

Rochester Institute of Technology

RIT Digital Institutional Repository

Theses

11-4-2011

Using mathematical models in the hotel industry: Maximizing revenues through discount strategies

Karin Meuwissen

Follow this and additional works at: <https://repository.rit.edu/theses>

Recommended Citation

Meuwissen, Karin, "Using mathematical models in the hotel industry: Maximizing revenues through discount strategies" (2011). Thesis. Rochester Institute of Technology. Accessed from

This Thesis is brought to you for free and open access by the RIT Libraries. For more information, please contact repository@rit.edu.

**USING MATHEMATICAL MODELS IN THE HOTEL INDUSTRY:
MAXIMIZING REVENUES THROUGH DISCOUNT STRATEGIES**

by

Karin M. Meuwissen

with

Dr. David Rosss

Dr. Darren Narayan

K. William Gunter

A Thesis Submitted in

Partial Fulfillment of the

Requirements for the Degree of

Master of Science

in

Applied and Computational Mathematics

at

The Rochester Institute of Technology

College of Science

Department of Computational and Applied Mathematics

November 4, 2011

Mathematical modeling is an effective tool when maximizing revenue in hotels. With over 120,000 different combinations of variables and elements to consider when maximizing revenue, the process of finding the most lucrative combination can be time consuming and costly to hotels. An effective mathematical model assists in reducing the guesswork involved. This study will demonstrate how the implementation of discounts reduces revenue loss by forcing pre-payment and increasing occupancy levels.

In an effort to reduce revenue loss, hotels have implemented many strategies such as using credit card guarantees, demanding pre-payment, and offering discounts, all of which are used to reduce the no-show rate. A careful balance must be found, as offering a large discount to reduce the no-show rate can result in as much revenue loss as a high no-show rate. A mathematical model, used in combination with good judgement and managerial expertise, can assist in finding the discount necessary to maximize a hotels daily revenue.

Karin Meuwissen
Masters Thesis
Using Mathematical Models in the Hotel Industry:
Maximizing Revenues Through Discount Strategies
kmm6051@rit.edu

1 Introduction

The purpose of our paper is to develop a mathematical model which can be used to maximize revenue in hotels by exploring the effects of discounts on no-show rates and occupancy levels. In an article by Weatherford and Bodily, the authors calculated an extraordinary 124,416 combinations of variables and elements to consider when maximizing revenue. [1]. The large number of combinations makes it necessary for a hotel to limit the variables being considered and to estimate the values to assign to these variables. Estimated values can be changed and recalculated to determine their relative effects on revenue outcome. This process can be time consuming and costly to hotels until the most lucrative combination is found. Mathematical models assist in reducing the guesswork involved. Our mathematical model will enable manipulation of discounts, and will study the effect that these manipulations have on other interdependent elements, to determine their optimal values in maximizing hotel revenue. We think that using the techniques of discounts to reduce the no-show rate and increase occupancy levels can reduce revenue loss.

When a customer does not show up at the hotel in spite of having reserved a room (such a customer is called a **no-show**) and when a customer departs earlier than he had indicated when he made his reservation, (such a customer is called an **early departure**) the hotel loses revenue.

In order to prevent revenue loss, hotels have implemented strategies such as using credit card guarantees, demanding pre-payment, offering discounts to reduce the no-show rate, and overbooking. The use of **credit card guarantees** allows a customer to cancel within a limited time period prior to his reservation date. Credit card guarantees may only allow the hotel to charge the customer for one night, not multiple nights, if the renter does not fulfill the reservation as expected. However, research has shown that the implementation of credit card guarantees has had a positive effect on the no-show rate. Credit card guarantees have resulted in a reduction in the no-show rate from an average of 15 percent to an average 4 percent [1]. A **pre-payment** is a discounted payment generally made 2 to 3 weeks in advance that cannot be refunded. **Overbooking** occurs when a hotel reserves or offers more rooms than it has available. Overbooking is often necessary to compensate for the existence of no-shows. A **stayover** is a hotel guest who remains in his room one

or more nights beyond the checkout date he specified when he reserved the room. There are also laws and industry standards that certain hotels follow. For example, most state laws maintain that on a day when a hotel has been overbooked, the customers who stay over will have access to hotel rooms before new arrivals, even if the new arrivals have reservations. [1].

Although the aforementioned strategies have been effective in decreasing the no-show rate and revenue loss, hotels are still seeking a more effective method to minimize revenue loss and maximize profit. In addition, some hotels may be offering a discount to fill up their rooms, not necessarily to overbook their rooms. Therefore, in order to balance a profitable business with laws, guidelines, and customer satisfaction, the creation and implementation of revenue models has become more prevalent in the hotel industry.

2 An Overbooking Model: Rex S. Toh and Frederick Dekay

One such model is proposed by Toh and Dekay [1] in an article in the Cornell Hotel and Restaurant Administration Quarterly' titled 'Hotel Room Inventory Management: An Overbooking Model. The authors hypothesize that an overbooking model is necessary to maximize hotel revenue. Toh and Dekay make two assumptions in the creation of their model. The first is that their model could be defined as Bernoulli, which considers two situations; either the guest shows up (success), or the guest does not (failure). In addition, the authors assumed that the sampling distribution of no-shows and early departures is a normal distribution, given that the sample size is significantly large (Toh and Dekay used a sample size of 40 days). This is a result of the Central Limit Theorem, which holds that if the size of the sample is sufficiently large (it is general practice that the sample size be greater than 30), then it is conceivable that the distributions of the mean values will be approximately normal. [1] This allows Toh and Dekay to use the concept of z-scores mentioned in their model. The variables used in the model are described below:

1. The variable σ is the standard error of proportion for no-shows or early departures. The formula for the standard error of proportions is $\sqrt{\frac{pq}{n}}$.
2. The variable p is either the probability of a stayover or an expected arrival, as the authors calculate and account for both situations.
3. The variable q equals $1 - p$, the probability that a stayover or expected arrival does not occur.
4. The variable n is how many people are expected to arrive or stayover.

5. The variable C is the working inventory of rooms, which is the number of rooms a hotel has, less the unexpected stayovers. The unexpected stayovers must be subtracted because they take precedence over any new arrivals, even if those arrivals have reservations.
6. The variable z is the z-score that corresponds to the customer satisfaction rate and indicates the number of standard deviations a value is above or below the mean in the normal distribution. The authors have stated that the z-score will be 1-tailed, as the hotel only cares about protecting against the lower proportion of no-shows to prevent overselling. As a result, a 90 percent customer satisfaction rate indicates that 90 percent of the data falls below the corresponding z-score of 1.28.
7. The variable X is the authorized booking level, which is the outcome after all other variables are plugged in. For this model, X essentially represents how many rooms the hotel can book. If the hotel has 1000 rooms available, X may be 1010, indicating that it is safe, within a certain level of confidence or customer satisfaction, to overbook by 10 rooms.

Because the authors assume that the probability distribution is normal as well as Bernoulli, they have used the formula for expected value to determine X . Their model is listed below.

$$(1 - [p - z \cdot \sqrt{\frac{pq}{n}}]) \cdot X = C \quad (1)$$

Using mathematical transformations, the authors rewrote equation 1 as:

$$(1 - p)^2 X^3 - [2C(1 - p) + z^2 pq] X^2 + C^2 X = 0 \quad (2)$$

However, we found several errors in their calculations. The first was that the variable n existed in the original model, but there was no such variable in the ending model. In addition, in order to make the model have an X^3 as its first term, we had to multiply all values by X , therefore making an additional root of zero that would eventually be factored out. Even so, we manipulated equation 1 to be:

$$(1 - p)^2 X^3 - [2C(1 - p) + z^2 X \frac{pq}{n}] X^2 + C^2 X = 0 \quad (3)$$

In our first attempt, we noted that our model mirrored the authors model as long as X/n turned out to be 1.

After contacting the authors of the paper, Toh and Dekay indicated that there was an error, and that σ should have been represented by $\sqrt{\frac{pq}{X}}$ not $\sqrt{\frac{pq}{n}}$. Therefore, we were able to confirm equation 2 by performing the following manipulations:

$$\begin{aligned}
(1 - [p - z\sqrt{\frac{pq}{X}}])X &= C \\
(1 - p + z\sqrt{\frac{pq}{X}})X &= C \\
(X(z\sqrt{\frac{pq}{X}}))^2 &= (C - X(1 - p))^2 \\
Xz^2pq &= C^2 - 2CX(1 - p) + X^2(1 - p)^2 \\
0 &= X^2(1 - p)^2 - X[2C(1 - p) + z^2pq] + C^2
\end{aligned}$$

At this point we noted that in order to create Toh and Dekay's exact model, which was equation 2, we would have to multiply through by X , and would obtain $(1 - p)^2 \cdot X^3[2C(1 - p) + z^2pq]X^2 + C^2X = 0$. Although this now matched the authors' revised model, we still were unsure as to why the original model went from having one solution (as there was only one X) to three solutions, one of which was zero. In the paper, the authors state that after factoring out the X , and using the quadratic equation, as well as the idea of the discriminant, that we will now have two distinct real roots, but that only one root satisfies the original equation. [1] Although we were able to verify their model, it is our belief that the same result could be accomplished using a more effective approach. We were able to find the one value of X that we were seeking without discarding any roots. This work is demonstrated below.

$$\begin{aligned}
(1 - [p - z\sqrt{\frac{pq}{X}}])X &= C \\
[(1 - p) + z\sqrt{\frac{pq}{X}}]X &= C \\
(1 - p)X + z\sqrt{pq}\sqrt{X} &= C \\
(1 - p)X - C &= -z\sqrt{pq}\sqrt{X} \tag{4}
\end{aligned}$$

Therefore, we can see that the solution to equation 4 will be the intersection of a line with slope $(1p)$ and y-intercept of C , and the lower portion of a parabola, or quadratic

function, $(z\sqrt{pq}\sqrt{X})$. The graph below models this situation.

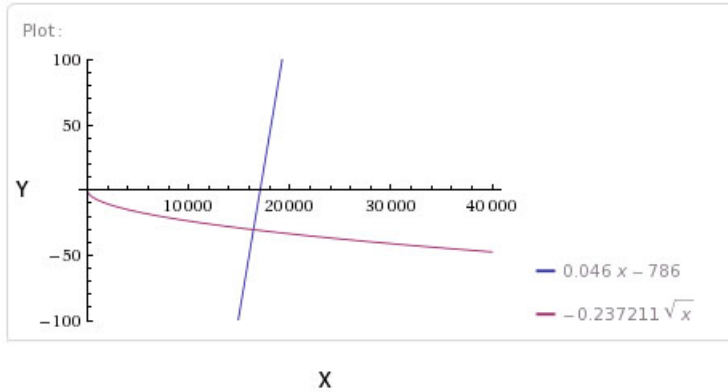


Figure 1: Shows the intersection of the linear function $(1 - p)X - C$ with the square root function $-z\sqrt{pq}\sqrt{X}$

When we substituted the values used by the authors for p , C , z , and q into equation 2, we were able to confirm the data, which showed that with 786 rooms in their inventory, the hotel can assume 813 rooms used with expected arrivals. When the authors allowed p to be the probability of a stayover as opposed to expected arrivals, the resulting X was 809. In both situations, they took the 786 available rooms and divided that number by the number of expected arrivals/stayovers to obtain a probability representing the number of rooms a hotel must remove from its working inventory.

For example, when $C = 786$ in the stayover model, the resulting X was 809 rooms to use with stayovers included, giving a value of .972. Therefore, .972 of a room must be removed from the working inventory of rooms for every one stayover booked. So, rather than remove one entire room, the hotel will only remove .972 of a room. Likewise, for the expected arrival model, the number of rooms removed for expected arrivals is now 786 out of 813, or .967. So, .967 of a room will be removed from the working inventory for every one expected arrival. However, what happens to X if the no-show rate can be reduced from 4 percent to a lower percentage? This concept will be explored later in the paper, but it seems relevant at this point to test this theory in the paper's model. Therefore, we will plug .03, .02, and .01 in for p (and .97, .98, and .99 in for q , respectively) in order to find how many rooms a hotel should set aside as the no-show rate gets reduced.

Since our only goal in plugging in different values of p is to see what happens to X ,

and therefore to the number of rooms removed from the working inventory, we will only examine equation 2 for expected arrivals. The resulting value of X will be the same as the model describing expected stayovers, as we are using the same value of p in both scenarios. After plugging $p = .03$ into the model for number of expected arrivals, the result was $X = 803.93$. The number 786 (the working inventory) was divided by $X = 803.93$ to get .978. So, at a 3 percent no-show rate, the hotel should be prepared to remove .978 of a room from its working inventory. Likewise, $p = .02$ was plugged into the model, resulting in $X = 796.88$. After dividing 786 by this value of X , it was found that the hotel should set aside .986 of a room from its working inventory when the no-show rate is reduced to 2 percent. The last no-show rate to consider, 1 percent, yielded an X value of 790.32, which when divided into 786 gave a result of .995. So, if the no-show rate is 1 percent, the hotel should set aside .995 of a room from its working inventory for every one arrival.

As the no-show rate decreases, the number of available rooms will also decrease. Therefore, the hotel must also reduce the number of rooms set aside for overbooking, as confirmed above. As far as a hotel is concerned, however, by reducing the no-show rate, seemingly profit will rise. What is the effect of the reduction of no-shows rate on profit? Is it possible for a hotel to obtain a maximum profit given all of these variables and conditions?

3 Exploring the Role of Discounts in the Creation of Revenue Models

One strategy that hotels have implemented is to offer a discount to those who pre-pay. A pre-payment reduces the revenue lost if the customer does not show because the pre-payment, although discounted, is non-refundable. One hotel, located in Miami FL, stated that they gave their customers a 5 percent discount to pre-pay, and that 15 percent of their customers took advantage of the offer. [3] Another hotel, located in Henrietta NY, gave a 15 percent discount, with 5 percent taking advantage of the discount. [2] The location of the hotels can account for the apparent discrepancy in the data. In Henrietta, due to its location and business climate in upstate New York, the hotel may have to offer a larger discount to fill up its rooms. In Miami, the rooms are more likely to fill up due to vacationers and a warm, ocean-front location.

Based on the data given by the hotels, we began to create two separate models for Henrietta and Miami exploring the relationship between percent of customers taking advantage of the discount (represented by x) and percent discount (represented by y). In order to accurately graph the different situations with only one coordinate point known for each, we worked under several assumptions. The first is that the plots were assumed to start

at the coordinate point (0,0), as nobody would want a discount of 0 percent. In addition, both plots were assumed to end at (1,1), working under the assumption that 100 percent of the people would take a discount of 100 percent. As a result, the first thought was that the function is linear. The following function demonstrates this idea:

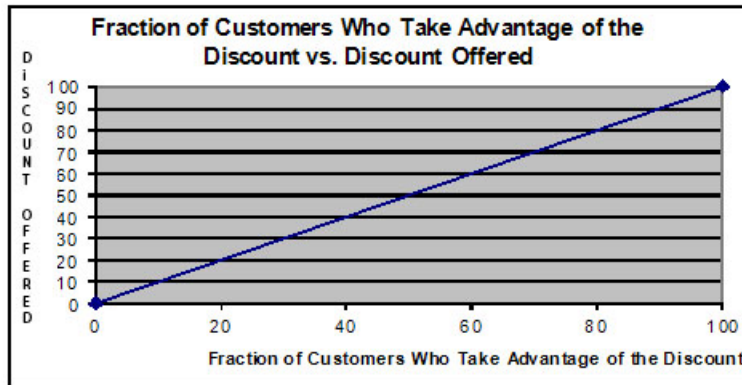


Figure 2: All models comparing percent of customers who take the discount and the discount offered are assumed to contain the point (0,0) (0 percent takes a 0 percent discount), and (1,1) (100 percent of the customers take a 100 percent discount.)

The second assumption was that at a certain percentage off, regardless of location, a significant number of people would take advantage of the discount. For both functions, we made the assumption that 60 percent of people would take advantage of an 80 percent discount and 75 percent of people would take advantage of a 90 percent discount. So, in addition to the points (0,0) and (1,1), the points (.75,.90) and (.60,.80) were added to both plots. However, one plot (Miami) had the point (.15,.05), and the other plot (Henrietta) had the point (.05, .15), as previously described. These points gave the following two graphs.

MIAMI PLOT 1

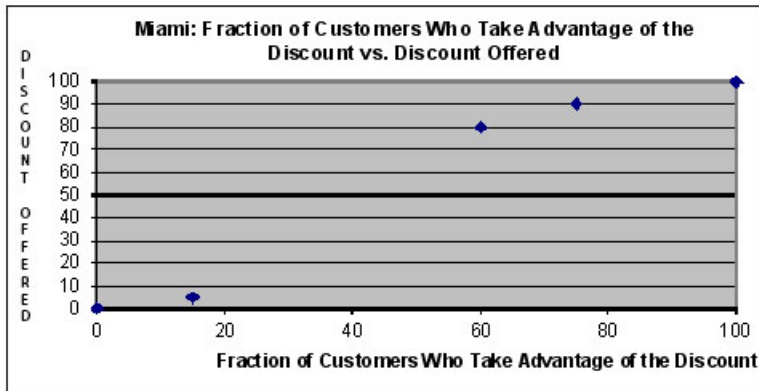


Figure 3: Data points (.60, .80) and (.75, .90) were assumed to be true for any hotel model. The point (.15, .05) is a known point for the hotel in Miami, FL.

HENRIETTA PLOT 1

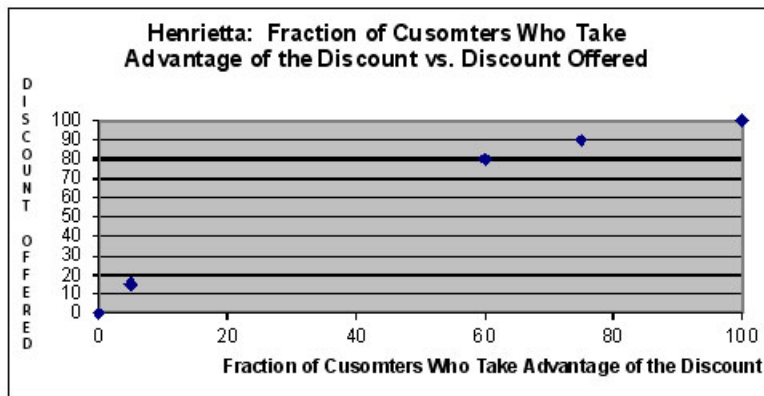


Figure 4: Data points (.60, .8) and (.75, .90) were assumed to be true for any hotel model. The point (.05, .15) is a known point for the hotel in Henrietta, NY.

In order to effectively fill in the rest of the points, several more assumptions were made about each plot. In Miami we assumed fewer people took advantage of the discount in the beginning and many people took advantage of the discount when a larger discount was offered. We also assumed that somewhere near (.50,.50), the function changed from concave up to concave down, and became S-like. Therefore, the points (.25,.10), (.40,.30), and (.50,.50) were filled in. This plot is shown below.

MIAMI FINAL FUNCTION

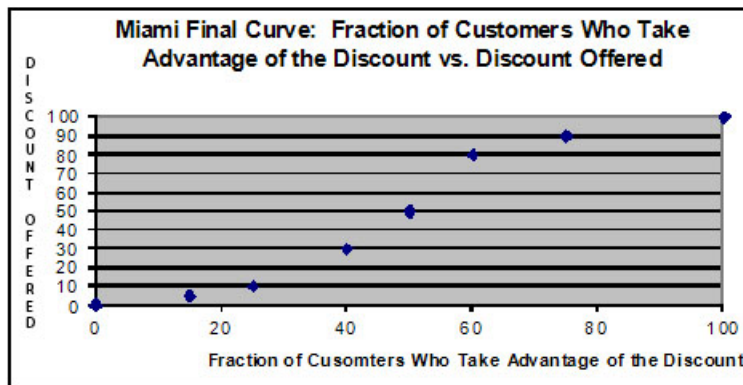


Figure 5: Due to location and climate, it is assumed that more people will take advantage of a smaller discount in Miami, FL.

In order to represent these data points as a function, we found a regression function that passes through these data points. We found that the data points were most accurately represented by the following function:

$$a_m(x) = -.71x^4 - 2.22x^3 + 4.49x^2 - .57x + .01 \quad (5)$$

This regression function has an R^2 value of .988. This function is graphed below with a domain $[0,1]$.

MIAMI FINAL PLOT AS FUNCTION

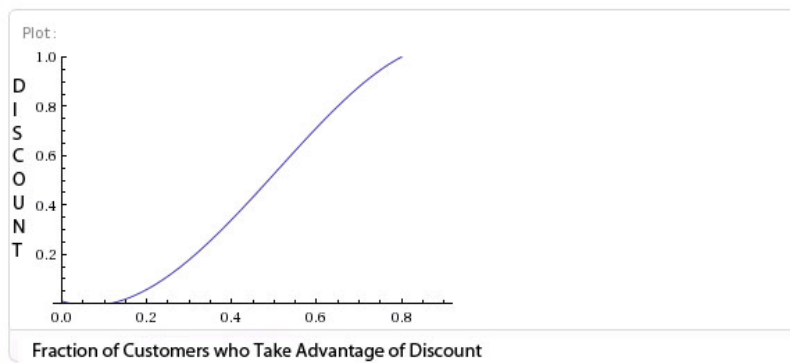


Figure 6: The data points from Figure 5 are shown as a function based on equation 5.

The same approach was taken to complete the Henrietta function. This function was assumed to be concave down for its entirety, therefore making it quadratic in nature. As a result, the points $(.25,.40)$, $(.40,.60)$, and $(.50,.70)$ were added. The function representing Henrietta is below.

HENRIETTA FINAL FUNCTION

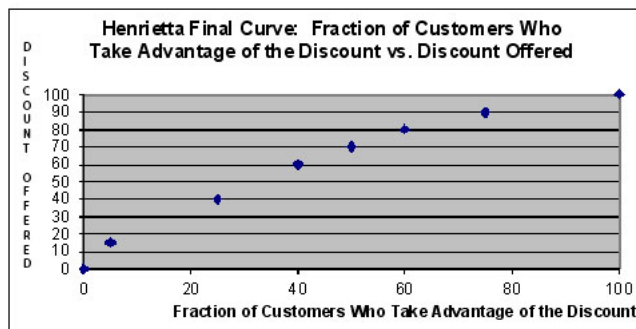


Figure 7: Due to location and climate, it is assumed that fewer people will take advantage of a larger discount in Henrietta, NY.

Again, using regression, we found the Henrietta function to be most accurately represented by the following function:

$$a_h(x) = -1.22x^4 + 2.44x^3 - 2.29x^2 + 2.04x + .01 \quad (6)$$

This regression function has an R^2 value of .998. This function is graphed below under the domain $[0,1]$.

HENRIETTA FINAL PLOT AS FUNCTION

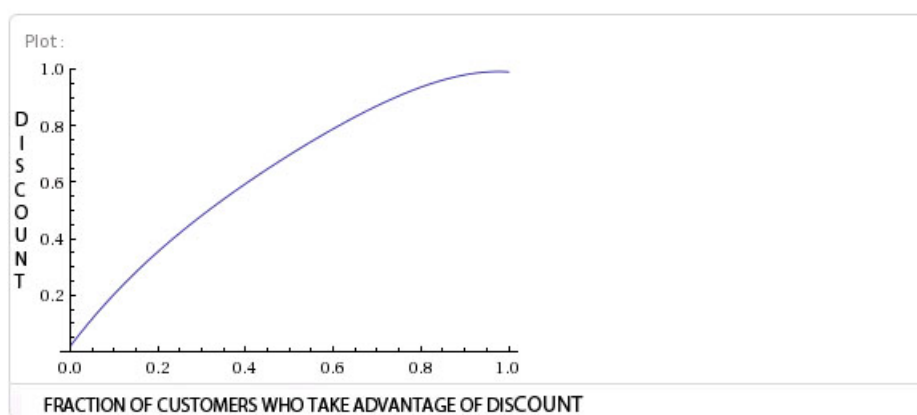


Figure 8: The data points from Figure 7 are shown as a function based on equation 6.

Given the two final functions, we wanted to study what would happen to the revenue as we moved along each graph. After creating a function to represent how the revenue was calculated, the revenue was then graphed based on that function. In order to do so, we had to make the following assumptions:

1. Four percent of the customers would still be no-shows. However, the percentage of people who took advantage of the discount was calculated before the four percent no-show rate was deducted.
2. Based on the information given in the article, hotels set about two percent of their rooms aside for stayovers. [1] As a result, we set the number of available rooms to be 98 percent of the number of total hotel rooms, or $.98H$.

3. The variables in the function are as follows: the variable H is the total number of hotel rooms in that hotel, the variable C is the cost of the hotel room per night, the variable x is the percentage of people that take advantage of the discount, and the variable y is the percent discount given. The function $r(x, y)$ is the revenue.
4. People renting the two percent of rooms that were set aside for stayovers will pay full price, as will the rest of the customers who did not take advantage of the discount. The four percent of no-shows will be considered lost revenue, and the percent who take advantage of the discount will pay the discounted price.

Given this information, we came up with the following function:

$$r(x, y) = C(H - A \cdot H \cdot x) + A \cdot H \cdot x \cdot (1 - y) \cdot C - C \cdot N \cdot A \cdot H \cdot (1 - x) \quad (7)$$

For equation 7, $C(H - A \cdot H \cdot x)$ represents those rooms that command full price. These rooms include the two percent set aside for stayovers, as well as the rooms occupied by those customers who did not book at a discounted rate. The section of the equation listed as $A \cdot H \cdot x \cdot (1 - y) \cdot C$ represents rooms occupied by patrons who took advantage of the discount. The section $C \cdot N \cdot A \cdot H \cdot (1 - x)$ represents the 4 percent of no-show rooms that remain vacant.

Note: In our case, the variable $A = .98$, as the hotel has set aside two percent of its rooms for stayovers. If a hotel sets aside more or less than two percent, then A should reflect that change as $A = (1 - \text{fraction of rooms set aside})$. In addition, the variable N can be adjusted to reflect the no-show rate of the particular hotel. For our purposes, $N = .04$.

Using equation 7, we plugged each of the points from the Henrietta final function and the Miami final function into the equation for revenue. For the sake of simplicity, we assumed each hotel had 1000 rooms. The given cost of the Miami hotel is \$299.00 per night, while the given cost of the Henrietta hotel is \$119.00 per night. [2][3] The following graphs were produced:

MIAMI REVENUE AT EACH POINT ON ITS PLOT

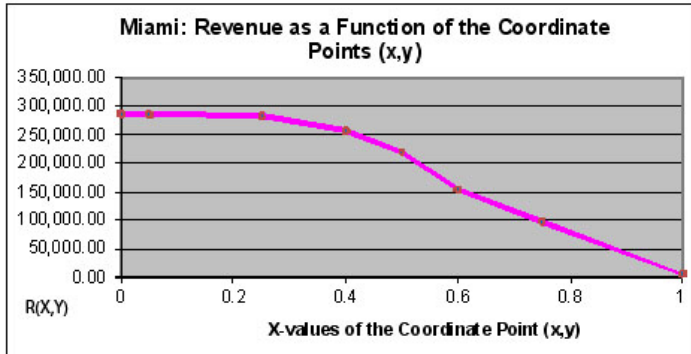


Figure 9: Each coordinate value (x, y) in the graph representing fraction of customers who take advantage of the discount and discount offered in Miami is substituted into $r(x, y)$, and the corresponding revenue is found.

HENRIETTA REVENUE AT EACH POINT ON ITS PLOT

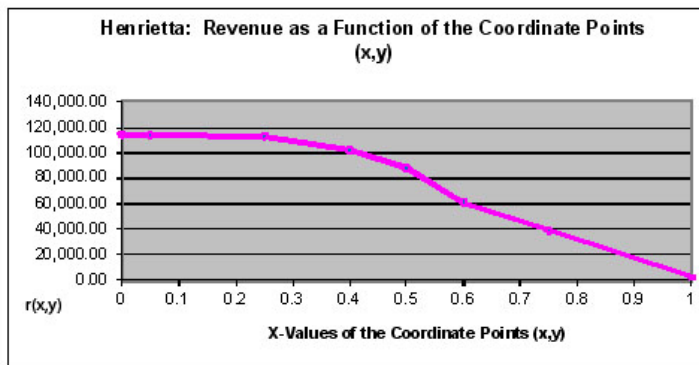


Figure 10: Each coordinate value (x, y) in the graph representing fraction of customers who take advantage of the discount and discount offered in Henrietta is substituted into $r(x, y)$, and the corresponding revenue is found.

4 Using Pre-Payment Discounts to Reduce No-Show Rates

As Figure 9 and Figure 10 show, the revenue is highest for each hotel when nobody takes advantage of the discount. Therefore, if the no-show rate stays at 4 percent, it would be more beneficial for the hotels to offer no discounts to their customers. However, it seems reasonable that the no-show rate will reduce if pre-payment is utilized, because some of those who may have been no-shows have now pre-paid, and will no longer count as no-shows.

We began to consider what would happen if the no-show rate does go down as the discount goes up. Given that each function, both for Miami (equation 5) and for Henrietta (equation 6), has eight points as shown in Figure 5 and 7, we reduced the no-show rate by assuming that as the discount increased more people would take advantage of it. Seemingly the people that would take advantage of it would come from both the people who were going to show up, and the people who were not. For example, if a hotel has 1000 rooms with a no-show rate of 4 percent, then 960 rooms will pay full price, and 40 rooms will have been booked for which payment has not been made. Let's assume that a discount of 25 percent is offered. This will entice some people to pre-pay. A pre-payment does not guarantee that a customer will show. As far as hotel revenue is concerned, this customer is no longer in the no-show category as the hotel has his/her money. Some of the 960 people who were paying full price will be enticed to take advantage of the discount, as will some of the 40 original no-shows. Now let us assume that of the 960 people, 100 take advantage of the discount and pay at a discounted rate. Also, 5 of the 40 no-shows now take advantage of the discount and pay at a discounted rate. Therefore, although 40 people may not show up, we still have only lost revenue for 35 people. This reduces the no-show rate (the non-payment rate) from 4 percent to 3.5 percent.

Using this logic, as we moved along both the Miami function (Equation 5, Figure 5) and the Henrietta function (Equation 6, Figure 7), and as the percent discount increased, we also assumed that the no-show rate decreased. However, we were not comfortable associating one point to one no-show rate. Since many assumptions are being made, we felt it safer to say a 4 percent no-show rate could happen at (0,0), (in other words, no discount is offered, so nobody has taken advantage of the discount, and as a result the no-show rate stays at 4 percent), or it could happen at (.15,.05), (even though a 5 percent discount is offered, and 15 percent of the people take it, this does not guarantee that the no-show rate will reduce). However, there is a chance that at (.15,.05) the no-show rate will in fact go down. Therefore, we also put the point (.15, .05) under a possibility of a 3 percent no-show rate. This practice of overlapping was used as we moved along both functions. We also made the assumption that the only time a no-show rate of 0 percent would occur is at the point (1,1), meaning that everybody has received a free room, and therefore there will be no no-shows. The organization of the points that we associated

with each no-show rate is shown below.

MIAMI

4 PERCENT NO SHOW RATE
(0,0), (.15,.05), (.25,.10)

3 PERCENT NO SHOW RATE
(.15, .05), (.25, .10), (.40, .30)

2 PERCENT NO SHOW RATE
(.40, .30), (.50, .50), (.60, .80)

1 PERCENT NO SHOW RATE
(.60, .80), (.75, .90)

0 PERCENT NO SHOW RATE
(1,1)

HENRIETTA

4 PERCENT NO SHOW RATE
(0,0), (.05, .15), (.25, .40)

3 PERCENT NO SHOW RATE
(.05, .15), (.25, .40), (.40, .60)

2 PERCENT NO SHOW RATE
(.40, .60), (.50, .70), (.60, .80)

1 PERCENT NO SHOW RATE
(.60, .80), (.75, .90)

0 PERCENT NO SHOW RATE
(1,1)

By plugging the points in for x and y , and replacing the no-show rate with the percent given, we were able to find revenues for each of these points using equation 7. For each percentage there was a range of revenues, as there were sometimes three points considered at each no-show rate. The graphs representing the revenues associated with the points are listed below.

MIAMI REVENUE AS NO-SHOW RATE DECREASES

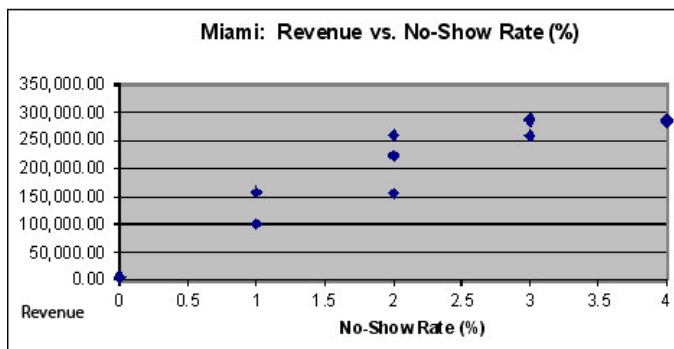


Figure 11: Shows the relationship between the discount and its effect on the no-show rate in Henrietta. These values were then plugged into the equation 7 and revenue was found.

HENRIETTA REVENUE AS NO-SHOW RATE DECREASES

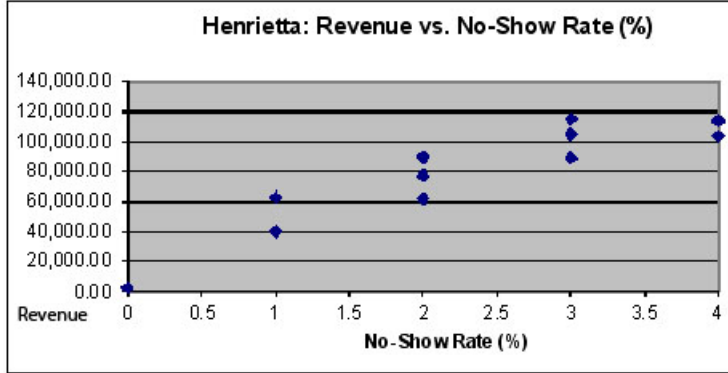


Figure 12: Shows the relationship between the discount and its effect on the no-show rate in Miami. These values were then plugged into equation 7, and $r(x, y)$, or revenue, was found.

Figures 11 and 12 show that the highest revenue occurs at a 3 percent no-show rate with a fairly small discount (5 percent for Miami and 15 percent for Henrietta) and with a small percentage of customers taking advantage of the discount (15 percent for Miami and 5 percent for Henrietta).

We also attempted to maximize revenue by minimizing the revenue loss. We simplified equation 7 through the following transformations:

$$\begin{aligned}
 r(x, y) &= C(H - .98Hx) + .98H \cdot x \cdot (1 - y) \cdot C - C \cdot N \cdot (.98H) \cdot (1 - x) \\
 r(x, y) &= CH[1 - .98x + .98x(1 - y) - .98N(1 - x)] \\
 r(x, y) &= CH[1 - .98(x - x(1 - y) + N(1 - x))] \tag{8}
 \end{aligned}$$

At this point, we turned both x and N into functions of y because we considered the percentage of people who take advantage of the discount, along with the no-show rate, to be dependent on the discount. This follows our previous assumptions that a larger discount encourages more people to take advantage of it, a greater number of people to pre-pay

and, therefore, a smaller number of no-shows. As a result, we set x equal to the function $a(y)$ ($a_m(y)$ for Miami and $a_h(y)$ for Henrietta), and we set N equal to the function $N(y)$ ($N_m(y)$ for Miami and $N_h(y)$ for Henrietta). Now, the equation from above is as follows:

$$r(a(y), y) = CH[1 - .98(a(y) - a(y)(1 - y) + N(y)(1 - a(y)))]$$

$$r(a(y), y) = CH[1 - .98(a(y)(1 - (1 - y)) + N(y)(1 - a(y)))]$$

$$r(a(y), y) = CH[1 - .98(y \cdot a(y) + N(y)(1 - a(y)))] \quad (9)$$

In order to find $a(y)$, we used the regression equations (previously labeled $a_m(x)$ and $a_h(x)$) that we had already found for the functions comparing percentage of customers who take advantage of the discount vs. percent discount for Miami and for Henrietta. However, the original functions (equations 5 and 6) were in terms of x , so these new regression functions were transformed to be functions in terms of y . Although these functions are not completely accurate, they are a reasonable place to start when plugging in for $a_m(y)$ and $a_h(y)$. The regression functions of best fit, now functions in terms of y , are listed below.

MIAMI

$$a_m(y) = .26y^4 + 3.08y^3 - 4.94y^2 + 2.58y + .02 \quad (10)$$

HENRIETTA

$$a_h(y) = 3.42y^4 - 5.97y^3 + 3.65y^2 - .10y + .0003 \quad (11)$$

In order to find $N(y) = N$, which is the function comparing the no-show rate with the percent discount, we used the premise that as the discount increases, more people will take advantage of it, and more people will pre-pay, thereby decreasing the no-show rate. Bascially, we split the graphs into four sections. As we move along the function and the discounts increase, the no-show rates decrease. By using the same overlapping no-show rates and discount rates as we did previously, we can create a new graph, and then find a regression equation for that graph, which will then represent $N(y)$.

By letting the independent variable be the discount, the dependent variable be the no-show rate, and the function be $N(\text{percent discount}) = \text{no-show rate}$, we have the following points to plot based on the previous graph.

MIAMI

$N(0) = .04$
 $N(.05) = .04$
 $N(.10) = .04$
 $N(.05) = .03$
 $N(.10) = .03$
 $N(.30) = .03$
 $N(.30) = .02$
 $N(.50) = .02$
 $N(.80) = .02$
 $N(.80) = .01$
 $N(.90) = .01$
 $N(1) = 0$

HENRIETTA

$N(0) = .04$
 $N(.15) = .04$
 $N(.40) = .04$
 $N(.15) = .03$
 $N(.40) = .03$
 $N(.60) = .03$
 $N(.60) = .02$
 $N(.70) = .02$
 $N(.80) = .02$
 $N(.80) = .01$
 $N(.90) = .01$
 $N(1) = 0$

The graphs for each city are shown below.

MIAMI REVENUE AS NO-SHOW RATE DECREASES - IN TERMS OF Y

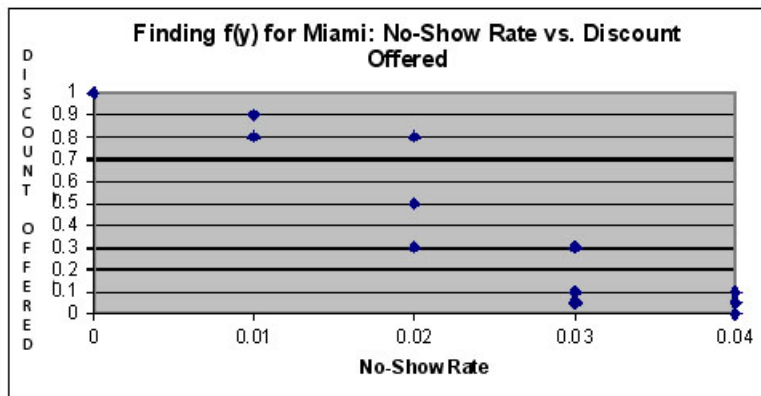


Figure 13: This graph is used to assist in the creation of a function representing relationship between the discount and its affect on the no-show rate in Miami in terms of the discount offered.

HENRIETTA REVENUE AS NO-SHOW RATE DECREASES - IN TERMS OF Y

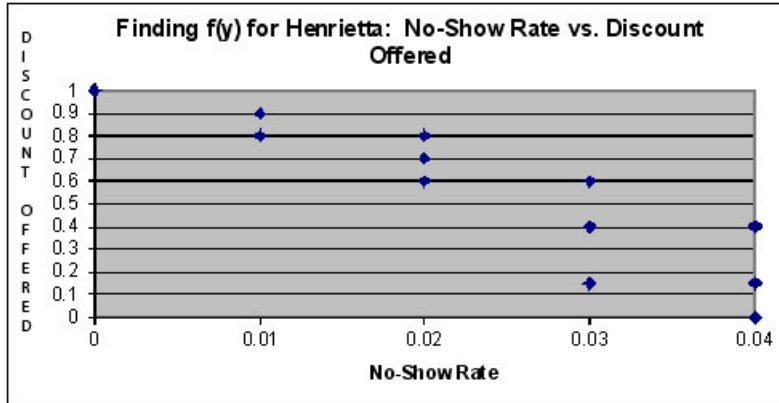


Figure 14: This graph is used to assist in the creation of a function representing relationship between the discount and its affect on the no-show rate in Henrietta in terms of the discount offered.

Using a cubic regression for Figures 13 and 14, $N(y)$ was found to be the following for each city:

MIAMI

$$N_m(y) = -.10y^3 + .15y^2 - .09y + .04 \quad (R^2 = .885) \quad (12)$$

HENRIETTA

$$N_h(y) = -.02y^3 - .01y^2 - .01y + .04 \quad (R^2 = .879) \quad (13)$$

Given the high R^2 values, it is safe to say that these functions are an acceptable fit for the data points. These functions are graphed below.

MIAMI REVENUE AS NO-SHOW RATE DECREASES - AS A FUNCTION $F(Y)$

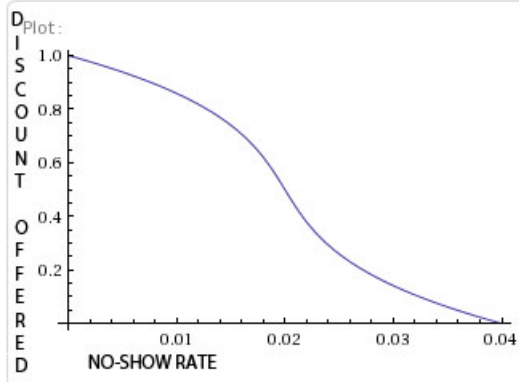


Figure 15: The data points from Figure 13 are graphed as equation 12

HENRIETTA REVENUE AS NO-SHOW RATE DECREASES - AS A FUNCTION $F(Y)$

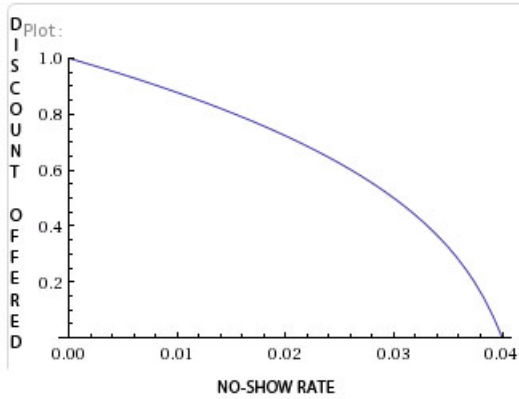


Figure 16: The data points from Figure 14 are graphed as equation 13.

Therefore, by using regression equations, we now know that we can plug in equations 10, 11, 12, and 13 for $a_m(y)$, $a_h(y)$, $N_m(y)$, and $N_h(y)$, respectively.

Using these regression equations in conjunction with equation 9 representing $r(a(y), y)$ above, we now have specific equations to calculate the revenue for Miami and Henrietta.

MIAMI

$$r(a(y), y) = CH[1 - .98(y \cdot (.26y^4 + 3.08y^3 - 4.94y^2 + 2.58y + .02) + (-.10y^3 + .15y^2 - .09y + .04(1 - (.26y^4 + 3.08y^3 - 4.94y^2 + 2.58y + .02)))))] \quad (14)$$

HENRIETTA

$$r(a(y), y) = CH[1 - .98(y \cdot (3.42y^4 - 5.97y^3 + 3.65y^2 - .10y + .0003) + (-.02y^3 - .01y^2 - .01y + .04(1 - (3.42y^4 - 5.97y^3 + 3.65y^2 - .10y + .0003)))))] \quad (15)$$

Starting with Henrietta, we first minimized the revenue loss (represented by $y \cdot (3.42y^4 - 5.97y^3 + 3.65y^2 - .10y + .0003) + (-.02y^3 - .01y^2 - .01y + .04(1 - (3.42y^4 - 5.97y^3 + 3.65y^2 - .10y + .0003)))$), and maximized equation 15 to find at what y value the revenue will be at its highest. We are letting C be \$119.00 and H is set equal to 1000. To start, we simplified both what needs to be minimized (revenue loss) and what needs to be maximized (revenue) to read as follows:

MINIMIZE REVENUE LOSS

$$.06y^7 - .06y^6 + 3.43y^5 - 6.09y^4 + 3.88y^3 - .25y^2 - .002y + 0.04$$

MAXIMIZE REVENUE MADE

$$r(a(y), y) = -7345y^7 + 7348y^6 - 399864y^5 + 710426y^4 - 452007y^3 + 29375y^2 + 226y + 114535 \quad (16)$$

Both the minimization of the revenue loss and maximization of the revenue should produce the same value for y .

In order to minimize the revenue loss, the equation was graphed with a domain of $[0,1]$. In addition, we took the derivative and found the zeros between 0 and 1 to verify that what the original graph had indicated was correct. This work is shown below.

HENRIETTA - MINIMIZING REVENUE LOSS

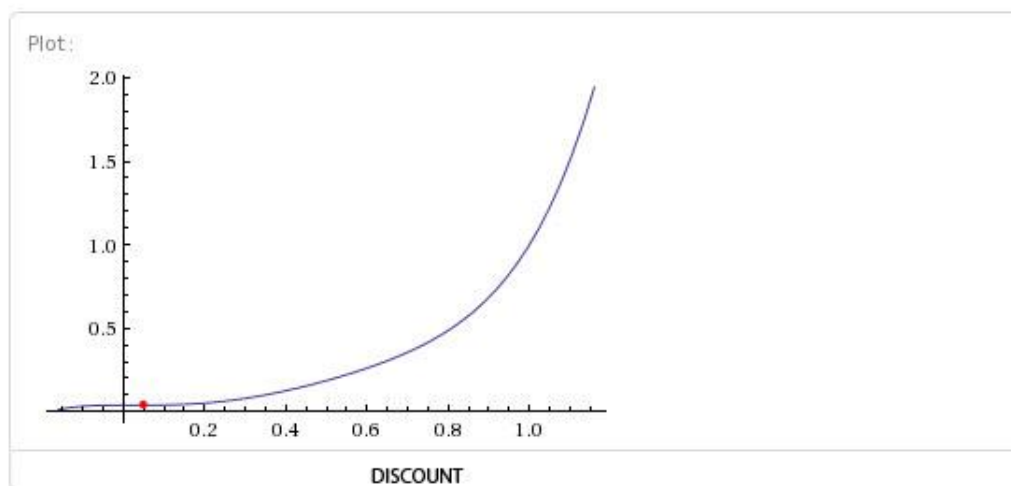


Figure 17: The red point on the graph indicates that the least amount of revenue loss occurs at a 5.2 percent discount in Henrietta.

It can be seen from the Figure 17 that the minimum amount of revenue loss will occur at a y value of approximately .0519, which would imply that Henreitta will make its maximum revenue when they offer a 5.2 percent discount. To verify this, we took the derivative of what we were trying to minimize ($.06y^7 - .06y^6 + 3.43y^5 - 6.09y^4 + 3.88y^3 - .25y^2 - .002y + 0.04$) and found it to be $.44y^6 - .38y^5 + 17.14y^4 - 24.37y^3 + 11.63y^2 - .50y - .002$. We then graphed this function from 0 to 1 to find its zeros. This graph is shown below.

HENRIETTA - DISCOUNT WHERE DERIVATIVE IS EQUAL TO 0

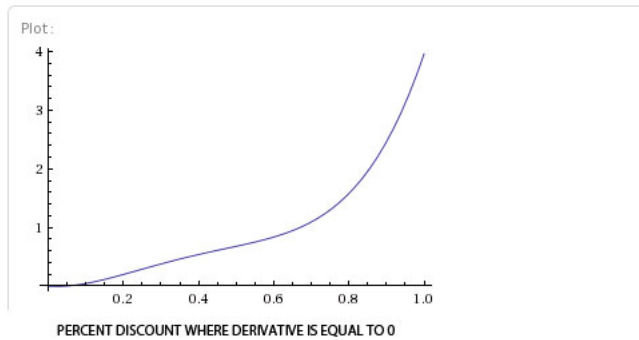


Figure 18: The graph of the derivative of $(.06y^7 - .06y^6 + 3.43y^5 - 6.09y^4 + 3.88y^3 - .25y^2 - .002y + 0.04)$ verifies that the optimal discount is 5.2 percent in Henrietta, NY.

Figure 18 verifies that a zero occurs at approximately .0519, indicating that the maximum revenue will occur in Henrietta when a 5.2 percent discount is offered.

Now that we can see what the discount will be, we want to maximize the revenue to find out how much money the hotel would make at the 5.2 percent discount. As a result, we then maximized equation 16 by graphing it between 0 and 1 and finding its highest point. The y value should remain the same at .052, but what is found for the $r(a(y), y)$ will tell us what the maximum revenue will be. This graph is shown below.

HENRIETTA MAXIMUM REVENUE

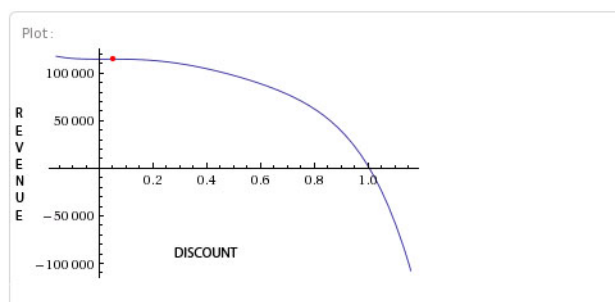


Figure 19: At 5.2 percent, this graph shows the maximum revenue to be \$114,568.00.

Figure 19 verifies that the y remained the same, and the maximum $r(a(y), y)$ value for Henrietta is \$114,567.71.

We will work with the Miami data in the same way, that is with the intention of minimizing the revenue loss and maximizing the revenue. We will minimize the revenue loss (represented by $[y \cdot (.26y^4 + 3.08y^3 - 4.94y^2 + 2.58y + .02) + (-.10y^3 + .15y^2 - .09y + .04(1 - (.26y^4 + 3.08y^3 - 4.94y^2 + 2.58y + .02)))]$), and maximize the revenue represented by the equation 14 when replacing C with \$299.00 and H with 1000. Both equations have been simplified to make substitutions easier.

MINIMIZE REVENUE LOSS

$$.03y^7 + .28y^6 - .70y^5 + 4.35y^4 - 5.98y^3 + 3.15y^2 - .17y + .04$$

MAXIMIZE REVENUE MADE

$$r(a(y), y) = -7770y^7 - 82350y^6 + 206126y^5 - 1274950y^4 + 1752870y^3 - 924298y^2 + 50034y + 287490 \quad (17)$$

Just as in the case with Henrietta, in order to minimize the revenue loss, we graphed the equation from 0 to 1. In addition, the derivative was found and the zeros between 0 and 1 were found to verify the results. This work is shown below.

MIAMI - MINIMIZING REVENUE LOSS

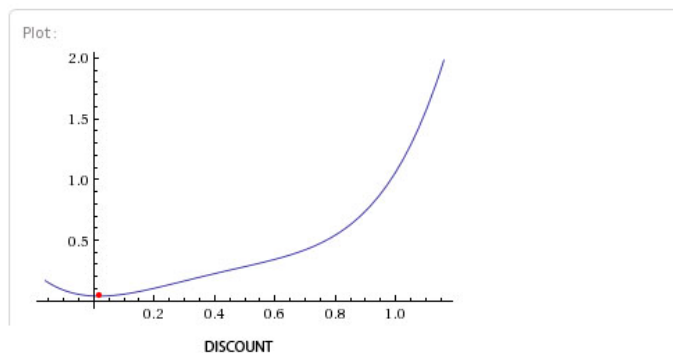


Figure 20: The red point on the graph indicates that the least amount of revenue loss occurs at a 2.9 percent discount in Miami.

Figure 20 shows the maximum value of y occurring at approximately .0294, which would imply that Miami should offer a discount of 2.9 percent in order to maximize its revenue. To verify this result, we took the derivative of what we were trying to minimize ($.03y^7 + .28y^6 - .70y^5 + 4.25y^4 - 5.98y^3 + 3.15y^2 - .17y + .04$) and found it to be $.19y^6 + 1.69y^5 - 3.52y^4 + 17.40y^3 - 17.95y^2 + 6.31y - .17$. At this point, we graphed this function from 0 to 1 to find its roots. This graph is shown below.

MIAMI - DISCOUNT WHERE DERIVATIVE IS EQUAL TO 0

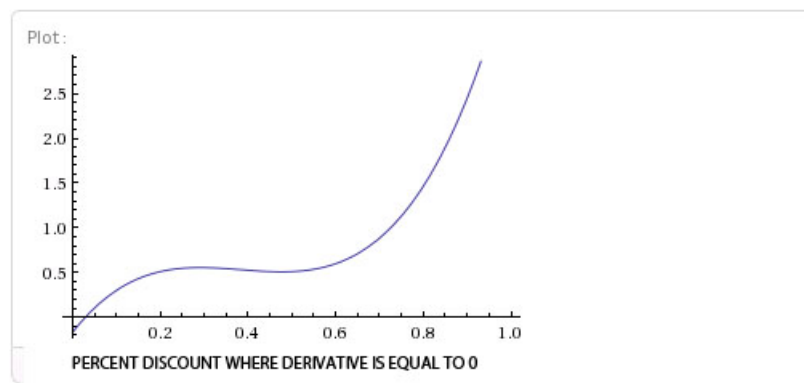


Figure 21: The graph of the derivative of $.03y^7 + .28y^6 - .70y^5 + 4.25y^4 - 5.98y^3 + 3.15y^2 - .17y + .04$ verifies that the optimal discount is 2.9 percent in Miami, FL.

Figure 21 verifies that a zero occurs at approximately .029, and therefore the revenue is maximized at 2.9 percent.

Now that we have found the 2.9 percent discount, we want to find the revenue of the hotel at that discount. As a result, we maximized the function represented by equation 17 by graphing it between 0 and 1 and finding its highest point. This graph is shown below.

MIAMI MAXIMUM REVENUE

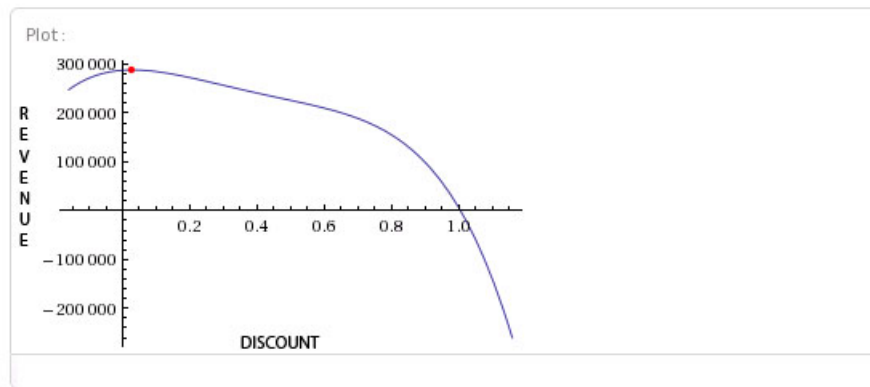


Figure 22: At 2.9 percent, this graph shows the maximum revenue to be \$288,205.68

It can be seen from Figure 22 that y remained .029, and the maximum $r(a(y), y)$ value for Miami is \$288,205.68.

Because the Miami functions $a_m(y)$, $N_m(y)$ and the Henrietta functions $a_h(y)$, $N_h(y)$ were different, there are different values for y when maximizing revenue. Since both the graph for Miami and the graph for Henrietta took into account a declining no-show rate as the discount increases, (this was done by the regression equations $a_m(y)$, $a_h(y)$, $N_m(y)$ and $N_h(y)$), it seems reasonable to predict that a relatively small discount rate will maximize revenue for both hotels.

Since working with equation 16 indicated that Henrietta's revenue is maximized at 5.2 percent during a normal business week, it seems likely that they are offering a 15 percent discount for reasons other than reducing the no-show rate. It is our assumption that this inflated discount was implemented to fill up empty hotel rooms as a result of reduced business.

5 Manipulating Discount Levels to Optimize Revenue

Next we wanted to consider what would happen to a hotel's revenue when a certain percentage of customers take a specific discount. For example, let us say that a hotel offers a 5 percent discount. What happens when 10 percent of the customers take the discount, or 50 percent, or 100 percent? In order to do this, we assumed the following:

1. We let the cost of the hotel room be \$299.00. However, the results will not change based on the cost of the hotel.
2. As more customers take advantage of the discount, the no-show rate will decrease. If an increased number of customers are paying a smaller rate, it seems reasonable to assume that more customers will pre-pay. Therefore the amount of customers who are considered no-shows, in other words customers who do not pay, will decrease. If a person pre-pays, but still does not show up, this person is no longer a no-show because the hotel has his/her money.
3. We decreased the no-show rate by one percentage each time we increased the percent of customers who take advantage of the discount by 25 percent. For example, when 0 - 24 percent of the customers taking advantage of the discount, we assumed a 4 percent no-show rate. In addition, when 25 - 49 percent of the customers paying the discounted rate, we assumed a 3 percent no-show rate, when 50 - 74 percent we assumed a 2 percent no-show rate, and when 75 - 99 percent we assumed a 1 percent no-show rate. The only time we assumed a 0 percent no-show rate was if every customer paid the discounted rate.

This process began by looking at what we considered to be a reasonable hotel discount. Since most discounts are done in 5 percent increments, we looked at the discount values of 5 and 10 percent first. Given that Figures 19 and 22, representing Henrietta and Miami, indicated that a smaller discount maximized their revenue (5.2 percent and 2.9 percent, respectively) we started by examining a 5 percent discount. After substituting this discount and the different no-show rates into the function for $r(a(y), y)$ (equation 9), and then dealing solely with equation 14 representing Miami for the sake of simplicity, we developed the following graph.

REVENUE AT 5 PERCENT DISCOUNT RATE

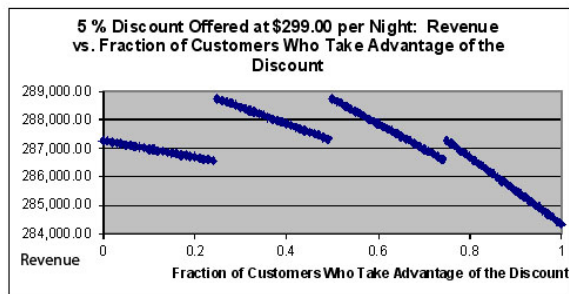


Figure 23: At a five percent discount, the highest revenue occurs at $y = .25$.

Figure 23 illustrates that the highest revenues will occur when 25 percent of the customers take advantage of the discount and when there is a 3 percent no-show rate, or when 50 percent of the customers take advantage of the discount and there is a 2 percent no-show rate. The revenue at these points, again with a rate of \$299.00 per night, was \$288,744.30. It can be seen that this graph has a parabolic shape. From here we looked at a 10 percent discount only to continue moving in increments of 5 percent. Again, we decreased the no-show rate as the percent of customers taking advantage of the discount increased. The following graph was produced.

REVENUE AT 10 PERCENT DISCOUNT RATE



Figure 24: At a ten percent discount, the graph indicates that the hotel makes the most revenue if nobody takes advantage of the discount. A ten percent discount is not advantageous to a hotel's revenue.

Figure 24 clearly shows that it does not benefit the hotel to offer a 10 percent discount, as the highest revenue of \$287,279.20 occurred when nobody took the discount. From this point, we wondered at what percentage it became a disadvantage for a hotel to offer a discount. For example, at a 5 percent discount the revenue increased, assuming that the no-show rate decreased (as seen in Figure 23) . However, at 10 percent, it was not advantageous for a hotel to offer a discount, regardless of the fact that the no-show rate decreased (as seen in Figure 24). We tested discounts of 1 percent, 2 percent, etc. until we were able to see at what percentage a discount was no longer advantageous to a hotel's revenue. We found that percentage to be an 8 percent discount. An 8 percent discount is the first time, regardless of a decreased no-show rate, that a hotel's revenue is maximized when nobody takes advantage of the discount. We then graphed all of these graphs together on one graph to illustrate what discounts are advantageous to a hotel.

REVENUE AT VARYING DISCOUNT RATES

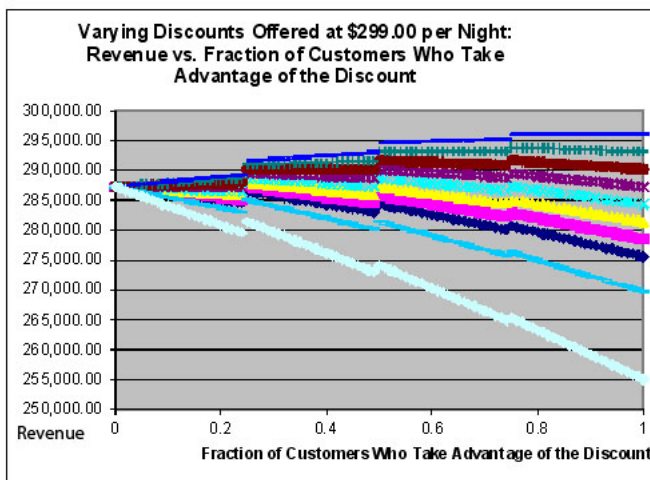


Figure 25: The pink line and the dark blue line directly below it indicate that at between 7 and 8 percent, it no longer benefits the hotel to offer a discount in the hopes of decreasing the no-show rate to gain revenue.

The highest revenue occurs at a small discount with a large amount of customers taking advantage of that discount. As the discount increased beyond this threshold, hotel revenue decreased. As a result of Figure 25, the conclusion can be drawn that if a hotel is trying to eliminate no-shows while keeping with the idea of maximizing revenue, the hotel should offer no discount greater than 7 percent, because at 8 percent the hotel will start to lose revenue if all rooms are booked. This seemed to verify what Figures 19 and 22 suggested; a smaller discount is more beneficial in maximizing revenue when the hotel's rooms are all filled.

Because the results suggested that a significant discount would cause a hotel's revenue to decrease, we began to explore why the hotel in Henrietta would offer a 15 percent discount. We also researched the highest discount offered by a hotel, finding that there have been discounts offered as high as 60 percent [4]. It seems reasonable that hotels offering such large discounts are not doing so in order to reduce their no-show rate, but rather to fill up empty rooms during a slow business week.

6 Using Discounts to Increase Occupancy Levels

When a hotel needs to fill its rooms given that a certain percentage of rooms are already filled, a hotel may offer a specific discount based on the percentage of rooms filled. Based on the amount of the discount, a certain percentage of new customers will take advantage of it, thereby filling up the empty rooms. As a result, we have assigned the following variables:

Let N = the no-show rate

Let y = the decimal discount offered by the hotel

Let θ = the number of rooms booked at the discounted rate

Let z = the fraction of rooms that were originally filled

We will continue to assume that H is the number of hotel rooms that a hotel has available to rent, and C is the cost of the room before any discount is given. To create a function that can be used to calculate revenue, we assumed that N , z , H , and C are all constants that a hotel would know. Of the remaining variables, y and θ , we have assumed that y is the independent variable, and θ is dependent, as we believe that the number of rooms booked at the discounted rate will depend on the size of the discount. It seems reasonable to assume that as the discount increases, the number of rooms booked at that discount will increase as well. Therefore, we have set $\theta = u(y) = ky$. The value of the parameter k may depend on the location of the hotel, among other factors. The function $r(y)$, used to calculate revenue, can be represented by:

$$r(y) = HC[z \cdot (1 - N)] + (1 - z) \cdot (u(y) = ky = \theta) \cdot (1 - y) \cdot C \quad (18)$$

Using the model representing revenue, we took the derivative of $r(y)$ in equation 18 and set it equal to zero to find at what discount value of y the hotel will maximize its revenue.

This work, and the resultant y , is shown below.

$$\begin{aligned} r'(y) &= (1 - z) \cdot k \cdot C - 2ky \cdot (1 - z) \cdot C = 0 \\ 2ky \cdot (1 - z) \cdot C &= (1 - z) \cdot k \cdot C \\ 2y &= 1 \end{aligned}$$

Therefore, the revenue is maximized at a discount of $y = .5$.

Using this information, we calculated the specific revenues for our two model cities, Miami, FL and Henrietta, NY. In order to find the function $u(y)$ for each city, we assumed that a hotel in Henrietta may be more inclined to offer larger discounts than one in Miami, due to location. We also assumed that even though a hotel in Henrietta may offer a larger discount, it would be more difficult to fill up vacant rooms than it would be for a hotel in Miami, and that the discount would depend on what percent of the rooms are filled. For example, if a hotel has filled only 30 percent of its rooms on a given night, its discount would be higher than if it has already filled 60 percent of its rooms. We again assumed that there were 1000 rooms in the hotel. Therefore, we filled in decimal percentages for y , θ , and z accordingly:

MIAMI

$$\begin{aligned} &\text{If } z = .3 \\ y = .30, u(y) &= 420 \\ y = .25, u(y) &= 350 \end{aligned}$$

For example, if 30 percent of the rooms in a Miami hotel are filled (which is 300 rooms for our model), then they may offer a 30 percent discount, of which 60 percent of the people may take advantage. This translates into 60 percent of the remaining 700 rooms being filled at the discounted rate, or 420 rooms. The hotel may also offer a 25 percent discount, for which 50 percent of the remaining 700 rooms may be filled at the discounted rate, or 350 rooms. Although the discount may not be large, the hotel's premium location may encourage more people to take advantage of the discount.

$$\begin{aligned} &\text{If } z = .4 \\ y = .25, u(y) &= 300 \\ y = .20, u(y) &= 240 \end{aligned}$$

$$\begin{aligned} &\text{If } z = .5 \\ y = .20, u(y) &= 200 \\ y = .15, u(y) &= 150 \end{aligned}$$

$$\begin{aligned} &\text{If } z = .6 \\ y = .15, u(y) &= 120 \\ y = .10, u(y) &= 80 \end{aligned}$$

$$\begin{aligned} &\text{If } z = .7 \\ y = .10, u(y) &= 60 \\ y = .05, u(y) &= 30 \end{aligned}$$

$$\begin{aligned} &\text{If } z = .8 \\ y = .05, u(y) &= 20 \\ y = 0, u(y) &= 0 \end{aligned}$$

We also went under the assumption that, for both locations, a hotel may not offer any discount if 80 percent or more of the rooms are filled.

HENRIETTA

$$\begin{aligned} &\text{If } z = .3 \\ y = .60, u(y) &= 210 \\ y = .50, u(y) &= 175 \end{aligned}$$

Due to location, in Henrietta a hotel may offer a larger discount, of which fewer customers may take advantage. If 30 percent of the rooms in a Henrietta hotel are filled (300 rooms filled, as we have assumed $H = 0$), then it may offer a 60 percent discount. This may result in 30 percent of the remaining 700 rooms being booked. For the same reason, the hotel could also offer a 50 percent discount which may result in 25 percent of the remaining 700 rooms being booked.

$$\begin{aligned} &\text{If } z = .4 \\ y = .50, u(y) &= 150 \\ y = .40, u(y) &= 120 \end{aligned}$$

$$\begin{aligned} &\text{If } z = .5 \\ y = .40, u(y) &= 100 \\ y = .30, u(y) &= 75 \end{aligned}$$

$$\begin{aligned} &\text{If } z = .6 \\ y = .30, u(y) &= 60 \\ y = .20, u(y) &= 40 \end{aligned}$$

$$\begin{aligned} &\text{If } z = .7 \\ y = .20, u(y) &= 30 \\ y = .10, u(y) &= 15 \end{aligned}$$

$$\begin{aligned} &\text{If } z = .8 \\ y = .10, u(y) &= 10 \\ y = 0, u(y) &= 0 \end{aligned}$$

Therefore, $u(y) = \theta$ will be different for Miami and Henrietta. For Miami, using the above assumptions, we have set $u(y) = \theta = 900y$, and for Henrietta we have set $u(y) = \theta = 225y$.

When we substituted these expressions for $u(y)$ into the equations for Miami and Henrietta, we found the following:

MIAMI

$$r(y) = HC[z \cdot (1 - N)] + (1 - z) \cdot (900y) \cdot (1 - y) \cdot C \quad (19)$$

HENRIETTA

$$r(y) = HC[z \cdot (1 - N)] + (1 - z) \cdot (225y) \cdot (1 - y) \cdot C \quad (20)$$

Note: H, C, z , and N are all constants that are most likely known by the specific hotel.

As shown by the above calculation, regardless of k , the revenue for each city will be maximized at a 50 percent discount.

Based on this information, we now utilized a 50 percent discount for the specific cases of Miami and Henrietta. For Miami, we set $C = \$299.00, H = 1000, N = .04$, and worked

with the same z values and the same functions for θ that were used above, with a range of .3 to .8.

For Henrietta, we set $C = \$119.00$, $H = 1000$, $N = .04$, and used z values from .3 to .8 again. As a result, maximum revenues for Miami and Henrietta were produced using equations 19 and 20.

Once the maximum revenues were found, each was graphically confirmed to occur at $x = .5$, or when a 50 percent discount is offered. We then compared these values with the revenues gathered by the hotels in each city if no discount were offered. For example, if 40 percent of the rooms were filled in Henrietta which translates into a hotel making 40 percent of its maximum profit, the hotel's revenue would be $.4CH(1 - N)$, or based on what we have been assuming, it would be $.4CH(.96)$.

Therefore, we calculated the revenue with the formula $r = z \cdot C \cdot H \cdot (1 - N)$. After comparing revenues, we found the percent change of the maximized revenues for different discount values compared to the revenue when no discount is offered. These results are:

MIAMI

Z	WITHOUT DISCOUNT	WITH DISCOUNT	PERCENT CHANGE
.3	\$86,112.00	\$133,204.50	54.7
.4	\$114,816.00	\$155,181.00	35.2
.5	\$143,520.00	\$177,157.50	23.4
.6	\$172,224.00	\$199,134.00	15.6
.7	\$200,928.00	\$221,110.50	10.0
.8	\$229,632.00	\$243,087.00	5.9

The graphs below illustrate the above data. The first graph compares the revenues. The purple dots represent the revenues with a discount given to increase customer sales in Miami. The blue dots represent a hotel's revenue in Miami with no new reservations at a hotel that is not fully booked. The second graph illustrates the percent change of a hotels revenue as the fraction of empty rooms increases from .3 to .8.

MIAMI COMPARING REVENUES - DISCOUNT VS. NON-DISCOUNT

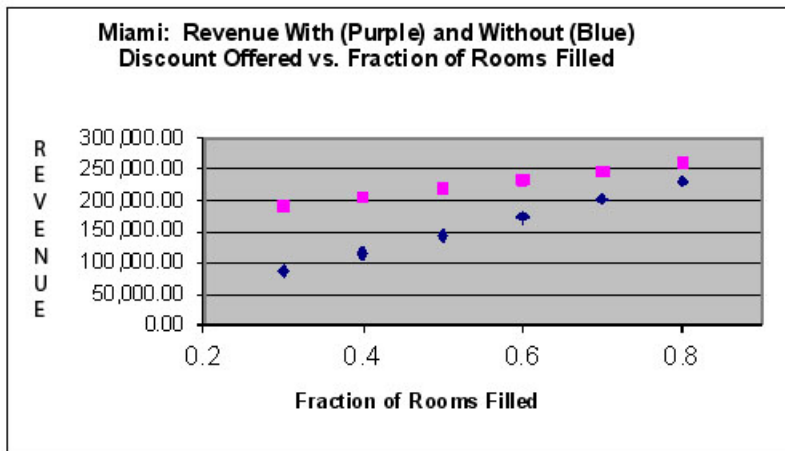


Figure 26: When a discount is offered to fill up vacant rooms, the hotel's revenue is higher than if no discount is offered and the rooms remain vacant.

MIAMI PERCENT CHANGE - DISCOUNT VS. NON-DISCOUNT

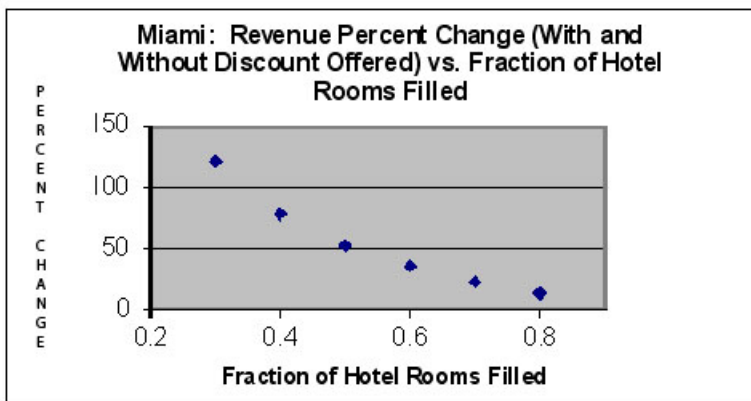


Figure 27: Percent change in the revenue when a discount is offered to fill vacant rooms and when a discount is not offered.

HENRIETTA

Z	WITHOUT DISCOUNT	WITH DISCOUNT	PERCENT CHANGE
.3	\$34,272.00	\$38,957.63	13.7
.4	\$45,696.00	\$49,712.25	8.8
.5	\$57,120.00	\$60,466.88	5.9
.6	\$68,544.00	\$71,221.50	3.9
.7	\$79,968.00	\$81,976.13	2.5
.8	\$91,392.00	\$92,730.75	1.5

Just as is the case with Miami, the graphs below illustrate the above data for Henrietta. The first graph compares the revenues, and the second graph illustrates the percent change of a hotel's revenue as the fraction of empty rooms increases from .3 to .8.

HENRIETTA COMPARING REVENUES - DISCOUNT VS. NON-DISCOUNT

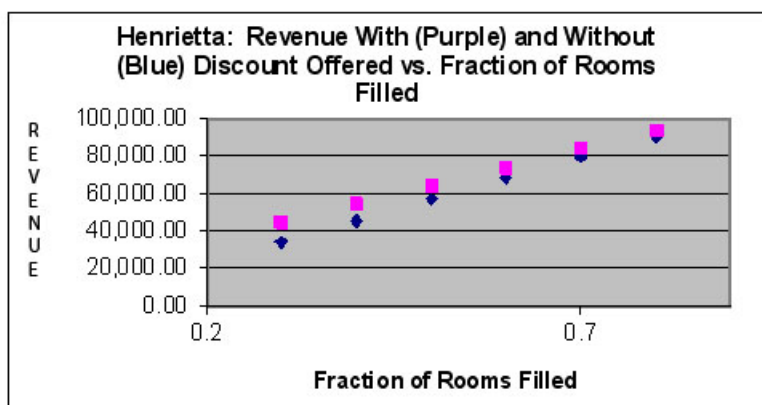


Figure 28: When a discount is offered to fill up vacant rooms, the hotel's revenue is higher than if no discount is offered and the rooms remain vacant.

HENRIETTA PERCENT CHANGE - DISCOUNT VS. NON-DISCOUNT

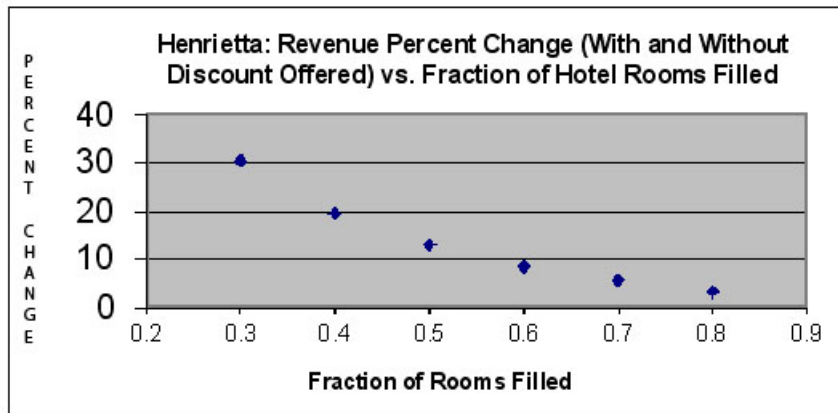


Figure 29: Percent change in the revenue when a discount is offered to fill vacant rooms and when a discount is not offered.

We assumed in equation 18 that the relationship between the discount and the number of people who take the discount was linearly proportional. This was an incorrect assumption, as a 50 percent discount is considered high in the hotel industry. It seems likely that at a 50 percent discount some customers will try to cancel their reservations, and then re-book a room at the same hotel at the new, reduced rate. Therefore, this new scenario must be examined. After completing some internet research, looking at several websites that discussed cancellation fees and cancellation penalties, it seems that the following two scenarios were the most common. [5]

1. A hotel offers a discount, but their cancellation fee is too high, and therefore customers do not cancel and subsequently re-book their reservations. For example, there is a window of time in which a hotel allows its patrons to cancel their reservations at no cost. If the reservation is cancelled after the window has lapsed, then the customer must pay either one or two night's stay. This may be more costly to the customer, and therefore the cancellation will not occur. However, these customers may demand some sort of compensation from the hotel. In these situations, hotels will often give a gift card, or a free meal, etc. for some given amount of money. This amount of money will seemingly be related to the cost of the room. It is important to note that the hotel demands that these customers pre-pay. If the hotel does not demand a pre-payment, then customers could simply just not show

up instead of cancelling, and then walk in and pay the discounted rate. By asking for a pre-payment, even if the customer chooses not to honor his/her reservation, the hotel still has the revenue from the pre-payment. Since pre-payment is required, the hotel will not lose any revenue from those customers who do not show up, and as a result there will not be a no-show rate.

2. A hotel will offer a discount, and this discount is offered within the amount of time that the customer may have to pay either a small fee, or no cancellation fee, for cancelling his/her reservation. Therefore, the customer cancels the existing reservation, and then re-books at the reduced rate.

The models for each situation are described below.

SITUATION 1 - COMPENSATION GIVEN TO THE CUSTOMER

In situation 1, the customer did not want to cancel his/her reservation as it was not cost effective. However, those who did not receive a discount may seek some sort of compensation. As a result, we have introduced two new variables. The first is s , which is the number of customers who will demand some sort of compensation. These are the customers who would actually search for more cost effective rates on the internet, and would find the new advertised discounted rate. The other new variable is G , which is the fractional amount of money given back to the customer in the form of a gift card. This situation is represented by the equation:

$$r(y) = HC[z \cdot (1 - N) - s \cdot G \cdot z \cdot (1 - N)] + \min\{(1 - z), \theta\} \cdot (1 - y) \cdot C \quad (21)$$

For equation 21, $H \cdot C \cdot z \cdot (1 - N)$ represents the customers who paid the original rate and booked early. The section $H \cdot C \cdot s \cdot G \cdot z \cdot (1 - N)$ represents those customers who found the discounted rate but decided it was not in their best interest to cancel their reservations. As a result, they demanded some sort of compensation. The section $\min\{(1 - z), \theta\} \cdot (1 - y) \cdot C$ represents those who booked at the discounted rate. This part of the function must be represented as the minimum of θ and $(1 - z)$ because a certain number of customers will seek out and find the discounted hotel room, regardless of how many rooms a hotel may have available. For example, if 100 customers find the discount, and a hotel only has 10 empty rooms, the hotel will not offer the discount to all 100 customers, but rather to the first 10 who book. However, if 100 customers find the discounted rate, and a hotel as 250 empty rooms, the hotel will offer the discounted rate to all 100 customers. As far as the minimum function is concerned, when $(1 - z) > \theta$, then a hotel has filled up some empty

rooms, but still has more available. When $(1 - z) < \theta$, then the hotel is overbooked and has no more empty rooms to offer at the discounted rate. When $(1 - z) = \theta$, then the hotel is completely booked, and offering a larger discount will only result in less revenue for the hotel. To prevent overbooking by a significant amount, the minimum value function is necessary.

Note: We set $s = \sigma \cdot y$, where the parameter σ can change based on many factors, such as discount, hotel chain, location, and time of year. Our thought process in making s dependent on y was that the amount of people who look for and take advantage of the discount will be related to how high or low the discount is.

A hotel does not want to overbook its rooms by a significant amount. If a hotel does over- book, it seems reasonable that it will do so based on the model described in section 1 of this paper. However, as described above, when $(1 - z) < \theta$, the hotel is overbooked, and may be overbooked by a significant amount. If 100 customers want the discount, and 10 rooms are available, our model will not allow the hotel to book 100 rooms at this new rate, but only the 10 rooms that are available. Therefore, once $(1 - z) = \theta$, our revenue will be maximized as all rooms will be filled. So, for our model, we need only consider when $(1 - z) \geq \theta$, as this describes when we are continuing to fill empty rooms or when the hotel is completely booked.

Therefore, since θ is the minimum value, the general equation can be rewritten as:

$$r(y) = HC[z(1 - N) - (s = \sigma y) \cdot G \cdot (1 - N) \cdot z] + (\theta = ky) \cdot (1 - y) \cdot C \quad (22)$$

At this point, we will take the derivative of this function and set it equal to zero to try to find the best discount to offer to maximize the revenue.

$$r'(y) = -CH \cdot \sigma \cdot G \cdot (1 - N) \cdot z + Ck - 2Cky = 0$$

$$-CH \cdot \sigma \cdot G \cdot (1 - N) \cdot z + Ck = 2Cky$$

$$y = \frac{k - H \cdot \sigma \cdot G \cdot (1 - N) \cdot z}{2k}$$

$$y = .5 - .5\left(\frac{H \cdot \sigma \cdot G \cdot (1 - N) \cdot z}{k}\right)$$

For the sake of simplification, we made a new parameter, γ , to represent $\frac{H \cdot \sigma \cdot (1 - N) \cdot z}{k}$. We

will discuss this parameter in greater detail at a later point in the paper.

Therefore, through substitution, we calculate that the revenue is maximized in situation 1 when $y = \frac{1}{2}(1 - G\gamma)$.

In order to find the actual maximized revenue, we must then plug this y -value back into the original equation, $r(y)$. The function $r(y)$ representing revenue for situation 1 is listed as equation 22. The work to find $r(\frac{1}{2}(1 - G \cdot \gamma))$ is shown below.

$$r(\frac{1}{2}(1 - G\gamma)) = HC[z(1 - N) - (\sigma(\frac{1}{2}(1 - G\gamma))) \cdot G \cdot (1 - N) \cdot z] + (k(\frac{1}{2}(1 - G\gamma))) \cdot (1 - (\frac{1}{2}(1 - G\gamma))) \cdot C$$

$$r(\frac{1}{2}(1 - G\gamma)) = HC[z \cdot (1 - N) - [\frac{\sigma \cdot G \cdot (1 - N) \cdot z - \sigma \cdot G^2 \cdot \gamma \cdot (1 - N) \cdot z}{2}] + (\frac{k}{4} - \frac{k \cdot G^2 \cdot \gamma^2}{4}) \cdot C$$

At this point, we multiplied the function by $\frac{k}{k}$, resulting in the k on the outside, and dividing every term on the inside by k . Therefore, $r(y)$ now becomes:

$$r(\frac{1}{2}(1 - G\gamma)) = k[\frac{HCz(1-N)}{k} - \frac{HC\sigma G(1-N)z}{2k} + \frac{\sigma G^2 \gamma(1-N)HCz}{2k} + \frac{C}{4} - \frac{G^2 \gamma^2 C}{4}]$$

Reminder: $\gamma = \frac{H \cdot \sigma \cdot G \cdot (1-N) \cdot z}{k}$. Therefore, through substitution and factoring out a C , we calculate that:

$$r(\frac{1}{2}(1 - G\gamma)) = kC(\frac{\gamma}{\sigma} - \frac{G\gamma}{2} + \frac{G^2\gamma}{2} + \frac{1}{4} - \frac{G^2\gamma^2}{4}) \quad (23)$$

This above representation of $r(\frac{1}{2}(1 - G\gamma))$ is the maximized revenue for scenario 1.

By analyzing the results from the first situation, we conclude the following:

1. As $G \rightarrow 0$, we notice that the hotel's revenue is maximized when $y = .5$.
2. The hotel will make the most money overall when $G = 0$, as the hotel will not be giving any compensation back to its customers. We can mathematically confirm that the revenue is at its highest when $G = 0$. The expressions of $r(\frac{1}{2}(1 - G\gamma)) =$

$kC(\frac{\gamma}{\sigma} - \frac{G\gamma}{2} + \frac{G^2\gamma}{2} + \frac{1}{4} - \frac{G^2\gamma^2}{4})$ that contain only G s are $(-\frac{G\gamma}{2} + \frac{G^2\gamma}{2} - \frac{G^2\gamma^2}{4})$. We factored out $\frac{G\gamma}{2}$, and the expression simplifies to be $\frac{G\gamma}{2} \cdot (-1 + G - \frac{G\gamma}{2})$. Since G will never be greater than 1, as the hotel will never give more back than full compensation, we can confirm that $\frac{G\gamma}{2} \cdot (-1 + G - \frac{G\gamma}{2})$ will always be negative, unless $G = 0$, at which point the expression becomes 0. Therefore, the hotel will always lose revenue unless G is set equal to 0.

However, this translates into poor customer service. Poor customer service could result in lost customers, and therefore future revenue loss. It may not be in the hotel's best interest to refuse compensation to its customers who paid full price.

3. As $G \rightarrow 1$, we can see that $y \rightarrow \frac{1}{2}(1 - \gamma)$.
4. As $z \rightarrow 0$, implying that the hotel is relatively empty, then $\gamma \rightarrow 0$, and $y \rightarrow \frac{1}{2}$.
5. As $z \rightarrow 1$, it seems reasonable that $\gamma \rightarrow 1$. There are two reasons for this assumption. The first is that $(1 - N) = 1$ as $N = 0$ because the customers must pre-pay. The second reason is that the number of people who look for a discount and the number of people who ask for compensation should be relatively equal, making that ratio in γ approximately 1. Therefore, we have made the assumption that $\gamma \approx z$. So, as $z \rightarrow 1$, and therefore $\gamma \rightarrow 1$, $y \rightarrow \frac{1}{2}(1 - G)$.
6. It is also safe to assume that $0 \leq G \leq .5$, as a hotel will most likely not compensate its customers for more than 50 percent of the cost of the room without losing significant revenue.
7. As $\sigma \rightarrow 0$, meaning nobody requests compensation, then the maximum revenue occurs at $\frac{CKzH(1-N)}{k} + \frac{Ck}{4}$.
8. As $k \rightarrow 0$, which means that nobody booked at the original discounted rate (a realistic situation), our model seems to break down at $\frac{G^2\gamma^2}{4}$ as r will then approach $-\infty$. However, the stipulation that $\theta \leq (1 - z)$, or in other words that $y = \frac{1}{2}(1 - G\gamma) \leq \frac{(1-z)}{k}$, prevents this situation from occurring. The fractional discount, y , must be positive. Therefore, we consider the first situation where y is not positive, which occurs when $y = 0$. The discount y becomes 0 when $G\gamma = 1$. If $G\gamma = 1$, then the $\frac{1}{4} - \frac{G^2\gamma^2}{4}$ will cancel out (as it becomes $\frac{1}{4} - \frac{1}{4}$), and the problem of the revenue approaching $-\infty$ no longer exists.

The maximum revenues for the equation 23 depend on G and γ . Therefore, when we graphed the first scenario we had to consider a graph where G varied and γ was set, and another where G was set and γ was varied.

As stated above, the revenue will be maximized when G is equal to 0, or when the customer receives no compensation from the hotel, although this may lead to poor customer service and future revenue loss. We expected our graphs to confirm this. However,

we still wanted to consider the revenues at different values of G . Therefore, we varied G from 0 to .5, as we felt comfortable assuming that a hotel would not give compensation any greater than 50 percent of the cost of the room.

In order to set γ at a fixed value, we had to consider the percent of rooms filled, the percent discount offered and number of customers taking advantage of that discount, and the fraction of customers who may demand compensation. Due to the large number of factors, we did not feel comfortable setting just one value of γ . Therefore, we made the following assumptions:

1. We chose what we felt was a reasonable discount (15 percent), and assumed that 100 people would take advantage of that discount. Since $\theta = ky$, and therefore $100 = k(.15)$, we set $k = 700$.
2. We assumed that the percentage of people who ask for compensation will depend on the discount offered. For the sake of simplicity, we set $\sigma = .5$.
3. We varied γ based on the fraction of rooms filled, which we have assumed to range from .3 to .8.
4. The number of hotel rooms, H , continues to be 1000.

The following γ values were produced as z varied from .3 to .8:

$z = .3$	$\gamma = .214$
$z = .4$	$\gamma = .286$
$z = .5$	$\gamma = .357$
$z = .6$	$\gamma = .429$
$z = .7$	$\gamma = .500$
$z = .8$	$\gamma = .571$

The graph will illustrate that as G increases, the hotel loses revenue. The varying γ values, based on the varying z values, are illustrated on the graph in different colors. This graph is shown below:

SCENARIO 1 - FIX GAMMA VALUES, GRAPH IN TERMS OF G

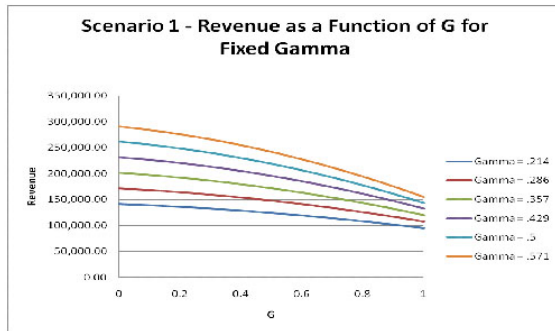


Figure 30: As G increases, the hotel will lose revenue. G represents the fraction of the cost of the hotel room given back to the customer as compensation.

We also wanted to consider at what value the revenue becomes zero. Graphically, we wanted to show where the quadratic functions intersected the G -axis. Therefore, we extended the G values until the revenue was shown to be 0. Algebraically, we used the quadratic formula on the function $r(\frac{1}{2}(1 - G\gamma)) = kC(\frac{\gamma}{\sigma} - \frac{G\gamma}{2} + \frac{G^2\gamma}{2} + \frac{1}{4} - \frac{G^2\gamma^2}{4})$ to see at what values of G the roots would occur. Both the graphic and algebraic representations are shown below:

SCENARIO 1 - FIX GAMMA/ADJUST G - REVENUE = 0

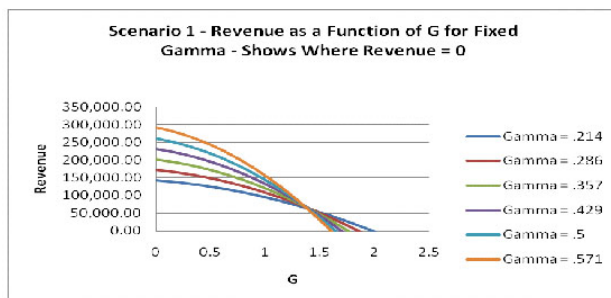


Figure 31: A compensation value between 1.5 and 2, depending on γ , translates into the hotel making a revenue of 0. A hotel can give back more than 100 percent of the cost of the room and still make money because of the customers who are paying full price.

Figure 31 indicates that the zeros occur at G values between 1.5 and 2. This would mean that the hotel has a revenue of 0 when it gives the customer compensation of 150 percent to 200 percent of the cost of the hotel room.

Using the quadratic formula, we can see that the revenue is zero at the following values:

$$0 = kC\left(\frac{\gamma}{\sigma} - \frac{G\gamma}{2} + \frac{G^2\gamma}{2} + \frac{1}{4} - \frac{G^2\gamma^2}{4}\right)$$

$$0 = G^2 \cdot \left(\frac{\gamma}{2} - \frac{\gamma^2}{4}\right) + G \cdot \left(\frac{-\gamma}{2}\right) + \left(\frac{1}{4} + \frac{\gamma}{\sigma}\right)$$

$$G = \frac{\frac{\gamma}{2} \pm \sqrt{\frac{\gamma^2}{4} - 4 \cdot \left(\frac{\gamma}{2} - \frac{\gamma^2}{4}\right) \cdot \left(\frac{1}{4} + \frac{\gamma}{\sigma}\right)}}{2 \cdot \left(\frac{\gamma}{2} - \frac{\gamma^2}{4}\right)}$$

Through simplification, G is rewritten as:

$$G = \frac{\frac{\gamma}{2} \pm \sqrt{\frac{\gamma^2}{2} - \frac{\gamma}{2} - \frac{2\gamma^2}{\sigma} + \frac{\gamma^3}{\sigma}}}{\left(\frac{2\gamma - \gamma^2}{2}\right)}$$

Graphing equation 23 by fixing G and adjusting γ proved to be a much more difficult task.

We started this process by graphing the function for different values of G , which we presumed would range anywhere from 0 to .6. We expected our result to be a quadratic function, and this was confirmed. However, the revenues produced were a concern. For the sake of simplicity, we chose $C = \$299.00$. If all 1000 rooms were booked at \$299.00 per night, the hotel's revenue would be \$299,000.00. However, when the graph varying γ was produced, the revenues were shown to be in the millions. In order to explain this result, we again look at the function $r\left(\frac{1}{2}(1 - G\gamma)\right) = kC\left(\frac{\gamma}{\sigma} - \frac{G\gamma}{2} + \frac{G^2\gamma}{2} + \frac{1}{4} - \frac{G^2\gamma^2}{4}\right)$. Recall that γ contains the value of k in its denominator. In order for γ to increase, k must decrease. However, as γ increases along its axis, the k on the outside ($r\left(\frac{1}{2}(1 - G\gamma)\right) = kC\left(\frac{\gamma}{\sigma} - \frac{G\gamma}{2} + \frac{G^2\gamma}{2} + \frac{1}{4} - \frac{G^2\gamma^2}{4}\right)$) remains constant. When there is a growth in γ , k should be decreasing. Therefore, since the k on the outside is not properly adjusted to reflect the growth of γ , the revenues blow up into unreasonable values. If k were to decrease, then the graph would demonstrate acceptable revenues.

The graph below illustrates the large revenue values for reasonable values of γ , as well as the quadratic nature of the equation 23 when graphed in terms of γ .

SCENARIO 1 - FIX G, GRAPH IN TERMS OF GAMMA

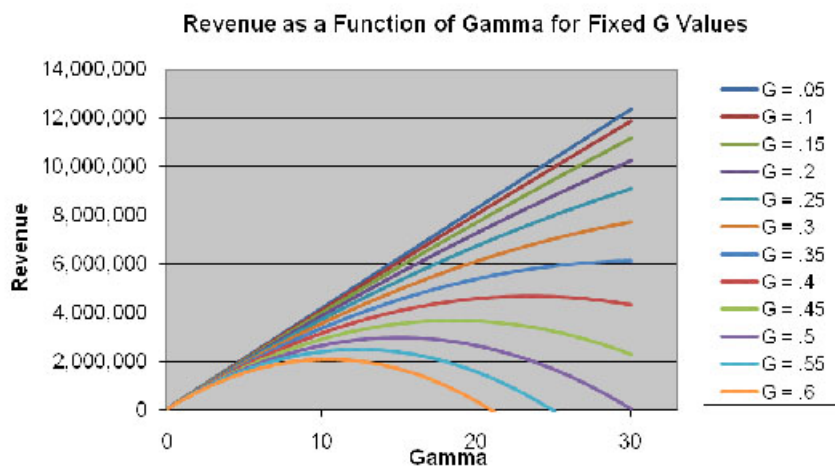


Figure 32: Demonstrates the function as quadratic. The values of the revenue are unreasonable because as γ grows, the k on the outside of the function is not decreased to reflect this change.

Figure 32 shows that as G increases, and the best value of γ is found, then the revenue will eventually decrease for each different value of G . This can be confirmed algebraically.

By taking the derivative of equation 23 in terms of γ , and setting that derivative equal to 0, we can see at what value of γ the revenue will be maximized. This work is shown below:

$$r'(\frac{1}{2}(1 - G\gamma)) = kC(\frac{1}{\sigma} - \frac{G}{2} + \frac{G^2}{2} - \frac{2G^2\gamma}{4})$$

We set this equation equal to 0, and then solve for γ .

$$0 = kC(\frac{1}{\sigma} - \frac{G}{2} + \frac{G^2}{2} - \frac{2G^2\gamma}{4})$$

$$0 = \frac{1}{\sigma} - \frac{G}{2} + \frac{G^2}{2} - \frac{2G^2\gamma}{4}$$

$$\frac{2G^2\gamma}{4} = \frac{1}{\sigma} - \frac{G}{2} + \frac{G^2}{2}$$

$$\gamma = \frac{2}{\sigma \cdot G^2} - \frac{1}{G} + 1$$

Therefore, when $\gamma = \frac{2}{\sigma \cdot G^2} - \frac{1}{G} + 1$, the revenue is maximized.

SITUATION 2 - CANCELLATION FEE ASSESSED

In situation 2, the customer did cancel his/her reservation and rebooked at the discounted rate, but was forced to pay a cancellation fee as a result. Similar to case one, s is used to represent the fraction of customers who have found the new discount, have cancelled, and then have re-booked their reservations. A new variable, F , is also introduced. F will be used to represent the fee paid to the hotel by the customer for cancelling his/her existing reservation. This fee may be as small as 0, but also could be as large as the hotel sees fit. Therefore, the equation below is used to represent scenario number 2:

$$r(y) = HC[z \cdot (1 - N) - (s = \sigma y) \cdot z \cdot (1 - N) + (s = \sigma y) \cdot (1 - N) \cdot z \cdot (1 - y)] + F \cdot z \cdot (1 - N) \cdot H \cdot (s = \sigma y) + \min\{\theta, (1 - z)\} \cdot (1 - y) \cdot C \quad (24)$$

In the above equation, $HC \cdot z \cdot (1 - N)$ represents the people who originally booked at full price, $HC \cdot (s = \sigma y) \cdot z \cdot (1 - N)$ represents those who cancelled their reservations, $HC \cdot (s = \sigma y) \cdot (1 - N) \cdot z \cdot (1 - y)$ represents the same people who now have re-booked at the new discounted rate, $F \cdot z \cdot (1 - N) \cdot H \cdot (s = \sigma y)$ represents the customers who have re-booked and now have to pay a fee F for their original cancellation, and $\min\{\theta, (1 - z)\} \cdot (1 - y) \cdot C$ represents the customers who booked at a discounted rate. The last part of the function includes a minimum value function for the same reason described in situation 1. We are again just going to focus on the scenario when $(1z) \geq \theta$, as this describes when the hotel is filling up empty rooms or is completely booked, but not overbooked.

NOTE: For scenario 2, it is important to note that if $F \geq (1 - y)C$, then the customers will not cancel their reservations, as they will not gain any money in the process. However, if $F < (1 - y)C$, then customers will be more inclined to cancel to save money.

Since θ is the minimum value when $(1z) \geq \theta$, the function now reads as follows:

$$r(y) = HC[z \cdot (1 - N) - (\sigma y) \cdot z \cdot (1 - N) + (\sigma y) \cdot z \cdot (1 - N) \cdot (1 - y)] + F \cdot z \cdot (1 - N) \cdot H \cdot (\sigma y) + (ky) \cdot (1 - y) \cdot C \quad (25)$$

At this point, we will take the derivative of equation 25 and set it equal to zero to find

the y value at which the revenue will be maximized.

$$r'(y) = -\sigma HCz(1-N) + HC\sigma z(1-N) - 2\sigma y HCz(1-N) + Fz(1-N)H\sigma + Ck - 2Cky = 0$$

$$Fz(1-N)H\sigma + Ck = (2Ck + 2\sigma HCz(1-N)) \cdot y$$

$$y = \frac{Fz(1-N)H\sigma + Ck}{2Ck + 2\sigma HCz(1-N)}$$

We divided each value in the fraction by C and k and factored out $\frac{1}{2}$ to calculate the following representation of y :

$$y = \frac{1}{2} \left(\frac{1 + \frac{F}{C} \frac{z(1-N)H\sigma}{k}}{1 + \frac{\sigma Hz(1-N)}{k}} \right)$$

In scenario 1, we had identified a new variable γ , which we had set equal to $\frac{z(1-N)H\sigma}{k}$. We will use the same representation of γ in scenario 2, and using substitution we calculate that y can be represented by:

$$y = \frac{1}{2} \left(\frac{1 + \frac{F}{C} \gamma}{1 + \gamma} \right) \quad (26)$$

Reminder: This scenario only applies when $\theta \leq (1-z)$, or if $y < \frac{(1-z)}{k}$.

In order to find the actual maximized revenue, we began by manipulating $r(y)$ to become a simplified expression in terms of γ :

$$r(y) = HC[z \cdot (1-N) - (\sigma y) \cdot z \cdot (1-N) + (\sigma y) \cdot z \cdot (1-N) \cdot (1-y)] + F \cdot z \cdot (1-N) \cdot H \cdot (\sigma y) + (ky) \cdot (1-y) \cdot C$$

Multiply the above equation by $\frac{k}{k}$

$$r(y) = k \cdot \left(\frac{H \cdot C \cdot z \cdot (1-N) \cdot \sigma}{k} - \frac{H \cdot C \cdot z \cdot (1-N) \cdot \sigma \cdot y}{k} + \frac{H \cdot C \cdot z \cdot (1-N) \cdot \sigma \cdot y \cdot (1-y)}{k} + \frac{F \cdot z \cdot (1-N) \cdot H \cdot \sigma \cdot y}{k} + \frac{k \cdot y \cdot (1-y) \cdot C}{k} \right)$$

We now will replace all values of $\frac{H \cdot z \cdot (1-N) \cdot \sigma}{k}$ with γ .

$$r(y) = k \cdot \left(\frac{C \cdot \gamma}{\sigma} - C \cdot y \cdot \gamma \cdot (1 - y) + F \cdot y \cdot \gamma + C \cdot y \cdot (1 - y) \right)$$

This can be simplified to read:

$$r(y) = k \cdot \left(\frac{C \cdot \gamma}{\sigma} - C \cdot y^2 \cdot \gamma + F \cdot y \cdot \gamma + C \cdot y - C \cdot y^2 \right) \quad (27)$$

To find the maximum revenue, we now replace all values of y in equation 27 with the value of y listed in equation 26, and the expression is simplified:

$$r\left(\frac{1}{2} \left(\frac{1 + \frac{F}{C} \gamma}{1 + \gamma} \right)\right) = k \cdot \left(\frac{C \gamma}{\sigma} - \left(\frac{\frac{F^2 \gamma^3}{C} + 2F \gamma^2 + C \gamma + \frac{F^2 \gamma^2}{C} + 2F \gamma + C}{4(1 + \gamma)^2} \right) + \left(\frac{\frac{F^2 \gamma^2}{C} + 2F \gamma + C}{2(1 + \gamma)} \right) \right) \quad (28)$$

The above representation of $r\left(\frac{1}{2} \left(\frac{1 + \frac{F}{C} \gamma}{1 + \gamma} \right)\right)$ is the maximized revenue for scenario 2.

By analyzing the results from equation 28, we conclude the following:

1. As $F \rightarrow C$, we notice that the hotel's revenue is maximized when $y = .5$.
2. As $F \rightarrow 0$, $x \rightarrow \frac{1}{2} \cdot (1 + \gamma)$.
3. As $z \rightarrow 1$, it seems reasonable that $\gamma \rightarrow 1$. There are two reasons for this assumption. The first is that although $N \neq 0$ in scenario 2, it is still a reasonably small value. We have assumed throughout the paper that the maximum N is $N = .04$. Therefore, $(1 - N) \approx 1$. In addition, the number of people who look for a discount initially and the number of people who cancel their existing reservations and re-book at the discounted rate should be relatively equal, making that ratio in γ approximately 1. Therefore, we have made the assumption that $\gamma \approx z$. So, as $z \rightarrow 1$, then $\gamma \rightarrow 1$, which causes $y \rightarrow \frac{\frac{F}{C} + 1}{4}$.
4. As $z \rightarrow 0$, implying that the hotel is virtually empty, then $\gamma \rightarrow 0$, and $y \rightarrow .5$.
5. As $\sigma \rightarrow 0$, meaning nobody cancels their reservations and re-books at the discounted rate, then the maximum revenue occurs at $C \cdot H \cdot z \cdot (1 - N) + \frac{C}{4}$.

6. As $k \rightarrow 0$, which means that nobody originally booked at the discounted rate, then the revenue is maximized at $C \cdot H \cdot z \cdot (1 - N) + \frac{F^2 \cdot H \cdot z \cdot (1 - N) \cdot \sigma}{4}$.
7. As F grows larger, the hotel's revenue will increase.

The maximum values for equation 28 depend on F and γ . To graph equation 28, we considered a graph where F was changing and γ was fixed, as well as one where F was fixed and γ was changing. The revenue will increase as F increases, as the customer is paying the hotel a greater fee to cancel. This hypothesis is confirmed by our graph. In order to vary F , we considered a range of cancellation fees from \$0 to \$300. Given that we are using $\$C = 299.00$ as we did in scenario 1, we only graphed until $F = \$300$ as a customer will likely not cancel if the fee is higher than the room rate.

We set γ at the same fixed rates as we did in scenario 1 depending on the fraction of rooms filled. As a reminder, these are listed below.

$z = .3$	$\gamma = .214$
$z = .4$	$\gamma = .286$
$z = .5$	$\gamma = .357$
$z = .6$	$\gamma = .429$
$z = .7$	$\gamma = .500$
$z = .8$	$\gamma = .571$

The graph below illustrates that as F increases, the revenue increases. The varying values of γ based on the varying values of z are illustrated on the graph in different colors.

SCENARIO 2 - FIX GAMMA, GRAPH IN TERMS OF F

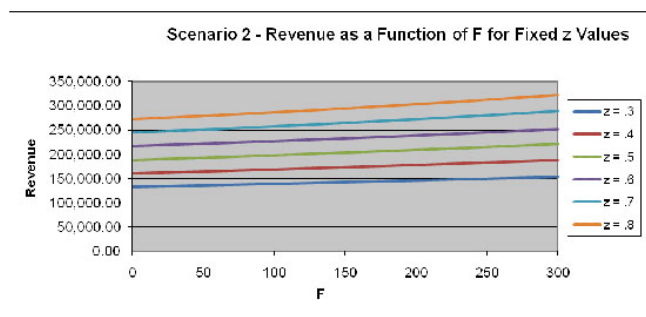


Figure 33: As F (cancellation fee) increases, the revenue increases.

Now that we have explained and analyzed the general equations for situation 1 and situation 2, we will apply these scenarios to our model cities, Miami and Henrietta.

Reminder: For Miami, $\theta = 900y$, and for Henrietta, $\theta = 225y$. For each city, we must also make assumptions about the value of s , which as of now has only been represented as σy .

In order to find an appropriate value for σ for both Miami and Henrietta, we assumed that the percentage of people who book at the discounted rate will depend on the discount. For the sake of simplicity in both scenarios, we let $\sigma = .5$.

The following equations represent the models for Miami and Henrietta after plugging in the respective functions for s and σ .

SITUATION 1 - COMPENSATION GIVEN TO THE CUSTOMER

MIAMI

$$r(y) = HC[z(1 - N) - (.5y)(G)(z)(1 - N)] + (900y)(1 - y)(C) \quad (29)$$

HENRIETTA

$$r(y) = HC[z(1 - N) - (.5y)(G)(z)(1 - N)] + (225y)(1 - y)(C) \quad (30)$$

SITUATION 2 - CANCELLATION FEE ASSESSED

MIAMI

$$r(y) = HC[z(1 - N) - (.5y)(z)(1 - N) + (.5y)(1 - N)(z)(1 - y)] + (F)(z)(1 - N)(H)(.5y) + (900y)(1 - y)(C) \quad (31)$$

HENRIETTA

$$r(y) = HC[z(1 - N) - (.5y)(z)(1 - N) + (.5y)(1 - N)(z)(1 - y)] + (F)(z)(1 - N)(H)(.5y) + (225y)(1 - y)(C) \quad (32)$$

We can see that, for Henrietta, $\sigma = .5$ and $k = 225$, and for Miami $\sigma = .5$ and $k = 900$.

Recall that for both scenario 1 and scenario 2, graphing the function in terms of γ was problematic. However, if we work with the function before it was written in terms of γ , we can find appropriate values for the revenue. Therefore, in order to maximize the revenue for each city for equations 29 - 32, we just have to substitute the appropriate variables and constants missing from each $r(y)$.

SITUATION 1 - COMPENSATION GIVEN TO THE CUSTOMER

We assume that the discount offered depends on the fraction of rooms filled. We also assume that the number of people taking advantage of the discount and the fraction of people demanding compensation depend on the discount offered. Therefore, for each different value of z , we will adjust the values of σ, k , and y accordingly. These values will then be substituted into the function, and a revenue produced. The values used are consistent with those listed earlier in the paper for each of our model cities.

MIAMI

Regardless of the z value listed, $H = 1000, C = \$299.00, \sigma = .5, G = .15$, and $N = 0$.

We then calculate the revenues using equation 29 given the following varying values of z, y , and k .

$z = .3$	$y = .275$	$k = 1400$	Therefore, $r(y) = \$171,308.31$
$z = .4$	$y = .225$	$k = 1200$	Therefore, $r(y) = \$180,147.50$
$z = .5$	$y = .175$	$k = 1000$	Therefore, $r(y) = \$190,705.94$
$z = .6$	$y = .125$	$k = 800$	Therefore, $r(y) = \$203,880.63$
$z = .7$	$y = .075$	$k = 600$	Therefore, $r(y) = \$221,745.87$
$z = .8$	$y = .025$	$k = 400$	Therefore, $r(y) = \$241,666.75$

Note: Now that the k values are being properly adjusted, the revenues produced are reasonable.

Once the revenues for Miami in scenario 1 were found, we then compared these values with the revenues gathered by the hotel in Miami if no discount was offered. We then calculated the percent change of the revenue when no discount was offered to that of the revenue in scenario 1. The table below shows this data.

MIAMI

Z	WITHOUT DISCOUNT	WITH DISCOUNT	PERCENT CHANGE
.3	\$86,112.00	\$171,308.31	98.9
.4	\$114,816.00	\$180,147.50	56.9
.5	\$143,520.00	\$190,705.94	32.9
.6	\$172,224.00	\$203,880.63	18.4
.7	\$200,928.00	\$221,745.87	10.4
.8	\$229,632.00	\$241,666.75	5.2

The graphs below illustrate the above data. The first graph compares the revenues before and after discount. The second graph illustrates the percent change of the hotel's revenue.

MIAMI SCENARIO 1 COMPARING REVENUES - DISCOUNT VS. NON-DISCOUNT

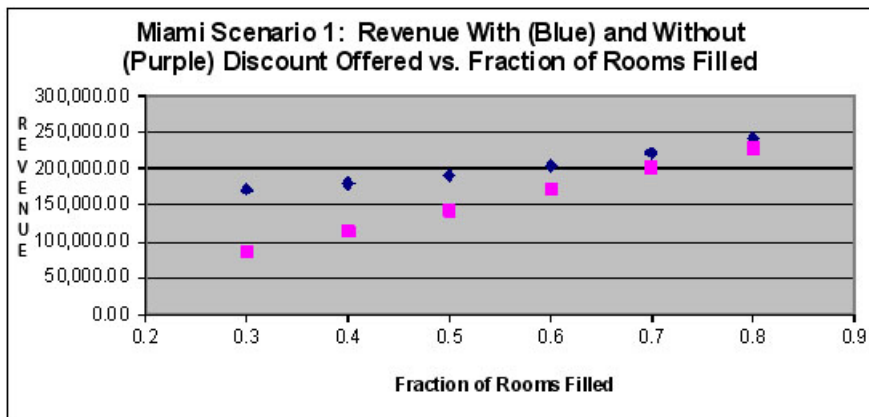


Figure 34: Within scenario 1 in Miami, when a discount is offered to fill up vacant rooms, the hotel's revenue is higher than if no discount is offered and the rooms remain vacant.

MIAMI SCENARIO 1 PERCENT CHANGE - DISCOUNT VS. NON-DISCOUNT

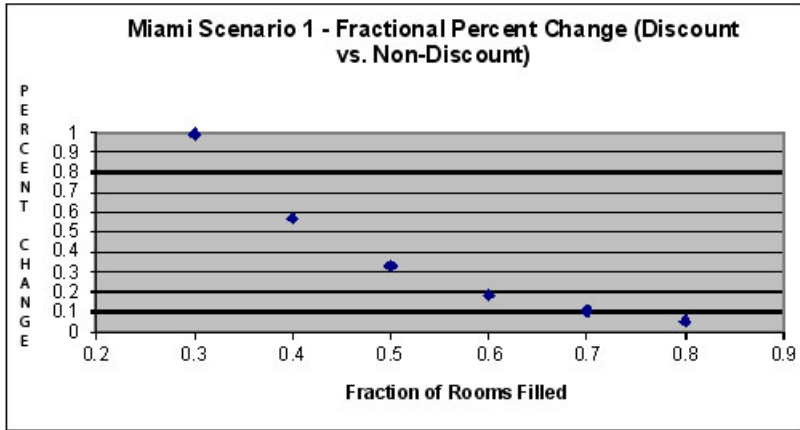


Figure 35: Within scenario 1 in Miami, percent change is shown for the revenue when a discount is offered to fill vacant rooms and when a discount is not offered.

The same process was repeated for Henrietta within the parameters of scenario 1.

Regardless of the z value listed, $H = 1000$, $C = \$119.00$, $\sigma = .5$, $G = .15$, and $N = 0$.

We then calculate the revenues using equation 30 given the following varying values of z , y , and k .

$z = .3$	$y = .55$	$k = 350$	Therefore, $r(y) = \$44,535.75$
$z = .4$	$y = .45$	$k = 300$	Therefore, $r(y) = \$54,829.25$
$z = .5$	$y = .35$	$k = 250$	Therefore, $r(y) = \$64,706.25$
$z = .6$	$y = .25$	$k = 200$	Therefore, $r(y) = \$74,523.75$
$z = .7$	$y = .15$	$k = 150$	Therefore, $r(y) = \$84,638.75$
$z = .8$	$y = .05$	$k = 100$	Therefore, $r(y) = \$95,408.25$

It is important to note that reasonable revenues are once again produced.

The percent change of the revenue when no discount was offered to that of the revenue in scenario 1 was calculated. The table below shows this data.

HENRIETTA

Z	WITHOUT DISCOUNT	WITH DISCOUNT	PERCENT CHANGE
.3	\$34,272.00	\$44,535.75	29.9
.4	\$45,696.00	\$54,829.25	20.0
.5	\$57,120.00	\$64,706.25	13.3
.6	\$68,544.00	\$74,523.75	8.7
.7	\$79,968.00	\$84,638.75	5.8
.8	\$91,392.00	\$95,408.25	4.4

The graphs below illustrate this data. The first graph compares the revenues before and after discount. The second graph illustrates the percent change of the hotels revenue.

HENRIETTA SCENARIO 1 COMPARING REVENUES - DISCOUNT VS. NON-DISCOUNT

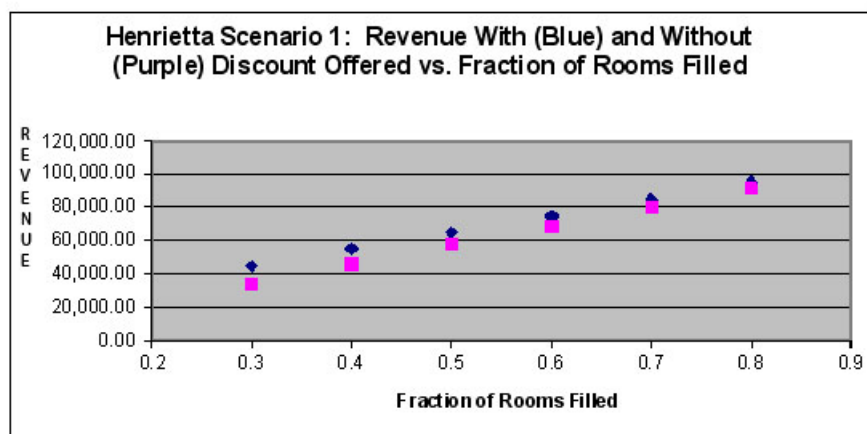


Figure 36: Within scenario 1 in Henrietta, when a discount is offered to fill up vacant rooms, the hotel's revenue is higher than if no discount is offered and the rooms remain vacant.

HENRIETTA SCENARIO 1 PERCENT CHANGE - DISCOUNT VS. NON-DISCOUNT

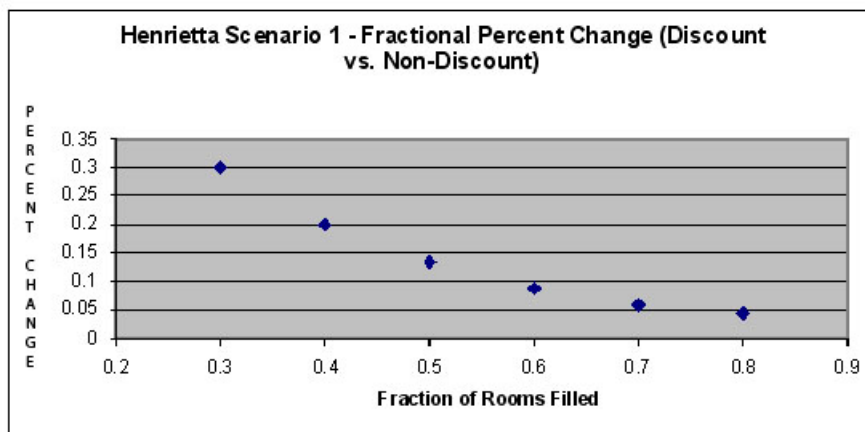


Figure 37: Within scenario 1 in Henrietta, percent change is shown for the revenue when a discount is offered to fill vacant rooms and when a discount is not offered.

SITUATION 2 - CANCELLATION FEE ASSESSED

For situation 2, we again assumed that the discount offered depends on the fraction of rooms filled, and that the number of people taking advantage of the discount, as well as the fraction of people who cancel and re-book at the discounted rate, depends on the discount. In addition, we have assumed that as the discount goes up, the cancellation fee also goes up. This assumption was made because, at a high discount, a hotel would want to discourage those customers who have already booked from cancelling and booking at a reduced rate, by stipulating a high cancellation fee. Therefore, for each different value of z , we will adjust the values of k , y , and F accordingly. The revenues listed below are produced:

MIAMI

Regardless of the z value listed, $H = 1000$, $C = \$299.00$, $\sigma = .5$, $G = .15$, and $N = 0$.

We then calculate the revenues using equation 31 given the following varying values of z , y , and k .

$z = .3$	$y = .275$	$k = 1400$	$F = \$225.00$	Therefore, $r(y) = \$171,308.31$
$z = .4$	$y = .225$	$k = 1200$	$F = \$195.00$	Therefore, $r(y) = \$180,147.50$
$z = .5$	$y = .175$	$k = 1000$	$F = \$165.00$	Therefore, $r(y) = \$190,705.94$
$z = .6$	$y = .125$	$k = 800$	$F = \$135.00$	Therefore, $r(y) = \$203,880.63$
$z = .7$	$y = .075$	$k = 600$	$F = \$105.00$	Therefore, $r(y) = \$221,745.87$
$z = .8$	$y = .025$	$k = 400$	$F = \$75.00$	Therefore, $r(y) = \$241,666.75$

Once the revenues for Miami in scenario 2 were found, they were then compared with the revenues when no discount was offered, and the percent change was again calculated. The table below shows this data.

MIAMI

Z	WITHOUT DISCOUNT	WITH DISCOUNT	PERCENT CHANGE
.3	\$86,112.00	\$175,224.30	103.5
.4	\$114,816.00	\$182,889.50	59.3
.5	\$143,520.00	\$191,420.50	33.4
.6	\$172,224.00	\$201,901.00	17.2
.7	\$200,928.00	\$215,454.80	7.2
.8	\$229,632.00	\$233,195.50	1.6

The graphs below illustrate this data. The first graph compares revenues before and after discount. The second graph illustrates the percent change of the hotel's revenue.

MIAMI SCENARIO 2 COMPARING REVENUES - DISCOUNT VS. NON-DISCOUNT

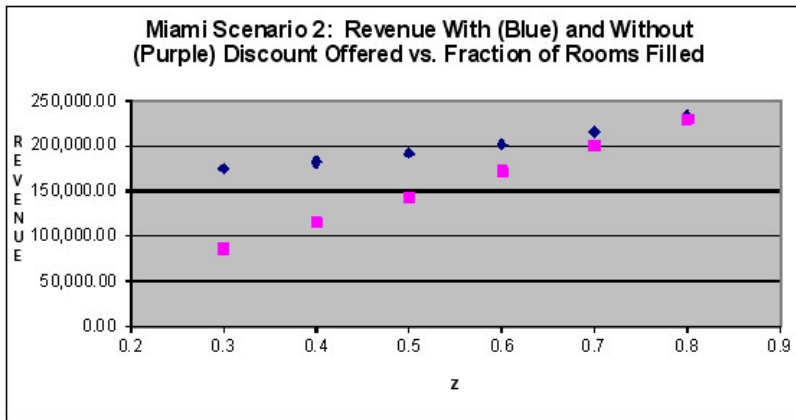


Figure 38: Within scenario 2 in Miami, when a discount is offered to fill up vacant rooms, the hotel's revenue is higher than if no discount is offered and the rooms remain vacant.

MIAMI SCENARIO 2 PERCENT CHANGE - DISCOUNT VS. NON-DISCOUNT

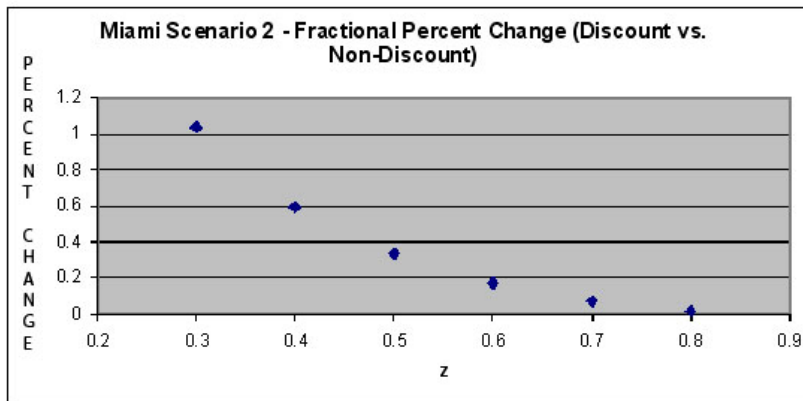


Figure 39: Within scenario 2 in Miami, percent change is shown for the revenue when a discount is offered to fill vacant rooms and when a discount is not offered.

The same process was repeated for Henrietta within scenario 2.

HENRIETTA

Regardless of the z value listed, $H = 1000$, $C = \$119.00$, $\sigma = .5$, $G = .15$, and $N = 0$.

We then calculate revenues using equation 32 given the following varying values of z , y , and k .

$z = .3$	$y = .555$	$k = 350$	$F = \$225.00$	Therefore, $r(y) = \$57,216.74$
$z = .4$	$y = .45$	$k = 300$	$F = \$195.00$	Therefore, $r(y) = \$66,753.03$
$z = .5$	$y = .35$	$k = 250$	$F = \$165.00$	Therefore, $r(y) = \$74,249.53$
$z = .6$	$y = .25$	$k = 200$	$F = \$135.00$	Therefore, $r(y) = \$80,584.50$
$z = .7$	$y = .15$	$k = 150$	$F = \$105.00$	Therefore, $r(y) = \$86,636.24$
$z = .8$	$y = .05$	$k = 100$	$F = \$75.00$	Therefore, $r(y) = \$93,283.01$

The difference in revenues, as well as the percent change, was calculated for revenues with and without discounts offered. The table below shows this data.

HENRIETTA

Z	WITHOUT DISCOUNT	WITH DISCOUNT	PERCENT CHANGE
.3	\$34,272.00	\$57,216.74	67.0
.4	\$45,696.00	\$66,753.03	46.1
.5	\$57,120.00	\$74,249.53	30.0
.6	\$68,544.00	\$80,584.50	17.6
.7	\$79,968.00	\$86,636.24	8.3
.8	\$91,392.00	\$93,283.01	2.1

The graphs below illustrate this data. The first graph compares the revenues, and the second graph illustrates the percent change of the hotels revenue.

HENRIETTA SCENARIO 2 COMPARING REVENUES - DISCOUNT VS. NON-DISCOUNT

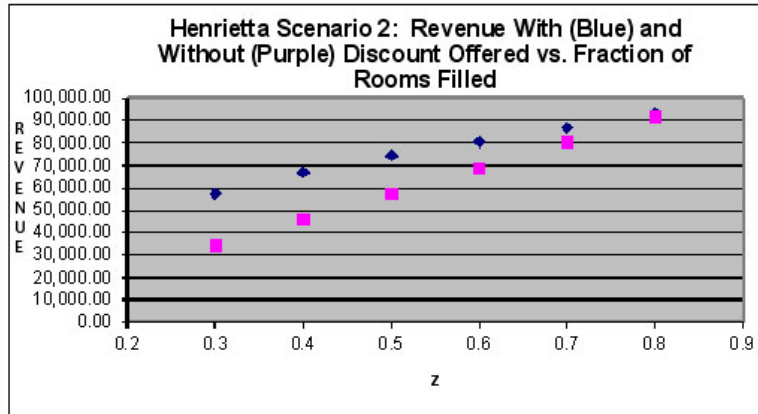


Figure 40: Within scenario 2 in Henrietta, when a discount is offered to fill up vacant rooms, the hotel's revenue is higher than if no discount is offered and the rooms remain vacant.

HENRIETTA SCENARIO 2 PERCENT CHANGE - DISCOUNT VS. NON-DISCOUNT

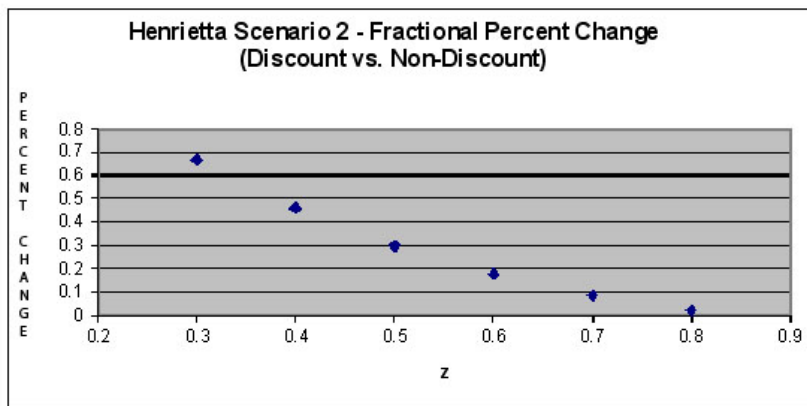


Figure 41: Within scenario 2 in Henrietta, percent change is shown for revenue when a discount is offered to fill vacant rooms and when a discount is not offered.

Regardless of whether we are considering compensation, cancellation fees, re-booking, or any combination, it can be concluded that the hotel will increase its revenue when a discount is offered to increase occupancy levels.

7 Conclusion

The process of maximizing hotel revenue depends on many factors. Our goal was to provide a mathematical model to reduce the guesswork involved. As stated in the introduction, there are 124,416 combinations of variables and elements that need be considered when maximizing revenue. The interdependent nature of these variables forced us to make many assumptions throughout the paper.

We chose to focus our paper on the effect that discount strategies have on maximizing revenue. Offering a discount impacts many other factors, such as how many customers take advantage of the discount, how many customers ask for compensation, how many customers cancel their reservations and then re-book at a discounted rate, how many vacant rooms are filled, the no-show rate, customer satisfaction, etc. Our first model indicated that revenue was shown to maximize at relatively small discount rates ranging between 2 percent and 8 percent. These numbers vary based on factors such as location and business climate. This model also indicates that requiring customers to pre-pay at a discounted rate will reduce the no-show rate, thereby increasing revenue as there are fewer instances of revenue loss.

We noted that hotels were offering discounts larger than 8 percent, which we assumed was done as a strategy to fill up empty rooms. Our second model was created to study the effects of this practice. Offering discounts to fill up vacant rooms also increases the hotel's revenue, as rooms that were empty are now filled, causing the revenue to grow. We discussed that many customers have found creative methods to minimize money spent on hotel rooms. We were then forced to consider two scenarios (compensation given to the customer, or the customer cancels his reservation and re-books at a reduced rate) when calculating the revenue generated by rooms that are no longer vacant. Regardless of which scenario was considered, offering a discount to fill up vacant rooms proved to be more lucrative to hotels than offering no discount at all. We noted that making a discount available only to those who pre-pay may be necessary to eliminate revenue loss.

Based on our findings, we can make the following recommendations.

1. Offering a discount to reduce the no-show rate is beneficial to hotels that are not in need of filling vacant rooms. The discount should be no more than 8 percent.
2. Offering a discount to fill up vacant rooms always increases revenue. When this practice is used in combination with requiring pre-payment for said discount, the effect is positive.
3. We recommend considering the effects of discounts on customer satisfaction. For example, when there is no compensation given back to the customer, or when a hotel requires a high cancellation fee, the hotel stands to make more money. However, the long-term effects of such a decision can be detrimental.
4. When maximizing revenue, it is important to note that the amount by which a hotel's revenue increases depends on the effect that the discount has on:
 - (a) The number of people who book at the discounted rate.
 - (b) The number of people who cancel and re-book at the discounted rate.
 - (c) The number of people who request compensation for not being offered the original discount.
 - (d) The amount of revenue loss.
5. Location and business climate greatly affect revenue, and should be considered when offering discounts.

In addition, we would like to make the following recommendations for the expansion of, or future study with, this paper:

1. We did not consider the effect that any of our assumptions have on customer satisfaction. Customer satisfaction is an integral part of the process of maximizing revenue, as poor customer service can translate into lost revenue.
2. There are many factors that we did not include in this paper, such as third party booking (for example Priceline or Expedia), group rates, and the state of the economy, to name a few.
3. The paper could be improved with more data points. We only had two data points from which to build our model (one from the hotel in Miami, FL and one from the hotel in Henrietta, NY). The small number of data points forced us to make many assumptions, some of which may be inaccurate.

In closing, this model demonstrates the positive effect that discounts have on maximizing revenue. However, a mathematical model alone is not enough, as there are too many factors for one model to consider. When used in conjunction with good judgement and managerial expertise, our mathematical model can increase a hotel's potential revenue through the implementation of discounts.

References

- [1] Rex S. Toh and Frederick Dekay, *Hotel Room Inventory Management: An Overbooking Model*, Cornell HRA Quarterly, Vol. 43, No. 4, pp. 79-90, Published by Elsevier Science (2002).
- [2] Gunther, K., Personal Interview with Dr. Darren Narayan, 2011
- [3] Managerial Staff at Conrad Hilton in Miami, FL, Personal Interview with Dr. Darren Narayan, 2011
- [4] ebookers.com, Ebookers Offers 60 Percent Discount on Hotels, Wikinews, <http://news.ebookers.com/news/ebookers-offers-60-percent-discount-on-hotels/125/>.
- [5] arvo.com, Hotel Minimum Night Stays and Cancellation Policy, <http://www.arvo.org/eweb/am2011/pdf/AM2011-Hotel-cancellation.pdf>