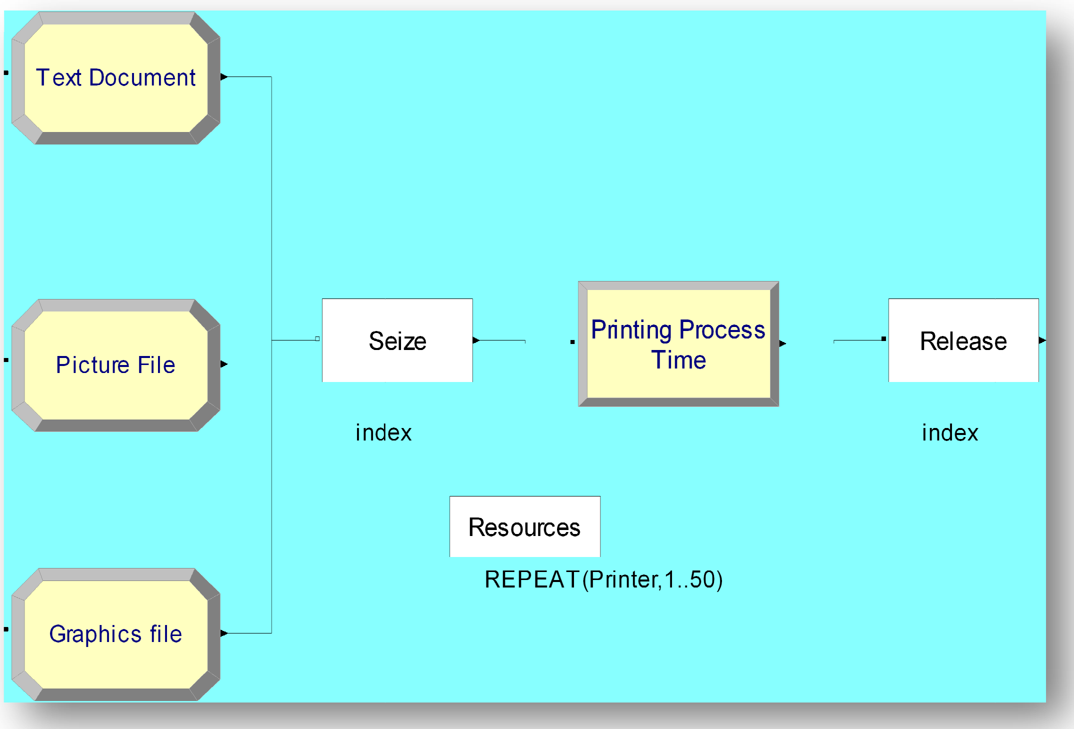

Figure 5.1 c: Simulating the printing process

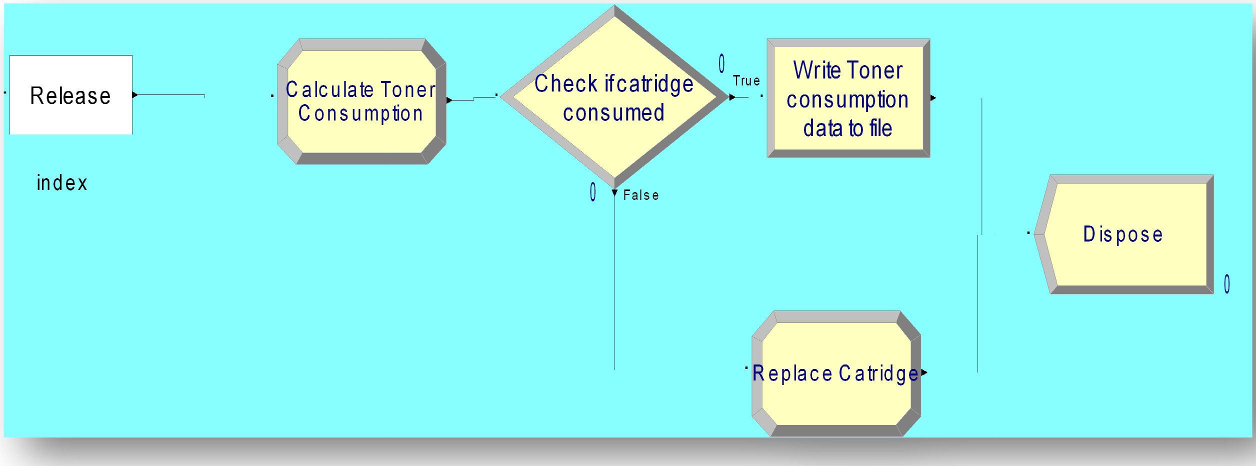

Figure 5.1 d: Consumption algorithm

The arrivals of print jobs are random and are specific to each printer. In the case of this example they are assumed to be random exponential, normal, lognormal, gamma, and erlang. Three types of print jobs are created, i.e. word document, picture file and graphics file. The created print jobs are then split based on their type and size. These arriving print jobs go through the printing process.

The printing process is considered as a seize delay release process in simulation. The process time to print is calculated based on the length of the print job and the set-up time. Before printing the received print job, the toner level in the cartridge is checked. In this model it is assumed that if there is not sufficient toner in the cartridge to complete the print job, then this job is discarded and the toner cartridge for the printer is replaced.

The consumption algorithm calculates the amount of toner used as a function of the print job's characteristics. This model calculates the simulated actual amount of toner consumed by each print job and the consumption algorithm utilizes the randomly

generated print job characteristics to estimate the amount of toner consumed per toner cartridge per printer.

The output comprising of the cumulative estimated amount of toner consumed, simulated actual amount of toner consumed and its time of occurrence for each print job for each printer's cartridge is made available in the form of an excel spreadsheet. Figure 5.2 shows the components of the simulated printer system. Appendix-A shows the sample output file for this simulation model.

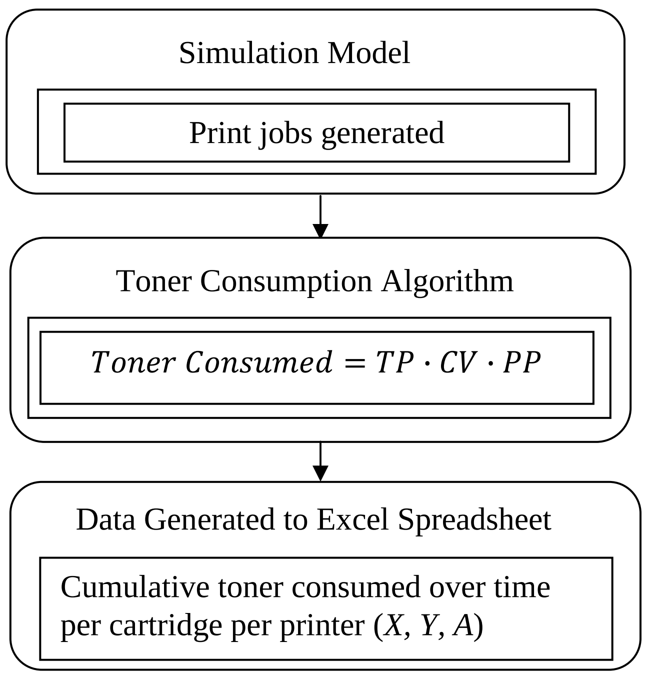

Figure 5.2: Components of the simulated printer system

The simulation model developed has a static nature, i.e., there is no feedback loop about the demand queue or the supplier lead time. Here, we assume that there is a continuous/infinite supply of toner cartridges to the printer fleet and the cartridge replacement times are recorded. So, the difference between the modeling detail and the real world function is that there should be a continuous input to the printer system from

the existing inventory levels to accommodate for the penalty due to waiting/ shortage of cartridges.

5.2 Demand Forecasting Method

The demand forecasting method extrapolates the historic and current toner consumption data from the consumption algorithm to calculate the predicted replacement time of the printer's toner cartridge in use as well as the subsequent replacement times. In this model there is a causal relationship between the toner consumed and time. A Linear Regression method of forecasting has been applied to the consumption curve. This model gives the predicted replacement date of the cartridge with reference to the current day. The output of the simulation model is the input to the demand forecasting method.

In this research, the demand forecasting method is developed using MATLAB software. Current date, number of printers, time horizon and the toner consumption data are the input parameters to the demand forecasting model. From the toner consumption data, the previous cartridge replacements for the printer till current date and the start date of the current cartridge in use are recorded. A linear regression line is fitted to this consumption data for each printer's cartridge in use to forecast the current replacement time and its subsequent replacement times. The confidence interval for each forecast is calculated over the specified time horizon and the probable demands for cartridges over the time horizon are calculated. The aggregate of this probable demand of cartridges over the time horizon for all the printers is output onto an excel spreadsheet. The forecast is reviewed continuously till the end of the time horizon. The output from the demand forecasting model serves as an input to the optimization model. Figure 5.3 shows the

components of the demand forecasting model. Appendix-D shows the output spreadsheet for the example problem. Appendix-B shows the formulation in MATLAB.

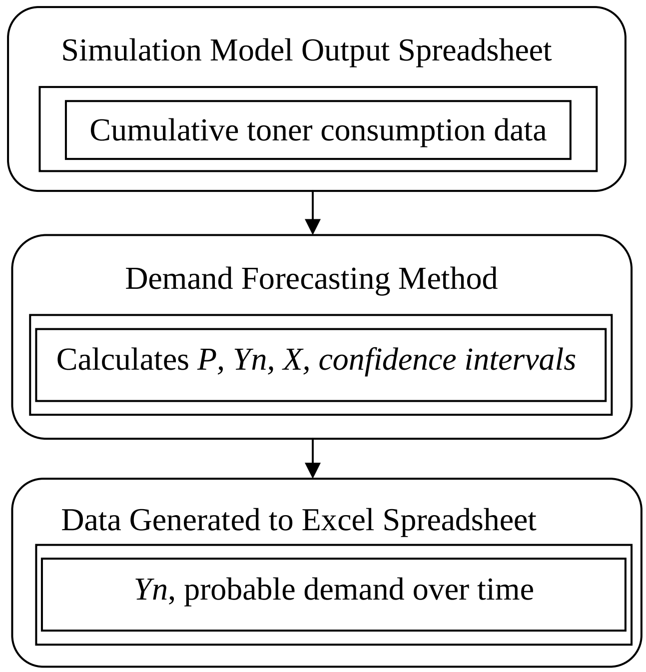

Figure 5.3: Components of the demand forecasting model

5.3 Optimization Model

The forecasted demand is the input to the optimization model. The mixed integer programming model is developed using a linear and integer programming software. Given the forecasted demands the objective of the optimization model is to calculate a minimal cost ordering strategy over a given time horizon for a specified service level. This optimization model is developed in ILOG CPLEX software. The corresponding ILOG CPLEX code is shown in APPENDIX – E. Figure 5.4 gives the component structure of the optimization model.

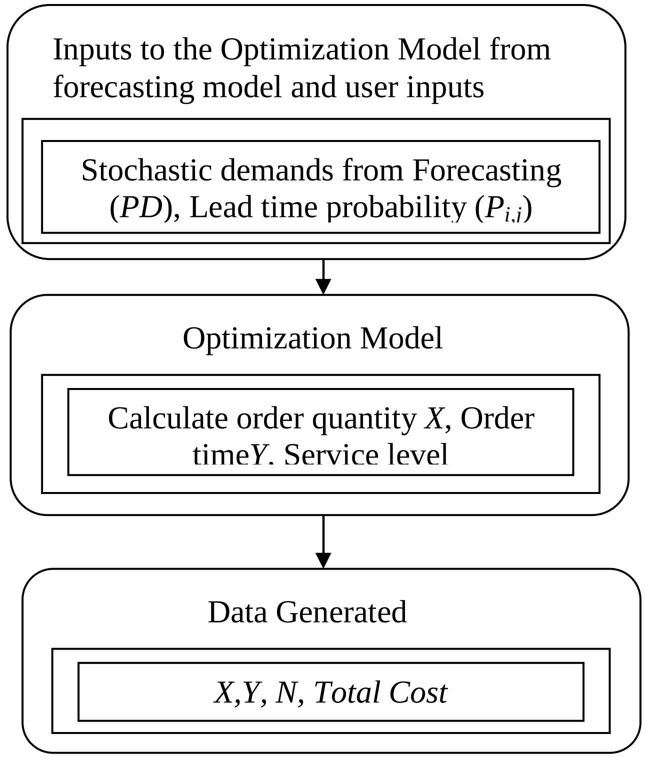

Figure 5.4: Components of the optimization model

5.4 Verification, Validation and Testing

In order to verify and validate the methods mentioned above different experiments are run to test whether the model operates the way it is intended to. These experiments help check if the algorithm represents the real world system accurately. The model is verified by conducting step by step evaluation of each component. The models are broken down into smaller segments and each segment is tested before adding it to the complete system.

In the simulation model the flow of entities is observed to be similar to the flow of print jobs in an office. The execution of the model is observed from the animation of the simulation. The sample means and variances of the print job arrival distributions are observed to be similar to the values computed manually.

The model is run for simple configurations such as a truncated normal distribution for print job arrivals and a constant process time, which help verify the output from the model. For the no-variability case the inter-arrival time between print jobs is set to be a constant of 5 minutes for all the printers. The ouput from the no-variability case is in Appendix C. A trace is run on the simulation to check the flow of logic. The number of entities entering the model is tallied with the number of entities exiting the model.

The demand forecasting method is tested by executing each logic loop within the program individually. The output from the demand forecasting method is verified by checking with the historic consumption data. The demand forecast is calculated for a simplified system configuration such as 100 printers and with no variability in demand and the observed forecast from the linear regression fit is similar to the expected forecast value. The output for the no variability case for the demand forecast model is in Appendix C.

The optimization model is tested by executing it for the no variability in demand and a constant lead time case. Here the forecasted cost is observed to be the same as the actual cost if the suggested policy is used. The output is validated by running the simplified model and comparing the results with those for an EOQ model and a (Q,r) model.

6 Experimental Evaluation

The goal of the research is to develop an algorithm to dynamically determine the optimal replenishment strategy over a rolling time horizon, for the inventory of toner cartridges in a system of a fleet of printers. It is an ongoing process where the user can specify the time horizon and will be able to calculate and update the reorder values dynamically over a rolling time horizon. To investigate the ability of the model to generate consistent results for various system configurations, it is necessary to conduct an experimental performance evaluation on the system. The experiments are designed to test the robustness of the developed inventory system. Two types of experiments are conducted on the system-

I. Comparison of the developed system with a traditional inventory system; and

II. Parameter performance evaluation of the developed system.

The experiments are conducted using the following case as the base scenario:

Scenario: *A printer fleet consisting of hundred printers is considered. Each printer in the group has its own set of print jobs arriving randomly. The print jobs are considered to be of three types – word document, picture file, and graphics file. The maximum toner in each printer's toner cartridge is considered to be 75 grams/cartridge. The holding and penalty costs are assumed to be $2 and $5 per unit per day and the ordering cost is $10 per order. An initial inventory of 22 units is assumed to be present at the start of the system run. The service level required is 90%. The order arrival probability is assumed to be [0.05, 0.90, 0.05]. Table 6.1 shows the historical cartridge demands generated for the 100 printer system.*

Table 6.1: Historic demand data for 100 printers over 30 days

Day	Demand
-30	9
-29	13
-28	9
-27	14
-26	14
-25	9
-24	14
-23	8
-22	4
-21	18
-20	10
-19	8
-18	8
-17	8
-16	15
-15	8
-14	11
-13	10
-12	11
-11	8
-10	11
-9	16
-8	15
-7	12
-6	13
-5	8
-4	6
-3	3
-2	2
-1	5

mean	10.00
std dev	3.85

6.1 Comparison of the developed system with a (Q,r) type periodic review inventory policy

A cost comparative study is conducted on the results from the developed algorithm versus those obtained by applying traditional inventory policies from literature to the forecasted demand data. A simplified configuration of the scenario described above is considered for comparing the costs of the ordering strategy suggested by the developed method in this research and the periodic review policies already existing in literature. Here, the system is evaluated for a printer network consisting of 100 printers and the lead time is assumed to be known and a constant of 2 days. The ordering policy calculation for the (Q,r) methodology is shown in Figure 6.1 below.

T=30 days
Average Lead time = 2 days/order
Demand distribution mean, μ_D = 10 units; standard deviation, σ_D = 3.85 units
Average demand during lead time, μ = 10 * 2 = 20 units
Standard deviation of demand during lead time, $\sigma = 3.85 * \sqrt{2} = 5.44$
Average daily demand during lead time, D = 10 units
Service level required, β = 90%
Holding cost, h = $2/unit/day
Penalty cost, pen = $5/unit/day
Ordering cost, K= $10/order

$$Q_0 = \sqrt{\frac{2KD}{h}} = \sqrt{\frac{2 * 10 * 10}{2}} = 10\ units$$
$$n(R_0) = Q_0\,(1-\beta) = 10 * (1 - 0.90) = 1.0$$
$$L(z_0) = \frac{n(R_0)}{\sigma} = \frac{1}{5.44} = 0.1838$$
$$z_0 = 0.55, \qquad 1 - F(R_0) = 0.2912$$
$$R_0 = \sigma z_0 + \mu = 5.44 * 0.55 + 20 = 22.99\ \cong 23$$
$$Q_1 = \frac{n(R_0)}{1-F(R_0)} + \sqrt{\frac{2KD}{h} + \left(\frac{n(R_0)}{1-F(R_0)}\right)^2} = \frac{1}{0.2912} + \sqrt{100 + \left(\frac{1}{0.2912}\right)^2} \cong 15$$
$$n(R_1) = Q_1\,(1-\beta) = 15 * 0.1 = 1.5$$
$$L(z_1) = \frac{n(R_1)}{\sigma} = \frac{1.5}{5.44} = 0.2757$$
$$z_1 = 0.28, \qquad 1 - F(R_1) = 0.3897$$
$$R_1 = \sigma z_1 + \mu = 5.44 * 0.28 + 20 = 21.52\ \cong 22$$

$$Q_2 = \frac{n(R_1)}{1-F(R_1)} + \sqrt{\frac{2KD}{h} + \left(\frac{n(R_1)}{1-F(R_1)}\right)^2} = \frac{1.5}{0.3897} + \sqrt{100 + \left(\frac{1.5}{0.3897}\right)^2} \cong 15$$

$$n(R_2) = Q_2\,(1-\beta) = 1.5$$

$$L(z_2) = n(R_2)/\sigma = 0.2757$$
$$z_2 = 0.28, \qquad 1 - F(R_2) = 0.3897$$
$$R_2 = \sigma z_2 + \mu = 5.44 * 0.28 + 20\ \cong 22$$

$$(\boldsymbol{Q}, \boldsymbol{R}) = (\mathbf{15}, \mathbf{22})$$

Figure 6.1: (Q, R) Policy calculation methodology

According to this policy, for a 90% service level to be obtained, an order of 15 units is placed every time the inventory falls below 22 units. For the same printer system given above, the ordering strategy is calculated using the policy suggested in the current research. Table 6.1.1 shows a comparison of the ordering policies for the demands using (Q, R) policy, static review and dynamic review of the developed algorithm. Tables 6.2 and 6.3 show the ordering strategy suggested using the developed algorithm in the current research. Figures 6.2 and 6.3 are the inventory on hand plus back orders and inventory position over the time horizon for the (Q, R) policy. Figures 6.4 and 6.5 show the inventory on hand plus backorders for the static and dynamic review case. Figures 6.6 and 6.7 show the inventory position over time using the developed algorithm for static and dynamic review cases.

From this comparison, it can be seen that the method suggested in this research provides a better minimal cost optimal ordering strategies over a time horizon and thus proves to be a more beneficial option for inventory systems for a fleet of printers.

Table 6.2: Comparison of ordering policies from (Q, R), Static review case and Dynamic review case

	Day	Demand	Quantity Ordered		
			(Q,R)	Static	Dynamic
	1	3	15	11	11
	2	5	0	0	0
	3	8	15	14	15
	4	5	15	0	0
	5	9	15	22	5
	6	10	15	0	0
	7	8	0	21	16
	8	7	15	0	17
	9	7	0	22	9
	10	10	0	0	0
	11	9	15	21	25
	12	9	0	0	11
	13	10	15	27	0
	14	11	15	0	25
	15	11	15	31	0
	16	22	15	0	33
	17	8	15	19	0
	18	5	15	0	19
	19	14	0	24	0
	20	8	0	0	20
	21	10	15	17	6
	22	8	15	0	0
	23	10	15	24	22
	24	9	15	0	0
	25	12	15	22	48
	26	18	15	0	0
	27	10	0	23	0
	28	13	0	0	0
	29	16	15	0	0
	30	8	15	0	0
Total expected cost			$1243	$1195	$890

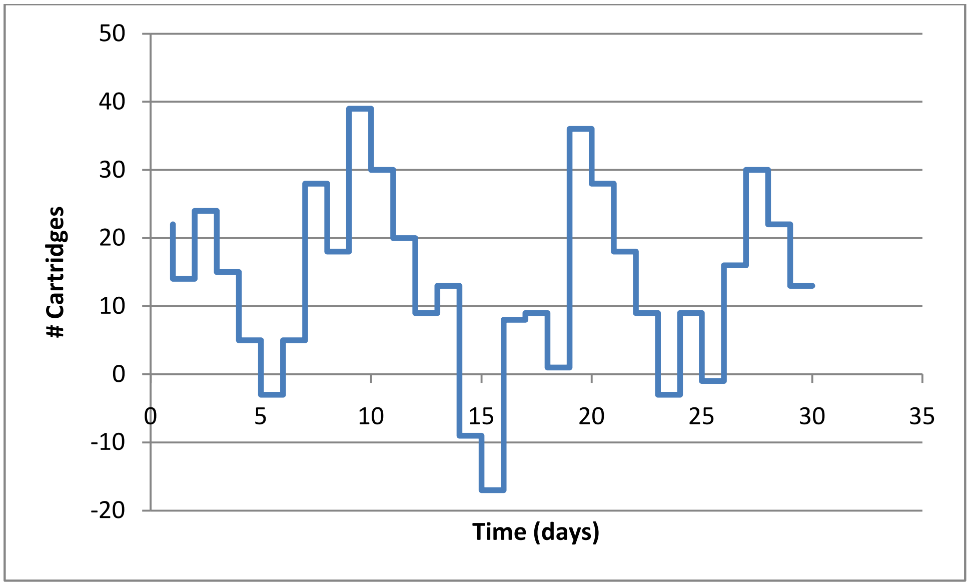

Figure 6.2: Base Case (Q, R) policy solution - Inventory on hand

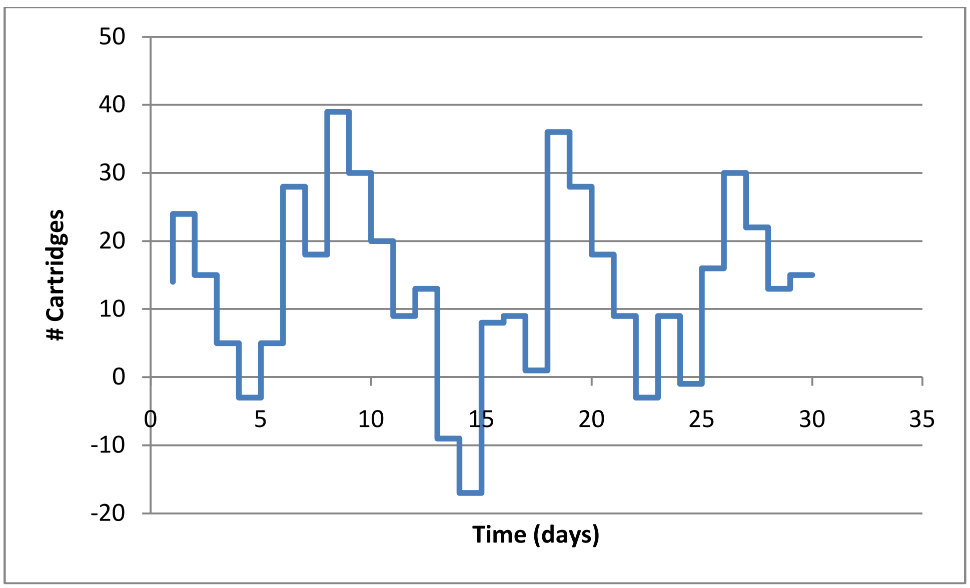

Figure 6.3: Base Case (Q, R) policy solution - Inventory position

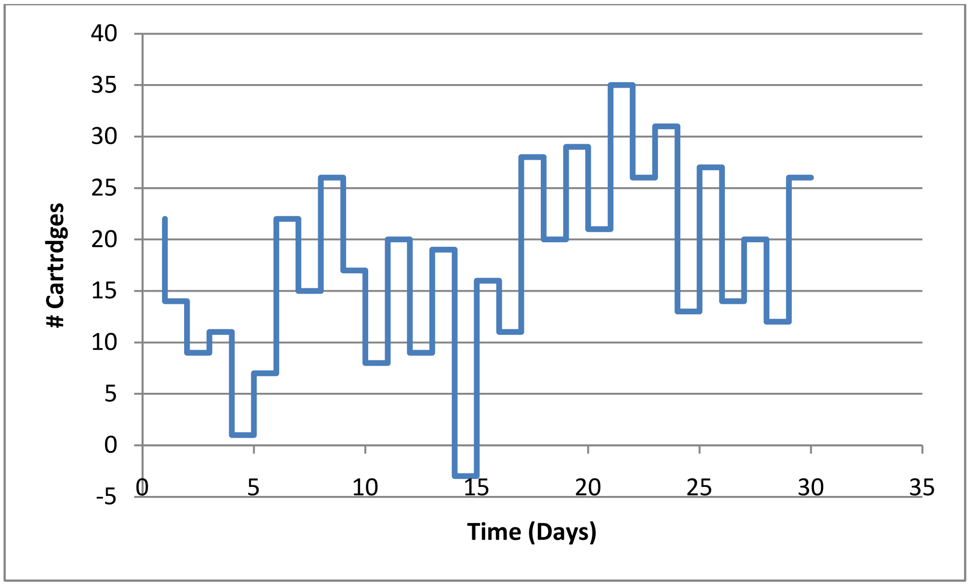

Figure 6.4: Base Case Static case solution - Inventory on hand

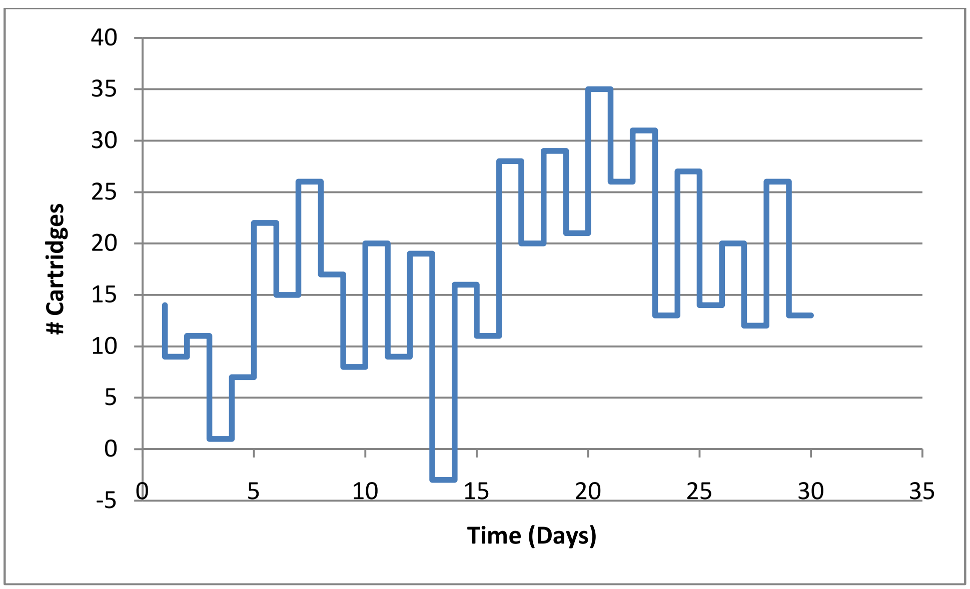

Figure 6.5: Base Case Static case solution - Inventory position

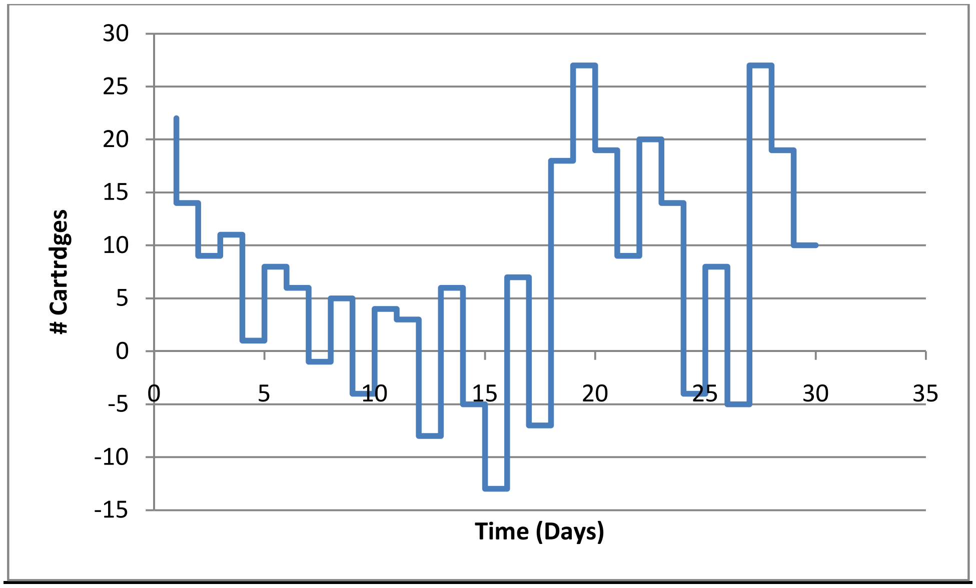

Figure 6.6: Base Case Dynamic policy solution - Inventory on hand

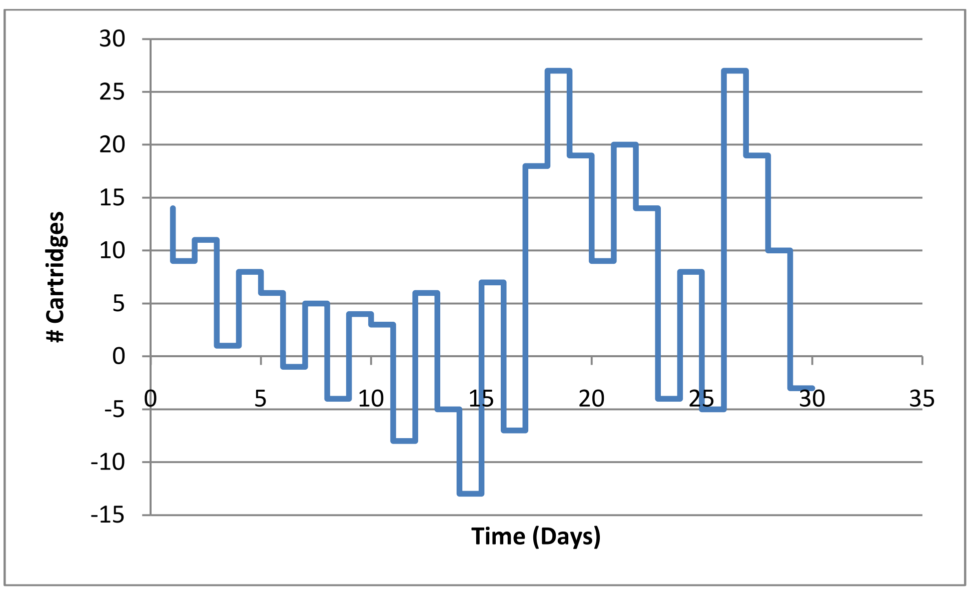

Figure 6.7: Base Case Dynamic solution - Inventory position

6.2 Performance evaluation of the system parameters

The factors considered for evaluation are: the number of printers in the network; relative variability in system costs; relative variability in the lead time; average length of the lead time; and different service levels.

Experimental test cases are designed to evaluate each of these factors. These cases are as follows:

- Case 1: Size of the printer network
    - A. A printer network with 100 printers
    - B. A printer network with 500 printers
- Case 2: Changes in system costs
    - A. A system with a relatively high holding cost, low ordering cost and a high penalty cost
    - B. A system with equal holding and penalty costs and a comparatively low ordering cost
    - C. A system with relatively high holding, low penalty cost and a high ordering cost
    - D. A system with relatively low holding cost, low penalty cost and a high ordering cost
- Case 3: Relative variability in lead time
    - A. A system with low variance in lead time ($\sigma^2 \leq 5\%$ of μ)
    - B. A system with high variance in lead time ($\sigma^2 \geq 50\%$ of μ)
- Case 4: Average length of lead time
    - A. A system with 2 days of average lead time

B. A system with 5 days of average lead time

- Case 5: Service level required

    A. A system with a required service level of 85%

    B. A system with a required service level of 97%

The methodology adopted for conducting the experiments is:

- Generate demand data sets for the printer fleet using the simulation model
- This generated data is sent as input to the demand forecasting model which is used to calculate the forecasted demands over the time horizon for all the printers
- This demand data is sent as input to the optimization model, the solution of which suggests an ordering strategy for a given time horizon. This is the Static solution case.
- The ordering policy is then reviewed periodically (daily) over the time horizon. This is the Dynamic solution case.
- Calculate the inventory replenishment strategy for the above generated demands set using the (Q, R) policy
- The suggested ordering policies from the Static Case, Dynamic Case and the (Q, R) policy are compared for each test case.

On testing the model for different cases as listed above the outputs observed are similar to the expected results. While testing the system, the effect of varying the output from each component on the entire system is also observed. The resulting optimal ordering strategies over the given time horizon for all the printer groups are validated by testing them for a known and constant parameters case.

6.2.1 Case 1: Size of the printer network

The model is tested for a varied number of printers starting from 2 printers to 500 printers. This section tests the system for 100 printers and 500 printer cases.

6.2.1.1 Case1A: A printer network having 100 printers

Consider a system with 100 printers where each printer has its own set of demands for print jobs and own set of demands for toner cartridges. Table 6.3 shows the comparison between the static case and the dynamic case ordering strategies using the suggested algorithm. Figure 6.8 shows the inventory on hand plus backorders at the beginning of the day for the time horizon of 30 days. Figure 6.9 shows the inventory position for the ordering policy. Figure 6.10 and Figure 6.11 show the inventory on hand and inventory position for the 100 printer case when the ordering policy is reviewed every day.

Table 6.3: Case 1A Comparison between static case and dynamic case ordering strategies

	Day	Demand	Quantity Ordered	
			Static	Dynamic
	1	3	41	41
	2	5	0	0
	3	8	0	0
	4	5	0	0
	5	9	4	0
	6	10	0	14
	7	8	24	0
	8	7	9	15
	9	7	0	14
	10	10	22	0
	11	9	10	27
	12	9	0	17
	13	10	30	0
	14	11	0	16
	15	11	9	0
	16	22	32	20
	17	8	0	0
	18	5	7	16
	19	14	24	0
	20	8	7	26
	21	10	0	0
	22	8	23	22
	23	10	0	15
	24	9	21	0
	25	12	0	22
	26	18	12	15
	27	10	23	0
	28	13	0	22
	29	16	0	0
	30	8	0	18
Total expected cost			$1430	$1055

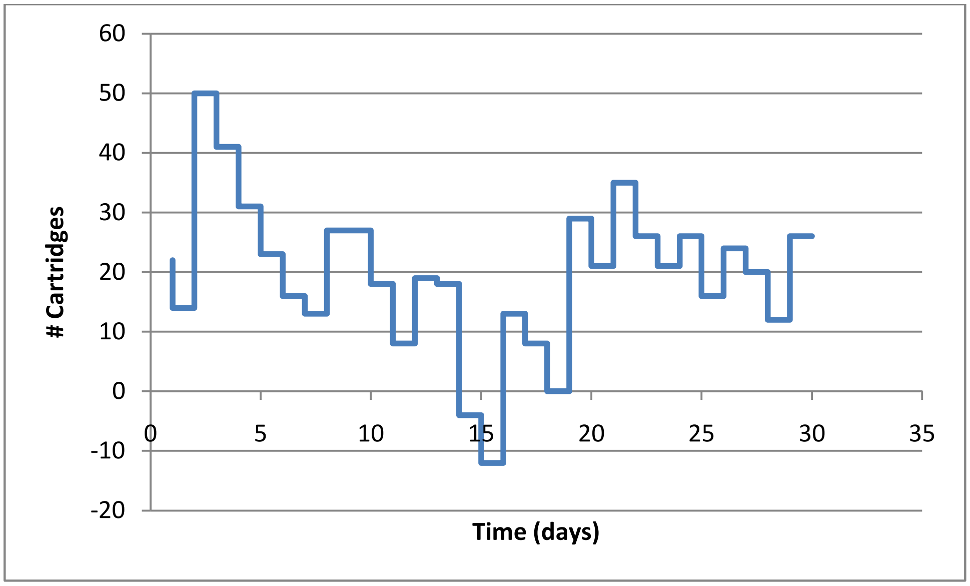

Figure 6.8: Case1A Static case solution - Inventory on hand

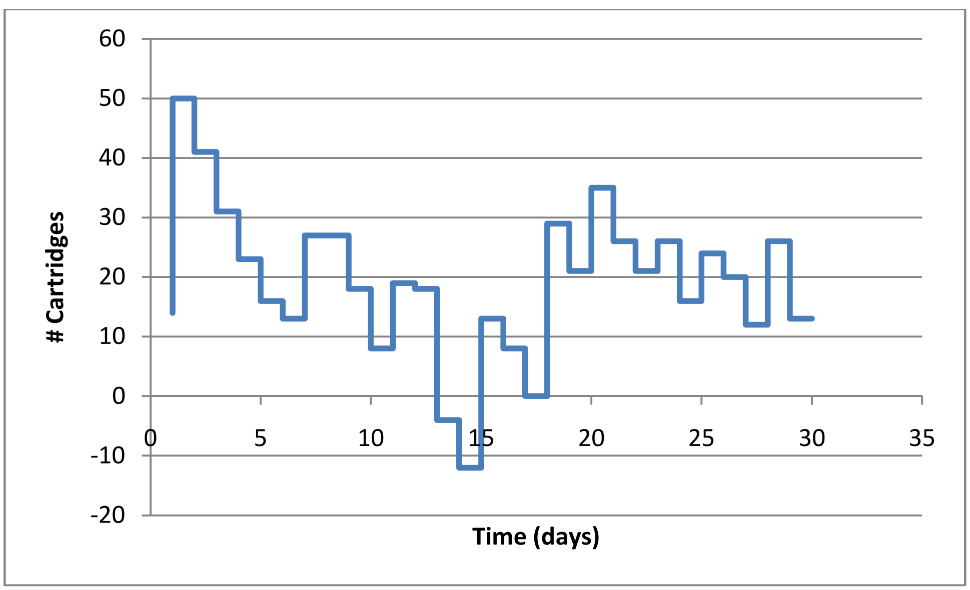

Figure 6.9: Case 1A Static case solution - Inventory position

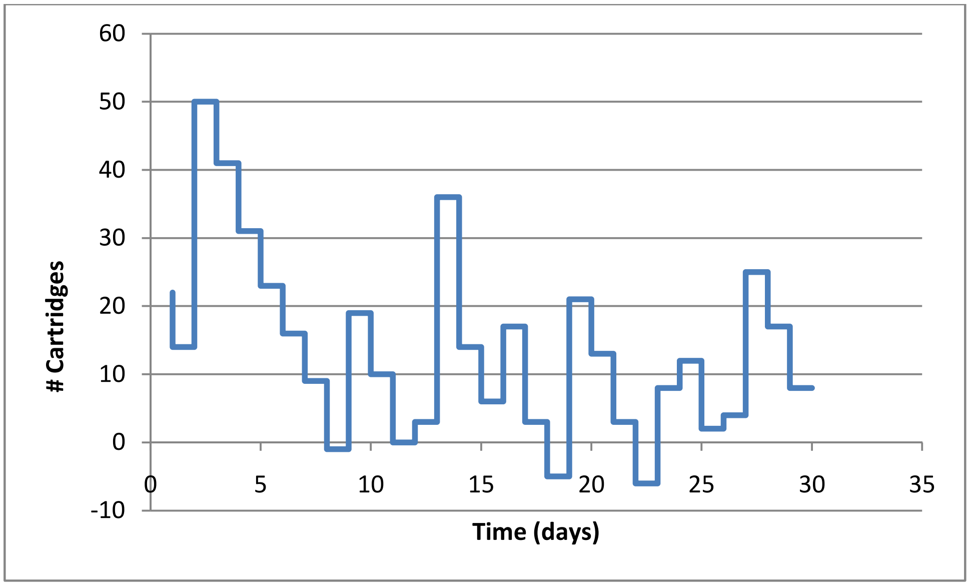

Figure 6.10: Case 1A Dynamic case solution - Inventory on hand

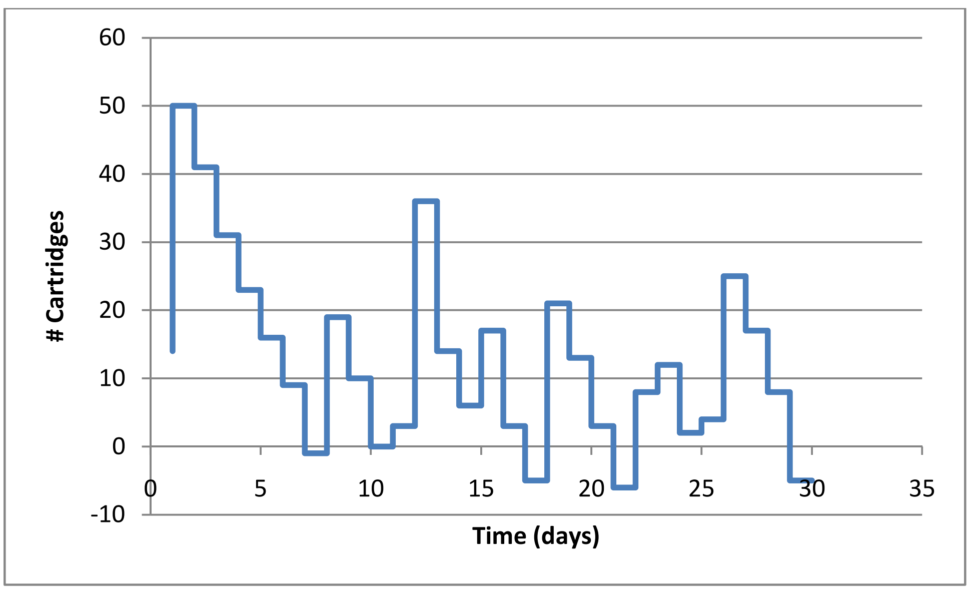

Figure 6.11: Case1A Dynamic case solution - Inventory position

6.2.1.2 Case1B: A printer network having 500 printers

A system consisting of 500 printers with different arrival distributions is considered. The time horizon is considered to be 30 days here. Table 6.4 shows the suggested ordering strategy for the static and dynamic cases. Figure 6.12 shows the inventory on hand plus backorders at the beginning of the day. Figure 6.13 shows the inventory position for the ordering policy. Figure 6.14 and Figure 6.15 show the inventory on hand and inventory position for the 500 printer case when the ordering policy is reviewed every day. From the Figures 6.14 and 6.15, it is clearly demonstrated how periodically reviewing the inventory ordering strategy over the time horizon helps attaining a lower overall cost and a better ordering policy in the case where a lot of variability is involved in the demands.

Table 6.4: Case 1B Comparison between static case and dynamic case ordering strategies

	Day	Demand	Quantity Ordered	
			Static	Dynamic
	1	0	8	8
	2	2	0	0
	3	2	8	0
	4	2	0	8
	5	2	9	0
	6	4	0	12
	7	5	19	0
	8	5	0	11
	9	5	21	0
	10	6	0	19
	11	3	26	0
	12	11	0	19
	13	7	30	0
	14	9	0	32
	15	14	40	0
	16	11	0	28
	17	16	36	17
	18	16	0	0
	19	17	46	33
	20	16	0	0
	21	11	46	41
	22	24	0	0
	23	14	35	33
	24	21	0	0
	25	17	39	56
	26	15	0	0
	27	20	39	54
	28	38	0	0
	29	23	0	44
	30	24	0	0
Total expected cost			$2238	$1001

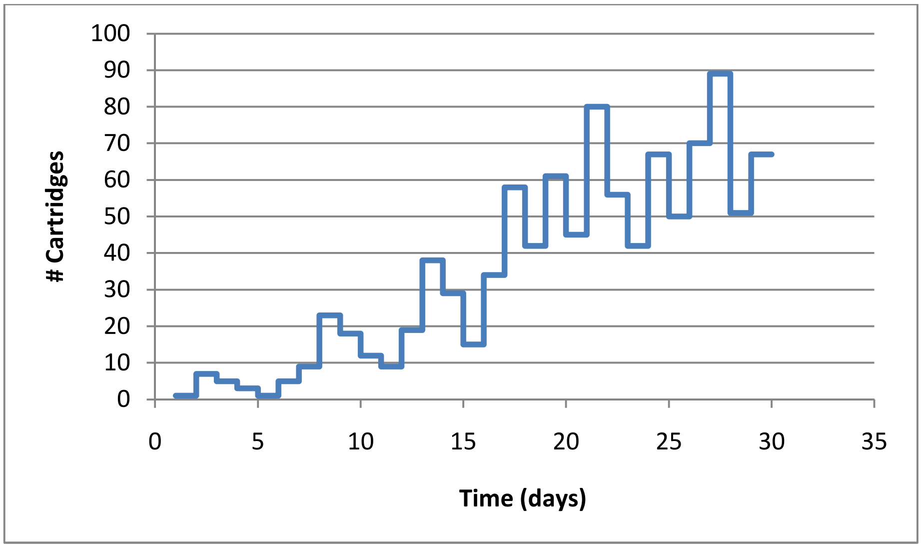

Figure 6.12: Case1B Static case solution - Inventory on hand

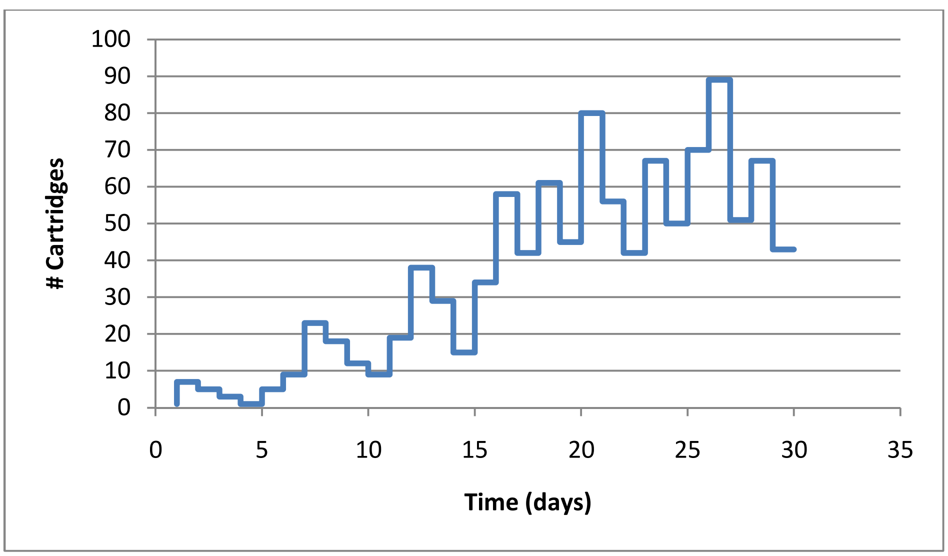

Figure 6.13: Case 1B Static case solution - Inventory position

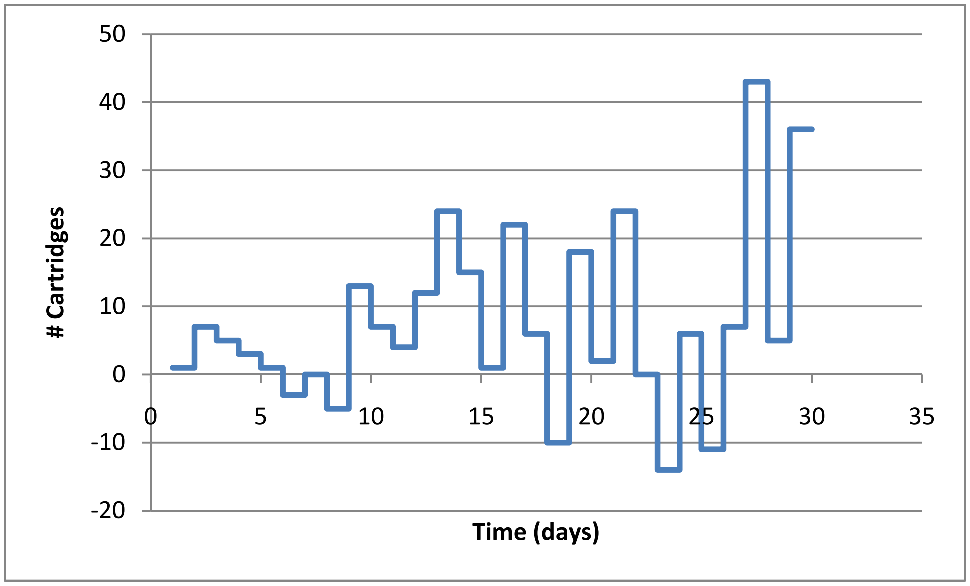

Figure 6.14: Case1B Dynamic case solution - Inventory on hand

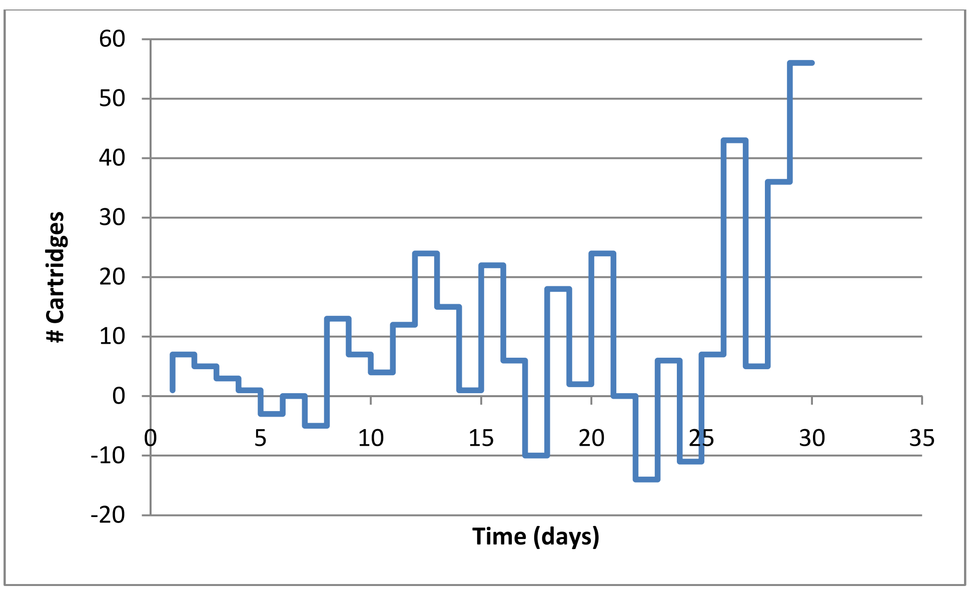

Figure 6.15: Case1B Dynamic case solution - Inventory position

6.2.2 Case 2: Changes in system costs

Varying the unit holding, penalty and ordering costs change the ordering strategy over the time horizon. Depending on the balance between the holding and penalty, the number of backorders or excess orders is decided. Also, increasing or decreasing the ordering cost leads to lesser or more number of orders placed.

6.2.2.1 Case 2A: A system with a relatively high holding cost, low ordering cost and a high penalty cost

Here, the holding cost is $2 per unit per day, penalty cost is $15 per unit per day and the ordering cost is considered to be $10 per order. The suggested inventory on hand and ordering strategies are shown in the figures below. Table 6.2.5 shows the suggested ordering strategy for the static and dynamic cases. Figure 6.2.16 shows the inventory on hand plus backorders at the beginning of the day. Figure 6.2.17 shows the inventory position for the ordering policy. Figure 6.2.18 and Figure 6.2.19 show the inventory on hand and inventory position for the 100 printer case when the ordering policy is reviewed every day.

Table 6.5: Case 2A Comparison between static case and dynamic case ordering strategies

	Day	Demand	Quantity Ordered	
			Static	Dynamic
	1	3	41	41
	2	5	0	0
	3	8	0	0
	4	5	0	0
	5	9	0	0
	6	10	14	14
	7	8	0	0
	8	7	24	15
	9	7	0	14
	10	10	19	0
	11	9	0	28
	12	9	28	16
	13	10	0	0
	14	11	24	15
	15	11	0	0
	16	22	29	20
	17	8	0	0
	18	5	22	17
	19	14	0	0
	20	8	20	27
	21	10	7	0
	22	8	0	8
	23	10	24	27
	24	9	0	0
	25	12	23	23
	26	18	0	15
	27	10	23	0
	28	13	0	23
	29	16	0	0
	30	8	0	18
Total expected cost			$1397	$1388

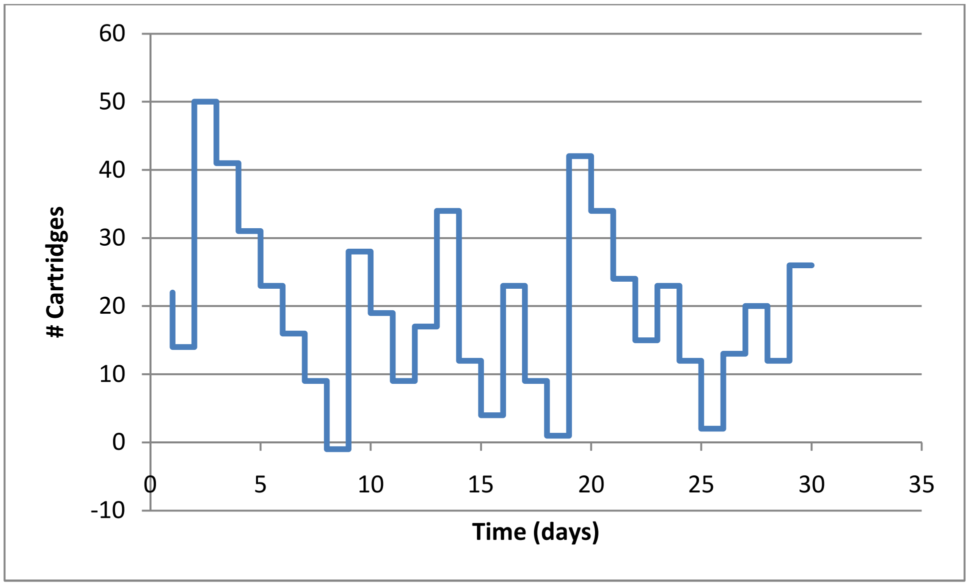

Figure 6.16: Case 2A Static case solution - Inventory on hand

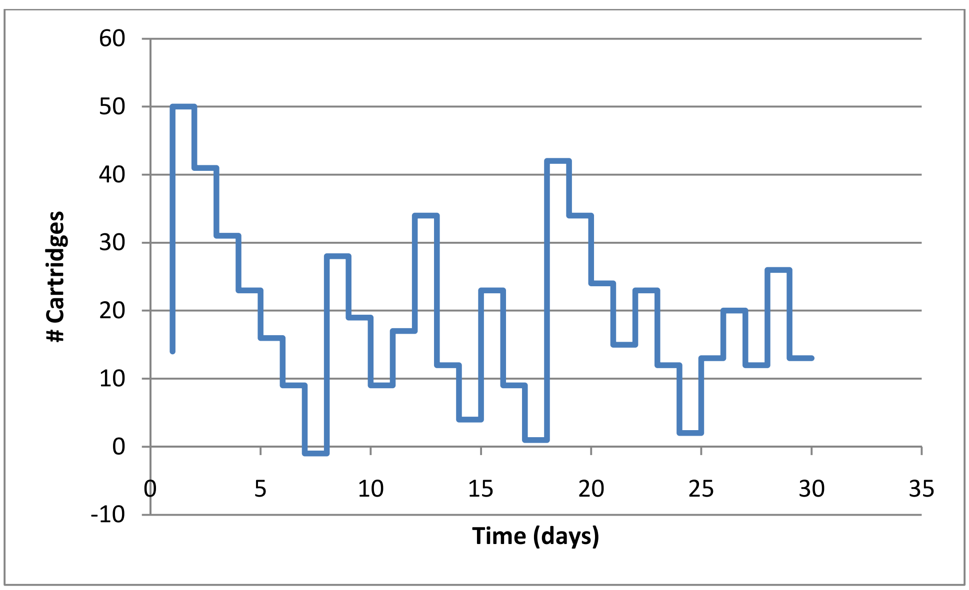

Figure 6.17: Case 2A Static case solution - Inventory position

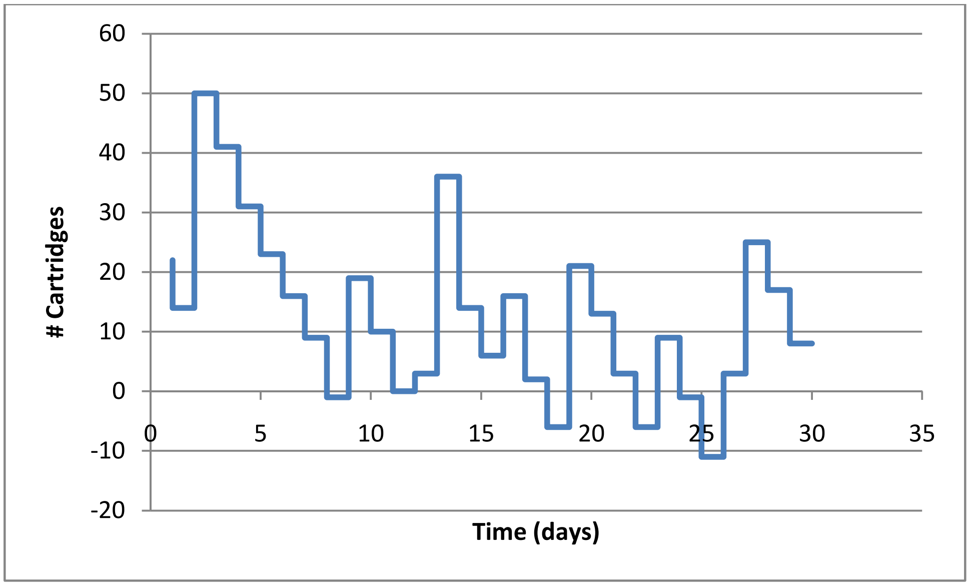

Figure 6.18: Case 2A Dynamic case solution - Inventory on hand

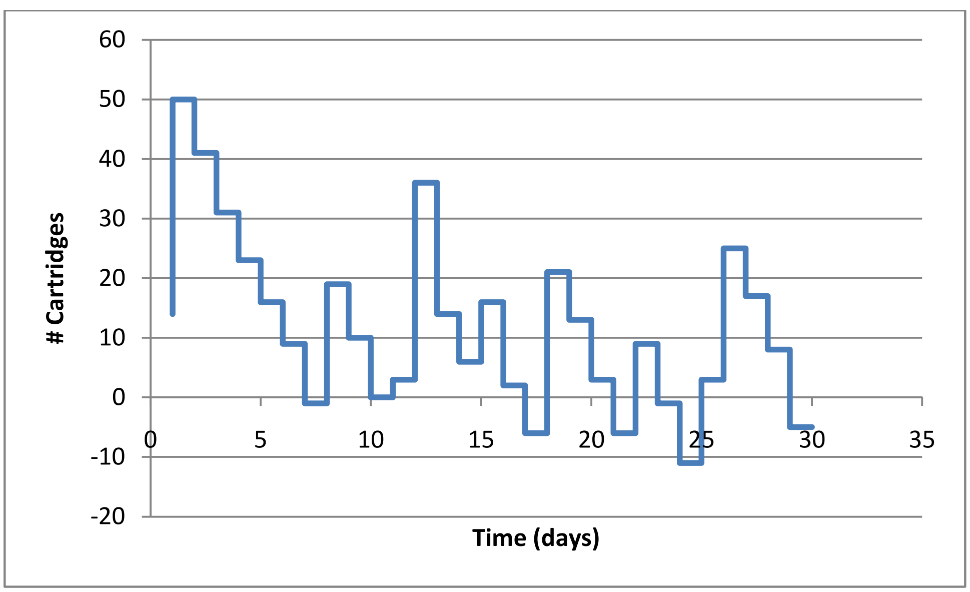

Figure 6.19: Case 2A Dynamic case solution - Inventory position

6.2.2.2 Case 2B: A system with equal holding and penalty costs and a comparatively low ordering cost

Here the ordering cost is $10 per order and the inventory holding and penalty costs are $2 per unit per day each. All other parameters are kept same as in the base case. Table 6.2.6 shows the suggested ordering strategy for the static and dynamic case. Figure 6.20 shows the inventory on hand plus backorders at the beginning of the day. Figure 6.21 shows the inventory position for the ordering policy. Figure 6.22 and Figure 6.23 show the inventory on hand and inventory position for the 100 printer case when the ordering policy is reviewed every day.

Table 6.2.6: Case 2B Comparison between static case and dynamic case ordering strategies

	Day	Demand	Quantity Ordered	
			Static	Dynamic
	1	3	41	41
	2	5	0	0
	3	8	0	0
	4	5	0	0
	5	9	0	0
	6	10	0	14
	7	8	28	0
	8	7	0	15
	9	7	19	15
	10	10	0	0
	11	9	23	24
	12	9	0	19
	13	10	28	0
	14	11	0	15
	15	11	30	0
	16	22	0	20
	17	8	19	0
	18	5	0	6
	19	14	23	26
	20	8	0	0
	21	10	17	20
	22	8	0	0
	23	10	25	29
	24	9	0	0
	25	12	22	20
	26	18	0	14
	27	10	23	0
	28	13	0	23
	29	16	0	7
	30	8	0	0
Total expected cost			$1,240	$962

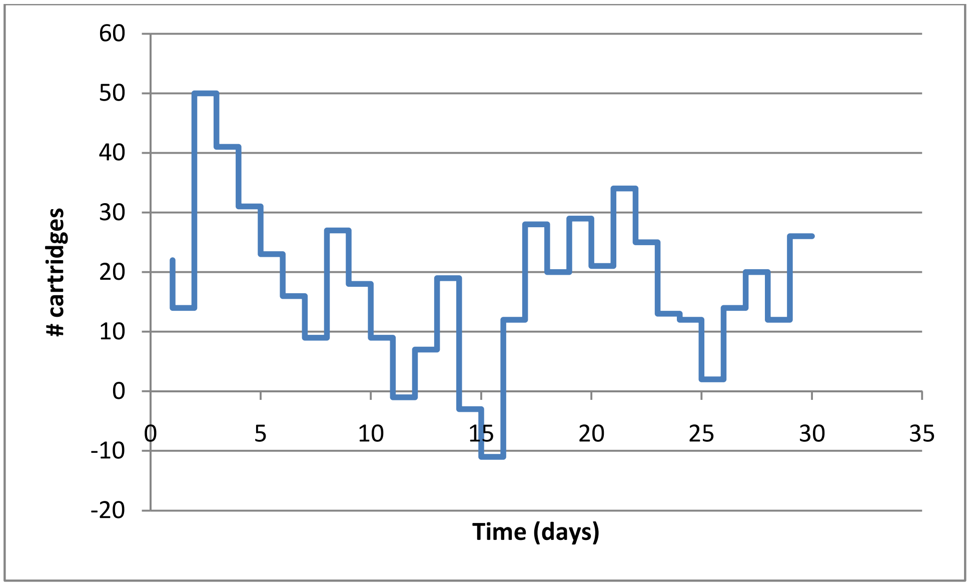

Figure 6.20: Case 2B Static case solution - Inventory on hand

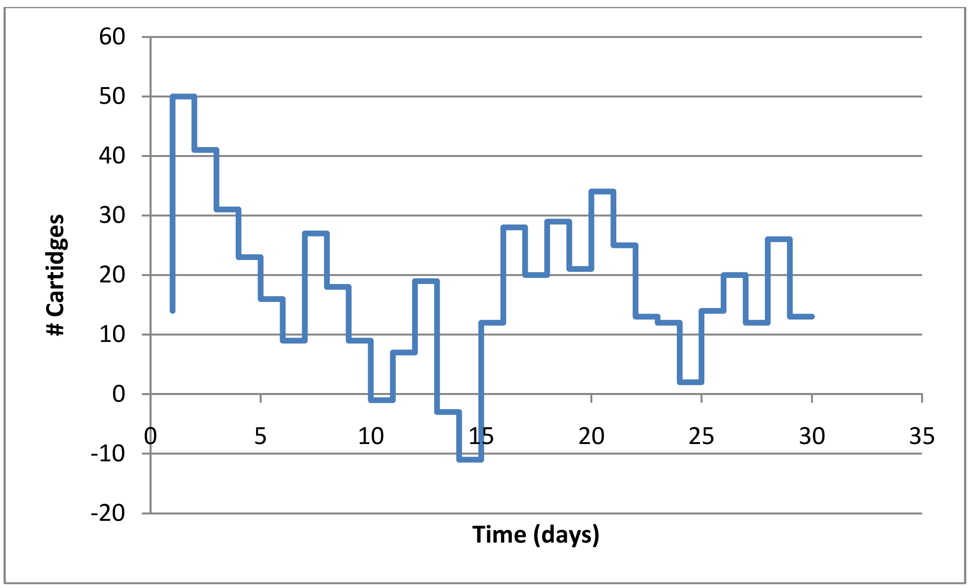

Figure 6.21: Case 2B Static case solution - Inventory position

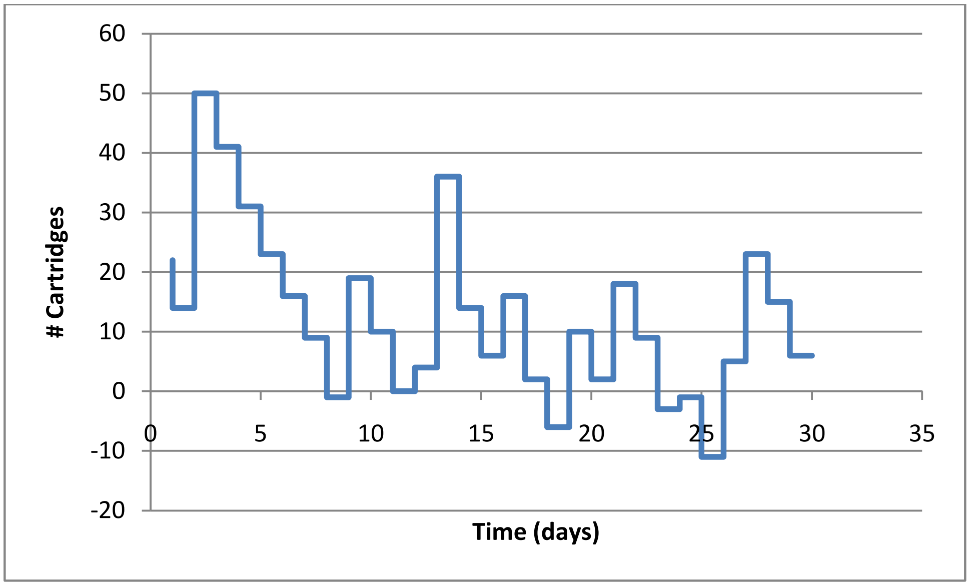

Figure 6.22: Case 2B Dynamic case solution - Inventory on hand

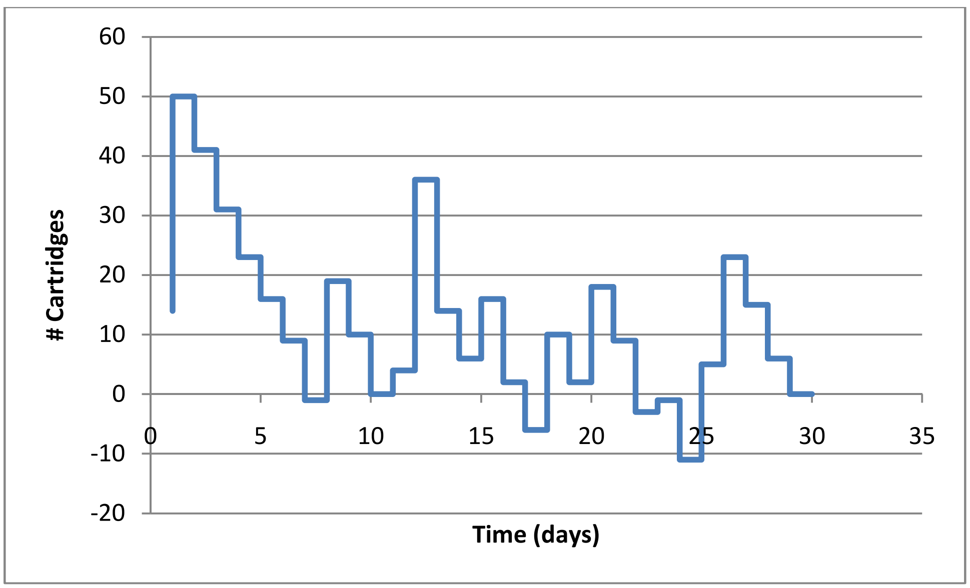

Figure 6.23: Case 2B Dynamic case solution - Inventory position

6.2.2.3 Case 2C: A system with relatively high holding cost, low penalty costs and a high ordering cost

Here the ordering cost is \$30 per order, the inventory holding cost is \$2 per unit per day and the penalty cost is \$5 per unit per day. Table 6.7 shows the suggested ordering strategy for the static and dynamic case. Figure 6.24 shows the inventory on hand plus backorders at the beginning of the day. Figure 6.25 shows the inventory position for the ordering policy. Figure 6.26 and Figure 6.27 show the inventory on hand and inventory position for the 100 printer case when the ordering policy is reviewed every day.

Table 6.2.7: Case 2C Comparison between static case and dynamic case ordering strategies

	Day	Demand	Quantity Ordered	
			Static	Dynamic
	1	3	41	41
	2	5	0	0
	3	8	0	0
	4	5	0	0
	5	9	0	3
	6	10	14	0
	7	8	0	28
	8	7	33	0
	9	7	0	0
	10	10	0	28
	11	9	23	9
	12	9	0	24
	13	10	37	0
	14	11	0	0
	15	11	0	26
	16	22	39	0
	17	8	0	0
	18	5	0	22
	19	14	32	0
	20	8	0	26
	21	10	0	0
	22	8	32	19
	23	10	0	0
	24	9	0	28
	25	12	26	0
	26	18	0	26
	27	10	21	0
	28	13	0	9
	29	16	0	20
	30	8	0	0
Total expected cost			$1,604	$1,475

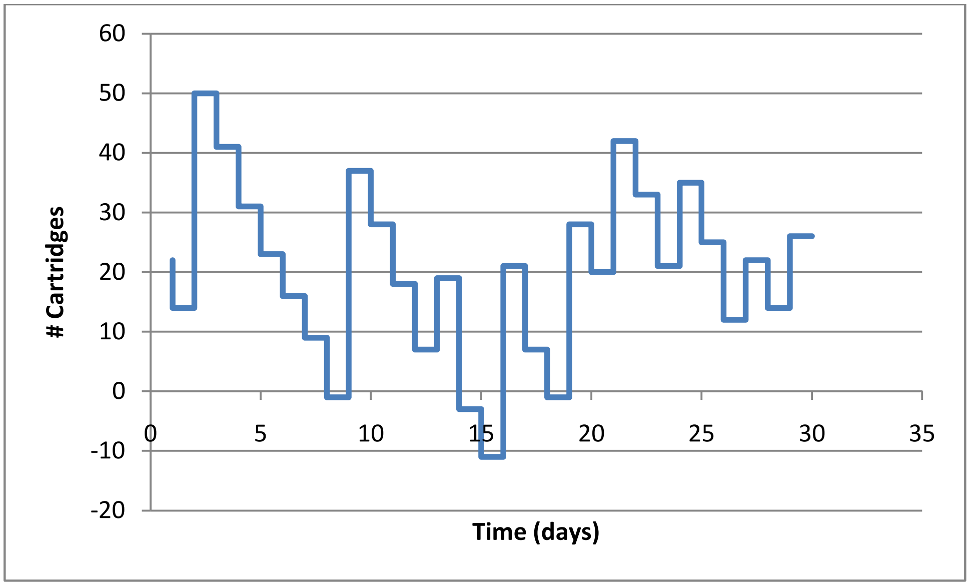

Figure 6.24: Case 2C Static case solution - Inventory on hand

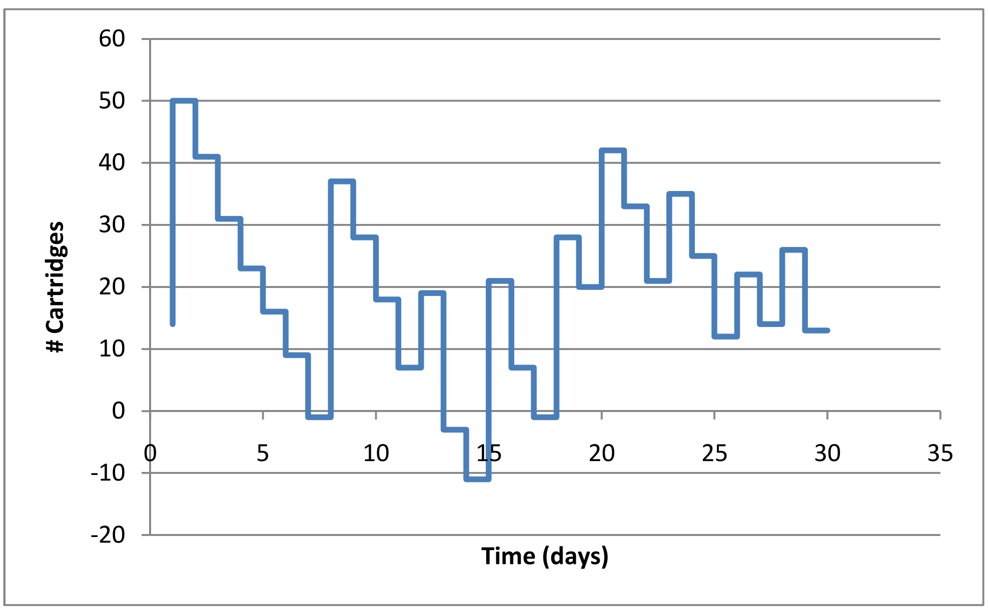

Figure 6.25: Case 2C Static case solution - Inventory position

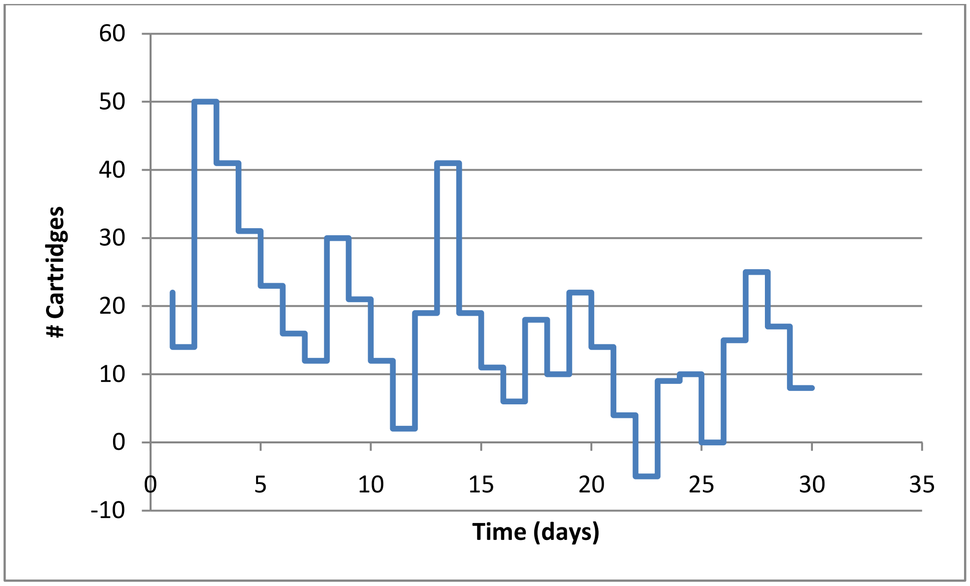

Figure 6.26: Case 2C Dynamic case solution – Inventory on hand

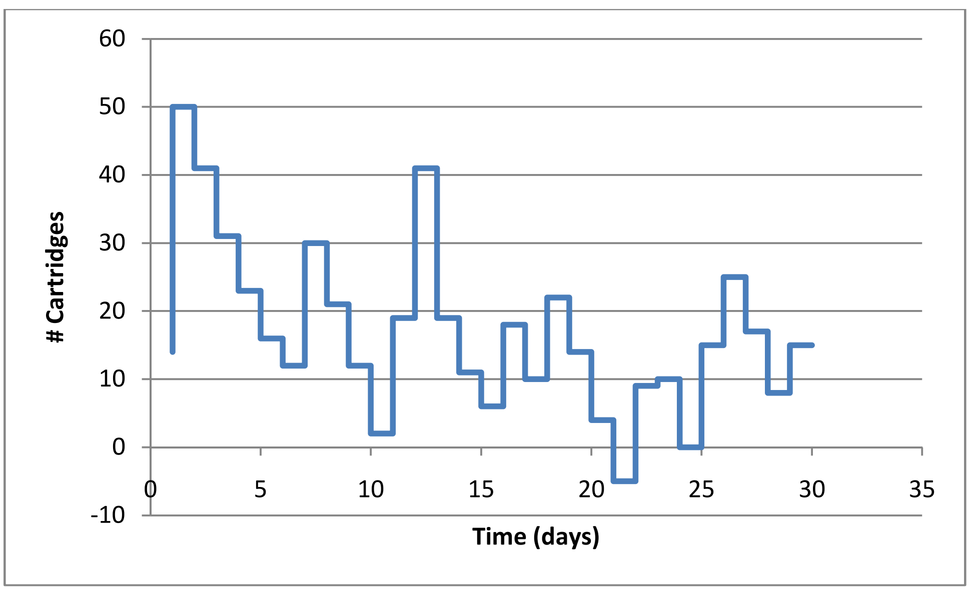

Figure 6.27: Case 2C Dynamic case solution - Inventory position

6.2.2.4 Case 2D: A system with relatively low holding cost, low penalty cost and high ordering cost

In this case, the holding cost is assumed to be $1/unit/day, penalty cost =$10/unit/day, and ordering cost is $50/order. This case is the most representative of the real-world situations. Table 6.8 shows the suggested ordering strategy for the static and dynamic case. Figure 6.28 shows the inventory on hand plus backorders at the beginning of the day. Figure 6.29 shows the inventory position for the ordering policy. Figure 6.30 and Figure 6.31 show the inventory on hand and inventory position for the 100 printer case when the ordering policy is reviewed every day. This case demonstrates better the differences between the static and dynamic ordering policies. As seen from the figures, reviewing the policy daily over the time horizon helps adjust the policy to account for the high variability in demand.

Table 6.8: Case 2D Comparison between static case and dynamic case ordering strategies

	Day	Demand	Quantity Ordered	
			Static	Dynamic
	1	3	45	45
	2	5	0	0
	3	8	0	0
	4	5	0	0
	5	9	0	0
	6	10	0	0
	7	8	42	27
	8	7	0	0
	9	7	0	0
	10	10	0	57
	11	9	49	0
	12	9	0	0
	13	10	0	0
	14	11	0	33
	15	11	51	0
	16	22	0	0
	17	8	0	0
	18	5	0	38
	19	14	40	0
	20	8	0	0
	21	10	0	0
	22	8	0	0
	23	10	34	54
	24	9	0	0
	25	12	0	0
	26	18	37	58
	27	10	0	0
	28	13	0	0
	29	16	0	0
	30	8	0	0
Total expected cost			$1,167	$1,074

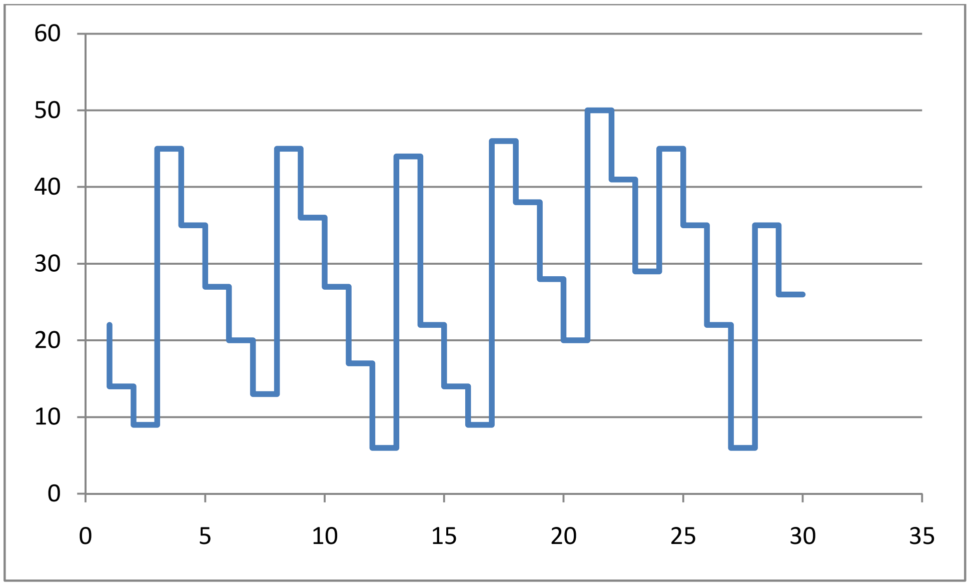

Figure: 6.28: Case 2D Static case solution - Inventory on hand

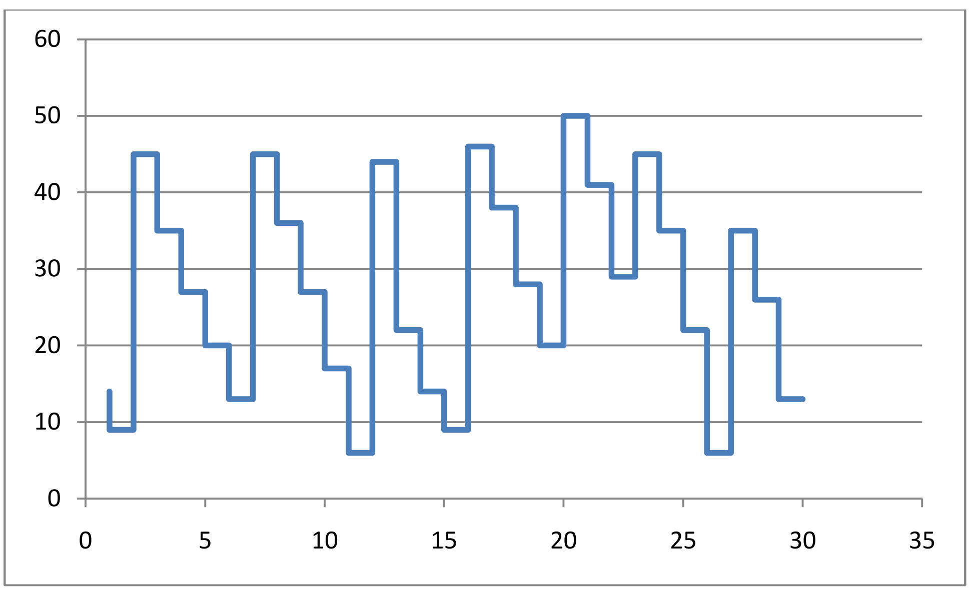

Figure 6.29: Case 2D Static case solution - Inventory position

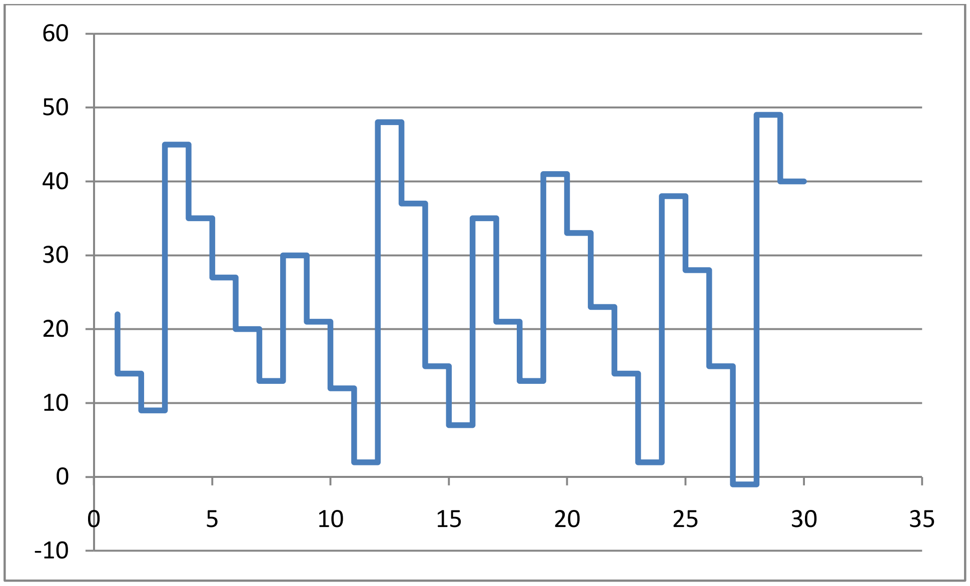

Figure 6.30: Case 2D Dynamic case solution - Inventory on hand

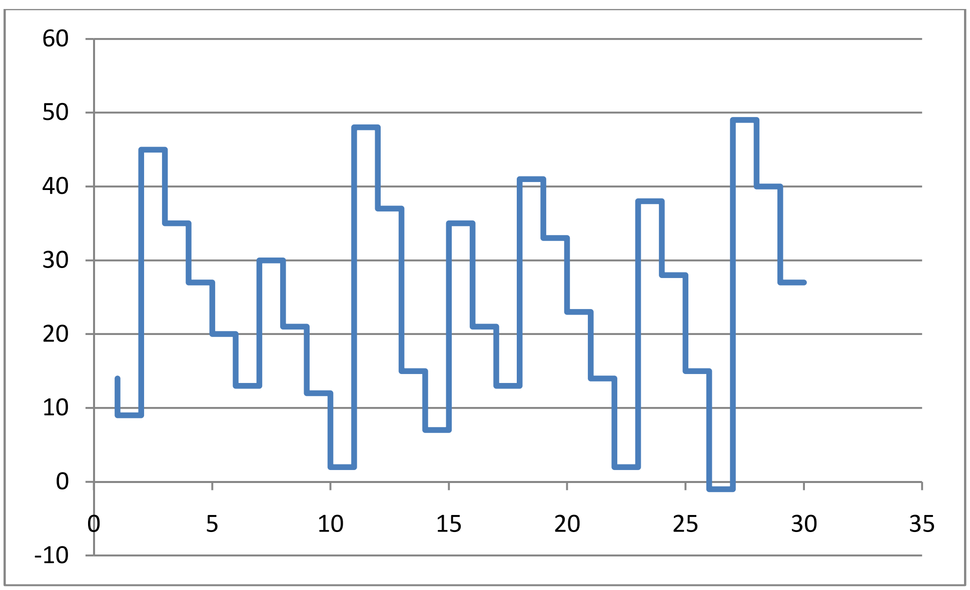

Figure 6.31: Case 2D Dynamic case solution - Inventory position

6.2.3 Case 3: Relative Variability in Lead Time

Variability in lead time is one of the key factors in deciding the ordering strategy for a give time period. To check the robustness of the model at various levels of variability in lead time, the model is tested for two levels of variance in lead time.

6.2.3.1 Case 3A: A system with low variance in lead time ($\boldsymbol{\sigma^2 \leq 5\%}$ of $\boldsymbol{\mu}$)

Here the probability of receiving the orders is set as [0, 0.02, 0.96, 0.02] in the optimization model. This implies that the probability of receiving an order 1 day after placing the order is 0.02, 2 days after placing the order is 0.96 and three days after placing the order is 0.02. Figure 6.32 shows the probabilities of receiving the receiving the orders on any particular day and the variance in lead time.

x (Day)	0	1	2	3
Probability	0	0.02	0.96	0.02

$E[x] = \mu = 0.02\ (1) + 0.96\ (2) + 0.02\ (3) = 2$

$\sigma^2 = E[x^2] - (E[x])^2 = 0.04$

$\sigma^2 / \mu = 0.02$

Figure 6.32: Probabilities of receiving orders over time

Table 6.9 shows the suggested ordering strategy for the static and periodic review case. Figure 6.33 shows the inventory on hand plus backorders at the beginning of the day. Figure 6.34 shows the inventory position for the ordering policy. Figure 6.35 and Figure 6.36 show the inventory on hand and inventory position for the 100 printer case when the ordering policy is reviewed every day.

Table 6.9: Case 3A Comparison between static case and dynamic case ordering strategies

	Day	Demand	Quantity Ordered	
			Static	Dynamic
	1	3	11	11
	2	5	0	0
	3	8	14	15
	4	5	0	11
	5	9	22	0
	6	10	0	18
	7	8	21	0
	8	7	0	15
	9	7	21	15
	10	10	0	0
	11	9	22	26
	12	9	0	16
	13	10	27	0
	14	11	0	16
	15	11	31	0
	16	22	0	20
	17	8	19	0
	18	5	0	18
	19	14	23	0
	20	8	0	30
	21	10	17	0
	22	8	0	16
	23	10	24	16
	24	9	0	0
	25	12	24	21
	26	18	0	15
	27	10	22	0
	28	13	0	23
	29	16	0	0
	30	8	0	18
Total expected cost			$1,141	$848

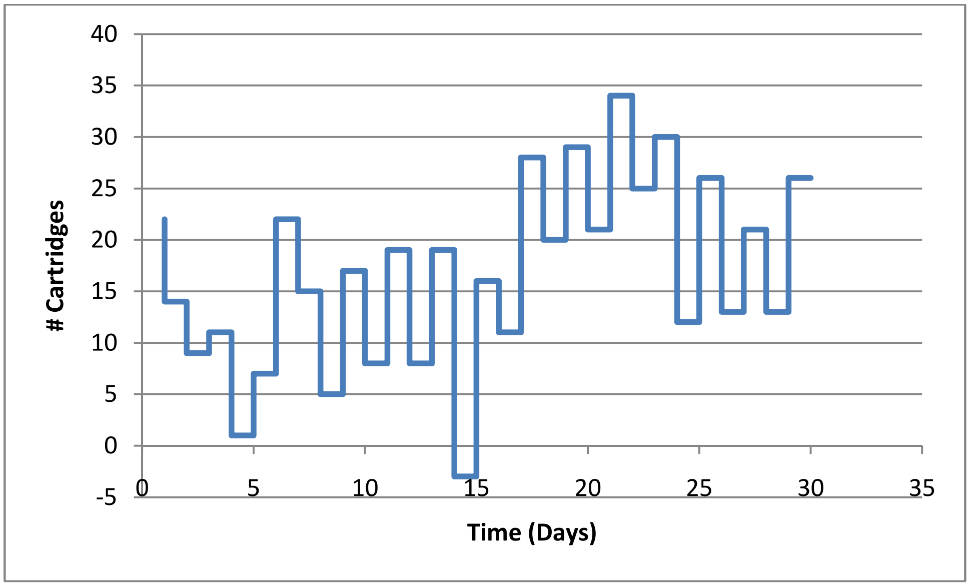

Figure 6.33: Case 3A Static case solution - Inventory on hand

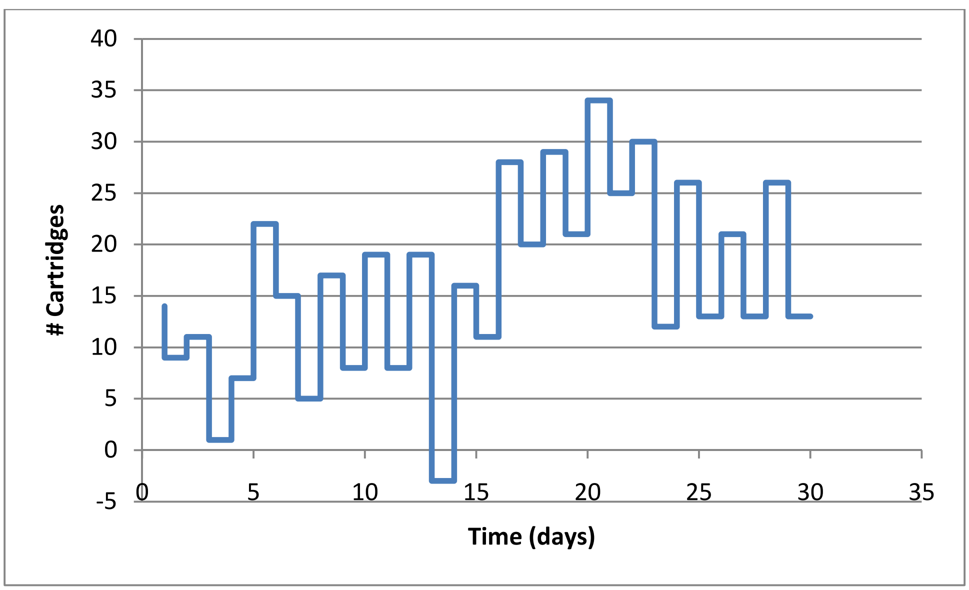

Figure 6.34: Case 3A Static case solution - Inventory position

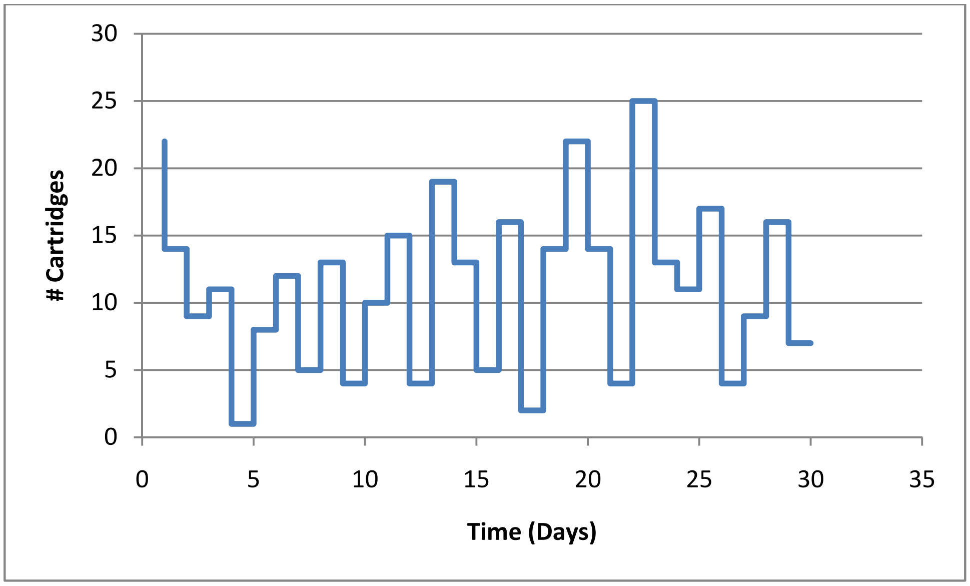

Figure 6.35: Case 3A Dynamic case solution - Inventory on hand

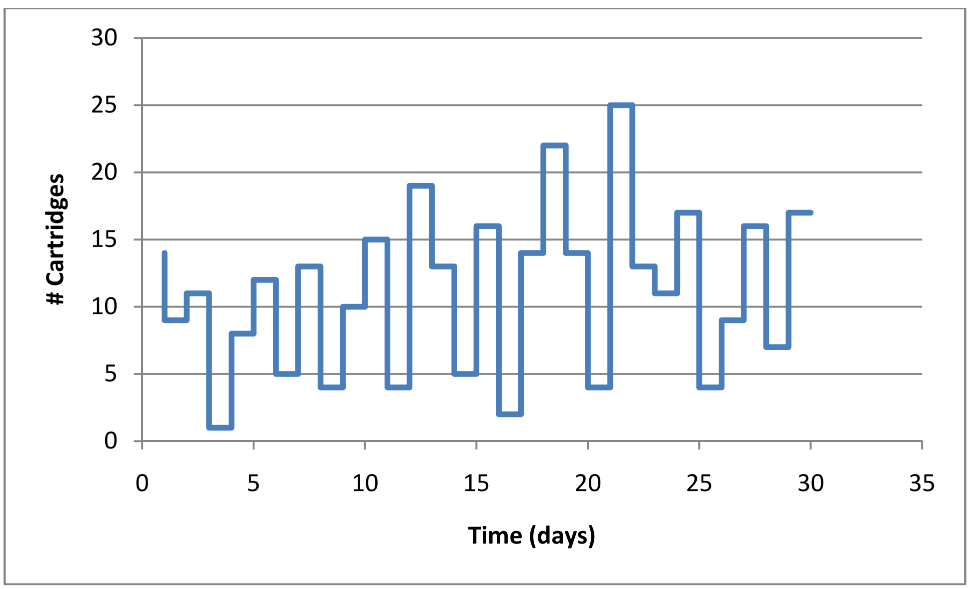

Figure 6.36: Case 3A Dynamic case solution - Inventory position

6.2.3.2 Case 3B: A system with high variance in lead time ($\sigma^2 \geq 50\%$ of μ)

Here the probability of receiving the orders is set as [0, 0.33, 0.33, 0.33] in the optimization model. This implies that the probability of receiving an order 1 day after placing the order is 0.33, 2 days after placing the order is 0.33 and three days after placing the order is also 0.33, that is, it is equally likely to receive the orders on the three days from order placement. Figure 6.37 shows the probabilities of receiving the receiving the orders on any particular day and the variance in lead time.

x (Day)	0	1	2	3
Probability	0	0.33	0.33	0.33

$E[x] = \mu = 0.33\ (1) + 0.33\ (2) + 0.33\ (3) = 2$

$\sigma^2 = E[x^2] - (E[x])^2 = 4.62$

$\sigma^2 / \mu = 2.31$

Figure 6.37: Probabilities of receiving the orders over time

Table 6.10 shows the suggested ordering strategy for the static and dynamic case. Figure 6.38 shows the inventory on hand plus backorders at the beginning of the day. Figure 6.39 shows the inventory position for the ordering policy. Figure 6.40 and Figure 6.41 show the inventory on hand and inventory position for the 100 printer case when the ordering policy is reviewed every day.

Table 6.10: Case 3B Comparison between static case and dynamic case ordering strategies

	Day	Demand	Quantity Ordered	
			Static	Dynamic
	1	3	26	26
	2	5	0	0
	3	8	0	0
	4	5	0	0
	5	9	0	25
	6	10	43	0
	7	8	0	0
	8	7	0	0
	9	7	0	45
	10	10	44	0
	11	9	0	0
	12	9	0	39
	13	10	0	0
	14	11	58	0
	15	11	0	0
	16	22	0	33
	17	8	0	0
	18	5	43	0
	19	14	0	33
	20	8	0	0
	21	10	0	0
	22	8	41	27
	23	10	0	0
	24	9	0	0
	25	12	0	46
	26	18	46	0
	27	10	0	0
	28	13	0	31
	29	16	0	0
	30	8	0	0
Total expected cost			$1,575	$1,142

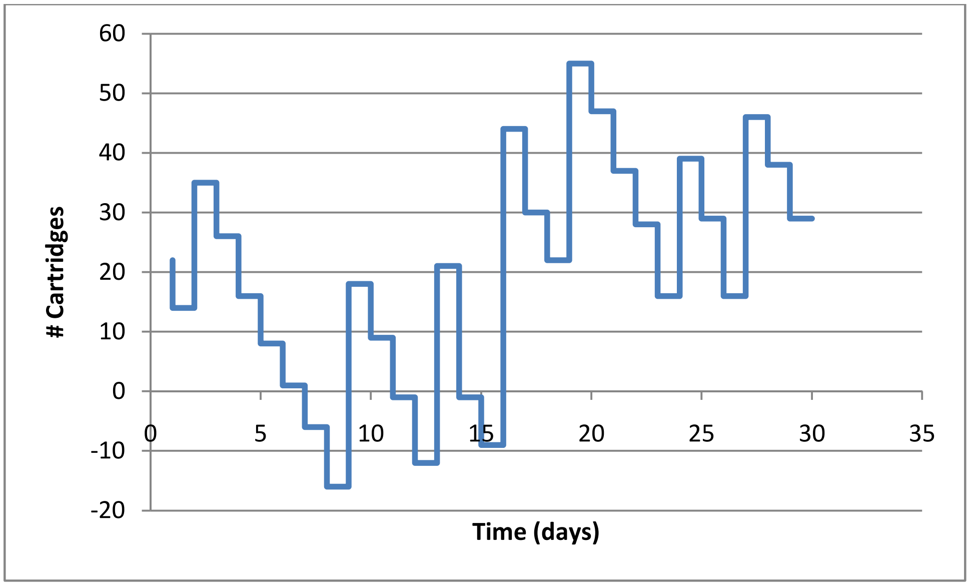

Figure 6.38: Case 3B Static case solution - Inventory on hand

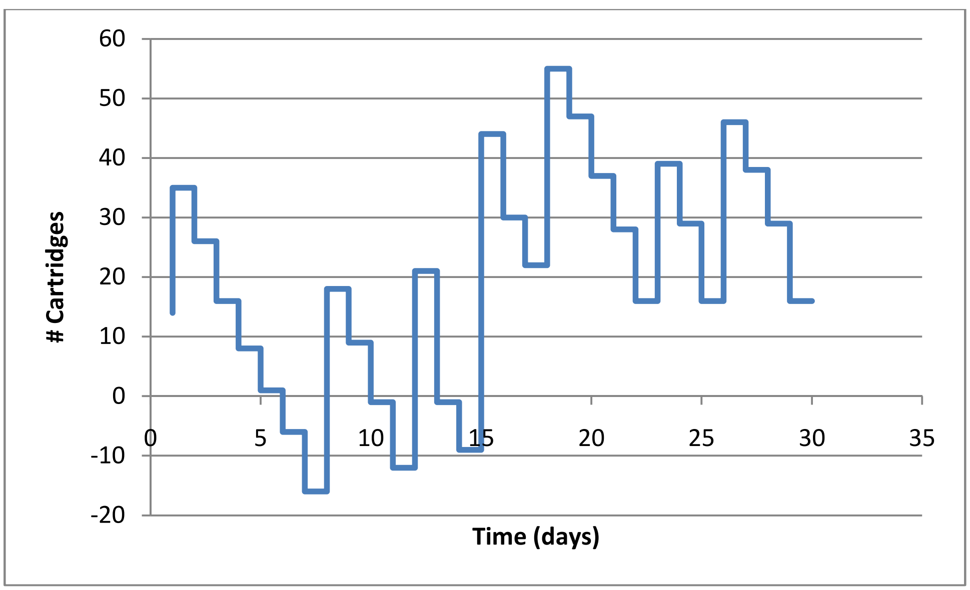

Figure 6.39: Case 3B Static case solution - Inventory position

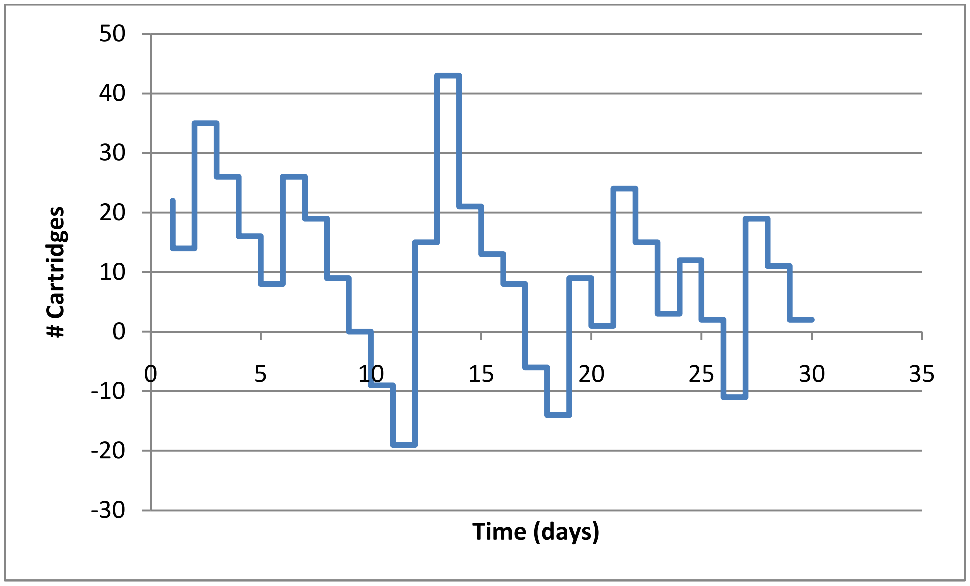

Figure 6.40: Case3B Dynamic case solution - Inventory on hand

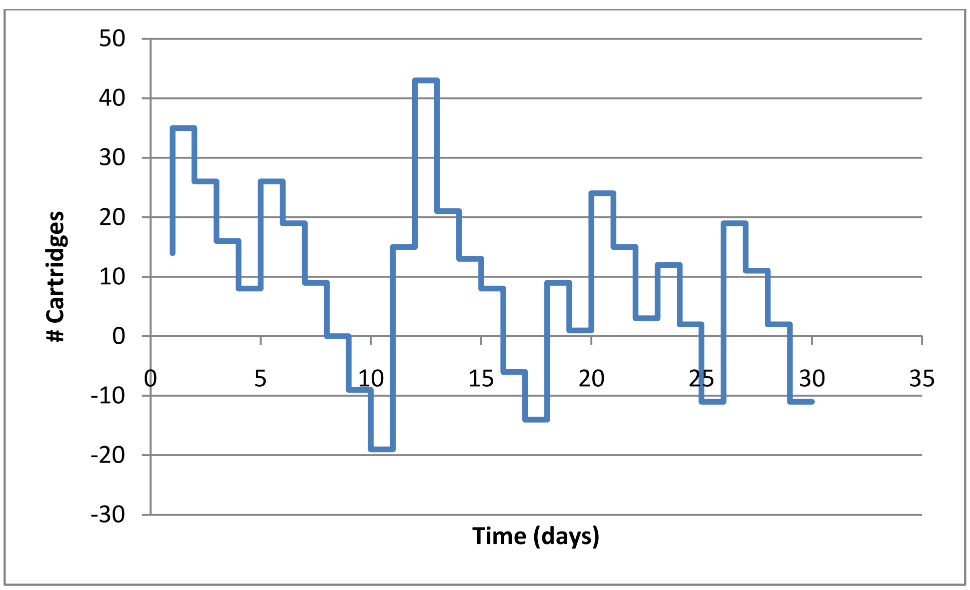

Figure 6.41: Case 3B Dynamic case solution - Inventory position

6.2.4 Case 4: Average length of lead time

In this case, the model is run for varied lead time durations while keeping the other system parameters same as the base case. Two cases have been discussed below to show the changes in ordering decisions over the time horizon.

6.2.4.1 Case 4A: A system with 2 days of average lead time

Here the probability of receiving the orders is set as [0, 0.25, 0.75] in the optimization model. This indicates that the probability of receiving an order 1 day after placing the order is 0.25, and 2 days after placing the order is 0.75. Table 6.11 shows the suggested ordering strategy for the static and dynamic case. Figure 6.42 shows the inventory on hand plus backorders at the beginning of the day. Figure 6.43 shows the inventory position for the ordering policy. Figure 6.44 and Figure 6.45 show the inventory on hand and inventory position for the 100 printer case when the ordering policy is reviewed every day.

Table 6.11: Case 4A Comparison between static case and dynamic case ordering strategies

	Day	Demand	Quantity Ordered	
			Static	Dynamic
	1	3	19	19
	2	5	0	0
	3	8	0	0
	4	5	0	13
	5	9	28	0
	6	10	0	17
	7	8	14	0
	8	7	0	16
	9	7	28	0
	10	10	0	34
	11	9	0	0
	12	9	38	29
	13	10	0	0
	14	11	0	16
	15	11	45	0
	16	22	0	19
	17	8	0	0
	18	5	30	0
	19	14	0	31
	20	8	19	0
	21	10	0	21
	22	8	0	0
	23	10	31	32
	24	9	0	0
	25	12	24	0
	26	18	0	32
	27	10	0	0
	28	13	33	0
	29	16	0	31
	30	8	0	0
Total expected cost			$1,271	$1,014

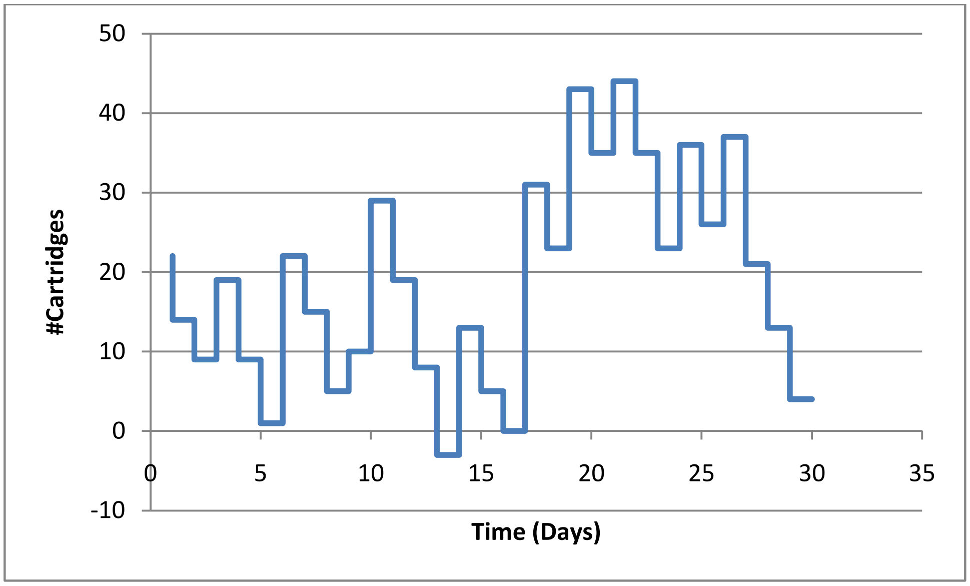

Figure 6.42: Case 4A Static case solution - Inventory on hand

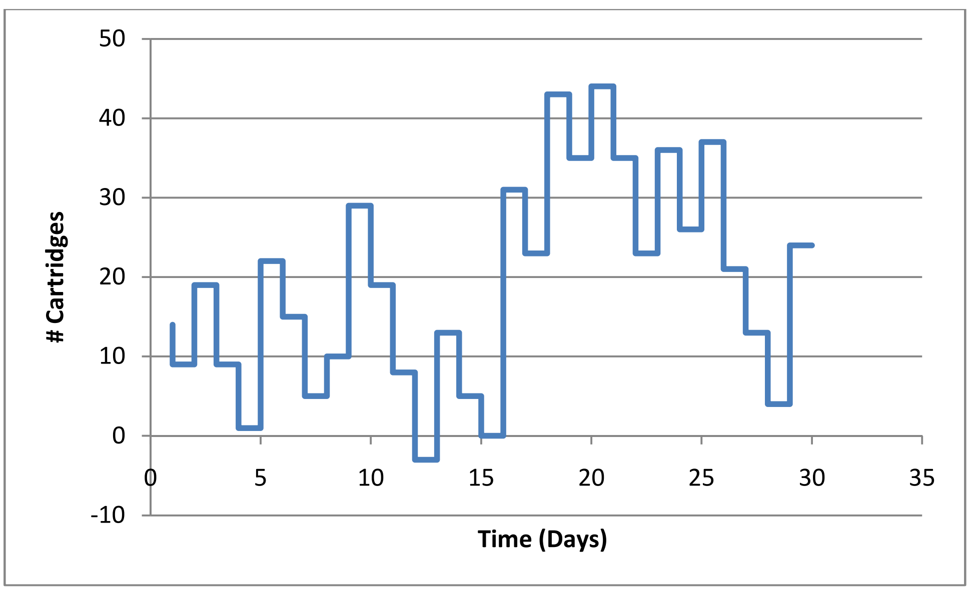

Figure 6.43: Case 4A Static case solution - Inventory position

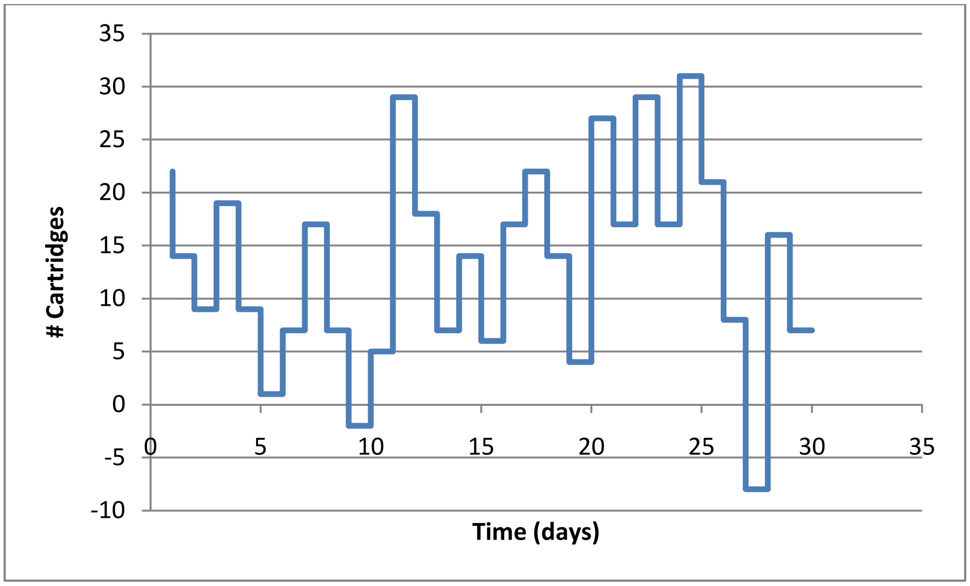

Figure 6.44: Case 4A Dynamic case solution - Inventory on hand

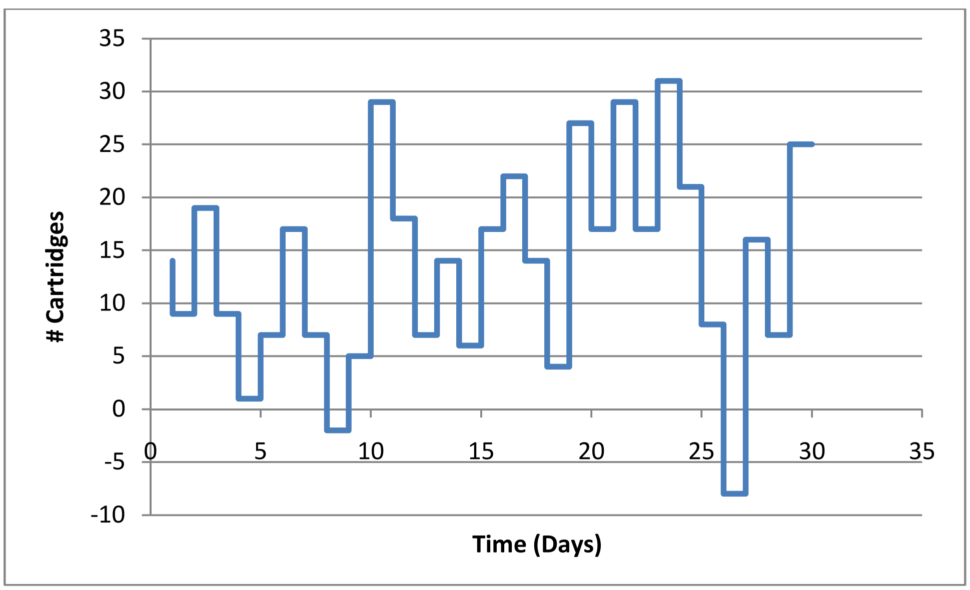

Figure 6.45: Case 4A Dynamic case solution - Inventory position

6.2.4.2 Case 4B: A system with 5 days of average lead time

Here the probability of receiving the orders is set as [0, 0.05, 0.15, 0.65, 0.10, 0.05] in the optimization model. This indicates the probability of receiving the orders 0, 1, 2, 3, 4, and 5 days after placing the order respectively. Table 6.12 shows the suggested ordering strategy for the static and dynamic case. Figure 6.46 shows the inventory on hand plus backorders at the beginning of the day. Figure 6.47 shows the inventory position for the ordering policy. Figure 6.48 and Figure 6.49 show the inventory on hand and inventory position for the 100 printer case when the ordering policy is reviewed every day.

Table 6.12: Case 4B Comparison between static case and dynamic case ordering strategies

	Day	Demand	Quantity Ordered	
			Static	Dynamic
	1	3	48	48
	2	5	0	0
	3	8	0	0
	4	5	0	0
	5	9	0	0
	6	10	2	0
	7	8	32	29
	8	7	0	0
	9	7	19	0
	10	10	1	34
	11	9	2	0
	12	9	37	30
	13	10	0	0
	14	11	0	0
	15	11	44	0
	16	22	4	39
	17	8	0	0
	18	5	25	0
	19	14	0	0
	20	8	9	47
	21	10	1	0
	22	8	27	0
	23	10	0	32
	24	9	3	0
	25	12	43	0
	26	18	0	38
	27	10	0	0
	28	13	0	0
	29	16	0	0
	30	8	0	0
Total expected cost			$1,282	$1,165

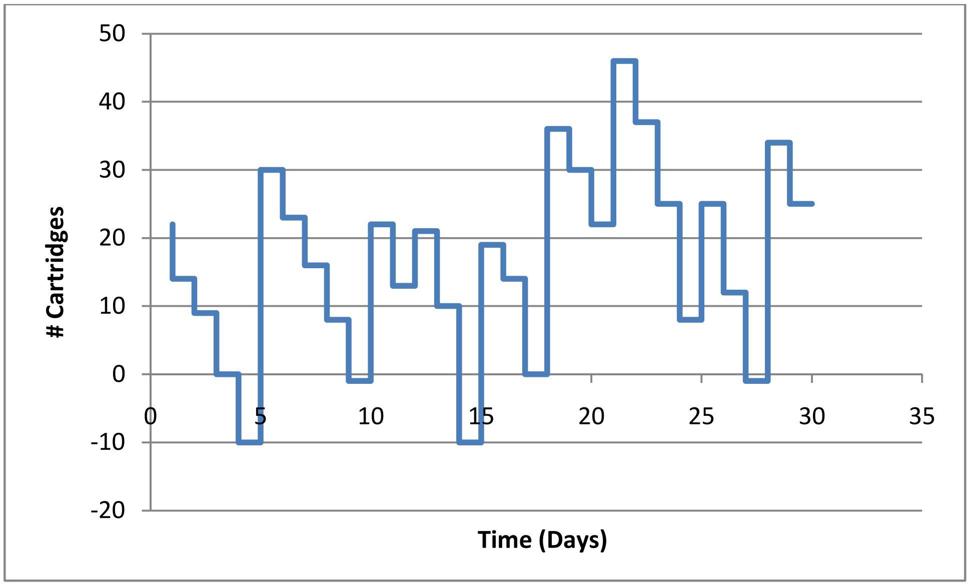

Figure 6.46: Case 4B Static case solution - Inventory on hand

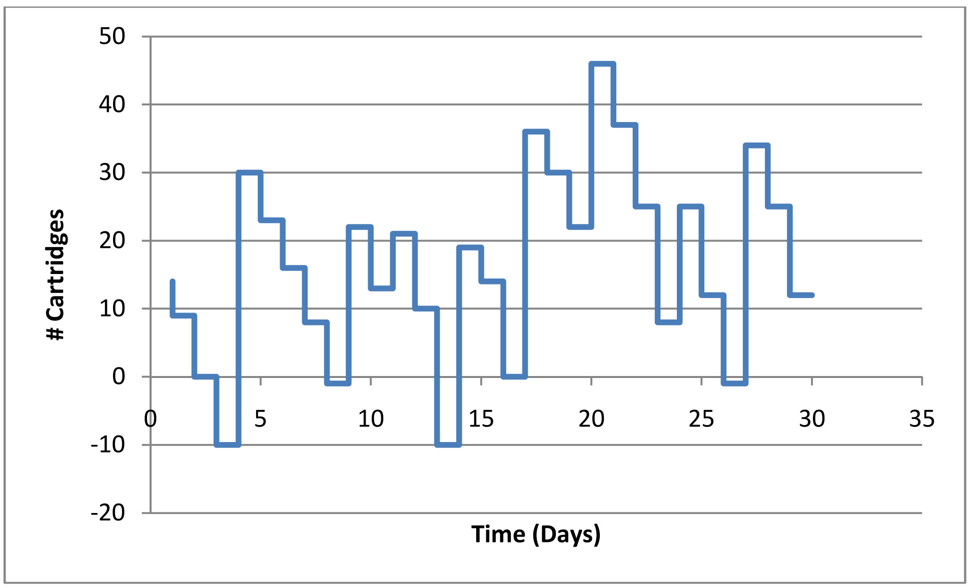

Figure 6.47: Case 4B Static case solution - Inventory position

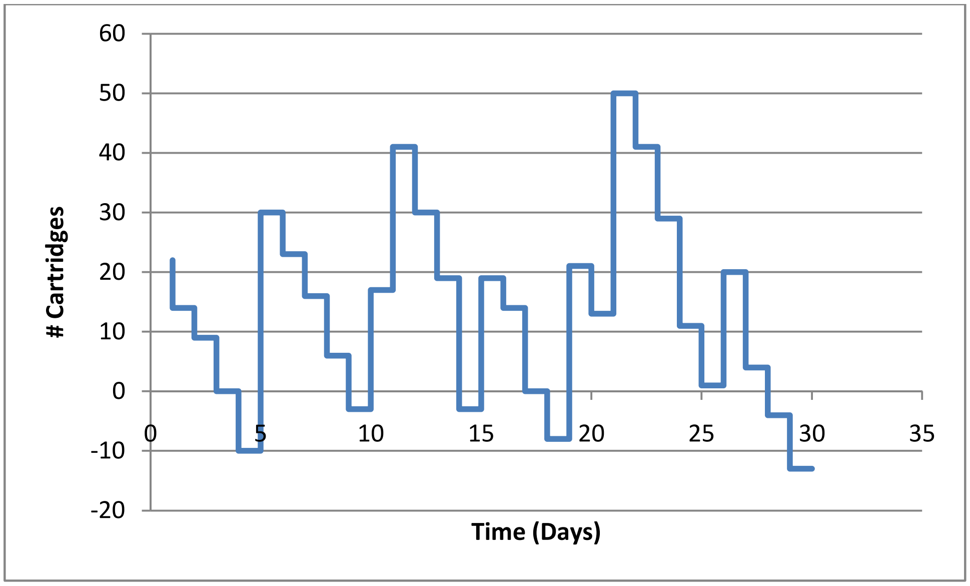

Figure 6.48: Case 4B Dynamic case solution - Inventory on hand

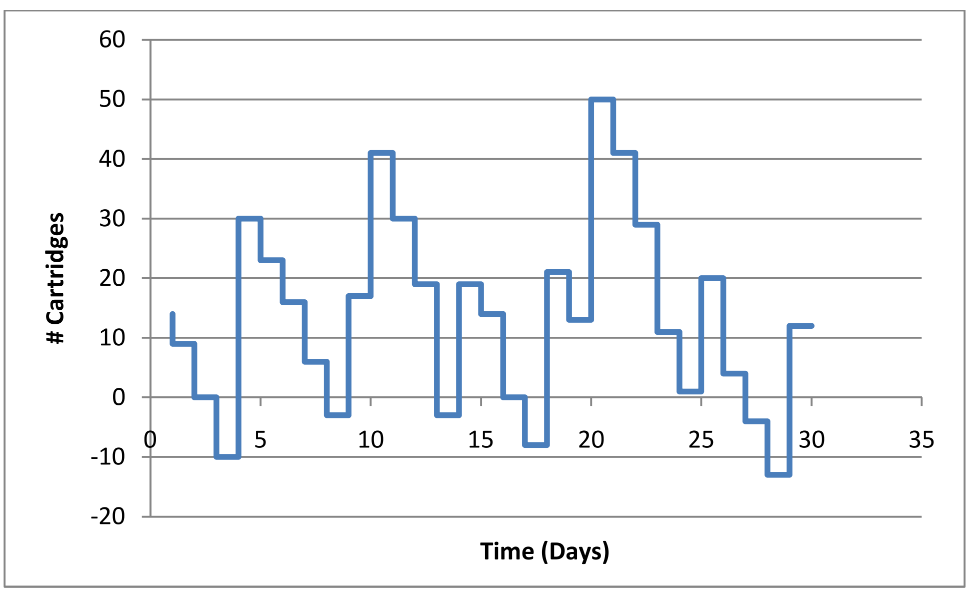

Figure 6.49: Case 4B Dynamic case solution- Inventory position

6.2.5 Case 5: Required service level

In this scenario, the service level is changed to observe the changes in inventory strategy due to higher or lower service level. Two cases are discussed to show the behavior of on hand inventory level with the change in service level.

6.2.5.1 Case 5A: A system with a required service level of 85%

In this case, the required service level for the system is set at 85% and the inventory ordering strategy suggested by the model for a time horizon of 30 days is shown in figures. Table 6.13 shows the suggested ordering strategy for the static and dynamic case. Figure 6.50 shows the inventory on hand plus backorders at the beginning of the day. Figure 6.51 shows the inventory position for the ordering policy. Figure 6.52 and Figure 6.53 show the inventory on hand and inventory position for the 100 printer case when the ordering policy is reviewed every day.

Table 6.13: Case 5A Comparison between static case and dynamic case ordering strategies

	Day	Demand	Quantity Ordered	
			Static	Dynamic
	1	3	12	12
	2	5	0	0
	3	8	12	14
	4	5	0	0
	5	9	22	20
	6	10	0	0
	7	8	22	0
	8	7	0	25
	9	7	20	14
	10	10	0	0
	11	9	24	25
	12	9	0	17
	13	10	27	0
	14	11	0	16
	15	11	31	0
	16	22	0	20
	17	8	18	0
	18	5	0	17
	19	14	23	0
	20	8	0	27
	21	10	18	0
	22	8	0	21
	23	10	24	15
	24	9	0	9
	25	12	22	0
	26	18	0	28
	27	10	23	0
	28	13	0	22
	29	16	0	0
	30	8	0	18
Total expected cost			$1,186	$858

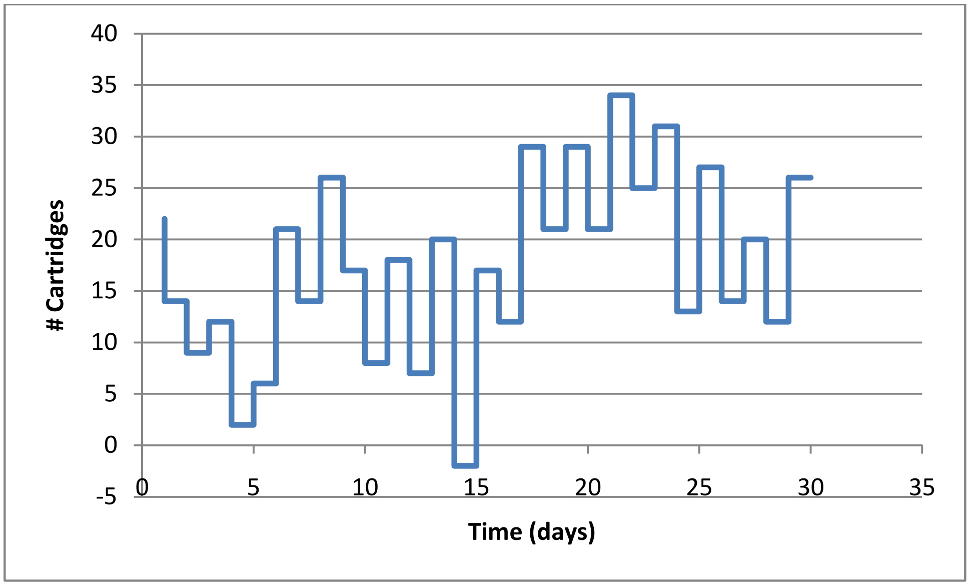

Figure 6.50: Case 5A Static case solution - Inventory on hand

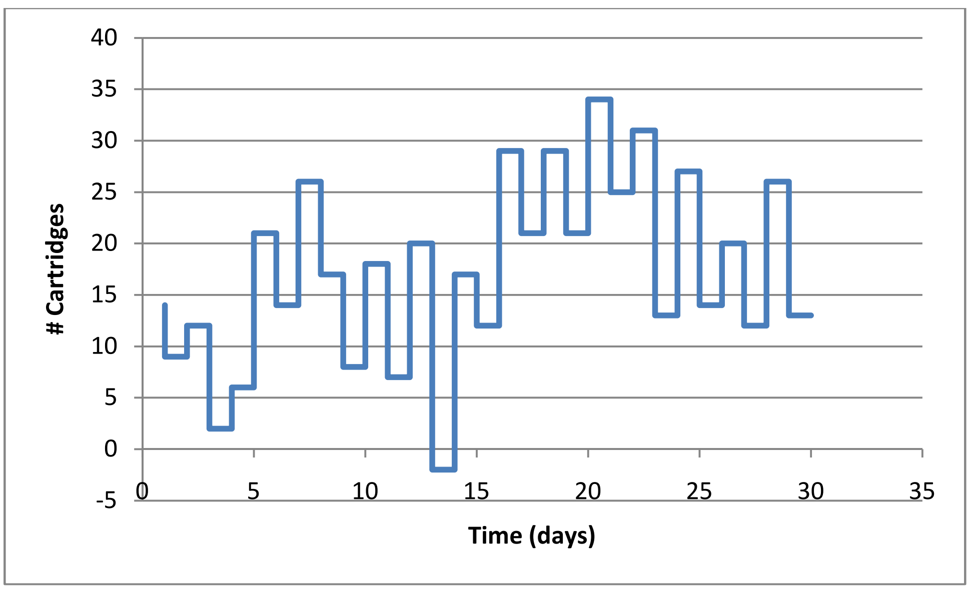

Figure 6.51: Case 5A Static case solution - Inventory position

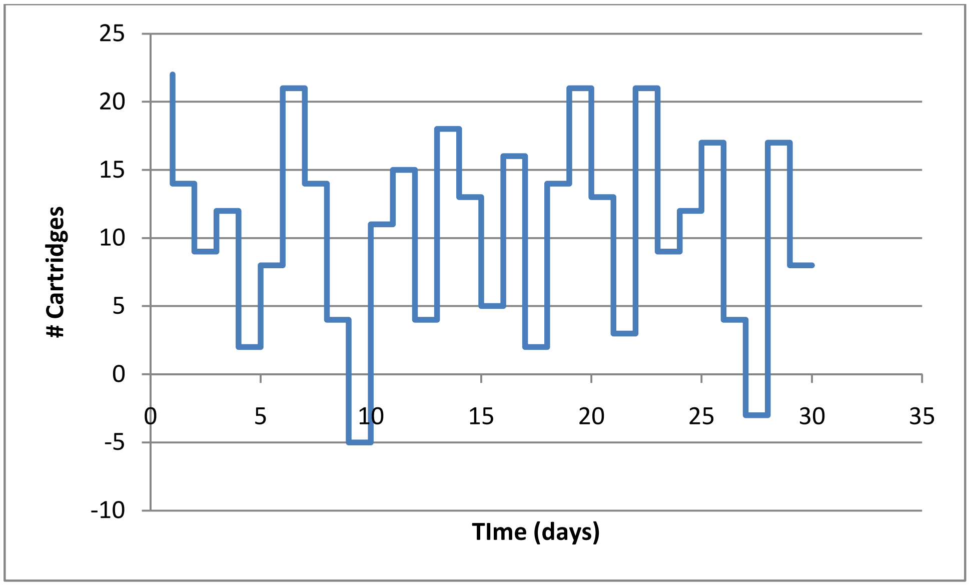

Figure 6.52: Case 5A Dynamic case solution - Inventory on hand

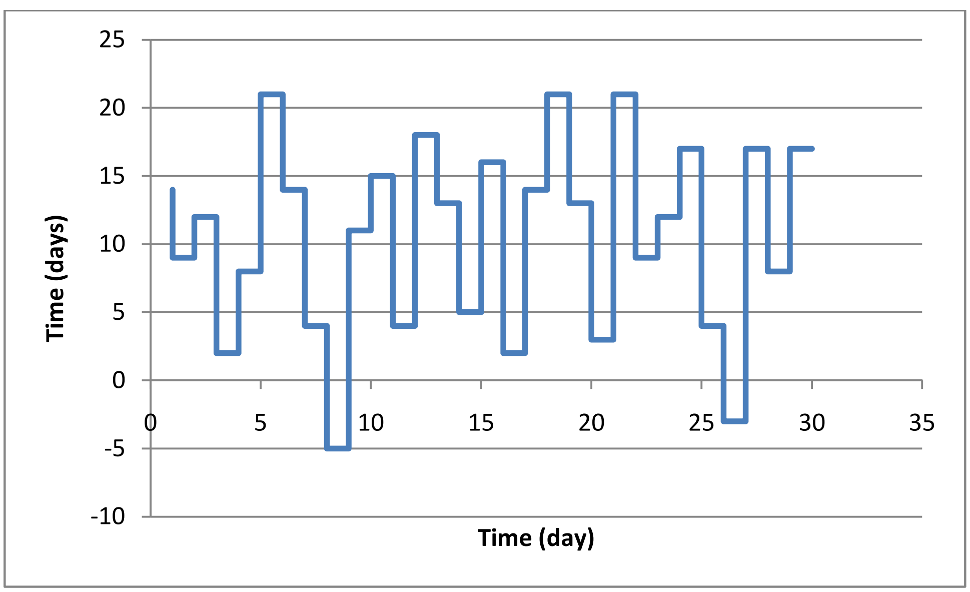

Figure 6.53: Case 5A Dynamic case solution - Inventory position

6.2.5.2 Case 5B: A system with a required service level of 97%

In this case, the required service level for the system is set at 97% and the inventory strategy suggested by the model for a time horizon of 30 days is shown in figures below. Table 6.13 shows the suggested ordering strategy for the static and dynamic case. Figure 6.54 shows the inventory on hand plus backorders at the beginning of the day. Figure 6.55 shows the inventory position for the ordering policy. Figure 6.56 and Figure 6.57 show the inventory on hand and inventory position for the 100 printer case when the ordering policy is reviewed every day.

Table 6.14: Case 5B Comparison between static case and dynamic case ordering strategies

	Day	Demand	Quantity Ordered	
			Static	Dynamic
	1	3	11	11
	2	5	0	0
	3	8	14	15
	4	5	0	0
	5	9	21	20
	6	10	0	0
	7	8	22	19
	8	7	0	0
	9	7	21	21
	10	10	0	0
	11	9	21	23
	12	9	0	18
	13	10	29	0
	14	11	0	17
	15	11	30	0
	16	22	0	20
	17	8	19	0
	18	5	0	16
	19	14	24	0
	20	8	0	25
	21	10	16	0
	22	8	0	23
	23	10	25	1
	24	9	0	25
	25	12	24	0
	26	18	0	25
	27	10	21	0
	28	13	0	23
	29	16	0	0
	30	8	0	18
Total expected cost			$1,198	$883

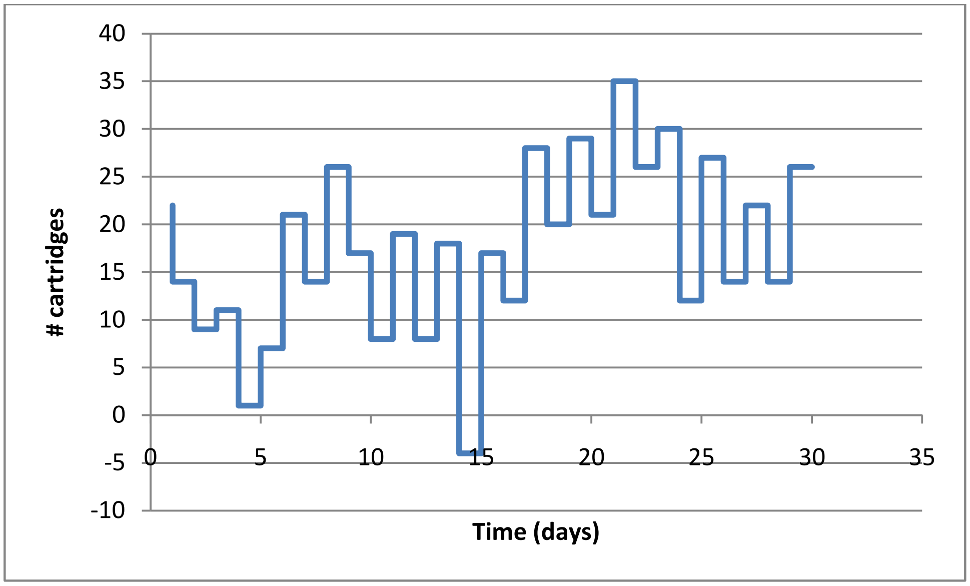

Figure 6.54: Case 5B Static case solution - Inventory on hand

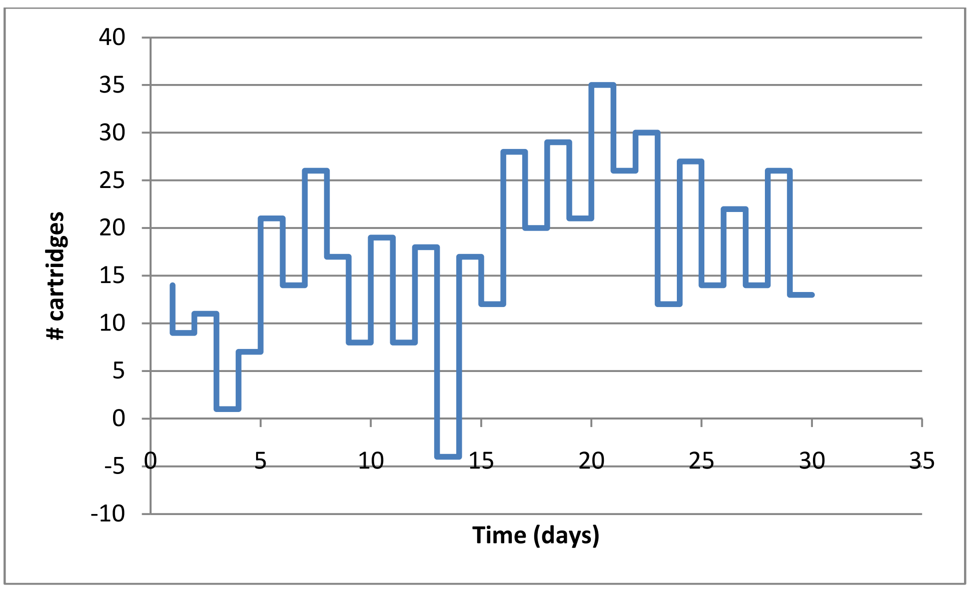

Figure 6.55: Case 5B Static case solution - Inventory position

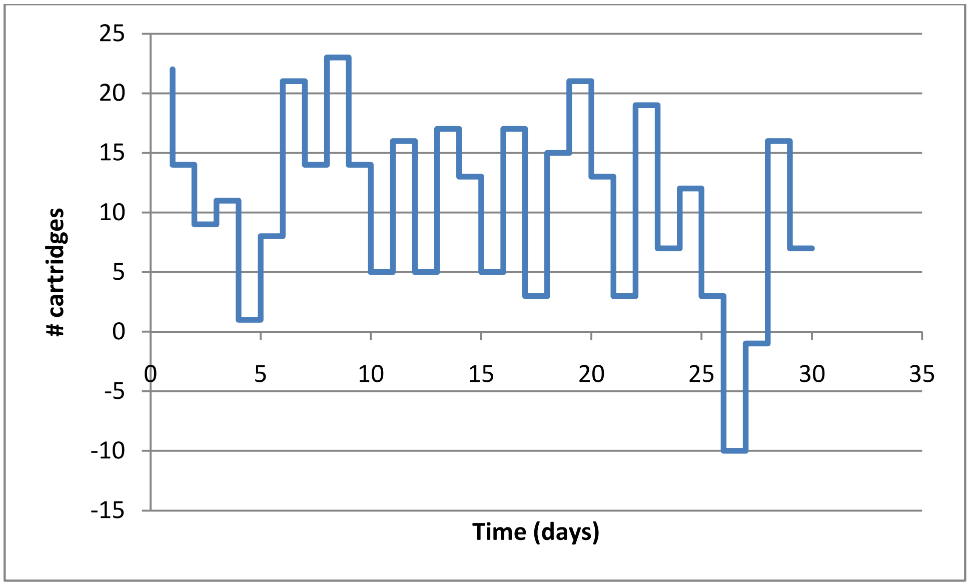

Figure 6.56: Case 5B Dynamic case solution - Inventory on hand

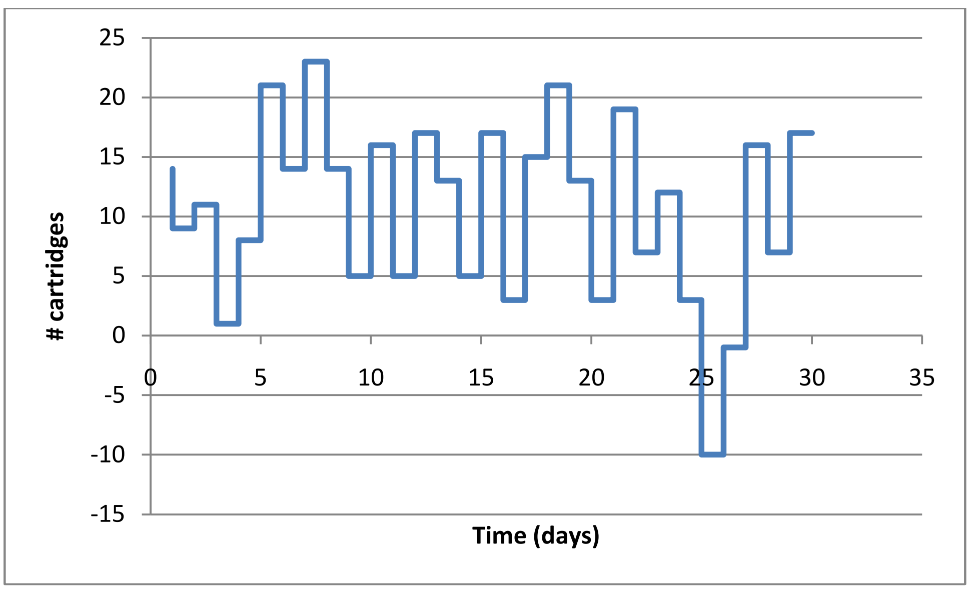

Figure 6.57: Case 5B Dynamic case solution - Inventory position

6.3 Discussion of Results

In sections 6.1 and 6.2, an extensive experimental evaluation of the inventory management system is conducted. The results of these experiments are described in this section. This section gives an overview of the experimental results describing the reasoning behind selecting the parameters for the experiments, the expected and unexpected results for the test parameters and the interpretation of the results obtained.

In section 6.1 a cost comparative study is conducted on the results from the developed algorithm versus those obtained by applying a traditional inventory policy from literature to the forecasted demand data. The comparison is made between the solution obtained using a (Q, R) type inventory policy and the inventory management strategy suggested in the research for the example scenario described in Chapter 6. In the (Q, R) policy the same quantity is ordered each time the inventory falls below a threshold value whereas using the developed algorithm the order quantities and order placement times vary over the time horizon. This is a result of the forecasting methodology based on the consumption algorithm which takes into account the real time status of the toner within a cartridge and not just what is in the inventory at that point of time. This also explains how the ordering strategy can be dynamically adjusted over the time horizon giving an optimal ordering strategy for systems with high variability in the demand forecast. A cost comparison of the policies shows that the total cost of the ordering policy using the (Q, R) type inventory policy is $1243 versus that using the developed inventory management policy is $890. From this comparison, it can be seen that the method suggested in this research can provide a better minimal cost optimal ordering strategies over a time horizon

and thus may be a more beneficial option for toner cartridge inventory systems for a fleet of printers.

In section 6.2 the experimental analysis involves testing the developed inventory management system for different cases with varying parameters. The criteria for testing were selected based on the impact of the parameters on the system. A case by case discussion of the results is shown below:

- Case 1: Size of the printer network –

  In this case the robustness and scalability of the inventory management system is tested for a system having different numbers of printers. The range of printers in the fleet is selected based on the practicality of the system size and a quality that would be a significant test of the performance of the inventory management system. A 100 and 500 printer system are studied. Another reason to consider systems with different numbers of printers a higher number of printers is to test the effect of variability of the demand on the system. Table 6.15 shows the test results for Case 1.

Table 6.15: Comparison of Case 1 test results

Case	# of Printers	Total Expected Cost - Static Case	Total Expected Cost - Dynamic Case
1A	100	$1430	$1055
1B	500	$2238	$1001

  It is observed that there is more variability in the demands and the demand forecasts of the 500 printer case (Case 1B) than the 100 printer case. Here, the periodically reviewed inventory policy shows a more stable pattern when compared to the static ordering strategy as the demand forecast gets smoothened as we

approach the cartridge replenishment times. As expected, in the case of high variability in demand, the static case shows more inventory build up towards the end of the time horizon whereas the dynamically reviewed ordering strategy shows a more balanced inventory and lower cost policy for the same demand.

- Case 2: Changes in system costs-

  System costs are one of the major parameters affecting the ordering strategy as minimum cost is the primary driver for the ordering strategy calculation. The developed algorithm is tested for four scenarios selected from a permutation of cartridge holding cost (low, high), cartridge penalty cost (low, high) and ordering cost (low, high). Table 6.16 shows a comparison of the parameters in the test cases and the number of orders for the dynamically reviewed ordering strategy in each case.

Table 6.16: Comparison of Case 2 test cases

	Case 2A	Case 2B	Case 2C	Case 2D
Holding cost , (*h*)	high	high, h=pen	high	low
Penalty cost, (*pen*)	high	high, h=pen	low	low
Ordering cost, (*K*)	low	low	high	high
# of Orders	16	16	14	7
Average Order Size	20	20	22	45
Total expected cost – Static Case	$1397	$1240	$1604	$1167
Total expected cost – Dynamic Case	$1388	$962	$1475	$1074

In comparison of Case 2C and 2D it is observed that the number of orders in case 2C is twice as many as in case 2D. As expected, the higher the ordering cost compared to the holding cost, the smaller the number of orders and vice-versa. Case 2C has 2 orders less than the Cases 2A and 2B because the penalty cost is lower and ordering cost is higher thereby allowing more shortages and a fewer number of

orders. Therefore, as expected, lower penalty cost allows more shortages. For cases 2A versus 2B, more inventory build-up is seen in Case 2B (between days 15 to 25) as service level constraint comes into play when the unit holding and penalty costs are the same. Varying the unit cost only affects the optimal ordering strategy decision but does not affect all the other components of the inventory management systems such as the print job demand, the toner consumption calculation or the demand forecast.

- Case 3: Variance in lead time –

In this case, the system is tested for the effects of low and high variance in the lead time on the ordering strategy. It is observed that for the case where the lead time variance is low, i.e. $\leq 5\%$ (Case 3A), the number of orders are twice as many as the the case where the variance is high, i.e. $\geq 50\%$ (Case 3B), 14 versus 7 orders. In Case B larger quantities are ordered fewer times since the lead time variance is higher. This result is in accordance with the expected result for a case with high variance in lead time, that is in order to cover for the shortages due to high variability in lead time, large quantities are ordered less frequently. In this case, there is an interaction between the variability in demand and the variability in lead time which affects the system such that not making the right inventory ordering decision would lead to a lot of backorders and shortages leading to a lower customer service level. Table 6.16 shows the cost comparison for the parameters in Case 3.

Table 6.17: Comparison of Case 3 test results

Case	Variance in Lead Time	Total Expected Cost - Static Case	Total Expected Cost - Dynamic Case
3A	$\sigma^2 \leq 5\%$ of μ	$1141	$848
3B	$\sigma^2 \geq 50\%$ of μ	$1575	$1142

- Case 4: Average lead time –

Here the system is tested for varying average length of lead time. Case 4A represents an average lead time of 2 days. In this case, the number of orders placed is 13. Case 4B represents an average lead time of 5 days. In this case, the number of orders placed is 8 which are much lower than Case 4A. As expected, this difference is because we order more to cover the variability in demand during lead time when the lead time is longer. Table 6.18 shows a comparison of the test results for Case 4.

Table 6.18: Comparison of Case 4 test results

Case	Average Lead Time	Total Expected Cost - Static Case	Total Expected Cost - Dynamic Case
4A	2 days	$1271	$1014
4B	5 days	$1282	$1165

- Case 5: Required service level -

The objective of the research is to develop a system to calculate the minimum cost ordering policy at the required service level. This being said, customer required service level plays an important role in deciding the inventory plan. The developed system is analyzed for service levels of 85% and 97% to test the robustness of the system for high service level requirements. Here, the service level is calculated using the formula:

$$ServiceLevel = 1 - \left(\frac{\sum_{i=0}^{(T-1)} PenP_i}{nP * T}\right)$$

Where, $PenP_i$ is the proportion of shortages over the time horizon, nP is the number of printers and T is the time horizon. Table 6.19 shows a comparison of the cumulative actual service levels obtained using this system.

Table 6.19: Cumulative service levels obtained for Case 5A and 5B

	Cumulative Service Level	
Day	Case 5A	Case 5B
1	100%	100%
2	100%	100%
3	100%	100%
4	100%	100%
5	100%	100%
6	100%	100%
7	100%	100%
8	100%	100%
9	99.4%	100%
10	99.5%	100%
11	99.5%	100%
12	99.6%	100%
13	99.6%	100%
14	99.6%	100%
15	99.7%	100%
16	99.7%	100%
17	99.7%	100%
18	99.7%	100%
19	99.7%	100%
20	99.8%	100%
21	99.8%	100%
22	99.8%	100%
23	99.8%	100%
24	99.8%	100%
25	99.8%	100%
26	99.8%	99.6%
27	99.7%	99.6%
28	99.7%	99.6%
29	99.7%	99.6%
30	99.7%	99.6%

From, this table it is observed that the required service level is not only achieved but is exceeded for Case A and Case B. Table 6.20 shows a cost comparison for the test cases.

Table 6.20: Comparison of Case 5 test results

Case	Required Service Level	Total Expected Cost - Static Case	Total Expected Cost - Dynamic Case
5A	85%	$1186	$858
5B	97%	$1198	$883

The algorithm balances the cost with the service level and based on the current scenario, under the parameters for these two cases, the optimal minimum cost is achieved at a service level that is higher than the minimum specified service level.

6.3.1 Summary

In systems where there is a lot of variability in demand and lead time such as the printer fleet toner cartridge inventory system, the suggested ordering strategy solution proves to be more cost and service level effective. Periodically reviewing the ordering policy over the time horizon helps to minimize the cost and service level penalties as the error in the demand forecast reduces as we go closer to the actual demand. Also, it helps prevent placing multiple orders while waiting to receive the previously placed orders when the variability in lead time in large. The dynamic review of the ordering strategy suggested in the research proves to be beneficial tool to aid in determining an optimal ordering policy with minimum cost and required customer service levels to all the purchasing managers and inventory stock keepers of companies with a fleet of printers.

7 Conclusions and Future Work

The objective of the research is to develop a dynamic inventory optimization algorithm applied to printer fleet management. For the stochastic inventory systems with large variability in demand, demand forecast and the lead time, the dynamically reviewed inventory system provides an optimal cost ordering strategy while catering to the required customer service level. This inventory management system is designed to function as a very useful tool to the toner cartridge purchasing managers and the cartridge inventory stock-keepers of companies with a large fleet of printers. The developed inventory management system is interactive and automated to help assist the purchasing managers with the day to day decision making regarding the ordering strategy for toner cartridge inventory systems. The system has the ability to incorporate the variability due to the demand, demand forecast and the lead time as the most current consumption of the cartridge and a dynamic refresh of the forecast is obtained using the toner consumption algorithm and the forecasting algorithm which directly feed this information into the optimization algorithm to calculate the revised ordering strategy based on the most up to date demand forecast and the lead time information. This periodic review of the inventory policy provides important information to the purchasing managers on the basis of when to order and how much to order to achieve the required service level at a minimum cost.

The stated goal of the research is to design and develop a method to minimize the total inventory cost of an inventory system subject to a specified service level over a finite time horizon by determining the optimal times of order placement and order quantities given forecasts for individual demand units under uncertainty associated with demand, the demand forecast, and order lead time. The research work concludes with the

accomplishment of the objectives and construction of the components of the system namely – the toner consumption algorithm, the demand forecasting algorithm and the optimization algorithm. The developed method, specifically for the printer application, takes inputs in terms of current inventory level and demand forecasts based on an extrapolation of the consumption algorithm data to give solutions for the order time and order quantity over a time horizon. To test the developed inventory management system, a simulation model is developed to generate the print jobs dynamically and to replicate the real-world printing scenario. The simulated printer system has tested the robustness of the inventory management system and the system can now be used with a real-world printer scenario. The toner used to print the each print job is calculated automatically using a toner consumption algorithm. This gives real time data about the status of the toner in the cartridge which helps generate an accurate demand forecast using the forecasting algorithm. The demand forecast is calculated by extrapolating the toner consumption data of the cartridge in use from the toner consumption algorithm for all the printers in the time horizon. The demand forecast is calculated dynamically over the time horizon and the error in the forecast is reduced as we go closer to the actual demand. The demand forecast algorithm is robust and can generate forecasts while considering the variability in the demand and the lead time. This forecasted demand along with the current inventory level is used to calculate an optimal minimum cost ordering strategy subject to a specified service level over a time horizon. An experimental performance evaluation is conducted to test the robustness and scalability of the developed inventory system. The experimental results upon review affirmed the effectiveness of the proposed inventory management system. This whole process of executing the system right from data collection, feeding the

toner consumption algorithm and the demand forecasting algorithm, the optimal ordering strategy calculation using the optimization algorithm to feeding it back to the inventory system and generating charts for the experimental analysis is all automated.

The system proves to be very efficient for stochastic inventory systems with variability in demand and lead time and also applies to the following potential supplements to the system:

- Extending the research to be applicable for conditions under continuous time horizons;
- Expanding the research model to a more generalized method that can be used for similar inventory systems having variable demands and lead times- For example in systems where there is variability in demand at two stages, i.e., variability of the unit as a whole and variability of units within the bigger unit, e.g., demand for crates of soda and the demand for the bottles of soda within a single crate. This can be compared to the current printer system where there is variability in demand for toner within one cartridge as well as demand for the cartridges for the printers ; and
- Improving the components of the inventory systems as newer and better models are available in literature.

The mathematical model presented in this research demonstrates the methodology for simplified system configurations using discrete time. The optimization method suggests a minimal cost replenishment strategy over a specified time horizon while considering uncertainty in demand, demand forecasts and lead time subject to a service level specification. The research is coupled with an extensive experimental performance evaluation to test the scope and validity of the model developed. The developed algorithm

serves as a tool to aid in determining an optimal ordering policy with minimum cost and required customer service levels to all the purchasing managers and inventory stock keepers of companies with a fleet of printers.

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Appendix

Appendix A : Sample Output file from Simulation

The spreadsheet in Figure A.1 below shows a sample output file generated from the simulation model. The output from the simulation model is the toner consumption per printer per cartridge and the cartridge replacement times.

	A	B	C	D	E	F
1	Printer NO.	Cumulative Consumption		Day	Actual Consumption	
2	30	1	0.951687715	1.66E-05	0.941813502	
3	48	1	0.155434614	7.17E-05	0.138140552	
4	38	1	1.001758645	8.52E-05	1.003966473	
5	9	1	0.146507458	1.12E-04	0.144023318	
6	20	1	0.726976172	1.63E-04	0.729607518	
7	13	1	2.056108942	1.67E-04	2.030875328	
8	25	1	0.855329101	1.74E-04	0.854317487	
9	19	1	0.170010593	2.39E-04	0.166868754	
10	12	1	0.165722635	2.47E-04	0.169641743	
11	43	1	0.171192644	2.64E-04	0.185188652	
12	15	1	0.91146055	2.76E-04	0.904213344	
13	24	1	1.199156323	2.80E-04	1.196516155	
14	2	1	0.236197228	2.82E-04	0.248250198	
15	42	1	0.845596813	2.86E-04	0.827758433	
16	23	1	2.332580534	3.01E-04	2.322336647	
17	49	1	2.299019201	3.07E-04	2.296280506	
18	34	1	2.175530804	3.22E-04	2.181864993	
19	31	1	0.75838516	3.24E-04	0.765696814	
20	14	1	0.134103499	3.25E-04	0.127344892	
21	18	1	0.964968642	3.36E-04	0.944804209	
22	6	1	0.708563065	3.46E-04	0.69172553	

Figure A.1: Sample output file generated by the simulation model

Appendix B: MATLAB Code for the Demand Forecasting Method

Listed below is the MATLAB code for the demand forecasting method described in Chapter 5.2.

```
clear;
clc;

sd= input('Enter current day to start execution');
T= input('Enter length of time horizon:');
ffile=xlsread('C:\Documents and
Settings\sss9625\Desktop\resmodelout.xls','A:E');
pq=max(ffile,[],1);
np=pq(1);
len=length(ffile);
for(printerno=1:1:np)
    ij=1;
for(i=1:1:len)
    index=ffile(i,1);
    if(index==printerno)
        j=2*index;
        k=j-1;
        X(ij,k:j)=ffile(i,2:3);
        Y(ij,index)=ffile(i,4);
        A(ij,index)=ffile(i,5);
        ij=ij+1;
    end;
end;
end;

for(b=sd:1:(T-1))
kk=1;
for(k=1:1:np)
pddli=0;
pddci=0;
MaxToner=75;
acc=0.9;
n=length(Y(:,k));
SSmodel(k)=0;
SStotal(k)=0;
c=1;
findex=1;
for(i=1:1:n)
    if(Y(i,k)>=b)
        cday(k)=i;
        break;
    end;
end;
fpv=0;
sv=1;

    for(a=c:1:cday(k))

        if (sum(A(c:a,k))>75)  %75 is only for sample example execution. In
the real model it will become MaxToner
            c=a;
```

```
            fp=floor(Y(c,k));
            sv=sv+1;
            if(fpv==fp)
            xx= PDD(fp,k);
            PDD(fp,k)=xx+1;
            else PDD(fp,k)=1;
                fpv=fp;
            end;
            findex=a;
        end;
    end;
if(findex==cday(k))
    loc=findex;
    Yn(k)=MaxToner.*Y(loc,k)./X(loc,kk+1);
    PDD(ceil(Y(loc,k)),k)=1;
    f=floor(T./Yn(k));
else loc=findex:cday(k);

inveX = inv((X(loc,kk:kk+1).')*X(loc,kk:kk+1));
XtY=X(loc,kk:kk+1).'*Y(loc,k);
P(:,k)=inveX*XtY;
PX=X(loc,kk:kk+1)*P(:,k);
error=Y(loc,k)-PX;
errorsqr=error.^2;
sumY= sum(Y(loc,k));
nsub=length(Y(loc,k));
MSE=sum(errorsqr)./(nsub);
Xn=[1; MaxToner];
cal = (Xn.')*(inveX)*(Xn);
Ssqr=MSE*((1/nsub) + cal);
S= sqrt(Ssqr);
%Goodness of fit
Ybar= sumY./nsub;
Yfit=PX;
for(i=1:1:nsub)
    SSmodel(k)=SSmodel(k)+((Yfit(i)-Ybar).^2);
end;
for(j=findex:1:cday(k))
    SStotal(k)=SStotal(k)+((Y(j,k)-Ybar).^2);
end;
Rsqr(k)=SSmodel(k)./SStotal(k);
% confidence interval calculation
Yn(k)=sum(P(:,k).*Xn);
tvalue(k)= tinv(acc,nsub);
llimit(k)=Yn(k)-tvalue(k)*S;
ulimit(k)=Yn(k)+tvalue(k)*S;
fllimit(k)=floor(llimit(k));
culimit(k)=ceil(ulimit(k));
f=floor(T./Yn(k));

PDD(fllimit(k),k)=0;

for(x=fllimit(k):1:culimit(k)-1)
    PDD(x+1,k)=normcdf(x+1,Yn(k),S)-normcdf(x,Yn(k),S);
    xpl=PDD(x+1,k);
    pddli=x+1;
```

```
end;
    PDD(culimit(k),k)=1-normcdf(culimit(k)-1,Yn(k),S);
    xpc=PDD(culimit(k),k);
    pddci=culimit(k);
end;
q=1;
while(Yn(k)<T)

        Xn=[1; (q+1)*MaxToner];
        cal = (Xn.')*(inveX)*(Xn);
        Ssqr=MSE*((1/nsub) + cal);
        S= sqrt(Ssqr);
        Yn(k)=sum(P(:,k).*Xn);
        tvalue(k)= tinv(acc,nsub);
        llimit(k)=Yn(k)-tvalue(k)*S;
        ulimit(k)=Yn(k)+tvalue(k)*S;
        fllimit(k)=floor(llimit(k));
        culimit(k)=ceil(ulimit(k));

         for(x=fllimit(k):1:culimit(k)-1)
            if(pddli==x+1)
                PDD(x+1,k)=xpl + normcdf(x+1,Yn(k),S)-normcdf(x,Yn(k),S);
            else
            PDD(x+1,k)= normcdf(x+1,Yn(k),S)-normcdf(x,Yn(k),S);
            xpl=PDD(x+1,k);
            pddli=x+1;
        end;
        end;
        if(pddci==culimit(k))
            PDD(culimit(k),k)=xpc+1-normcdf(culimit(k)-1,Yn(k),S);
        else
            PDD(culimit(k),k)=1-normcdf(culimit(k)-1,Yn(k),S);
            xpc=PDD(culimit(k),k);
            pddci=culimit(k);
        end;
        q=q+1;
end;
kk=kk+2;
end;
for(k=1:1:np)
    for(i=1:1:length(PDD(:,k)))
        PD(i,b+1-sd)=sum(PDD(i,:));
        ld=i;
    end;
end;
for(k=1:1:np)
    PDD(:,k)=0;
end;
for(i=ld:1:T)
    PD(i,b+1-sd)=0;
end;
end;
xlswrite('C:\Documents and Settings\sss9625\Desktop\DataOut.xls',PD);
```

Appendix C: Sample Output file from the Demand Forecasting Method

The Figure C.1 below shows a sample output from the demand forecasting algorithm. The row number indicates the day and the columns represent the successive day. The cells contain the toner cartridge demand and the demand is read in vertical (column-wise) order. The portion highlighted in yellow is the historical actual demand that occurred prior to the current day under consideration. The portion below the highlighted one shows the current demand forecast over the time horizon.

	Forecast on Day #									
Day	4	5	6	7	8	9	10	11	12	13
1	0	0	0	0	0	0	0	0	0	0
2	3	3	3	3	3	3	3	3	3	3
3	5	5	5	5	5	5	5	5	5	5
4	8.57	8	8	8	8	8	8	8	8	8
5	7.94	6.73	5	5	5	5	5	5	5	5
6	8.50	9.23	12.06	9	9	9	9	9	9	9
7	8.13	8.53	7.00	12.24	10	10	10	10	10	10
8	7.89	3.22	7.28	8.27	6.85	8	8	8	8	8
9	6.05	13.55	8.69	7.45	11.01	6.63	7	7	7	7
10	11.23	6.32	10.25	8.88	7.07	11.09	8.84	7	7	7
11	10.54	10.01	8.69	11.49	10.03	10.37	9.13	8.91	10	10
12	9.20	9.25	8.09	5.32	5.89	9.19	10.48	9.89	11.62	9
13	11.72	9.04	9.73	15.48	11.07	11.78	10.73	11.96	11.48	9.74
14	10.57	8.75	8.03	9.39	11.68	9.87	9.36	7.80	10.53	13.35
15	10.81	10.29	14.45	11.16	8.92	11.78	10.17	12.73	9.24	9.78

Figure C.1: Sample output from the demand forecasting model

Appendix D: Sample Output for No-Variability Case

The Figure D.1 below shows a sample output from the demand forecasting algorithm for a no variability case i.e., the demand and lead time are assumed to be constant. The row number indicates the day and the columns represent the successive day. The cells contain the toner cartridge demand and the demand is read in vertical (column-wise) order. The portion highlighted in yellow is the historical actual demand that occurred prior to the current day under consideration. The portion below the highlighted one shows the current demand forecast over the time horizon. This case is to show the model works for a no-variability case and hence proving that it works for a case with variability.

	Forecast on Day #									
Day	2	3	4	5	6	7	8	9	10	11
1	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0
4	13	13	13	13	13	13	13	13	13	13
5	6	6	6	6	6	6	6	6	6	6
6	6	6	6	6	6	6	6	6	6	6
7	9	9	9	9	9	9	9	9	9	9
8	8	8	8	8	8	8	8	8	8	8
9	7	7	7	7	7	7	7	7	7	7
10	9	9	9	9	9	9	9	9	9	9
11	6	6	6	6	6	6	6	6	6	6
12	10	10	10	10	10	10	10	10	10	10
13	6	6	6	6	6	6	6	6	6	6
14	9	9	9	9	9	9	9	9	9	9
15	9	9	9	9	9	9	9	9	9	9

Figure D.1: Sample output from demand forecasting model for no-variability case

Appendix E : ILOG Code for the Optimization Method

Listed below is the program for the optimization algorithm described in Chapter 5.3 coded using ILOG Cplex software. The input file to the model is listed under the "Data" section and the algorithm is listed under the "Model" section.

Model File :

```
/*********************************************
 * OPL 6.3 Model
 * Author: SURYA SHEETAL SARIPALLI (RIT Student)
 * Creation Date: 11/23/2010 at 4:19 PM
 *********************************************/

int T=...;
range t=0..(T-1);
range tt=1..(T-1);
range st=0..(T-2);
range r =0..T;
float I0=...;
int M=...;
int al=...; // average lead time
range alr=0..(al-1);
int nP=...;
int no=...;  // number of orders placed in the previous period
range order=1..no;
int ndpo[order]=...; // number of days prior to the current day that the
previous order was placed

float k=...;
float h=...;
float pen=...;

float d[t]=...;
float P[t]=...;
float gamma=...;
int XP[order]=...;

dvar int N;
dvar int+ X[t];
dvar float+ R[t][t];
dvar float+ ER[t];
dvar int+ Y[t] in 0..1;
dvar int+ z[t] in 0..1;
dvar int+ zz[t] in 0..1;
dvar float I[r];
dvar float+ HP[t];
dvar float+ HM[t];
dvar float+ PenP[t];
dvar float+ PenM[t];
dvar float hold[t];
dvar float penalty[t];
```

```
dvar float+ ServiceLevel;
dvar float+ SLD[t];
dvar float+ Cost;

dvar float+ PP[order][t];

dvar float+ delta in 0..1;

minimize sum(i in t)(k*Y[i]+h*HP[i]+pen*PenP[i])+ 1000000*delta ;

subject to
{
    I[0]==I0;
    forall(o in 1..no,jj in al..(T-1))PP[o][jj]==0;
    forall(o in 1..no, j in alr:ndpo[o]==1) PP[o][j]==P[j+1];
    forall(j in 0..(al-1),o in 1..no:ndpo[o]==2)
PP[o][j]==P[j+2]/(P[2]+P[3]);
    forall(o in 1..no:ndpo[o]==3)
      {
      PP[o][0]==1;
      PP[o][1]==0;
      PP[o][2]==0;
    };

    forall(j in t:j>=0) R[0][j]== sum(o in order)(XP[o]*PP[o][j]) +
X[0]*P[j];
    forall(i in tt, j in t:j>=i) R[i][j] == X[i]*P[j-i];
    forall(i in t, j in t:j<i) R[i][j] == 0;

    forall(i in 1..al)X[T-al+i-1]==0;

    forall(j in t) ER[j]==sum(i in t)R[i][j];

    forall(j in t) I[j]+ER[j]-d[j]==I[j+1];

      forall(i in t) hold[i]==I[i+1]-ER[i];
      forall(i in t) hold[i]==HP[i]-HM[i];
      forall(i in t) penalty[i]==-I[i+1];
      forall(i in t) penalty[i]==PenP[i]-PenM[i];

    forall(i in t)
    {
        HP[i] <= M * (1-z[i]);
        HM[i] <= M * z[i];
    };
    forall(i in t)
    {
        PenP[i] <= M * (1-zz[i]);
        PenM[i] <= M * zz[i];
    };

    forall(i in t)
    {
        X[i]<=M*Y[i];
        X[i]>=Y[i];
    };
    N==sum(i in t)Y[i];
```

```
    forall(j in t)
    {
      sum(i in 0..j)(PenP[i])<= (1-gamma+delta)*nP*(j+1);
      SLD[j]== 1-(sum(i in 0..j)(PenP[i])/(nP*(j+1)));
    };
    ServiceLevel == 1-((sum(i in t)PenP[i])/(nP*T));
    Cost == sum(i in t)(k*Y[i]+h*HP[i]+pen*PenP[i]);

};

};
```

Data File :

```
/*********************************************
 * OPL 6.3 Data
 * Author: sss9625
 * Creation Date: Nov 23, 2010 at 12:14:17 PM
 *********************************************/

SheetConnection sheetS("configurationfile.xlsm");
T from SheetRead(sheetS, "Sheet1!A2");
I0 from SheetRead(sheetS, "Sheet1!B2");
no from SheetRead(sheetS, "Sheet1!C2");
ndpo from SheetRead(sheetS, "Sheet1!D2:D4");
XP from SheetRead(sheetS, "Sheet1!E2:E4");
d from SheetRead(sheetS, "Sheet1!G2:G31");
M=10000;
al=3;
nP=100;
k=10;
h=2;
pen=5;
P=[0 0.05 0.90 0.05 0];
gamma=0.90;
SheetConnection sheetData("configurationfile.xlsm");
d to SheetWrite(sheetData, "PolicyOut!A2:A31");
X to SheetWrite(sheetData, "PolicyOut!B2:B31");
ER to SheetWrite(sheetData, "PolicyOut!C2:C31");
I to SheetWrite(sheetData, "PolicyOut!D2:D32");
HP to SheetWrite(sheetData, "PolicyOut!E2:E31");
PenP to SheetWrite(sheetData, "PolicyOut!F2:F31");
```

Appendix F: Sample Output from the Optimization Model

Listed below is a sample output generated from the optimization model using ILOG Cplex.

The description of the model file is given is Appendix G.

```
// solution (integer optimal, tolerance)
Y = [0 0 0 0 0 0 0 0 1 0 0 0 1 0 0
         1 0 0 1 0 0 0 1 0 0 0 1 0 0 1 0 0 1 0 0 1 0 0 0 0 0];
IP = [1 1 1 1 1 1 1 1 1 0.6869 0 0.16798 1.0059 0.57021 0.63262 1.3425 0 0
1.3147
         0.80793 0 1.5366 1.858 0 0 0.86322 1.316 0 0 0.96484 0.5865 0 1.8811
         1.6721 0 0.80422 0.93729 0.080841 1.5437 1.6776 0.079437];
IM = [0 0 0 0 0 0 0 0 0 0 0.012781 0 0 0 0 0 0.093 0.50844 0 0 0.28672 0 0
0.038716
         0.50157 0 0 0.11869 0.41443 0 0 0.28861 0 0 0.14598 0 0 0 0 0 0];
I = [1 1 1 1 1 1 1 1 1 0.6869 -0.012781 0.16798 1.0059 0.57021 0.63262 1.3425
         -0.093 -0.50844 1.3147 0.80793 -0.28672 1.5366 1.858 -0.038716 -
0.50157
         0.86322 1.316 -0.11869 -0.41443 0.96484 0.5865 -0.28861 1.8811
1.6721
         -0.14598 0.80422 0.93729 0.080841 1.5437 1.6776 0.079437];
R = [[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
             0 0 0 0 0]
             [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
             0 0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
             0 0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
             0 0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
             0 0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
             0 0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
             0 0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
             0 0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0 0 0.8 2 1.2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
             0 0 0 0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
             0 0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
             0 0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
             0 0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0 0 0 0 0 0 2.2 5.5 3.3 0 0 0 0 0 0 0 0 0 0 0 0
             0 0 0 0 0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
             0 0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
             0 0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1.8 4.5 2.7 0 0 0 0 0 0 0 0 0
             0 0 0 0 0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
```

```
          0 0 0 0 0 0 0 0 0]
          [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
          0 0 0 0 0 0 0 0 0]
          [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1.6 4 2.4 0 0 0 0 0 0 0
          0 0 0 0 0 0 0 0 0 0 0]
          [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
          0 0 0 0 0 0 0 0 0]
          [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
          0 0 0 0 0 0 0 0 0]
          [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
          0 0 0 0 0 0 0 0 0]
          [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1.2 3 1.8 0 0 0
          0 0 0 0 0 0 0 0 0 0 0]
          [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
          0 0 0 0 0 0 0 0 0]
          [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
          0 0 0 0 0 0 0 0 0]
          [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
          0 0 0 0 0 0 0 0 0]
          [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1.4 3.5
          2.1 0 0 0 0 0 0 0 0 0 0]
          [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
          0 0 0 0 0 0 0 0 0]
          [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
          0 0 0 0 0 0 0 0 0]
          [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 2.4
          6 3.6 0 0 0 0 0 0 0]
          [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
          0 0 0 0 0 0 0 0 0]
          [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
          0 0 0 0 0 0 0 0 0]
          [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
          0 0 1.6 4 2.4 0 0 0 0]
          [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
          0 0 0 0 0 0 0 0 0]
          [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
          0 0 0 0 0 0 0 0 0]
          [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
          0 0 0 0 0 1.4 3.5 2.1 0]
          [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
          0 0 0 0 0 0 0 0 0]
          [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
          0 0 0 0 0 0 0 0 0]
          [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
          0 0 0 0 0 0 0 0 0]
          [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
          0 0 0 0 0 0 0 0 0]];
X = [0 0 0 0 0 0 0 0 4 0 0 0 11 0 0 9 0 0 8 0 0 0 6 0 0 0 7 0 0 12 0 0 8 0 0
         7 0 0 0 0];
ER = [0 0 0 0 0 0 0 0 0 0.8 2 1.2 0 2.2 5.5 3.3 1.8 4.5 2.7 1.6 4 2.4 0 1.2
         3 1.8 0 1.4 3.5 2.1 2.4 6 3.6 1.6 4 2.4 1.4 3.5 2.1 0];
z = [0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 1 1 0 0 1 0 0 1 1 0 0 1 1 0 0 1 0 0 1 0
         0 0 0 0 0];
N = 9;
ServiceLevel = 0.9988;
```

Appendix G: MACRO to automate the excel spreadsheets

Purpose: To automate the data collection process in excel spreadsheets from the demand file, optimization model and the cost calculations.

Inputs to the macro: Length of the time horizon, current day, forecasted demands generated by the simulation model, and the suggested ordering strategy from the optimization model for the time horizon.

Outputs from the macro: Inventory on hand at the beginning of the current day, Number and quantity of previously placed and not yet received orders, and the demand forecast for the time horizon.

```
Option Explicit

Sub MasterMacro()

Call DemandCopy
Call PrevOrderCopy
Call InvOnHand

End Sub
```

```
Option Explicit

Sub DemandCopy()

    Dim RowIndex As Integer
    Dim ColumnIndex As Integer
    Dim RowTargetIndex As Integer
    Dim ColumnTargetIndex As Integer
    Dim CurrentDay As Integer
    Dim TimeHorizon As Integer
    Dim Counter As Integer
    Dim TempData As Double
    Sheets("Sheet1").Select
    CurrentDay = Cells(2, 6)
    TimeHorizon = Cells(2, 1)
    RowIndex = CurrentDay
    ColumnIndex = CurrentDay - 1
    RowTargetIndex = 2
    ColumnTargetIndex = 7
    Counter = 1
```

```
    While (Counter <= TimeHorizon)
        Sheets("DataOut").Select
        TempData = Cells(RowIndex, ColumnIndex)
        Sheets("Sheet1").Select
        Cells(RowTargetIndex, ColumnTargetIndex) = TempData
        RowIndex = RowIndex + 1
        RowTargetIndex = RowTargetIndex + 1
        Counter = Counter + 1
    Wend

End Sub

Sub PrevOrderCopy()

    Dim CurrentDay As Integer
    Dim TempData As Integer
    Dim i As Integer
    Sheets("Sheet1").Select
    CurrentDay = Cells(2, 6)
    For i = 1 To 3
        Sheets("CalcSheet").Select
        TempData = Cells(CurrentDay + 1, 8 + i)
        Sheets("Sheet1").Select
        Cells(i + 1, 5) = TempData
    Next i
End Sub
```

```
Option Explicit

Sub InvOnHand()

    Dim CurrentDay As Integer
    Dim TempData As Integer
    Sheets("Sheet1").Select
    CurrentDay = Cells(2, 6)
    Sheets("CalcSheet").Select
    TempData = Cells(CurrentDay + 1, 12)
    Sheets("Sheet1").Select
    Cells(2, 2) = TempData

End Sub
```

```
Option Explicit

Sub StepChart()
'
' StepChart Macro
'
' Keyboard Shortcut: Ctrl+a
'
    Dim ColumnIndex As Integer
    Dim RowIndex As Integer
    ColumnIndex = 2

    For RowIndex = 2 To 65
        If (Cells(RowIndex, ColumnIndex) <> Cells(RowIndex + 1, ColumnIndex)) Then
        Range(Cells(RowIndex + 1, ColumnIndex - 1), Cells(RowIndex + 1, ColumnIndex)).Select
        Application.CutCopyMode = False
        Selection.Insert Shift:=xlDown, CopyOrigin:=xlFormatFromLeftOrAbove
        Cells(RowIndex + 1, ColumnIndex - 1) = Cells(RowIndex, ColumnIndex - 1)
        Cells(RowIndex + 1, ColumnIndex) = Cells(RowIndex + 2, ColumnIndex)
        End If
    Next RowIndex

End Sub
```

Appendix H: Description of Calculations in “CalcSheet” tab in “configurationfile”

Table H.1 below describes the notations used in the Calculation sheet of the macro described in Appendix G. This “CalcSheet” tab in “configurationfile” spreadsheet is used to populate the ordering data automatically.

Table H.1: Notations used in the “CalcSheet” tab in “configurationfile”

Notation	Description
P_j	Probability of receiving an order “*j*” days after placing an order
L	Average lead time
(Actual Received)$_i$	$\sum_i (X_{L-i+1} * P_{L-i+1}) \ \forall \mathrm{i} \ \in (1, \mathrm{L})$
AR_j	Quantity of order placed “*j*” days from current day and not received yet = $[X_{day-j} * P_{j,day-j}]$
IOH_i *at the beginning of the day*	IOH_{i-1} + Qty. $Received_{i-1}$ –Actual $Demand_{i-1}$

Appendix I: User Guide for Developed System

CONTENTS:

H.I Information about the User Guide

This user manual is a step by step approach to using the developed algorithm. The manual describes the methodology to open/ review the developed models, change inputs and calculate outputs for a given inventory system using the developed algorithm.

H.II How to navigate through the User Guide

The procedure is to follow all the steps from 1 to 15. The guide contains the experimentation procedures for both the static case and dynamic case ordering strategy calculations. For the static case, skip steps 10, 11 and for the dynamic case/ period review, skip step #9.

H.III A step by step guide to use the developed algorithm

1. Run the simulation model to generate cartridge toner consumption data for the printer fleet. The output from this model is written into an Excel spreadsheet named "Resmodelout.xls".

2. This above spreadsheet is sent as input to the demand forecasting model developed using MATLAB software. The output from this model is exported to another excel spreadsheet named "DataOut.xls".

3. This output data from the forecasting algorithm is copied into the "DataOut" tab in another spreadsheet called "configurationfile". This file contains a macro listed in Appendix G.

4. To begin experimentation, in the “configurationfile” spreadsheet goto “Sheet1” tab. Set the current day to 1. Set all the previous order quantities to 0. Run the macro “demandInvCopy” (shortcut: ctrl+d). This macro copies the forecasted demands for the given time horizon into the Sheet1 tab. Save and close the spreadsheet.

5. Goto “CalcSheet” tab in “configurationfile” spreadsheet and change the lead time/ order arrival probabilities based on the assumptions for the test case.

6. This “Sheet1” tab in the “configurationfile” is read as input to the optimization model developed using ILOG software. The output from the optimization model namely, the order placement quantities and times, is written into “configurationfile” “PolicyOut” tab.

7. Once, the optimization model finishes execution open the “configurationfile” spreadsheet’s “Sheet 1” tab.

8. In this sheet increment the current day by 1 and run the macro named “MasterMacro” (shortcut: ctrl+m) as listed in Appendix G. Save and close the file.

9. Copy the suggested ordering strategy over the time horizon from Column B in “PolicyOut” Sheet to the column C in “CalcSheet” tab.

10. Run the optimization model for the new input data.

11. Repeat steps 6 – 8, 10 for the entire time horizon. Skip step# 9 for periodic review of the algorithm.

12. Open “configurationfile” spreadsheet and goto “CalcSheet” tab. This spreadsheet shows the inventory on hand, inventory position and, cost calculations.

13. Copy the days and inventory data for the given time horizon into a new tab.

14. Run the macro named “StepChart” (shortcut: ctrl+a) listed in Appendix G. This macro is used to create a step chart to illustrate the calculated inventory levels over time for the suggested ordering strategy.

15. Compare the solutions from the suggested inventory optimization algorithm with that of the (Q, R) policy.

Appendix J: DVD contents

The DVD contains the programs for executing the simulation model in ARENA, demand forecasting algorithm in MATLAB, the optimization model in ILOG Cplex and the results of all the experiment listed in Chapter 6. There are four folders namely – Simulation Model, Demand Forecasting Model, Optimization Model, and Experiments Spreadsheets. The Simulation model folder contains the following files:

- "newmodelJun23withvariability_100 printers.doe" – ARENA simulation model;
- "resmodeloutwrite.xls" – Excel spreadsheet which contains the output from the simulation model, i.e., cartridge consumption data and the cartridge replacement times for printers;
- "rejectdata.xls" – Contains the rejected print jobs data as generated from the simulation model; and
- Input Analyzer folder – Contains the files generated using the Input Analyzer program in ARENA to validate the input print job inter-arrival distributions to the model.

The Demand Forecasting Model folder contains the following files:

- "code25Jun10withvariability.m" – The MATLAB code for calculating the demand forecast for the case with variability in demand and lead time;
- "code4oct10novariability.m" – MATLAB code for the no-variability case where the demand and lead time are constant;

- "DataOut.xls" – Excel spreadsheet contains the output from the demand forecasting model.

The Optimization Model folder contains the following files:

- "Model23Nov.mod" – The ILOG Cplex code for calculating the optimal ordering strategy;
- "configurationfile.xlsm" – Contains the inputs to the optimization model, output from the optimization algorithm and the macro to automate the data collection process.

The Experiments Spreadsheets folder contains the experimental results for all the test cases explained in Chapter 6.