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Determining Photoconductor
Mobilities

Carol Panepinto
August 2005

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Determining Photoconductor
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A Numerical Method for Determining Photoconductor
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By

Carol Panepinto

Report submitted for final fulfillment of
requirements for Masters Degree in Applied and
Industrial Math

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August 2005

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ABSTRACT

A photoconductor's mobility is a measure of the speed at which electrons migrate through the material under the influence of an electric field. The mobility determines how long a packet of charge takes to go through the photoconductor. It also determines how much and in what manner the E-field changes during the packet transit. The problem in which we are interested is inferring the mobility of a photoconductor from time-of-flight measurements, that is, from measurements of the current produced per unit time by a known charge packet. Mathematically, the problem is an initial-boundary value problem for a nonlinear, non-local, hyperbolic conservation law that characterizes the E-field in the photoconductor. In this paper, we discuss the mathematical formulation of this problem, its solution using the method of characteristics, and the application of the solution to the problem of inferring mobilities.

INTRODUCTION

In this project, we are interested in finding the mobility of a specific photoconductor material. Mobility is a property that indicates how easily electrons flow within a photoconductor. Mobility is velocity/E-field, whose units are $\text{cm}^2/\text{V-sec}$. Because of the range of values in this project, the units are in $\mu\text{m}^2/\text{V-msec}$. A photoconductor is a material that conducts charge when light is present but acts as a dielectric or insulator in the dark.

Electric field and charge density inside the photoconductor can be represented by a partial differential equation that depends on mobility, and mobility itself depends on electric field. By solving the PDE for electric field for different mobilities, and by comparing to measured transit time and current, we can infer the mobility of a particular material.

GLOSSARY OF VARIABLES AND THEIR UNITS

β Beta

Constant used in calculation of mobility, unique to the material

Units: $\sqrt{\mu m/V}$

Typical value for this project: $.0015\sqrt{\mu m/V}$

Charge Density

Measure of charges per unit length in this project. Since the charges form a sheet inside the photoconductor initially, and the direction of travel is in only one dimension, the units are charges per length and not volume.

Units: coulombs/micron

Range for photoconductors: 10^2 to 10^3 coulombs /cm or .01 coulombs /micron to .1 coulombs /micron

$C(t)$

$\frac{\partial E(x,t)}{\partial t}$, for all x. Net rate of increase in E-field relative to time.

Units: V/micron-msec

Current Density j

Amount of current produced at the charge side of the material per unit time. The measured current occurs when the charges reach the far side of the material. Current density can be measured and is proportional to the charge density at the charge side.

Units: coulombs/micron-msec

E-field or E

Measure of voltage drop per unit length in this project

Units: Volts/micron

Range for photoconductors: If the initial voltage is 500V over 25 microns, the E-field before light injection is 20 volts/micron.

ϵ Relative Permittivity

A measure of the ability of a material to resist the formation of an electric field within it.

Typical value for photoconductor: Relative permittivity = $3\epsilon_0$;

ϵ_0 = permittivity in free space = 8.85×10^{-12} (F/m).

L or length

Measure of the width of photoconductor material or length that the charge packet must travel

Units: microns

Range for photoconductor: 25-30 microns

μ Mobility

Measure of how easily electrons travel through a material under the influence of an electric field

Units: $\text{cm}^2/\text{V-sec}$

Range for photoconductors: $10^{-5}\text{cm}^2/\text{volt-second}$ or $1 \text{ microns}^2/\text{volt-msec}$.

μ_0 Mobility Value for Material

Starting value for mobility which is unique to the material, and independent of E-field

Units: $\text{microns}^2/\text{V-msec}$

Typical value for photoconductor: $.0001\text{cm}^2/\text{volt-sec}$ or $1\text{microns}^2/\text{V-msec}$

Transit Time

The time it takes the charge packet to travel across the photoconductor.

Units: $L/(\mu(E(x,t))E(x,t))=L^2/(\mu(E(x,t))V_0)$ = milliseconds

Typical time for this project: Less than 300 msec usually 1-5 msec.

V or Voltage

Measure of charge potential

Units: Volts or millivolts

Typical Initial Voltage for this project: 500V

PROBLEM DEFINITION AND PHYSICAL DESCRIPTION

Figure 1 is a schematic of a photoconductor that has a constant voltage at one plate and is grounded at the other plate. The material receives energy from a light flash. As a result of the energy input, a sheet of electrons is injected at the ground side of the material. Initially the sheet of electrons defines the charge density inside the material as a delta function at the ground side.

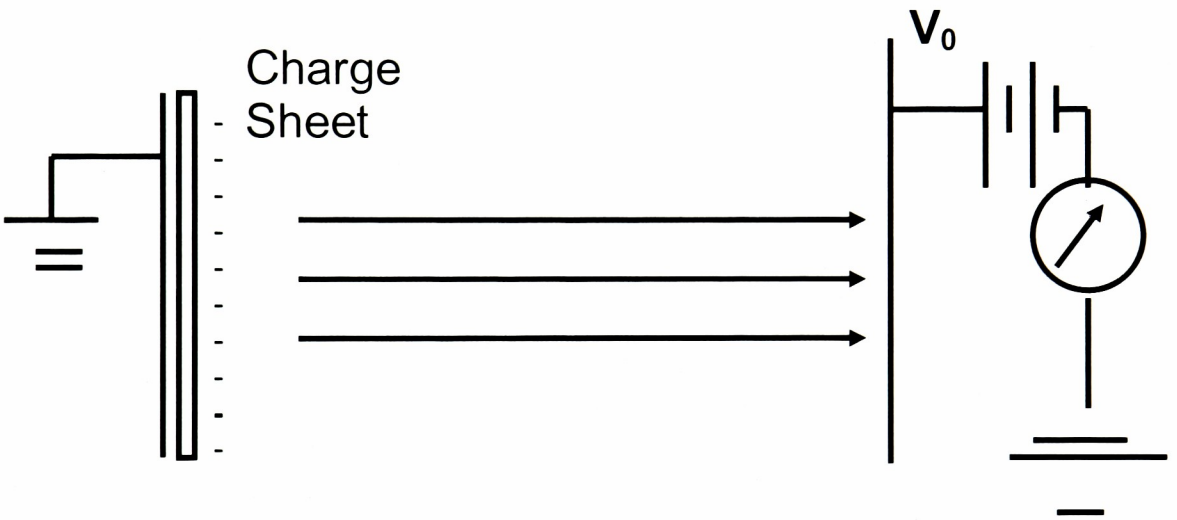


FIG 1: *Photoconductor just after light injects charge*

The charge sheet is driven across the material by the electric field between the two plates. As the electrons travel across the material, they spread apart due to mutual electrostatic repulsion. The charge density in charges/length, denoted by $n(x,t)$, decreases as the sheet moves and broadens, since the electrons become less tightly packed. Since the time of flight is short enough that recombinations do not occur, the same amount of electrons are occupying a longer length, resulting in a lower charge density. (Nelson [1])

Figure 2 illustrates the charge packet widening as it moves across the material. Since the total number of charges in the packet remains constant while inside the material, the charge density, $n(x,t)$, decreases over time.

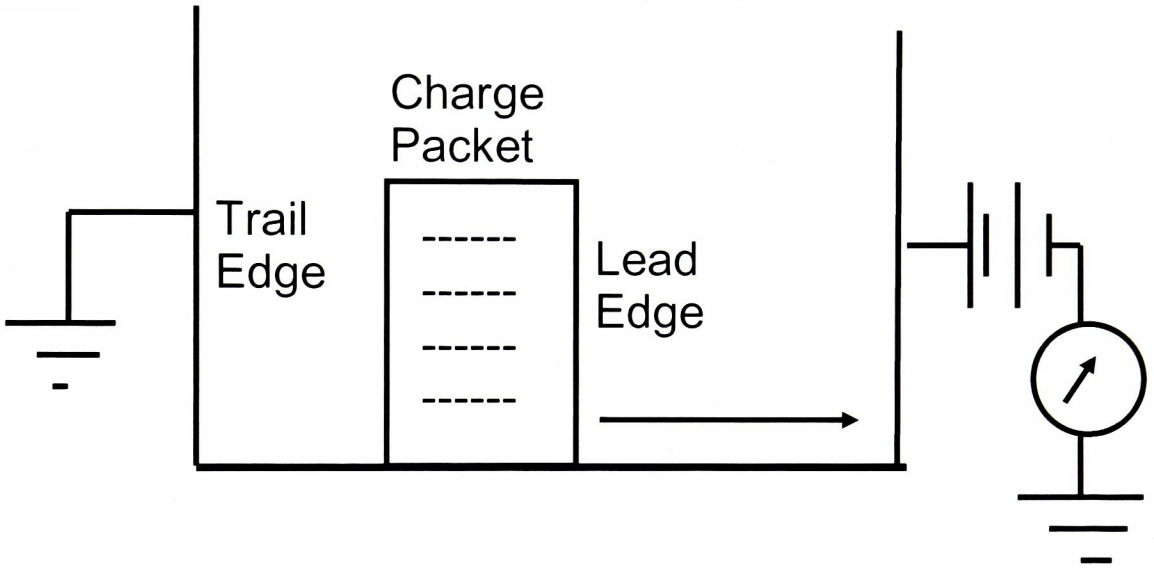


Figure 2: Photoconductor after charge injected at some later time when charge packet has moved and broadened.

In a standard xerographic printer system, the far side of the photoconductor initially has a uniform voltage and light is introduced only in the areas where an image is desired. The voltage is not held constant at the far side. When the injected charge reaches the far side, the areas corresponding to the image become neutral, or less positive. Positively charged toner is introduced and is attracted to these less positive areas. The toner is deposited in these areas and forms an image on the photoconductor. Paper is then placed in proximity to the

photoconductor and by introducing a negative charge to the back of the paper, the toner then gets drawn away from the photoconductor and is deposited on the paper in the desired image areas. After heating the paper and toner, usually under pressure, the toner melts on and into the paper and a permanent image is achieved.

ELECTRIC FIELD

For this one-dimensional problem, charge density, $n(x,t)$, is the spatial derivative of the E-field. Poisson's equation states that $\nabla^2 V = kn$. Since this is a one-dimensional problem, $\frac{\partial^2 V}{\partial x^2} = \frac{\partial E(x,t)}{\partial x} = kn(x,t)$, which is the one-dimensional version of Poisson's equation $\nabla^2 V = kn(x,t)$ with $k=1$.

E-field, which is the change in voltage/unit distance, is constant before energy is introduced to the material. After the sheet of electrons is injected, E-field becomes a step function at the ground side of the material and the original E-field is changed. Figure 3 illustrates this behavior.

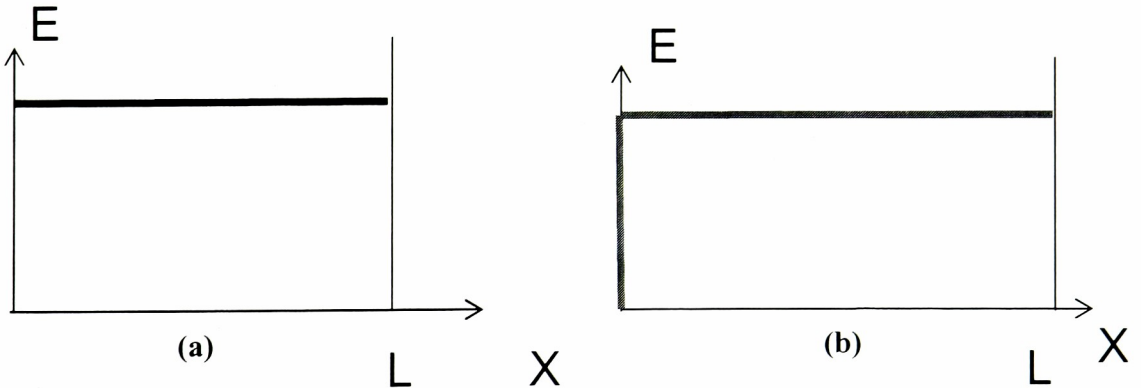


FIG 3: E -field before charge injection (a) and just after charge injection (b).

E -field is the spatial derivative of voltage, $\frac{\partial V}{\partial x} = E(x,t)$, and charge density is the spatial derivative of E -field,

$n(x,t) = \frac{E(x,t)}{\partial x}$. Before the light flash, the voltage drop across the photoconductor is constant 4(a) and so E-field is constant 4(b). There is no charge density. This relationship is shown in figure 4.

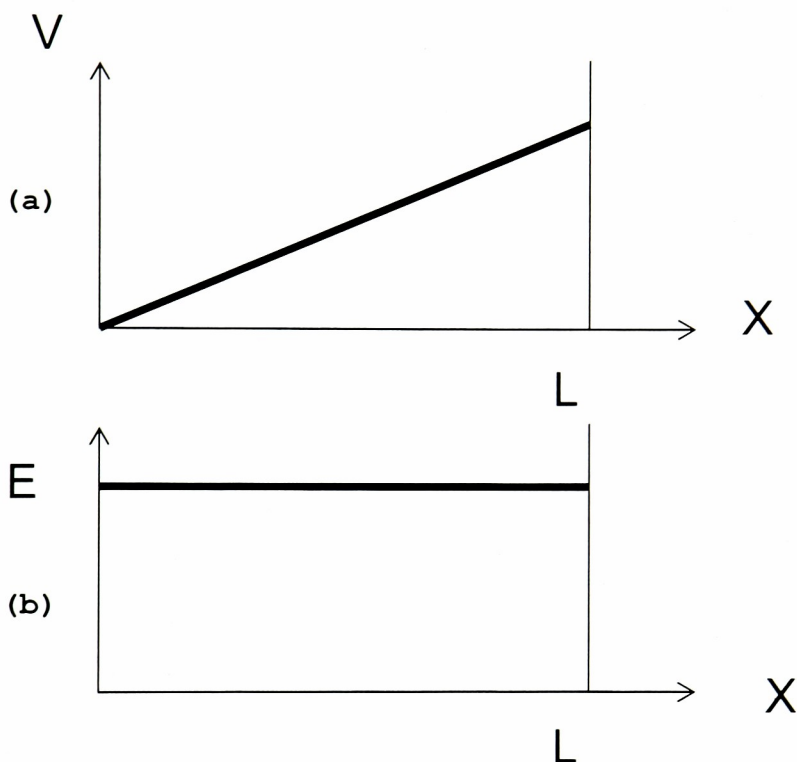


FIG 4: Voltage(a) and E-field (b) before charge injection.

After the electrons are injected and have traveled across the material for some time, the voltage drop is not a linear function anymore; it has become parabolic in the region of the charge as in figure 5(a). The E-field is changed; its graph in figure 5(b) has a constant slope in the region of the charge. Outside the charge area, the E-field is constant. Figure 5(c) shows the charges have

spread from the initial delta function and the charge density is lower.

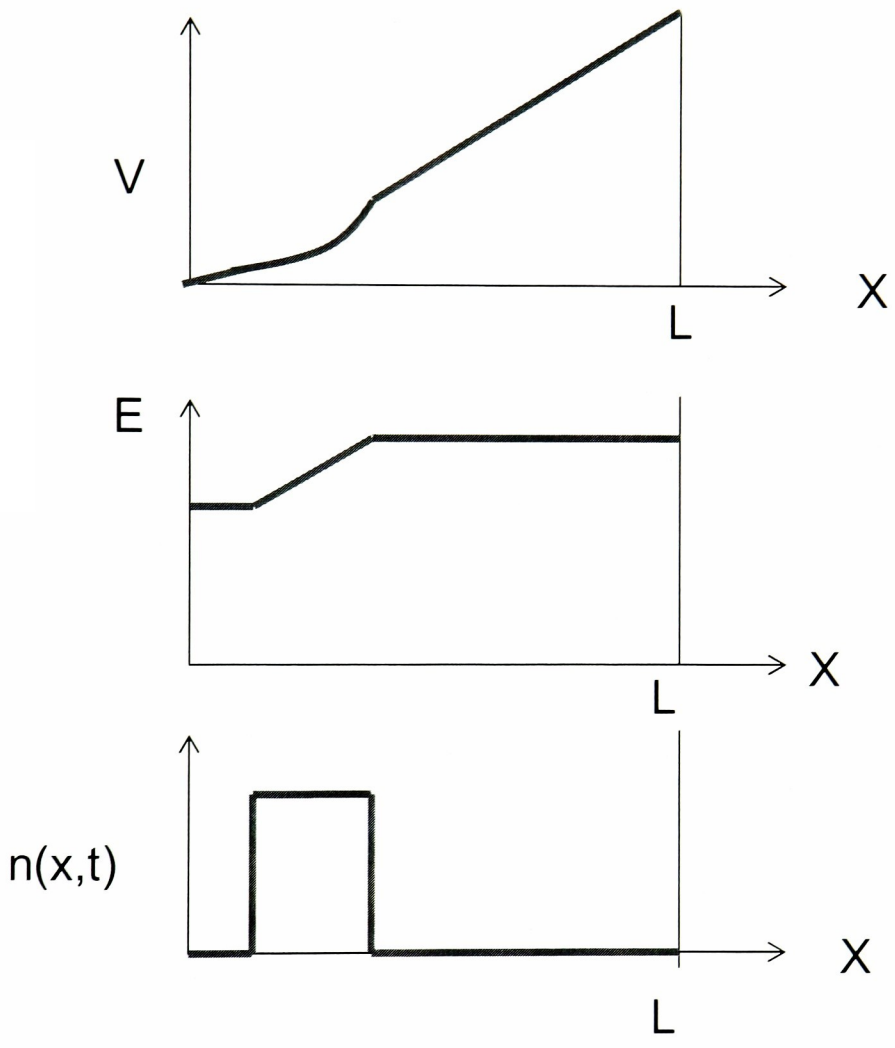


FIG 5: Voltage(a) E -field (b) and charge density (c) after charge injection.

Note that voltage is the integral of E-field with respect to position between the two plates of the photoconductor and is constant since the total change in voltage is constant. This relationship is represented by the equation

$$\int_0^L E(x,t)dx = V(t)$$

MATHEMATICAL DESCRIPTION

When the charges are contained within the photoconductor material, the number of charges between the plates is given by the equation:

$$\int_0^L n(x,t)dx = \text{number of charges}$$

The instantaneous change over time in the number of charges in a small region, Δx , is given by,

$$\frac{\partial}{\partial t}(n(x,t)\Delta x) = -(\mu(E(x,t))E(x,t)(n(x+\Delta x,t) - n(x,t)))$$

where $\mu(E(x,t))E(x,t)$ is the average velocity of the charges. Dividing both sides by Δx and rearranging, the result is the conservation law:

$$\frac{\partial n(x,t)}{\partial t} + \frac{\partial}{\partial x} \mu(E(x,t))E(x,t)n(x,t) = 0 \quad (1)$$

When some charges reach the far side of the photoconductor, an external current occurs. The total electrostatic force on a charge depends on the location and velocity of the charge. (Feynman [2]). The current density, \mathbf{j} in charge/mic²-msec, moves through a surface that is perpendicular to current flow.

Current density, \mathbf{j} , is equal to charge density times velocity, $n(x,t)\mathbf{v}$. Current is the integral of the normal component of flow through all the elements of the surface and is represented by

$$I = \int_S \mathbf{j} \cdot \mathbf{m} dS \quad (2)$$

where \mathbf{m} is the unit vector normal to the surface. If there is a net current out of a closed surface, the charge inside must decrease by the same amount because of the conservation of charge. Therefore

$$\int_s \mathbf{j} \cdot \mathbf{m} dS = -\frac{\partial q(\text{inside})}{\partial t} \quad (3)$$

The charge inside is equal to $\int_{\text{Length}} n dx$. Since $\mathbf{j} = n\mathbf{v}$ and the average drift velocity is constant, applying the del operator in the direction of current flow gives

$$\nabla \cdot \mathbf{j} = \nabla \cdot n\mathbf{v} = -\frac{\partial n}{\partial x} v = -\frac{\partial n}{\partial x} \frac{\partial x}{\partial t} = -\frac{\partial n}{\partial t} \quad \text{or} \quad v \frac{\partial n}{\partial x} = -\frac{\partial n}{\partial t}. \quad (4)$$

Velocity is the product of mobility and E-field; $\mathbf{v} = \mu(E(x,t))E(x,t)$. Rearranging terms yields the same conservation law as in the condition where all the charges are contained within the material:

$$\frac{\partial n(x,t)}{\partial t} + \frac{\partial}{\partial x} \mu(E(x,t))E(x,t)n(x,t) = 0$$

In Eq. (1), $\mu(E(x,t))$ is the mobility of the material, which can be thought of as velocity/unit E-field. The Poole-Frenkel equation states $\mu(E(x,t)) = \mu_0 e^{\beta \sqrt{E(x,t)}}$, where μ_0 and β are constants that are unique to the material. (Borsenberger [11])

Since $\frac{\partial E(x,t)}{\partial x} = n(x,t)$, after integrating both sides of Eq.(1) from zero to x with respect to x , the equation can be re-written as

$$\frac{\partial E(x,t)}{\partial t} + \mu(E(x,t))E(x,t) \frac{\partial E(x,t)}{\partial x} = c(t) \quad (5)$$

This equation can be integrated term-by-term with respect to x from 0 to L , where L is the width of the photoconductor material:

$$\frac{\partial}{\partial t} \int_0^L E(x,t) dx + \int_0^L \mu(E(x,t)) E(x,t) \frac{\partial E}{\partial x} dx = c(t)L \quad (6)$$

Since the integral of $E(x,t)$ with respect to x is $V(t)$, or voltage, the equation then becomes:

$$\frac{\partial}{\partial t} V(t) + \int_0^L \mu(E(x,t)) E(x,t) \frac{\partial E}{\partial x} dx = Lc(t) \quad (7)$$

Since voltage does not change with time the first term drops out and the equation then reduces to:

$$\int_0^L \mu(E(x,t)) E(x,t) \frac{\partial E}{\partial x} dx = Lc(t) \quad (8)$$

Equation (5) is similar to a standard initial boundary value problem for a hyperbolic conservation law:

$$\frac{\partial E(x,t)}{\partial t} + \mu(E(x,t)) E(x,t) \frac{\partial E(x,t)}{\partial x} = 0, \quad 0 \leq t, \quad 0 \leq x \quad (9)$$

with the initial condition:

$$E(x,0) = g(x)$$

and the boundary condition:

$$\frac{\partial E}{\partial t}(0,t) = 0.$$

MOTIVATION FOR USING

The problem for this project is similar to the above but has an extra unknown, $c(t)$, and an extra non-local condition.

The result is this problem, which appears to be well posed:

$$\frac{\partial E(x,t)}{\partial t} + \mu(E(x,t))E(x,t) \frac{\partial E(x,t)}{\partial x} = c(t), \quad 0 \leq t, \quad 0 \leq x$$

with initial condition:

$$E(x,0) = g(x)$$

and the boundary condition:

$$\frac{\partial E}{\partial t}(0,t) = c(t)$$

and the extra non-local condition:

$$\int_0^L E(x,t) dx = V(t).$$

MOTIVATION FOR USING METHOD OF CHARACTERISTICS

This problem is best solved using the method of characteristics. Finite difference and finite element methods work well with smooth functions, but this function is not smooth. Finite difference and finite element methods would set up a grid for time and space and try to break the wave into spatial increments and treat the entire E-field as a function of position at each time step. Because the initial E-field is a step function, no matter how large or small the pieces of E-field are sliced, Δx is zero initially. The derivative of E-field with respect to position using these initial values is undefined and charge density is a delta function. Δx changes over time as the charges spread out and cannot be quantified. The method of characteristics uses the change in x with respect to time to determine characteristic curves. Then along the curves, E-field changes linearly with respect to time. The error using finite difference and finite element methods is proportional to

$$\frac{\partial^2 V}{\partial x^2} = \frac{\partial n}{\partial x} = \text{charge density.}$$

Initially, charge density is a delta function at the ground plate, which would make the error large and the error bound infinite using finite difference or finite element methods. The method of characteristics looks at pieces of charge only within the charge packet, on the characteristics, and solves each section independently over time. Outside of the charge packet we know that the charge density is zero and the E-field is constant. Therefore we only need to solve for E-field in the area corresponding to the charge packet.

METHOD OF CHARACTERISTICS: MATHEMATICAL DETAILS

By using the method of characteristics, we reduce the original partial differential equation to a large system of ordinary differential equations with t as the independent variable. We define characteristic curves, along which the E-field changes only with respect to time. The slopes of these characteristic curves equal the velocities of the charges as they travel through the material. On the characteristic curves the PDE for E-field reduces to an ODE.

In equation 8 we solved for $c(t)$:

$$c(t) = \frac{1}{L} \left[\int_0^L \mu_0 e^{\beta \sqrt{E}} E \frac{\partial E}{\partial x} dx \right] = \frac{1}{L} \left[\mu_0 \int_{E(x=0)}^{E(x=L)} E e^{\beta \sqrt{E}} dE \right]$$

If we make the substitution,

$$u = \sqrt{E}$$

the equation becomes

$$c(t) = \frac{1}{L} \left[\mu_0 \int 2u^3 e^{\beta u} du \right] \quad (10)$$

Integration by parts results in

$$c(t) = \frac{\mu_0}{L} \left[\frac{2}{\beta^3} e^{\beta u} \left\{ \frac{-6}{\beta} + 6u - 3\beta u^2 + \beta^2 u^3 \right\} \right] = \frac{\mu_0}{L} \left[\frac{2}{\beta^3} e^{\beta \sqrt{E}} \left\{ \frac{-6}{\beta} + 6u - 3\beta u^2 + \beta^2 u^3 \right\} \right] \quad (11)$$

We then define the characteristic curves by,

$$\frac{dx}{dt} = \mu E \quad (12)$$

Along these characteristic curves,

$$\frac{dE}{dt} = \frac{\partial E}{\partial t} + \frac{dx}{dt} \frac{\partial E}{\partial x} = \frac{\partial E}{\partial t} + \mu E \frac{\partial E}{\partial x} = c(t) \quad (13)$$

Then along such a curve,

$$\frac{dE}{dt} = c(t) = \frac{1}{L} \left[\frac{2}{\beta^3} e^{\beta\sqrt{E}} \left\{ \frac{-6}{\beta} + 6u - 3\beta u^2 + \beta^2 u^3 \right\} \right] \bigg|_{E(\text{tail})}^{E(\text{lead})} \quad (14)$$

The problem defines a rarefaction wave; the characteristic curves do not intersect because their slopes increase as a function of mobility and E-field.

The system of differential equations to be solved is then:

$$\left\{ \begin{array}{l} \frac{dx_i}{dt} = \mu E_i = \mu_0 e^{\beta\sqrt{E_i}} E_i \\ \frac{dE_i}{dt} = \begin{cases} \frac{1}{L} \frac{2}{\beta^3} e^{\beta\sqrt{E}} \left(-\frac{6}{\beta} + 6\sqrt{E} - 3\beta E + \beta^2 \sqrt{E}^3 \right) \bigg|_{E(x=0)}^{E(x=L)}, & 0 \leq x \leq L \\ \frac{\mu_0}{L} \frac{2}{\beta^3} e^{\beta\sqrt{E}} \left(-\frac{6}{\beta} + 6\sqrt{E} - 3\beta E + \beta^2 \sqrt{E}^3 \right) \bigg|_{E(x=0)}^{E(x=k-1)} + \\ \quad \frac{\mu_0}{L} e^{\beta\sqrt{E_k}} (E_k - E_{k-1}) \left(\frac{L - x_k}{x_k - x_{k-1}} \right), & x_k > L \end{cases} \\ \frac{\partial E_i}{\partial t}(0, t) = c(t) \\ E_i(x, 0) = V(t) = \frac{V_L - V_0}{L} \\ \int_0^L E(x, t) dx = V(t) = \frac{V_L - V_0}{L} \end{array} \right.$$

SOLUTION

To solve the system of ODEs, we used Matlab's ODE45 Solver, which integrates the individual differential equations over time simultaneously. This solver uses a Runge-Kutta 4-5 method with variable time step. The output was set for evenly spaced time intervals.

By solving for E-field over time with a range of mobility values, results can be compared to measured current and time of flight to determine the mobility that matches the measured results.

In this project, the parameters varied were: amount of initial E-field displacement caused by charge injection, and the mobility constants μ_0 and β . The program was run initially with nominal values of $\mu_0=(1.0\text{micron}^2/\text{volt-msec})$ and $\beta=(0.0015\sqrt{\mu m/V})$, and an E-field displacement from 20V/micron to 0V/micron at the ground side. The ranges for the variable changes are shown in Table 1:

Sample	Initial E-high V/mic	Initial E-low (V/mic)	μ_0 (micron ² /volt- msec)	β ($\sqrt{\mu m/V}$)
1	20	0	1.0	0.0015
2	20	10	1.0	0.0015
3	20	0	1.0	0.02
4	20	0	1.2	0.0015

Table 1: Sample variable combinations investegated

Mobility was increased slightly by changing β from 0.0015 to 0.02 with very little change in the duration of time of flight or the magnitude of E-field during that time. Much larger changes in β would be necessary change mobility enough to see much of a difference in behavior. μ_0 was then changed from 1.0 to 1.2, creating a larger change in mobility, and creating a greater change in response. The time of flight decreased as expected with higher mobility, and the charges spread more as their movement within the material was less restricted.

See **Appendix A** for sample results.

RESULTS & CONCLUSIONS

Using the method of characteristics, the initial partial differential equation was reduced to a system of ordinary differential equations. This system of ODEs was then solved numerically using Matlab's ode45 solver.

As expected, when mobility values increase, the time it takes the charge packet to reach the far side of the photoconductor is reduced. As more energy is introduced resulting in a larger disturbance of E-field, the time for the packet to travel to the other side increases. When less energy is introduced, the packet takes less time to travel across, and does not broaden as much since there are fewer electrons injected.

SUMMARY

For this problem E-field can be determined at any position and time by solving the PDE with the method of characteristics. Mobility can be inferred by looking at a range of solutions for various mobilities and finding a match with measured experimental values. Further research on this problem might include:

- Looking at materials that are not homogeneous
- Looking at the problem in 2 or 3 dimensions

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APPENDIX A SAMPLE RESULTS

SAMPLE 1

Nominal Conditions

Results with $\mu_0 = 1 \text{ microns}^2/\text{V-msec}$ and $\beta = .0015$

Initial E field after charge injection 0-20 V/msec

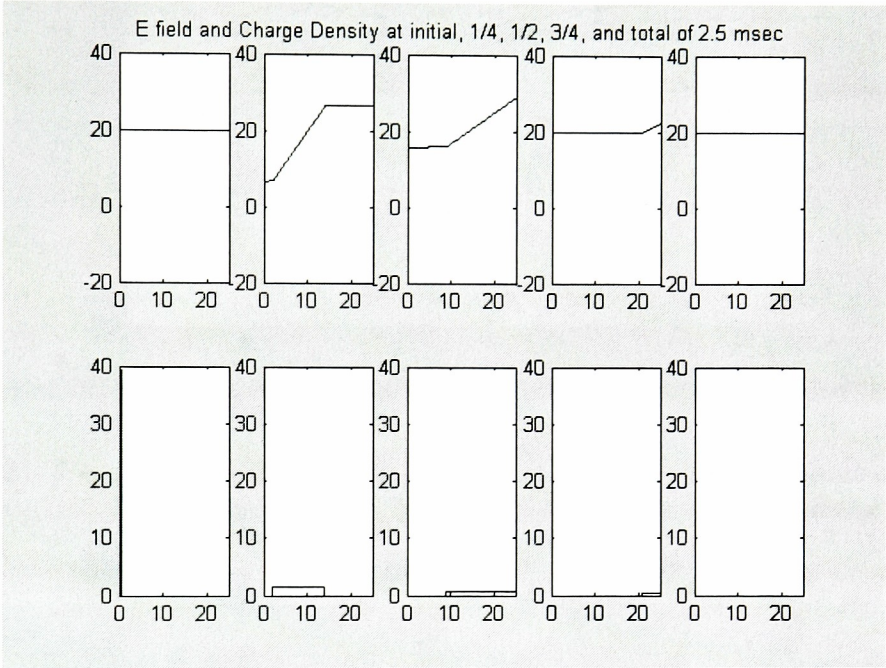


FIG. A1: *E-field and Charge density at several time steps as charge travels*

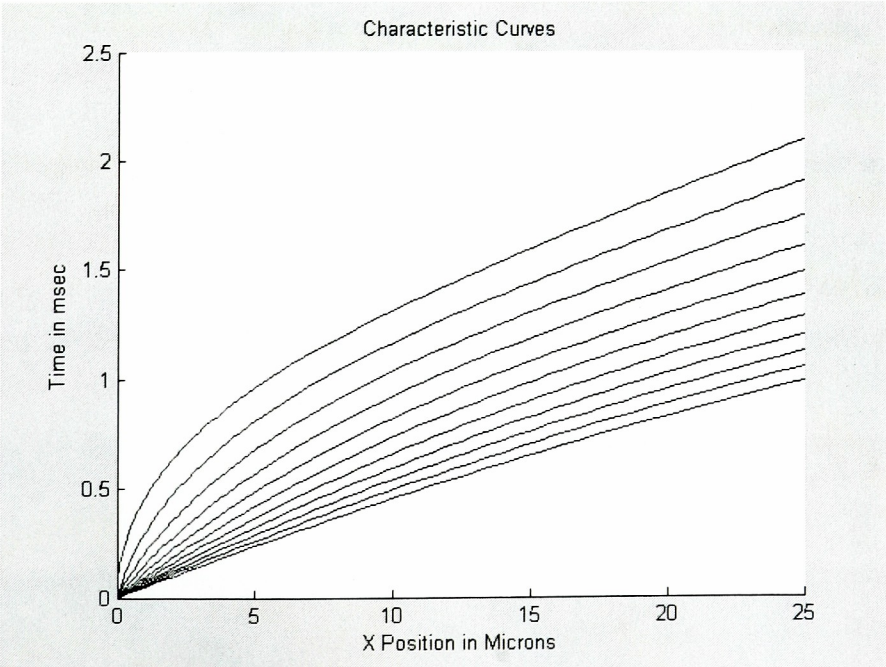


FIG. A1.1: *The characteristic curves form a rarefaction fan.*

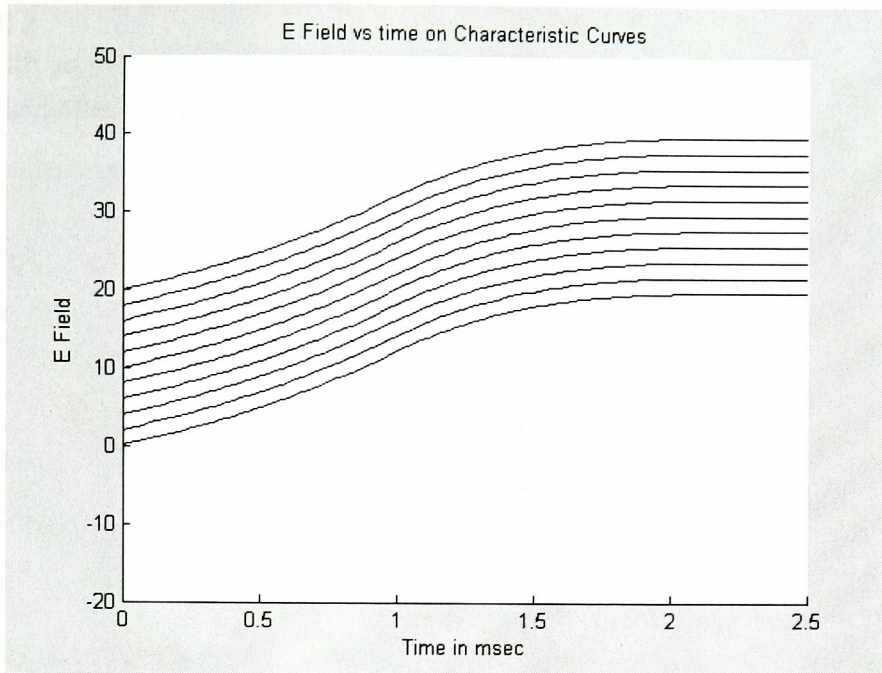


FIG. A1.2: *The change in E-field as a function of time is a constant which depends on time. Therefore, at each time step the change in E-field is the same, resulting in parallel curves.*

SAMPLE 2

Results with $\mu_0 = 1 \text{ microns}^2/\text{V-msec}$ and $\beta = .0015$

Initial E field after charge injection 10- 20V/msec

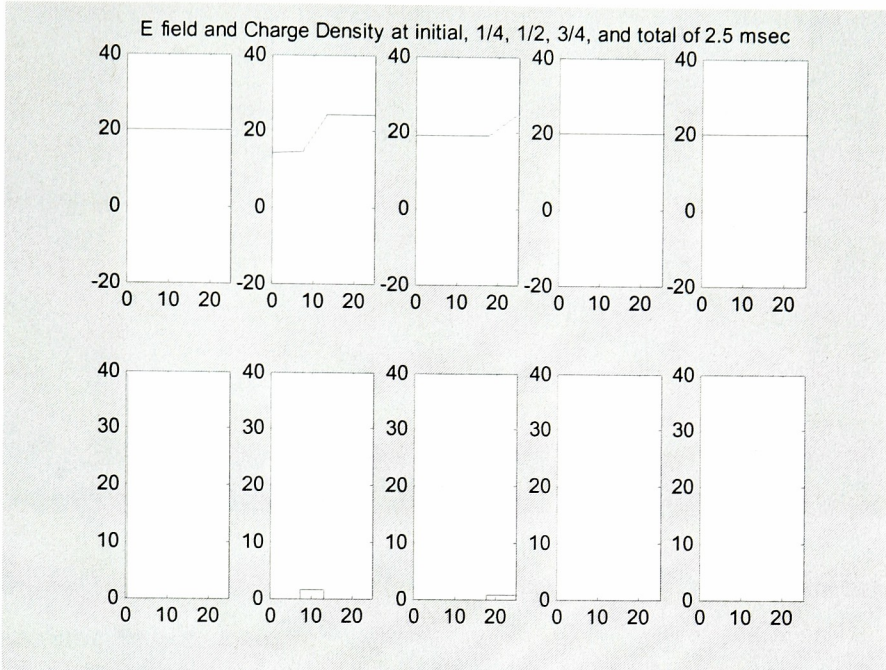


FIG A2: Less disturbance of initial E-field results in a shorter time for charges to move through the material than FIG. A1. Mobility coefficients are same as FIG A1

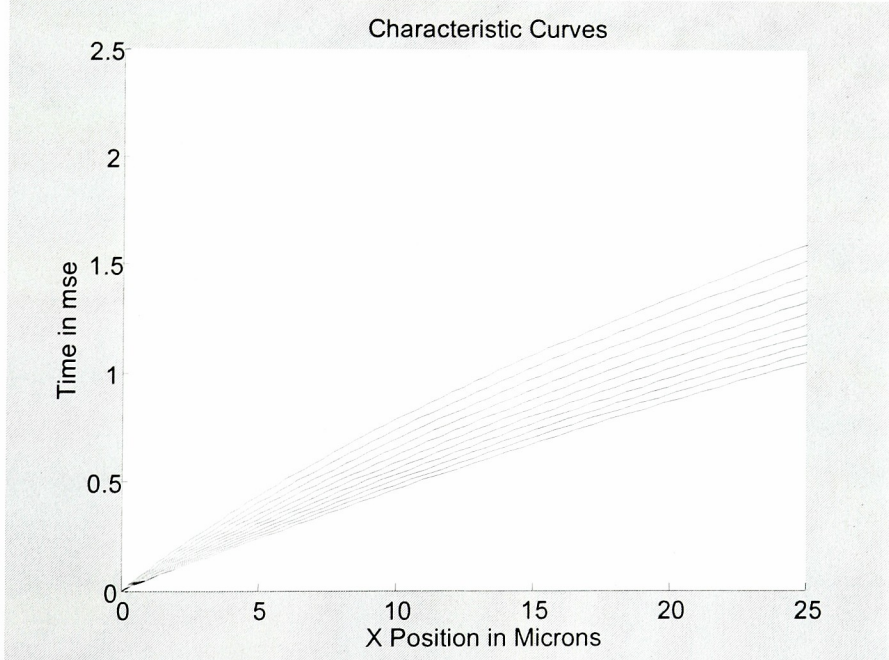


FIG. A2.1: With less disturbance in the initial E-field, the characteristic curves become shallower because position is changing more in the same time span.

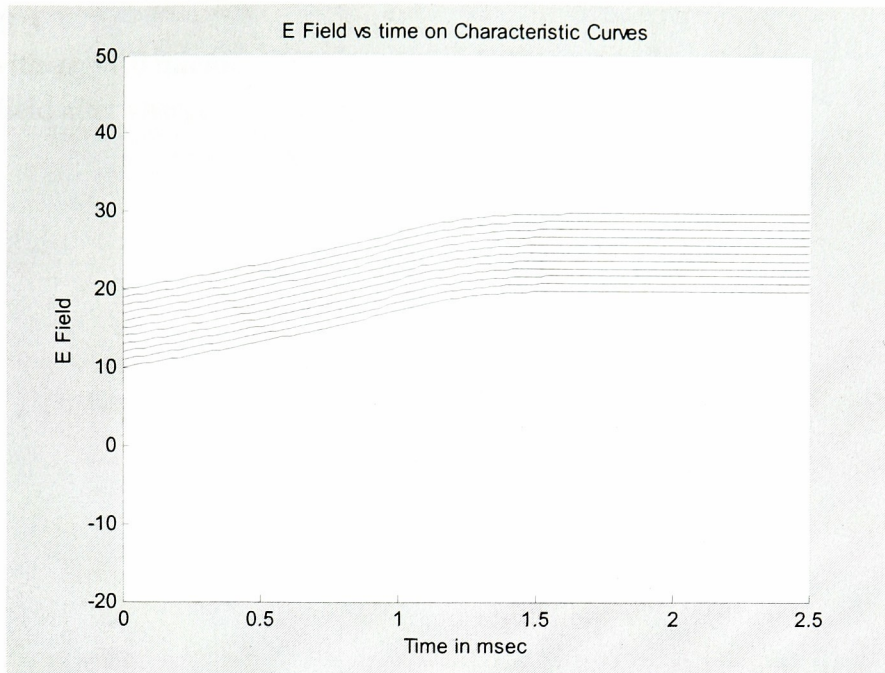


FIG. A2.2: *E-field changes less as in time since there is less initial disturbance and the charges reach the far side sooner.*

SAMPLE 3

Results with $\mu_0=1.0 \text{ microns}^2/\text{V-msec}$ and $\beta=0.02$

Initial E field after charge injection 0–20 V/msec

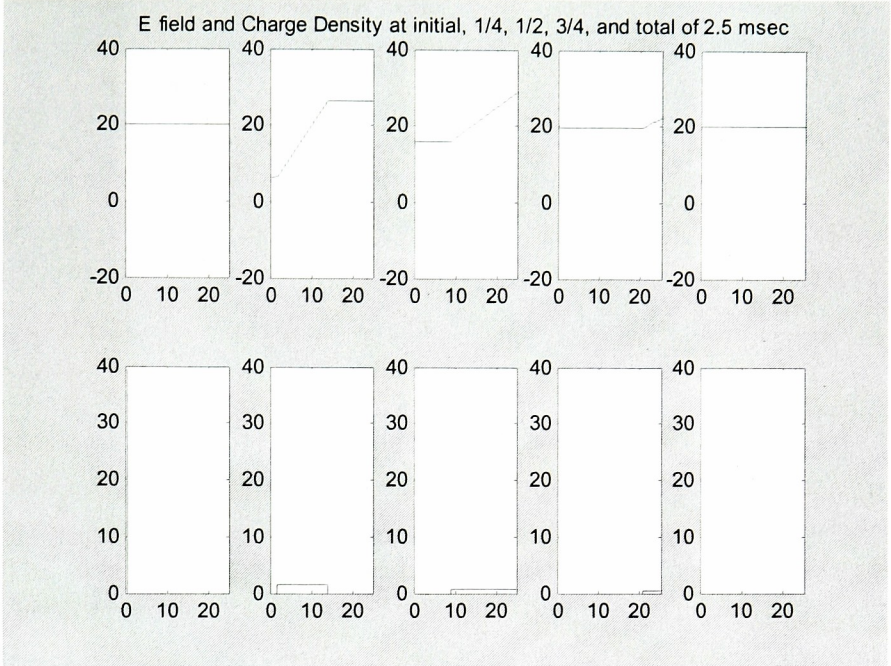


FIG A3: Results are almost the same as FIG A1 since mobility is not changed much by this change in β

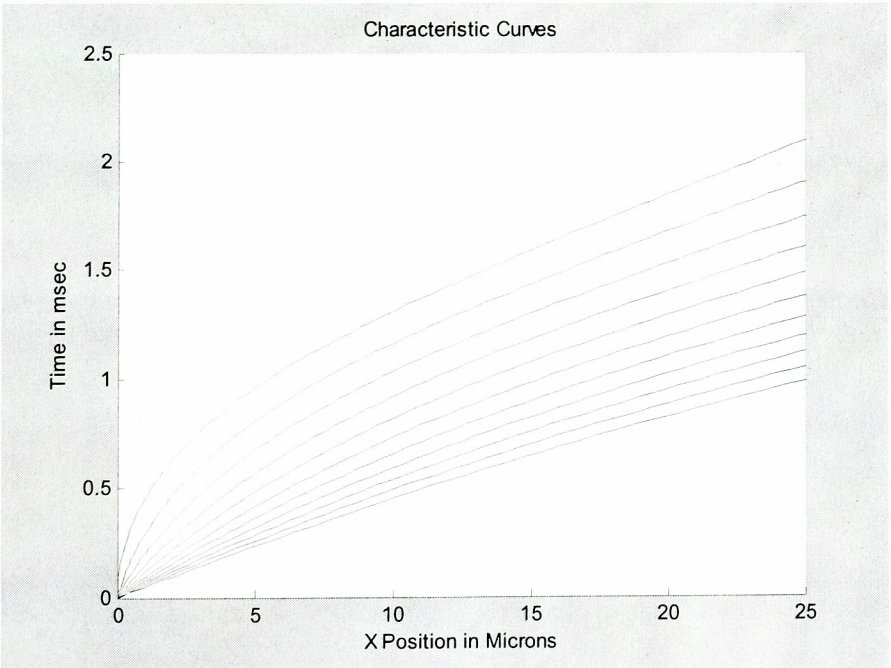


FIG A3.1: Rarefaction fan is similar to nominal condition fan because change in mobility is small

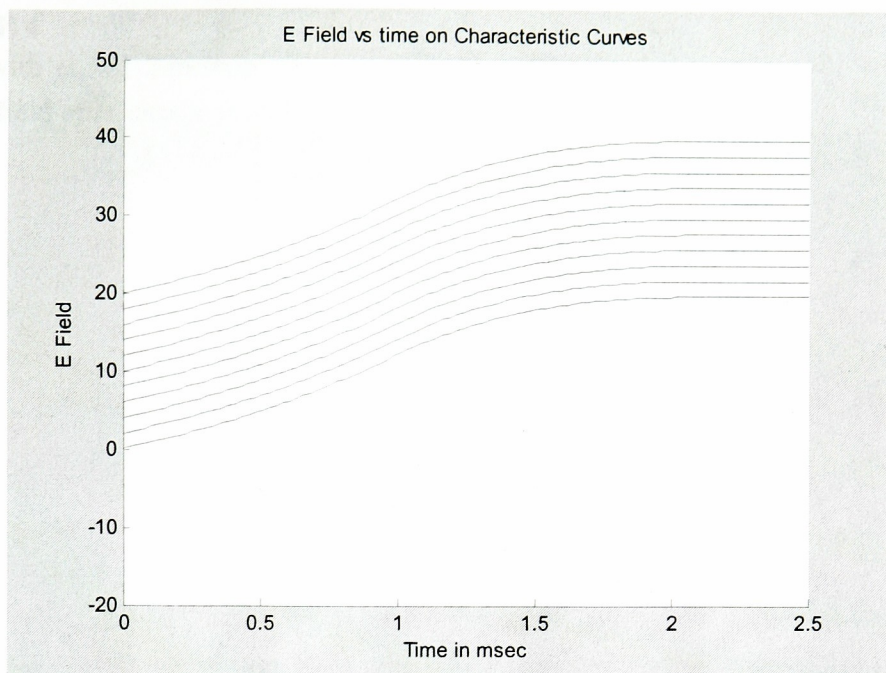


FIG A3.2: *E field shows similar change as a function of time as nominal conditions since mobility change is small*

SAMPLE 4

Results with $\mu_0 = 1.2 \text{ microns}^2/\text{V-msec}$ and $\beta = .0015$

Initial E field after charge injection 0-20V/msec

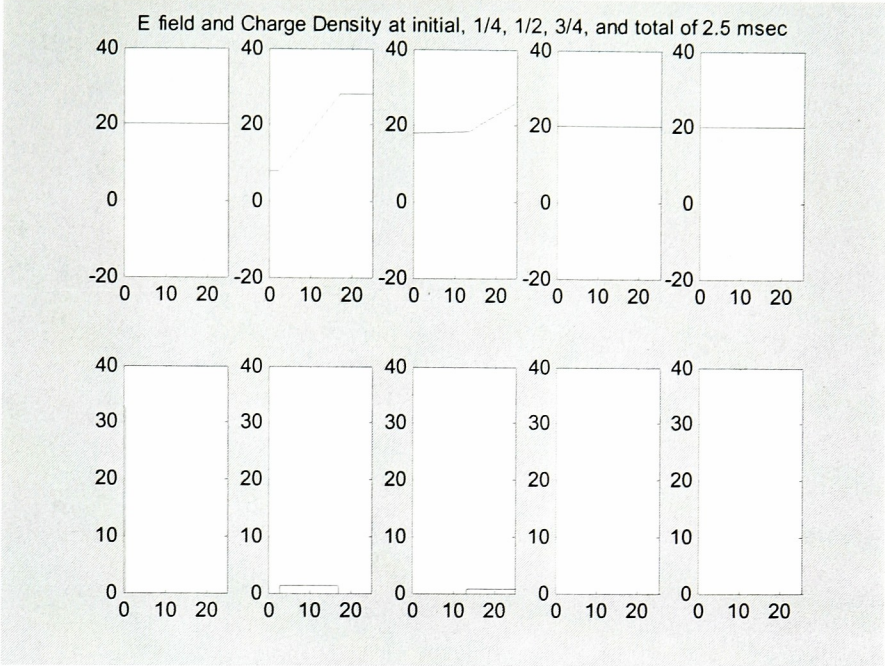


FIG. A4: Mobility increased by changing μ_0 from 1.0 to 1.2

Otherwise initial conditions are the same as FIG. A1. Increase in mobility results in less time for charge to reach the far side of the photoconductor. Charges spread more traveling across material

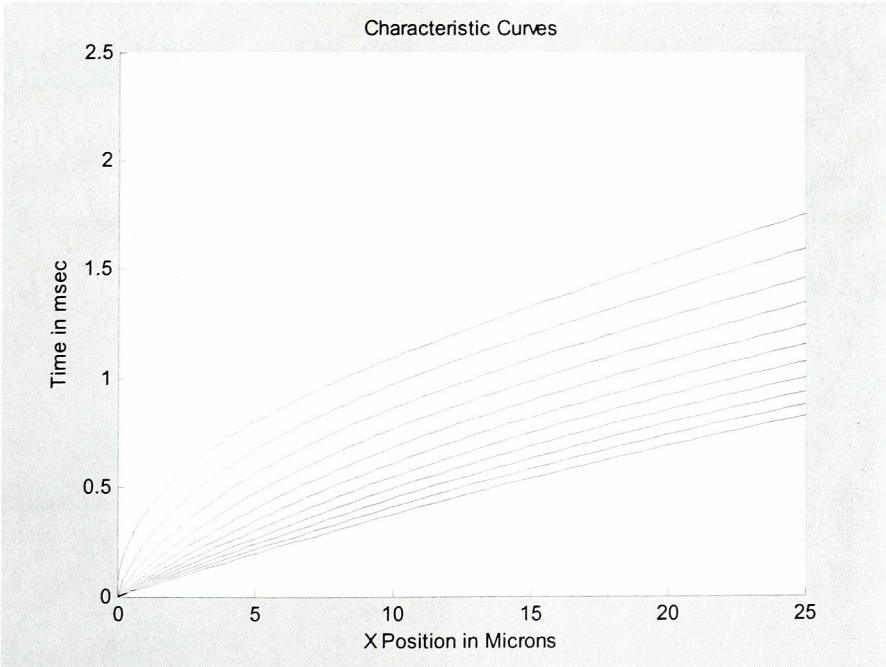


FIG. A4.1: Characteristic curves are shallower with higher mobility due to increased drift velocity.

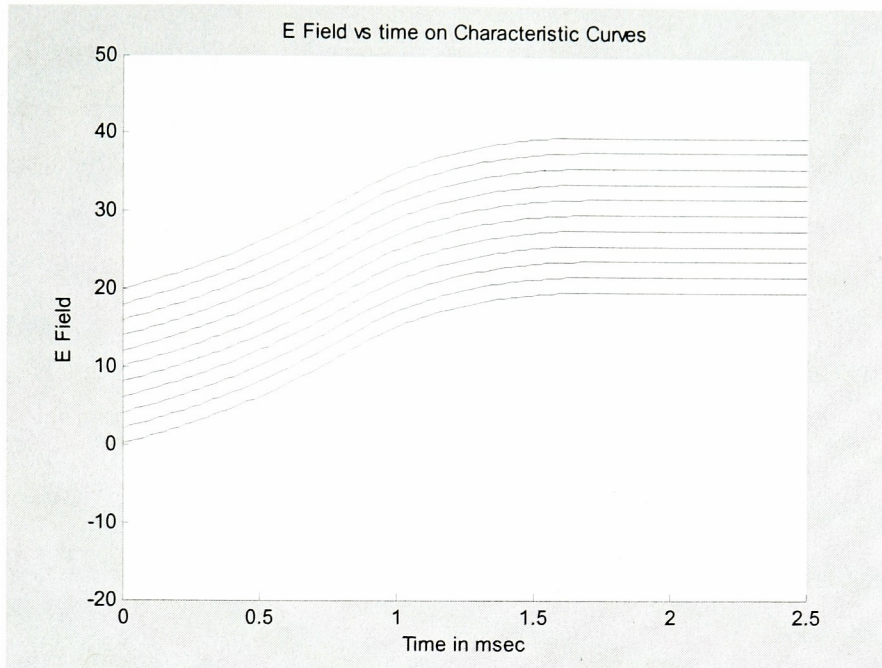


FIG. A4.2: *E-field changes faster with respect to time due to increase in mobility.*

APPENDIX B MATLAB CODE

Matlab's ODE solvers require an initial condition, the type of solver to be used, the variable of integration, and a function that defines the differential equations to be solved. Since this problem is looking at pieces of the charge sheet over time, each piece is solved for individually.

Before the charge sheet is generated, the E-field is constant and is equal to the known voltage divided by the photoconductor width. After the light flash, the E-field is disturbed at the ground-side of the material, proportionally to the energy introduced. For initial values, there are a set of zeros representing the ground position for each piece of charge and a set of evenly spaced values for E-field spanning the range achieved with a known energy introduced. The initial values for the solver are stored in an array containing values of zero for x and a range of corresponding E values.

The solution at each time step is a row in the solution matrix containing the next set of x values and E values. After the initial condition, the E-field between the ground side and the trail edge of the charge packet is the same as the value at the trailing edge. Before the leading edge of the packet reaches the charge side of the material, the E-field between the leading edge and the charge side is the same as the value at the leading edge.

When the trailing edge of the packet reaches the charge side, all the charges are neutralized and the E-field is back where it started before the introduction of energy. Charge density is calculated by numerically taking the derivative of E with respect to x.

The first file, **proj.m**, is the code to run the program. It calls the ODE solver, creates initial values, and generates the output graphs. The second file, **Projsol.m**, defines the system of ODEs and integrates them for each time step.

M file proj.m

```
%  
% Carol Panepinto  MS Project  August 2005  
% This file contains the set of commands that defines initial  
% conditions in the matrix,  
% calls the ODE solver, and generates the output graphs  
%  
  
% clear variable values before running code  
  
clear all  
  
%  
%initials:  elements representing x when the charge is injected at time  
%zero  
%  
  
initials = zeros(1,201);  
  
%  
%set up row vector for loop size  
%  
  
index = linspace(1,201,201);  
  
%  
%enter values for initial E field range  
%  
  
highE = input('input highest initial value for E when charge is  
injected in Volts/micron-enter number only ')  
lowE = input('input lowest initial value for E when charge is injected  
in Volts/micron-enter number only ')
```

```

%
% Set up loop to create the evenly spaced range of initial E values
%

for i = index

initials2(i) = lowE + i*((highE-lowE)/201); %initials2: elements
representing E when the charge is injected

end

%
% combine initial x and E values into one column vector called inits
%

inits=[initials,initials2]'; % inits: initial x and E values

%
% input total time and calculate evenly spaced time steps for output
%

time = input('input total time for the charges to travel in msec=? ');
tspan = linspace(0, time,100) ; %tspan: evenly spaced time
increments for output based on total time

%
% call the function solver using
% inputs of initial x and E and values and the timespan. store the
output in a matrix called
% sol with a column of t(time) values and columns for the calculated x
and E values
% each column of x and E values corresponds to one charge piece over
time
%

[t,sol]=ode45(@Projsol, tspan, [inits]);

% retrieve values for last time step and plot x vs. E -fill in values
after
% trail edge and before lead edge if necessary

figure(5)

hold on

sol100=[0 sol(100,1:201) 25 sol(100,202) sol(100,202:402)
sol(100,402)];
subplot(2,5,5), plot(sol100(1:203),sol100(204:406))
axis([0 25 -20 40])

```

```
% retrieve values for first time step and plot x vs. E-fill in values
after
% trail edge and before lead edge if necessary
```

```
figure(5)
```

```
hold on
```

```
sol1=[0 sol(1,1:201) 25 sol(1,202) sol(1,202:402) sol(1,402)];
subplot(2,5,1), plot(sol1(1:203), sol1(204:406))
axis([0 25 -20 40])
```

```
% retrieve values for 1/4 time and plot x vs. E-fill in values after
% trail edge and before lead edge if necessary
```

```
figure(5)
```

```
hold on
```

```
sol25=[0 sol(25,1:201) 25 sol(25,202) sol(25,202:402) sol(25,402)];
subplot(2,5,2), plot(sol25(1:203), sol25(204:406))
axis([0 25 -20 40])
```

```
% retrieve values for 1/2 total time and plot x vs. E-fill in values
after
% trail edge and before lead edge if necessary
```

```
figure(5)
```

```
hold on
```

```
sol50=[0 sol(50,1:201) 25 sol(50,202) sol(50,202:402) sol(50,402)];
subplot(2,5,3), plot(sol50(1:203), sol50(204:406))
axis([0 25 -20 40])
```

```
% retrieve values for first time step and plot x vs. E-fill in values
after
% trail edge and before lead edge if necessary
```

```
figure(5)
```

```
hold on
```

```
sol75=[0 sol(75,1:201) 25 sol(75,202) sol(75,202:402) sol(75,402)];
subplot(2,5,4), plot(sol75(1:203), sol75(204:406))
axis([0 25 -20 40])
```



```

%
% set up index to plot position vs. time at some time steps
% to see some of the characteristic lines
%

index10=linspace(1,201,11);
for q = index10
    % plot x vs. t to get dx/dt = mu*E
    figure(2)
    hold on
    plot(sol(:,q),t) % plot x vs. t
    axis([0 25 0 time])
end

% set up index and plot time vs. E field at some time steps
% to see E field change over time on the characteristic curves
for r = index10
    % plot t vs. E
    figure(4)
    hold on
    plot(t, sol(:,(r+201)))
    axis([0 time lowE-(highE-lowE) highE+1.5*(highE-lowE)])
end

%
% Label Graphs
%
figure(2)
xlabel('X Position in Microns')
ylabel('Time in msec')
title('Characteristic Curves')
hold off

figure(4)
xlabel('Time in msec')
ylabel('E Field')
title('E Field vs. time on Characteristic Curves')
hold off

%
% numerically calculate the derivative of E with respect to x to get
charge
% density at first time step
%

figure(5)
dE=diff(sol(2,202:402))./diff(sol(2,1:201));
xd=sol(2,2:201);

%

```

```

'' % set up matrix containing x values corresponding to positions
% between the photoconductor plates and charge density values with zero
% values outside the charge packet positions
%

d1=[0 xd(1) xd xd(200) 25 0 0 dE 0 0];

% Plot Derivative (Charge Density) vs. Position at initial time step
subplot(2,5,6), plot(d1(1:204),d1(205:408))
axis([0 25 0 40])

figure(5)
% numerically calculate the derivative of E with respect to x to get
charge
% density at 1/4 way through total time
dE25=diff(sol(25,202:402))./diff(sol(25,1:201));
xd25=sol(25,2:201);

%
% set up matrix containing x values corresponding to positions
% between the photoconductor plates and charge density values with zero
% values outside the charge packet positions
%

d25=[0 xd25(1) xd25 xd25(200) 25 0 0 dE25 0 0];

% Plot Derivative (Charge Density) vs. Position 1/4 way through total
time
%
subplot(2,5,7), plot(d25(1:204),d25(205:408))
axis([0 25 0 40])

figure(5)
% numerically calculate the derivative of E with respect to x to get
charge
% density at 1/2 way through total time
dE50=diff(sol(50,202:402))./diff(sol(50,1:201));
xd50=sol(50,2:201);

d50=[0 xd50(1) xd50 xd50(200) 25 0 0 dE50 0 0];
% Plot Derivative (Charge Density) vs. Position at 1/2 way through
time
subplot(2,5,8), plot(d50(1:204),d50(205:408))
axis([0 25 0 40])
% %

% %

figure(5)
% numerically calculate the derivative of E with respect to x to get
charge
% density at 3/4 way through total time
dE75=diff(sol(75,202:402))./diff(sol(75,1:201));
xd75=sol(75,2:201);

```

```

d75=[0 xd75(1) xd75 xd75(200) 25 0 0 dE75 0 0];
% Plot Derivative (Charge Density) vs. Position at 3/4 way through
time
subplot(2,5,9), plot(d75(1:204),d75(205:408))
axis([0 25 0 40])
% %
% % % %calculate derivative of E with respect to x to get n at final
time
% step
% %
figure(5)
% numerically calculate the derivative of E with respect to x to get
charge
% density at final time step
dE500=diff(sol(100,202:402))./diff(sol(100,1:201));
xd500=sol(100,2:201);

d500=[0 xd500(1) xd500 xd500(200) 25 0 0 dE500 0 0];
% Plot Derivative (Charge Density) vs. Position at final time step
subplot(2,5,10), plot(d500(1:204),d500(205:408))
axis([0 25 0 40])

%
% Label Large Graph containing subplots of E-field and
% charge density at different time steps
%

figure(5)
subplot(2,5,3)
title(['E field and Charge Density at initial, 1/4, 1/2, 3/4, and
total of ' num2str(time), ' msec'])

% make movie of E field vs. position at each time step
set(gca,'nextplot','replacechildren')
for s=1:100
    figure(6);
    solmov=[0 sol(s,1:201) 25 sol(s,202) sol(s,202:402) sol(s,402)];
    plot(solmov(1:203),solmov(204:406));
    axis([0 25 lowE-(highE-lowE) highE+(highE-lowE)]);
    xlabel('Position in Microns')
    ylabel('E-field Volts/Micron')
    title(['E-field as Charges Move Over ' num2str(time), ' msec'])
    EFrame(s)=getframe;
end

%
% Run Movie of E-field over all time steps
%

movie(EFrame,1)

% make movie of charge density vs. position at each time step
set(gca,'nextplot','replacechildren')

```

```

' for t=1:100
    figure(7);
    warning off MATLAB:divideByZero
    dEmov=diff(sol(t,202:402))./diff(sol(t,1:201));
    xdmov=sol(t,2:201);

    dmov=[0 xdmov(1) xdmov xdmov(200) 25 0 0 dEmov 0 0];

    plot(dmov(1:204),dmov(205:408));
    axis([0 25 0 highE*2]);
    xlabel('Position in Microns')
    ylabel('Charge Density Coulombs/Micron')
    title(['Density as Charges Move Over ' num2str(time), ' msec'])
    DenFrame(t)=getframe;
end
% Run movie of charge density at all time steps

movie(DenFrame,1)

```

File Projsol.m

```

%
% Carol Panepinto MS Project August 2005
% This file defines the function that describes the derivatives with
% respect to time:
%  $dx/dt = \mu \cdot E$ , and  $dE/dt = c(t)$  It then stores
% the results of integration in a matrix called sol
%

function charge = projsol2(t,sol)

% L is the width of the photoconductor
L=25;

% b is one of the constants of the material and  $\mu_0$  is the other
% constant
% per the Poole Frenkel equation

b=.0015;

 $\mu_0=1.0$ ;

%
% the following constants for the calculation of  $dE/dt$  are pre
% calculated to
% make the final code easier to type and read
%

 $c_0 = (\mu_0/L) * (2/(b^3))$ ;       $\% (\mu_0/L) (2/(b^3))$ 

```

```

c1=(-6)/b;           % -6/b
c2=(-3)*b;           %-3b
c3=b^2;              %b^2

% set N initially equal to the index corresponding to the lead edge
% of the E field
N=402;

%
% set up loop to check for the first time x>L and if this occurs,
% set the corresponding index for E equal to one less than the index
% for E
% where x>L and call this index N
%

index3 = [1:201];
for m=index3
    if sol(m)>=L
        N=m+201-1;

        break
    else

    end
end

%
% set up an array containing dx/dt and dE/dt
% if N is less than or equal to the index for the first E
value(trailing edge),
% then the trail edge has gone past the width of the material
% let the x values continue but set the corresponding E values to zero
% since all charges are gone from the material
%

if N <= 202

charge=[mu0.*(sol(202:402)).*exp(b.*sqrt(sol(202:402)))];[zeros(201,1)
]];

%
% if N is greater than the index of the first E value(trail edge) and
less than
% the index of the last(lead edge) E value, this means that the lead
edge
% has entered the far side and the E field will be calculated for the
% charge area between the far side and the trail edge. Since N
corresponds
% to a region where x is slightly less than L, the short area between
this
% charge area and the charge plate is added to what is calculated for
the
% whole charge area with the derived equations. The rate of change in
each section

```



```
(c0*(exp(b*sqr t(sol(202))))*(cl+6*sqr t(sol(202))+c2*sol(202)+c3*sqr t(so  
l(202)^3)));  
(mu0*exp(b*sqr t(sol(N))))*(sol(N)-sol(N-1))*(L-sol(N-201))/(sol(N-  
201)-sol(N-  
202))/L+(c0*(exp(b*sqr t(sol(N))))*(cl+6*sqr t(sol(N))+c2*sol(N)+c3*sqr t  
(sol(N)^3)))-  
(c0*(exp(b*sqr t(sol(202))))*(cl+6*sqr t(sol(202))+c2*sol(202)+c3*sqr t(so  
l(202)^3)));  
(mu0*exp(b*sqr t(sol(N))))*(sol(N)-sol(N-1))*(L-sol(N-201))/(sol(N-  
201)-sol(N-  
202))/L+(c0*(exp(b*sqr t(sol(N))))*(cl+6*sqr t(sol(N))+c2*sol(N)+c3*sqr t  
(sol(N)^3)))-  
(c0*(exp(b*sqr t(sol(202))))*(cl+6*sqr t(sol(202))+c2*sol(202)+c3*sqr t(so  
l(202)^3)));  
(mu0*exp(b*sqr t(sol(N))))*(sol(N)-sol(N-1))*(L-sol(N-201))/(sol(N-  
201)-sol(N-  
202))/L+(c0*(exp(b*sqr t(sol(N))))*(cl+6*sqr t(sol(N))+c2*sol(N)+c3*sqr t  
(sol(N)^3)))-  
(c0*(exp(b*sqr t(sol(202))))*(cl+6*sqr t(sol(202))+c2*sol(202)+c3*sqr t(so  
l(202)^3)));  
(mu0*exp(b*sqr t(sol(N))))*(sol(N)-sol(N-1))*(L-sol(N-201))/(sol(N-  
201)-sol(N-  
202))/L+(c0*(exp(b*sqr t(sol(N))))*(cl+6*sqr t(sol(N))+c2*sol(N)+c3*sqr t  
(sol(N)^3)))-  
(c0*(exp(b*sqr t(sol(202))))*(cl+6*sqr t(sol(202))+c2*sol(202)+c3*sqr t(so  
l(202)^3)));  
(mu0*exp(b*sqr t(sol(N))))*(sol(N)-sol(N-1))*(L-sol(N-201))/(sol(N-  
201)-sol(N-  
202))/L+(c0*(exp(b*sqr t(sol(N))))*(cl+6*sqr t(sol(N))+c2*sol(N)+c3*sqr t  
(sol(N)^3)))-  
(c0*(exp(b*sqr t(sol(202))))*(cl+6*sqr t(sol(202))+c2*sol(202)+c3*sqr t(so  
l(202)^3)));  
(mu0*exp(b*sqr t(sol(N))))*(sol(N)-sol(N-1))*(L-sol(N-201))/(sol(N-  
201)-sol(N-  
202))/L+(c0*(exp(b*sqr t(sol(N))))*(cl+6*sqr t(sol(N))+c2*sol(N)+c3*sqr t  
(sol(N)^3)))-  
(c0*(exp(b*sqr t(sol(202))))*(cl+6*sqr t(sol(202))+c2*sol(202)+c3*sqr t(so  
l(202)^3)));  
(mu0*exp(b*sqr t(sol(N))))*(sol(N)-sol(N-1))*(L-sol(N-201))/(sol(N-  
201)-sol(N-  
202))/L+(c0*(exp(b*sqr t(sol(N))))*(cl+6*sqr t(sol(N))+c2*sol(N)+c3*sqr t  
(sol(N)^3)))-  
(c0*(exp(b*sqr t(sol(202))))*(cl+6*sqr t(sol(202))+c2*sol(202)+c3*sqr t(so  
l(202)^3)));
```


[illegible]


```
(c0*(exp(b*sqr t(sol(202))))*(cl+6*sqr t(sol(202))+c2*sol(202)+c3*sqr t(so  
l(202)^3))));  
    ((mu0*exp(b*sqr t(sol(N))))*(sol(N)-sol(N-1))*(L-sol(N-201))/(sol(N-  
201)-sol(N-  
202)))/L+(c0*(exp(b*sqr t(sol(N))))*(cl+6*sqr t(sol(N))+c2*sol(N)+c3*sqr t  
(sol(N)^3))-  
(c0*(exp(b*sqr t(sol(202))))*(cl+6*sqr t(sol(202))+c2*sol(202)+c3*sqr t(so  
l(202)^3))));  
    ((mu0*exp(b*sqr t(sol(N))))*(sol(N)-sol(N-1))*(L-sol(N-201))/(sol(N-  
201)-sol(N-  
202)))/L+(c0*(exp(b*sqr t(sol(N))))*(cl+6*sqr t(sol(N))+c2*sol(N)+c3*sqr t  
(sol(N)^3))-  
(c0*(exp(b*sqr t(sol(202))))*(cl+6*sqr t(sol(202))+c2*sol(202)+c3*sqr t(so  
l(202)^3))));  
    ((mu0*exp(b*sqr t(sol(N))))*(sol(N)-sol(N-1))*(L-sol(N-201))/(sol(N-  
201)-sol(N-  
202)))/L+(c0*(exp(b*sqr t(sol(N))))*(cl+6*sqr t(sol(N))+c2*sol(N)+c3*sqr t  
(sol(N)^3))-  
(c0*(exp(b*sqr t(sol(202))))*(cl+6*sqr t(sol(202))+c2*sol(202)+c3*sqr t(so  
l(202)^3))));  
    ((mu0*exp(b*sqr t(sol(N))))*(sol(N)-sol(N-1))*(L-sol(N-201))/(sol(N-  
201)-sol(N-  
202)))/L+(c0*(exp(b*sqr t(sol(N))))*(cl+6*sqr t(sol(N))+c2*sol(N)+c3*sqr t  
(sol(N)^3))-  
(c0*(exp(b*sqr t(sol(202))))*(cl+6*sqr t(sol(202))+c2*sol(202)+c3*sqr t(so  
l(202)^3))));  
    ((mu0*exp(b*sqr t(sol(N))))*(sol(N)-sol(N-1))*(L-sol(N-201))/(sol(N-  
201)-sol(N-  
202)))/L+(c0*(exp(b*sqr t(sol(N))))*(cl+6*sqr t(sol(N))+c2*sol(N)+c3*sqr t  
(sol(N)^3))-  
(c0*(exp(b*sqr t(sol(202))))*(cl+6*sqr t(sol(202))+c2*sol(202)+c3*sqr t(so  
l(202)^3))));  
    ((mu0*exp(b*sqr t(sol(N))))*(sol(N)-sol(N-1))*(L-sol(N-201))/(sol(N-  
201)-sol(N-  
202)))/L+(c0*(exp(b*sqr t(sol(N))))*(cl+6*sqr t(sol(N))+c2*sol(N)+c3*sqr t  
(sol(N)^3))-  
(c0*(exp(b*sqr t(sol(202))))*(cl+6*sqr t(sol(202))+c2*sol(202)+c3*sqr t(so  
l(202)^3))));  
    ((mu0*exp(b*sqr t(sol(N))))*(sol(N)-sol(N-1))*(L-sol(N-201))/(sol(N-  
201)-sol(N-  
202)))/L+(c0*(exp(b*sqr t(sol(N))))*(cl+6*sqr t(sol(N))+c2*sol(N)+c3*sqr t  
(sol(N)^3))-  
(c0*(exp(b*sqr t(sol(202))))*(cl+6*sqr t(sol(202))+c2*sol(202)+c3*sqr t(so  
l(202)^3))));
```



```

202)))/L+(c0*(exp(b*sqrt(sol(N))))*(c1+6*sqrt(sol(N))+c2*sol(N)+c3*sqrt
(sol(N)^3)))-
(c0*(exp(b*sqrt(sol(202))))*(c1+6*sqrt(sol(202))+c2*sol(202)+c3*sqrt(so
l(202)^3)));
((mu0*exp(b*sqrt(sol(N))))*(sol(N)-sol(N-1))*(L-sol(N-201)))/(sol(N-
201)-sol(N-
202)))/L+(c0*(exp(b*sqrt(sol(N))))*(c1+6*sqrt(sol(N))+c2*sol(N)+c3*sqrt
(sol(N)^3)))-
(c0*(exp(b*sqrt(sol(202))))*(c1+6*sqrt(sol(202))+c2*sol(202)+c3*sqrt(so
l(202)^3)));
((mu0*exp(b*sqrt(sol(N))))*(sol(N)-sol(N-1))*(L-sol(N-201)))/(sol(N-
201)-sol(N-
202)))/L+(c0*(exp(b*sqrt(sol(N))))*(c1+6*sqrt(sol(N))+c2*sol(N)+c3*sqrt
(sol(N)^3)))-
(c0*(exp(b*sqrt(sol(202))))*(c1+6*sqrt(sol(202))+c2*sol(202)+c3*sqrt(so
l(202)^3)));
((mu0*exp(b*sqrt(sol(N))))*(sol(N)-sol(N-1))*(L-sol(N-201)))/(sol(N-
201)-sol(N-
202)))/L+(c0*(exp(b*sqrt(sol(N))))*(c1+6*sqrt(sol(N))+c2*sol(N)+c3*sqrt
(sol(N)^3)))-
(c0*(exp(b*sqrt(sol(202))))*(c1+6*sqrt(sol(202))+c2*sol(202)+c3*sqrt(so
l(202)^3)));
((mu0*exp(b*sqrt(sol(N))))*(sol(N)-sol(N-1))*(L-sol(N-201)))/(sol(N-
201)-sol(N-
202)))/L+(c0*(exp(b*sqrt(sol(N))))*(c1+6*sqrt(sol(N))+c2*sol(N)+c3*sqrt
(sol(N)^3)))-
(c0*(exp(b*sqrt(sol(202))))*(c1+6*sqrt(sol(202))+c2*sol(202)+c3*sqrt(so
l(202)^3)));
((mu0*exp(b*sqrt(sol(N))))*(sol(N)-sol(N-1))*(L-sol(N-201)))/(sol(N-
201)-sol(N-
202)))/L+(c0*(exp(b*sqrt(sol(N))))*(c1+6*sqrt(sol(N))+c2*sol(N)+c3*sqrt
(sol(N)^3)))-
(c0*(exp(b*sqrt(sol(202))))*(c1+6*sqrt(sol(202))+c2*sol(202)+c3*sqrt(so
l(202)^3)));
((mu0*exp(b*sqrt(sol(N))))*(sol(N)-sol(N-1))*(L-sol(N-201)))/(sol(N-
201)-sol(N-
202)))/L+(c0*(exp(b*sqrt(sol(N))))*(c1+6*sqrt(sol(N))+c2*sol(N)+c3*sqrt
(sol(N)^3)))-
(c0*(exp(b*sqrt(sol(202))))*(c1+6*sqrt(sol(202))+c2*sol(202)+c3*sqrt(so
l(202)^3)))]];

```

%

% otherwise N is still the value corresponding to the trail edge and means that

% the charges still all reside within the material

[illegible]

[illegible]


```
(c0*(exp(b*sqrt(sol(N))))*(c1+6*sqrt(sol(N))+c2*sol(N)+c3*sqrt(sol(N)^3)))-
(c0*(exp(b*sqrt(sol(202))))*(c1+6*sqrt(sol(202))+c2*sol(202)+c3*sqrt(sol(202)^3)));
```

[illegible]


```
(c0*(exp(b*sqrt(sol(N))))*(c1+6*sqrt(sol(N))+c2*sol(N)+c3*sqrt(sol(N)^3)))-
(c0*(exp(b*sqrt(sol(202))))*(c1+6*sqrt(sol(202))+c2*sol(202)+c3*sqrt(sol(202)^3)));
```

```
(c0*(exp(b*sqrt(sol(N))))*(c1+6*sqrt(sol(N))+c2*sol(N)+c3*sqrt(sol(N)^3)))-
(c0*(exp(b*sqrt(sol(202))))*(c1+6*sqrt(sol(202))+c2*sol(202)+c3*sqrt(sol(202)^3)));
```

```
(c0*(exp(b*sqrt(sol(N))))*(c1+6*sqrt(sol(N))+c2*sol(N)+c3*sqrt(sol(N)^3)))-
(c0*(exp(b*sqrt(sol(202))))*(c1+6*sqrt(sol(202))+c2*sol(202)+c3*sqrt(sol(202)^3)));
```

```
(c0*(exp(b*sqrt(sol(N))))*(c1+6*sqrt(sol(N))+c2*sol(N)+c3*sqrt(sol(N)^3)))-
(c0*(exp(b*sqrt(sol(202))))*(c1+6*sqrt(sol(202))+c2*sol(202)+c3*sqrt(sol(202)^3)));
```

```
(c0*(exp(b*sqrt(sol(N))))*(c1+6*sqrt(sol(N))+c2*sol(N)+c3*sqrt(sol(N)^3)))-
(c0*(exp(b*sqrt(sol(202))))*(c1+6*sqrt(sol(202))+c2*sol(202)+c3*sqrt(sol(202)^3)));
```

```
(c0*(exp(b*sqrt(sol(N))))*(c1+6*sqrt(sol(N))+c2*sol(N)+c3*sqrt(sol(N)^3)))-
(c0*(exp(b*sqrt(sol(202))))*(c1+6*sqrt(sol(202))+c2*sol(202)+c3*sqrt(sol(202)^3)));
```

```
(c0*(exp(b*sqrt(sol(N))))*(c1+6*sqrt(sol(N))+c2*sol(N)+c3*sqrt(sol(N)^3)))-
(c0*(exp(b*sqrt(sol(202))))*(c1+6*sqrt(sol(202))+c2*sol(202)+c3*sqrt(sol(202)^3)));
```

```
(c0*(exp(b*sqrt(sol(N))))*(c1+6*sqrt(sol(N))+c2*sol(N)+c3*sqrt(sol(N)^3)))-
(c0*(exp(b*sqrt(sol(202))))*(c1+6*sqrt(sol(202))+c2*sol(202)+c3*sqrt(sol(202)^3)));
```

```
(c0*(exp(b*sqrt(sol(N))))*(c1+6*sqrt(sol(N))+c2*sol(N)+c3*sqrt(sol(N)^3)))-
(c0*(exp(b*sqrt(sol(202))))*(c1+6*sqrt(sol(202))+c2*sol(202)+c3*sqrt(sol(202)^3)));
```

```
(c0*(exp(b*sqrt(sol(N))))*(c1+6*sqrt(sol(N))+c2*sol(N)+c3*sqrt(sol(N)^3)))-
(c0*(exp(b*sqrt(sol(202))))*(c1+6*sqrt(sol(202))+c2*sol(202)+c3*sqrt(sol(202)^3)));
```

```
(c0*(exp(b*sqrt(sol(N))))*(c1+6*sqrt(sol(N))+c2*sol(N)+c3*sqrt(sol(N)^3)))-
(c0*(exp(b*sqrt(sol(202))))*(c1+6*sqrt(sol(202))+c2*sol(202)+c3*sqrt(sol(202)^3)));
```

$$c_0 \left(\exp(b \sqrt{\text{sol}(N)}) \right) \left(c_1 + 6 \sqrt{\text{sol}(N)} + c_2 \text{sol}(N) + c_3 \sqrt{\text{sol}(N)}^3 \right) -$$

[illegible]

[illegible]

```

(c0*(exp(b*sqrt(sol(202))))*(c1+6*sqrt(sol(202))+c2*sol(202)+c3*sqrt(sol(202)^3)));

(c0*(exp(b*sqrt(sol(N))))*(c1+6*sqrt(sol(N))+c2*sol(N)+c3*sqrt(sol(N)^3))) -
(c0*(exp(b*sqrt(sol(202))))*(c1+6*sqrt(sol(202))+c2*sol(202)+c3*sqrt(sol(202)^3)));

(c0*(exp(b*sqrt(sol(N))))*(c1+6*sqrt(sol(N))+c2*sol(N)+c3*sqrt(sol(N)^3))) -
(c0*(exp(b*sqrt(sol(202))))*(c1+6*sqrt(sol(202))+c2*sol(202)+c3*sqrt(sol(202)^3)));

(c0*(exp(b*sqrt(sol(N))))*(c1+6*sqrt(sol(N))+c2*sol(N)+c3*sqrt(sol(N)^3))) -
(c0*(exp(b*sqrt(sol(202))))*(c1+6*sqrt(sol(202))+c2*sol(202)+c3*sqrt(sol(202)^3)));

(c0*(exp(b*sqrt(sol(N))))*(c1+6*sqrt(sol(N))+c2*sol(N)+c3*sqrt(sol(N)^3))) -
(c0*(exp(b*sqrt(sol(202))))*(c1+6*sqrt(sol(202))+c2*sol(202)+c3*sqrt(sol(202)^3)));

(c0*(exp(b*sqrt(sol(N))))*(c1+6*sqrt(sol(N))+c2*sol(N)+c3*sqrt(sol(N)^3))) -
(c0*(exp(b*sqrt(sol(202))))*(c1+6*sqrt(sol(202))+c2*sol(202)+c3*sqrt(sol(202)^3))))];

end

```

APPENDIX C TOOLS UTILIZED

MathWorks Matlab version 6.5

Matlab ode45 solver

Microsoft XP Operating System

Microsoft Word 2002

Microsoft Media Player version 9

Laptop Computer - HP/Compaq Armada 78000 Intel Pentium 2

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PRESENTATION SLIDES

MS Project Presentation

*A Numerical Method for Determining
Photoconductor Mobilities*

Carol Panepinto

May 11, 2005

Outline of Presentation

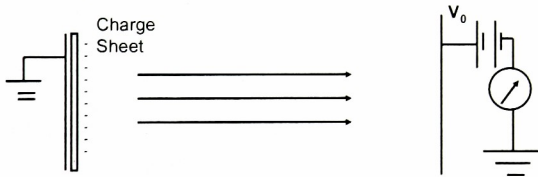
- Introduction
- Problem Definition and Physical Description
- Electric Field
- The Math
- Comparison to a Standard Problem
- Method of Characteristics
- Solution
- Summary

Introduction

- We are interested ultimately in finding the mobility (measure of how easily electrons flow: velocity/E field) within a photoconductor.
- Electric field and charge density inside the photoconductor can be represented by a PDE that is dependent on mobility, which itself is dependent on electric field.
- By solving the PDE for electric field for different mobilities, and by comparing to measured transit time and current, the mobility can be inferred.

Problem under Investigation

- A semiconductor with a constant voltage on one side receives energy from a light flash and a sheet of electrons is injected at the ground side of the material

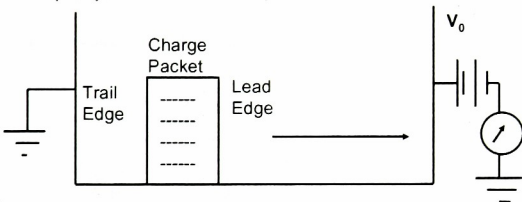


What Happens Next?

- The charge sheet is attracted to the opposite side and repelled by the ground side.
- The electrons travel across the material, increasing speed as they approach the other side, and spreading due to mutual repulsion.
- The charge density, n , decreases as the sheet moves and broadens, since the electrons are less tightly packed.

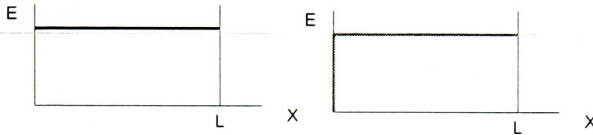
Physical Representation

- The charge packet widens as it moves across the material. The total charge in the packet remains constant as density, $n(x,t)$, decreases.



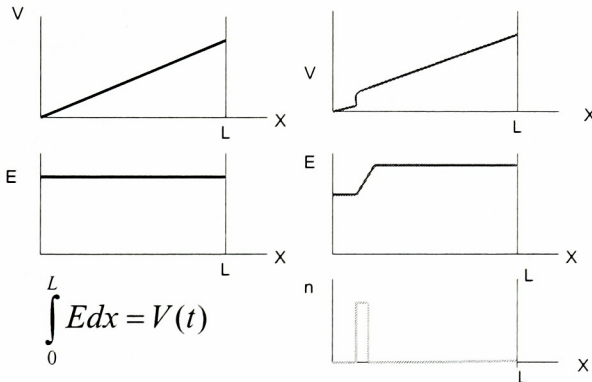
Electric Field

- Charge density, n , is the partial spatial derivative of the E field $\frac{\partial E(x,t)}{\partial x} = n(x,t)$, Poisson's equation.
- E field, Δ voltage/unit distance, is constant before energy is introduced to the material. After the sheet of electrons is injected, E field becomes a step function at the ground side of the material and the original E field is changed.



Electric Field

E field is the spatial derivative of voltage and charge density, $n(x,t)$, is the spatial derivative of E



The Math

- Charge is conserved during packet transit time,

$$\frac{\partial n(x,t)}{\partial t} + \frac{\partial}{\partial x} \mu(E(x,t))E(x,t)n(x,t) = 0 \quad (1)$$

- μ is the mobility of the material(vel./unit E field) and the Poole-Frankel equation states $\mu = \mu_0 e^{\beta \sqrt{E(x,t)}}$ where μ_0, β are constant material properties.
- Since $\frac{\partial E(x,t)}{\partial x} = n(x,t)$, after integrating (1) from zero to x with respect to x, the equation can be re-written as

$$\frac{\partial E(x,t)}{\partial t} + \mu(E(x,t))E(x,t) \frac{\partial E(x,t)}{\partial x} = c(t) \quad (2)$$

$$\frac{\partial E(x,t)}{\partial t} + \mu(E(x,t))E(x,t) \frac{\partial E(x,t)}{\partial x} = c(t) \quad (2)$$

- This left side of this equation can be split into 2 terms and integrated with respect to x from 0 to L (L=material width):

$$\frac{\partial}{\partial t} \int_0^L E(x,t) dx + \int_0^L \mu(E(x,t))E(x,t) \frac{\partial E}{\partial x} dx = c(t)L$$

- Since the integral of E with respect to x is V(t), voltage, the equation then becomes:

$$\frac{\partial}{\partial t} V(t) + \int_0^L \mu(E(x,t))E(x,t) \frac{\partial E}{\partial x} dx = L * c(t)$$

- Since voltage does not change with respect to time the equation then reduces to

$$\int_0^L \mu(E(x,t))E(x,t) \frac{\partial E}{\partial x} dx = L * c(t) \quad (3)$$

Comparing with Another Problem

(A Standard Problem)

$$\frac{\partial E}{\partial t} + \mu(E)E \frac{\partial E}{\partial x} = 0 \quad \text{Scalar Hyperbolic Conservation Law}$$

$$0 \leq t, \quad 0 \leq x$$

$$E(x,0) = g(x) \quad \text{Initial Condition}$$

$$\frac{\partial E}{\partial t}(0,t) = 0 \quad \text{Boundary Condition}$$

This Problem

(A More Complicated but Well-Posed Problem)

$$\frac{\partial E}{\partial t} + \mu(E)E \frac{\partial E}{\partial x} = c(t) \quad \text{Extra unknown}$$

$$0 \leq t, \quad 0 \leq x$$

$$E(x,0) = g(x) \quad \text{Initial Condition}$$

$$\frac{\partial E}{\partial t}(0,t) = c(t) \quad \text{Boundary Condition}$$

$$\int_0^L E(x,t) dx = V(t) \quad \text{Extra non-local condition}$$

Why Use Characteristics?

- Finite difference and finite element methods would break the wave into spatial increments. These methods work well with smooth functions.
- This function is not smooth and has corners.
- The error with FD and FE methods is proportional to $\frac{\partial^2 E}{\partial x^2} = \frac{\partial n}{\partial x}$
- For the initial delta function, the error would be large and the error bound infinite using FD and FE methods.
- The method of characteristics looks at pieces of charge over time only within the charge packet.

Method of Characteristics

- By using the method of characteristics, the original partial differential equation can be reduced to a large system of ordinary differential equations with t as the independent variable.
- On the characteristic curves the PDE for E reduces to an ODE.

Method of Characteristics

- $$c(t) = \frac{1}{L} \left[\int_0^L \mu_0 e^{\beta \sqrt{E}} E \frac{\partial E}{\partial x} dx \right] = \frac{1}{L} \left[\mu_0 \int_{E(x=0)}^{E(x=L)} E e^{\beta \sqrt{E}} dE \right]$$
- By making the substitution, $u = \sqrt{E}$, the equation becomes
$$\frac{1}{L} \left[\mu_0 \int 2 u^3 e^{\beta u} du \right] = c(t)$$
- Evaluating this integral, the solution is
$$c(t) = \frac{\mu_0}{L} \left[\frac{2}{\beta^3} e^{\beta u} \left(-\frac{6}{\beta} + 6u - 3\beta u^2 + \beta^2 u^3 \right) \right]$$
- Substituting \sqrt{E} back in for u ,

$$c(t) = \frac{\mu_0}{L} \left[\frac{2}{\beta^3} e^{\beta \sqrt{E}} \left(-\frac{6}{\beta} + 6\sqrt{E} - 3\beta E + \beta^2 \sqrt{E}^3 \right) \right] \bigg|_{E(\text{trailing edge of packet})}^{E(\text{leading edge packet})} \quad (4)$$

Method of Characteristics

$$\frac{\partial E(x,t)}{\partial t} + \mu(E(x,t)) E(x,t) \frac{\partial E(x,t)}{\partial x} = c(t) \quad (2)$$

- Along the characteristic curves,

$$\frac{dx}{dt} = \mu E$$

$$\frac{dE}{dt} = \frac{\partial E}{\partial t} + \frac{dx}{dt} \frac{\partial E}{\partial x} = \frac{\partial E}{\partial t} + \mu E \frac{\partial E}{\partial x}$$

- Then along such a curve

$$\frac{dE}{dt} = c(t) = \frac{1}{L} \left[\frac{2}{\beta^3} e^{\beta \sqrt{E}} \left(-\frac{6}{\beta} + 6\sqrt{E} - 3\beta E + \beta^2 \sqrt{E}^3 \right) \right] \bigg|_{E(\text{trail})}^{E(\text{lead})} \quad (4)$$

Solution

- Initial conditions at the ground side are stored in an array containing values of zero for x and a range of E values where x is zero.
- Matlab's ODE45 Solver integrates the differential equations over time simultaneously using values from each previous time iteration starting with the initial values.

$$\frac{dx_i}{dt} = \mu E_i = \mu_0 e^{\beta \sqrt{E_i}} E_i$$

$$\frac{\partial E_i}{\partial t} = c(t) = \frac{1}{L} \left[\frac{2}{\beta^2} e^{\beta \sqrt{E_i}} \left(-\frac{6}{\beta} + 6\sqrt{E_i} - 3\beta E_i + \beta^2 \sqrt{E_i}^3 \right) \right] \quad \left| \begin{array}{l} L(x=L) \\ L(x=0) \end{array} \right.$$

Solution

- The solution at each time step is an array containing the next set of x values and E values.
- After the initial condition, the E field between the ground side and the trail edge of the charge packet is the same as the value at the trailing edge.
- Before the leading edge of the packet reaches the charge side of the material, the E field between the leading edge and the charge side is the same as the value at the leading edge.

Solution

- When the trailing edge of the packet reaches the charge side, all the charges are neutralized and the E field is back where it started before the introduction of energy.
- Charge density is calculated by numerically taking the derivative of E with respect to x .

Summary

- For this problem E field can be determined at any position and time by solving the PDE with the method of characteristics.
- Mobility can be inferred by looking at a range of solutions for various mobilities.
- Further research:
 - Spatial heterogeneity
 - Higher dimensions
 - Shock formation?

Thank You

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