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Extension of evaluating the operating characteristics for dependent mixed variables-attributes sampling plans to large first sample size

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Extension of Evaluating the Operating Characteristics for Dependent Mixed Variables-Attributes Sampling Plans To Large First Sample Size

by

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ABSTRACT

In evaluation of the operating characteristics of mixed sampling plans (σ known), the probability function, $F_n(x)$, of the difference between the extreme value and the mean is important. Because it is an $n^{\underline{th}}$ multiple integral where n is the first sample size of a mixed dependent sampling plan, the normal way of evaluating this probability is the recursive quadrature method. Using this method, as the first sample size increases, the amount of computation increases exponentially, so that this probability function becomes a bottleneck in computation. This thesis presents a Taylor-expansion method for handling the probability function for large first sample sizes with satisfactory precision, for an arbitrary given x. In such a way, the evaluation of the dependent mixed plans is extended to a larger first sample size than previously available in the literature.

Theoretically, the computational method presented here could be used for evaluating mixed plans of very large first sample size. Technically, however, this method is limited, by the computer memory resources.

Using the computational approach described above, a FORTRAN program that can compute the operating characteristics of mixed plans for first sample sizes of up to 50 was developed. A series of mixed dependent sampling plans were then evaluated with an accuracy of seven decimal places.

I. Introduction

Mixed variables-attributes sampling plans combine the advantage of variables inspection and attributes inspection. They can be widely used in the fields of process acceptance inspection and in the lot acceptance procedures. The procedure for evaluating dependent mixed plans has been developed by Schilling and Dodge [1966a]. Joint probabilities, acceptance probability (Pa), average outgoing quality level (AOQ), and average sampling number (ASN), for a series of mixed plans of first sample size up to 10, have been computed. Since the mixed plans might be used under different sampling requirements, extending the evaluation of mixed plans to larger first sample size is necessary.

The procedure developed by Schilling and Dodge introduces the probability distribution function of the extreme deviate from the mean for computing the joint probability of $\bar{x} > A$ and failure number $i < c$. The probability distribution function $F_n(X)$ is a multiple recursive integral:

$$\begin{cases} F_n(X) = \dfrac{n\sqrt{n}}{\sqrt{n-1}} \displaystyle\int_0^X \frac{1}{\sqrt{2\pi}} e^{-\frac{n}{2(n-1)}x^2} F_{n-1}\left(\frac{n}{n-1}x\right) dx , \quad \text{for } n>1 \\ F_1(X) = 1 \end{cases}$$

where, if $S=\{ y_1, y_2, \ldots, y_n \}$ is a sample of size n drawn from a standard normal distributed population, and $\{ x_1, x_2, \ldots x_n \}$ are the order statistics of S, and $\bar{x}$ is the sample mean, then $P(x_n - \bar{x} < X) = F_n(X)$ (see Mackay [1935], Nair [1948], and Grubbs [1950]).

There is no analytical closed-form expression for computing $F_n(x)$. Grubbs' [1950] tabulated $F_n(x)$ from n=2 to 25, for x=0 to 5, with the

interval 0.05, by using the numerical quadrature method. The previous evaluation of mixed plans applied interpolation to the tabulated data.

This thesis presents a successive Taylor-expansion method for computing $F_n(x)$. Using this method, the Taylor expansion was first successively applied to compute the $F_n(x)$ values for x=0 to 5 in specific small intervals, for n=2 to 50, to establish an $F_n(x)$ matrix. Secondly, the $F_n(x)$ value for given n and x could then be expressed as a Taylor series in the neighborhood of a point of the $F_n(x)$ matrix under a given accuracy requirement. Based on this algorithm and the procedure for evaluating mixed dependent plans, a FORTRAN program for evaluating the OC curves, AOQ, and ASN of mixed sampling plans, in which the first sample size is less than or equal to 50, was written and several mixed plans were then evaluated.

II. Mixed Variables-Attributes Sampling Plans

1. The Nature of Mixed Variables-Attributes Sampling Plans and Historical Review

In acceptance sampling, there are two alternative methods of sampling inspection: variables inspection and attributes inspection. A variables inspection utilizes the sample mean and sample standard deviation, and some assumption (normality, etc.) must be made about the distribution of the quality characteristic being tested. In attributes sampling, the sample items are classified as defective or non-defective with respect to the quality characteristic(s). The attributes plan involves no assumption about the form of the distribution of the quality characteristic(s).

Because variables inspection utilizes the information about the population distribution, samples of smaller sizes are needed for a given degree of reliability in determining the acceptability of a lot. Variables inspection is prefered because of its smaller sample size, particularly when the testing is expensive or destructive. It is possible, however, that a sufficient number of individuals are close enough to the specification limit of the quality characteristic to cause the sample mean to fail the test criterion even though the lot contains no defective units. The combination of the variables inspection and attributes inspection in sampling plans, called Mixed Variables-Attributes Sampling Plans, has been suggested as a method of retaining the small sample size and yet insuring that defective units will be present if the lot is rejected.

An advantage of a mixed variables-attributes plan over a variables plan is the protection provided for sampling inspection from truncated and non-normal distributions. Under the mixed variables-attributes plan, a lot can not be rejected by poor variables results alone; rejection is always based on the result of the attributes inspection. Hence, there will always

be defectives in a rejected sample to show to the producer.

Compared to using an attributes plan alone, the mixed plan reduces the sample size for the same degree of discrimination for acceptance. Also, the variables inspection aspect of the mixed plan provides a more careful analysis of the distribution of quality characteristics of the product population being sampled. Since the control charts can provide the information for variable inspection, they can be used with mixed plans.

Furthermore, according to Schilling [1966], nonnormlity concerns can be minimized using a mixed plan "...by designing the plan in such a way as to accept on a variables basis only product with distribution locates far enough from the specification limit so that reasonable changes in the shape of the distribution will not cause appreciable changes in percent defective. In this way, a tight variables criterion could be employed to minimize the effect of changes in shape of distribution on the operating characteristic curve of the plan."

Dodge [1932] suggested that variables criteria be used in the first stage of a double-sampling plan "... for judging the results of a first sample and for determining when a second, substantially larger sample should be inspected before rejecting the lot." A double-sampling procedure involves variables inspection in the first sample and subsequent attributes inspection in the second sample or in both.

Mixed plans are of two types: "independent" and "dependent". For independent mixed plans, decisions on the two samples are independent. Dependent mixed plans combine the results of the first and second samples in making a rejection decision if indeed the second sample for attributes inspection is even necessary. The procedure of independent mixed plans was first discussed by Bowker, and Goode [1952]. In 1966, Schilling provided procedures for deriving independent mixed plans given two points on the OC curve. Gregory and Resnikoff [1955a] examined Dependent Mixed Plans, and provided an asymptotic expansion for approximating $P(Z_u\text{-}Z|c)$, they also discussed the case where the standard deviation is unknown. That same year, the general procedure of dependent mixed plans was proposed by Savage

[1955b] and is summarized here as follows:

1. Take first sample.
2. Test first sample against a given variables criterion and:
   a) If the test meets the variables criterion, Accept.
   b) If the test fails the variables criterion:
      (1) Reject, if the number of defectives in the sample exceeds a given attributes criterion.
      (2) Otherwise, take a second sample.
3. Obtain a second sample if 2 b)(2).
4. Test number defective of the first and second samples together against the given attributes criterion, and accept or reject as indicated by the test.

Schilling and Dodge [1966]&[1969] have developed a procedure for evaluating the operating characteristic of dependent mixed plans. More recently, Adams and Mirkhani [1976] have derived an approach to handle cases in where the standard deviation is unknown and c=0 and they examined the effect of nonormality on mixed plans.

2. Dependent Mixed Variables-Attributes Sampling Plans

Since the lower specification can be treated in the same manner as the upper specification limit, this discussion on upper specification limit has generality. For a given single upper specification limit U, known σ, and production with a normally distributed quality characteristic, the dependent mixed plan has been generalized by Schilling and Dodge [1969] as follows:

Let

N = lot size

n_1 = first sample size

n_2 = second sample size

A = Acceptance limit on sample mean

c_1 = attributes acceptance number for first sample

c_2 = attributes acceptance number on combined second sample

Note that $A = U-k\sigma$, for upper specification limit and standard variables factor k.

Then, the generalized plan would be carried out as follow:

1. Determine the parameters of the mixed plan: n_1, n_2, A, c_1, c_2.
2. Take a random sample of size n_1 from the lot.
3. If the sample average $\bar{x} \leq A$, accept the lot.
4. If the sample average $\bar{x} > A$, examine the first sample for the number of defectives d_1 therein.
5. If $d_1 > c_1$, reject the lot.
6. If $d_1 \leq c_1$, take a second random sample of size n_2 from the lot and determine the number of defectives d_2 therein.
7. If in the combined sample of $n = n_1 + n_2$, the total number of defectives $d = d_1 + d_2$ is such that $d \leq c_2$, accept the lot.
8. If $d > c_2$, reject the lot.

3. MIL-STD-414 Dependent Mixed Plans

In MIL-STD-414 [1957], Dependent Mixed Plans are specified in paragraphs A9.2.2 to A9.4.2 as:

A9.2.2 Mixed Variables-Attributes Inspection.

Mixed variables-attributes inspection is inspection of a sample by attributes, in addition to inspection by variables already made of a previous sample, before a decision as to acceptability or rejectability of a lot can be made.

A9.3 Selection of Sampling Plans.

The mixed variables-attributes sampling plan shall be selected in accordance with the following:

A9.3.1 Select the variables sampling plan in accordance with section B, C, or D.

A9.3.2 Select the attributes sampling plan from MIL-STD-105, paragraph 10, using a single sampling plan and tightened inspection. The same AQL value(s) shall be used for the attributes sampling plan as used for the variables plans of paragraph A9.3.1.
(Additional sample items may be drawn, as necessary, to satisfy the requirements for sample size of the attributes sampling plan. Count as a defective each sample item falling outside of specification limits(s).)

A9.4 Determination of Acceptability.
A lot meets the acceptability criterion if one of the following conditions is satisfied:

Condition A. The lot complies with the appropriate variables acceptability criterion of section B, C, or D.

Condition B. The lot complies with the acceptability criterion of paragraph 11.1.2 of MIL-STD-105.

A9.4.1 If Condition A is not satisfied, proceed in accordance with the attributes sampling plan to meet Condition B.

A9.4.2 If Condition B is not satisfied, the lot does not meet acceptability criterion.

The sample sizes of variables inspection in MIL-STD-414 are within the range 3-200. The most frequently used sample sizes are 10 to 50 (the corresponding lot sizes: 26 to 3200).

4. Mixed Plans with Control Charts

Variables control charts have been widely used in process control. They provide information on the variability and stability of a product from lot

to lot. From the aspect of variables inspection with mixed plans, variables control charts can be used in conjunction with dependent mixed plans.

Because of their descriptive nature, mixed plans used with variables control charts, would increase the power of traditional process control technique for evaluating the product being produced. In using mixed plans with control charts in process control, the result from the control chart becomes the result of the first sample of the mixed plan. The procedure is as follows:

1. Suppose the $\bar{x}$-R or $\bar{x}$-s charts with subgroup size n from lot size N are kept on the production data. Determine a mixed plan with parameters $n_1=n$, n_2, A, c_1, c_2 for an expected AOQL level for a given fraction defective. Use A as the control limit.
2. If the control chart indicates a point out of control, examine the corresponding subgroup data to find out the number of defectives d_1 therein.
3. If $d_1 > c_1$, reject the lot,adjust the process, screen the lot.
4. If $d_1 \leq c_1$, take a second random sample of n_2 from the lot and determine the number of defectives d_2 therein.
5. If, in the combined sample of $n_1 + n_2$, the total number of defectives $d = d_1 + d_2$ is such that $d \leq c_2$, accept the lot.
6. If $d > c_2$, reject the lot, adjust the process, and screen the lot.

Control charts used in quality control are essentially of two types--those that are applied to bring a process under control and those that are employed for maintaining control. The mixed plans can be used with the both types of control charts.

5. First Sample Size of Mixed Plans

From 3 and 4 above, the possible first sample size for mixed sampling plans is varied from a small number to a large number according to the requirement of the application. Hence, extending the evaluation of the operating characteristics for mixed plans to large first sample size is meaningful.

III. Evaluation of Mixed Sampling Plans

1. Evaluation of Operating Characteristics for Mixed Plans

The formulation of the procedure developed by Schilling and Dodge [1966] for evaluating the joint probability, the acceptance probability (Pa), the Average Outgoing Quality (AOQ), and the Average Sampling Number (ASN) is shown below:

Notation:

n_1 = first sample size

n_2 = second sample size

c_1 = attributes acceptance number on first sample

c_2 = attributes acceptance number on second sample

p = population fraction defective

μ = population mean

σ = population standard deviation

A = acceptance limit on sample mean

$P_{n_1}(c,\bar{x}>A)$ = The joint probability of $\bar{x}>A$ and the number of failures less than or equal to c

$$z_A = \frac{A-\mu}{\sigma} \tag{1.1}$$

z_u is defined in such a way that $\int_{z_u}^{\infty} \phi(0,1)dx = p$ (1.2)

where $\phi(0,1)$ is the standard normal distribution function

$k = z_u - z_A$ (k factor)

$F_n(X)$: the probability of the extreme deviation from the sample mean (in terms of the population σ-standard normal)

$P(i,n)$: the probability of having i failures in sample of size n.

and, $P(i,n) = \binom{n}{i} p^i(1-p)^{n-i}$

Pa: Probability of Acceptance

AOQ: Average Outgoing Quality Level

ASN: Average Sampling Number

1.a. The Joint Probability :

$$P_{n_1}(0,\bar{x}>A) = \int_{z_A}^{z_u} \frac{\sqrt{n1}}{\sqrt{2\pi}} e^{-\frac{n_1\bar{z}^2}{2}} F_{n_1}(z_u-\bar{z})\, d\bar{z}\ , \quad \text{for } c_1=0. \tag{1.3}$$

$$P_{n_1}(c_1,\bar{x}>A) = \binom{n_1}{i} \int_{z_u}^{\infty} \int_{\frac{n_1 z_A - i\bar{z}_2}{n_1-i}}^{z_u} \frac{\sqrt{i(n_1-i)}}{2\pi} e^{\frac{1}{2}[i\bar{z}_2{}^2+(n_1-i)\bar{z}_1{}^2]}$$

$$F_i(\bar{z}_2-z_u)\ F_{n_1-i}(z_u-\bar{z}_1)\ d\bar{z}_1 d\bar{z}_2\ , \quad \text{for } c_1 > 0 \tag{1.4}$$

1.b. The Pa, ASN, and AOQ :

When the variables inspection is used alone on the first sample for acceptance:

$$Pa = P(\bar{x}\le A) + \sum_{i=0}^{c_1}\sum_{j=0}^{c_2-i} P_{n_1}(i,\bar{x}>A)\ P(j,n_2)\ , \tag{1.5}$$

where $P(\bar{x}\le A) = \int_{-\infty}^{A} \phi(0,1)dx$

When both variables and attributes inspections are employed on the first sample for acceptance:

$$\begin{aligned} Pa &= \sum_{i=0}^{c1} P(i,\bar{x}\le A) + \sum_{i=0}^{c_1}\sum_{j=0}^{c_2-i} P_{n_1}(i,\bar{x}>A)\ P(j,n_2) \\ &= \sum_{i=0}^{c_1} P(i,n_1) - \sum_{i=0}^{c_1} P_{n_1}(i,\bar{x}>A) + \sum_{i=0}^{c_1}\sum_{j=0}^{c_2-i} P_{n_1}(i,\bar{x}>A)\ P(j,n_2) \end{aligned} \tag{1.6}$$

$$ASN = n_1 + n_2 \sum_{i=0}^{c_1} P_{n_1}(i,\bar{x}>A) \tag{1.7}$$

$$AOQ = p\ P_a \tag{1.8}$$

1.c. $F_n(x)$:

$$\begin{cases} F_n(X) = \dfrac{n\sqrt{n}}{\sqrt{n-1}} \displaystyle\int_0^X \frac{1}{\sqrt{2\pi}} \, e^{-\frac{n}{2(n-1)}x^2} F_{n-1}\left(\frac{n}{n-1}x\right) dx \ , \text{ for } n>1 \\ F_1(X) = 1 \end{cases} \quad (1.9)$$

2. <u>The</u> <u>Taylor-Expansion</u> <u>Method</u> <u>for</u> <u>Computation</u> <u>of</u> $F_n(x)$:

According to the Taylor's Theorem, given an arbitrary real function $f(x)$ with continuous $i^{\underline{th}}$ $(i=1,2,\ldots,n+1)$ derivate in the neigborhood of x_0 (i.e. $x_0 \pm \delta$), then for all x in the interval $(x_0-\delta,\ x_0+\delta)$,

$$f(x) = \sum_{i=0}^{n} \frac{f^{(i)}(x_0)}{i!} (x-x_0)^i + R_{n+1}(x)$$

$$\text{where} \quad R_{n+1}(x) = \frac{1}{n!} \int_{x_0}^{x} (x-x_0)^n \ f^{(n+1)}(t)dt \ , \quad (2.1)$$

$$\text{or} \quad R_{n+1}(x) = \frac{(x-x_0)^{n+1}}{(n+1)!} \ f^{(n+1)}(\xi) \ , \quad (2.2)$$

and ξ is a value between x_0 and x.

The form (2.1) is called the **integral form** of the remainder, and the form (2.2) is called the **Lagrange form** of the remainder.

2.a. The $1^{\underline{st}}$, $2^{\underline{nd}}$, $3^{\underline{rd}}$, $4^{\underline{th}}$ Derivates of $F_n(x)$:

In (1.9), by setting :

$$X = \frac{1}{r} v_r \ , \text{ for } r = 2,3,\ldots,n$$

The cumulative distribution function $F_n(X)$ may be expressed as:

$$F_n(x) = \frac{\sqrt{n}}{(\sqrt{2\pi})^{n-1}} \int_0^{nx} \int_0^{v_n} \int_0^{v_{n-1}} \cdots \int_0^{v_5} \int_0^{v_4} \int_0^{v_3} \exp\left[-\frac{1}{2}\sum_{i=2}^{n} \frac{v_i^2}{i(i-1)}\right] dv_2 dv_3 \ldots dv_n \tag{2.3}$$

Define a function $G_n(x)$ such that:

$$\begin{cases} G_n(x) = \int_0^{x} e^{-\frac{t^2}{2n(n-1)}} G_{n-1}(t)dt, \text{ for } n>1 \\ G_1(x) = 1 \end{cases} \tag{2.4}$$

then, we have

$$F_n(x) = \frac{\sqrt{n}}{(\sqrt{2\pi})^{n-1}} G_n(nx), \text{ for } n=2,3,4,\ldots \tag{2.5}$$

and

$$F_n^{(i)}(x) = \frac{d^{(i)}}{dx^{(i)}} F_n(x)$$

$$= \frac{n^i\sqrt{n}}{(\sqrt{2\pi})^{n-1}} \frac{d^{(i)}}{d(nx)^{(i)}} G_n(nx) . \tag{2.6}$$

From (2.4) we have

$$G_n^{(1)}(t) = e^{-\frac{1}{2n(n-1)}t^2} G_{n-1}(t) \tag{2.7}$$

$$G_n^{(2)}(t) = e^{-\frac{1}{n(n-2)}t^2} G_{n-2}(t) - \frac{t}{n(n-1)} e^{-\frac{1}{2n(n-1)}t^2} G_{n-1}(t) \text{ , for } n>2$$

(2.8a)

$$G_n^{(2)}(t) = -\frac{t}{n(n-1)}\,e^{-\frac{1}{2n(n-1)}t^2}\,G_{n-1}(t), \quad \text{for } n=2$$

(2.8b)

$$G_n^{(3)}(t) = e^{-\frac{3}{2n(n-3)}t^2}\,G_{n-3}(t) - \left[\frac{t}{n(n-1)} + \frac{2t}{n(n-2)}\right] e^{-\frac{1}{n(n-2)}t^2}\,G_{n-2}(t)$$
$$+ \left[\frac{t^2}{n^2(n-1)^2} - \frac{1}{n(n-1)}\right] e^{-\frac{1}{2n(n-1)}t^2}\,G_{n-1}(t), \quad \text{for } n>3$$

(2.9a)

$$G_n^{(3)}(t) = -\left[\frac{t}{n(n-1)} + \frac{2t}{n(n-2)}\right] e^{-\frac{1}{n(n-2)}t^2}\,G_{n-2}(t)$$
$$+ \left[\frac{t^2}{n^2(n-1)^2} - \frac{1}{n(n-1)}\right] e^{-\frac{1}{2n(n-1)}t^2}\,G_{n-1}(t), \quad \text{for } n=3$$

(2.9b)

$$G_n^{(3)}(t) = \left[\frac{t^2}{n^2(n-1)^2} - \frac{1}{n(n-1)}\right] e^{\frac{1}{2n(n-1)}t^2}\,G_{n-1}(t), \quad \text{for } n=2$$

(2.9c)

$$G_n^{(4)}(t) = e^{-\frac{2}{n(n-4)}t^2}\,G_{n-4}(t) \quad \left[\frac{t}{n(n-1)} + \frac{2t}{n(n-2)} + \frac{3t}{n(n-3)}\right] e^{-\frac{3}{2n(n-3)}t^2}\,G_{n-3}(t)$$
$$+ \left[\frac{t^2}{n^2(n-1)^2} + \frac{2t^2}{n^2(n-1)(n-2)} + \frac{4t^2}{n^2(n-2)^2} - \frac{2}{n(n-1)} - \frac{1}{n(n-2)}\right] e^{-\frac{1}{n(n-2)}t^2}\,G_{n-2}(t)$$
$$+ \left[\frac{3t}{n^2(n-1)^2} - \frac{t^3}{n^3(n-1)^3}\right] e^{-\frac{1}{2n(n-1)}t^2}\,G_{n-1}(t), \quad \text{for } n>4$$

(2.10a)

$$G_n^{(4)}(t) = -\left[\frac{t}{n(n-1)} + \frac{2t}{n(n-2)} + \frac{3t}{n(n-3)}\right] e^{-\frac{3}{2n(n-3)}t^2} G_{n-3}(t)$$

$$+ \left[\frac{t^2}{n^2(n-1)^2} + \frac{2t^2}{n^2(n-1)(n-2)} + \frac{4t^2}{n^2(n-2)^2} - \frac{2}{n(n-1)} - \frac{1}{n(n-2)}\right] e^{-\frac{1}{n(n-2)}t^2} G_{n-2}(t)$$

$$+ \left[\frac{3t}{n^2(n-1)^2} - \frac{t^3}{n^3(n-1)^3}\right] e^{-\frac{1}{2n(n-1)}t^2} G_{n-1}(t), \quad \text{for } n=4$$

(2.10b)

$$G_n^{(4)}(t) =$$

$$\left[\frac{t^2}{n^2(n-1)^2} + \frac{2t^2}{n^2(n-1)(n-2)} + \frac{4t^2}{n^2(n-2)^2} - \frac{2}{n(n-1)} - \frac{1}{n(n-2)}\right] e^{-\frac{1}{n(n-2)}t^2} G_{n-2}(t)$$

$$+ \left[\frac{3t}{n^2(n-1)^2} - \frac{t^3}{n^3(n-1)^3}\right] e^{-\frac{1}{2n(n-1)}t^2} G_{n-1}(t), \quad \text{for } n=3$$

(2.10c)

$$G_n^{(4)}(t) = \left[\frac{3t}{n^2(n-1)^2} - \frac{t^3}{n^3(n-1)^3}\right] e^{-\frac{1}{2n(n-1)}t^2} G_{n-1}(t), \quad \text{for } n=2$$

(2.10d)

Use the relation between $F_n(x)$ and $G_n(t)$ (2.4),

Substitute (2.4) into (2.7), (2.8), (2.9), and (2.10) to obtain the derivate of $F_n(x)$.

From (2.7),

$F_n^{(1)}(x)$:

$$F_n^{(1)}(x) = \frac{n\sqrt{n}}{\sqrt{n-1}} \frac{1}{\sqrt{2\pi}} e^{-\frac{n}{2(n-1)}x^2} F_{n-1}\left(\frac{n}{n-1}x\right) \qquad (2.11)$$

$\mathbf{F_n^{(2)}(x)}$:

From (2.8a), for n>2,

$$F_n^{(2)}(x) \;=\; \frac{n^2\sqrt{n}}{\sqrt{n-2}}\,\frac{1}{2\pi}\,e^{-\frac{1}{(n-2)}x^2}\,F_{n-2}(\tfrac{n}{n-2}x)$$

$$-\;\frac{n^2x\sqrt{n}}{\sqrt{n-1}\,\sqrt{2\pi}}\,e^{-\frac{n}{2(n-1)}x^2}\,F_{n-1}(\tfrac{n}{n-1}x) \qquad (2.12a)$$

From (2.8b), for n=2:

$$F_n^{(2)}(x) \;=\; -\,\frac{n^2x\sqrt{n}}{\sqrt{n-1}\,\sqrt{2\pi}}\,e^{-\frac{nx^2}{2(n-1)}}\,F_{n-1}(\tfrac{n}{n-1}x) \qquad (2.12b)$$

$\mathbf{F_n^{(3)}(x)}$:

From (2.9a), for n>3:

$$F_n^{(3)}(x) \;=\; n^3\{\frac{\sqrt{n}}{\sqrt{n-3}\,(\sqrt{2\pi})^3}\,e^{-\frac{3n}{2(n-3)}x^2}\,F_{n-3}(\tfrac{n}{n-3}x)$$

$$-\;[\frac{x}{(n-1)} + \frac{2x}{(n-2)}]\frac{\sqrt{n}}{\sqrt{n-2}}\,\frac{1}{2\pi}\,e^{-\frac{n}{n-2}x^2}\,F_{n-2}(\tfrac{n}{n-2}x)$$

$$+\;[\frac{x^2}{(n-1)^2} \quad \frac{1}{n(n-1)}]\;\frac{\sqrt{n}}{\sqrt{n-1}}\;\frac{1}{\sqrt{2\pi}}\;e^{-\frac{n}{2(n-1)}x^2}\,F_{n-1}(\tfrac{n}{n-1}x)\} \qquad (2.13a)$$

From (2.9b), for n=3:

$$F_n^{(3)}(x) \;=\; n^3\{-[\frac{x}{(n-1)} + \frac{2x}{(n-2)}]\frac{\sqrt{n}}{\sqrt{n-2}}\,\frac{1}{2\pi}\,e^{-\frac{n}{n-2}x^2}\,F_{n-2}(\tfrac{n}{n-2}x)$$

$$+\;[\frac{x^2}{(n-1)^2} - \frac{1}{n(n-1)}]\;\frac{\sqrt{n}}{\sqrt{n-1}}\;\frac{1}{\sqrt{2\pi}}\;e^{-\frac{n}{2(n-1)}x^2}\,F_{n-1}(\tfrac{n}{n-1}x)\} \qquad (2.13b)$$

From (2.9c), for n=2:

$$F_n^{(3)}(x) = n^3 \left[\frac{x^2}{(n-1)^2} - \frac{1}{n(n-1)}\right] \frac{\sqrt{n}}{\sqrt{n-1}} \frac{1}{\sqrt{2\pi}} e^{-\frac{n}{2(n-1)}x^2} F_{n-1}\left(\frac{n}{n-1}x\right)\} \qquad (2.13c)$$

$F_n^{(4)}(x)$:

From (2.10a), for n>4:

$$F_n^{(4)}(t) = n^4\{\frac{\sqrt{n}}{\sqrt{n-4}\,(2\pi)^2} e^{-\frac{2n}{(n-4)}x^2} F_{n-4}\left(\frac{n}{n-4}x\right)$$

$$\left[\frac{x}{(n-1)} + \frac{2x}{(n-2)} + \frac{3x}{(n-3)}\right]\frac{\sqrt{n}}{\sqrt{n-3}\,(\sqrt{2\pi})^3} e^{-\frac{3n}{2(n-3)}x^2} F_{n-3}\left(\frac{n}{n-3}x\right)$$

$$+ \left[\frac{x^2}{(n-1)^2} - \frac{2x^2}{(n-1)(n-2)} + \frac{4x^2}{(n-2)^2} \quad \frac{2}{n(n-1)} - \frac{1}{n(n-2)}\right]\frac{\sqrt{n}}{\sqrt{n-1}\,(2\pi)} e^{-\frac{n}{(n-2)}x^2} F_{n-2}\left(\frac{n}{n-2}x\right)$$

$$+ \left[\frac{3x}{n(n-1)^2} - \frac{x^3}{(n-1)^3}\right] \frac{\sqrt{n}}{\sqrt{n-1}} \frac{1}{\sqrt{2\pi}} e^{-\frac{n}{2(n-1)}x^2} F_{n-1}\left(\frac{n}{n-1}x\right)\} \qquad (2.14a)$$

From (2.10b), for n=4:

$$F_n^{(4)}(t) = n^4\{ - \left[\frac{x}{(n-1)} + \frac{2x}{(n-2)} + \frac{3x}{(n-3)}\right]\frac{\sqrt{n}}{\sqrt{n-3}\,(\sqrt{2\pi})^3} e^{-\frac{3n}{2(n-3)}x^2} F_{n-3}\left(\frac{n}{n-3}x\right)$$

$$+ \left[\frac{x^2}{(n-1)^2} + \frac{2x^2}{(n-1)(n-2)} + \frac{4x^2}{(n-2)^2} - \frac{2}{n(n-1)} - \frac{1}{n(n-2)}\right]\frac{\sqrt{n}}{\sqrt{n-1}\,(2\pi)} e^{-\frac{n}{(n-2)}x^2} F_{n-2}\left(\frac{n}{n-2}x\right)$$

$$+ \left[\frac{3x}{(n-1)^2} - \frac{x^3}{(n-1)^3}\right]\frac{\sqrt{n}}{\sqrt{n-1}} \frac{1}{\sqrt{2\pi}} e^{-\frac{n}{2(n-1)}x^2} F_{n-1}\left(\frac{n}{n-1}x\right)\} \qquad (2.14b)$$

From (2.10c), for n=3 :

$$F_n^{(4)}(t) =$$

$$n^4\{[\frac{x^2}{(n-1)^2} + \frac{2x^2}{(n-1)(n-2)} + \frac{4x^2}{(n-2)^2} - \frac{2}{n(n-1)} - \frac{1}{n(n-2)}] \frac{\sqrt{n}}{\sqrt{n-1}\ (2\pi)} e^{-\frac{n}{(n-2)}x^2} F_{n-2}(\frac{n}{n-2}x)$$

$$+ [\frac{3x}{(n-1)^2} - \frac{x^3}{(n-1)^3}]\frac{\sqrt{n}}{\sqrt{n-1}} \frac{1}{\sqrt{2\pi}} e^{-\frac{n}{2(n-1)}x^2} F_{n-1}(\frac{n}{n-1}x)\} \qquad (2.14c)$$

From (2.10d), for n=2 :

$$F_n^{(4)}(t) = n^4 [\frac{3x}{(n-1)^2} - \frac{x^3}{(n-1)^3}]\frac{\sqrt{n}}{\sqrt{n-1}} \frac{1}{\sqrt{2\pi}} e^{-\frac{n}{2(n-1)}x^2} F_{n-1}(\frac{n}{n-1}x) \qquad (2.14d)$$

In summarizing (2.11), (2.12), and (2.13),

$$F_n^{(1)}(x) = a_1 F_{n-1}(\frac{n}{n-1}x) \qquad (2.15)$$

$$F_n^{(2)}(x\ ) = \begin{cases} b_1 F_{n-1}(\frac{n}{n-1}x) + b_2 F_{n-2}(\frac{n}{n-2}x), & \text{if } n>2 \\ b_1 F_{n-1}(\frac{n}{n-1}x), & \text{if } n=2 \end{cases}$$

$$F_n^{(3)}(x) = \begin{cases} c_1 F_{n-1}(\frac{n}{n-1}x)+c_2 F_{n-2}(\frac{n}{n-2}x)+c_3 F_{n-3}(\frac{n}{n-3}x), & \text{if } n>3 \\ c_1 F_{n-1}(\frac{n}{n-1}x)+c_2 F_{n-2}(\frac{n}{n-2}x), & \text{if } n=3 \\ c_1 F_{n-1}(\frac{n}{n-1}x), & \text{if } n=2 \end{cases}$$

Where a_1, b_1, b_2, c_1, c_2, c_3 are those corresponding coefficients.

2.b. Establishing the $F_n(x)$ Matrix :

From (1.9), in the domain of x>0, $F_n(x)$ has continuous derivative of

arbitrary order for $n \geq 2$. Hence Taylor Expansion could be applied.

Using only the first four terms Taylor series expansion of $F_n(x_{i+1})$ given that $F_n(x_i)$ is known,

let $x_{i+1}-x_i = \delta x_1$. Then,

$$F_n(x_{i+1}) = F_n(x_i) + F_n^{(1)}(x_i)\delta x_1 + \frac{F_n^{(2)}(x_i)}{2}\delta x_1^2 + \frac{F_n^{(3)}(x_i)}{6}\delta x_1^3 + R_4(x_{i+1}) \qquad (2.17)$$

Since the $1^{\underline{st}}$, $2^{\underline{nd}}$, and $3^{\underline{rd}}$ derivate of $F_n(x_i)$ are the functions of $F_{n-1}(\frac{n}{n-1}x_i)$, $F_{n-2}(\frac{n}{n-2}x_i)$, and $F_{n-3}(\frac{n}{n-3}x_i)$, when $F_n(x_i)$, $F_{n-1}(\frac{n}{n-1}x_i)$, $F_{n-2}(\frac{n}{n-2}x_i)$, and $F_{n-3}(\frac{n}{n-3}x_i)$ are known, then $F_n(x_{i+1})$ can be computed with error $R_{n+1}(x_{i+1})$. According to the earlier calculation by Grubbs [1950], for $x>5$, $F_n(x)=1$; hence, in this investigation, the computation are focued on the interval [0,5].

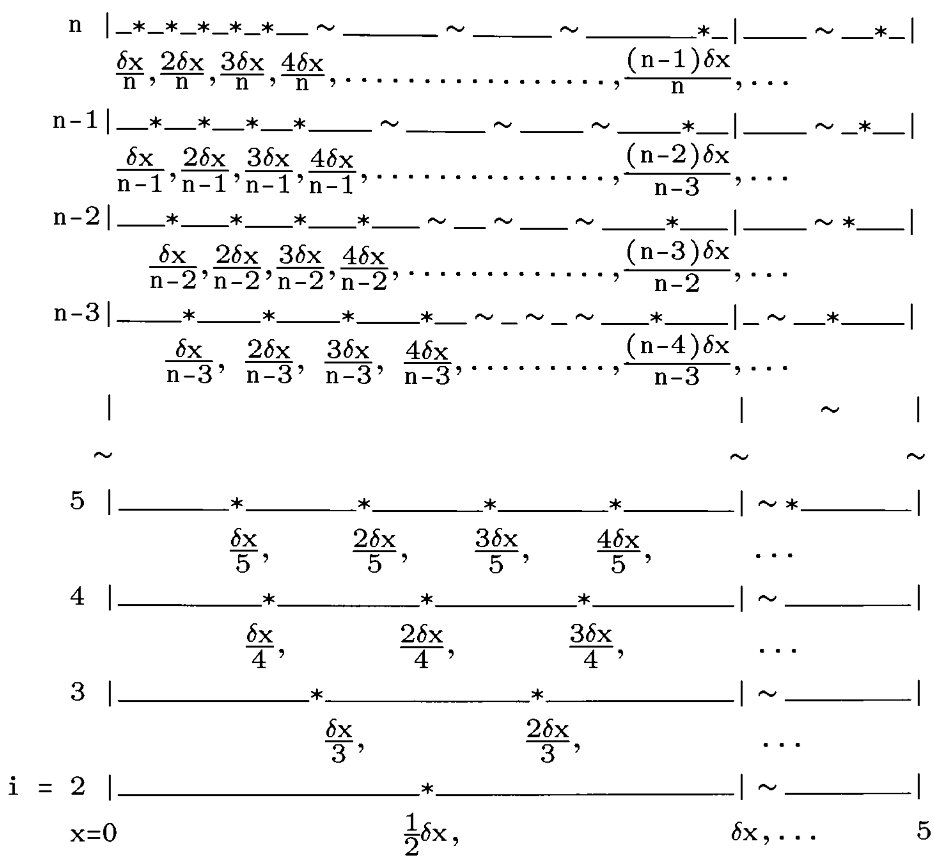
In order to compute $F_n(x)$ successively, let us construct a series partition of [0,5] for each value of n, so that the computation of $F_n(x)$ at those partition points x_i, for $i=2,3,4,\ldots,n$ can be based on those known values of

$$F_n(x_{i-1}),\ F_{n-1}(\tfrac{n}{n-1}x_{i-1}),\ F_{n-2}(\tfrac{n}{n-2}x_{i-1}),\text{ and } F_{n-3}(\tfrac{n}{n-3}x_{i-1}),$$

where $i = 2,3,4,\ldots,n$. Now, let $N \times \delta x$ be a partition of [0,5], such that $N = \frac{5}{\delta x}$ is an integer, and construct a series of partitions of [0,5] for $i=2,3,4,\ldots,n$, determined by

$$(N \times i) \times (\tfrac{\delta x}{i}). \qquad (2.18)$$

The procedure for constructing the partitions of [0,5] is shown in the figure of bellow:



* These partitions satisfy the described requirement, and this can be demonstrated accordingly:

For a given $i \le n$, and arbitrary integer $j<iN$,

$$x_j = \frac{j}{i}\delta x \text{ and } x_{j-1} = \frac{j-1}{i}\delta x$$

$$\frac{i}{i-1}x_{j-1} = (j-1)\frac{\delta x}{i-1} \text{ , for } i-1>0.$$

$$\frac{i}{i-2}x_{j-1} = (j-1)\frac{\delta x}{i-2} \text{ , for } i-2>0.$$

$$\frac{i}{i-3}x_{j-1} = (j-1)\frac{\delta x}{i-3} \text{ , for } i-3>0.$$

Using the partition (2.18), if the values of $F_i(\frac{\delta x}{i})$, for i=2,3,4,...,n, are known, then the $F_i(x)$ values at the partition points $\{x_j = \frac{j\delta x}{i} \mid j = 2, 3,4,5,...,iN\}$ can be computed successively by

$$F_n(x_j) = F_n(x_{j-1}) + F_n{}^{(1)}(x_{j-1})\frac{\delta x}{i} + \frac{F_n{}^{(2)}(x_{j-1})}{2}(\frac{\delta x}{i})^2 + \frac{F_n{}^{(3)}(x_{j-1})}{6}(\frac{\delta x}{i})^3 , \tag{2.19}$$

with error equal to $\frac{(\delta x)^4}{24i^4} F_4{}^{(4)}(\xi)$, $x_{j-1} < \xi < x_j$.

Nair [1948] presented an approximation formula for $F_n(x)$ in case x is very small:

$$F_n(x) \approx \frac{\sqrt{n}}{(n-1)!}(\frac{nx}{\sqrt{2\pi}})^{n-1}\{ 1- \frac{n(n-1)x^2}{2(n+1)}\} \tag{2.20}$$

Verification by the Guassian quadrature method, shows this formula to be very accurate for x<0.1.

Equations (2.11), (2.12),and (2.13) show that for n=2, the $F_n{}^{(1)}(x)$, $F_n{}^{(2)}(x)$,$F_n{}^{(3)}(x)$ are directly computable. Based on the previous discussion, given n, an $F_n(x)$ matrix, for i=2 to n, can be constructed according to the following procedure:

1. Select a small $\delta x<0.1$ such that $N = \frac{5}{\delta x}$ is an integer.

2. Use (2.20) to compute $F_i(\frac{\delta x}{i})$, for i=2,3,4,...,n.

3. Use (2.17) and (2.11) through (2.13c), successively, to compute the $F_i(j\times\frac{\delta x}{i})$, for j=2,3,...,iN, beginning with i=2, and stepping up to n.

2.c. Computation of $F_n(x)$ for a Given x

Within the $F_n(x)$ matrix that has been constructed, given arbitrary x>0 and i<n, the following principles apply:

1) if $x \geq 5$, then $F_i(x) = 1$.

2) if $x<5$, from the $i^{\underline{th}}$ row of the $F_n(x)$ matrix, find the nearest point x_j such that $|x_j-x|< \frac{1}{2i}\delta x$, and use that x_j to do a three-term Taylor expansion, where the values of $F_n(x_j)$, $F_n^{(1)}(x_j)$, $F_n^{(2)}(x_j)$, and $F_n^{(3)}(x_j)$ can be determined from within the matrix according to

$$F_n(x) = F_n(x_j)+ F_n^{(1)}(x_j)(x-x_j)+ \frac{F_n^{(2)}(x_j)}{2}(x-x_j)^2+ \frac{F_n^{(3)}(x_j)}{6}(x-x_j)^3$$

with error term $R_4(x)$.

3. The Computer Program

3.a The Algorithm

Set $\delta x=0.01$ and $n=50$, and follow the procedure discussed above to construct an $F_n(x)$ matrix with $2 \leq n \leq 50$. Then, use formula (1.1) to (1.8) to evaluate the given mixed plan for the specified nonconforming level(s).

For the joint probability (1.3), the program employs the IMSL subroutine QDAG to do the integration. This subroutine applies a (2k+1)-points Gauss-Kronrod rule to estimate the integral over each subinterval. The error for each subinterval is approximated by comparing the integral estimate obtained by the k-point Gaussian quadrature rule. The subinterval with the largest estimated error is then bisected, and the same procedure is applied to both halves. The bisection process is continued until either the error criterion is satisfied, roundoff error is detected, the subinterval becomes too small, or the maximum number of subintervals allowed is reached. In calling this subroutine, the absolute accuracy desired was set to 0 and the relative accuracy desired has been set to 0.0001. The choice of Gauss-Kronrod rule was made to 10-21 points.

For the joint probability (1.4), the program uses the IMSL subroutine TWODQ to do the two dimensional integration. Sharing the same characteristics as the QDAG, the TWODQ uses the Gauss-Kronrod rule for evaluation. The desired absolute and relative error setting are the same

as that of the QDAG. while the choice of the Gauss-Kronrod rule was put at 25-51 points. The $F_i(x)$ $(2 \leq i \leq 50)$ values for the given x's required by QDAG or TWODG were computed by using the Taylor-Expansion method with the $F_n(x)$ matrix values.

3.b. The Computational Accuracy in Evaluating the $F_n(x)$:

a. The error term.

In constructing the $F_n(x)$ matrix, use the Lagrange form of the remainder, given i<n and $x_j = j \times \frac{\delta x}{i} < 5$ such that, $j \in \{2,3,4,\ldots,iN\}$. Then,

$$R_4(x) = \frac{(\delta x)^4}{24 i^4} F_i^{(4)}(\xi) \xrightarrow[\delta x = 10^{-2}]{} \frac{10^{-8}}{24 i^4} F_i^{(4)}(\xi) \ ,$$

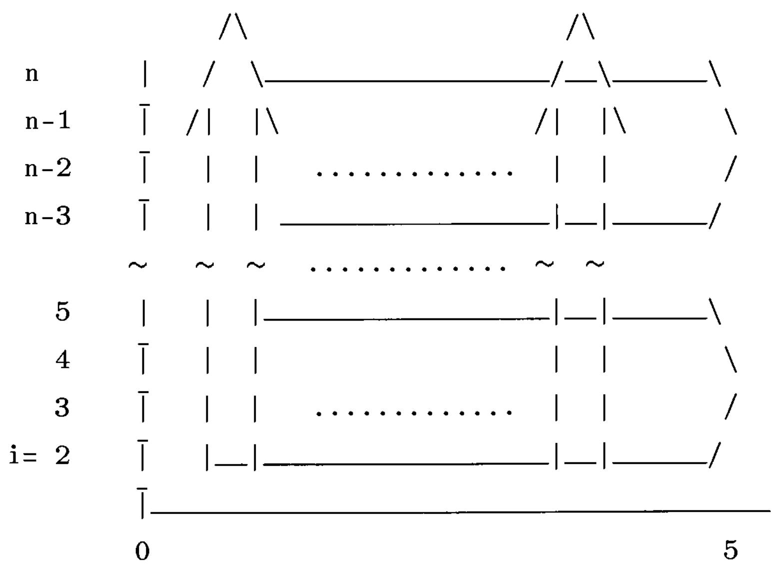
where $x_{j-1} < \xi < x_j$.

From (2.14a-d), $MAX(F_i^{(4)}(\xi)) \sim O(i^4)$, so

$$MAX(R_4(x)) \sim O(\frac{10^{-8}}{24}) = O(10^{-9})$$

b. The cumulative error of the procedure:

Since the procedure computes the $F_i(x_j)$ successively, the two possible ways of cumulating error are with respect to i or with respect to j. These two "paths" are illustrated pictorially in the figure shown below:



With i held constant (cumulating error horizontally), the integral form of the remainder (2.1) results in a cumulated error E for x_k given by:

$$E = \frac{1}{3!}\sum_{j=3}^{k}\int_{x_{j-1}}^{x_j}\left(\frac{\delta x}{i}\right)^3 F_i^{(4)}(t)dt \approx \frac{1}{6}\left(\frac{\delta x}{i}\right)^3\int_0^{x_j} F_i^{(4)}(t)dt \qquad (2.21)$$

Since $F_i(x)$ is a probability function and, for all $x > 5$, $F_i(x)=1$, it follows that

$$\int_5^x F_i^{(1)}(t)\ dt = 0, \text{ hence } \lim_{x\to 5}\int_0^x F_i^{(1)}(t)\ dt = 1 \ , \quad \text{and}$$

at the final point of the cumulation for $x \geq 5$,

$$E = \frac{\delta x^3}{6i^3}\int_0^5 F_i^{(4)}(t)dt = \lim_{x^-\to 5}\frac{d^{(3)}}{dt^3}\left[\frac{\delta x^3}{6i^3}\left(\int_0^5 F_i^{(1)}(t)dt + \int_5^x F_i^{(1)}(t)dt\right)\right] = 0 \ .$$

Note that the horizontally cumulated error will not grow infinitely, but rather it will end up at zero when x=5.

In the case of vertically cumulated error, since the error form i-1, i-2, and i-3 are weighted by at least 10^{-2}, 10^{-4}, and 10^{-6}, respectively, this error can be ignored. In short, the successive computation procedure has an error-digest characteristic that suppresses error growth. Since from (2.13a-c), the MAX $(F_i^{(3)}(x)) \sim O(i^3)$,

$$\text{MAX}[F_i^{(3)}(x)] = \text{MAX}\left(\int_0^x F_i^{(4)}(x)dx\right) \sim O(i^3) \ .$$

From (2.21), a conservative estimate of the error is

$$\text{MAX(ERROR)} \sim O(10^{-7}) \ .$$

Values in the $F_n(x)$ matrix values were compared to the tabulated $F_n(x)$ values[1] in Grubbs' [1950] paper; the first five decimal places are the same.

---

[1] - In Grubbs' paper, the tabulated $F_n(x)$ values are of five decimal places and $n \leq 19$.

A determination of the error in evaluating $F_n(x)$ matrix was discussed above, for the $F_i(x)$ ($\forall x$ beyond the matrix points x_j) computed by using the $F_n(x)$ matrix, According to the earlier discussion, $|x_j-x| < \frac{1}{2i}\delta x$; hence $|x-x_j| < \frac{\delta x}{i}$. So, the accuracy here is the same as that of the $F_n(x)$ matrix.

3.c. The Operation of the Program and an Example :

The program is written in FORTRAN-77, utilizing many IMSL subroutines for handling the integrations, probability functions, and inverse probability functions. (see Appendix for the code.)

To create an executable file in a VAX/VMS environment, the file containing the source code must first be compiled. The resulting "object" file then linked with the IMSL library. Specifically, the "compile" and "link" commands are:

```
$ FOR filename
$ LINK filename,LIB_IMSL/LIBRARY
```

To run the executable file,type in:

```
$ RUN executable-file
```

The required parameters are input at the keyboard as the prompts come up on the screen. (The program is an interactive program which asks the user for the parameters of the evaluated mixed plan(s) and the desired nonconforming level(s).

The program executes the first step in the execution to establish the $F_n(x)$ matrix, this requires approximately 5 to 10 minutes. The $F_n(x)$ matrix is then stored in memory until the user stops the execution. The working array for the matrix is of size $25 \times 51 \times 500$ (real variables).

To make efficient use of the execution time, evaluation of more than one mixed plan per run is recommended. The results are stored in a file named OC.DAT; this is a standard ASCII file that can be either printed out or be edited. On the RITVAX, an executable version of this program is in a

file name MIXED.EXE, available in the account: 800dept1.

EXAMPLE:

```
$ RUN MIXED
  Establishing the F-Matrix,it Takes about 5 to 10 minutes
  Please wait.....
  ***********************************************
  *       MIXED DEPENDENT SAMPLING PLANS         *
  *             EVALUATION PROGRAM               *
  *               ( Sigma Known)                 *
  *                                              *
  *         Maximum first sample size=50         *
  *  Maximum Number of nonconforming Levels=100  *
  *   Minimum nonconforming level=0.0000002867   *
  *          (corresponding Z value=5.0)         *
  ***********************************************
  First Sample Size:
  6
  Second Sample Size:
  20
  K Factor:
  3.0
  Use (1)Attri.and Variables on First Sample (0)Variables Only:
  1
  Attributes Criteria for First Sample C1:
  0
  Attributes Criteria for Second Sample C2:
  0
  Number of Nonconforming Levels:
  9
  Nonconforming Level         1:
  0.0000317
  Nonconforming Level         2:
  0.00135
```

```
Nonconforming Level         3:
0.005
Nonconforming Level         4:
0.008
Nonconforming Level         5:
0.01
Nonconforming LeveL         6:
0.02
Nonconforming LeveL         7:
0.05
Nonconforming LeveL         8:
0.1
Noncomforming LeveL         9:
0.2
Evaluating Another Mixed Plan?(y)Yes(n)No
n
The results are on file:OC.DAT
```

The output list:

```
K=3.00 n1= 6 n2= 20 c1= 0 c2= 0

(Use Only Variables Criteria on First Sample)
```

p	Pa	ASN	AOQ
3.1700001E-05	0.9999546	6.142521	3.1698564E-05
1.3500000E-03	0.9793874	15.85072	1.3221730E-03
4.9999999E-03	0.8922800	22.42426	4.4614002E-03
8.0000004E-03	0.8225970	23.58508	6.5807765E-03
9.9999998E-03	0.7791264	23.84226	7.7912640E-03
2.0000000E-02	0.5948130	23.51278	1.1896259E-02
5.0000001E-02	0.2638116	20.69294	1.3190578E-02
0.1000000	6.4622492E-02	16.62864	6.4622494E-03
0.2000000	3.0224004E-03	11.24292	6.0448010E-04

IV. The Tables of Operating Characteristics of A Series of Mixed Plans

$K=3.00$ $n_1= 6$ $n_2= 20$ $c_1= 0$ $c_2= 0$

(Use Only Variables Criterion on First Sample)

p	Pa	ASN	AOQ
3.1700001E-05	0.9999546	6.142521	3.1698564E-05
1.3500000E-03	0.9793874	15.85072	1.3221730E-03
4.9999999E-03	0.8922800	22.42426	4.4614002E-03
8.0000004E-03	0.8225970	23.58508	6.5807765E-03
9.9999998E-03	0.7791264	23.84226	7.7912640E-03
2.0000000E-02	0.5948130	23.51278	1.1896259E-02
5.0000001E-02	0.2638116	20.69294	1.3190578E-02
0.1000000	6.4622492E-02	16.62864	6.4622494E-03
0.2000000	3.0224004E-03	11.24292	6.0448010E-04

$K=3.00$ $n_1= 6$ $n_2= 20$ $c_1= 0$ $c_2= 0$

(Use Attributes and Variables Criteria on First Sample)

p	Pa	ASN	AOQ
3.1700001E-05	0.9998052	6.142521	3.1693828E-05
1.3500000E-03	0.9787983	15.85072	1.3213777E-03
4.9999999E-03	0.8920375	22.42426	4.4601876E-03
8.0000004E-03	0.8224644	23.58508	6.5797158E-03
9.9999998E-03	0.7790328	23.84226	7.7903275E-03
2.0000000E-02	0.5947874	23.51278	1.1895747E-02
5.0000001E-02	0.2638054	20.69294	1.3190269E-02
0.1000000	6.4618625E-02	16.62864	6.4618625E-03
0.2000000	3.0204309E-03	11.24292	6.0408621E-04

$K=3.00$ $n_1= 7$ $n_2= 25$ $c_1= 0$ $c_2= 0$

(Use Attributes and Variables Criteria on First Sample)

p	Pa	ASN	AOQ
3.1700001E-05	0.9997749	7.101397	3.1692864E-05
1.3500000E-03	0.9742672	19.28688	1.3152608E-03
4.9999999E-03	0.8671802	27.87382	4.3359012E-03
8.0000004E-03	0.7840371	29.16388	6.2722969E-03
9.9999998E-03	0.7332559	29.37048	7.3325592E-03
2.0000000E-02	0.5263122	28.55002	1.0526244E-02
5.0000001E-02	0.1938290	24.45436	9.6914526E-03
0.1000000	3.4335975E-02	18.95744	3.4335975E-03
0.2000000	7.9069362E-04	12.24292	1.5813873E-04

$K=3.00$ $n_1= 8$ $n_2= 30$ $c_1= 0$ $c_2= 0$

(Use Attributes and Variables Criteria on First Sample)

p	Pa	ASN	AOQ
3.1700001E-05	0.9997441	8.069755	3.1691889E-05
1.3500000E-03	0.9697720	22.71347	1.3091922E-03
4.9999999E-03	0.8425862	33.37830	4.2129308E-03
8.0000004E-03	0.7470464	34.71965	5.9763715E-03
9.9999998E-03	0.6899102	34.83461	6.8991021E-03
2.0000000E-02	0.4657591	33.41195	9.3151815E-03
5.0000001E-02	0.1424400	27.90092	7.1220011E-03
0.1000000	1.8244842E-02	20.91411	1.8244842E-03
0.2000000	2.0636749E-04	13.03321	4.1273499E-05

$K=3.00$ $n_1= 8$ $n_2= 30$ $c_1= 1$ $c_2= 1$

(Use Attributes and Variables Criteria on First Sample)

p	Pa	ASN	AOQ
3.1700001E-05	0.9999999	8.070344	3.1699998E-05
1.3500000E-03	0.9991919	22.99930	1.3489091E-03
4.9999999E-03	0.9855893	34.52605	4.9279467E-03
8.0000004E-03	0.9639659	36.52988	7.7117276E-03
9.9999998E-03	0.9455926	37.06862	9.4559258E-03
2.0000000E-02	0.8244267	37.57857	1.6488533E-02
5.0000001E-02	0.4272107	36.28102	2.1360537E-02
0.1000000	9.5289364E-02	32.39333	9.5289368E-03
0.2000000	2.1769092E-03	23.09962	4.3538187E-04

$K=3.00$ $n_1= 9$ $n_2= 35$ $c_1= 0$ $c_2= 0$

(Use Attributes and Variables Criteria on First Sample)

p	Pa	ASN	AOQ
3.1700001E-05	0.9997132	9.046919	3.1690910E-05
1.3500000E-03	0.9653122	26.13071	1.3031715E-03
4.9999999E-03	0.8183568	38.91494	4.0917839E-03
8.0000004E-03	0.7115777	40.23196	5.6926222E-03
9.9999998E-03	0.6489988	40.21930	6.4899880E-03
2.0000000E-02	0.4122388	38.10257	8.2447752E-03
5.0000001E-02	0.1046890	31.05809	5.2344506E-03
0.1000000	9.6946750E-03	22.55982	9.6946751E-04
0.2000000	5.3462139E-05	13.69766	1.0692428E-05

$K=3.00$ $n_1= 9$ $n_2= 35$ $c_1= 1$ $c_2= 1$

(Use Attributes and Variables Criteria on First Sample)

p	Pa	ASN	AOQ
3.1700001E-05	0.9999998	9.047380	3.1699994E-05
1.3500000E-03	0.9989343	26.49874	1.3485613E-03
4.9999999E-03	0.9808336	40.41370	4.9041682E-03
8.0000004E-03	0.9527347	42.58939	7.6218778E-03
9.9999998E-03	0.9292767	43.12260	9.2927665E-03
2.0000000E-02	0.7806041	43.46206	1.5612083E-02
5.0000001E-02	0.3470832	41.50707	1.7354161E-02
0.1000000	5.7102043E-02	36.11967	5.7102046E-03
0.2000000	6.4995012E-04	24.26739	1.2999003E-04

$K=3.00$ $n_1=10$ $n_2= 40$ $c_1= 0$ $c_2= 0$

(Use Attributes and Variables Criteria on First Sample)

p	Pa	ASN	AOQ
3.1700001E-05	0.9996819	10.03106	3.1689920E-05
1.3500000E-03	0.9608873	29.53889	1.2971979E-03
4.9999999E-03	0.7945634	44.46654	3.9728167E-03
8.0000004E-03	0.6776573	45.68778	5.4212585E-03
9.9999998E-03	0.6104596	45.51635	6.1045955E-03
2.0000000E-02	0.3649278	42.62824	7.2985552E-03
5.0000001E-02	7.6947525E-02	33.94936	3.8473762E-03
0.1000000	5.1508904E-03	23.94725	5.1508908E-04
0.2000000	1.3527533E-05	14.29500	2.7055066E-06

K=3.00 n_1=10 n_2= 40 c_1= 1 c_2= 1

(Use Attributes and Variables Criteria on First Sample)

p	Pa	ASN	AOQ
3.1700001E-05	0.9999999	10.03141	3.1699998E-05
1.3500000E-03	0.9986441	29.99802	1.3481696E-03
4.9999999E-03	0.9754882	46.36046	4.8774406E-03
8.0000004E-03	0.9403922	48.65811	7.5231381E-03
9.9999998E-03	0.9115930	49.16683	9.1159297E-03
2.0000000E-02	0.7360350	49.29797	1.4720700E-02
5.0000001E-02	0.2794307	46.55446	1.3971536E-02
0.1000000	3.3779543E-02	39.44421	3.3779542E-03
0.2000000	1.8987860E-04	25.03250	3.7975722E-05

K=3.00 n_1=11 n_2= 45 c_1= 0 c_2= 0

(Use Attributes and Variables Criteria on First Sample)

p	Pa	ASN	AOQ
3.1700001E-05	0.9996506	11.02031	3.1688924E-05
1.3500000E-03	0.9564967	32.93814	1.2912705E-03
4.9999999E-03	0.7712578	50.01943	3.8562890E-03
8.0000004E-03	0.6452783	51.07842	5.1622265E-03
9.9999998E-03	0.5742002	50.72143	5.7420013E-03
2.0000000E-02	0.3230958	46.99535	6.4619151E-03
5.0000001E-02	5.6559663E-02	36.59610	2.8279831E-03
0.1000000	2.7363522E-03	25.12159	2.7363523E-04
0.2000000	3.1305265E-06	14.86550	6.2610530E-07

K=3.00 n_1=11 n_2= 45 c_1= 1 c_2= 1

(Use Attributes and Variables Criteria on First Sample)

p	Pa	ASN	AOQ
3.1700001E-05	0.9999998	11.02057	3.1699994E-05
1.3500000E-03	0.9983221	33.49704	1.3477349E-03
4.9999999E-03	0.9695835	52.35210	4.8479172E-03
8.0000004E-03	0.9270707	54.72571	7.4165659E-03
9.9999998E-03	0.8927719	55.19443	8.9277187E-03
2.0000000E-02	0.6914741	55.08421	1.3829481E-02
5.0000001E-02	0.2232645	51.41499	1.1163226E-02
0.1000000	1.9775193E-02	42.38132	1.9775194E-03
0.2000000	5.3767719E-05	25.49562	1.0753544E-05

K=3.00 n_1=12 n_2= 50 c_1= 0 c_2= 0

(Use Attributes and Variables Criteria on First Sample)

p	Pa	ASN	AOQ
3.1700001E-05	0.9996191	12.01317	3.1687927E-05
1.3500000E-03	0.9521398	36.32864	1.2853887E-03
4.9999999E-03	0.7484755	55.56303	3.7423773E-03
8.0000004E-03	0.6144090	56.39867	4.9152719E-03
9.9999998E-03	0.5401093	55.83307	5.4010926E-03
2.0000000E-02	0.2860946	51.21035	5.7218918E-03
5.0000001E-02	4.1574549E-02	39.01818	2.0787276E-03
0.1000000	1.4534112E-03	26.12158	1.4534112E-04
0.2000000	5.3376948E-07	15.43600	1.0675389E-07

$K=3.00$ $n_1=12$ $n_2= 50$ $c_1= 1$ $c_2= 1$

(Use Attributes and Variables Criteria on First Sample)

P	Pa	ASN	AOQ
3.1700001E-05	0.9999998	12.01335	3.1699994E-05
1.3500000E-03	0.9979697	36.99582	1.3472592E-03
4.9999999E-03	0.9631594	58.37747	4.8157969E-03
8.0000004E-03	0.9128947	60.78531	7.3031578E-03
9.9999998E-03	0.8730109	61.20143	8.7301088E-03
2.0000000E-02	0.6474931	60.81902	1.2949863E-02
5.0000001E-02	0.1772474	56.08236	8.8623725E-03
0.1000000	1.1477420E-02	44.95038	1.1477420E-03
0.2000000	1.4305724E-05	25.74399	2.8611448E-06

$K=3.00$ $n_1=13$ $n_2= 55$ $c_1= 0$ $c_2= 0$

(Use Attributes and Variables Criteria on First Sample)

P	Pa	ASN	AOQ
3.1700001E-05	0.9995875	13.00847	3.1686926E-05
1.3500000E-03	0.9478164	39.71049	1.2795521E-03
4.9999999E-03	0.7262409	61.08878	3.6312044E-03
8.0000004E-03	0.5850053	61.64529	4.6800426E-03
9.9999998E-03	0.5080708	60.85120	5.0807074E-03
2.0000000E-02	0.2533578	55.27901	5.0671557E-03
5.0000001E-02	3.0560087E-02	41.23402	1.5280044E-03
0.1000000	7.7171047E-04	26.98036	7.7171047E-05
0.2000000	0.0000000E+00	16.02368	0.0000000E+00

$K=3.00$ $n_1=13$ $n_2=55$ $c_1=1$ $c_2=1$

(Use Attributes and Variables Criteria on First Sample)

p	Pa	ASN	AOQ
3.1700001E-05	0.9999998	13.00860	3.1699994E-05
1.3500000E-03	0.9975855	40.49429	1.3467404E-03
4.9999999E-03	0.9562461	64.42736	4.7812304E-03
8.0000004E-03	0.8979819	66.83205	7.1838559E-03
9.9999998E-03	0.8524908	67.18531	8.5249087E-03
2.0000000E-02	0.6045492	66.50043	1.2090983E-02
5.0000001E-02	0.1399433	60.55208	6.9971657E-03
0.1000000	6.6132741E-03	47.17423	6.6132745E-04
0.2000000	3.1229997E-06	25.85063	6.2459998E-07

$K=3.00$ $n_1=14$ $n_2=60$ $c_1=0$ $c_2=0$

(Use Attributes and Variables Criteria on First Sample)

p	Pa	ASN	AOQ
3.1700001E-05	0.9995560	14.00542	3.1685926E-05
1.3500000E-03	0.9435260	43.08386	1.2737601E-03
4.9999999E-03	0.7045686	66.59013	3.5228427E-03
8.0000004E-03	0.5570126	66.81690	4.4561005E-03
9.9999998E-03	0.4779643	65.77709	4.7796434E-03
2.0000000E-02	0.2243836	59.20702	4.4876719E-03
5.0000001E-02	2.2463288E-02	43.26073	1.1231644E-03
0.1000000	4.0937195E-04	27.72618	4.0937197E-05
0.2000000	0.0000000E+00	16.63884	0.0000000E+00

$K=3.00$ $n_1=14$ $n_2=60$ $c_1=1$ $c_2=1$

(Use Attributes and Variables Criteria on First Sample)

p	Pa	ASN	AOQ
3.1700001E-05	0.9999998	14.00551	3.1699994E-05
1.3500000E-03	0.9971731	43.99244	1.3461837E-03
4.9999999E-03	0.9488825	70.49461	4.7444124E-03
8.0000004E-03	0.8824347	72.86289	7.0594777E-03
9.9999998E-03	0.8313783	73.14487	8.3137834E-03
2.0000000E-02	0.5629756	72.12659	1.1259513E-02
5.0000001E-02	0.1099633	64.82130	5.4981629E-03
0.1000000	3.7866705E-03	49.07801	3.7866706E-04
0.2000000	1.0731310E-07	25.87480	2.1462620E-08

$K=3.00$ $n_1=15$ $n_2=80$ $c_1=0$ $c_2=0$

(Use Attributes and Variables Criteria on First Sample)

p	Pa	ASN	AOQ
3.1700001E-05	0.9995244	15.00424	3.1684922E-05
1.3500000E-03	0.9303783	53.70654	1.2560107E-03
4.9999999E-03	0.6375602	85.23027	3.1878012E-03
8.0000004E-03	0.4714075	85.04711	3.7712599E-03
9.9999998E-03	0.3873716	83.44626	3.8737161E-03
2.0000000E-02	0.1468082	74.07636	2.9361644E-03
5.0000001E-02	7.6478431E-03	52.06358	3.8239217E-04
0.1000000	4.3745313E-05	31.47138	4.3745313E-06
0.2000000	0.0000000E+00	17.81476	0.0000000E+00

K=3.00 n_1=15 n_2= 80 c_1= 1 c_2= 1

(Use Attributes and Variables Criteria on First Sample)

p	Pa	ASN	AOQ
3.1700001E-05	0.9999998	15.00432	3.1699994E-05
1.3500000E-03	0.9955726	54.98831	1.3440230E-03
4.9999999E-03	0.9208788	90.78292	4.6043941E-03
8.0000004E-03	0.8249592	93.61668	6.5996740E-03
9.9999998E-03	0.7551180	93.86728	7.5511793E-03
2.0000000E-02	0.4312202	92.16380	8.6244037E-03
5.0000001E-02	4.5901433E-02	81.32435	2.2950717E-03
0.1000000	5.1562110E-04	58.92376	5.1562110E-05
0.2000000	0.0000000E+00	28.37014	0.0000000E+00

K=3.00 n_1=20 n_2= 85 c_1= 0 c_2= 0

(Use Attributes and Variables Criteria on First Sample)

p	Pa	ASN	AOQ
3.1700001E-05	0.9993660	20.00032	3.1679901E-05
1.3500000E-03	0.9213320	60.75444	1.2437982E-03
4.9999999E-03	0.6006484	94.47337	3.0032420E-03
8.0000004E-03	0.4322478	92.04307	3.4579828E-03
9.9999998E-03	0.3488215	89.41437	3.4882152E-03
2.0000000E-02	0.1198834	76.74612	2.3976681E-03
5.0000001E-02	4.5788921E-03	50.47149	2.2894461E-04
0.1000000	1.5229762E-05	30.33405	1.5229762E-06
0.2000000	0.0000000E+00	20.97998	0.0000000E+00

$K=3.00$ $n_1=20$ $n_2=85$ $c_1=1$ $c_2=1$

(Use Attributes and Variables Criteria on First Sample)

p	Pa	ASN	AOQ
3.1700001E-05	0.9999996	20.00033	3.1699987E-05
1.3500000E-03	0.9945675	62.47609	1.3426662E-03
4.9999999E-03	0.9045860	102.1625	4.5229299E-03
8.0000004E-03	0.7952086	103.7126	6.3616689E-03
9.9999998E-03	0.7175540	103.4576	7.1755396E-03
2.0000000E-02	0.3767596	99.90828	7.5351908E-03
5.0000001E-02	2.9893408E-02	82.54678	1.4946704E-03
0.1000000	1.9684926E-04	53.29863	1.9684927E-05
0.2000000	0.0000000E+00	25.87991	0.0000000E+00

$K=3.00$ $n_1=25$ $n_2=90$ $c_1=0$ $c_2=0$

(Use Attributes and Variables Criteria on First Sample)

p	Pa	ASN	AOQ
3.1700001E-05	0.9992076	25.00002	3.1674881E-05
1.3500000E-03	0.9123802	67.77600	1.2317133E-03
4.9999999E-03	0.5679121	102.9081	2.8395604E-03
8.0000004E-03	0.3978217	98.49096	3.1825735E-03
9.9999998E-03	0.3150196	94.97218	3.1501960E-03
2.0000000E-02	9.7945288E-02	79.31225	1.9589057E-03
5.0000001E-02	2.7423652E-03	49.96511	1.3711826E-04
0.1000000	5.5283349E-06	31.46107	5.5283351E-07
0.2000000	1.2812853E-08	25.34001	2.5625706E-09

$K=3.00 \; n_1=25 \; n_2= 90 \; c_1= 1 \; c_2= 1$

(Use Attributes and Variables Criteria on First Sample)

p	Pa	ASN	AOQ
3.1700001E-05	0.9999995	25.00003	3.1699983E-05
1.3500000E-03	0.9934801	69.96064	1.3411981E-03
4.9999999E-03	0.8879855	112.8502	4.4399276E-03
8.0000004E-03	0.7655303	113.3323	6.1242427E-03
9.9999998E-03	0.6805757	112.6496	6.8057571E-03
2.0000000E-02	0.3278239	107.0228	6.5564788E-03
5.0000001E-02	1.9342395E-02	82.81404	9.6711976E-04
0.1000000	7.5048956E-05	49.40854	7.5048956E-06
0.2000000	7.8444323E-08	27.46507	1.5688865E-08

$K=3.00 \; n_1=30 \; n_2= 95 \; c_1= 0 \; c_2= 0$

(Use Attributes and Variables Criteria on First Sample)

p	Pa	ASN	AOQ
3.1700001E-05	0.9990491	30.00000	3.1669857E-05
1.3500000E-03	0.9035298	74.76512	1.2197653E-03
4.9999999E-03	0.5381454	110.8032	2.6907271E-03
8.0000004E-03	0.3667246	104.6002	2.9337965E-03
9.9999998E-03	0.2847849	100.2597	2.8478485E-03
2.0000000E-02	8.0038451E-02	81.82027	1.6007690E-03
5.0000001E-02	1.6433699E-03	50.39058	8.2168495E-05
0.1000000	2.1601390E-06	34.02713	2.1601390E-07
0.2000000	7.1021042E-09	30.11760	1.4204209E-09

K=3.00 n_1=30 n_2= 95 c_1= 1 c_2= 1

(Use Attributes and Variables Criteria on First Sample)

p	Pa	ASN	AOQ
3.1700001E-05	0.9999993	30.00000	3.1699979E-05
1.3500000E-03	0.9923136	77.43720	1.3396235E-03
4.9999999E-03	0.8710361	123.0985	4.3551801E-03
8.0000004E-03	0.7358762	122.6606	5.8870097E-03
9.9999998E-03	0.6442202	121.5534	6.4422023E-03
2.0000000E-02	0.2842012	113.5470	5.6840247E-03
5.0000001E-02	1.2450807E-02	82.58610	6.2254036E-04
0.1000000	2.9806004E-05	47.45088	2.9806004E-06
0.2000000	6.7080045E-08	30.99963	1.3416009E-08

K=3.00 n_1=35 n_2=100 c_1= 0 c_2= 0

(Use Attributes and Variables Criteria on First Sample)

p	Pa	ASN	AOQ
3.1700001E-05	0.9988908	35.00000	3.1664837E-05
1.3500000E-03	0.8947746	81.72619	1.2079456E-03
4.9999999E-03	0.5106120	118.3213	2.5530600E-03
8.0000004E-03	0.3382615	110.4682	2.7060919E-03
9.9999998E-03	0.2575167	105.3398	2.5751665E-03
2.0000000E-02	6.5395370E-02	84.30705	1.3079074E-03
5.0000001E-02	9.8411471E-04	51.60825	4.9205737E-05
0.1000000	8.4554222E-07	37.50314	8.4554223E-08
0.2000000	2.8231542E-09	35.04057	5.6463084E-10

$K=3.00$ $n_1=35$ $n_2=100$ $c_1= 1$ $c_2= 1$

(Use Attributes and Variables Criteria on First Sample)

p	Pa	ASN	AOQ
3.1700001E-05	0.9999990	35.00000	3.1699972E-05
1.3500000E-03	0.9910635	84.91083	1.3379358E-03
4.9999999E-03	0.8537353	133.0582	4.2686765E-03
8.0000004E-03	0.7062946	131.7754	5.6503569E-03
9.9999998E-03	0.6086196	130.2084	6.0861963E-03
2.0000000E-02	0.2455581	119.5263	4.9111629E-03
5.0000001E-02	7.9722535E-03	82.20239	3.9861267E-04
0.1000000	1.1502120E-05	47.23755	1.1502120E-06
0.2000000	2.7942548E-08	35.39550	5.5885097E-09

$K=3.00$ $n_1=40$ $n_2=105$ $c_1= 0$ $c_2= 0$

(Use Attributes and Variables Criteria on First Sample)

p	Pa	ASN	AOQ
3.1700001E-05	0.9987324	40.00000	3.1659816E-05
1.3500000E-03	0.8861131	88.65961	1.1962528E-03
4.9999999E-03	0.4848839	125.5544	2.4244194E-03
8.0000004E-03	0.3120780	116.1381	2.4966241E-03
9.9999998E-03	0.2328692	110.2413	2.3286920E-03
2.0000000E-02	5.3426836E-02	86.79892	1.0685367E-03
5.0000001E-02	5.8922742E-04	53.49372	2.9461371E-05
0.1000000	3.7617858E-07	41.55198	3.7617859E-08
0.2000000	1.3242332E-09	40.01396	2.6484664E-10

$K=3.00$ $n_1=40$ $n_2=105$ $c_1= 1$ $c_2= 1$

(Use Attributes and Variables Criteria on First Sample)

p	Pa	ASN	AOQ
3.1700001E-05	0.9999988	40.00000	3.1699965E-05
1.3500000E-03	0.9897336	92.38194	1.3361403E-03
4.9999999E-03	0.8361215	142.8100	4.1806074E-03
8.0000004E-03	0.6769146	140.7012	5.4153171E-03
9.9999998E-03	0.5739295	138.6219	5.7392954E-03
2.0000000E-02	0.2115325	125.0021	4.2306492E-03
5.0000001E-02	5.0819926E-03	81.90168	2.5409964E-04
0.1000000	4.5628067E-06	48.44967	4.5628067E-07
0.2000000	1.3620923E-08	40.15353	2.7241847E-09

$K=3.00$ $n_1=45$ $n_2=110$ $c_1= 0$ $c_2= 0$

(Use Attributes and Variables Criteria on First Sample)

p	Pa	ASN	AOQ
3.1700001E-05	0.9985741	45.00000	3.1654799E-05
1.3500000E-03	0.8775431	95.56675	1.1846832E-03
4.9999999E-03	0.4606801	132.5613	2.3034005E-03
8.0000004E-03	0.2879436	121.6334	2.3035486E-03
9.9999998E-03	0.2105845	114.9828	2.1058447E-03
2.0000000E-02	4.3654110E-02	89.31683	8.7308220E-04
5.0000001E-02	3.5322661E-04	55.93834	1.7661330E-05
0.1000000	1.8886512E-07	45.96006	1.8886512E-08
0.2000000	5.2023191E-10	45.00479	1.0404638E-10

K=3.00 n_1=45 n_2=110 c_1= 1 c_2= 1

(Use Attributes and Variables Criteria on First Sample)

p	Pa	ASN	AOQ
3.1700001E-05	0.9999986	45.00000	3.1699958E-05
1.3500000E-03	0.9883241	99.85191	1.3342376E-03
4.9999999E-03	0.8182065	152.4030	4.0910323E-03
8.0000004E-03	0.6478705	149.4451	5.1829643E-03
9.9999998E-03	0.5403087	146.7932	5.4030870E-03
2.0000000E-02	0.1817508	130.0160	3.6350163E-03
5.0000001E-02	3.2307492E-03	81.84492	1.6153746E-04
0.1000000	2.0689797E-06	50.76038	2.0689798E-07
0.2000000	5.9372192E-09	45.05869	1.1874438E-09

K=3.00 n_1=50 n_2=115 c_1= 0 c_2= 0

(Use Attributes and Variables Criteria on First Sample)

p	Pa	ASN	AOQ
3.1700001E-05	0.9984158	50.00000	3.1649783E-05
1.3500000E-03	0.8690767	102.4372	1.1732535E-03
4.9999999E-03	0.4378724	139.3634	2.1893620E-03
8.0000004E-03	0.2657169	126.9635	2.1257356E-03
9.9999998E-03	0.1904557	119.5768	1.9045565E-03
2.0000000E-02	3.5674181E-02	91.87904	7.1348361E-04
5.0000001E-02	2.1236934E-04	58.84851	1.0618467E-05
0.1000000	1.0920922E-07	50.59267	1.0920923E-08
0.2000000	1.9008449E-10	50.00164	3.8016899E-11

K=3.00 n_1=50 n_2=115 c_1= 1 c_2= 1

(Use Attributes and Variables Criteria on First Sample)

p	Pa	ASN	AOQ
3.1700001E-05	0.9999983	50.00000	3.1699947E-05
1.3500000E-03	0.9868453	107.3091	1.3322412E-03
4.9999999E-03	0.8001091	161.8458	4.0005455E-03
8.0000004E-03	0.6192904	157.9981	4.9543232E-03
9.9999998E-03	0.5078889	154.7169	5.0788885E-03
2.0000000E-02	0.1557889	134.6130	3.1157774E-03
5.0000001E-02	2.0487155E-03	82.13408	1.0243578E-04
0.1000000	1.0403573E-06	53.88531	1.0403573E-07
0.2000000	2.5174856E-09	50.02216	5.0349713E-10

K=3.00 n_1=50 n_2=120 c_1= 1 c_2= 2

(Use Attributes and Variables Criteria on First Sample)

p	Pa	ASN	AOQ
3.1700001E-05	0.9999983	50.00000	3.1699947E-05
1.3500000E-03	0.9970875	109.8008	1.3460681E-03
4.9999999E-03	0.9324139	166.7087	4.6620695E-03
8.0000004E-03	0.8233624	162.6936	6.5868990E-03
9.9999998E-03	0.7349869	159.2698	7.3498688E-03
2.0000000E-02	0.3205662	138.2918	6.4113247E-03
5.0000001E-02	7.5734942E-03	83.53122	3.7867471E-04
0.1000000	3.5150667E-06	54.05424	3.5150669E-07
0.2000000	2.5175104E-09	50.02312	5.0350207E-10

V. Conclusion

According to the discussion above and the computation of a series of mixed plans, the computation method presented in this thesis is of high accuracy and high efficiency. The need to extend the evaluation of the operating characteristic of dependent mixed plans to large first sample sizes has been satisfied. In fact, the FORTRAN program could be modified for evaluating mixed plans with larger first sample sizes(>50) if more computer memory resources were available.

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Appendix. COMPUTER PROGRAM LIST

```
      integer m,n,n1,n2,i,j,c1,c2,attri
      character sw
      real k,p(100)
      real f(25,51,500)
      common /a2/p,/a1/f

      open(6,file='oc.dat',status='new')
      print *,'  Establishing the F-Matrix,it Takes about
    + 5 to 10 minutes'
      print *,'  Please wait.....'
      call bfmatrix()
1     write(*,2)
2     format(1x,/
    + 1x,'************************************************',/,
    + 1x,'*       MIXED DEPENDENT SAMPLING PLANS         *',/,
    + 1x,'*             EVALUATION PROGRAM               *',/,
    + 1x,'*               ( Sigma Known)                 *',/,
    + 1x,'*                                              *',/,
    + 1x,'*        Maximum first sample size=50          *',/,
    + 1x,'*  Maximum Number of nonconforming Levels=100  *',/,
    + 1x,'*   Minimum nonconforming level=0.0000002867   *',/,
    + 1x,'*          (corresponding Z value=5.0)         *',/,
    + 1x,'************************************************',/)

        print *,'First Sample Size:'
        read *,n1
        print *,'Second Sample Size:'
        read *,n2
        print *,'K Factor:'
        read *,k
```

```
3         print *,'Use (1)Attri.and Variables on First Sample
     + (0)Variables Only:'
          read *,attri
          if(attri.gt.1.or.attri.lt.0) goto 3
          print *,'Attributes Criterion for First Sample C1:'
          read *,c1
          print *,'Attributes Criterion for Second Sample C2:'
          read *,c2
          print *,'Number of Nonconforming Levels:'
          read *,m
          do i=1,m
           print *,'Nonconforming Level',i,':'
           read *,p(i)
          enddo
          call plan(n1,n2,k,c1,c2,m,attri)
4       print *,'Evaluate Another Mixed Plan? (y)Yes (n)No :'
        read(*,5)sw
        if(sw.ne.'y' .and.sw.ne.'n') goto 4
        if(sw.eq.'y') goto 1
        close(6)
        print *,'The Results are on file "OC.DAT" '
5       format(a1)
        end

        subroutine plan(n,n21,k,c1,c2,m,attri)

        integer c1,c2,n11,n21,attri,i,j,k1
        real n1,n2,sum,sum2
        real k,za(100),zu(100),p1(100)
        real p2(100),pa(100),p(100)
        real abs,a,errabs,errest,error,errrel,b,pp
        real anorin, anordf, bindf, h,g
        real asn(100),aoq(100),f0,f1
        intrinsic abs,exp,sqrt
        external qdag,twodq,anorin,anordf,bindf,h,g,f1,f0
```

```
      common /a2/p,/a3/za,/a3/zu,/a4/i,/a4/j,/a5/n11

      n11=n
      n1=real(n11)
      n2=real(n21)
      do 202,i=1,m
         p1(i) = 0
         pp=1-p(i)
         zu(i)=anorin(pp)
         za(i)=zu(i)-k
         if(attri.eq.0) then
           p1(i) = anordf(sqrt(n1)*za(i))
          else
           do 201,j=0,c1
            p1(i) = p1(i)+binpr(j,n11,p(i))
201        continue
          endif
202    continue
       errabs=0.0
       irule = 2
       b = 0.0

c
       do 4,i=1,m
        sum = 0.0
        sum2= 0.0
         do 400,j=0,c1
           if(j.eq.0.0) then
             goto 120
            else
             irule = 5
             errrel=0.0001
             call twodq(f1,zu(i),10.0,g,h,errabs,errrel,irule,
     +       p2(j+1),errest)
```

```
              p2(j+1)=p2(j+1)*Comb(n1,j)*sqrt(real(j)*
     +        (n1-real(j)))/(2*3.14159)
             endif
             goto 130
120          errrel=0.0001
             call qdag(f0,za(i),zu(i),errabs,errrel,irule,p2(j+1)
     +       ,errest)
130          continue
            sum2 = sum2 + p2(j+1)
            do 401,k1=0,c2-j
              pp=binpr(k1,n21,p(i))
              sum=sum+p2(j+1)*pp
401         continue
400       continue
403       if(attri.eq.0) then
            pa(i) = p1(i) + sum
           else
            pa(i) = p1(i)   sum2 + sum
          endif
          if(pa(i).lt.0) pa(i)=0.0
         asn(i) = n1 + n2 * sum2
         aoq(i) = p(i)*pa(i)
  4     continue

        write(6,999)k,n11,n21,c1,c2
        if(attri.eq.0)then
         write(6,998)
998      format(/,1x,'  (Use Only Variables Criterion on First
     + Sample)    ')
         else
         write(6,997)
997      format(1x,'  (Use Attributes and Variables Criteria
     + on First Sample)    ',/)
        endif
        write(6,1000)
```

```fortran
 999      format(/,1x,4x,'K=',f4.2,' n1=',I2,' n2=',I3,' c1=',I2,
     +' c2=',I2,/)
1000      format(1x,1x,'p     ',10x,'Pa   ',11x,'ASN ',11x,'AOQ'/)
          do 5,i=1,m
           write(6,*)p(i),pa(i),asn(i),aoq(i)
1001       format(1x,1x,f13.11,2x,f9.7,6x,f13.7,3x,f9.7/)
   5      continue
          return
          end
c
          real function comb(x,y)
          real x,s1,s2
          integer y,l

          s1 = 1.0
          s2 = 1.0
          do 200 l=int(x)-y+1,int(x)
            s1=s1*real(l)
 200      continue
          do 201 l=1,y
            s2=s2*real(l)
 201      continue
            comb = s1/s2
          return
          end
c
          real function g(x)
          integer i,j
          real x,n1,za(100),zu(100)
          common /a3/za,/a3/zu,/a4/i,/a4/j,/a5/n11
          n1=real(n11)
          g = (n1*za(i)-j*x)/(n1-j)

          return
          end
```

```
c
      real function h(x)
      integer i,j,n11
      real x,za(100),zu(100)
      common /a4/i,/a4/j,/a3/za,/a3/zu

      h = zu(i)
      return
      end
c
      real function f1(x,y)

      real x,y,z,fa,fb
      integer i,j,n11
      real n1,za(100),zu(100)
      common /a3/za,/a3/zu,/a4/i,/a4/j,/a5/n11

      n1=real(n11)
      z = zu(i)-y
      call fn(z,n11-j,fa)

      z = x - zu(i)
      call fn(z,j,fb)
      f1 = exp(-0.5*(x**2*j+(n1-j)*y**2))*fa*fb

      return
      end

c
      real function f0(x)
      integer i,n11
      real x,f,n1
      real za(100),zu(100)
      common /a3/za,/a3/zu,/a4/i,/a4/j,/a5/n11
```

```
n1=real(n11)
call fn(zu(i)-x,n11,f)

f0= sqrt(n1/(2*3.14159))*exp(-(n1*x**2)/2)*f
return
end

subroutine fn(x,n11,fr)
real x,mx1,fr,f0,f1,f2,f3,n1
integer i,j,k,n11

n1=real(n11)
if(x.lt.0.01/n1)then
  m1=1
  mx1=0.01/n1
 else
  m1=int(n1*x/0.01+.5)
  mx1=real(m1)*0.01/n1
endif
call fd(mx1,m1,n11,f0,f1,f2,f3)
fr=f0+f1*(x-mx1)+f2*(x-mx1)**2/2+
f3*(x-mx1)**3/6
return
end

subroutine fd(x,m,n,ff0,ff1,ff2,ff3)
integer i,j,k,m,n,m1,n1,m2,n2,m3,n3,m4,n4
real ff0,ff1,ff2,ff3,f1,f2,f3,n0
real f(25,51,500),x
common /a1/f

n0=real(n)

if(n.gt.1)then
m1=m/n
```

```
n1=mod(m,n)
call nf(m1,n1,n,i,j,k)
ff0=f(i,j,k)
else
 if(n.eq.1) ff0=1.0
 if(n.lt.1) ff0=0.0
endif

if(n-1.gt.0)then
m2=m/(n-1)
n2=mod(m,n-1)
call nf(m2,n2,n-1,i,j,k)
if(n-2.eq.0)then
  f1=1.0
 else
  if(n-2.gt.0.and.k.lt.501) f1=f(i,j,k)
  if(n-2.gt.0.and.k.gt.500) f1=1.0
endif
ff1=n0*sqrt(n0/((n0-1)*2*3.14159))*exp(-n0/
(2*(n0-1))*x**2)*f1
else
 if(n-1.lt.1) ff1=0
endif

if(n-2.gt.0)then
 m3=m/(n-2)
 n3=mod(m,n-2)
 call nf(m3,n3,n-2,i,j,k)
endif
if(n-3.eq.0)then
  f2=1.0
 else
  if(n-3.gt.0.and.k.lt.501) f2=f(i,j,k)
  if(n-3.gt.0.and.k.gt.500) f2=1.0
endif
```

```
      if(n-2.gt.0)then
      ff2=n0**2*(sqrt(n0/(n0-2))/(2*3.14159)*exp(-n0/
     +   (n0-2)*x**2)*f2-x*sqrt(n0/((n0-1)*2*3.14159))/
     +   (n0-1)*exp(-n0/(2*(n0-1))*x**2)*f1)
      else
      if(n-1.gt.0)then
      ff2=n0**2*x*(-sqrt(n0/((n0-1)*2*3.14159))/
     +   (n0-1)*exp(-n0/(2*(n0-1))*x**2)*f1)
      else
      ff2=0.0
      endif
      endif
      if(n-3.gt.0)then
       m4=m/(n-3)
       n4=mod(m,n-3)
       call nf(m4,n4,n-3,i,j,k)
      endif
      if(n-4.eq.0) then
        f3=1.0
       else
        if(n-4.gt.0.and.k.lt.501) f3=f(i,j,k)
        if(n-4.gt.0.and.k.gt.500) f2=1.0
      endif
      if(n-3.gt.0)then
      ff3=n0**3*(sqrt(n0/(n0-3))/(sqrt(2*3.14159))**3
     +   *exp(-3*n0*x**2/(2*(n0-3)))*f3-(x/(n0-1)+2*x/
     +   (n0-2))*sqrt(n0/(n0-2))/(2*3.14159)*exp(-n0*x**2
     +   /(n0-2))*f2+((x/(n0-1))**2-1/(n0*(n0-1)))*sqrt(n0
     +   /((n0-1)*2*3.14159))*exp(-n0*x**2/(2*(n0-1)))*f1)
      else
      if(n-2.gt.0)then
      ff3= n0**3*(-(x/(n0-1)+2*x/
     +   (n0-2))*sqrt(n0/(n0-2))/(2*3.14159)*exp(-n0*x**2
     +   /(n0-2))*f2+((x/(n0-1))**2-1/(n0*(n0-1)))*sqrt(n0
     +   /((n0-1)*2*3.14159))*exp(-n0*x**2/(2*(n0-1)))*f1)
```

```fortran
      else
      if(n-1.gt.0)then
      ff3= n0**3*(((x/(n0-1))**2-1/(n0*(n0-1)))*sqrt(n0
     +   /((n0-1)*2*3.14159))*exp(-n0*x**2/(2*(n0-1)))*f1)
      else
      ff3=0.0
      endif
      endif
      endif

      return
      end
c
      subroutine bfmatrix()
      integer i,j,k,m,n,o,k1,j1,a,b,c
      real f(25,51,500)
      real ff1,ff2,ff3,fn,v,mx,x
      common /a1/f
      external ft

      do i=2,50
        v=real(i)
        x=0.01/i
        if(i.lt.31)then
        fn=sqrt(v)/(sqrt(2*3.14159))**(i-1)*
     +    (i*x)**(i-1)/ft(i-1)*(1-v*(v-1)*x**2/
     +    (2*(v+1)))
        else
        fn=0.0
        endif
        call nf(0,1,i,a,b,c)
        f(a,b,c)=fn
      enddo
```

```
        do 503,i=2,50
         do 504,j=0,499
          if(j.eq.0) then
            o=2
           else
            o=0
          endif
          do 505,k=o,i-1
           if (k.eq.0) then
              k1=i-1
              j1=j-1
            else
              k1=k-1
              j1=j
           endif
           x = 0.01*(real(j1)+real(k1)/real(i))
           mx=0.01/real(i)
           m=j1*i+k1
           call fd(x,m,i,ff0,ff1,ff2,ff3)
           call nf(j,k,i,i3,j3,k3)
           f(i3,j3,k3)=ff0+ff1*mx+
     +     ff2/2*mx**2+ff3/6*mx**3
c           if (i.eq.19) then
c            print *,x+mx,f(i3,j3,k3)
c           endif
505       continue
504      continue
503     continue
        return
        end
c
        subroutine nf(l0,m0,n0,l1,m1,n1)
        integer l0,m0,n0,l1,m1,n1

        n1=l0+1
```

```
if(n0.lt.26) then
   l1=n0
   m1=m0+1
  else
   l1=50-n0+1
   m1=m0+l1+1
endif
return
end

real function ft(i)
integer i,j

ft=1.0
do j=1,i
 ft=ft*real(j)
enddo
return
end
```