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An Agent based model of a two good economy on a network

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Abstract

This thesis studies the relationship between people in a two good economy on a social network. Each person in the network is allotted a certain number of firewood and a certain number of candy bars. Each person tries to increase his or her happiness through trading. Each person in the network knows at least one other person in the network. The people in the network can trade with the people they know to increase their happiness. The goal of the thesis is to be able to predict how each person's happiness is affected, just by knowing who knows whom within the network. That is, is there a network importance metric that is a good predictor of happiness?

The thesis presents many trading simulations with different networks, through a MATLAB code that was created using an agent-based model. The size of the network is varied through the experiments, and the probability that people know each other within the network is also varied. Data is collected from all the trading simulations in order to understand clearly what different factors affect the networks. Most importantly what affects happiness within the networks is studied. Three different standard measures of centrality are studied to determine which is the best indicator of happiness. The three centrality measures include: degree, clustering coefficients, and eigenvalue centrality. Throughout many different trading simulations, each person's centrality measurement is compared to his or her ending happiness, in order to determine which standard measure of centrality is the best predictor of happiness.

1. Introduction

This paper presents an agent-based model to investigate the trading of two types of goods between individuals connected together on a social network seeking to maximize their happiness, or reach equilibrium. The scenario in question is the trading of firewood and candy bars within a social network to increase the happiness of the people in the network. This example, of the trading of firewood and candy bars, stems from an example in Jeffrey Perloff's Microeconomics text (Perloff, 2011). Firewood and candy bars were chosen because they are neither substitute nor complementary goods. When goods are supplementary, each good can be substituted for the other, or they serve the same purpose. For example, Tide detergent and Gain detergent are supplementary goods. When goods are complementary, when one of the goods is acquired the other good becomes more desired to have. For example, peanut butter and jelly are complementary goods. Firewood and candy bars have no relationship to one another, and were a viable option in this network.

An example of experiments involving real people trading in a two good economy can be found in the literature (Crocket, 2008). The agents in the agent-based model presented here simulate real people trading in a pure exchange economy. The hypothesis tested in the Crocket, 2008, paper involves whether or not the traders will reach equilibrium on price. In contrast to that paper, this thesis determines which centrality measure is the best happiness predictor. As well, in this thesis the (simulated) people are connected in a social network as compared to the pairs of people in the Crocket, 2008, paper.

The social network is made up of individual people, who may or may not know one another. People who know each other are considered connected. Each person in the network has initial quantities of both firewood and candy bars in tons. Individuals can only trade with the people they know and only if they would benefit from trading with them, if the happiness of both of them increase (or doesn't decrease). Throughout all the trades, prices vary for both firewood and candy bars for each individual. Before each potential trade each person decides a specific price for the firewood and candy bars based on how scarce each good is to them individually. The trading of firewood and candy bars may continue to occur as long as at least two individuals can benefit from trading. To model the behavior of people, the individuals are considered as a collection of autonomous decision-making entities connected together on a network. Each person in the social network individually evaluates their situation and makes an assessment based on a certain set of guidelines that are set in place, whether or not they will benefit from trading. Thus throughout the paper, an agent-based modeling technique is being carried out.

The most common use of agent-based modeling systems is in social networks, such as traffic control and financial markets. Examples of agent-based modeling applied to economics can be found in Markose, Arifovic, and Sunder (2007). There are many benefits of agent-based models when modeling social situations, and since the basis of this paper is a social network, an agent-based model was chosen to simulate trading. The benefit of the agent-based modeling technique is having to model only the behavior of a single agent, the people trading, and the system over which they interact, the social network. This is in contrast to a top down approach, (Kultti, 2000), such as a system of

differential equations that attempts to model the entire system at once; agent-based modeling is a bottom up approach. The ability to design a heterogeneous network of people with an agent-based model is significant because it is useful in describing discontinuity of individual behavior, which is difficult when using differential equations. "Individual behavior is complex. Although hypothetically any process can be explained by an equation, the complexity of differential equations increases exponentially as the complexity of behavior increases. Describing complex individual behavior with equations can therefore become intractable" (Castle, 2006). Agent-based modeling is also very flexible. In the MATLAB code created for this paper, for example, if any equation for a parameter needs to be changed, only one line of code needs to be changed and the rest is still valid.

Section 2 shows how traders set prices of their goods based on scarcity. Section 3 explains how each individual's net worth is calculated. Section 4 describes the derivation of the happiness equation. Section 5 gives an example on how trades can increase happiness. Section 6 shows how the networks are formed. Section 7 explains how goods are initially distributed to all the individuals. Section 8 demonstrates an example of a complete trading simulation. Section 9 shows the relation between the degree of each person in the network and the ending price of each person in the network. Section 10 provides a comparison of average happiness throughout all the attempted trades. Section 11 shows the variance of happiness and p, (the random graph connection parameter) and the variance of degree and p. Section 12 will show the relationship between the variance of eigenvalue centrality and p. Section 13 compares the three standard measures of centrality (degree, eigenvalue centrality, and clustering coefficients) against the ending

happiness in an Erdos Renyi random network. Section 14 explains the difference between the Erdos Renyi random graphs and scale free networks. Section 15 compares the three standard measures of centrality (degree, eigenvalue centrality, and clustering coefficients) against the ending happiness in a Scale free network. Degree, eigenvalue centrality, clustering coefficients, and p will be defined later on in the thesis.

In "Price formation on the Marseille fish market: Evidence from a network analysis" by Vignes and Etienne, a homogeneous seller-seller network is set up, where two sellers are linked when they share one or more buyer at a time. Even though in Vignes and Etienne's paper the network distinguishes between sellers and buyers, and in this paper there are no distinctions, prices are determined in a similar way where sellers who share the most buyers with competitors have the highest prices. In this paper, the more of a good in hand and directly available to you, the higher the price is for that good. Vignes and Etienne also adopted a pure 'nomad' or 'loyal' strategy, where 'loyal' buyers pay lower prices than 'nomads.' So here, "traders prefer to deal exclusively with the same partner, with the consequence that, over time, bilateral relationships come to dominate the market" (Vignes & Etienne, 2011). This is not true in this paper; no one has preferential treatment based on past trades.

In Klaus Kultti's paper "A model of random matching and price formation," buyers and sellers meet randomly and are not set in a fixed social network, where the people each individual knows is constant (Kultti, 2000). Agents can decide to either search for the best deal with people they randomly know at that time, or wait to see if anything better comes along later. In this paper, however, trades happen randomly and individuals do not have control over when they trade; happiness levels is the only factor

that is considered if people trade or not. People cannot hold off on trading, unless a trade negatively affects their happiness. Also, Kultti's model is formed where not everybody meets everyone else, but they only have knowledge of people they know. The total number of goods in a network is not known, since there is no network. Throughout this paper, even if people do not know each other, the total number of goods in the network is known and trades are affected by that knowledge. Individuals in Kultti's paper have to play a guessing game to determine when the best time to trade is; if they wait they may have missed a great trading opportunity and not be able to get that deal again.

2. Derivation of Price Formulas

Prices are determined based on scarcity. "When the quantity of any commodity which is brought to market falls short of the effectual demand, all those who are willing to pay... cannot be supplied with the quantity which they want... Some of them will be willing to give more. A competition will begin among them, and the market price will rise... When the quantity brought to market exceeds the effectual demand, it cannot be all sold to those who are willing to pay the whole value of the rent, wages and profit, which must be paid in order to bring it thither... The market price will sink..." (Smith, 1776).

The scarcity of firewood is determined for each person individually depending on the quantity of firewood they possess and how much firewood people they know possess compared to the total firewood in the entire network. Scarcity for candy bars is based on the same information as firewood, how many candy bars they have and how many candy bars people they are connected to have, compared to how many candy bars there are in

the entire social system. As trades go on, between connected individuals, scarcity of both goods for individuals, who are trading and the individuals connected to the people who are trading, will change. Therefore, the prices, for both goods, for those same individuals will also change as a result.

A trader measures the scarcity of a good by the fraction of the total number of that good that is available to him or her. The number of a good that is available to a trader is defined as the number that trader possesses plus half the number present in their neighborhood. The neighborhood of a trader includes only the people the trader is connected to. A bird in the hand is worth two in the bush (Aesop, 600 BCE). The fraction of a half was chosen as the most conservative weighting of the value of the goods of neighbors. The price of a good is proportional to its scarcity (meaning scarcity for everyone else since the more a person has the higher the price they demand). The prices of the goods are determined by the following equations.

Price of Firewood

$$Pf(x) = \frac{tons\ of\ firewood\ for\ person\ x + \frac{tons\ of\ firewood\ connected\ to\ person\ x}{2}}{total\ tons\ of\ firewood\ in\ the\ network}$$

Price of Candy Bars

$$Pb(x) = k \cdot \frac{tons\ of\ candy\ bars\ for\ person\ x + \frac{tons\ of\ candy\ bars\ connected\ to\ person\ x}{2}}{total\ tons\ of\ candy\ bars\ in\ the\ network}$$

(k is a parameter, where k has no units)

In Crockett, Spear, Sunder, 2008, they investigate the effects of the memory and intelligence of the agents in determining the prices. Here, the agents have no memory but

do have the knowledge about the holdings of the goods of others, including the total tonnage of each of the two types of goods in the social network.

In this paper, candy bars are three times more desirable in terms of utility than firewood and therefore $k = 3$. There is no currency and the firewood and candy bars are valued in terms of one another; it is a pure exchange economy. Therefore one of the proportionality constants can be set to 1, which in this paper is firewood. Firewood is then called the numeraire, (Perloff, 2011). The paper "Modeling Price Pressure in Financial Markets" by Elena Asparouhova and Peter Bossaerts influenced the price equation above. In that paper, it was shown that the aspiration levels determine prices, which each individual expects to attain. Pf(x) and Pb(x) were therefore formed based off that idea. The numerator of each equation is the potential goods each person can obtain. People can only potentially own what they already have and what people connected to them have. They can only aspire to have the goods they think they can obtain. What the individuals' neighbors have contributes to their price only half as much as the goods they already hold. The price is then calculated by taking the potential number of either firewood or candy bars and dividing it by the total number of that good in the entire network. For the purpose of the paper, it is assumed that the individuals know how many of each good are in the whole network; otherwise scarcity would not be known. That value gives you the price based from what each person can potentially obtain compared to the total in the network. Since the price is also based off of what each individual cannot obtain, it is said the price is based upon scarcity of the goods. The price for candy bars is scarcity multiplied by k=3 as discussed above. After each trade the aspirations of each individual can possibly change. The number of firewood and candy bars owned by,

themselves and their neighbors combined may have changed, and in result the numerator in the price equation may vary. Therefore, after every trade there must be a re-evaluation of the prices for every individual for both firewood and candy bars. In the paper the total number of goods doesn't change so the denominators don't change. However, the total number of goods was kept in the model to allow for future work where the total does change. The pricing scheme is admittedly nonstandard; in the standard economics texts the prices are determined by the ratio of the marginal utilities of the two trading individuals.

A person with more connections will demand higher prices since they will have a large numerator. The network structure might have an effect on price stability. An individual connected to few others might experience a greater fluctuation in prices as compared to a more connected person because the more connected person will have more people to smooth out the numerator.

Other pricing schemes are possible and have been explored in other papers. In Ke Shi's master's thesis, if any individual has a monopoly, that person can basically charge whatever they desire. In his paper, if an individual has more than 50% of the total number of firewood or candy bars then that individual is considered to have a monopoly of that good. The model chosen in this paper has a somewhat similar approach, but instead prices rise not only by having a lot of a good but by knowing people who have a lot of that good. Another way to determine price is to follow Walrasian tatonnment theory that builds on the premise that price is determined by the excess demand of each individual. Despite the other options, scarcity was chosen to determine price for this paper.

3. Net worth

Before any trading occurs, the initial net worth of each individual needs to be calculated. Each individual has a given number of firewood and a given number of candy bars to start with. The initial quantities of goods are then used to determine the prices of firewood and candy bars for each person in the social network. To determine each person's initial net worth, I(x), use the following equation:

$$I(x) = \\ (initial\ \#\ of\ firewood\ of\ person\ x)(initial\ price\ of\ firewood\ of\ person\ x) \\ + \\ (initial\ \#\ of\ candy\ bars\ of\ person\ x)(initial\ price\ of\ candy\ bars\ of\ person\ x)$$

I(x) is determined before any trading occurs and does not ever change throughout trading. I(x) is a constant for each individual throughout future trading.

After trading occurs, there is a current net worth, C(x) that needs to be calculated. Since the firewood and candy bar distribution changes after trading, so do the prices. The 4 variables in the net worth equation change after each trade, therefore, the new net-worth must be re-calculated. To calculate the current net worth use the following equation:

$$C(x) = \\ (current\ \#\ of\ firewood\ of\ person\ x)(current\ price\ of\ firewood\ of\ person\ x) \\ + \\ (current\ \#\ of\ candy\ bars\ of\ person\ x)(current\ price\ of\ candy\ bars\ of\ person\ x)$$

Unlike I(x), C(x) changes overtime.

To determine how well each individual is doing after trades have occurred calculate the following:

$$C(x) - I(x)$$

If this value is positive the individual is doing well and their net worth has increased through trade. If this value is negative, the individual is doing badly and their net worth has decreased through the trade. If this value is zero, the individual has neither benefitted from trading or was harmed by trading. The goal in this paper is to investigate how, each individual benefits as trading occurs. However, the trades can harm people since trades not only affect the two individuals doing the trading, but also the people surrounding them. This is because their prices may change as a result of trades, not involving them, but involving their neighbors. If their prices can be affected it is also true that their net worth's can be affected, and maybe in a negative way, since only the people who trade are guaranteed to not be negatively affected.

4. Derivation of the Happiness Function

The goal of individuals in trading is to optimize happiness and not to optimize only net worth or their utility. There are a couple of parts to the happiness function and the first is the utility function, which is as follows:

$$U(f,b) = 1 - e^{-a*(current\ \#of\ firewoof\ of\ person\ x) - b*(current\ \#\ of\ candy\ bars\ of\ person\ x)}$$

(a and b are both parameters with units $\frac{1}{tons}$)

In this paper, $a = 0.001$ and $b = 0.003$. The value for b is three times greater than a, because candy bars are three times more important than firewood, for all intents and purposes. This is a nonstandard utility function chosen because the standard form becomes infinitely large as the amount of either good increases. The utility function in this thesis approaches a maximum of 1 as the amount of either good increases.

Now, let's take a look at the partial derivative of the utility function with respect to the number of firewood. The partial derivative is as follows:

$$\frac{\partial U}{\partial(current\ \#\ of\ firewood\ of\ person\ x)} = a * (1 - U)$$

The potential for the utility of firewood is 1. $(1 - U)$ is the potential that remains. The 'a' is just a proportionality constant. Now looking at a real life example, let's say a person has 10 pieces of firewood and 10 candy bars. With those initial goods they also have an initial utility. Then that same person acquires an additional piece of firewood on top of the 10 pieces they already have. That same person will then have a certain change in their utility based on the one additional piece of firewood they obtained. Using the initial value of the utility, the change in the number of firewood, and the new value of utility, the value for 'a' can be calculated. If 'a' is high, the utility gained from one extra piece of firewood is significant. If 'a' is a low number, the utility gained from one extra piece is not as substantial. In this paper, both 'a' and 'b' are set to very small values because then there needs to be a very high number of goods before the utility function is maxed out, and therefore it takes a large number of each good for a significant change in utility to occur. In the standard form of the utility function, the ratio of the partial derivatives is

a function of the number of each good but in this case the ratio is a constant. This is not a problem because the people are trying to improve their happiness not only their utility.

The utility people acquire from goods follows the law of diminishing returns. For example, the first candy bar someone has is much more important than the second candy bar, and the second is more important than the third. This trend goes on and on. The utility gained from each candy bar will keep decreasing the more an individual obtains, and eventually the utility added when gaining another candy bar will be negligible.

When choosing a function to represent utility, the function must have a maximum value. More specifically, a utility function with a maximum value of 1 is desired, because then the utility everyone has is a fraction of the utility that can be potentially gained. So,

$$U(\infty, b) = U(f, \infty) = 1$$

People also cannot have negative utility, because people cannot have a negative number of goods. Therefore the following is true:

$$U(0,0) = 0$$

The utility of goods follows the law of diminishing returns and therefore, the second partial derivatives of the utility function must be negative.

$$\frac{\partial^2 U}{\partial f^2} < 0 \quad \& \quad \frac{\partial^2 U}{\partial b^2} < 0$$

Since the second partials must be negative the utility function has to be concave down.

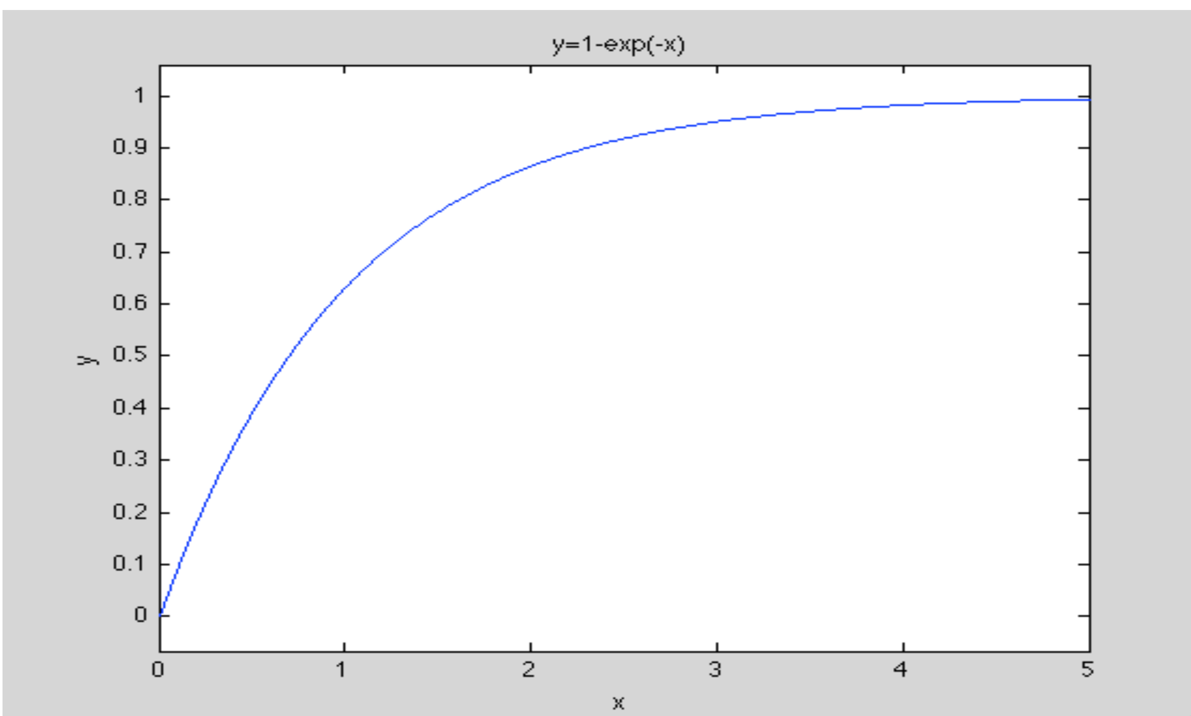
As the number of firewood or candy bars increase for a person their utility functions must also increase. That means that the partial derivatives must be positive.

$$\frac{\partial U}{\partial f} > 0 \quad \& \quad \frac{\partial U}{\partial b} > 0$$

Therefore, the utility function must be an increasing function.

Taking into consideration all the requirements for the utility function: being an increasing function, being concave down, having a maximum at 1, and having a minimum at 0 the following function was chosen:

$$y = 1 - e^{-x}$$



Graph representing the equation $y = 1 - e^{-x}$

Taking the partial derivatives of the utility equation:

$$U(f,b) = 1 - e^{-a*(current\ \#of\ firewoof\ of\ person\ x) - b*(current\ \#\ of\ candy\ bars\ of\ person\ x)}$$

$$\frac{\partial U}{\partial f} = a * e^{-a*(current\ \#of\ firewoof\ of\ person\ x) - b*(current\ \#\ of\ candy\ bars\ of\ person\ x)}$$

$$= a * (1 - U)$$

The partial derivative is proportional to how much utility person x can potentially gain, $(1 - U)$, to a, where a=0.001. Similarly,

$$\frac{\partial U}{\partial b} = b * e^{-a*(current\ \#of\ firewoof\ of\ person\ x) - b*(current\ \#\ of\ candy\ bars\ of\ person\ x)}$$

$$= b * (1 - U)$$

Here the partial derivative is proportional to how much utility person x can potentially gain, $(1 - U)$, to b, where b=0.003.

The direct correlation between potential utility and the parameters put in place makes it evident that those parameters are important to the function. That is why the value of the parameters chosen was a direct correlation between the importance of firewood and the importance of candy bars.

Keep in mind that the utility function is not affected by price and is only affected by the number of firewood and candy bars each person has. Utility is only one of two parts of the happiness function, and the second part is affected by the price of the goods.

The second part of the happiness function includes net worth, and more specifically the gain or loss in net worth from before trading till after trading. Therefore,

$$C(x) - I(x)$$

Individual's happiness is affected by how much net worth has changed. Individuals compare what they have now to what they used to have. If net worth increases, happiness will be positively affected and if net worth decreases happiness will be negatively affected. This models the observed real behavior in Crocket, 2008.

Now putting the two parts together, the happiness function is as follows:

$$H(x) = U(x) + d * (C(x) - I(x))$$

(d is a parameter, where d has no units)

Throughout the paper d=1. If d is set to a value lower than 1, the importance of the change in net-worth would decrease. If d is set to a value higher than 1, the importance of the change in net-worth would increase. The parameter d can be thought to measure how much happiness is derived from the potential trading value of the stored goods as weighted against the actual utility of those goods.

As price goes to infinity, the current net worth goes to infinity as well and in turn makes happiness go to infinity. Therefore, why is it expectable to represent the change in net worth linearly? Why isn't there a cap on how happy a person can get from improving their wealth? The reason a linear representation is used is because it is the simplest and

most conservative way of representing the difference in net worth. The linear representation is also acceptable because the extreme cases do not need to be taken into account; only a linear approximation of the truth is necessary. The chances of prices being infinitely/extremely high are negligible.

The goal in all the trading is to maximize the happiness function. Since the happiness function is dependent on the number of firewood and the number of candy bars, the happiness function is a two-dimensional function. To maximize a two-dimensional function the gradient needs to be calculated, since the gradient will indicate the direction of greatest increase.

5. How Trades Can Increase Happiness

To show that happiness can increase with trading, a concrete example with two people (s and t) is given below:

Initial Firewood, $F(s) = 500$ & $F(t)= 200$

Initial Candy Bars, $B(s) = 45$ & $B(t)=120$

Initial Price of Firewood, $Pf(s)= 0.8571$ & $Pf(t)0.5$

Initial Price of Candy Bars, $Pb(s)=1.9091$ & $Pb(t) =2.5901$

Initial Net-Worth, $I(s) = 514.4595$ & $I(t) =410.812$

Initial happiness, $H(s) = 0.4701$ & $H(t) = 0.4288$

First, a relation must be made between price of candy bars for person s and price of firewood for person t. That relationship is as follows:

$$Candy\ Bar\ Firewood\ Exchange\ Rate\ A = \frac{Current\ price\ of\ candy\ bars\ for\ person\ s}{Current\ price\ of\ firewood\ for\ person\ t}$$

$$Candy\ Bar\ Firewood\ Exchange\ Rate\ A = \frac{1.9091}{0.5} = 3.8194$$

Second, a relation must be made between price of candy bars for person t and price of firewood for person s. That relationship is as follows:

$$Candy\ Bar\ Firewood\ Exchange\ Rate\ B = \frac{Current\ price\ of\ candy\ bars\ for\ person\ t}{Current\ price\ of\ firewood\ for\ person\ s}$$

$$Candy\ Bar\ Firewood\ Exchange\ Rate\ B = \frac{2.5901}{0.8571} = 3.0219$$

The reason for looking at these two relationships is so there is a direct correlation between the prices of both goods that are being traded. Remember, there is no currency throughout the paper (pure exchange economy) and the value of the goods are based off each other.

The gradient for the number of firewood and the gradient for the number of candy bars must then be calculated. Remember the gradient is necessary to maximize the happiness function, since it is a multi-variable function. The gradient is as follows:

$$Gs = [0.001 * e^{-0.001*F(s)-0.003*B(s)} - Pf(s) \quad 0.003 * e^{-0.001*F(s)-0.003*B(s)} - Pb(s)]$$

$$Gs = [-0.8566 \quad -1.9075]$$

$$Gt = [0.001 * e^{-0.001*F(t)-0.003*B(t)} - Pf(t) \quad 0.003 * e^{-0.001*F(t)-0.003*B(t)} - Pb(t)]$$

Gt = [-0.4994   -2.5884]

Let's have person s trade firewood for one ton of person t's candy bars. Then to ensure that when a trade takes place the gradient is pointing in a positive direction, the following inequalities need to hold true:

$$Gt(1) * (Candy\ Bar\ Firewood\ Exchange\ Rate\ A) + Gt(2) * (-1) \geq 0$$

$$(-0.4994) * (3.8194) + (-2.5884) * (-1) = 0.681 \geq 0$$

&

$$Gs(1) * (-(Candy\ Bar\ Firewood\ Exchange\ Rate\ A)) + Gs(2) * 1 \geq 0$$

$$(-0.8566) * (-3.8194) + (-1.9075) * 1 = 1.3642 \geq 0$$

The next step is to ensure that there are a sufficient number of goods to trade. In this case, person s trading their firewood for one ton of person t's candy bars, the following must be true:

$$F(s) \geq Candy\ Bar\ Firewood\ Exchange\ Rate\ A$$

$$F(s) = 500 \geq 3.8194 = Candy\ Bar\ Firewood\ Exchange\ Rate\ A$$

The following inequality must also be true:

$$B(t) \geq 1$$

$$B(t) = 120 \geq 1$$

Since both these inequalities are also true the trade can take place.

If person t was trading firewood for one ton of person s' candy bars, instead of the way in the example, different inequalities would have to be true for trades to occur. And they are as follows:

$$Gs(1) * (Candy\ Bar\ Firewood\ Exchange\ Rate\ B) + Gs(2) * (-1) \geq 0$$

$$Gt(1) * (-(Candy\ Bar\ Firewood\ Exchange\ Rate\ B)) + Gt(2) * 1 \geq 0$$

$$F(t) \geq Candy\ Bar\ Firewood\ Exchange\ Rate\ B$$

$$B(s) \geq 1$$

Back to the example where person s trades firewood for one ton of person t's candy bars: Now that all conditions for trading to happen are checked and met, the trade can occur. First the number of firewood and candy bars for each person changes slightly as a result of a trade.

$$F(s) = F(s) - (Candy\ Bar\ Firewood\ Exchange\ Rate\ A) = 496.1806$$

$$B(s) = B(s) + 1 = 46$$

$$F(t) = F(t) + (Candy\ Bar\ Firewood\ Exchange\ Rate\ A) = 203.8194$$

$$B(t) = B(t) - 1 = 119$$

Now the new prices must be calculated based on the new distribution of goods. The prices are as follows:

$$Pf(s) = 1.1962\ \ \&\ \ Pf(t) = 0.9038$$

$$Pb(s) = 1.9182\ \ \&\ \ Pb(t) = 2.5818$$

The current net-worth of person s and person t is as follows:

$$C(s) = 681.7684\ \ \&\ \ C(t) = 491.4462$$

Both net-worth values have increased since before the trade. Now calculate the current happiness function to get:

$$H(s) = 167.7785 \quad \& \quad H(t) = 81.0635$$

As this process shows, the happiness of both individuals can increase through trading. The current net worth, C(x), has more of an effect on the happiness function than the utility function, U(x). U(x) is bounded between 0 and 1, where C(x) has no lower or

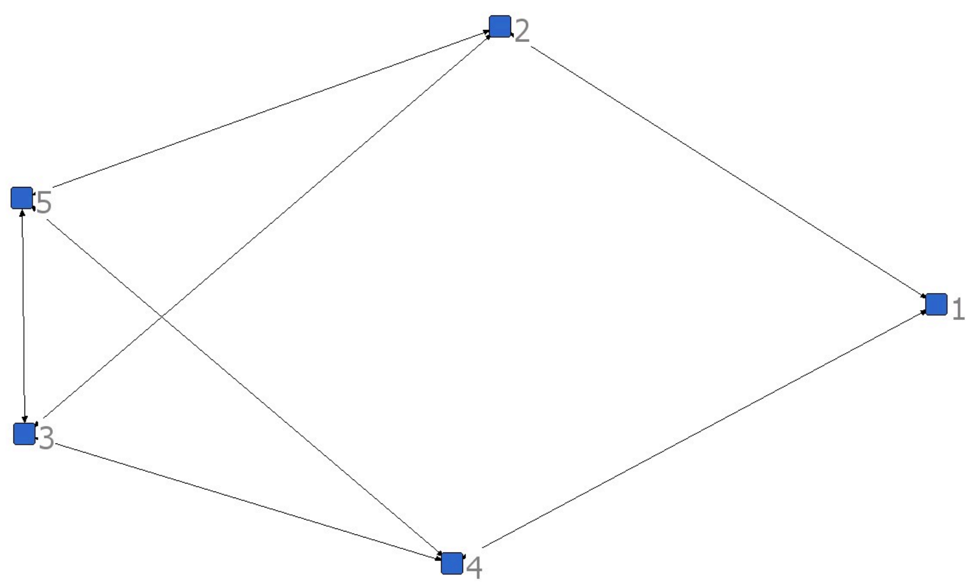
upper bound. Since the parameter d is equal to 1 in the happiness equation, shown in Section 4, the difference between the initial and current net-worth is not scaled down and therefore, has a greater affect than U(x). If d was set to a much lower value, U(x) could be more influential, but since it isn't C(x) remains the dominant force in happiness. In this example there was a significant increase from just one trade. To maximize the happiness of person s and t, continue this process as long as all the trading conditions hold. In a network of more than two people, there are many more trading possibilities since some individuals will most likely be able to trade with more than one person.

6. The Network

Before any trading can occur a network must be created to determine which individuals can trade with whom. The number of people in the social network is n. To represent the networks, an n-by-n adjacency matrix is used. An adjacency matrix, A(x,y), is a means of representing which individuals (or nodes) know one another, that is, are connected, in the social network. The adjacency matrix is symmetric and is made up of only 1's and 0's. First, all the individuals (or nodes) need to be numbered. If two individuals are connected, let's say person s and t, then a 1 needs to be placed in the A(s,t) and A(t,s) spots in the matrix. If the same two individuals were not connected there needs to be a 0 placed in the A(s,t) and A(t,s). The adjacency matrix is a symmetric matrix since A(s,t)=A(t,s) whether or not s and t are connected. All the values on the diagonal must be 0's since an individual cannot be connected to himself or herself. An example is shown below where n=5:

$$
A = \begin{matrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{matrix}
$$

The adjacency matrix represents the network below:



Picture of a 5-person network based on the adjacency matrix above (Ucinet)

Each individual in a network is connected to a certain number of people. That number represents the degree of that individual. Say a person is connected, or knows, 3 other people. That same person then has a degree of 3. For example, in the above network, person 2, 3, 4, and 5 have a degree of 3. Person 1 only has a degree of 2. Besides the degree, another way to represent the nodes in a network is by the clustering coefficient.

The clustering coefficient is the probability that two friends of a node are connected to one another (Newman, 2010). To calculate the clustering coefficient of each node, the following formula is used:

$$Clustering\ Coefficient = \frac{number\ of\ three\ step\ paths\ from\ the\ node}{(degree\ of\ the\ node) * ((degree\ of\ the\ node) - 1)}$$

Therefore, the clustering coefficient is a measure of degree in which the people/nodes in the network have a tendency to cluster together. Ultimately, the higher the clustering coefficient is the better the network can withstand the effect of link removal, which can fragment the network. The last standard measure of centrality looked at will be eigenvalue centrality, (Newman, 2010).

Eigenvalue centrality assigns relative values to all the nodes in the network. This value is based on the notion that connections to high-valued nodes contribute more to the value of the centrality of the node in question than equal connections to low-valued nodes. To calculate eigenvalue centrality, first, solve for the eigenvalues of the adjacency matrix, A. Then find the eigenvector associated with the highest eigenvalue, and the eigenvector gives the centrality for all the nodes in the network in number order. The question is how is this eigenvector associated with the max eigenvalue significant; where did this come from? An example can be shown to answer this.

First take a seven-person network and assume that all people/nodes in the network are of equal importance, where the total importance adds up to 1. Assume each person has an importance of 1/7. Let's take the initial importance of people, without taking any other factors into account, and display it in a vector below:

$$\overrightarrow{V_0} = \begin{matrix} 1/7 \\ 1/7 \\ 1/7 \\ 1/7 \\ 1/7 \\ 1/7 \\ 1/7 \end{matrix}$$

What needs to be taken into account is that a connection to others influences their importance as well. To account for connections to other nodes a new importance vector must be calculated, by doing the following.

$$\overrightarrow{V_1} = A\overrightarrow{V_0}$$

This takes into account all the edges from one person to another, but does not take into account when people are two people away or three people away, etc. To take all paths into account, the importance vector needs to be multiplied by the adjacency matrix, A, over and over again until when you multiply the importance vector by A, the resulting vector is just a scale factor of that importance vector. In other words, the new importance vector is just a scale factor of the old importance vector times the adjacency matrix. The process is shown below:

$$\overrightarrow{V_2} = A\overrightarrow{V_1}$$

$$\overrightarrow{V_3} = A\overrightarrow{V_2}$$

$$\vdots$$

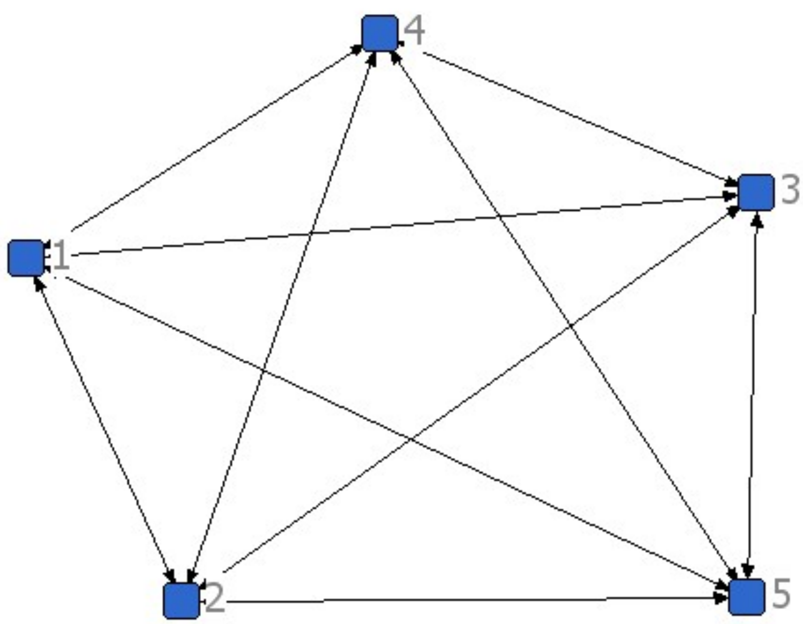
$$\overrightarrow{\lambda V_n} = A\overrightarrow{V_n}$$

The ending result is the equation for an eigenvector, where the λ, is just the max eigenvalue. This long method can then be skipped and the eigenvector of the largest eigenvalue can be computed directly.

In this paper, there are two different networks used. In the MATLAB code that was written for this paper (see the appendix), the networks being tested are randomly generated and are called Erdos Renyi random networks. These random networks are used throughout the entire paper, except in Section 15. Scale Free networks will be discussed later. To generate the Erdos Renyi random graph there are a few steps to be taken. First, the size of the adjacency matrix is selected and not randomly; this determines how many people there are in the matrix. Next, 1's and 0's are randomly placed inside the top half of the matrix, but remember the diagonal must be all 0's. The rest of the matrix then can be filled in, since the adjacency matrix must be symmetric. To create a matrix that is more dense with 1's than with 0's, all that needs to be done is make the probability that a 1 is placed in the matrix greater than the probability that a 0 is placed in the matrix. To do this a value p is introduced into the code; p represents the probability of attaining a 1. If p=0.5, the probability of both a 1 and a 0 is the same. If p=0.8, the probability of getting a

1 is 0.8 and the probability of getting a 0 is 0.2. If p=1, the matrix is made up of all 1's, except the diagonal that must be comprised of 0's, this results in a complete graph. An adjacency matrix for a complete network and the corresponding diagram are represented below:

$$A = \begin{matrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{matrix}$$



Picture of a complete 5-person network (Ucinet)

In the Erdos Renyi networks, the expected degree for each node follows a binomial distribution and is as follows:

$$Expected\ Degree = p(n-1)$$
$$= (probability\ of\ nodes\ being\ connected)(\#\ of\ other\ nodes\ in\ the\ network)$$

The degree distribution will also be expected to form a binomial curve, which is affected by the p value (probability nodes are connected) and the number of people in the network. Lastly, the clustering coefficients of the nodes are expected to equal p, because the clustering coefficient is just the probability that two nodes connected to the node in question are also connected. In other words, the clustering coefficient is the probability that two friends of a certain person are also friends with one another.

Those connections between the individuals that know each other are called edges. If p=1 for example we have a complete graph and the number of edges is as follows:

$$Number\ of\ Edges = \frac{n * (n-1)}{2}$$

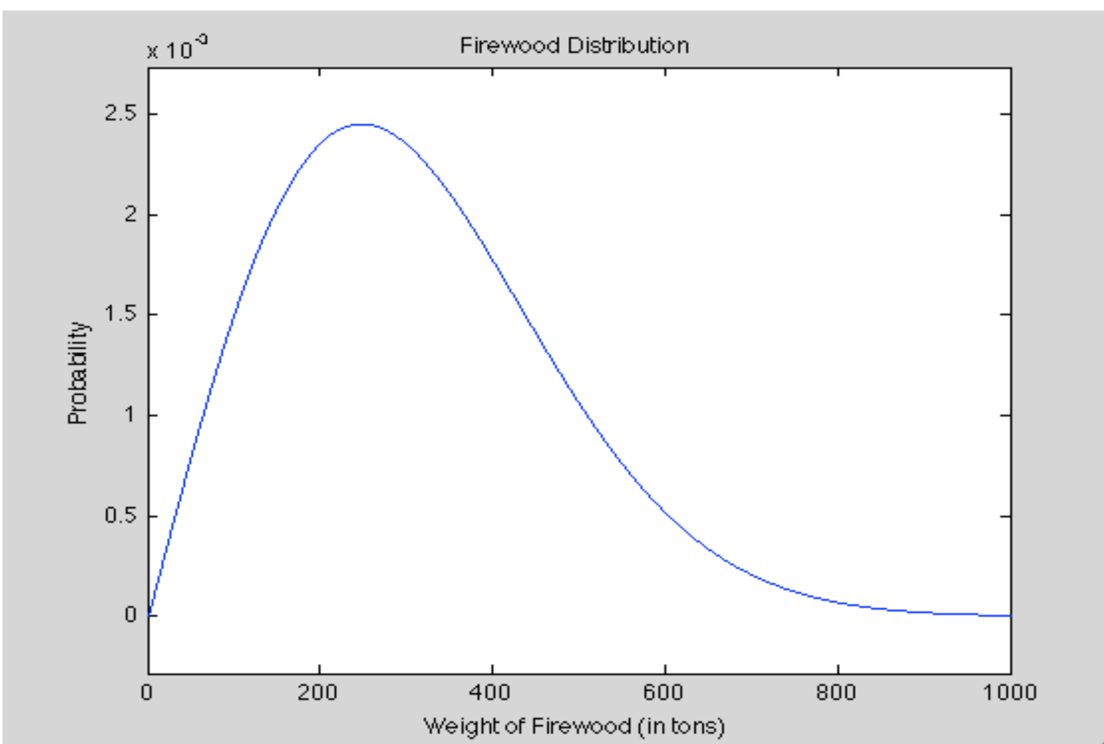
In the networks in the paper, the edges are numbered before trading can occur. Once the edges are numbered, one of the edges is randomly selected, and that number edge refers to a connection between two people in the adjacency matrix. Those two people then have the opportunity to trade firewood and candy bars with each other, based on whether they both benefit from a trade or not. This method ensures that only people that are connected are given a chance to trade.

7. Random Initialization of Goods

The initial weight of firewood and candy bars that each person has, before any trading occurs, is also randomized. Unlike the randomization of the matrix, which used uniform distribution, the randomization of the initial quantity of goods is based on a Weibull distribution, which is given by:

$$W(x) = \alpha\beta^{-\alpha}x^{\alpha-1}e^{-(\frac{x}{\beta})^{\alpha}}$$

The α and β values must be chosen to represent the distribution of goods desired. The following graph represents the Weibull distribution that randomly distributes the firewood for each person where $\alpha = 2$ and $\beta = 350$.



The Weibull distribution representing the initial distribution of firewood (in tons)

The distribution guarantees that no one is given a negative weight (which might occur if a normal distribution was used) of firewood and ensures that the probability for someone to be given a really high weight of firewood is negligible. Also, the average weight of firewood that people are given is close to 300 tons, which was the desired average. The exact mean is 303.038 tons. Therefore the probability distribution with $\alpha = 2$ and $\beta = 350$ is:

$$W_f(x) = 2 * 350^{-2} * x^1 * e^{-(\frac{x}{350})^2}$$

Now that the probability distribution function is known, the cumulative distribution of $W_f(x)$ must be calculated to be able to randomize the initial weight of firewood. To calculate the cumulative distribution, do as follows:

$$y_1(x) = \int_0^x W_f(x)dx = \int_0^x \left[2 * 350^{-2} * x^1 * e^{-(\frac{x}{350})^2}\right] dx$$

$$= 1 - e^{-\frac{x^2}{122,500}}$$

Then switch the x and y_1 values and solve for y_1 to find the inverse equation. Only consider the most positive solution.

$$y_1 = 350 * [-\ln(1 - x)]^{\frac{1}{2}}$$

The last step is to uniformly pick a number from 0 to 1 and plug that number in for x in the inverse cumulative distribution equation. That gives the number of tons of firewood

for an individual. Do this for all of the nodes in the network to get the initial weight of firewood for all of the people.

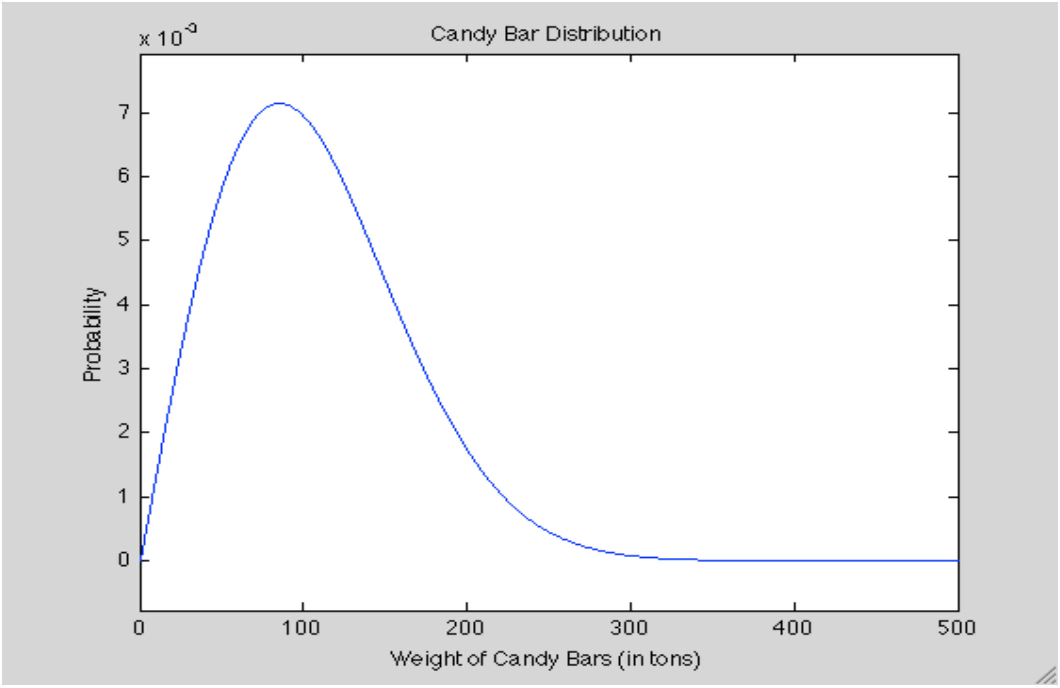
The inversion equation above can be made simpler. Since x is randomly uniformly distributed from 0 to 1 then 1-x is randomly uniformly distributed from 0 to 1 as well. Therefore, an x can replace the 1-x in the equation.

$$y_1 = 350 * [-\ln(x)]^{\frac{1}{2}}$$

The initial weight of candy bars dispersed was also determined by a Weibull distribution. Since a ton of candy bars is more useful than a ton of firewood, one-third the weight will be initially dispersed. Therefore, the α and β values chosen should result in a mean of about 100 candy bars. Therefore, $\alpha = 2$ and $\beta = 120$ to get a mean of 103.899, which is approximately one-third of the mean of firewood. The Weibull distribution for candy bars is as follows:

$$W_b(x) = 2 * 120^{-2} * x^1 * e^{-(\frac{x}{120})^2}$$

With the corresponding graph as follows:



The Weibull distribution representing the initial distribution of candy bars (in tons)

The cumulative distribution must also be calculated and is as follows:

$$y_2(x) = \int_0^x \left[2 * 120^{-2} * x^1 * e^{-(\frac{x}{120})^2}\right] dx$$

$$= 1 - e^{-\frac{x^2}{14400}}$$

Then switch the x and $y_2$values and solve for $y_2$to find the inverse equation. Only consider the most positive solution and re-write the answer as we did above for firewood to get:

$$y_2 = 120 * [-\ln(x)]^{\frac{1}{2}}$$

Now, uniformly pick a number from 0 to 1 for every node, and plug that number in for y_2 to get the initial value of candy bars for each person.

8. Flow Chart

Before trading can occur, a network must be formed. To form a network, the first thing to do is choose its size, or n value. The second thing to do is to decide the probability of people being connected in the network, or choose the p value. Then every person in the network is randomly given an initial amount of firewood and candy bars using the inversion method described above. The connections, or edges, between people are numbered and randomly chosen. If both people benefit from a trade, trading occurs. There are a set number of attempts taken and once they are completed, the final happiness values for all people are calculated. On the following page is the flow chart of the whole process:

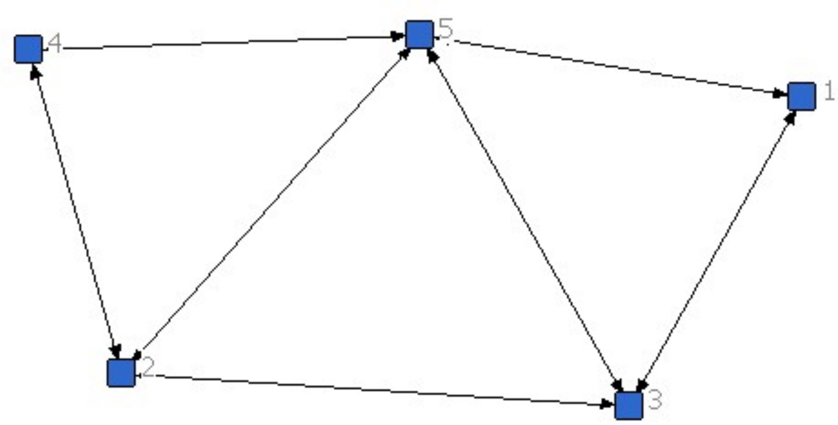

Create a Random Network/Adjacency Matrix and make a list of all the Edges
Give each person a random number of firewood and candy bars
Calculate the total firewood and candy bars in the network
Calculate the initial price of both goods for each person in the network
Calculate the initial net-worth and utility of each person in the network
Calculate the initial happiness values for each individual
Start a counter for the # of trade attempts
Pick an edge at random and see if those two people benefit from trading
Check if the gradient of the happiness function for both people are not negative for at least one trading possibility (who gets firewood and who gets candy Bars)
If one trade is possible, check if there are a sufficient # of goods to trade.
Make the trade, changing the number of candy bars and firewood for person s and person t.
If Not
Check counter for possible trades. If the counter is up stop trading and if the counter is NOT up try to trade again
Re-calculate the prices of firewood and candy bars for all individuals
Calculate the final happiness functions

Below is a concrete example using the flow chart through the MATLAB code:

The number of people in a network was chosen to be 5. Let's make p=0.75, so the probability of a 1 being chosen in the adjacency matrix for a network is 0.75 and the probability a 0 being chosen is 0.25. The matrix for the network was then randomly formed to be:

$$\mathbf{A} = \begin{matrix} \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{0} \end{matrix}$$

The above adjacency matrix represents the network shown below:



Picture of the 5-person Erdos Renyi random network based on the adjacency matrix above.

The next step is to distribute random quantities of firewood and candy bars to all 5 people based on the probability distribution discussed in Section 7.

Person/Node	Weight of Firewood (tons)	Weight of Candy Bars (tons)
1	494.1018	484.1139
2	217.9144	596.1202
3	293.4244	140.7334
4	451.2499	266.2807
5	293.4966	94.5592

Now that random quantities are distributed to the 5 people in the network the total quantities in the system can be calculated, and they are:

$$Total\ firewood\ in\ the\ network = 1{,}750.2\ tons$$

$$Total\ candy\ bars\ in\ the\ network = 1{,}581.8\ tons$$

The next step is to calculate the initial prices for all people in the system, using the equation discussed in Section 2. The initial prices are as follows:

Person/Node	Initial Price of Firewood	Initial Price of Candy Bars
1	0.4500	1.1413
2	0.4211	1.6062
3	0.4549	1.3809
4	0.4039	1.1600
5	0.5838	1.5897

Now calculate the initial net worth of each individual, discussed in Section 3, and then calculate each person's initial Happiness, discussed in Section 4.

Person/Node	Initial Net-worth	Initial Happiness
1	774.8	1.0000
2	1049.3	1.0000
3	327.8	0.9992
4	491.2	1.0000
5	321.7	0.9969

Then it is time to start trading. An edge must be chosen uniformly at random and the two people connected need to be checked for trading compatibility.

The code has chosen person 3 and 5 to see if they can benefit from trading with each other, where s = 1 and t = 5. Now, we need to calculate the following:

$$Candy\ Bar\ Firewood\ Exchange\ Rate\ A = \frac{Current\ price\ of\ candy\ bars\ for\ person\ s}{Current\ price\ of\ firewood\ for\ person\ t}$$

$$= 2.3653$$

$$Candy\ Bar\ Firewood\ Exchange\ Rate\ B = \frac{Current\ price\ of\ candy\ bars\ for\ person\ t}{Current\ price\ of\ firewood\ for\ person\ s}$$

$$= 3.4945$$

$$Gs = [-0.4549 \quad -1.3809]$$

$$Gt = [-0.5838 \quad -1.5896]$$

These values will be used to check and see if any trades can take place.

Now we must use the conditions on trading reviewed in Section 5. The following inequalities must hold to make a trade where person t trades firewood for one ton of candy bars from person s. If the first two inequalities are true, that means that the happiness of both individuals involved in the trade do not decrease. If the first inequality doesn't hold true, the trade negatively affects person s, meaning their happiness decreases from the trade. The same is true with inequality (2), for person t. Inequality (3) and (4) just ensures that person s and person t has a sufficient number of goods for the trade to take place.

$$(1) \quad Gs(1) * (Candy\ Bar\ Firewood\ Exchange\ Rate\ B) + Gs(2) * (-1) \geq 0$$

$$(2) \quad Gt(1) * (-(Candy\ Bar\ Firewood\ Exchange\ Rate\ B)) + Gt(2) * 1 \geq 0$$

$$(3) \quad F(t) \geq (Candy\ Bar\ Firewood\ Exchange\ Rate\ B)$$

$$(4) \quad B(s) \geq 1$$

So, let's check to see if that trade is possible.

$$(1)\ Gs(1) * (Candy\ Bar\ Firewood\ Exchange\ Rate\ B) + Gs(2) * (-1) = -0.2087 \leq 0$$

Since the first inequality does not hold, there is no need to check the rest. The trade cannot be made since it will make person s less happy than they are currently. Now, we need to check the one other trading possibility, which is person s trading for one ton of candy bars from person t. This time these 4 inequalities must be true for a trade:

$$(1) \quad Gt(1) * (Candy\ Bar\ Firewood\ Exchange\ Rate\ A) + Gt(2) * (-1) \geq 0$$

$$(2) \quad Gs(1) * (-(Candy\ Bar\ Firewood\ Exchange\ Rate\ A)) + Gs(2) * 1 \geq 0$$

$$(3) \quad F(s) \geq (Candy\ Bar\ Firewood\ Exchange\ Rate\ A)$$

$$(4) \quad B(t) \geq 1$$

Now let's check to see if these inequalities hold:

$$(1) \quad Gt(1) * (Candy\ Bar\ Firewood\ Exchange\ Rate\ A) + Gt(2) * (-1) = 0.2087 \geq 0$$

The first inequality holds, now checking the second:

$$(2) \quad Gs(1) * (-(Candy\ Bar\ Firewood\ Exchange\ Rate\ A)) + Gs(2) * 1 = -0.3049 \leq 0$$

The second inequality does not hold and therefore there is no point in checking the remaining two. There are no trades possible between person 3 and 5 at this time.

Now another edge must be chosen to attempt at a trade. This time the computer picked person 2 and person 5. The same values need to be calculated again before a trade can be attempted, where s=2 and t=5.

$$Candy\ Bar\ Firewood\ Exchange\ Rate\ A = \frac{Current\ price\ of\ candy\ bars\ for\ person\ s}{Current\ price\ of\ firewood\ for\ person\ t}$$

$$= 2.7511$$

$$Candy\ Bar\ Firewood\ Exchange\ Rate\ B = \frac{Current\ price\ of\ candy\ bars\ for\ person\ t}{Current\ price\ of\ firewood\ for\ person\ s}$$

$$= 3.7751$$

$$Gs = [-0.4211 \quad -1.6062]$$

$$Gt = [-0.5838 \quad -1.5896]$$

The same inequalities must now be checked. First check the inequalities in order for person t to trade firewood for one ton of candy bars from person s.

$$(1) \quad Gs(1) * (Candy\ Bar\ Firewood\ Exchange\ Rate\ B) + Gs(2) * (-1) = 0.1347 \geq 0$$

$$(2) \quad Gt(1) * (-(Candy\ Bar\ Firewood\ Exchange\ Rate\ B)) + Gt(2) * 1 = 0.4505 \geq 0$$

$$(3) \quad F(t) = 293.4966 \geq 3.4945 = (Candy\ Bar\ Firewood\ Exchange\ Rate\ B)$$

$$(4) \quad B(s) = 596.1202 \geq 1$$

All four of the inequalities are true and therefore person t trades 3.4945 tons of firewood for 1 ton of candy bars. The distribution of firewood and candy bars is therefore changed and in turn so are prices of both. The new values are as follows:

Number Person/Node	Weight of Firewood (tons)	Weight of Candy Bars (tons)
1	494.1018	484.1139
2	221.6895	595.1202
3	293.4244	140.7334
4	451.2499	266.2807
5	289.7215	95.5592

Person	New Price of Firewood	New Price of Candy Bars
1	0.4500	1.1413
2	0.4211	1.6062
3	0.4549	1.3809
4	0.4039	1.1600
5	0.5838	1.5897

Now using that new data the new happiness needs to be calculated, by using the formula in Section 4.

Person	Initial Happiness	Ending Happiness
1	1.0000	1.0000
2 (person s)	1.0000	1.0165
3	0.9992	0.9992
4	1.0000	1.0000
5 (person t)	0.9969	1.6113

As shown, happiness has increased for both people involved in the trade, person s and person t.

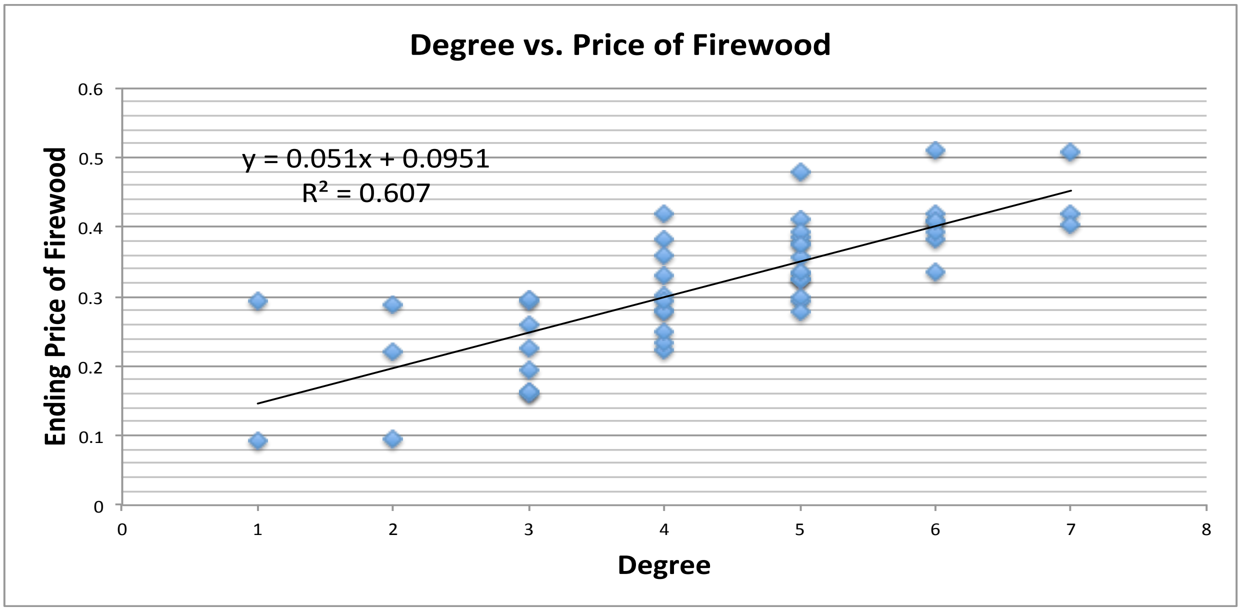
To continue trading and increase happiness further, randomly pick another edge and try again. Keep doing this until no more trades can take place. When trades are no longer possible the network is said to be in equilibrium, the happiest the network can be.

9. Degree vs. Price

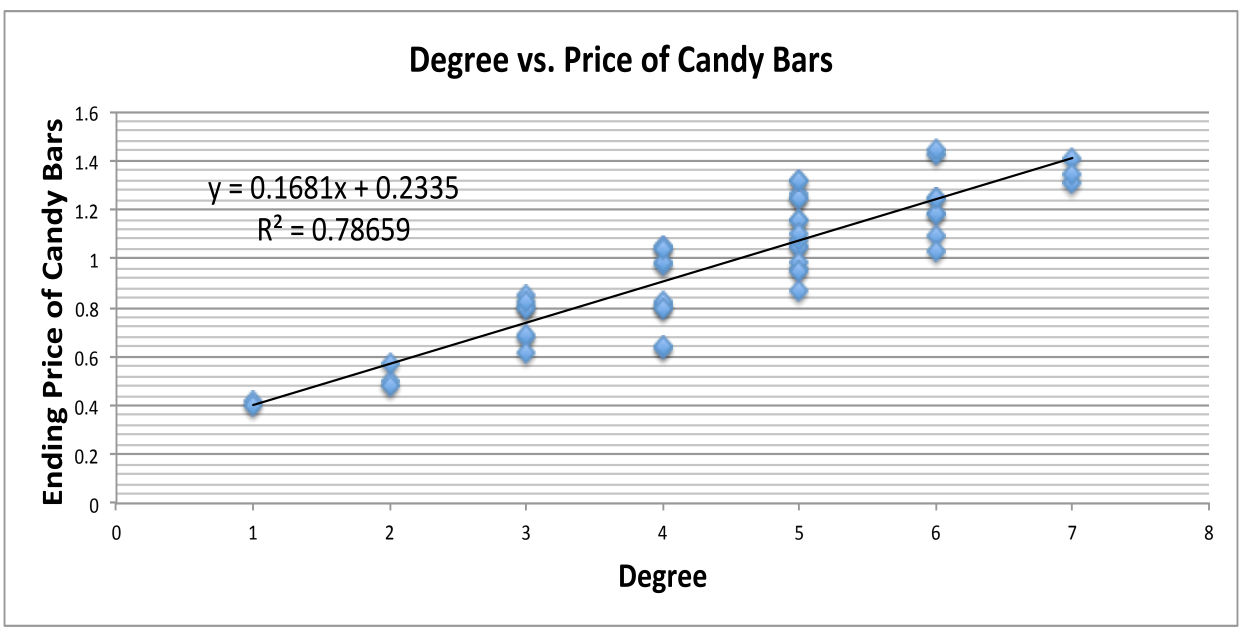
Now that the network is set-up and the rules for trading are set, experiments must be conducted. The experiments need to help determine what factors determine how long it takes for equilibrium to occur, which node benefits the most in a network, the best indicator of happiness, and various other results. Remember, equilibrium is when happiness is maximized and no other trades are able to occur in the network.

As discussed in Section 2, a person with more connections will demand higher prices. To show this to be true, degree will be compared to the price of firewood and the price of candy bars. The network size of n=10 and the connectivity of p=0.5 was chosen

to illustrate this. Below are the two different graphs comparing degree to the two different goods.



Graph representing the relationship between the degree of a node and the ending price of firewood of a node



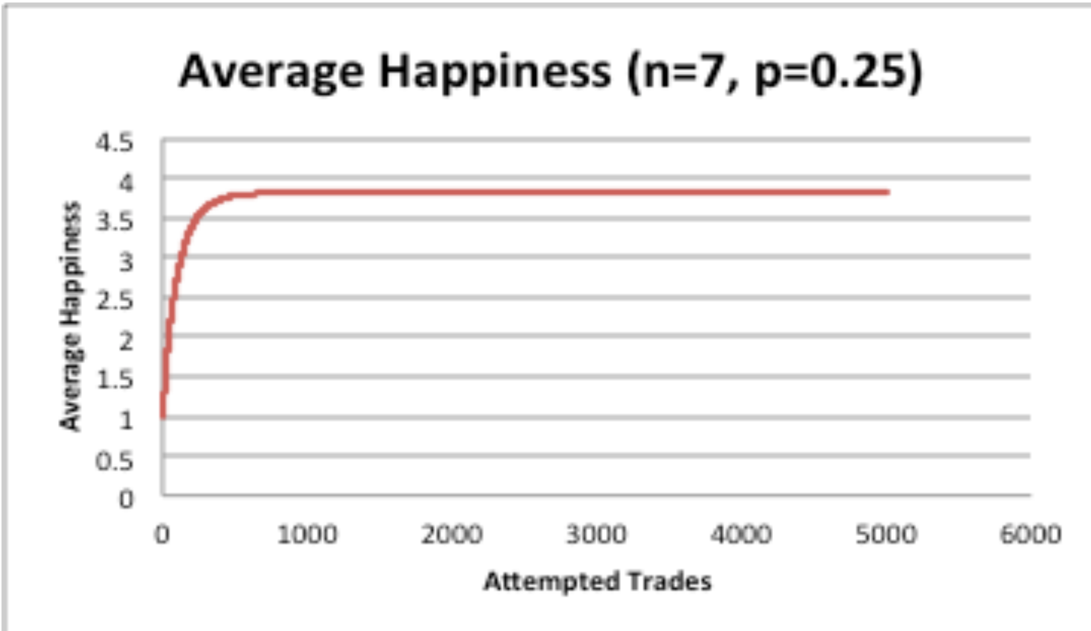
Graph representing the relationship between the degree of a node and the ending price of candy bars of a node

Both graphs show the relationship that was discussed earlier, that the more people someone is connected to the more that person can charge for a good. The slope of the best-fit line for candy bars is approximately three times the slope of the best-fit line for firewood, which makes sense, because the price of candy bars is three times the price of firewood.

10. Average Happiness vs. Time

The second experiment conducted compares average happiness with time, or the number of trades attempted in the network. This experiment needs to be conducted with different p values, the probability that nodes are connected to each other when making a network, and different sized matrices, or different n values. The p values being tested are as follows: p=0.25, 0.5, 0.75, 1. The matrix sizes being tested are as follows: n=7, 10, and 15. All combinations of these values will be used; therefore, there will be 12 different sets of experiments of average happiness vs. time.

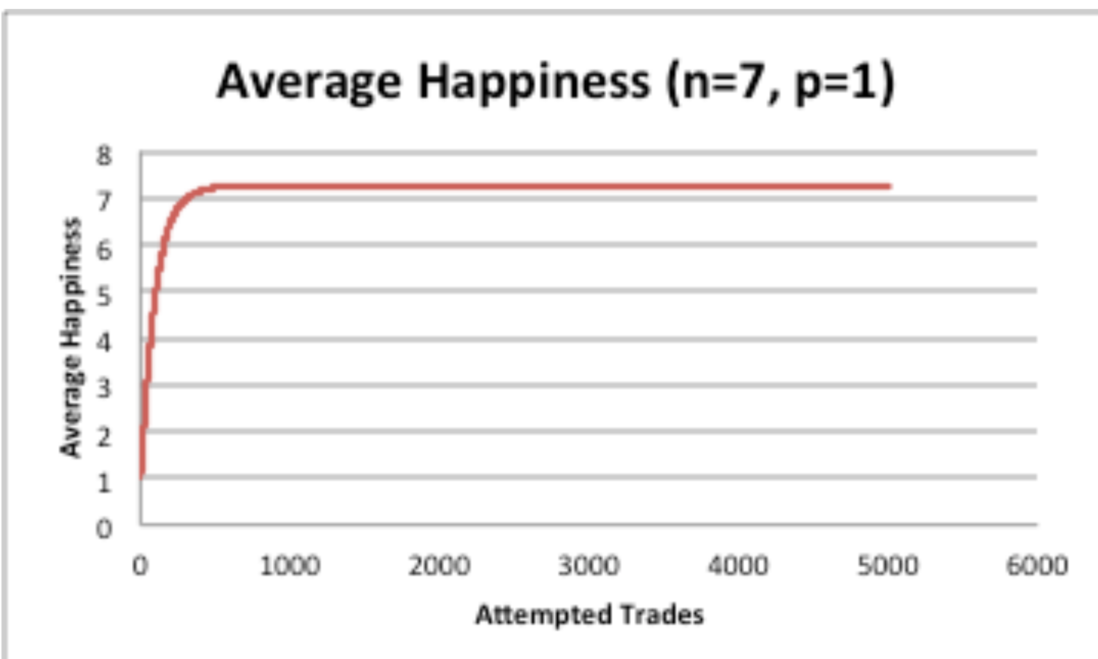
First, all the matrices where n=7 will be compared. All simulations will go through 5000 trade attempts. For each p value the code is run 100 times and the ending happiness is averaged for each trade attempt. The average of the average happiness is then compared to the time, or trade attempt. Below are the four different p values with a network of seven people, n=7.



Graph representing the relationship between the average happiness throughout all the trades within a network when n=7 and p=0.25 (Erdos Renyi Random Graph)

Graph representing the relationship between the average happiness throughout all the trades within a network when n=7 and p=0.5 (Erdos Renyi Random Graph)

Graph representing the relationship between the average happiness throughout all the trades within a network when n=7 and p=0.75 (Erdos Renyi Random Graph)



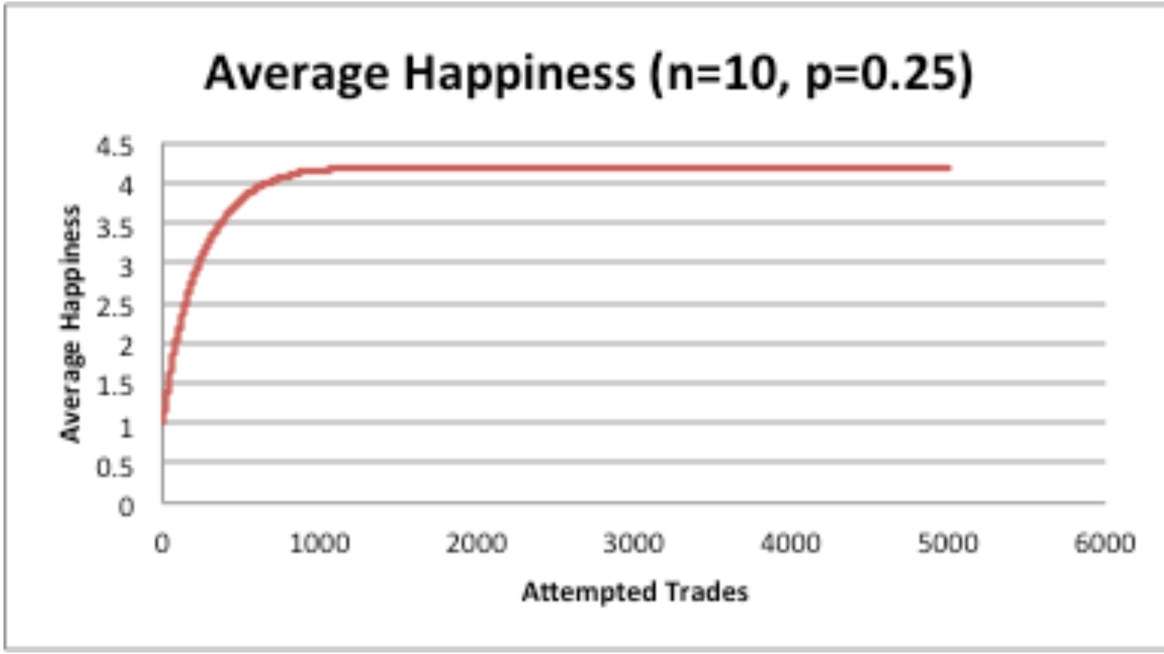
Graph representing the relationship between the average happiness throughout all the trades within a network when n=7 and p=0.25 (Erdos Renyi Random Graph)

Now here is a graph of all four p values to compare them with each other.

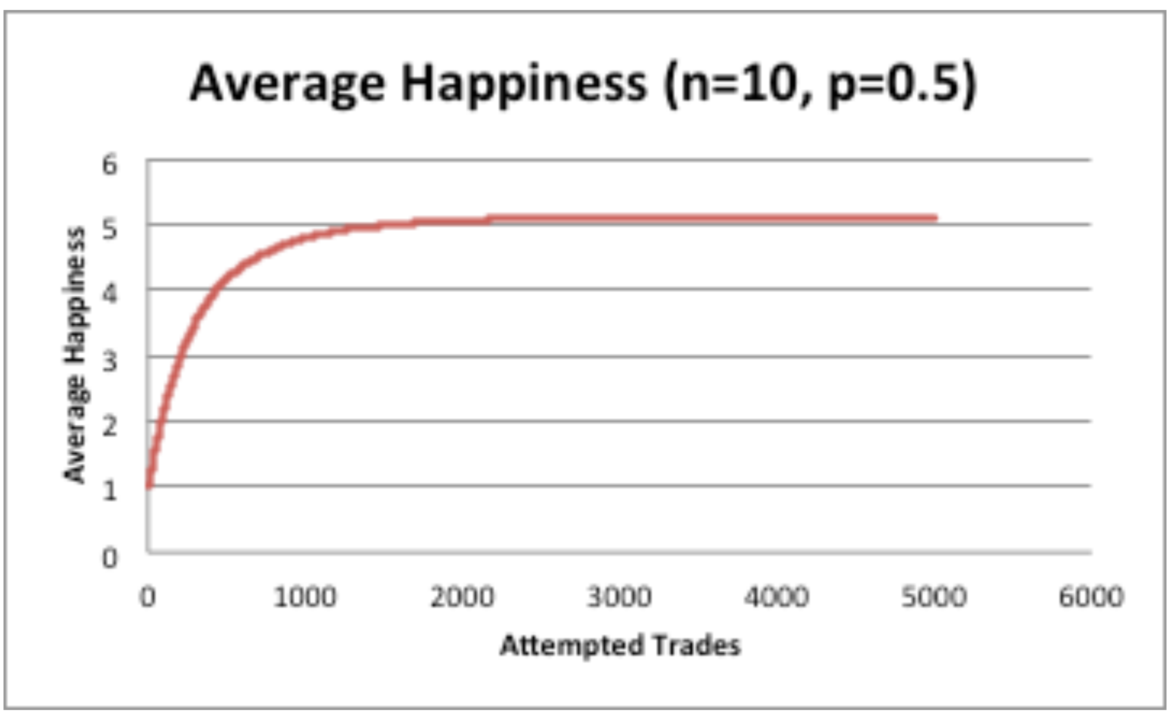
Graph comparing the relationship between the average happiness throughout all the trades within a network when n=7 for different p values (Erdos Renyi Random Graph)

From the graph above, it is evident that equilibrium is reached near 1000 attempted trades for all p values. Also as the p value increases so does the ending average happiness. The equilibrium happiness when p=0.75 is 3.812, when p=0.5 is 5.074, when p=0.75 is 6.660, and when p=1 is 7.256. Every time the p value was increased so did the average happiness in the network.

Now let's look at the n=10 graphs.

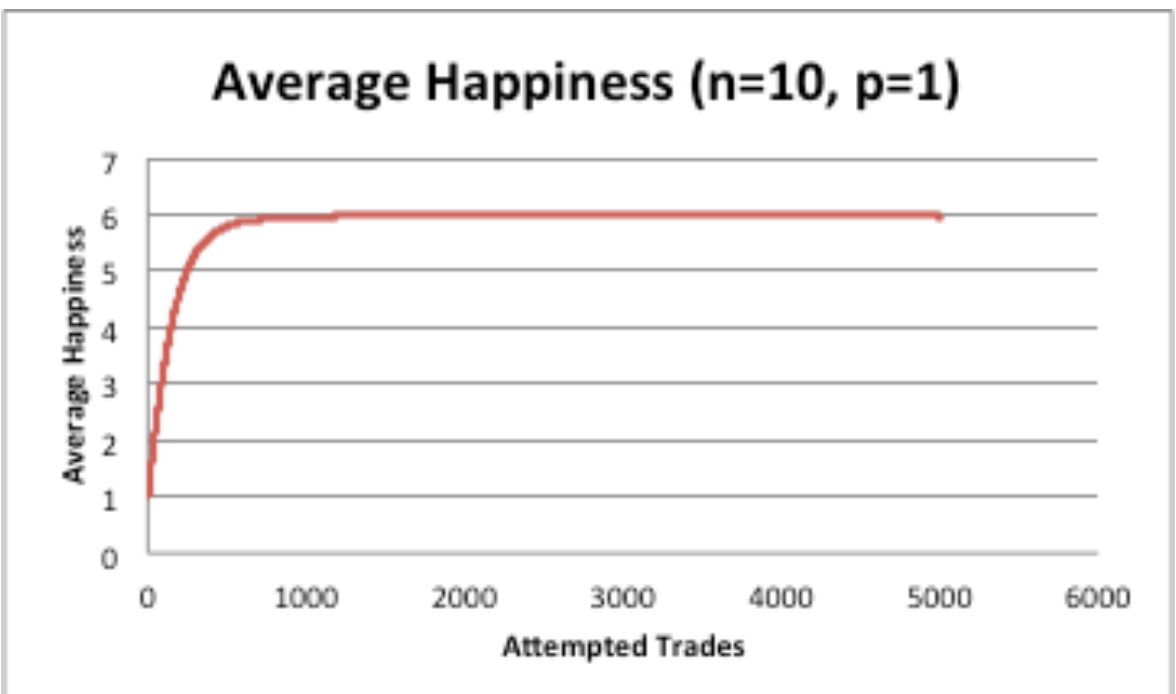


Graph representing the relationship between the average happiness throughout all the trades within a network when n=10 and p=0.25 (Erdos Renyi Random Graph)



Graph representing the relationship between the average happiness throughout all the trades within a network when n=10 and p=0.5 (Erdos Renyi Random Graph)

Graph representing the relationship between the average happiness throughout all the trades within a network when n=10 and p=0.75 (Erdos Renyi Random Graph)



Graph representing the relationship between the average happiness throughout all the trades within a network when n=10 and p=1 (Erdos Renyi Random Graph)

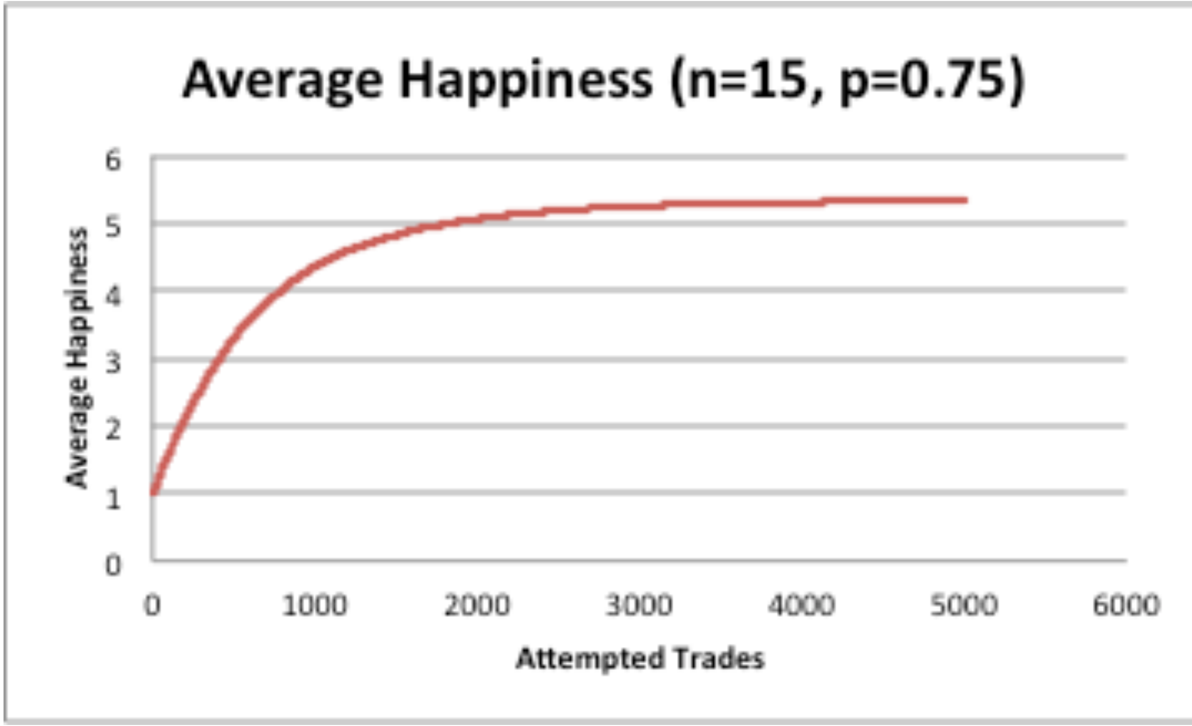
Graph comparing the relationship between the average happiness throughout all the trades within a network when n=10 for different p values (Erdos Renyi Random Graph)

Based on the graph above, it can be seen that on average it takes 1500 attempted trades to reach equilibrium and, as before, as the p value increases so does final average happiness.

Below are the graphs comparing the p values for a matrix of size n=15.

Graph representing the relationship between the average happiness throughout all the trades within a network when n=15 and p=0.25 (Erdos Renyi Random Graph)

Graph representing the relationship between the average happiness throughout all the trades within a network when n=15 and p=0.5 (Erdos Renyi Random Graph)



Graph representing the relationship between the average happiness throughout all the trades within a network when n=15 and p=0.75 (Erdos Renyi Random Graph)

Graph representing the relationship between the average happiness throughout all the trades within a network when n=15 and p=1 (Erdos Renyi Random Graph)

Graph comparing the relationship between the average happiness throughout all the trades within a network when n=15 for different p values (Erdos Renyi Random Graph)

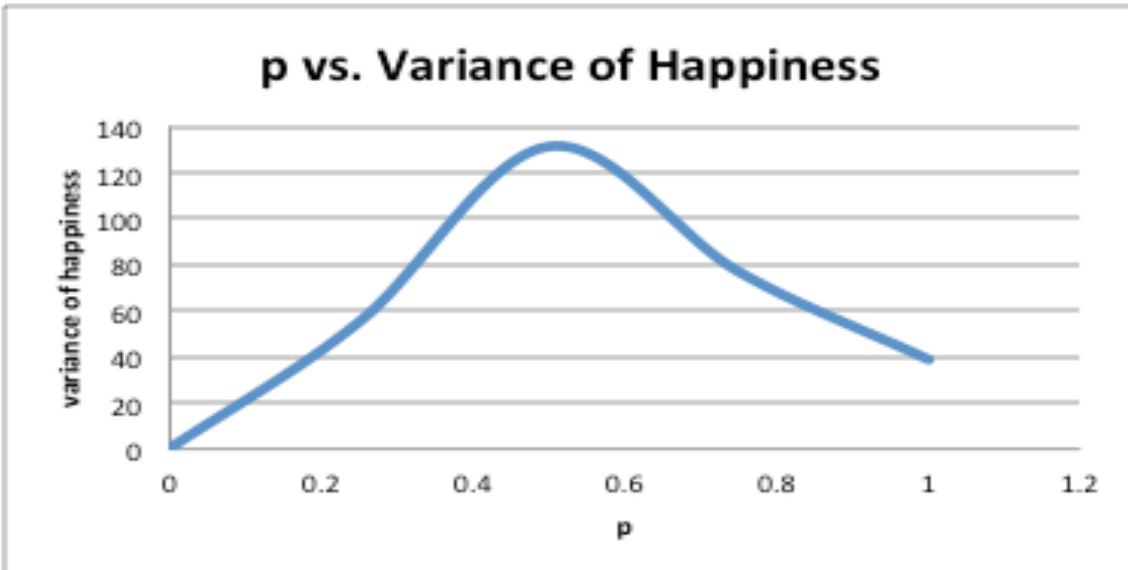
In the above graph it can be seen that it takes more time to reach equilibrium when n=15 than the other two network sizes, when n=7 and n=10. It takes around 3000-4000 trades to reach equilibrium. Once again as the p value increases so does the final average happiness.

Looking at all three of the n values, it can be seen that as the n value increases so does the time to reach equilibrium. The more people there are in a network, the more potential trades there can be. Therefore, equilibrium takes longer to be attained. It is also clear that as the p value, or connectivity, of the matrix is higher so is the final average happiness.

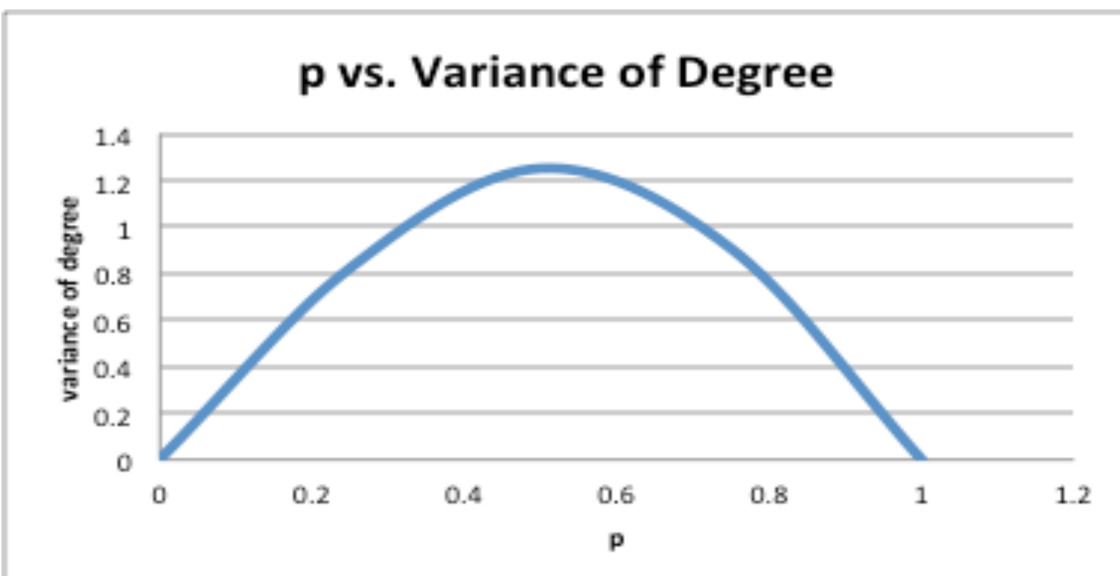
11. p vs. Variance of Happiness and Degree

It is important to know what happens when p, probability of people in the network being connected, increases. The first experiment is to see how the variance of happiness is affected by a change in the value of p. Remember variance is a measure of how far a certain set of numbers is spread out, and in particular how far the happiness levels of the different people in the network are spread out. At the same time, Variance of Degree and the value of p is compared.

Let's first compare p v. variance of happiness and p v. variance of degree when n=7.



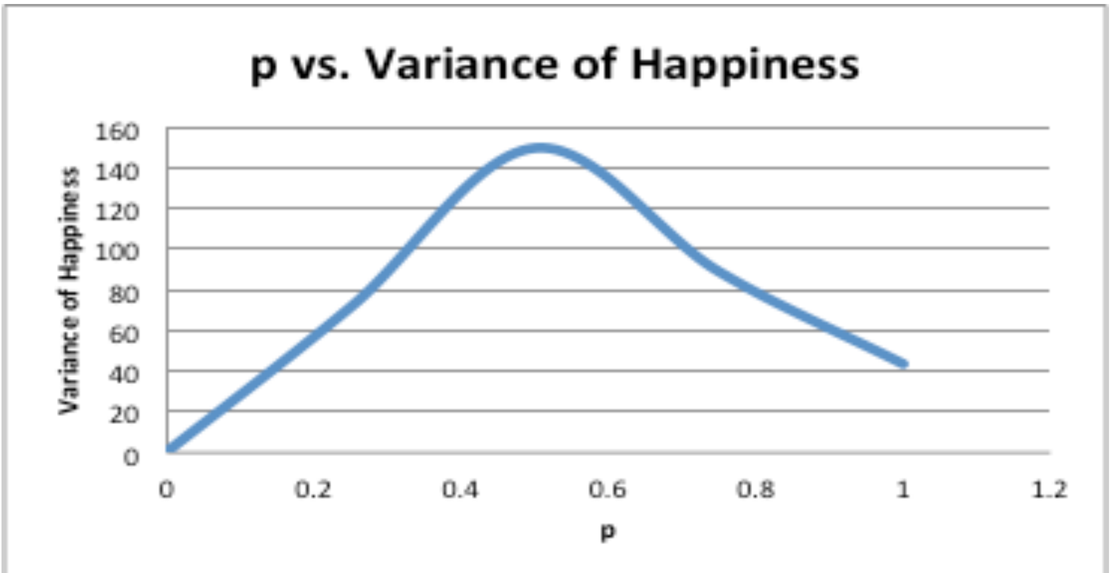
Graph representing the relationship between p and the variance of happiness when n=7 (Erdos Renyi Random Graph)



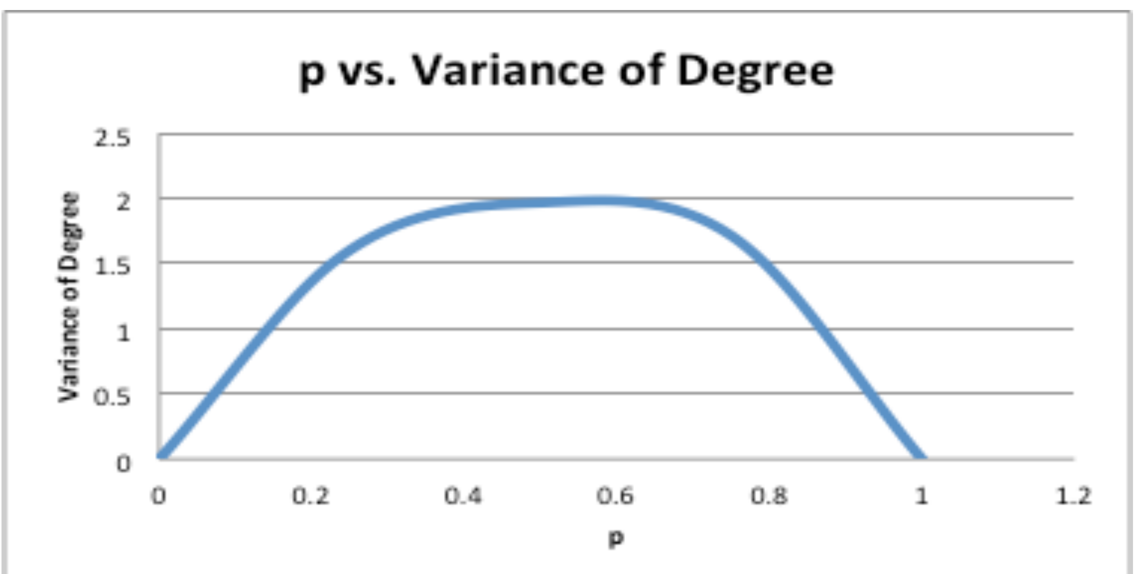
Graph representing the relationship between p and the variance of degree when n=7 (Erdos Renyi Random Graph)

It can be seen that for both graphs the variance is greatest when p=0.5.

Now, let's make the same comparisons when n=10.



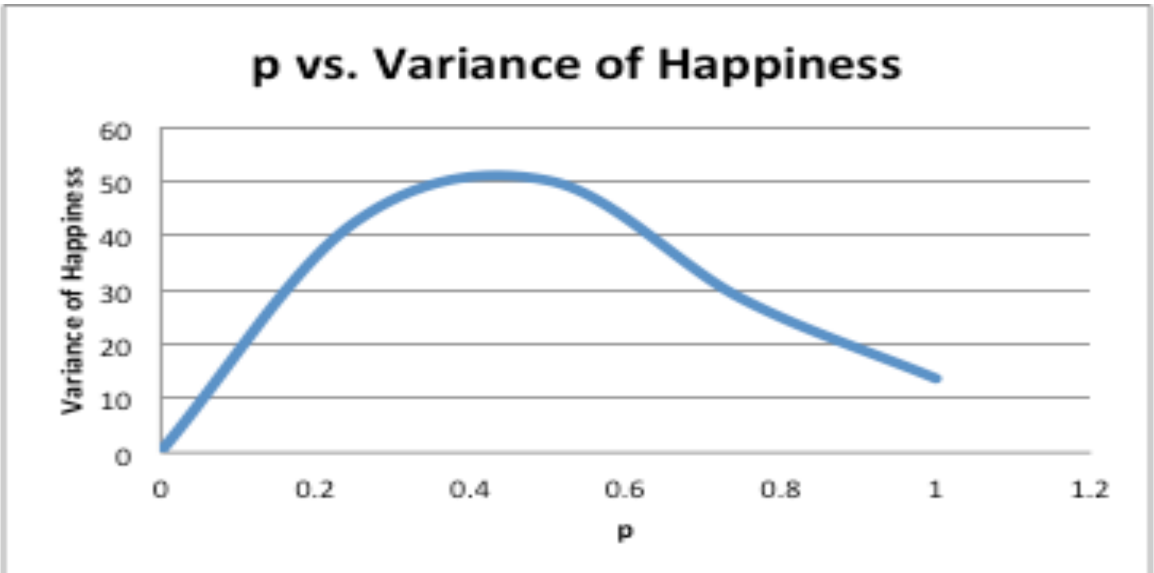
Graph representing the relationship between p and the variance of happiness when n=10 (Erdos Renyi Random Graph)



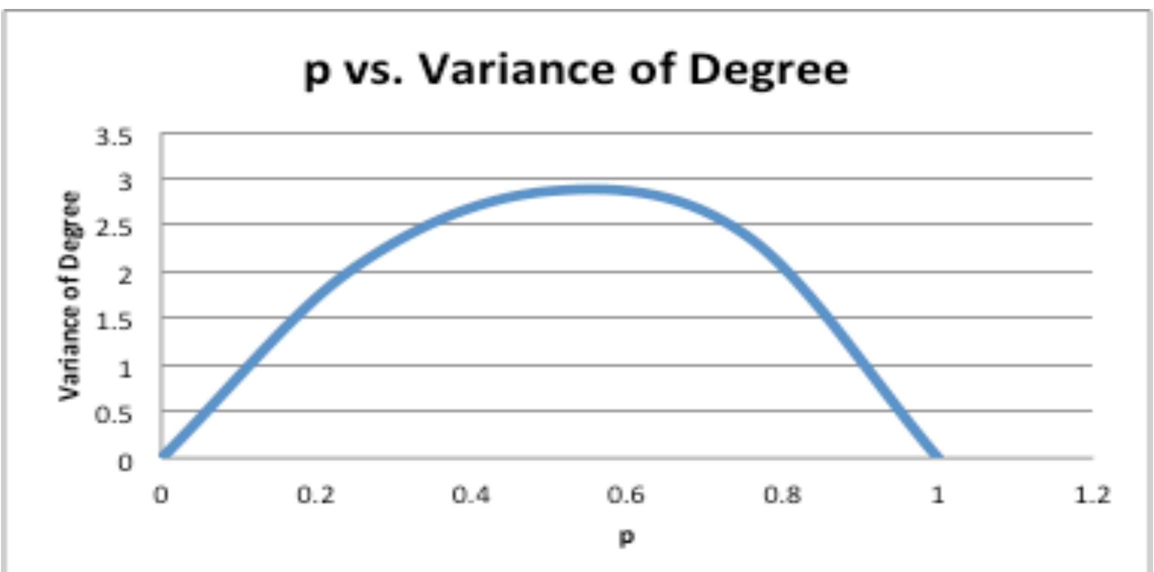
Graph representing the relationship between p and the variance of degree when n=10 (Erdos Renyi Random Graph)

For n=10, the variance of happiness and the variance of degree is greatest when p=0.5.

Lastly let's look at the relationships of variance and p when n=15.



Graph representing the relationship between p and the variance of happiness when n=15 (Erdos Renyi Random Graph)



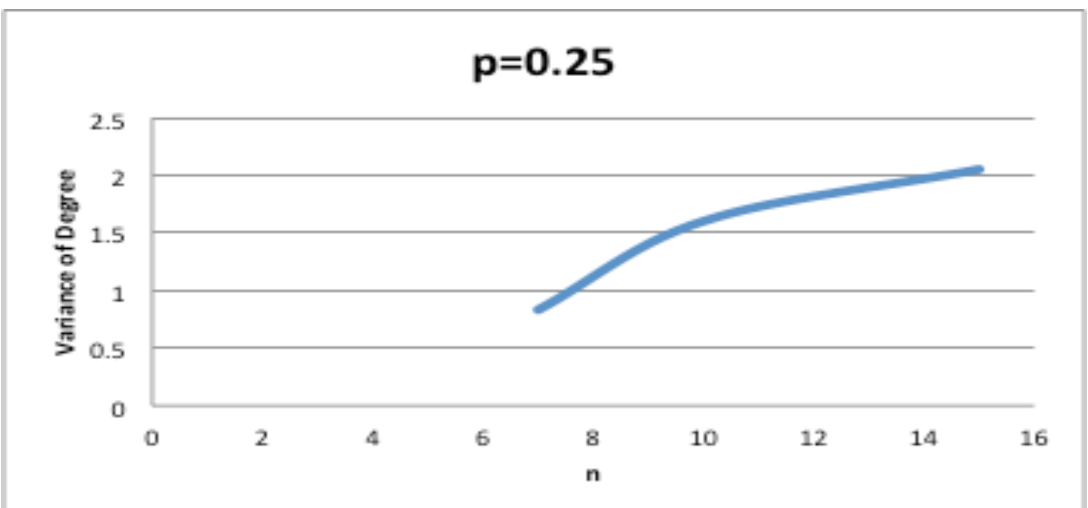
Graph representing the relationship between p and the variance of degree when n=15 (Erdos Renyi Random Graph)

When n=15, variance is greatest when p=0.5

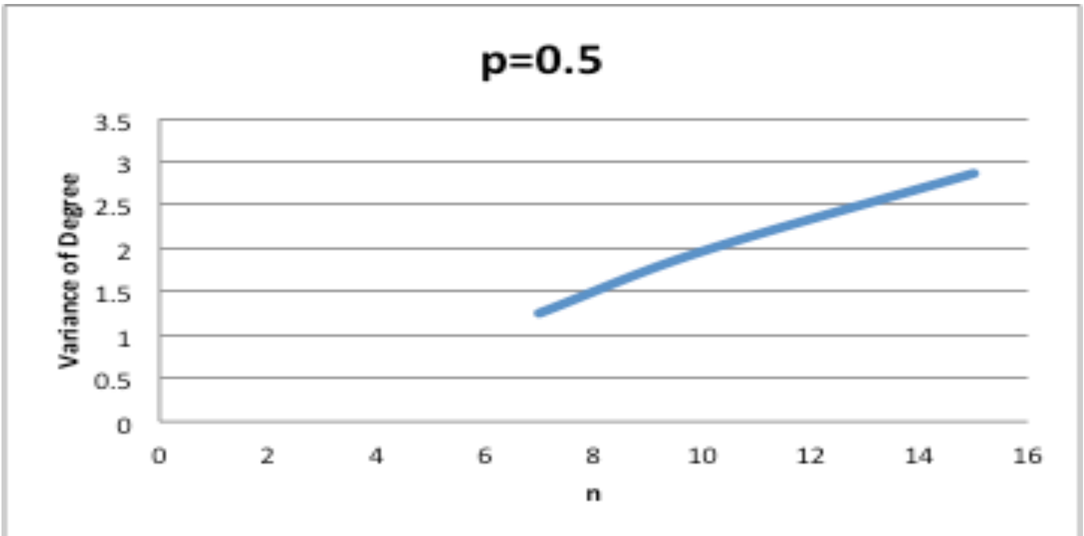
It can be seen that no matter how many people are in the network, the variance of happiness and degree is greatest when p=0.5. Since the variance of a binomial probability is $np(1-p)$ then the variance in happiness is related to the binomial variance which also peaks at p=0.5. Since variance of degree is also the highest when p=0.5, this seems to say that the variance of degree is closely related to the variance of happiness.

The variance of degree can also be compared to the size of the network. When p=0 and when p=1 the variance of degree is 0, therefore only when p=0.25, 0.5, and 0.75 can variance in degree be compared.

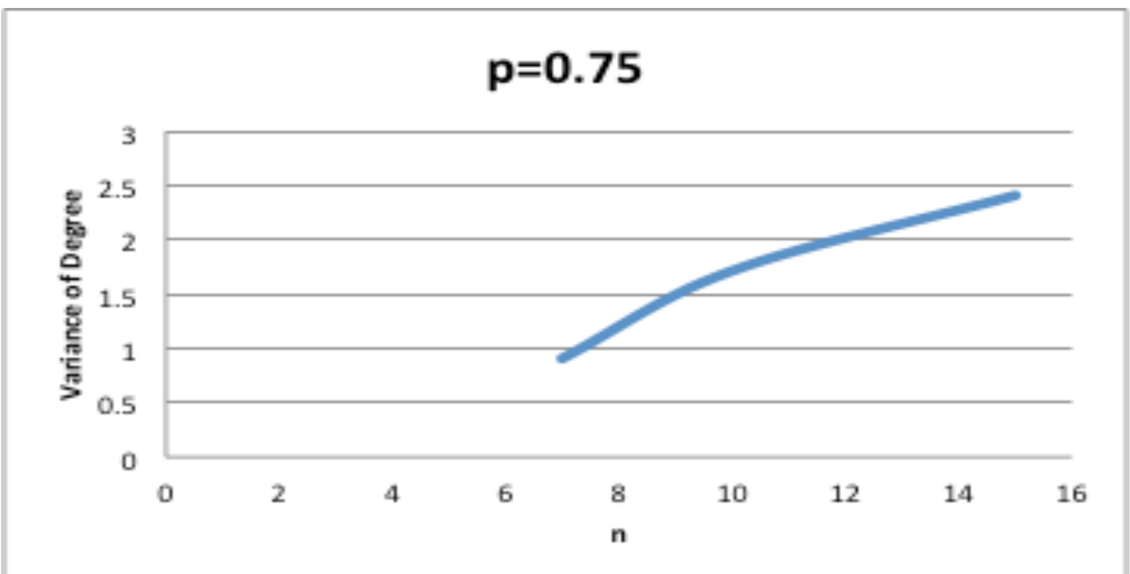
The graphs comparing n vs. variance of degree are below:



Graph representing the relationship between variance of degree and the size of the network, n, when p=0.25 (Erdos Renyi Random Graph)



Graph representing the relationship between variance of degree and the size of the network, n, when p=0.5 (Erdos Renyi Random Graph)

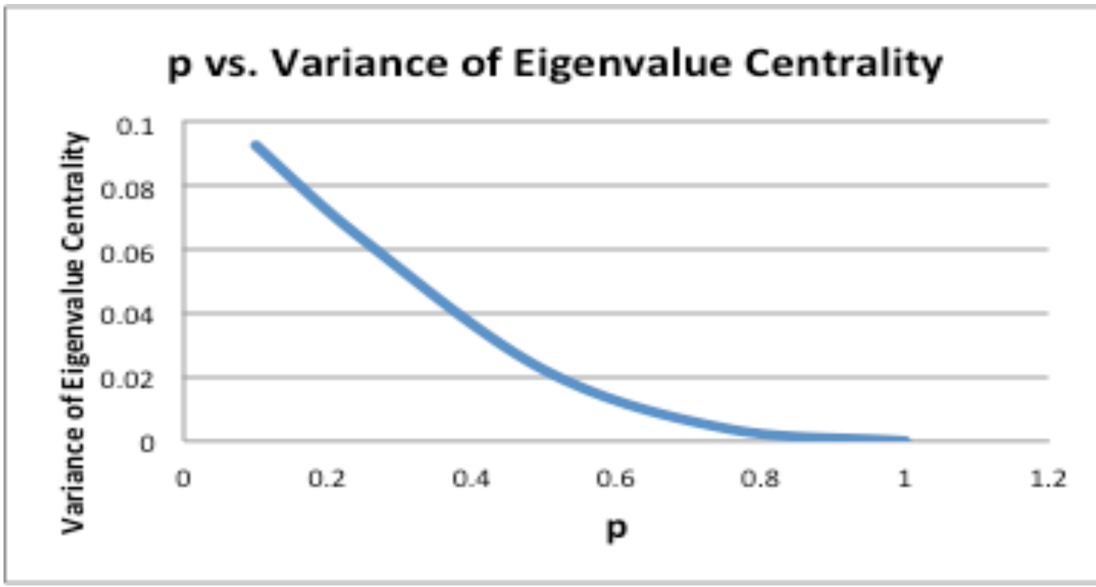


Graph representing the relationship between variance of degree and the size of the network, n, when p=0.75 (Erdos Renyi Random Graph)

All the graphs show that when n is higher so is the variance of degree. This makes complete sense since there are more people in the network and more of a chance to either be or not be connected to someone else in the network. The three graphs above are verifying the equation $np(1-p)$, the variance of a binomial distribution, and how the variance is linearly proportional to the size of the sample set (n in this case).

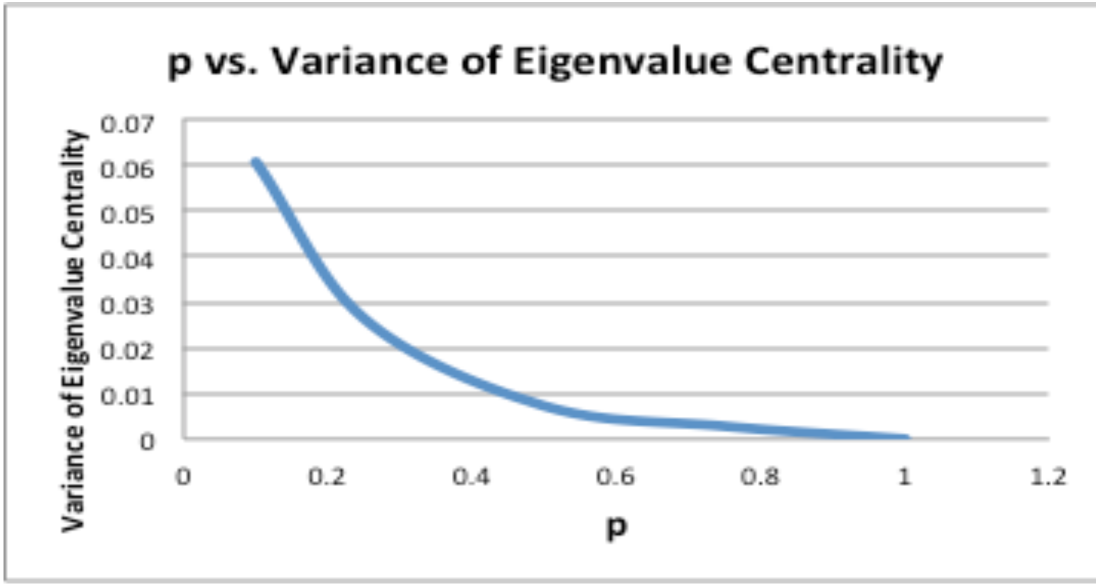
12. p vs. Variance of Eigenvalue Centrality

Now that we have determined the relationship of p to the variance of degree and the variance of happiness, p will now be related to the variance of eigenvalue centrality. Below is the graph when n=7 relating p to the variance of eigenvalue centrality.



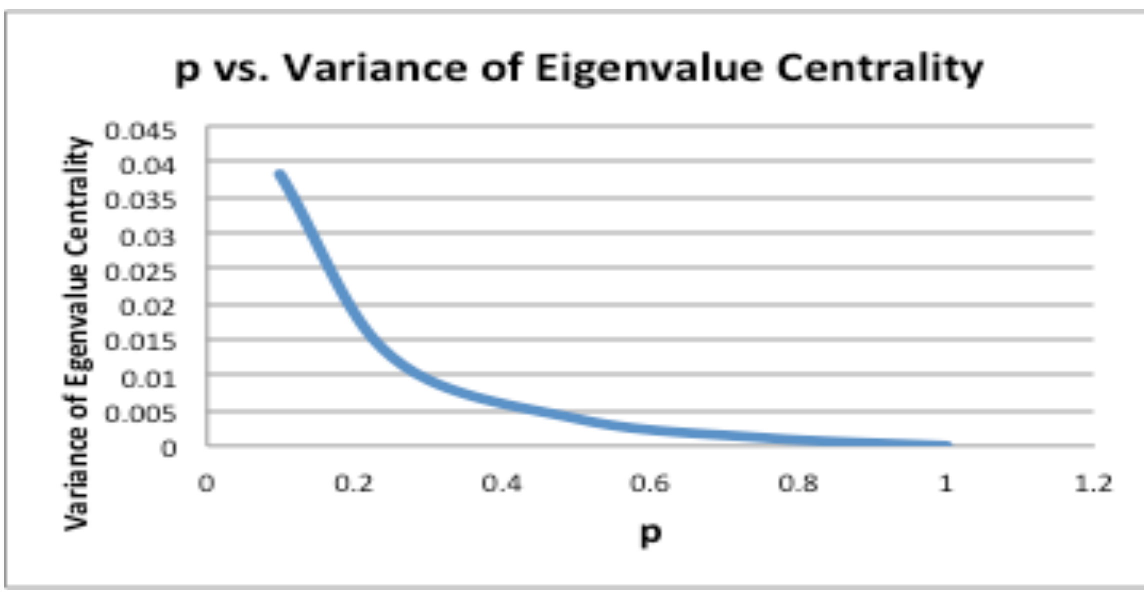
Graph representing the relationship between the variance of eigenvalue centrality and p when n=7 (Erdos Renyi Random Graph)

Now when n=10:



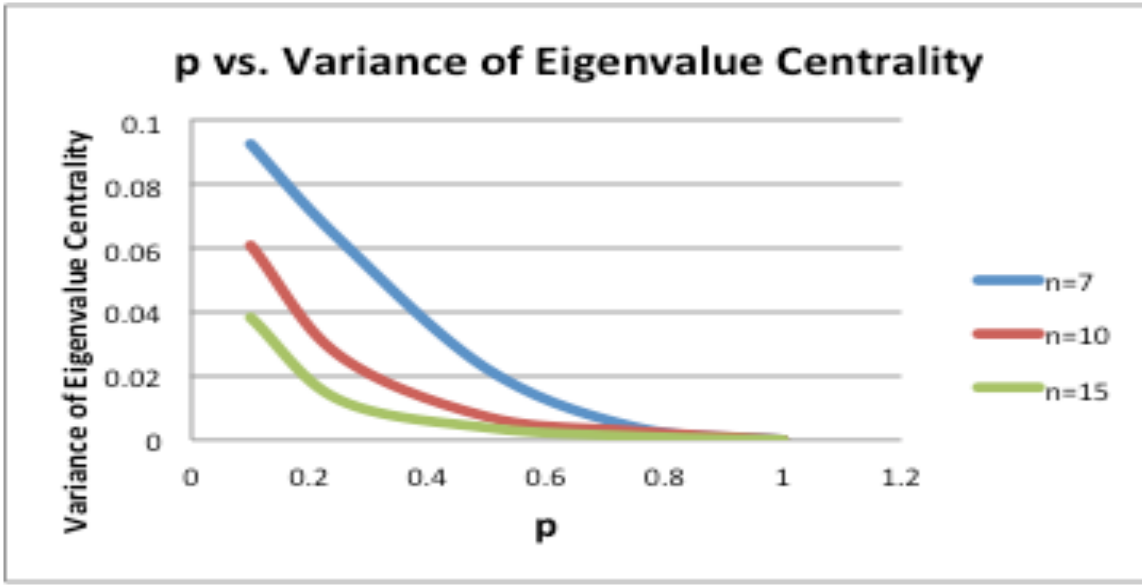
Graph representing the relationship between the variance of eigenvalue centrality and p when n=10 (Erdos Renyi Random Graph)

And now when n=15,



Graph representing the relationship between the variance of eigenvalue centrality and p when n=15 (Erdos Renyi Random Graph)

All three graphs show a similar relationship between p and eigenvalue centrality. Now let's see the relationship between all the different n values.



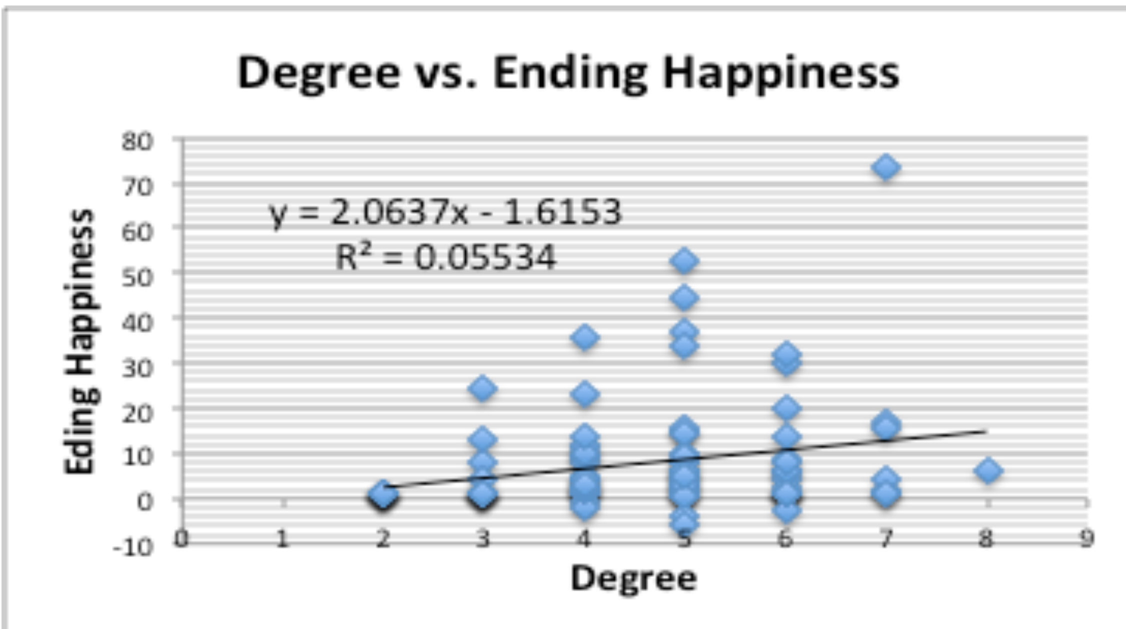
Graph comparing the relationship between the variance of eigenvalue centrality and p for the 3 different network sizes, n values (Erdos Renyi Random Graph)

The graph above indicates that as the n value, size of the matrix, increases, the variance of eigenvalue centrality decreases. However, the curves must go back down to zero at p=0. This would happen very fast and abruptly. The majority of the relationship, however, is depicted by this graph.

From the previous section we found that p vs. variance of degree and p vs. variance of happiness have similar representations of their relationships, a binomial curve. However, p vs. variance of eigenvalue centrality does not show the same relationship. This only says that the variance of happiness is more closely related to the variance of degree and does not say that degree is the best predictor of happiness. To determine the best predictor of happiness more experiments must be conducted.

13. Centrality vs. Ending Happiness in a Random Network

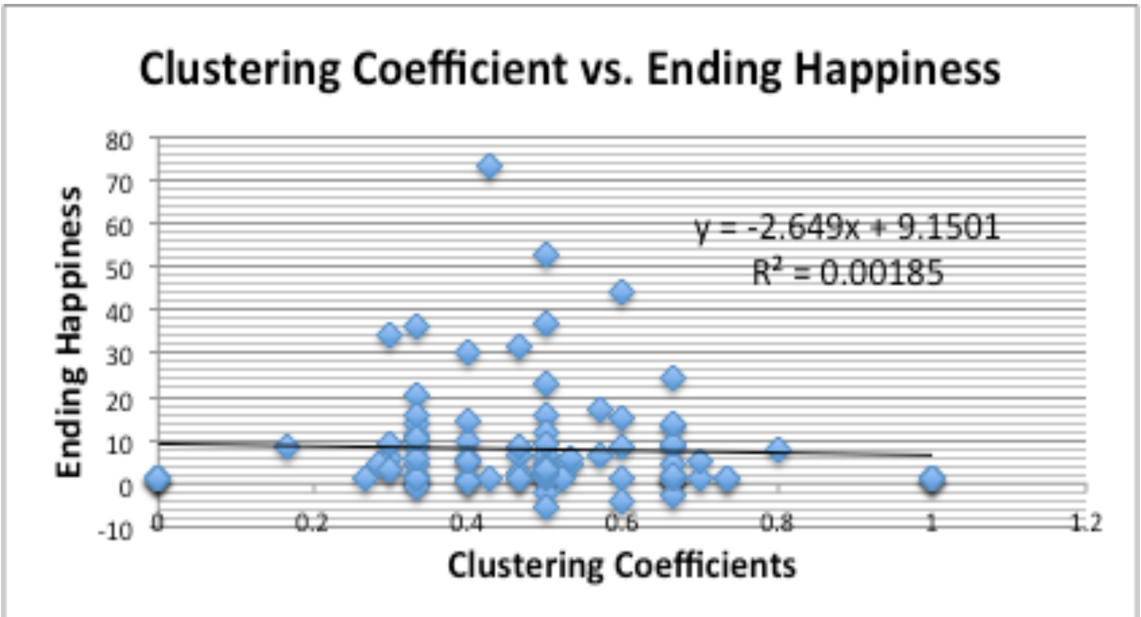
To get an idea of the best predictor of happiness, degree, clustering coefficient, and eigenvalue centrality will be compared to ending happiness. All comparisons will be made in these experiments when n=10 and p=0.5. The graphs represent the relationship between the ending happiness and the three standard measures of centrality over ten different random networks. The first graph is degree vs. ending happiness.



Graph representing the relationship between the degree and ending happiness of a node in Erdos Renyi Random Networks

The R^2 value is very low and therefore there is no evident linear relationship between degree and ending happiness. It can be seen, however, that there is the highest potential when the degree of a node is in the middle. The variance of a middle-valued degree is the highest and therefore it is hardest to predict those nodes' ending happiness. The lower and higher degrees are easier to predict; those people are likely to have a low ending happiness.

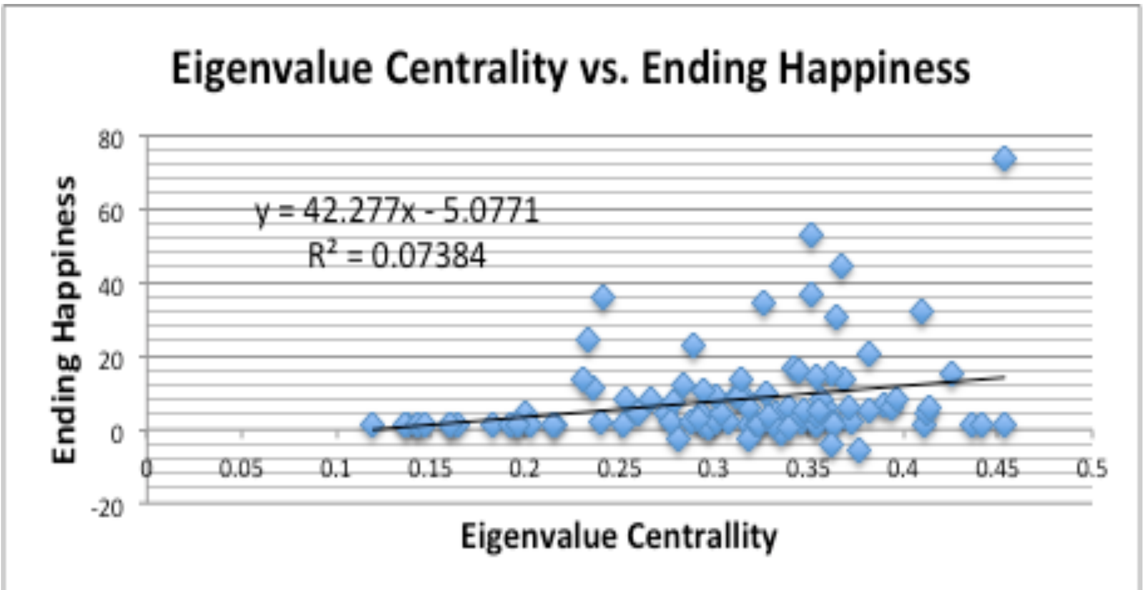
The second graph is clustering coefficient vs. ending happiness.



Graph representing the relationship between the clustering coefficients and ending happiness of a node in Erdos Renyi Random Networks

It can be seen that, like degree vs. ending happiness, there is the highest potential when the clustering coefficient's value of a node is in the middle. The variance of a middle-valued clustering coefficient is the highest and therefore it is hardest to predict those nodes' ending happiness. The lower and higher clustering coefficients are easier to predict; those people are likely to have a low ending happiness.

The third and last graph is eigenvalue centrality vs. ending happiness.



Graph representing the relationship between eigenvalue centrality and ending happiness of a node in Erdos Renyi Random Networks

There is also no trend evident in the graph of eigenvalue centrality vs. ending happiness. Therefore, the relationship is also not clear between eigenvalue centrality and ending happiness.

By looking at the three graphs above, there is not a linear relationship between the standard measures of centrality and ending happiness, since the R^2 values are so low. There are no other evident relationships between eigenvalue centrality and ending happiness. This could be due to the random initialization of goods in the beginning of the code. It cannot be determined which standard measure of centrality is the best indicator of happiness because, even though some guesses can be made with degree and clustering coefficients, the middle values cannot be predicted. To try and get a better idea of how the standard measures of centrality affect happiness a scale free network will be used

instead of an Erdos Renyi random graph, which has been used throughout the rest of the paper.

14. Erdos Renyi Random Network vs. Scale Free Network

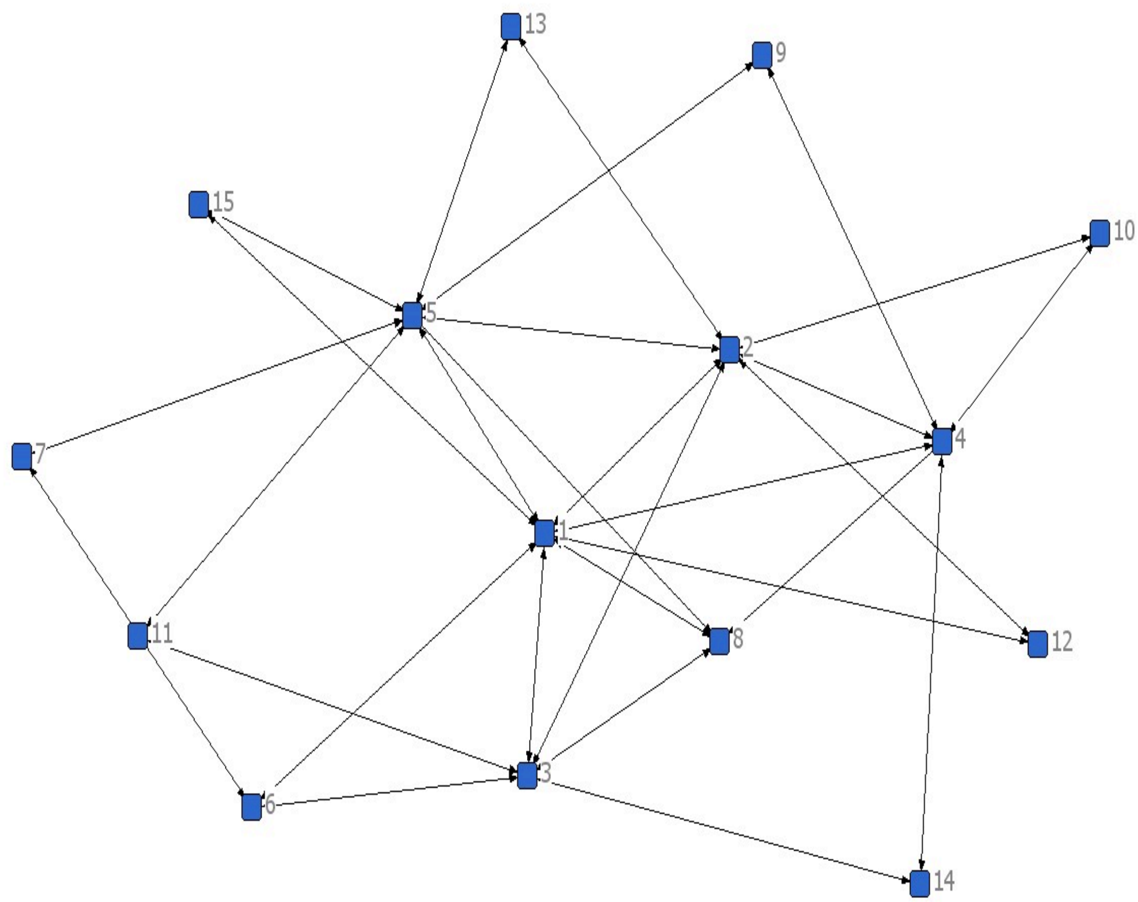
In the next section a different kind of random graph will be generated to complete a few experiments. It is called a scale free network/graph. A scale free network has a few nodes with a high degree and many nodes with a small degree, thus in a scale free network; degree distribution follows a power law, unlike the Erdos Renyi random graph, whose degree distributions follow a binomial distribution. Scale free networks are randomly created through a preferential attachment method where new joining nodes connect to established nodes proportional to the degree of the established nodes. The nodes with the highest degree are called hubs. Many real networks display scale free characteristics. An example of the trading model on a scale free network will be shown in Section 15.

15. Centrality vs. Ending Happiness in a Scale free Network

Since the graphs of standard measures of centrality vs. ending happiness did not show clear relationships using a randomly generated Erdos Renyi graph, the same experiments will be done with a scale free network. The scale free network, with size n=15, is as follows:

$$A = \begin{matrix}
0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\
1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{matrix}$$

Which looks like,

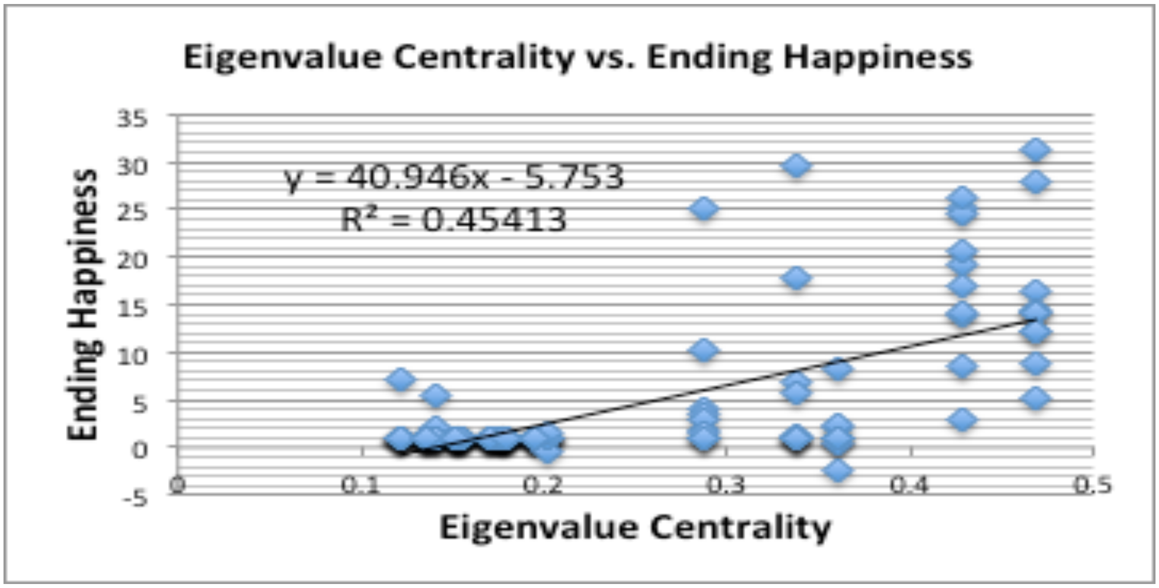


Picture of the scale free 15-person network based on the adjacency matrix above (Ucinet)

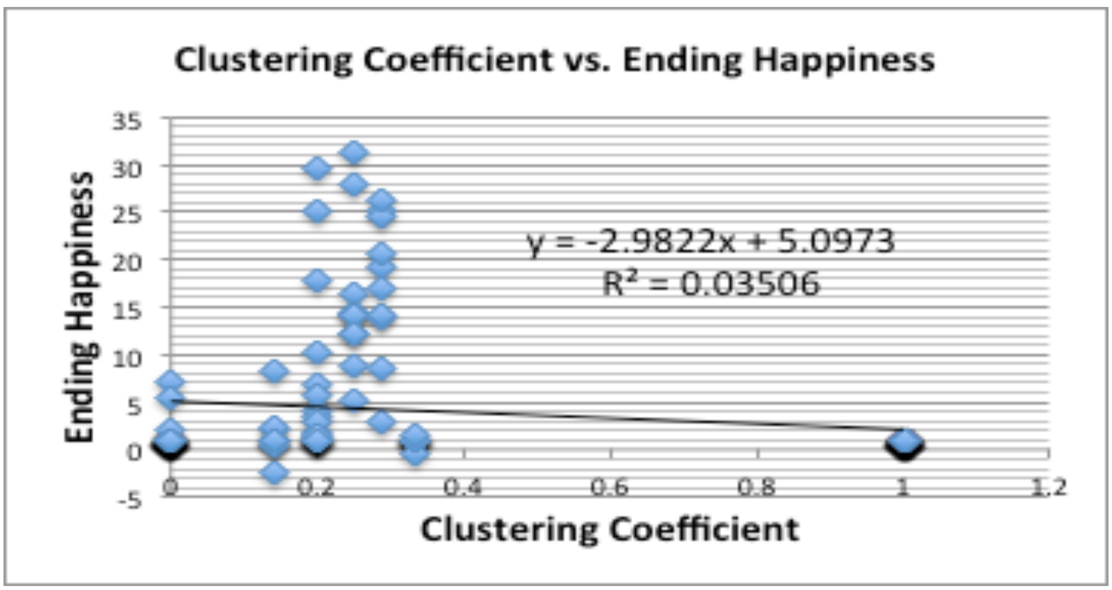
This network was made using the Albert Barabasi method (Barabasi, 2003). The method will now be explained. The network starts off with 2 connected nodes. Then, new nodes are added to the network one at a time where a new node is connected to a certain number of already existing nodes with a probability that is proportional to the number of links that the existing nodes already have. Therefore, there is a larger probability that the new nodes are connected to existing nodes that have many connections than existing nodes that have only a small amount.

Using the above network and running the MATLAB code ten times, the graphs comparing the standard measures of centrality and ending happiness are as follows:

Graph representing the relationship between the degree and ending happiness of a node in a Scale free Network



Graph representing the relationship between eigenvalue centrality and ending happiness of a node in a Scale free Network

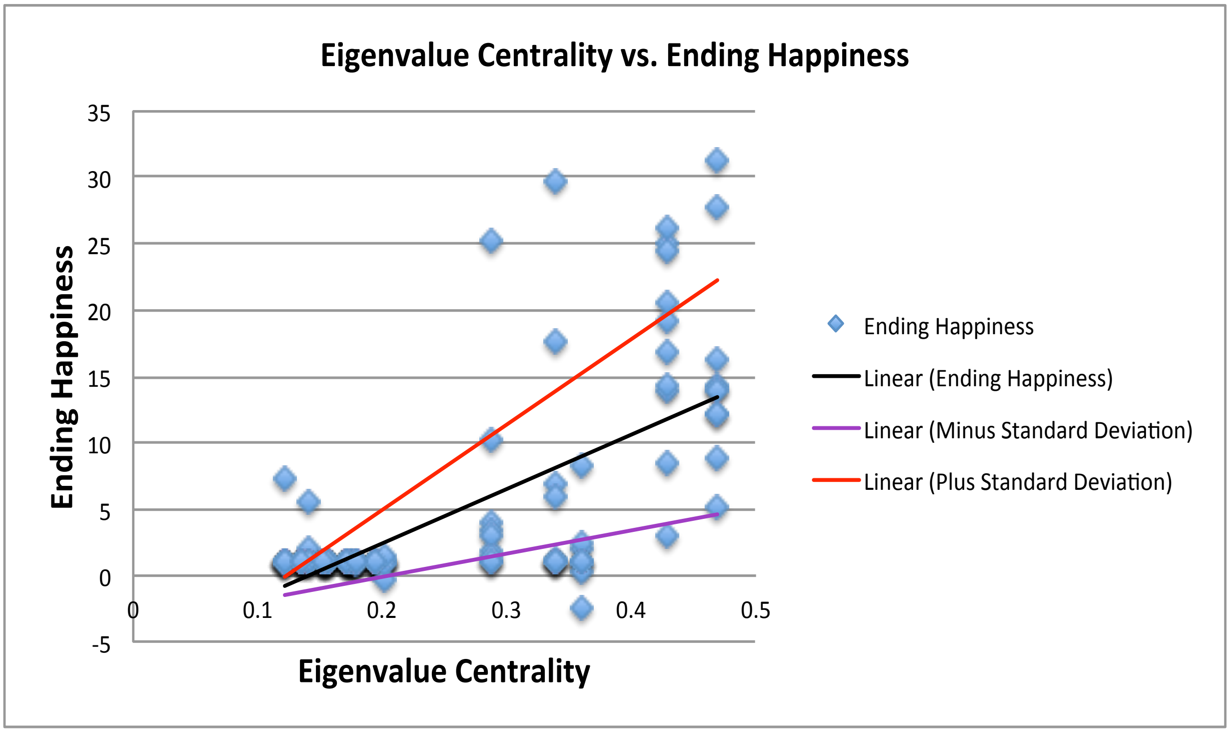


Graph representing the relationship between the clustering coefficients and ending happiness of nodes in a Scale free Network

The graph comparing clustering coefficients with a scale free network, like the one using an Erdos Renyi random graph, does not give any conclusive relationship to ending happiness. That is not true for degree and eigenvalue centrality. Although, the R^2 values

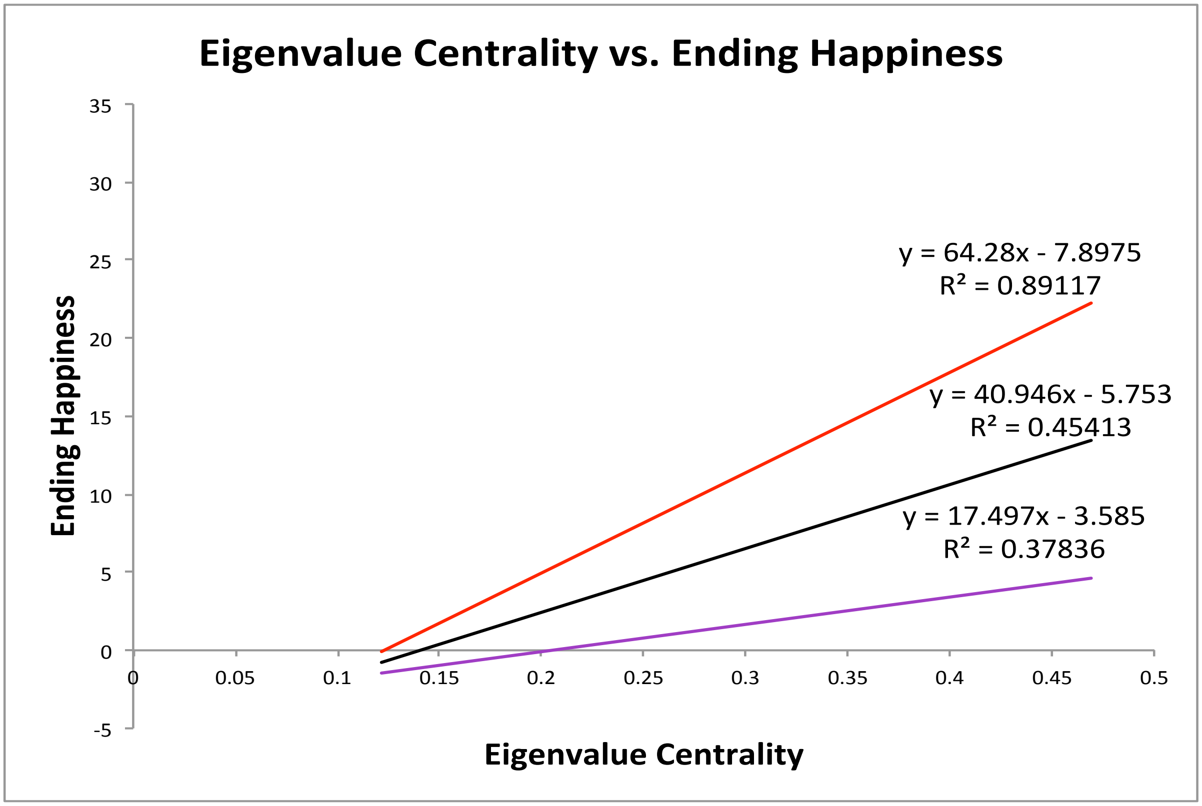
are not that close to 1, they are significantly better when using the scale free network. Once again, there is no control over the distribution of the initial goods; they are still randomly distributed using a Weibull distribution. However, the scale free network results are more conclusive. It can be seen that as degree increases so does the ending happiness, unlike the graph using an Erdos Renyi random network, which showed that middle valued degrees had the highest potential for happiness. Also, as eigenvalue centrality increases, it is the case that ending happiness increases, which was not clear from the Erdos Renyi random network. The exact relationship between degree and ending happiness and eigenvalue centrality and ending happiness cannot be determined but the general relationship is clearer. It can be determined, however, that since the R^2 value for eigenvalue centrality is closer to 1 than the R^2 value for degree, eigenvalue centrality seems to be a better predictor of happiness than degree.

Below represents the same graph of eigenvalue centrality vs. ending happiness as above, but here a confidence interval was added. All the values of ending happiness for each major eigenvalue centrality rate were taken, and then the standard deviation was derived for each value. The lines surrounding the original best fit line for eigenvalue centrality vs. ending happiness, represents one standard deviation away from that best fit line, using the standard deviation just calculated. This area between the two outer lines is called the confidence interval. One standard deviation away from the best-fit line lies within the confidence interval.



Graph displaying the confidence interval for ending happiness for different quantities of eigenvalue centrality in a Scale free Network

To get a better idea of how the confidence interval works, here is an example. First let's take any eigenvalue centrality that would lie on the graph, say 0.35. To find the interval in which the ending happiness should lie, the value on the red and purple best-fit lines, when eigenvalue centrality equals 0.35, must be determined. The red and purple lines both represent the best-fit lines of data a standard deviation away from the best-fit line of eigenvalue centrality and ending happiness, the black line.



The best-fit line of eigenvalue centrality vs. ending happiness surrounded by the 2 lines representing the confidence interval

The value on the best-fit line of eigenvalue centrality and ending happiness is as follows:

$$y(x) = 40.946x - 5.753$$

$$y(0.35) = 40.946(0.35) - 5.753 = 8.5781$$

This is just to have an idea of the middle ground of what can be expected. Now to get the actual interval the eigenvalue centrality, 0.35, must be plugged into the best-fit line formulas for standard deviation, the red and purple lines. The low value in the interval, plugging into the line equation a standard deviation lower than the original best-fit line is as follows:

$$y(x) = 17.497x - 3.5853$$

$$y(0.35) = 17.497(0.35) - 3.5853 = 2.53865$$

The high value in the interval, plugging into the line equation a standard deviation higher than the original best-fit line is as follows:

$$y(x) = 64.28x - 7.8975$$

$$y(0.35) = 64.28(0.35) - 7.8975 = 14.6005$$

Therefore, the interval when the eigenvalue centrality is 0.35 is predicted to be:

$$[2.53865, 14.6005]$$

This same process can be done for any eigenvalue centrality value within the values given in the network.

Conclusion

This paper presents an agent-based model to investigate the trading of two types of goods between individuals connected together on a social network. All the people in the network are seeking to maximize their happiness, or reach equilibrium. Throughout the process of studying such a network, many different results about the social network were found. One of those results is as follows: as degree, number of people each

individual knows, increases so does the price of the goods they hold. They have more selling power and are able to sell their goods for a higher price. It was also found that as the price increases so does the final average happiness.

There were other relationships found as well. It was also found that as the probability of people knowing each other is p=0.5, the variance of happiness and the variance of degree is the greatest. Variance of degree and variance of happiness versus p shows a binomial relation. This was definitely to be expected for variance of degree, and it can now be concluded that variance of happiness has a similar relationship to p as variance of degree. It can be inferred that as the variance of degree increases, the variance of happiness will also likely increase. The relationship of p and variance of eigenvalue centrality is different to that of happiness and degree. That does not mean that degree is the best indicator of happiness, but just means the variance of happiness and the variance of eigenvalue centrality are not related.

From Section 13 and Section 15, it was shown that if a network is an Erdos Renyi random network then the ending happiness is not easily predicted. Scale free networks lead to better predictions of happiness than the Erdos Renyi networks do. Based on the results from Section 15, the graphs displaying the relationships between the standard measures of centrality and ending happiness, it was concluded that eigenvalue centrality is the best predictor of happiness.

Eigenvalue centrality showed the best results, surpassing both degree and clustering coefficients when compared to average ending happiness. A confidence interval was determined, where if the eigenvalue centrality is known the ending happiness will most likely lie within that confidence interval. As discussed earlier,

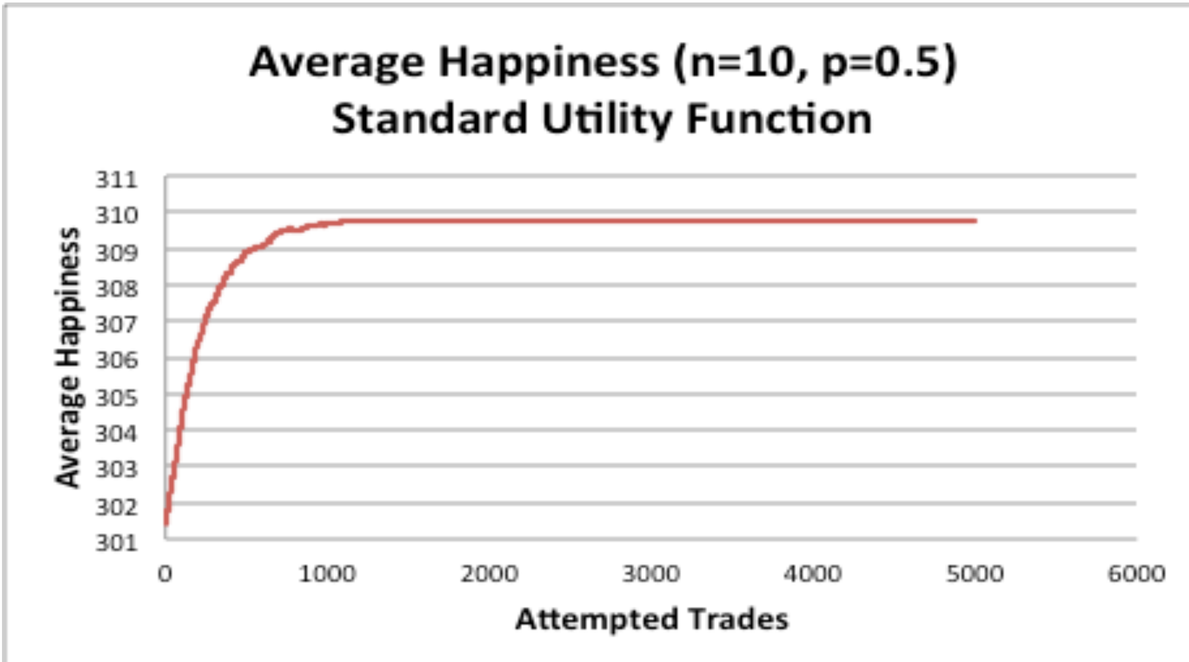
eigenvalue centrality can be seen as the influence each person has on the network. This influence, or eigenvalue centrality, is proportional to the total influence of the people to whom he or she is connected. Here we have determined that the higher eigenvalue centrality rate a person has, the happier they will be at the end of trading. Even though eigenvalue centrality is the best indicator, the degree may be used to estimate the ending happiness. This is because, degree is easier to calculate than eigenvalue centrality and is only a little inferior as a predictor than eigenvalue centrality. For degree, all that needs to be determined is how many connections each node has or how many friends each person has. On the other hand, to calculate eigenvalue centrality, eigenvalues and eigenvectors need to be determined and that is way more time consuming than determining degree.

Since a non-standard utility function was used throughout the thesis, the results need to be verified with the standard utility function, by checking the relationship between average happiness and time to see if it is the same. The utility function in the thesis cannot be the only part to the happiness function, since the gradients would be constant. By adding the net worth to the happiness function, the gradient problem is taken care of. To verify that the happiness function used in the thesis is valid, the comparison must be made between the thesis happiness function and the happiness function that is considered standard. The standard happiness function is just the standard utility since net worth is not considered.

The standard utility/happiness function is as follow:

$$U(t) = F(t)^{\frac{1}{4}} \cdot B(t)^{\frac{3}{4}}$$

The corresponding graph of average happiness vs. time, where n=10 and p=0.5, using the above happiness function is as follows:



This graph shows the relationship between average happiness and time using the standard utility function.

Compare the graph above, to the graph in section 10, where n=10 and p=0.5.

Both graphs show similar relationships of average happiness vs. time and therefore, the happiness function used in this paper is validated.

For future work on this topic, there are many different opportunities. In this paper, it was assumed that individuals know how many of each good are in the entire network. What if this was not the case? If people only knew the goods they possessed and the goods their neighbors possessed, a different price equation would have to be developed (such as in Kultti, 2000). This price equation could also be based on scarcity, but on a smaller scale. Scarcity would have to be determined by the goods of an individual compared to the goods of their neighbors alone; no more other information would be known.

Another example of changes that can be made to further research this subject is as follows. Throughout all trades, the number of firewood and the number of candy bars was constant. Therefore, in the price equations, the denominators, the total number of each good, don't change. What if the total number of both firewood and candy bars could change throughout the trading process. For example, people could cut down their own firewood or make their own candy bars. It can also be said that firewood can rot and candy bars can spoil (such as in Vignes & Etienne, 2011). If an increase or decrease occurred for any person after any trade, the denominator of their associated price equation would not be constant and would either increase or decrease. The question is, what would determine if a person was to make a candy bar or that their firewood would rot or any of the other possibilities? How would this factor into the code?

In Vignes and Etienne's paper, fish is the good to be traded, where fish is a perishable good. The longer fish waits to be sold, the less fresh it is, and the lower the price becomes. In this paper, candy bars and firewood are not seen as perishable and can last forever. If candy bars and firewood lost their value over time, then the price functions would have to be adjusted to take time into consideration. Once the price of goods go to zero, the quantity of goods would also have to be adjusted since they would be un-tradable.

Other standard measures of centrality, like closeness and betweenness can be compared to ending happiness like degree, eigenvalue centrality, and clustering coefficients were. In this thesis, it was determined that eigenvalue centrality was the best predictor of happiness. But who is to say that closeness, betweenness, or any other measure of centrality are not as good of a predictor? To find out if another measure of

centrality can predict happiness more accurately, more experiments and relationships need to be determined.

Appendix

Below is the matlab code I used throughout the entire thesis.

```
function val = Code_in_Thesis

Fraction=.25; % the p value

n=7;          % Size of the network

%------------------------------------------------------------

A = zeros(n,n);  %Random adjacency matrix of the network


            for j=1:n
                for k=1:n
                    if k==j
                        A(j,k)=0;
                    end
                    if k~=j
                        c=rand(1,1);
                            if c>Fraction
                                b=0;
                            end
                            if c<=Fraction
                                b=1;
                            end
                    A(j,k)=b;
                    A(k,j)=b;
                end
            end
        end

%------------------------------------------------------------

% Numberering the edges in vector form

Number_Ones=0;

for s=1:n
    for t=s:n
        if A(s,t)==1
            Number_Ones=Number_Ones +1;
            Adjacency_List(Number_Ones, 1)=s;
            Adjacency_List(Number_Ones, 2)=t;
        end
    end
end
```

```
%-----------------------------------------------------------

% Randomly distributing the initial amounts of firewood and
candy bars

random_firewood=rand(1,n);

random_candy_bars=rand(1,n);

for b=1:n

Firewood(b) = 350*(-log(random_firewood(b)))^(1/2);

end

for c=1:n

Candy_Bars(c) = 350*(-log(random_candy_bars(c)))^(1/2);

end

% L = 500;
% Firewood=3*floor(L*rand(1,n)); %Random # of Firewood for
each person
% Candy_Bars=floor(L*rand(1,n)); %Random # of Canybars for
each Person

Fo=Firewood;     %Initial Firewood
Bo=Candy_Bars;   %Initial Candybars

F=Fo;   %Variable Firewood
B=Bo;   %Variable Candybars

V=ones(1,n);
Ftotal=dot(Fo,V);  %Total Firewood in network
Btotal=dot(Bo,V);  %Total Candybars in network

C=A*F';    %Total Firewood Available to each person
D=A*B';    %Total Candybars Available to each person

%-----------------------------------------------------------

% Determining the price of goods

for y=1:n
    Pf(y) = 1*(F(y)+C(y)/2)/(Ftotal);
    Pb(y) = 3*(B(y)+D(y)/2)/(Btotal);
```

```
end

for S=1:n
    Pfo(S) = 1*(Fo(S)+C(S)/2)/(Ftotal);
    Pbo(S) = 3*(Bo(S)+D(S)/2)/(Btotal);
end

%Initial value of goods for each person

for r=1:n
    M(r) = Fo(r)*Pfo(r)+Bo(r)*Pb(r);
end

%Constant accounting for the difference in price for
firewood
a=.01;
%Constant accounting for the difference in price for
candy bars
b=.03;

%Utility Function

for m=1:n

   U(m) = 1-exp(-a*Fo(m)-b*Bo(m));

end

%Happiness Function

x=1;

% parameter to measure how much happiness is derived from
the potential trading value of the stored goods as
weighted against the actual utility of those goods.

for v=1:n

    H(v) = U(v) + x*(M(v)-(Pfo(v)*Fo(v)+Pbo(v)*Bo(v)));

end

%-----------------------------------------------------------

% Trading

Number_Attempts=5000;
```

```
Average_Happiness=zeros(1,Number_Attempts);

Average_Happiness(1) = dot(H,V)/n;

Trade_Attempts=0;
trades=0;

while Trade_Attempts<=Number_Attempts;

    Trade_Attempts = Trade_Attempts + 1; % Counter

    % Choosing a random edge
    Random_Trade=randi(Number_Ones, 1);

    s=Adjacency_List(Random_Trade, 1);
    t=Adjacency_List(Random_Trade, 2);

    % Price changes for every trade

    for L=1:n
        Pf(L) = 1*(F(L)+C(L)/2)/(Ftotal);
        Pb(L) = 3*(B(L)+D(L)/2)/(Btotal);
    end

    EXA = Pb(s)/Pf(t);  %Candy bar firewood exchange rate A
    EXB = Pb(t)/Pf(s);  %Candy bar firewood exchange rate B

    %Gradient

    Gs = [a*exp(-a*F(s)-b*B(s))-x*Pf(s)  b*exp(-a*F(s)-
b*B(s))-x*Pb(s)];
    Gt = [a*exp(-a*F(t)-b*B(t))-x*Pf(t)  b*exp(-a*F(t)-
b*B(t))-x*Pb(t)];

    % Restrictions on trading

    if (F(s)>=EXA && B(t)>=1 && (Gt(1)*EXA+Gt(2)*(-1))>=0
&& (Gs(1)*(-EXA)+Gs(2)*1)>=0)
          F(s) = F(s) - EXA;
          B(s) = B(s) + 1;
          F(t) = F(t) + EXA;
          B(t) = B(t) - 1;
          trades = trades +1;
    end

    if (F(t)>=EXB && B(s)>=1 && (Gs(1)*EXB+Gs(2)*(-1))>=0
&& (Gt(1)*(-EXB)+Gt(2)*1)>=0)
        F(s) = F(s) + EXB;
```

```
        B(s) = B(s) - 1;
        F(t) = F(t) - EXB;
        B(t) = B(t) + 1;
        trades = trades +1;
    end

    %Utility Function

    for i=1:n

        U(i) = 1-exp(-a*F(i)-b*B(i));

    end

    %Happiness Function

    for k=1:n

        H(k) = U(k) + x*(M(k)-(Pf(k)*F(k)+Pb(k)*B(k)));

    end

    Average_Happiness(1+Trade_Attempts) = dot(H,V)/n;

end

% Trading is over

%-----------------------------------------------------------

% Final happiness information

Average_Happiness';

Ending_Happiness = H';

Variance_of_Happiness = var(H);

%-----------------------------------------------------------

%Finding information about the degree

degree = A*V';
Variance_of_degree = var(degree);

%-----------------------------------------------------------

% Finding information about eigenvalue centrality
```

```
Eigenvalues = eig(A);
[V,D] = eig(A);
%# The maximum eigenvalue and its index
[maxValue,index] = max(diag(D));
%# The associated eigenvector in V
Eigenvalue_Centrality = V(:,index);

Variance_of_Eigenvalue_Centrality =
var(Eigenvalue_Centrality);

%-----------------------------------------------------------

% Finding information about clustering coefficients

number_of_three_step_paths = A^3;

for cc=1:n

    Clustering_Coefficient(cc) =
(number_of_three_step_paths(cc,cc))/((degree(cc))*(degree(c
c)-1));

end

Clustering_Values = Clustering_Coefficient';

Variance_of_Clustering_Coefficients =
var(Clustering_Values);
```

References

Aesop. “The Hawk and The Nightingale.” Aesop’s Fables.

Asparouhova, Elena; Bossaerts, Peter. “Modeling price pressure in financial markets.” Journal of Economic Behavior and Organization 72, 119-130.

Barabasi, Albert-Laszlo; Bonabeau, Eric. “Scale-Free Networks.” Scientific American. 288 (5): 60-9. May 2003.

Borgatti, S.P.; Everett, M.G; Freeman, L.C. Ucinet for Windows: Software for Social Network Analysis. Harvard, MA: Analytic Technologies. 2002

Castle, Christian J.E; Crooks, Andrew T. “Principles and Concepts of Agent-Based Modeling for Developing Geospatial Simulations.” UCL Working Papers Series. Paper 110. Pages. 12-15. September, 2006.

Crockett, Sean. “Learning competitive equilibrium in laboratory exchange economies.” Economic Theory 34. Pages 157-180. 2008.

Crockett, Sean; Spear, Stephen; Sunder, Shyam. “Learning competitive equilibrium.” Journal of Mathematical Economics 44. Pages 651-671. 2008.

Erdos, Paul; Renyi, Alfred. “On the evolution of random graphs.” Publications of the Mathematical Institute of the Hungarian Academy of Sciences 5. Pages 17-61. 1960.

Fowler, H. James; Christakis, Nicholas A. “Dynamic spread of happiness in a large social network: longitudinal analysis over 20 years in the Framingham Heart Study.” British Medical Journal. September 10, 2008.

Kultti, Klaus. “A Model of random matching and price formation.” European Economic Review 44. Pages 1841-1856. 2000.

Markose, Sheri; Arifovic, Jasmina; Sunder, Shyam. "Advances in experimental and Agent-based modeling: Assets markets, economic networks, computational mechanism design, and evolutionary game dynamics." Journal of Economic Dynamics and Control 31. Pages 1801-1807. 2007.

Newman, M. E. J. "Networks: An Introduction." Oxford: Oxford University Press. 2010.

Nicholson, Walter; Snyder, Chritopher. Microeconomic Theory Basic Principle and Extentions. Tenth Edition. Chapter 13. Thomson South-Western. 2008.

Perloff, M. Jeffrey. Microeconomics (The Pearson Series in Economics). Edition 6. Prentice Hall. January 13, 2011.

Tore Opsahl. "Clustering in Two-mode Networks." Conference and Workshop on Two-Mode Social Analysis. September 30 – October 2, 2009.

Smith, Adam. "An Inquiry into the Nature and Causes of the Wealth of Nation." London: Methuen and Co., Ltd., ed. Edwin Cannan, 1904. Fifth edition. Book 1, Chapter 7, para 9.

Vignes, Annick; Etienne, Jean-Michel. "Price formation on the Marseille fish market: Evidence from a network analysis." Journal of Economic Behavior & Organization. Volume 80, Issue 1. Pages 50-67. 2011.