9-29-2005

The importance of mental calculation skills: a review of the literature

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The Importance of Mental Calculation Skills: A Review of the Literature

Master's Project

Submitted to the Faculty
of the Master of Science Program in Secondary Education
of Students who are Deaf or Hard of Hearing

National Technical Institute for the Deaf
ROCHESTER INSTITUTE OF TECHNOLOGY

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In Partial Fulfillment of the Requirements
for the Degree of Master of Science

Rochester, New York May 7, 2002

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Abstract

This paper reviewed research related to phonological awareness in deaf and hearing individuals. A number of studies showed that individuals skilled in memory tasks and reading achievement utilize a phonological voice. Often referred to as inner voice or the articulatory loop, phonology was used by both hearing and deaf individuals considered to be skilled readers. The evidence also shows that the use of phonology for coding information is related to the ability to use memory efficiently.

As reading achievement is very important in a deaf child's education, so is mathematics. Compared to hearing students, deaf students do not perform at the same academic level in mathematics. While the importance of phonology is known in terms of reading achievement and memory, it is probable that use of the articulatory loop is important in mathematics. This logic led to another potentially important factor of mathematical achievement – mental calculation. Thus, a review of the literature was conducted on the importance of mental calculation and recommended key strategies for teaching mental arithmetic skills.

The Importance of Mental Calculation Skills: A Review of the Literature

Introduction

Phonological coding or “inner voice” is the process by which hearing people organize cognitive thought processes. This method of coding information serves as the foundation for reading comprehension and tasks involving short-term memory. In a more general sense, phonological coding is the fundamental method by which hearing people organize thoughts regardless of whether related to reading, deciding what to prepare for dinner, or solving a mathematical computation. This articulatory code, as it is often referred, is based on the sounds of speech. When hearing people read, recall a recent idea, or estimate their change from the grocery store, the information accessed is based on knowledge of speech sounds corresponding to the associated words or phrases.

For coding information in short-term memory, hearing people use phonological coding. For memory coding, similarly, deaf individuals use phonological coding as supported by the evidence. Conrad (1972) was among the first to investigate code use by deaf individuals during short-term memory recall. He found evidence for the existence of a speech based memory code in hearing subjects but not in deaf individuals. Conrad’s research, however, was overwhelmingly refuted by later studies.

To obtain a better understanding of the qualitative aspects of immediate memory of deaf individuals, MacSweeney, Campbell and Donlan (1996) used the technique of interference of encoding and rehearsal of lists by concurrent activity. To study memory coding, subjects were shown a series of pictures in conjunction with interference and the resulting evidence would reveal clues to memory coding strategies of deaf individuals.

Results showed that deaf individuals do indeed use an articulatory loop for memory coding.

Hanson and Lichtenstein (1990) addressed the question, "Is serial recall language based or linked to phonological awareness?" They compared performance of hearing and deaf subjects on recall of formationally similar and formationally dissimilar word lists. Words that are considered "formationally similar" correspond to signs that share similar features such as movement and handshape such as MOTHER and FATHER. Words that correspond to signs that are "formationally dissimilar" do not share this likeness. For example, the signs MOTHER and KIND have different handshapes, locations and movements. If a decrement in performance was noted for the deaf subjects' recall of formationally similar lists compared to hearing peers, this would support the theory that coding was language based (visual, in this case) and not speech based. This holds true because studies have shown that hearing people, when presented with phonetically similar word lists, will perform worse on serial recall tasks when compared to performance on tasks involving word lists containing phonetically dissimilar words. In other words, if the words are acoustically similar, hearing people will not recall as many words as if they were acoustically dissimilar. Therefore, if deaf subjects use a visual code, they would not perform as well with words corresponding to visually similar signs when compared to performance with word lists corresponding to signs visually dissimilar. Moreover, when comparing the performance of hearing and deaf individuals on the same tasks with formationally similar word lists, if deaf subjects use a language based or visual code, they would perform worse than their hearing peers. However, recall of formationally similar lists was analogous for hearing and deaf participants of

this study suggesting that deaf individuals use phonology during short-term memory recall.

Hanson (1982) wanted to investigate if a speech based code or sign based code was used for recall of ordered items and free items. She presented subjects with three sets of printed words. The first set was formationally similar, the second was phonetically similar (similar sound) and the third list contained words with letters and letter combinations that make parts of the words share a similar sound (i.e. *f*ork, *ph*one and roug*h* – each word has a letter or combination of letters to represent a similar phoneme /*f*/). The most significant result involved the phonetically similar set of printed words. Ability for ordered recall was reduced suggesting the use of a speech based code used by deaf individuals. This is true because research has shown that performance on such recall tasks is hindered by the use of phonological coding in hearing people. Such findings suggest that deaf individuals make use of the articulatory loop for memory coding.

Clearly, deaf and hearing people use phonological coding for memory tasks but the scope of phonological code use extends beyond activities involving memory. Evidence has shown that, logically, skilled hearing readers make use of phonological coding. Deaf individuals, who have achieved advanced levels of reading ability, also use the articulatory loop for coding printed text.

Reynolds (1986) investigated the educational and research potential of the Degrees of Reading Power (DRP) tests using deaf college students. One hundred deaf and hard of hearing college students were tested; the degree of hearing loss ranged from 27 to 120 dB loss. Test results showed a significant difference in scores among

prelingually and postlingually deaf subjects suggesting a correlation between reading ability and phonological awareness. Skilled readers seemed to have had increased input of spoken language.

Using word pairs, Hanson and Fowler (1987) studied whether skilled deaf readers were able to access phonological information about words under similar conditions as skilled hearing readers. Her results did not significantly differ for hearing and deaf participants suggesting that deaf and hearing readers utilize phonological information in a similar manner.

Hanson, Goodell and Perfetti (1991) tested the reaction times (RTs) of deaf participants' attempts to determine the semantic acceptability of sentences based on reading tasks in which the sentences were preceded by a phonologically similar set of numbers. They found that RTs were slower for judgments of sentences that contained phonologically similar sets of numbers when compared to the control. These results also provide support for the utilization of phonological information in skilled deaf readers.

Together, these reports provide evidence for the use of the articulatory loop in hearing and deaf individuals with respect to memory tasks and reading achievement. Daneman, Nemeth, Stainton and Huelsmann (1995) studied the relationship between memory and reading ability. They examined storage capacity by way of administration of a series of recall tests. They wanted to examine individual differences in the ability to coordinate the processing and storage functions of working memory and determine if variation in ability correlates to reading achievement among deaf children. They used five tests to asses working memory capacity: reading span test, listening span test, visual shape span test, word identification and passage comprehension test. They found that all

measures of working memory capacity were strongly correlated with reading achievement. The results of the regression analysis showed that working memory capacity was a good predictor of deaf children's word skills, and an even better predictor of their passage comprehension skills.

As stated, the articulatory loop is important for memory tasks and reading achievement in deaf and hearing people. Hearing people use this code for virtually all cognitive thought processes and awareness of this code is vital to an individuals' success in an academic environment regardless of hearing status. Just as phonological coding is academically important to reading achievement and memory, it is also important to mathematical ability. For mental mathematical calculations and thinking through problems, research has shown that hearing individuals use phonological coding to perform these activities.

The mathematical abilities of deaf individuals are not equivalent to hearing peers. Traditionally, deaf students have performed worse in the mathematical environment when compared to their hearing counterparts. One plausible explanation is that the lower levels of math achievement displayed by deaf individuals is linked to a lack of phonological awareness. However, what other factors may be associated with the dissimilar levels of mathematical achievement among deaf and hearing students? Perhaps the answer may be partially explained by how they function with mental calculation skills. A thorough understanding of mental calculation skills may provide insight into the reasons why deaf individuals do not perform on an equitable level with hearing peers in the mathematical arena and supply educators with tools to help narrow the gap. This information, in conjunction with what is known about the importance of

phonological awareness can perhaps provide educators and researchers a different perspective on teaching deaf students mathematics. Deaf students, with an efficient, internal coding system at their disposal and relevant mental calculation strategies to deal with mathematical problems, may be able to maximize their learning potential in mathematics. This review of the literature will attempt to answer the following research questions:

1) What is the importance of mental arithmetic skills?
2) What are the key instructional strategies for developing mental arithmetic skills?

Literature Review

Deaf students' difficulties with mathematical word problems are well documented. Difficulties with word problems are often attributed to delayed language development. Barham and Bishop (1991) state that decreased performance levels of deaf students occur because students are introduced to new, confusing words such as "multiply" or "rectangle" and they have trouble with connectives such as "if" and "because." Serrano Pau (1995) looked at the ability of deaf individuals to handle mathematical word problems by examining the extent to which reading comprehension influenced problem solving skills of students when assigned various types of linguistically formulated arithmetic tasks. His subjects included 12 prelingually and profoundly deaf schoolchildren between the ages of 8 and 12 years of age. The results of this study showed that for 10 of 12 participants, reading comprehension was 2-5 years below grade level. Students had trouble with comprehending crucial words directly linked to the ability to solve the problem. These words/terms included "greater than," "less than," "together" and "gave." For example, the deaf student mistranslated "more"

in the phrase "more than" as an addition sign even though the correct operation to solve the problem was to subtract. Also, subjects tended to simplify text. For example, they interpreted "have more than" to mean, "have." Difficulty with mathematics for deaf students is not limited to word problems.

Titus (1995) investigated deaf and hard of hearing students' understanding of fractional numbers as measured by their ability to determine order and equivalence of fractional numbers. The concept of fractional numbers is important because knowledge of rational number topics such as fractions, decimals and percents are concepts that have been quoted as being "the capstone of elementary math" and "the cornerstone of high school math." Subjects included 21 deaf or hard of hearing students and 26 hearing students between ages 10 and 16. Both samples of hearing and deaf students were separated based on age resulting in one younger and one older group of deaf students and one younger and one older group of hearing students. The results showed that older hearing students significantly outperformed all other groups. Essentially, there were no differences identified between the younger hearing students and both age groups of deaf and hard of hearing students. In terms of performance, both age groups of deaf students performed similarly to the younger hearing students. This suggests that deaf and hard of hearing students may develop rational number concepts in manner similar to hearing students but development is delayed. Interestingly, Titus claims that the results are primarily related to English language development delay and difficulty with mathematical terminology, even though his materials were mainly comprised of fractions without the confusing terminology often responsible for difficulties with word problems.

Performance in mental arithmetic or mental calculation ability has not been thoroughly studied in the deaf population. However, examination of this particular domain of mathematics provides an exceptionally interesting area of study. This is true because of the mental nature of mental calculation. Compared to studies involving text comprehension such as word problems or reading passages, performance in mental calculation is least influenced by a persons' knowledge of the English language. Therefore, since the nature of mental calculation minimizes the influence of English language knowledge as a variable of performance, a more accurate picture of the true capabilities of the deaf student is obtainable. More likely than not, knowledge of the articulatory loop will help with performance on mental calculation tasks. However, by eliminating the influence of confusing terminology characteristic of word problems, the window into the true mathematical ability of the deaf student becomes a bit more clear. Language known to be confusing for deaf students such as "more" "if" and "because" do not factor in a persons' ability to calculate 51 x 49.

Mental Arithmetic Skills – Definition

Hall (1954) defines mental arithmetic as…

1) Arithmetic problems which arise
    a) in an oral manner
    b) in written form, or
    c) "in the head" of the person who needs to solve the problem;
2) Problems in which pencil an paper and other mechanical devices, such as calculators, are not used to record the intermediate steps between the statement of the problem and its answer;

3) Problems in which pencil and paper are used, and problems in which they are not used, to record the answer;
4) Problems in which quick estimations are made which either may or may not be verified by a written response (p. 353).

Development of Skills

Development of mental calculation skills stimulates higher-level mathematical thinking skills (Reys, 1985). Hazekamp (1986) discussed some of the benefits of teaching children mental calculation strategies. He claimed that the use of mental calculation strategies could provide children with the ability to recognize and work with numbers that are multiples of powers of ten. Also, students gain mental flexibility that allows them to think of numbers in many ways and forms (Beishuizen, 1997). Hope (1986a) and Schall (1973) stress the importance of mental calculation in that mental strategies help students to better understand the number system and number relationships. These authors assert that mental calculation strategies can help children become independent of techniques that were learned via memorization. Having acquired such techniques, mental strategies give children more options for solving problems and they are not locked into mentally attempting to use standard written algorithms for a problem that could easily and efficiently be solved with the appropriate mental technique. Academically, the benefits of mental calculation skills extend beyond the mathematics classroom and weakness in this area can profoundly influence material comprehension in other subjects. Flournoy (1959) noted that "The incidence of quantitative statements seems to run very high in social studies and science materials and it seems highly

probably that such statements are not understood at all by many pupils; in fact, the learner may hardly be conscious of their presence."(p. 135)

The beneficial characteristics resulting from proficiency in mental calculation are very important. Use of mental computation strategies will benefit students in math class and in other educational areas. As important as it is to teach mental calculation strategies for these reasons, perhaps the most important incentive to teach mental calculation should be directed at the acquisition of more pragmatic, fundamental abilities for students.

To have the ability to mentally calculate arithmetic problems requires mathematical understanding. Relying on skills obtained via drill and practice exercises involving standard written algorithms will not make the grade when trying to mentally calculate 8 x 999. Knowledge of techniques to calculate 8 x 999 mentally by changing the problem to 8 x (1000 – 1) demonstrates an understanding of the distributive law (Hope & Sherrill, 1987). As cited in French (1987), Cockcroft states, "We may note that, although it is possible to practise written methods of computation as routines with little understanding of the underlying method, good mental methods have to be based on understanding…"(p. 39). Aze (1988) also supports the view that mental methods of calculation require mathematical understanding.

Everyday life demands a practical need for mental calculation skills for children and adults. Sauble (1955) states:

> They need to realize that the better they understand numbers, the more skill they possess in the operations of arithmetic, and the oftener they put these understandings and skills to productive use, the more adequately they will be able to meet the steadily increasing quantitative demands of daily living (p. 33).

Children and adults alike are always in situations where a quick mental calculation will prove a most handy and efficient tool to support or refute an estimation. Children at the checkout counter of the candy store or dad attempting to estimate the cost of topsoil to cover the lawn will undoubtedly attempt to figure out the estimation by way of mental means. Hope (1986a) cited evidence based on interviews from Wandt and Brown (1957) showing that when people encountered a situation requiring a calculation, people performed routine mental calculations 75 % of the time compared to the use of written computations which were performed 25 % of the time. These data were recorded over a 24 hour time period and the calculations were off-the-job calculations. Schall (1973) also cited the practical need of mental calculation. He discussed how children and adults encounter many daily activities requiring simple mental calculations such as making estimates, interpreting quantitative data, terms and statements.

Finally, use of mental calculation strategies helps students refine their problem solving skills. Hall (1954) described the relationship between mental calculation and problem solving. Atweh (1982) maintained that a mathematics curriculum that includes mental arithmetic should emphasize problem solving: “Developing skills in mental computation calls for the use of algorithms that are often rejected in paper-and-pencil computations (p. 55). In many instances, mental algorithms do not exist or are forgotten. Hence, mental computation also falls under the category of problem solving” (p. 55).

It is very important to teach children techniques in mental calculation. Certainly, there is an academic need. Mental methods will help children in the math classroom and other classrooms as well. The development of these skills will serve as a springboard for more complex arithmetic procedures they will encounter in their adolescent years.

Mental methods of calculation require that math principles are clearly understood and acquisition of these methods offer children an additional bank of means to find the exact answers or estimations to problem solving tasks. Moreover, there is a practical everyday need for the ability to calculate mentally and this need exists long into an individuals' twilight years. Clearly, mental calculation skills are important and applicable to many academic and practical situations. However, what does it mean to be considered "skilled" in mental calculation? How are these skills acquired? Do children need to be simply taught various strategies or is there more involved? What makes someone proficient in mental calculation?

Characteristics of Expert Mental Calculators

Interestingly, many experts seemed to have acquired their skills by way of a fascination with numbers or an exceptional interest in a game or task involving numbers. When asked how he remembered that 36^2 equaled 1296, Hope and Sherrill (1987) reported that one of their participants' answer related to his knowledge of probability and dice. Quoting the boy, Hope and Sherrill state, "In rolling dice you have a chance of one sixth of rolling a 1. So if you do that 4 times in a row, it would be one sixth to the 4th or 1 in 1296." (p. 109). Hope (1986b) provided additional data to support the opinion that the development of mental methods of calculation extends beyond classroom instruction. He provided various examples of how children develop their skills. For example, one boy, while thinking about numbers to pass the time on his bus ride home, discovered the following pattern which led to the ability to square very large numbers:

$9 \times 11 = 100 - 1 = 10^2 - 1^2$

$8 \times 12 = 100 - 4 = 10^2 - 2^2$

$7 \times 13 = 100 - 9 = 10^2 - 3^2$

and so on….until,

$1 \times 19 = 100 - 81 = 10^2 - 9^2$

Another characteristic shared by expert calculators relates to the way they manage memory. Hitch (1977) conducted experiments confirming that mental arithmetic errors occur because of rapid forgetting of temporary information. He noted that calculations consist of discrete stages, ordered depending on an individuals' particular strategy. The strategy determines the interval over which the initial and interim information must be stored. The "units, tens, hundreds" strategy (calculations are performed left to right) of multiplication is most taxing on short-term memory and therefore quite prone to error. The most efficient strategies are the ones that minimize the effects of rapid forgetting. The use of efficient strategies is a characteristic shared by skilled mental calculators.

To observe strategies and characteristics of skilled and unskilled individuals of mental calculation, Hope and Sherrill (1987) conducted a research study involving 30 high school students. Fifteen students were deemed skilled in mental calculation and 15 were unskilled. The results showed that the skilled students used greater variation of strategies to accomplish the calculation tasks presented to them. Also, skilled students frequently used methods that they felt "minimize forgetting." Skilled students recalled many more numerical equivalents. Numerical equivalents are automatic operation conversions into numbers. For example, if an individual knows that 5 x 5 is equal to 25, for that person 5 x 5 is a numerical equivalent to 25. A person who knows that 25 x 25 is equal to 625 knows the numerical equivalent of 25 x 25. The answer is known instantaneously and essentially requires no thought. The skilled students eliminated,

when possible, the carry operation. Another characteristic of the skilled students was the fact that they calculated left to right rather than the contrary which has shown to be more demanding on short-term memory. Finally, the skilled students tended to incorporate the interim calculations into a single unit (continuously retrieving a sum and updating it by adding a newly calculated product). Again, keeping a "running total" was less demanding on memory. Hunter (1978) provided information related to using memory efficiently. He states, "Experts have, through years of experience, acquired a vastly large fund of numerical equivalents upon which they can draw with speed and accuracy. Experts have also acquired a large repertoire of calculative methods which enables them to calculate in ways that make maximally effective use of the fund of equivalents and which keep interrupted working memory to a minimum. In short, experts build up vast resources of long-term memory, and they shift the burden of calculation onto long-term memory while, at the same time, minimizing their reliance on temporary memory" (p. 340). Calculations are done in large blocks or prefabricated segments which are handled by long-term memory and only minimal actual calculations are necessary.

Many experts have demonstrated an added incentive to understand numbers or number relationships. The have developed a self-interest that is, plain and simple, based on having fun with numbers. Expert calculators have developed a large repertoire of strategies to custom fit the task at hand. Different problems have different methods required to solve them and expert calculators have, at their disposal, many different strategies to efficiently solve a variety of problems. They are not locked into one laborious, inefficient method of trying to perform diverse calculations via the traditional written algorithm. They employ strategies that utilize memory in a most efficient way by

eliminating the carry operation, performing calculations from left to right and using a large store of numerical equivalents.

Mental Calculation Strategies

Thompson (1999), in an effort to support/advise teachers to meet the changing curricular demands of increased emphasis on the development of a child's mathematical mental calculation abilities, provided a comprehensive list of the most commonly used mental calculation strategies used for one digit number operations for primary school aged children. Thompson (2000) also produced a list of the most commonly used strategies for two digit number operations. Thompson (1997) used these strategies in his research study in which he attempted to determine if written methods could actually reflect mental calculation strategies. The focus of Thompson's work involved addition and subtraction operations only.

The research of Hope and Sherrill (1987) revealed a number of calculation strategies. As previously noted, the study participants included 15 students skilled in mental computation and 15 who were considered unskilled. Analysis of the student reports indicated that 3 methods and 4 strategies per method were used to solve the calculation tasks.

1) Mental pencil and paper – doing the same mental processes in one's head as he or she would conventionally do on paper.
    a) No partial product retrieved - i.e. 25 x 25 → "five times 5 is 25, carry 2, 5 x 2 = 10 plus 2 = 12….etc."

b) One partial product retrieved – i.e. – "5 x 48 is 5 x 8 = 40, carry 4, 24, 240. And 2 x 48 is 96, etc. NOTE: 240 was calculated digit by digit but 96 was retrieved as numerical equivalent.

c) Two partial products retrieved as numerical equivalents. 12 x 250. 2 x 250 = 500, 1 x 250 is 250, move over one, 3000!

d) Stacking – each partial product was completed digit-by-digit and visualized as a stacked arrangement. i.e. 8 x 999 is 72,72 and 72 right across.

2) Distribution- transforming one or more factors into a series of sums or differences.

   a) Additive distribution – each partial product is added successively to produce a running sum. i.e. 8 x 4211 = 8 x 4000 = 32000, 8 x 200 = 1600 and 8 x 11 is 88. Answer: 33688.

   b) Fractional distribution: applied when factor contained a "5" as a unit digit. i.e. 15 x 48 was calculated as 10 x 48 = 480 and half of that is 240 so the answer is 720.

   c) Subtractive distribution – used when students thought expressing the numbers as a difference made the calculation more tractable. i.e. 8 x 999 is the same as 8 x (1000 – 1) = 8000 – 8 = 7992.

   d) Quadratic distribution- The algebraic identity for the difference of squares $(x - y)(x + y) = x^2 - y^2$. So, students solved by the problem 49 x 51 by changing it to $50^2 - 1$.

3) Factoring – one or more factors in the task were transformed into a series of products or quotients rather than a series of sums/differences.

    a) General – factoring one or more of the factors before applying the multiplication law. i.e. 25 x 48 = (5 x 5) x 48. 5 times 48 = (5 x 40) + (5 x 8) and 5 x 240 is 1200.

    b) Half and double - This strategy is used when one factor is a multiple of 2. i.e. 12 x 15 equals 6 times (1/2) 30 (double) = 180.

    c) Aliquot parts – transforming one factor into a quotient.

    d) Exponential factoring – used to calculate products of power through the exponential rule. i.e. 32 x 32. 32 is 2 to the 5^{th} power, squaring this is two to the tenth power, "which I just know is 1024." For this person 2^{10} is a numerical equivalent of 1024.

Some of these strategies have been described in the work of Hazekamp (1986) and Atweh (1982). However, these authors identified several strategies unique to their respective reports. Hazekamp discussed the rules of multiplying by 5, 50 and 100:

"By 5 – Divide by 2, multiply by 10.	5 is 10 ÷ 2
By 50 – Divide by 2, multiply by 100.	50 is 100 ÷ 2
By 500 – Divide by 2, multiply by 1000.	500 is 1000 ÷ 2" (p. 119).

For example, the problem 364 x 50 is solved by dividing 364 by 2, yielding 182. Next, multiply by 100 to give 18,200.

Atweh provided an interesting strategy for a multiplication calculation that can be performed assuming two criteria are met. First, the units digits must add to ten. Second, the tens digits must be the same. If both of these criteria hold true for a problem, the

problem can be solved by multiplying the units digits to yield the last two digits of the product. Then, increase one of the tens digits by one, keep the other the same and multiply. The result is the first two digits of the product. i.e. 64 x 66… 6 x 4 = 24, the last two digits in the product….and 7 x 6 = 42…is the first two digits in the product. Therefore, the answer is 4224.

Clearly, many mental calculation strategies exist. A person does not have to be an expert to partake in the many available benefits of possessing the ability to skillfully calculate mentally. Note, however, knowledge of many strategies is one of the most fundamental characteristics of expert mental calculators. Although a person might have characteristics of experts, such as efficient use of memory or a vast store of numerical equivalents, those skills are irrelevant if a person does not posses knowledge of various strategies. Some people invent or acquire strategies on their own through number experiences, others learn them in school.

Considerations for Teaching Mental Arithmetic

Mental calculation strategies must be taught and the work must be an integral part of the instructional program (Atweh, 1982). Ideally, mental calculation should be taught early as to avoid students becoming dependent upon the use of standard written algorithms. Hazekamp (1986) states, "Special mental multiplying procedures need to be taught early and integrated into the multiplication program before students have almost mastered the written algorithm" (p. 124). Children need to be encouraged to think without the use of pencil and paper. Teachers need to try to intrinsically motivate students to want to understand meaning behind the statements they are trying to interpret (Flournoy, 1959). French (1987) claims, "Surely it is important that children are

encouraged to think about efficient and effective ways of doing such calculations, rather than encumbering them with half understood written algorithms or making them totally dependent on a calculator" (p. 39).

Teachers need to make themselves aware of the many different existing strategies for mental computation. They need to recognize that they, themselves, have strategies for mental calculation and so could their students. Knowledge of the diverse strategies that exist and the prospect of class discussions and sharing in the experiences of other students is, for teachers, a key aspect in providing all students with optimal chances to become skilled mental calculators. First, however, they must understand the influence they have on their students' individual learning styles. Bills (1999) reported that children form mental images, which they use for mental calculation, from the actions, words and materials of teachers. Mental calculation is a learned "skill" and it needs to be developed over time. While each child will develop his/her own style of what information to attend to during instruction, much similarity (of the way they form mental images) will exist because children form their images based on the representations of the teacher. This implies that teachers need to be aware of the images *individual* kids are using to exploit, build upon, or enhance the imagery that is particular to each student to allow the student to receive maximized benefit from their individual method. In other words, teachers must pay close attention to the needs of each student to know what individual strategy works best for each student.

When the teacher becomes familiar with the learning styles and calculation strategies used by all students, class discussions can provide an invaluable resource for teaching mental calculation. French (1987) cited recommendations for teachers to help

children develop their mental calculation skills. One recommendation is based on the value of class discussion:

> The variety of methods that children and adults use in doing mental calculations is very valuable, not to produce a "best method", but to encourage a flexible approach and make explicit the advantages and insights that come from considering alternatives. Much of the value of doing mental mathematics arises through discussing the methods used. If children are simply given a set of questions and then told the answers, a valuable opportunity for learning has been lost, if nothing more is said (p. 39).

The value of class discussions and participation from all students as key components of instruction in mental calculation techniques has been highlighted by other researchers (Aze, 1988; Hazekamp, 1986; Thompson, 2000).

One thread common to many skilled mental calculators is the fact that they all seem to share increased interest in something related to numbers or number relationships. Therefore, it would behoove teachers to try to instruct with the goal of stimulating interest. They must provide experiences that will mean something, something that will truly matter to the student. Students need to relate the language of mathematics to something that is tangible to them, to their lives. The information they receive must be practical. What is the significance of 34 x 57? Without context, the problem itself is of very little significance. However, what is the significance of "Do I have enough ingredients and money to double the recipe?" Or, what is the significance of "I wonder if I could ride my dirt bike to my friends house on one tank of gas?" Presented in a context based on reality, students will likely be more apt to desire to learn mental calculation

skills. On a daily basis, there are many opportunities for students to practice their skills. Teachers should make students aware of these opportunities and alert them to some of the possible places and times these events could take place such as in the lunch line or the mall. The data of Sauble (1955) included an interesting example of the application of mental methods to everyday life:

> "John (in the 4th grade) gave this illustration of a situation in which he estimated: 'I wanted to pay for movie tickets for two friends and myself. The tickets were 49¢ each. I had two dollar bills and a half dollar in my purse. I gave the ticket seller a dollar bill and the half dollar because I knew that 3 x 50¢ was $1.50.'" (p. 36).

Another way teachers can foster the development of mental calculation skills is to provide many diverse experiences and give children ample opportunities to practice their skills to help to keep them sharp. Atweh (1982) pointed out the need for experience and practice.

> "The need to develop the skills related to estimation and mental abilities has some very important implications to the practice of teaching middle school mathematics. Probably the most obvious implication is for the teacher to plan appropriate instruction. Just as in any other skill to be developed, students should have ample experiences and practice in situations using these skills. In other words, students need to be taught the different methods. It is not sufficient to hope that students would develop those skills when they need them" (p. 54).

In support of this position, Hazekamp (1986) provided examples involving calculations with items or characters that students could easily relate to such as: "Each bus carries 48

pupils. How many can four buses carry? Each box contains two dozen pencils. Will 25 boxes be enough to give 640 pupils one pencil each?" (p. 126-127).

Texts can serve as a powerful learning tool to assist children to develop their techniques. The degree to which texts provide experiences and practice in estimation, interpretation and computation varies. By way of an examination of six fifth grade arithmetic texts, the data from Flournoy (1959) revealed the following:

> Each of the six books provided some computation practice with estimating answers, some practice in estimating answers to word problems, and practice in interpreting graphs and scales. Those books providing similar types of exercises varied greatly in the amount of practice provided. For example, in providing computation practice in estimating answers, one book was found to provide nine pages with exercises of this kind. Three other books included only one page each on this type of practice. Only one textbook included exercises on interpreting quantitative statements using "reference measures" (p. 136).

Although these data were provided some time ago, the emphasis on mental calculation has not changed dramatically and, as a result, the texts have not improved in terms of providing sufficient experiences and practice in mental computation techniques. However, texts can be an invaluable tool as a means to expose children to situations for which mental methods are the logical means to solve the problem.

Finally, another practice teachers need to employ to help students develop mental calculation techniques and keep skills sharp is testing on a regular basis. Reys (1985) identified six considerations for assessing mental computation skills:

1) Keep tests short, probably ten to twenty problems.

2) Start with a narrow focus. In the beginning, concentrate only on one operation (addition, subtraction, multiplication, or division) with specific numbers (whole numbers, decimals, or fractions).
3) Emphasize the mental nature of these tests. For students accustomed to written algorithms, mental computation is a new experience.
4) Use different testing formats. For example, you might read each problem aloud and allow a brief interval (five to fifteen seconds, depending on the complexity of the problems) between problems. Another approach would be to write the problems on a transparency and display them individually for a few seconds as the mental computation is completed.
5) Build in some "nested" problems. These "nests" are groups of problems that are related to each other. For example, 4 x 8, 4 x 80, and 40 x 80 are "nested."
6) Use mental computation tests regularly. These tests don't take much to give or score. They keep students sharp on basic facts and remind them that mental computation is valued. Often the problems on the tests lead to additional learning as the problems are discussed.

When including instruction in mental computation strategies in a mathematics curriculum, there are many things to consider. Teachers need to know their students. They need to know individual learning styles. If the children have begun to develop calculation strategies on their own, teachers need to be aware of these strategies in order to legitimize them. Sharing strategies used by all classmates makes for good discussions and additional learning opportunities for all students. Students will have the opportunity to pick-up different strategies, perhaps a strategy that will lead them to more success in

their mental computation practice. Children need to “think mentally.” Teachers must stress the mental nature of these tasks so students realize that what they are doing is not connected to the standard written algorithms they have been used to. Students need many chances to practice mental computation. Teachers must direct them to situations that call for mental calculation strategies. Perhaps, as they perform successful calculations from time to time, they will find motivation within. It is important that teachers provide experiences that students can relate to and practice on their own. Students must be kept interested, as exceptional interest is a trait shared by skilled mental calculators. Teachers need to provide many, diverse experiences for their students and students need to know why and when mental computation is the best, most efficient option for the said calculation. Mental computation techniques must be frequently utilized so they are kept sharp and not forgotten. Finally, assessments must be given on a regular basis. Testing serves as a continuous reminder that mental computation is a valuable technique and these skills will need to be continually developed. Students need to know that, in the same way that sports or playing an instrument is a skill, so is the ability to appropriately employ techniques in mental calculation. Regardless of the activity, to become skilled requires a person to participate in learning experiences and practice. Dedication and a concerted effort to succeed will give students a practical, valuable asset to utilize in many everyday situations.

Discussion

The skill of mental calculation is a valuable trait to possess and it is useful in mathematics as well as other educational areas. Mental calculation provides the foundation for the development of higher level thinking skills. It provides children with a

new way to think about the number system and number relationships. Successfully performing efficient and effective mental calculations requires a thorough understanding of mathematic principles. It is not a mechanical, rote method of solving problems. Instead, students learn the principles behind mathematics. Children will achieve an understanding that addition, subtraction, multiplication and division are simply methods to combine or separate numbers. The successful application of mental methods requires a person to be skilled at solving problems and not simply adept at memorization of mathematical formulas.

There is a practical need for mental calculation strategies in everyday life. Throughout an individuals' lifetime, to a lesser or greater extent, the need for knowledge of mental calculation strategies will always exist. In terms of practical application, one of the most valuable skills children can take away from their elementary education is proficiency in mental calculation. Chances are, regardless of income or social status, there will be food to buy and bills to pay. The skill of mental computation is a handy, applicable asset. It is important that educators focus efforts toward early skill development and consistent instruction in mental calculation techniques.

Mental calculation strategies must be taught and they should be taught early so children do not become reliant on standard written algorithms as a means to solve the majority of problems they encounter. If skills are taught early in a child's education, children will not become dependent on the single, standard method. Instead they will be aware of a multitude of calculation options and they will appreciate the traditional method of calculation when they encounter it in the upper elementary grades.

Teachers need to be aware, and make all students aware of the many calculation strategies that exist. They must be in a position to comment and provide feedback for the strategies volunteered by all students. They have to encourage class discussions and pupil to pupil interaction so all students can benefit from the diverse strategies that exist.

Teachers must provide students with a large, diverse number of opportunities to practice their mental calculation skills. As argued in this paper, one of the most beneficial aspects of mental calculation ability is attributed to its practical application. Teachers must exploit this aspect of mental calculation because options to practice this skill occur everyday in the lives of students and if not utilized, a powerful learning opportunity is wasted. Therefore, teachers must make students aware of the myriad of ways they can practice this skill. This, in turn, will foster interest which is one trait common to many skilled mental calculators. Teachers can use practical examples in and out of the classroom. They can give students brief assignments to employ outside the classroom when they know they will be in a position to practice their skills.

Textbooks can provide students with good practice problems for mental calculation strategies. Since the emphasis on mental calculation ability has just recently begun to gain increased popularity, teachers need to examine textbooks closely. The research has shown that examples in texts for computation, estimation or interpretation varies greatly. Therefore, to ensure students receive ample chances to practice and develop their skills, teachers must analyze their materials in detail.

Finally, to help children develop their skills and keep them sharp, testing of these skills should be done on a regular basis. Testing helps students develop their skills but it also lets students know that this is a valued aspect of their mathematical education and

they should expect consistent work in this area and be prepared for testing. Testing is one of the most effective ways of assessing performance of all students.

The information presented here has implications for deaf students. Most of literature reviewed for this paper has been from studies involving hearing participants. Because of this fact, it is difficult to draw conclusions within the deaf population. However, it is not likely that the different mathematical achievement levels among hearing and deaf students can be attributed to lack of instruction in mental calculation strategies for the deaf students. This is true because, only recently has the emphasis (for education in general) begun to shift toward an increased focus on learning techniques in mental calculation. The research has shown the importance of teaching mental arithmetic. It is clear that children attain many benefits from mental arithmetic inside and outside the classroom. Also, it is clear that mental methods of calculation need to be taught. Therefore, deaf students certainly have much to gain from instruction in mental calculation strategies. Deaf students can, most likely, partake in the same benefits acquired by hearing students. However, this remains speculation in the absence of research specific to deaf students and mental calculation. Future research should include instruction in mental calculation for the deaf population. Comparison of deaf students instructed in mental strategies to students who were not instructed would be interesting. This would answer the question, "Do deaf students receive similar benefit as their hearing peers?" Future research should include an evaluation of students who received instruction in mental methods and who are considered to have an awareness of phonology (based on reading and memory skill) to a sample of students who also received instruction but are considered not to have a firm knowledge of phonology. This type of

study could lend insight into the influence of phonology on mental calculation ability. Perhaps, since instruction would be minimally influenced by language (compared to word problems or tasks more intimately linked to the English language), both groups would perform similarly. Naturally, many other variables would require careful consideration such as communication mode during instruction and mathematical abilities of the student sample. Nonetheless, this would make for a very interesting analysis because many studies have been focused on tasks in which it is easy to identify delayed language development as the catch all reason for imbalanced educational abilities among deaf and hearing. True, the mathematical information is coded via the articulatory loop for hearing individuals, but the mental nature of mental calculation strategies makes for an environment in which the impact of phonology is reduced compared to other areas of study such as reading comprehension and mathematical word problems. Data from future research will continue to provide clues to the reasons for the gap between achievement levels among deaf and hearing students. Regardless of the contributions of future research, it seems logical that deaf students can benefit from instruction in mental computation strategies. With proper instruction of mental calculation techniques, experiences and motivation, deaf students can raise their level of performance in mathematics. In turn, the skills acquired can be utilized for learning in other classes. These skills can be used in practical situations on a daily basis. Teaching children skills in mental computation is a very profitable investment for all students and professionals in the field of education.

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