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Analysis of supersaturated designs

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by

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ABSTRACT

In today's fiercely competitive marketplace, successful and profitable companies distinguish themselves by bringing new products to market before their competitors. The cycle time to develop and launch new products largely depends on a company's ability to study large numbers of factors and to separate, or detect, the significant factors from the insignificant factors. Most ordinary experimental situations with many variables are easily satisfied with the use of a saturated, or nearly-saturated, fractional factorial experimental designs. However, there are occasions where the cost of running a statistically designed experiment can be so great as to prohibit the use of these techniques, forcing the experimenter to resort to other, riskier, experimental techniques.

Theory suggests that a relatively new class of designs, systematic supersaturated designs, may prove to be even more effective at identifying significant factors than saturated, or nearly-saturated, fractional factorial designs. For the purpose of continuous improvement in the monetary and cycle time expenditures for new product design, new process launch, and new manufacturing process launch, supersaturated designs may provide the experimenter with a viable solution to the problem of studying more factors than permitted in a saturated design.

Although, much has been written about creating supersaturated designs, little has been written regarding the analysis of these designs. This paper examines three test statistics which one might consider using when analyzing a supersaturated design. These test statistics are studied for four different supersaturated designs. The simulations and mathematical justifications presented in this paper suggest that it is not in the best interest of the experimenter to use these test statistics with these designs on a regular basis.

INTRODUCTION

Research in the area of supersaturated experimental designs has primarily focused on how to create such designs (e.g., Booth and Cox (1962), Voelkel(1986)). Recent ground-breaking work in this area by Lin (1993, 1993a), and Wu (1993) has created a large and useful class of supersaturated designs which uses half-fractions of Hadamard matrices. While much has been written on the topic of how to create supersaturated designs, little has been written on how to analyze these designs. (Two recent papers, presented at the Institute of Mathematical Statsitics 1994 Chapel Hill Meeting by Filliben and Westfall and Young, present two methods for analyzing superstaurated designs. These papers were requested, however, a response from the authors was not received until after the research had been completed.) Of particular interest when analyzing these designs is which method produces the best estimates and how good are these estimates compared to their fractional-factorial counterparts.

This paper examines three test statistics which seem naturally suited to analyzing supersaturated designs. All test statistics studied identify the significance of only the largest (in absolute value) half-effect. This largest absolute half-effect is intended to indicate the factor with the largest influence. The first two statistics utilize a half-normal plot methodology to identify the significance of the largest half-effect. The third method uses a stepwise regression approach to identify the largest half-effect.

For simulation purposes, three hypotheses were considered for each of the three test statistics, as shown below:

Ho: No active effects ($\Delta=0$)

H_{1a}: One active effect

H_{1b}: Two active effects.

Based on simulations conducted under the null hypothesis, critical values for each test statistic were determined at selected alpha levels (0.10, 0.05, and 0.01). Simulations were then conducted under each of the two alternative hypothesis, and the power of each test statistic was determined. Hypotheses are discussed in greater detail in the Simulation Methodology and Results sections of the paper.

The method of creating the supersaturated designs is detailed in the Historical Review section of the paper. The major findings of the research can be found in the Simulation Methodology and Results section of the paper for each respective hypothesis. Power curves for each of the test statistics are also included for each design in the Results section of the paper. The computer code used to create many of the tables found in the paper can be found in the Appendix section of the paper. The Discussion and Conclusion sections present opportunities for further research.

HISTORICAL REVIEW

The supersaturated designs studied here are a natural extension of unreplicated, saturated two-level designs. For a given number n of experimental runs, saturated fractional factorial designs are limited to at most n-1 factors (for example 11 factors in 12 runs). Supersaturated designs break this barrier by relaxing the usual orthogonality property of fractional factorial designs. Supersaturated designs permit examining more than n-1 factors in n runs, so that the number of factors $k>n-1$.

The designs considered in this paper were generated using the technique developed by Lin(1993, 1993a). This technique starts with an unreplicated, saturated, Plackett-

Burman design (Hadamard matrix), that is not from the 2^{k-p} series of designs. These designs include the n=12, 20, 24 and 36 designs.. An example is shown in Table 1.

Table 1: Unreplicated, Saturated, Plackett-Burman Design for 11 Factors

Run	A	B	C	D	E	F	G	H	J	K	L
1	+1	+1	-1	+1	-1	-1	-1	+1	+1	+1	-1
2	+1	+1	+1	-1	+1	+1	-1	+1	-1	-1	-1
3	-1	+1	+1	+1	-1	+1	+1	-1	+1	-1	-1
4	-1	-1	+1	+1	+1	-1	+1	+1	-1	+1	-1
5	+1	-1	-1	-1	+1	+1	+1	-1	+1	+1	-1
6	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
7	+1	-1	+1	-1	-1	-1	+1	+1	+1	-1	+1
8	-1	+1	+1	-1	+1	-1	-1	-1	+1	+1	+1
9	+1	-1	+1	+1	-1	+1	-1	-1	-1	+1	+1
10	+1	+1	-1	+1	+1	-1	+1	-1	-1	-1	+1
11	-1	-1	-1	+1	+1	+1	-1	+1	+1	-1	+1
12	-1	+1	-1	-1	-1	+1	+1	+1	-1	+1	+1

The design is separated into two groups using any factor and its factor level subscripts as the separating criterion as shown in Table 2.

Table 2: Sorted and Split Unreplicated Saturated Plackett-Burman Design

Run	A	B	C	D	E	F	G	H	J	K	L	Group
1	+1	+1	-1	+1	-1	-1	-1	+1	+1	+1	-1	1
2	+1	+1	+1	-1	+1	+1	-1	+1	-1	-1	-1	1
3	-1	+1	+1	+1	-1	+1	+1	-1	+1	-1	-1	1
4	-1	-1	+1	+1	+1	-1	+1	+1	-1	+1	-1	1
5	+1	-1	-1	-1	+1	+1	+1	-1	+1	+1	-1	1
6	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1
7	+1	-1	+1	-1	-1	-1	+1	+1	+1	-1	+1	2
8	-1	+1	+1	-1	+1	-1	-1	-1	+1	+1	+1	2
9	+1	-1	+1	+1	-1	+1	-1	-1	-1	+1	+1	2
10	+1	+1	-1	+1	+1	-1	+1	-1	-1	-1	+1	2
11	-1	-1	-1	+1	+1	+1	-1	+1	+1	-1	+1	2
12	-1	+1	-1	-1	-1	+1	+1	+1	-1	+1	+1	2

After removing the factor which was used to separate the design (which has now become a constant), both halves of the Plackett-Burman design become supersaturated designs, with k-1 factors and n/2 runs as is shown in Table 3.

Table 3: A Supersaturated Design For 10 Factors

Run	A	B	C	D	E	F	G	H	J	K
1	+1	+1	-1	+1	-1	-1	-1	+1	+1	+1
2	+1	+1	+1	-1	+1	+1	-1	+1	-1	-1
3	-1	+1	+1	+1	-1	+1	+1	-1	+1	-1
4	-1	-1	+1	+1	+1	-1	+1	+1	-1	+1
5	+1	-1	-1	-1	+1	+1	+1	-1	+1	+1
6	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1

Four Plackett-Burman designs (Plackett and Burman (1946)) were used to create the supersaturated designs studied in this paper. Table 4 shows the sequence of signs (for n =12, 20, 24, and 36) which were used to create these designs. Plackett-Burman design creation followed standard generation steps including the addition of a row of minus signs as the final step. Lin's technique was then used to generate the four supersaturated designs shown in Table 5. Each of the designs shown in Table 5 was studied in this paper.

In Lin's paper, comparisons were made between the supersaturated designs given by Satterthwaite (1959) and Booth and Cox (1962). Using Booth and Cox's method to measure non-orthogonality, $E\left(s^2\right) = \frac{\sum s_{mj}^2}{\binom{k}{2}}$ where $s_{mj} = \Sigma c'_m c_j$ (the sum of the cross-products of any two columns), Lin (1993) showed that half-fractions of Hadamard Matrices where n=6, 10, 12, 14, 18, 22, 24, 26, and 30 are superior to the methods presented by either Satterthwaite or Booth and Cox.

Table 4: Signs Used to Create Plackett-Burman Designs

k=11, n=12	k=19, n=20	k=23, n=24	k=35, n=36
1	1	1	-1
1	1	1	1
-1	-1	1	-1
1	-1	1	1
1	1	1	1
1	1	-1	1
-1	1	1	-1
-1	1	-1	-1
-1	-1	1	-1
1	1	1	1
-1	-1	-1	1
	1	-1	1
	-1	1	1
	-1	1	1
	-1	-1	-1
	-1	-1	1
	1	1	1
	1	-1	1
	-1	1	-1
		-1	-1
		-1	1
		-1	-1
		-1	-1
			-1
			-1
			1
			-1
			1
			-1
			1
			1
			-1
			-1
			1
			-1

Lin (1993a) states that "One simple way to measure the degree of non-orthogonality between two columns, c_i and c_j, is to consider their correlation, $r_{ij}=c'_i c_j/n$." Table 6 displays the pairwise correlation coefficients between pairings of columns. Figure 7 displays the pairwise correlation coefficients for each of the designs in the form of a frequency histogram. As can be seen in both Table 6 and Figure 7, the structure of the pairwise correlation coefficients is different for each design. The importance of this note will be addressed later in this paper.

Table 5: Supersaturated Designs Studied

k=10, n=6

1	2	3	4	5	6	7	8	9	10
1	1	-1	1	-1	-1	-1	1	1	1
1	1	1	-1	1	1	-1	1	-1	-1
-1	1	1	1	-1	1	1	-1	1	-1
-1	-1	1	1	1	-1	1	1	-1	1
1	-1	-1	-1	1	1	1	-1	1	1
-1	-1	-1	-1	-1	-1	-1	-1	-1	-1

k=18, n=10

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1	-1	1	1	-1	-1	-1	-1	1	-1	1	-1	1	1	1	1	-1
-1	1	1	-1	1	1	-1	-1	-1	-1	1	-1	1	-1	1	1	1	1
1	1	1	1	-1	-1	1	1	-1	1	1	-1	-1	-1	-1	1	-1	1
1	-1	1	1	1	-1	-1	1	-1	1	1	-1	1	-1	-1	-1	-1	1
1	-1	1	-1	1	1	1	1	-1	-1	1	1	-1	1	1	-1	-1	-1
-1	1	-1	1	-1	1	1	1	1	-1	-1	-1	1	1	-1	1	1	-1
-1	-1	1	-1	1	-1	1	1	1	1	-1	-1	-1	1	1	-1	1	-1
-1	-1	-1	1	-1	1	-1	-1	-1	1	1	1	1	-1	-1	1	1	1
1	1	-1	-1	-1	-1	1	-1	1	-1	1	1	1	1	-1	-1	1	1
-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1

k=22, n=12

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
1	1	1	1	1	-1	-1	-1	-1	1	-1	1	-1	-1	1	1	-1	-1	1	1	-1	1
1	-1	1	1	1	1	1	-1	-1	-1	-1	1	-1	1	-1	-1	1	1	-1	-1	1	1
1	1	-1	1	-1	1	1	1	1	-1	-1	-1	-1	1	-1	1	-1	-1	1	1	-1	-1
-1	1	1	-1	1	-1	1	1	1	1	1	-1	-1	-1	-1	1	-1	1	-1	-1	1	1
1	1	-1	-1	1	1	-1	1	-1	1	1	1	1	1	-1	-1	-1	-1	1	-1	1	-1
-1	1	1	-1	-1	1	-1	1	-1	1	1	1	1	1	-1	-1	-1	-1	1	-1	1	1
1	-1	-1	1	1	-1	-1	1	1	-1	1	-1	1	1	1	1	1	-1	-1	-1	-1	1
1	-1	1	-1	-1	1	1	-1	-1	1	1	-1	1	-1	1	1	1	1	1	-1	-1	-1
-1	1	-1	1	-1	-1	1	1	-1	-1	1	1	-1	1	-1	1	1	1	1	1	-1	-1
-1	-1	1	-1	1	-1	1	1	-1	1	1	-1	1	-1	1	1	1	1	1	1	-1	1
-1	-1	-1	1	-1	1	-1	-1	1	1	-1	-1	1	1	-1	1	-1	1	1	1	1	1
-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1

k=34, n=18

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
1	-1	-1	1	-1	-1	1	1	-1	1	-1	1	-1	-1	-1	-1	1	-1	-1	1	1	1	-1	1	1	1	1	1	-1	-1	-1	1	1	1
1	1	1	-1	1	-1	-1	1	-1	-1	1	1	-1	1	-1	1	-1	-1	-1	-1	1	-1	-1	1	1	1	-1	1	1	1	1	1	-1	-1
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-1	-1	1	1	1	-1	1	-1	-1	1	-1	-1	1	1	-1	1	-1	1	-1	-1	-1	-1	1	-1	-1	1	1	1	-1	1	1	1	1	1
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1	-1	1	-1	-1	-1	-1	1	-1	-1	1	1	1	-1	1	1	1	1	1	-1	-1	-1	1	1	1	-1	1	-1	-1	1	-1	-1	1	1
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-1	1	1	-1	1	-1	1	-1	-1	-1	-1	1	-1	-1	1	1	1	-1	1	1	1	1	1	-1	-1	-1	1	1	1	-1	1	-1	-1	1
1	-1	-1	1	1	-1	1	-1	-1	-1	-1	-1	-1	1	-1	-1	1	1	1	-1	1	1	1	1	1	-1	-1	-1	1	1	1	-1	1	-1
-1	1	-1	-1	1	1	-1	1	-1	1	-1	-1	-1	-1	1	-1	-1	1	1	1	-1	1	1	1	1	1	-1	-1	-1	1	1	1	-1	1
-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1

Table 6: Correlation Coefficients for Supersaturated Designs Studied

k=10, n=6

	Var1	Var2	Var3	Var4	Var5	Var6	Var7	Var8	Var9
Var2	0.333								
Var3	-0.333	0.333							
Var4	-0.333	0.333	0.333						
Var5	0.333	-0.333	0.333	-0.333					
Var6	0.333	0.333	0.333	-0.333	0.333				
Var7	-0.333	-0.333	0.333	0.333	0.333	0.333			
Var8	0.333	0.333	0.333	0.333	0.333	-0.333	-0.333		
Var9	0.333	0.333	-0.333	0.333	-0.333	0.333	0.333	-0.333	
Var10	0.333	-0.333	-0.333	0.333	0.333	-0.333	0.333	0.333	0.333

k=18, n=10

	Var1	Var2	Var3	Var4	Var5	Var6	Var7	Var8	Var9	Var10	Var11	Var12	Var13	Var14	Var15	Var16	Var17
Var2	0.200																
Var3	0.200	-0.200															
var4	0.200	0.200	-0.200														
Var5	0.200	-0.200	0.600	-0.200													
Var6	-0.200	-0.200	0.200	0.200	0.200												
Var7	0.200	0.200	0.200	-0.200	-0.200	-0.200											
Var8	-0.200	-0.200	0.200	0.200	-0.200	0.200	0.600										
Var9	-0.200	-0.200	-0.200	0.200	-0.200	0.200	0.200	0.200									
Var10	0.200	-0.200	0.200	0.600	0.200	-0.200	-0.200	0.200	0.200								
Var11	0.200	0.200	0.200	-0.200	-0.200	0.200	0.200	0.200	-0.200	-0.200							
Var12	0.600	0.200	-0.200	0.200	0.200	0.200	0.200	-0.200	0.200	-0.200	-0.200						
Var13	-0.200	0.200	0.200	-0.200	0.200	0.200	0.200	-0.200	0.600	-0.200	-0.200	0.200					
Var14	0.200	-0.200	-0.200	-0.200	0.200	-0.200	0.200	0.200	0.200	0.200	0.200	0.200	-0.200				
Var15	-0.200	0.200	-0.200	0.200	0.200	0.600	-0.200	0.200	-0.200	-0.200	0.200	0.200	-0.200	0.200			
Var16	-0.200	0.600	0.200	0.200	0.200	-0.200	0.200	0.200	-0.200	0.200	-0.200	-0.200	0.200	-0.200	0.200		
Var17	-0.200	0.200	-0.200	-0.200	0.200	-0.200	-0.200	-0.200	0.200	0.200	0.200	-0.200	0.200	0.600	0.200	0.200	
Var18	0.200	0.200	0.200	0.200	-0.200	0.200	-0.200	-0.200	0.200	0.200	0.600	-0.200	0.200	-0.200	-0.200	-0.200	0.200

k=22, n=12

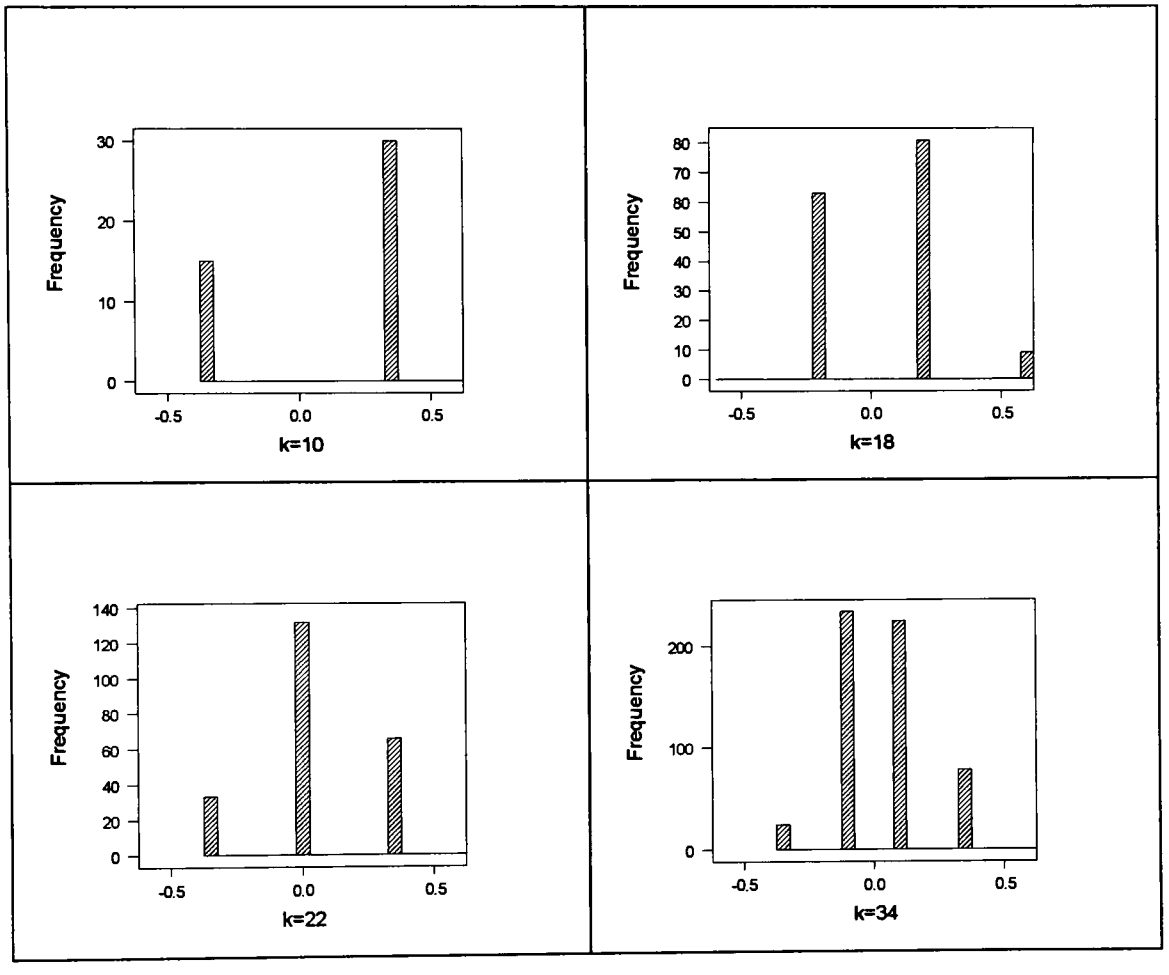
	Var1	Var2	Var3	Var4	Var5	Var6	Var7	Var8	Var9	Var10	Var11	Var12	Var13	Var14	Var15	Var16	Var17	Var18	Var19	Var20	Var21
Var2	0.000																				
Var3	0.000	0.000																			
Var4	0.333	0.000	-0.333																		
Var5	0.333	0.000	0.333	0.000																	
Var6	0.333	0.000	0.000	0.000	-0.333																
Var7	0.000	0.333	0.333	0.000	-0.333	0.333															
Var8	0.000	0.333	-0.333	0.000	0.333	-0.333	0.000														
Var9	-0.333	0.000	0.000	0.000	0.000	0.000	0.000	0.333													
Var10	0.333	0.333	0.000	0.000	0.000	0.333	0.000	0.000	0.000												
Var11	0.000	0.333	0.000	-0.333	0.000	0.000	0.333	0.333	0.000	0.000											
Var12	0.000	0.333	0.333	0.000	0.333	0.000	0.000	0.000	-0.333	-0.333	0.000										
Var13	0.000	-0.333	0.000	-0.333	0.000	0.333	-0.333	0.000	0.333	0.000	0.333	0.000									
Var14	0.000	0.000	-0.333	0.333	0.000	0.333	0.000	0.000	0.000	-0.333	0.333	0.333	0.333								
Var15	0.333	0.000	0.333	0.000	0.000	0.000	0.000	0.000	0.333	0.000	0.000	0.000	0.333	-0.333							
Var16	0.000	0.000	0.000	0.333	0.000	-0.333	0.000	0.000	0.000	0.333	0.333	-0.333	0.000	0.000	0.000						
Var17	0.333	-0.333	0.000	0.333	0.000	0.000	0.333	0.333	0.000	-0.333	0.000	0.000	0.000	0.000	0.333	0.000					
Var18	-0.333	-0.333	0.333	0.000	0.000	0.000	0.333	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.333	0.333	0.333				
Var19	0.000	0.000	0.000	0.000	0.000	0.000	-0.333	0.000	-0.333	0.333	0.000	0.333	0.333	0.000	0.000	0.333	0.000	0.333			
Var20	-0.333	0.333	0.000	0.333	-0.333	0.000	0.000	0.000	0.333	0.000	-0.333	0.333	0.000	0.000	0.333	0.000	0.000	0.000	0.333		
Var21	0.000	0.000	0.000	0.000	0.333	0.333	0.000	0.333	0.333	0.333	-0.333	0.000	0.000	0.000	-0.333	-0.333	0.000	0.333	0.000	0.000	
Var22	0.000	0.000	0.333	0.333	0.333	0.000	0.000	-0.333	0.333	0.000	0.000	0.000	0.000	0.333	0.000	0.333	-0.333	0.000	-0.333	0.000	0.000

k=34, n=18

	Var1	Var2	Var3	Var4	Var5	Var6	Var7	Var8	Var9	Var10	Var11	Var12	Var13	Var14	Var15	Var16	Var17	Var18	Var19	Var20	Var21	Var22	Var23	Var24
Var2	-0.111																							
Var3	-0.111	0.333																						
Var4	0.111	-0.111	-0.111																					
Var5	0.111	0.333	0.111	-0.111																				
Var6	-0.111	0.333	-0.111	-0.111	-0.111																			
Var7	-0.333	-0.111	0.111	0.111	0.111	-0.111																		
Var8	0.111	-0.111	0.111	-0.111	-0.111	0.111	0.111																	
Var9	0.111	0.111	-0.111	0.333	0.111	0.333	0.111	-0.111																
Var10	-0.111	-0.111	-0.111	0.111	0.111	0.333	0.111	0.111	0.111															
Var11	0.333	-0.111	-0.111	-0.111	-0.111	0.111	-0.333	0.111	0.111	0.111														
Var12	-0.111	-0.111	0.111	-0.111	-0.111	-0.111	0.111	0.333	-0.111	-0.111	0.333													
Var13	-0.111	-0.111	0.333	-0.111	-0.111	0.111	0.111	-0.111	0.333	0.111	0.333	0.111												
Var14	0.111	-0.111	0.111	0.111	-0.111	0.111	0.333	0.111	0.111	-0.111	0.111	-0.333	0.111											
Var15	-0.111	0.333	0.111	-0.111	-0.333	0.333	-0.111	0.111	0.111	-0.333	-0.111	-0.111	0.111	0.111										
Var16	0.333	0.111	0.333	0.111	0.111	-0.111	-0.333	-0.111	-0.111	0.111	0.333	-0.111	-0.111	0.111	0.111									
Var17	0.333	-0.111	-0.111	-0.111	0.111	0.111	0.111	-0.111	0.333	-0.111	-0.111	0.333	0.111	-0.333	0.111	-0.111								
Var18	-0.111	-0.111	-0.111	0.111	0.111	0.333	0.111	0.111	-0.111	0.333	0.111	-0.111	-0.111	0.111	-0.111	0.111	-0.111							
Var19	-0.111	0.111	-0.111	-0.111	0.111	-0.111	0.111	-0.111	-0.111	0.333	-0.111	0.111	0.111	-0.111	0.111	-0.111	-0.111	0.333						
Var20	0.111	0.111	-0.111	0.111	0.111	0.111	0.111	0.333	-0.111	0.111	0.111	0.111	-0.111	-0.111	0.111	-0.111	-0.111	-0.111	0.333					
Var21	0.111	0.111	-0.333	0.111	-0.111	-0.111	0.333	-0.111	0.111	-0.111	0.111	0.111	-0.333	0.333	0.111	0.111	0.111	-0.111	0.111	-0.111				
Var22	0.111	0.111	-0.111	0.111	-0.111	0.111	-0.111	-0.333	-0.111	0.111	-0.111	-0.111	0.111	0.111	0.333	0.111	0.333	0.111	0.111	0.111	-0.111	-0.111		
Var23	0.111	-0.333	-0.111	-0.111	0.333	-0.111	0.111	0.111	0.111	-0.111	-0.111	-0.111	0.111	0.111	0.333	0.111	0.333	0.111	0.111	0.111	-0.111	-0.111		
Var24	0.333	0.333	0.111	-0.111	0.333	-0.111	0.111	0.333	0.111	0.111	-0.111	-0.111	-0.111	-0.111	-0.111	-0.111	0.111	0.111	0.111	0.111	-0.111	0.111	-0.111	-0.111
Var25	0.111	-0.111	0.111	-0.111	-0.333	0.111	-0.111	0.333	-0.111	0.111	-0.111	0.111	-0.111	0.111	0.111	-0.111	0.111	0.111	-0.111	-0.333	0.111	0.333	-0.111	0.333
Var26	0.111	0.111	-0.111	0.333	0.111	0.111	-0.111	0.333	-0.111	-0.111	0.111	-0.111	0.111	0.111	0.111	-0.111	-0.111	-0.111	-0.333	0.333	-0.111	-0.111	0.111	-0.111
Var27	-0.111	0.111	0.111	0.333	-0.111	0.111	0.111	-0.111	-0.111	0.111	-0.111	0.333	-0.111	-0.333	0.111	0.333	0.333	0.333	-0.111	-0.111	0.111	-0.111	-0.111	-0.111
Var28	0.111	0.111	0.111	-0.333	0.111	0.111	0.333	-0.111	-0.333	0.111	0.111	0.111	0.111	0.333	-0.111	0.111	-0.111	-0.111	0.111	0.333	0.111	-0.111	0.111	-0.111
Var29	-0.111	0.111	0.333	0.333	0.111	-0.111	0.111	0.111	0.333	-0.111	0.111	0.333	-0.111	0.111	-0.111	0.111	-0.111	-0.111	0.111	0.111	0.111	0.111	-0.111	-0.111
Var30	-0.111	0.111	-0.111	0.111	0.111	-0.111	-0.111	-0.111	-0.111	-0.333	0.111	0.111	0.111	0.111	0.111	-0.111	-0.111	-0.111	0.333	0.333	-0.333	0.111	-0.111	0.111
Var31	-0.111	-0.111	0.111	-0.111	0.333	0.111	-0.111	-0.111	-0.111	0.111	-0.111	0.111	-0.111	0.111	-0.111	0.111	0.111	0.111	-0.111	-0.111	-0.111	0.333	0.333	-0.333
Var32	-0.111	0.111	-0.111	0.111	0.111	-0.111	-0.111	-0.111	0.111	0.333	0.111	0.111	0.333	-0.111	-0.111	-0.111	-0.111	-0.333	-0.111	0.111	0.111	-0.111	0.111	0.111
Var33	0.333	-0.111	0.333	0.333	-0.111	-0.111	0.111	-0.111	-0.111	-0.111	-0.111	-0.111	0.333	0.111	-0.111	-0.111	0.111	0.111	0.111	0.111	-0.333	0.333	-0.111	0.111
Var34	-0.111	-0.111	0.111	0.111	-0.111	-0.333	0.111	0.111	-0.111	0.333	-0.111	-0.111	0.111	-0.111	0.333	0.333	-0.111	-0.111	0.111	0.111	0.111	0.111	0.333	0.111

	Var25	Var26	Var27	Var28	Var29	Var30	Var31	Var32	Var33
Var26	-0.111								
Var27	-0.111	0.111							
Var28	-0.111	0.111	0.111						
Var29	0.111	-0.111	-0.111	-0.333					
Var30	0.111	0.333	0.111	-0.111	-0.111				
Var31	0.333	0.111	-0.111	0.111	0.333	0.111			
Var32	0.111	0.333	-0.111	0.111	-0.111	0.333	0.111		
Var33	0.111	0.111	0.111	0.111	-0.111	0.111	-0.111	-0.111	
Var34	0.111	-0.111	0.111	-0.111	-0.111	-0.111	-0.111	0.333	-0.111

Figure 7: Frequency Histograms of Correlation Coefficients for Supersaturated Designs Studied



As previously mentioned, methods for analyzing these designs have received little attention in the literature. Since supersaturated designs are a natural extension of saturated designs, it is reasonable to consider extensions of saturated design analysis methods for analyzing supersaturated designs. The key difficulty in analyzing supersaturated designs is the lack of orthogonality inherent in these designs. Thus, a key part of this investigation was to extend analysis ideas to non-orthogonal designs, and to derive properties of these analysis methods to see whether these designs can be analyzed with reasonable confidence. Some of these properties can be derived in closed, analytic form; however, since these designs are non-orthogonal, some properties were investigated through simulation.

METHODS OF ANALYSIS

Four methods of analysis were initially considered for analyzing these designs. The first method of analysis considered is based on the most commonly employed analysis technique used for unreplicated, saturated, two-level designs. This method employs half-normal (or normal) probability plots (Daniel (1959), and Zahn(1961)) to decide which factors are important and which are not, with the usual assumptions on the errors (uncorrelated, normally distributed, and homogeneous variance). The graphical form of the half-normal plot is created by calculating k=n-1 half-effects (b_1, b_2, ..., b_k), which can be shown to be independent and identically distributed normal random variables with equal variances and with mean 0 under the null hypothesis and taking their absolute values. The absolute value of the half-effects are represented in this paper as (u_1, u_2, u_3, ..., u_k), where $u_i = |b_i|$ and i is the index of the random variable. (Under the null hypothesis u_i are independent and identically distributed half-normal random variables, see Daniel (1959), and Zahn(1961).) These u_i are then rank ordered such that $u_{(1)} < u_{(2)} < u_{(3)} < \ldots < u_{(k)}$.

These order statistics are then plotted against P[(i-0.5)/k] where P[(i-0.5)/k] represents the corresponding percentile of the standard half-normal distribution, to form a so-called half-normal plot. Active effects are distinguished from inactive effects by passing a line through the origin (0, 0) and through the inactive effects. Active effects are found to the right of this line. Theory for this extremely powerful, yet simple, method has been developed to test the significance of the random variables; however, in most cases the corresponding graphical selection of key variables is used. The ability of the half-normal plot to distinguish active effects from inactive ones is of interest in the supersaturated case. Rather than utilizing the graphical approach, this paper examines corresponding non-graphical methods of identifying active half-effects. For simplicity, this paper restricts itself to deciding only whether the largest absolute half-effect corresponds to an active effect.

Birdbaum (1959) investigated this scenario by examining the ratio of the largest absolute half-effect to the absolute value of the closest (0.683k+0.5) order-statistic. The (0.683k+0.5) order-statistic was chosen because it most closely estimates σ in a null experiment. We extended and empirically studied the ratios:

$$t^{(\Delta)}_{k,(0.5)} = \frac{u_{(k)}}{u_{(0.5)}} \qquad (1.1)$$

and

$$t^{(\Delta)}_{k,(0.7)} = \frac{u_{(k)}}{u_{(0.7)}} \qquad (1.2)$$

where $u_{(k)}$ is the largest of the k ordered absolute half-effects and $u_{(0.5)}$ and $u_{(0.7)}$ are the order statistics from the k ordered absolute half-effects that are numbered nearest to (0.5*k) and (0.7*k), respectively. Table 8 displays the order statistics used for each test statistic, for each of the supersaturated design cases studied in this paper.

Table 8: Order Statistics For Each Supersaturated Design Studied

	k=10, n=6	k=18, n=10	k=22, n=12	k=34, n=18
0.50	$u_{(5)}$	$u_{(9)}$	$u_{(11)}$	$u_{(17)}$
0.70	$u_{(7)}$	$u_{(12)}$	$u_{(15)}$	$u_{(23)}$

The symbol Δ represents the magnitude of a shift from 0 of an active half-effect of size Δσ, where Δ equals 0, 1, 2, 4, 6, 8, or 10. $t_{k,(0.5)}^{(\Delta)}$ and $t_{k,(0.7)}^{(\Delta)}$ are then the ratios of the absolute largest half-effect to the nearest absolute (0.5*k) and (0.7*k) largest half-effects, respectively. Intuitively, $t_{k,(0.5)}$ and $t_{k,(0.7)}$ should be fairly sensitive in the case where there is one active effect. In the case where there is more than one active effect, we would expect $t_{k,(0.7)}$ to become less sensitive before $t_{k,(0.5)}$ loses sensitivity.

The second method, used in the same situation as the first (unreplicated, saturated, two-level designs) , involves the use of stepwise regression to estimate which factors are important. The ratio being considered here is $t_{k,s}^{(\Delta)} = u_{(k)}/s$, where s is the root mean square error one would calculate after the first iteration of stepwise regression procedure or $\sqrt{\sum_{i=1}^{k-1}(Y_i-\hat{Y})^2/n-2}$.

The third method of analysis considered is used when there is some replication available, e.g. several center points. These center points are used as an estimate of error

which is then used to determine which factors are important. Although this method is attractive, and appears to provide a more "objective" answer than half-normal plots, usually there are too few degrees of freedom for the estimate of error to be reliable. In addition, for supersaturated designs, in which runs are at a premium, it would usually be more efficient to replace replicated runs with non-replicated runs.

The fourth method of analysis considered relies on a past estimate of experimental error. This method has the least to offer the experimenter, since such a past estimate is usually either unavailable or is based on a past experiment that does not quite match the current experiment. Neither the third nor the fourth methods were studied for the reasons given.

SIMULATION METHODOLOGY AND RESULTS NULL HYPOTHESIS

The first objective, prior to testing the effectiveness of $t_{k,(0.5)}$, $t_{k,(0.7)}$ and $t_{k,s}$ was to determine critical values for selected alpha levels under the null hypothesis, (no active effects present, or $\Delta = 0$). SAS statistical software was used to conduct the simulations needed to determine the critical values for each of the test statistics. SAS was originally chosen for its ability to create and manipulate large data sets. (This choice of software turned out to be less than optimal. SAS was inefficient, due to the method in which the data was created and manipulated. This resulted in very long program run times; several days on a dedicated SUN Cluster, and SUN SPARC 10 workstation for each supersaturated design. In hindsight, any lower level computer package with matrix manipulation capability would have been preferable.)

One thousand sets of standard random normal deviates were generated through simulation for each of the designs shown in Table 5. Using the inner product method, half-effects were calculated for each factor. The three test statistics, $t_{k,(0.5)}^{(0)}$, $t_{k,(0.7)}^{(0)}$, and $t_{k,s}^{(0)}$, were then calculated for each of the one thousand sets of random data. (Refer to Figure I in the Appendix for a simplified flow diagram of the steps followed.) Appendix I presents an example of the SAS simulation program used to create each of the test statistics under the null hypothesis. Figure 10 presents the histograms of the test statistics for each of the four designs studied. Based on the distribution of these test statistics, critical values were then determined at alpha levels 0.10, 0.05 and 0.01. Table 9 presents critical values at selected alpha levels for each of the test statistics, for each of the designs.

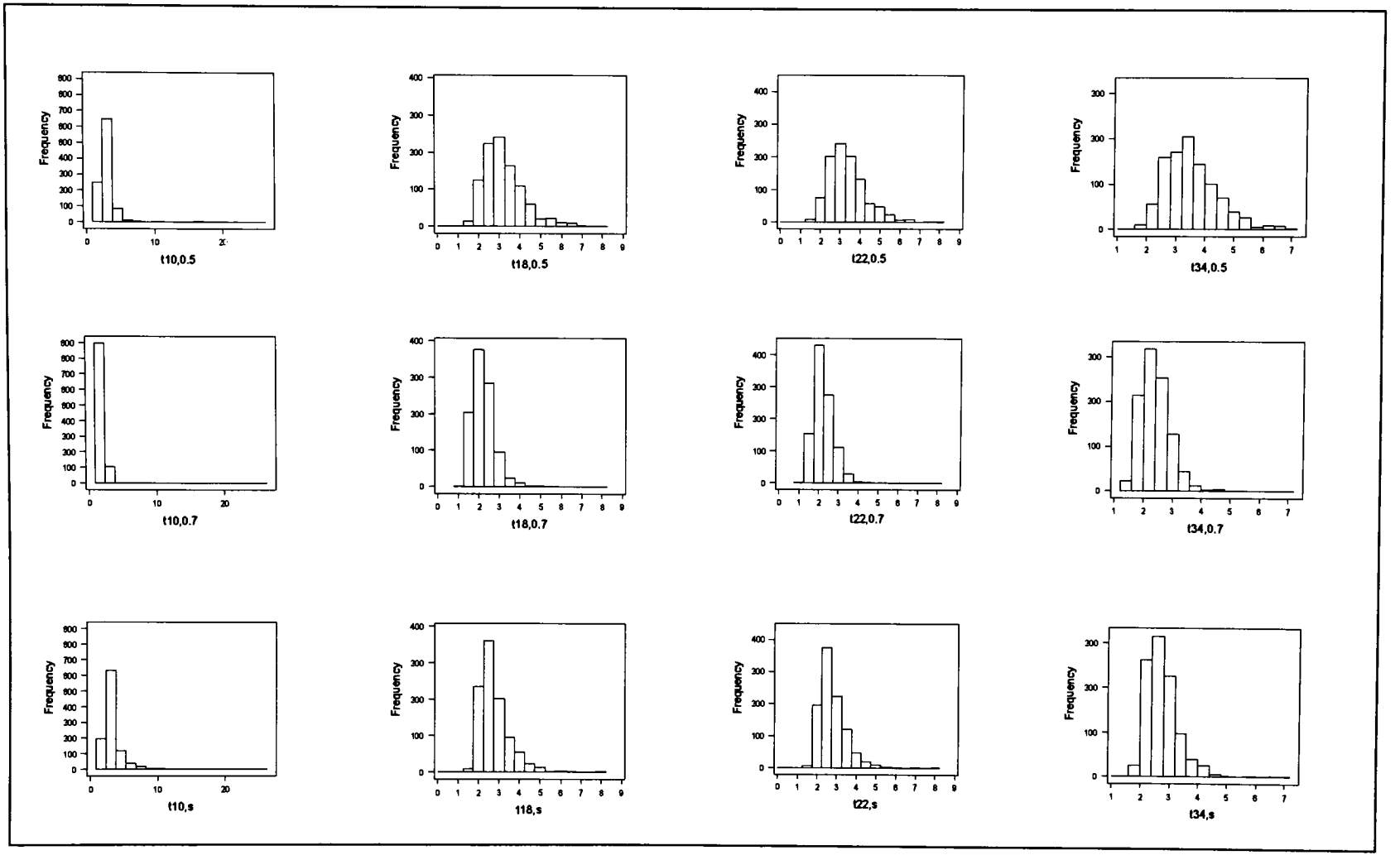
Table 9: Critical Values For The Test Statistics at Selected Alpha Levels

Test Statistic	0.10	0.05	0.01
$t_{10,0.5}$	3.800	4.415	7.220
$t_{10,0.7}$	2.259	2.425	2.637
$t_{10,s}$	4.501	5.599	8.069
$t_{18,0.5}$	4.398	5.133	6.282
$t_{18,0.7}$	2.842	3.164	3.777
$t_{18,s}$	3.753	4.144	5.043
$t_{22,0.5}$	4.582	5.085	6.036
$t_{22,0.7}$	2.891	3.090	3.608
$t_{22,s}$	3.638	4.033	5.032
$t_{34,0.5}$	4.686	5.177	6.291
$t_{34,0.7}$	3.070	3.350	3.971
$t_{34,s}$	3.445	3.766	4.367

Ninety-five percent confidence intervals were calculated for each of the critical values. Confidence intervals ranged from $\pm$ 0.042 to $\pm$ 0.54 depending on the test statistic and the design. No overlapping occurred between confidence intervals, so we concluded that the critical values were estimated reasonably well.

Due to uniqueness of the non-orthogonality patterns in the supersaturated designs (see Table 6 and Figure 7), comparisons between critical values for different designs are not possible. For example, one might make the observation that for an alpha level of 0.05, the critical values for $t_{10,(0.5)}$, $t_{18,(0.5)}$, $t_{22,(0.5)}$, and $t_{34,(0.5)}$ are 4.415, 5.133, 5.085, and 5.177, respectively, and might question why $t_{22,(0.5)}$, does not conform to the apparently increasing pattern of critical values. One must therefore take care to use the appropriate critical values for the associated design. Appendix III presents an example of the SAS programs used to simulate the conditions specified by the first and second alternative hypotheses.

Figure 10: Histograms of Test Statistics For Each of The Four Designs Studied Under The Null Hypothesis



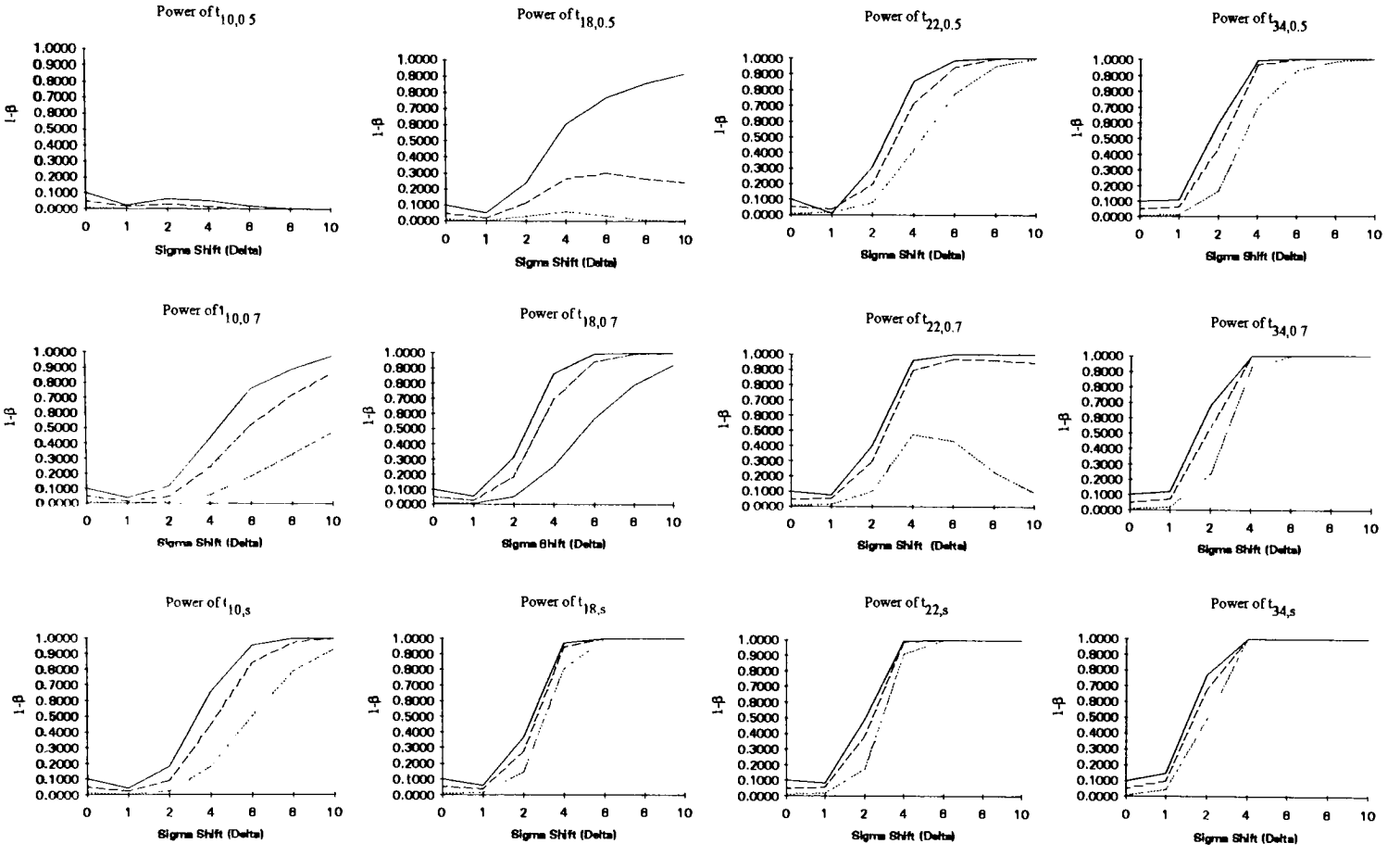
SIMULATION METHODOLOGY AND RESULTS FIRST ALTERNATIVE HYPOTHESIS

Simulations were conducted for the case of the first alternative hypothesis where one active effect was present. SAS statistical software was used to conduct the simulations needed to determine the critical values for each of the test statistics. One hundred sets of standard random normal deviates were generated for each of the designs shown in Table 5. The total number of deviates needed for each design is based on the number of runs in each design. For example, the design with 10 factors and 6 runs required 100 sets each containing 6 deviates. Therefore, this design required a total of 600 deviates. Each factor was independently introduced into the scenario described by the first alternative hypothesis by shifting the set of deviates according to the following model $Y= \Delta * X_j + \varepsilon$, where ε = a random standard normal deviate N(0,1), Δ = 1, 2, 4, 6, 8, and 10, and j = 1, 2, ..., k. (Refer to Figure 1 in the Appendix for a simplified flow diagram of the steps followed.) Using the inner product method, half-effects were calculated for each factor. Each of the three test statistics were then calculated for each of the kx100 sets of random data. (For example, in the case where n=6, 10x100=1000 sets of data were created.) In this scenario the test statistics should detect one active effect of magnitude Δ, and the factor that is detected should correspond to X_j. (Since comparisons were not to be made between the critical values there is no concern regarding the independence.) The graphs in Figure 11 summarize the percent of correctly identified active half-effects, or the power, of each statistic for each of the three alpha levels and for each design.

One can see in Figure 11 that some of the ratios studied for some of the designs namely, $t_{10,(0.5)}^{(\Delta)}$, $t_{18,(0.5)}^{(\Delta)}$, $t_{10,(0.7)}^{(\Delta)}$, $t_{18,(0.7)}^{(\Delta)}$, and $t_{22,(0.7)}^{(\Delta)}$, exhibit poor properties. For example, the test statistic $t_{k,(0.5)}$ calculated for the 10 factor design,

shows that as the shift (Δ) increases from 1σ to 10σ, the power of the test decreases. A theoretical explanation for why the power drops is included in the Theory First Alternative Hypothesis section of the paper. This loss of power does not occur in the orthogonal case. Namely, as an introduced shift gets larger, the power of the test statistic would increase. The inconsistent ability to detect significant shifts that $t_{k,(0.5)}^{(\Delta)}$ and $t_{k,(0.7)}^{(\Delta)}$ display represent undesirable properties for a test statistic. Conversely, the power of $t_{k,s}$ increases as the magnitude of the shift Δ increases, making $t_{k,s}$ the most useful test statistic for this case.

Figure 11: Power Curves For Each Test Statistic and Design For The First Alternative Hypothesis
Alpha levels 0.10, 0.05, 0.01, are identified by a solid, a dashed, and a dotted line, respectively.



THEORY
FIRST ALTERNATIVE HYPOTHESIS

As previously mentioned, the loss of power experienced by $t_{k,(0.5)}$ and $t_{k,(0.7)}$ is an undesirable property for a test statistic. The following definitions are necessary prior to discussing the reasons why the power of the tests drop.

Let Δ equal the magnitude of a shift from 0 of an active half-effect, $\Delta = 0, 1, 2, 4, 6, 8,$ or 10
X_m equal the column vector of ± 1's for the m^{th} factor in the design,
$X_{(m)}$ equal a matrix of the remaining factors in the design excluding X_m,
β_m equal the size of the half-effect for factor m,
$\beta_{(m)}$ equal the column vector of the remaining half-effects,
X_j equal the column vector of ± 1's for the *active* factor,
β_j equal the size of the *active* half-effect,
ε be a column of errors ε_i, where $\varepsilon_i \sim$ i.i.d. $N(0,\sigma^2)$, for $i = 1, 2, 3, \ldots, n$, and
Y be the response column.

Note that if $m = j$ then $\beta_m = \Delta$, otherwise $\beta_m = 0$. Hence, the model is

$$\begin{aligned} Y &= X_j\beta_j + \varepsilon \\ &= X_j\Delta + \varepsilon. \end{aligned} \tag{2.1}$$

Alternatively the actual model, for any m, can then be written as

$$Y = X_m\beta_m + X_{(m)}\beta_{(m)} + \varepsilon, \tag{2.2}$$

and our estimate of β_m is

$$b_m = (X_m'X_m)^{-1}X_m'Y. \tag{2.3}$$

So for the case where $m = j$, or the half-effect being estimated corresponds to the factor that has been shifted, then the expected value of b_m is:

$$E(b_m) = (X_j'X_j)^{-1}X_j'E(Y)$$
$$= (X_j'X_j)^{-1}X_j'X_j'\beta_j = \beta_j = \beta_m = \Delta. \qquad (2.4)$$

If, however, $m \neq j$ then b_m is biased as shown below.

$$E(b_m) = (X_m'X_m)^{-1}X_m'E(Y)$$
$$= (X_m'X_m)^{-1}X_m'(X_m\beta_m + X_{(m)}\beta_{(m)})$$

Since $\beta_m = 0$, this becomes

$$(X_m'X_m)^{-1}X_m'X_{(m)}\beta_{(m)}.$$

Since only β_j is active this further reduces to

$$\frac{X_m' X_j}{n} * \Delta = r_{mj}\Delta, \qquad (2.5)$$

where $r_{mj}\Delta$ is the correlation coefficient of the columns X_m and X_j (see Table 6) and Δ is the magnitude of the shift. In other words, b_j is biased by the product of the pairwise correlation coefficient and the shift Δ. Consider now what happens to $t_{k,(0.5)}^{(\Delta)}$ as the introduced shift Δ approaches infinity.

Let $O_p(1)$ (read as: big O of 1 in probability) represent any random variable Z_1 that is bounded in probability as Δ approaches infinity: For any $\delta > 0$ there is a c such that $P(|Z_1| > c) < \Delta$ as Δ approaches infinity,

$o_p(1)$ (read as: little o of 1 in probability) represent any random variable that converges in probability to 0 as Δ approaches infinity: For any $\delta > 0$, $P(|Z_2| > \delta)$ approaches 0 as Δ approaches infinity.

Note that if Z_1 is $O_p(1)$, then $\frac{Z_1}{\Delta}$ is $o_p(1)$. Therefore,

$$\hat{\beta}_m = E\left(\hat{\beta}_m\right) + O_p(1). \tag{2.6}$$

First, as Δ approaches infinity, (2.5) and (2.6) imply that

$$P\left(\left|\hat{\beta}_j\right| = \text{Max}\left\{\left|\hat{\beta}_m\right|\right\}, m = 1, 2, 3, ..., k\right) \rightarrow 1. \tag{2.7}$$

Secondly, for large Δ, for probability approaching one,

$$t_{k,(0.5)}^{(\Delta)} = \frac{\left|\hat{\beta}_j\right|}{\text{Median}\left\{\left|\hat{\beta}_m\right|, m \neq j\right\}}$$

$$= \frac{\left|\Delta + O_p(1)\right|}{\text{Median}\left\{\left|r_{mj}\Delta + O_p(1)\right|, m \neq j\right\}}$$

$$= \frac{\left|1 + o_p(1)\right|}{\text{Median}\left\{\left|r_{mj} + o_p(1)\right|, m \neq j\right\}}.$$

So as Δ approaches infinity,

$$t_{k,(0.5)}^{(\Delta)} \xrightarrow{P} \frac{1}{\text{Median}\left\{\left|r_{mj}\right|, m \neq j\right\}}, \tag{2.8}$$

the inverse of the median pairwise correlation coefficient. Similarly, $t_{k,(0.7)}$ converges in probability to the inverse of the seventieth percentile of the pairwise correlation coefficients:

$$t_{k,(0.7)}^{(\Delta)} \xrightarrow{P} \frac{1}{70^{th}\ \text{percentile}\left\{\left|r_{mj}\right|\right\}}. \qquad (2.9)$$

So, unless at least half (or 70 percent, respectively) of the correlation's r_{mj}, $m \neq j$ are 0 the ratio converges in probability to a constant for large Δ. (If more than half (70%) of these correlation's are 0, then the ratio converges in probability to infinity.) So, if the constant is larger than the critical values then the power approaches 1, otherwise it will approach 0.

For example, for the design with k=10 and n=6, all pairwise correlation coefficients are either +1/3 or -1/3. So for this design, as the shift Δ approaches infinity, the statistics $t_{10,(0.5)}^{(\Delta)}$ and $t_{10,(0.7)}^{(\Delta)}$ converge in probability to 3. It is now evident why the power curves for the test statistics are as they appear. The critical values for the statistic $t_{10,(0.5)}^{(\Delta)}$ are all greater than 3, but $t_{10,(0.5)}^{(\Delta)}$ converges in probability to 3 as the shift approaches infinity. Hence, as the effect of the active factor gets larger, the power of the test decreases, which is what is observed in Figure 11. Conversely, the critical values for the statistic $t_{10,(0.7)}^{(\Delta)}$ are all less than 3 therefore, the power curve increases as the shift increases. In general, these test statistics provide less than acceptable results. Due to the undesirable nature of these test statistics, $t_{10,(0.5)}^{(\Delta)}$ and $t_{10,(0.7)}^{(\Delta)}$ were not considered during simulations for the second alternative hypothesis (where there is more than one known active effect).

Consider now what happens to $t_{k,s}^{(\Delta)}$ as the introduced shift Δ approaches infinity. For large Δ with probability approaching one,

$$t_{k,s}^{(\Delta)} = \frac{\left|\hat{\beta}_j\right|}{s} = \frac{\Delta + O_p(1)}{s} \tag{2.10}$$

But

$$\begin{aligned}(n-2)s^2 &= \sum\left(Y_i - \hat{Y}_i\right)^2 \\ &= \sum\left(X_{ji}\beta_j + \varepsilon_i - X_j\hat{\beta}_j\right)^2 \\ &= \sum\left(X_{ji}\beta_j + \varepsilon_i - X_{ji}\beta - O_p(1)\right)^2 \\ &= O_p(1). \end{aligned} \tag{2.11}$$

Therefore $s = O_p(1)$, and so

$$t_{k,s}^{(\Delta)} \xrightarrow{P} \infty \tag{2.12}$$

as Δ approaches infinity. This shows that the power corresponding to this test statistic goes to 1 as Δ approaches infinity. For the first alternative hypothesis, $t_{k,s}^{(\Delta)}$ is a reasonable way to test for a large effect.

SIMULATION METHODOLOGY AND RESULTS SECOND ALTERNATIVE HYPOTHESIS

Since the test statistic $t_{k,s}$ utilizes all but the largest half effect to estimate s, the worst case second alternative hypothesis, would be shifting both factors an equal magnitude. Simulations were conducted for the case of the second alternative hypothesis where two active effect were present. As before one hundred sets of standard random normal deviates were generated for each of the designs shown in Table 5. Pairs of consecutive factors were selected and introduced into the scenario described by the first alternative hypothesis by shifting the set of deviates according to the following model $Y= \Delta*X_j + \Delta*X_{j+1} + \varepsilon$, where ε = a random standard normal deviate, Δ = 1, 2, 4, 6, 8, and 10, and j = 1, 2, ..., k-1. Consecutive factors were chosen to simplify the simulation computer programs. (Refer to Figure I in the Appendix for a simplified flow diagram of the steps followed.) As shown below, choosing the same Δ for both active effects creates the "worse case" scenario for the two active effects case. This scenario was chosen to see how well $t_{k,s}^{(\Delta)}$ could detect an active effect when two active effects are present. Using the inner product method, half-effects were calculated for each factor, the largest half-effect in absolute value was chosen, and s was calculated as before. The test statistic $t_{k,s}^{(\Delta)}$ was again calculated for each of the (k-1)x100 sets of random data. (For example, in the case where n=6, 9x100=900 sets of data were created.) The graphs in Figure 12 summarize the percent of correctly identified known active factors, or the power, of each statistic for each of the three alpha levels and for each design.

One can see in Figure 12 that $t_{k,(s)}^{(\Delta)}$ exhibits poor properties. For example, the test statistic $t_{k,(s)}$ calculated for the 10 factor design, shows that as the shift (Δ) increases from 1σ to 10σ, the power of the test decreases. A theoretical explanation for why the power drops is included in the Theory Second Alternative Hypothesis section of the paper.

As mentioned earlier, this loss of power does not occur in the orthogonal case. The inconsistent ability to detect significant shifts that this test statistics displays represents an undesirable property for a test statistic.

THEORY
SECOND ALTERNATIVE HYPOTHESIS

Since only the largest half-effect is identified using $t_{k,s}$, and since two factors have been shifted, s will be inflated. In addition to the definitions used previously, the following definitions are necessary prior to discussing the reasons why the power of the tests drop.

Let $X_{j'}$ equal a column vector of the *second active* factor, and
$\beta_{j'}$ equal the size corresponding to the *second active* half-effect.

Note from previous definitions that if m = j or m=j' then $\beta_m = \Delta$, otherwise $\beta_m = 0$. The actual model can then be written as:

$$Y = X_j\beta_j + X_{j'}\beta_{j'} + \varepsilon \qquad (3.1)$$

where our estimate of β_m is

$$b_m = (X_m'X_m)^{-1}X_m'Y. \qquad (3.2)$$

Consider now what happens to $t_{k,s}^{(\Delta)}$ under the second alternative hypothesis as the introduced shift Δ approaches infinity. For large Δ, either the $\text{Max}\left\{\left|\hat{\beta}_m\right|\right\}$ corresponds to the order statistic with index j, with probability p, or to j', with probability (1-p). In either event

$$t_{k,s}^{(\Delta)} = \frac{\text{Max}\left\{\left|\hat{\beta}_m\right|\right\}}{s} = \frac{\Delta + O_p(1)}{s}.$$

But

$$(n-2)s^2 = \sum\left(Y_i - \hat{Y}_i\right)^2$$

$$= \sum\left(X_{ji}\beta_j + X_{j'i}\beta_{j'} + \varepsilon_i - X_{ij}\hat{\beta}_j\right)^2$$

$$= \sum\left(X_{ji}\beta_j + X_{j'i}\beta_{j'} + \varepsilon_i - X_{ji}\beta_j - O_p(1)\right)^2$$

$$= \sum\left(X_{j'i}\beta_{j'}\right)^2 + O_p(1)$$

$$= n\Delta^2 + O_p(1)$$

Hence

$$s = \Delta\sqrt{\frac{n}{n-2}} + O_p(1) \tag{3.3}$$

and so

$$t_{k,s}^{(\Delta)} \xrightarrow{P} \sqrt{\frac{n-2}{n}} \tag{3.4}$$

as Δ approaches infinity. Since $t_{k,s}^{(\Delta)}$ converges in probability to a number less than 1 for all the designs, and all critical values are greater than 1, the power of all these designs approaches 0 as Δ approaches infinity. (Apparently Δ is just not large enough for the n=18 design.) For the second alternative hypothesis, $t_{k,s}^{(\Delta)}$ is not a reasonable way to test for the largest effect. Since the test statistic $t_{k,s}$ utilizes all but the largest half effect to estimate s, shifting two factors an equal magnitude will inflate the estimate of standard deviation.

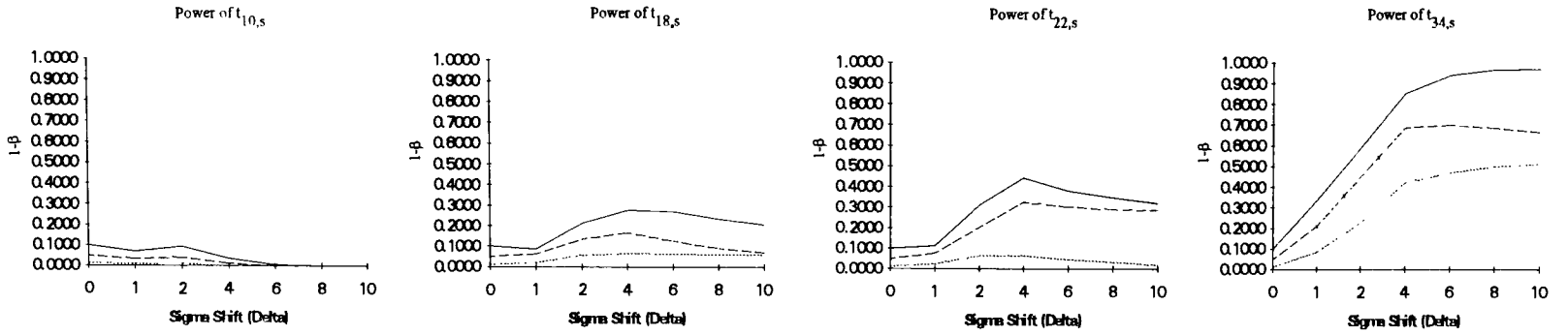
DISCUSSION

It appears, based on these simulations, that these methods of analysis are not as useful as previously thought. There are however, several other possible test statistics, and test strategies that could be evaluated. For example, one could start with the f largest half-effects, and then use backward elimination to determine the active effects.

CONCLUSION

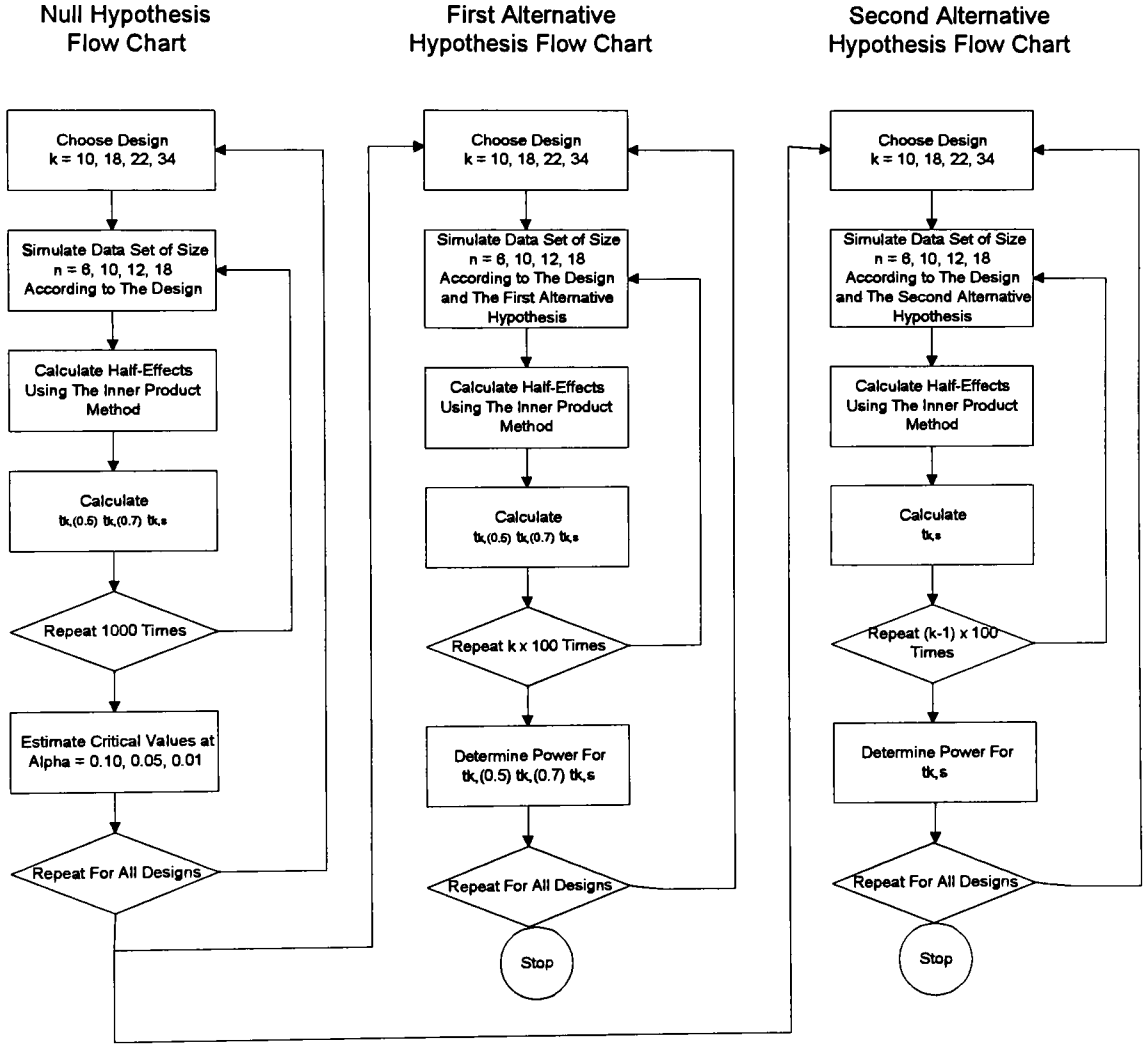
Based on the simulations and mathematical justifications, it is not in the best interest of the experimenter to use these test statistics with these designs on a regular basis. If, however, an individual is faced with the scenario presented in this paper and has limited statistical resources as well as time, selected designs and test statistics could be used as a preferable experimental strategy to experimenting using the one factor at a time method. However, choosing one of these test statistics with one of these designs carries with it many assumptions, see Theory section for details.

Figure 12: Power Curves For Each Test Statistic and Design For The Second Alternative Hypothesis
Alpha levels 0.10, 0.05, 0.01, are identified by a solid, a dashed, and a dotted line, respectively.



APPENDIX

Figure I: Flow Chart Describing General Steps Followed in Paper



APPENDIX I: Sample SAS Program Used to Create Distributions of Test Statistics

Program: Trial.SAS

```
/*****************************************************************/
/* Creates random sets of data, reads in the supersaturated design */
/* from the file DESIGN.DAT which is created by SIM*.*.  The macro */
/* variables RUNS, and FACTORS need to be changed according to the */
/* Plackett-Burman design being used in SIM*.*.  For example, if:  */
/*   k (the number of factors) = m, then FACTORS = (m-1)           */
/*   N (the number of runs) = p, then RUNS = (p-1)                 */
/*****************************************************************/

OPTIONS LS=72;
%LET DSGN=4;  *** Indicates which design matrix is to be used;
%LET REPS=10;  ***  Controls the number of sets of random variables;
%LET RUNS=18;  ***  Indicates the number of runs (T.C.'s) in the design;
%LET FACTORS=34;  ***  Indicates the number of factors under study;
%LET FIFTY=%EVAL((&FACTORS*5)/10);  ***  Marks the 50% of the factors;
%LET SEVENTY=%EVAL((&FACTORS*7)/10); ***  Marks the 70% of the factors;
FILENAME RATIOS&DSGN 'RATIOS.DAT';

%MACRO DESIGNMT;  ***  Read in the design matrix (generated by SS**PB.SAS);
INFILE DESIGN&DSGN;
INPUT VAR1-VAR&FACTORS #35;
%MEND DESIGNMT;

%MACRO RANDAT;  ***  Reads in design array and creates sets of random numbers;
DATA TWO;
SET ONE;
DROP I;
DO I=1 TO &RUNS;
  RND=RANNOR(-&RUNS);  ***  Negative sign signals time signature for seed;
END;
OUTPUT;
%MEND RANDAT;

%MACRO ANALYSIS;  ***  Performs analysis on each data set;

%LET NEW=AHE;  ***  New data set which will contain main half-effects;
%LET OLD=TWO;  ***  Original data set containing variables and random numbers;
FILENAME HE 'HE.SAS';
%INC HE; * Calculates mean and half-effects using inner-product method;

%LET IP=&NEW; ***  I/P dataset;
%LET VAR=HALFEFF; ***  Var to get half-normal scores from;
%LET CODE='*'; ***  Code to appear on plot;
FILENAME HALFNORM 'HALFNORM.SAS';
```

```
%INC HALFNORM;

PROC SORT DATA=HALFNORM OUT=HALFNORM;  ***  Sort the abs(half-effects);
BY ABSVAL;

DATA CRITVAL;
ARRAY ABSVALS{&FACTORS} ABSVAL1-ABSVAL&FACTORS;
RETAIN R1 R2 MAXFACT MEAN HALFEFF E2 0 ABSVAL&FACTORS;
KEEP R1 R2 RS;
IF _N_=1 THEN DO;
  DO N=1 TO &FACTORS;
    SET HALFNORM (RENAME=(FACTOR=MAXFACT));
    ABSVALS(N)=ABSVAL;
  END;
  R1=ABSVAL&FACTORS/ABSVAL&FIFTY; *** Largest Abs(HE)/ &fifty Abs(HE);
  R2=ABSVAL&FACTORS/ABSVAL&SEVENTY; ** Largest Abs(HE)/ &seventy Abs(HE);
  END;
***  Now maxfact of the variable corresponding to the largest Abs(HE);
***  Read in mean, and halfeff corresponding to largest Abs(HE);
IF _N_=1 THEN SET AHE (KEEP=MEAN HALFEFF) POINT=MAXFACT;

***  Now read in data to find s;
SET TWO END=EOF;
ARRAY VARS {&FACTORS} VAR1-VAR&FACTORS;
E=RND-MEAN-HALFEFF*VARS{MAXFACT};
E2=E2+E*E;
IF EOF THEN DO;
  S=SQRT(E2/(&RUNS-2));  ***  Calculate s for 1-(&factors-1) order stats;
  RS=ABSVAL&FACTORS/(S/SQRT(&RUNS));  ***  Ratio of largest Abs(HE) to s;
  OUTPUT;
  END;
RUN;

PROC APPEND BASE=RATIOS DATA=CRITVAL;  ***  Retain ratios;
%MEND ANALYSIS;

%MACRO BEGIN;  ***  Macro to loop[B through data generation and analysis;
%DO REPP=1 %TO &REPS;
 %RANDAT;  ***  Generate new set of random numbers;
 %ANALYSIS;  ***  Analyze the data set;
%END;
%MEND BEGIN;

DATA ONE;  ***  Read in design array;
%DESIGNMT;
%BEGIN;

DATA NEW;
SET RATIOS;
FILE RATIOS&DSGN;
PUT R1 R2 RS;
```

```
PROC UNIVARIATE PLOT DATA=RATIOS;  ***  Now analyze the ratios;
RUN;
```

Sample Output

```
4.0996821091   2.6563276229   2.9157518982
4.476756569    2.7513464185   3.0795334365
3.3774640357   2.259580332    2.979022799
2.6155171115   1.7793158905   2.1960255079
4.64317006     2.6073082937   2.9625305285
3.9011504372   2.391511376    2.9189275348
3.8107647837   1.9223253115   2.2531538822
2.4696096965   1.6541517506   2.1047508979
3.4614300858   2.7217352166   3.1710922076
3.022975137    2.2009933437   2.564261952
3.3390706876   2.4009418992   2.2954544046
2.1638165124   1.6184699936   2.1227314213
2.6871347192   1.7131569819   2.1018401297
2.5641581001   2.1956388695   2.5609299583
2.5068041927   1.9104687434   2.1494091611
4.2496318721   2.3544162431   3.2408659515
5.0366884435   3.4707182549   3.0189571112
4.6599480712   3.1382481569   3.2426675971
3.0486380872   1.8551177653   2.2931620617
1.8672749128   1.304139361    1.7721172825
...            ...            ...
```

APPENDIX II: Sample SAS Program Used to Identify Critical Values

Program: Crval.SAS

```
OPTIONS LS=72;

%MACRO GEN;
DATA R1;
SET ONE;
KEEP R1;

PROC RANK TIES=LOW DATA=R1 OUT=R1;
RANKS R1RANK;
VAR R1;
PROC SORT DATA=R1 OUT=R1;
BY DESCENDING R1RANK;

DATA R2;
SET ONE;
KEEP R2;

PROC RANK TIES=LOW DATA=R2 OUT=R2;
RANKS R2RANK;
VAR R2;
PROC SORT DATA=R2 OUT=R2;
BY DESCENDING R2RANK;

DATA RS;
SET ONE;
KEEP RS;

PROC RANK TIES=LOW DATA=RS OUT=RS;
RANKS RSRANK;
VAR RS;
PROC SORT DATA=RS OUT=RS;
BY DESCENDING RSRANK;

DATA TWO;
MERGE R1 R2 RS;
CV1=INT(1000*.99+(1.96*SQRT(1000*.99*.01)))+1;
CV2=(1000*.99);
CV3=INT(1000*.99-(1.96*SQRT(1000*.99*.01)))+1;
CV4=INT(1000*.95+(1.96*SQRT(1000*.95*.05)))+1;
CV5=(1000*.95);
CV6=INT(1000*.95-(1.96*SQRT(1000*.95*.05)))+1;
CV7=INT(1000*.90+(1.96*SQRT(1000*.90*.10)))+1;
CV8=(1000*.90);
CV9=INT(1000*.90-(1.96*SQRT(1000*.90*.10)))+1;
```

```
ARRAY CV(9) CV1-CV9;
DO CRIT=1 TO 9;
IF CV(CRIT)=R1RANK THEN OUTPUT;
END;

PROC PRINT DATA=TWO;
VAR R1RANK R1 R2 RS;

%MEND GEN;

DATA ONE;
INFILE 'RATIOS1.DAT';
INPUT R1 R2 RS;
%GEN;

DATA ONE;
INFILE 'RATIOS2.DAT';
INPUT R1 R2 RS;
%GEN;

DATA ONE;
INFILE 'RATIOS3.DAT';
INPUT R1 R2 RS;
%GEN;

DATA ONE;
INFILE 'RATIOS4.DAT';
INPUT R1 R2 RS;
%GEN;

RUN;
```

APPENDIX III: Sample SAS Program Used to Generate Data Used in Creating Power Curves

Program: Model4.SAS

```
/*********************************************************************/
/* This program calculates ratios for the first alternative        */
/* hypothese where there is only one significant effect.  The      */
/* file ONEFFE10.DAT contains the ratios produced by this program. */
/*********************************************************************/

OPTIONS LS=72;
%LET DSGN=4;    *** The experimental design to be used;
%LET REPS=10;   *** The number of times to simulate experiment;
%LET RUNS=18;   *** The number of runs in the design;
%LET FACTORS=34; *** The number of factors in the design;
%LET B=5;    *** Largest delta shift to consider;
%LET TRIALS=5;  *** Number of times delta is used with predictors;
%LET SIZE=%EVAL(&FACTORS+1);   *** Used for design input;
%LET FIFTY=%EVAL((&FACTORS*5)/10);   ***  Marks the 50% of the factors;
%LET SEVENTY=%EVAL((&FACTORS*7)/10);    ***  Marks the 70% of the factors;
FILENAME HE 'HE.SAS';   *** File that creates half effects;
FILENAME ONEFFECT 'ONEFFE10.DAT';   ***Ratio output filename;

%MACRO RANDAT;  ***  Reads in design array and creates sets of random numbers;
DATA ONE;
INFILE DESIGN&DSGN;
ARRAY VARS{&FACTORS} VARS1-VARS&FACTORS;
INPUT VARS1-VARS&FACTORS #&SIZE;
DROP I;
DO I=1 TO &RUNS;
  RND=RANNOR(-&RUNS);  ***  Negative sign signals time signature for seed;
END;
ONEFF=RND+&BETA*VARS(&I);
OUTPUT;
RUN;
%MEND RANDAT;

%MACRO HALF;
DATA TWO;
SET ONE END=LASTREC;
RETAIN HEFF1-HEFF&FACTORS FCTR1-FCTR&FACTORS;
KEEP MEAN HALFEFF FACTOR;
ARRAY HEFF{&FACTORS} HEFF1-HEFF&FACTORS;
ARRAY FCTR{&FACTORS} FCTR1-FCTR&FACTORS;
ARRAY VARS{&FACTORS} VARS1-VARS&FACTORS;
MEAN+ONEFF;
DO I=1 TO &FACTORS;
```

```
 HEFF{I}+VARS{I}*ONEFF;
 FCTR(I)=I;
 END;
IF LASTREC THEN DO;
 MEAN=MEAN/_N_;
 DO I=1 TO &FACTORS;
  HALFEFF=(HEFF{I})/&RUNS;
  FACTOR=FCTR(I);
  OUTPUT;
  END;
 END;
RUN;
%MEND HALF;

%MACRO ANALYSIS;
%LET BETA=0;
%DO B=1 %TO &B;
%LET BETA=%EVAL(&BETA+1);
  %LET I=0;
  %DO X=1 %TO &FACTORS;
   %LET I=%EVAL(&I+1);
   %DO R=1 %TO &REPS;
   %RANDAT;
   %HALF;
   %LET IP=TWO;
   %LET VAR=HALFEFF;
   %LET CODE='*';
   FILENAME HALFNORM 'HALFNORM.SAS';
   %INC HALFNORM;
   PROC SORT DATA=HALFNORM OUT=HALFNORM;
    BY ABSVAL;

  DATA CRITVAL;
   ARRAY ABSVALS{&FACTORS} ABSVAL1-ABSVAL&FACTORS;
   RETAIN R1 R2 MAXFACT MEAN HALFEFF E2 0 ABSVAL&FACTORS;
   KEEP TRUBETA TRUFACT LFE R1 R2 RS;
   TRUBETA=&BETA;
   TRUFACT=&I;
   IF _N_=1 THEN DO;
    DO N=1 TO &FACTORS;
     SET HALFNORM (RENAME=(FACTOR=MAXFACT));
     ABSVALS(N)=ABSVAL;
    END;
    R1=ABSVAL&FACTORS/ABSVAL&FIFTY;
    R2=ABSVAL&FACTORS/ABSVAL&SEVENTY;
    END;
   IF _N_=1 THEN SET TWO (KEEP=MEAN HALFEFF) POINT=MAXFACT;
   SET ONE END=EOF;
   ARRAY VARS{&FACTORS} VARS1-VARS&FACTORS;
   E=ONEFF-MEAN-HALFEFF*VARS{MAXFACT};
   E2=E2+E*E;
```

```
     IF EOF THEN DO;
       S=SQRT(E2/(&RUNS-2));
       RS=ABSVAL&FACTORS/(S/SQRT(&RUNS));
       LFE=MAXFACT;
       OUTPUT;
       END;

    PROC PRINT;
     TITLE 'Beta= ' &BETA 'Factor Number= ' &I;

    PROC APPEND BASE=ONEFFECT DATA=CRITVAL;

        DATA NEW;
        SET ONEFFECT;
        FILE ONEFFECT;
        PUT TRUBETA TRUFACT LFE R1 R2 RS;
     %END;
   %END;
  %END;
%MEND ANALYSIS;
%ANALYSIS;
RUN;
```

Sub-Program: HALFNORM.SAS

```
/*********************************************************************/
/* Creates halfnormal scores and a halfnormal plot                   */
/* from the SAS dataset &IP, using the variable &VAR                 */
/* and the coding variable &CODE for the plot codes.                 */
/* For example, to get halfnormal plots of EFFECTS in the            */
/* SAS dataset DOE, using codes CODES, say:                          */
/* %let var=effects; %let ip=doe; %let code=codes; %inc halfnorm;    */
/*********************************************************************/

data halfnorm;
  set &ip;
  absval=abs(&var);
  output;
  absval=-1+0*absval  /*= -1 or . */;
  output;
proc rank normal=blom out=halfnorm;
  var absval;
  ranks halfnorm;
data halfnorm;
  set;
  label halfnorm='Halfnormal Scores' absval="Abs(&var)";
  if absval > -.5;
/*
proc plot;
```

```
  plot halfnorm*absval=&code/hpos=40 vpos=25 vzero hzero;
  run;
*/
```

Sub-Program: HE.SAS

```
/*************************************************************************/
/* Creates half-effects and calculates the mean of the response data */
/*************************************************************************/

DATA &NEW;
SET &OLD END=LASTREC;
RETAIN HEFF1-HEFF&FACTORS FCTR1-FCTR&FACTORS;
KEEP MEAN HALFEFF FACTOR;
ARRAY HEFF {&FACTORS} HEFF1-HEFF&FACTORS;
ARRAY VARS {&FACTORS} VAR1-VAR&FACTORS;
ARRAY FCTR {&FACTORS} FCTR1-FCTR&FACTORS;
MEAN+RND;
DO I=1 TO &FACTORS;
 HEFF{I} + VARS{I}*RND;
 FCTR(I)=I;
 END;
IF LASTREC THEN DO;
 MEAN=MEAN/_N_;
 DO I=1 TO &FACTORS;
   HALFEFF=(HEFF{I})/&RUNS;
   FACTOR=FCTR(I);
   OUTPUT;
   END;
 END;
RUN;
```

Sample Output

1	1	1	2.9768905084	2.1797568889	4.4512875578
1	1	2	2.2790259746	1.4885609062	2.110497891
1	1	10	3.1304284504	1.7245647205	2.6262124743
1	1	4	2.4427598617	2.054907379	3.1176824802
1	1	9	2.6920910679	1.7085947037	2.8278967695
1	1	1	1.4931511403	1.2228486212	1.8535410653
1	1	1	3.1818592882	2.0685127503	6.4649955437
1	1	1	3.3922556793	2.1799159464	4.6465872806
1	1	10	1.7953084116	1.4219810943	1.7702986521
1	1	1	1.9714202859	1.4856449084	2.2329723708
1	1	1	2.2776447964	1.638132689	2.4980056921
1	1	3	3.1891796688	2.250666948	4.0991418059
1	1	6	2.0871479446	1.7548871919	2.7778063511
1	1	1	2.9297869382	1.5323461795	2.6700765226

1	1	1	2.754178602	2.2977223131	5.1920145397
1	1	4	2.6968414587	1.8646534134	3.3411668921
1	1	1	2.691220481	2.1857968391	4.4812594248
1	1	1	2.565030396	2.3358473071	6.284356537
1	1	2	2.7660534063	1.7101865416	2.5674997876
1	1	7	3.6072906994	2.0230107276	4.1010709713
1	1	4	2.7833119269	2.3659008447	4.6586502922
...	...	...	...	...	...

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