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School of Printing Management and Sciences
Rochester Institute of Technology
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MASTER'S THESIS

This is to certify that the Master's Thesis of

Jui-lin Hsu

With a major in Printing Technology
has been approved by the Thesis Committee
as satisfactory for the thesis requirement for the
Master of Science degree at the convocation of

May 1989

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**A STUDY OF INK TRAPPING
COMPARING GRAVIMETRIC AND DENSITOMETRIC
METHODS OF MEASUREMENT**

by
Jui-lin Hsu

A thesis submitted in partial fulfillment of the
requirements for the degree of Master of Science in the
School of Printing Management and Sciences in the
College of Graphic Arts and Photography of the
Rochester Institute of Technology

May, 1989

Thesis Advisor: Mr. Sven Ahrenkilde

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ABSTRACT

In 1958, Preucil suggested an equation to calculate ink trap based on measurements of a printed sheet with a densitometer. Preucil's equation was based on the additivity rule and the proportional rule. However, these rules do not hold in all cases. Several alternative equations have been suggested. This study intends to examine the effectiveness of three ink trap equations using densitometry by means of comparing the calculated values to those measured by the gravimetric method. An attempt is also made to estimate the value of maximum printable density, D_m , in Hamilton's equation.

The theoretical basis of each equation is briefly introduced. The causes attributing to the failure of additivity and proportionality are discussed as well. Also reviewed are several methods for measuring ink trap, such as colorimetric, spectrophotometric, and magnetic methods, and factors involved in the efficiency of ink trapping.

In the experiment, two newsprint and one coated paper were printed with cyan, magenta, and yellow ink in a simulated wet-on-wet

condition on the IGT printability tester. Each ink had a specific tack. The second-down inks were transferred on the first ink layers with low, medium, and high thickness. The percent ink trapping on printed strips were then calculated by both the densitometric methods and the gravimetric method. The factors causing poor trapping and back-trap in the experiment are discussed.

Preucil's equation was found to correspond better with gravimetric trap values than Brunner's equation. However, both underestimated ink trap in the majority of the tested conditions. As the ink trap measured gravimetrically increased, the discrepancy of ink trap calculated by Preucil's equation increased. This applied to Brunner's equation as well, but not obviously to Hamilton's equation. Hamilton's equation proved to reduce the differences between gravimetric ink trap and densitometric ink trap significantly. A D_m value of 1.59 for newsprint A, 1.99 for newsprint B, and 2.5 for coated paper resulted in the greatest accuracy over the conditions tested. When the D_m value approaches infinity, Hamilton's equation becomes the equivalent of Preucil's equation.

The experiment suggests that a range of D_m values between 1.5 and 2.5 was suitable for the tested conditions. For future research, a larger selection of various types of papers and inks are needed to specify a D_m value for general conditions. It might also be useful to specify a theoretical maximum printable density for general

conditions, with which the maximum printable density of a given paper can be compared. Thus, the reproduction quality of the paper can be predicted before the printing is conducted.

Since the effects of back-trap and ink contamination influence the accuracy of Hamilton's equation in this experiment, studies for compensating this inaccuracy are suggested.

CHAPTER ONE

INTRODUCTION

In multiple-color printing, the control of primary color overlap is critical since any change in ink transfer will cause changes in the hue, saturation, and lightness of the overprint colors.¹ An accurate and convenient way of measuring ink trap could help control the printed color. However, the current methods used in measuring ink trap give different trap values.

Ink trap is defined as a ratio--expressed in percentage--in which the ink film thickness of the overprint on a previously printed ink layer is compared to the ink film thickness directly on the substrate.

Rather than using physical ink film thickness measurements, however, the most commonly used method in the printing industry is measuring ink trapping by densitometry. Due to time and cost considerations, ink trapping is easier to calculate from the densities of solid ink areas by using a reflection densitometer.

Ink transfer takes place from the inked printing element or blanket to the print substrate. The substrate may or may not already be covered with wet or dry ink film. The term, ink trap, as defined in Jorgensen's article², describes the amount of ink transferred onto wet or dry ink films already present on the print substrate.

Usually, the second and subsequent inks are transferred onto the preceding ink before it is dry. This is called "wet-on-wet ink trapping." Similarly, the transfer of wet ink to dry ink is called "wet-on-dry ink trapping".

The purpose of this study is to compare densitometric measurements of ink trapping to that of gravimetric measurements of ink trapping. The densitometric methods to be compared include Preucil's equation, Brunner's equation, and Hamilton's equation. The gravimetric method measures the physical ink trapping and that will be the point of reference in this investigations because it represents realistic ink trapping. Since Hamilton suspects the paper property has an effect on the accuracy of calculating ink trap by Preucil's equation, it is expected that Hamilton's equation will either estimate ink trap close to that measured by the gravimetric method or that it can be further modified for better correlation to it.

Preucil³ was the first person to write an ink trap equation based on densitometric measurements on the printed sheet. According to

Jorgensen⁴, there are two reasons for this approach, rather than expressing the ink trap as a percentage of the ink film on the blanket before impression. The first reason is that no method exists to measure the ink film on the blanket. The second reason is that there is no simple relationship between the film's thickness on the substrate and its reflection density due to various other factors.

Preucil's equation was based on two assumptions. The first assumption was that the density of the combined ink areas is equal to the sum of the densities of the individual inks. This is also known as the additivity rule. The second assumption was that the ink film thickness is proportional to its density.

Since these assumptions do not hold in all cases, many alternative equations for calculating ink trapping have been suggested. Several studies have examined these equations theoretically. Yet, the actual effectiveness of the alternative equations is still unknown.

This study will concentrate on the latest of these equations, Hamilton's equation, which takes into account the maximum printable density on the given substrate.⁵ This author believes that additivity failure in Preucil equation should be compensated for the substrate's characteristic. In his article, Hamilton explained this.⁶ Lawphongpanich in his master's thesis also indicates that paper

properties have a large effect on ink trap measurements.⁷ The literature is discussed in Chapter Three.

In addition to Hamilton's equation, this study will also compare Preucil's equation and Brunner's equation with the gravimetric method, which measures the physical amount of ink trap. The gravimetric method weighs the printing plate before and after printing to find the weight of ink transferred onto the substrate. Then the weight of ink transferred to the substrate is converted to ink film thickness for calculating ink trap.

Several methods found in the literature regarding measuring ink trap will be discussed in Chapter Two. Also discussed will be the factors involved in additivity failure and ink trap. Chapter Three reviews each densitometric equation. The hypotheses and methodology for this study are outlined in Chapter Four and Chapter Five. In Chapter Six and Seven, the results and conclusions are discussed.

FOOTNOTES FOR CHAPTER ONE

1. Jorgensen, George W., "Control of Color on Press: Overprints", GATF Research Project Reports, No.118, 1983, p. 3-4
2. Ibid
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4. Jorgensen, George W., "Review of Ink Trap Studies", GATF Research Reports, 1981, p. 49
5. Yule, John A. C., Principles of Color Reproduction, John Wiley & Sons, Inc., 1967, p. 230 231
6. Hamilton, John F., "A New Ink-trap Formula for Newsprint", TAGA Proceedings, 1986, p. 158-165
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CHAPTER TWO

THEORETICAL BASIS

This chapter discusses the variables contributing to the additivity failure, the methods for measuring ink trapping, and the factors contributing to it.

Variables Causing Additivity Failure

Using reflection density of the overlap and single layer ink areas to calculate percent ink trap is based on two assumptions. These assumptions are that the density of a combination of inks is equal to the sum of the densities of the individual inks, and that the ratios of two ink film thicknesses and their densities are proportional. In practice, these two assumptions are not true. Yule and Clapper¹ found that reflection density is not additive. Nor is the density of an ink layer proportional to its film thickness². This is why ink trap measured with a reflection densitometer is in error. A brief summary of the optical factors causing this problem is described below.

1. First Surface Reflections. When light strikes paper fully concentrated with ink, some of the light usually is reflected in several directions. For a matte surface, light is diffused in all directions so that part of it reaches the eye or densitometer's photocell, thereby limiting the obtainable reflection density considerably. Therefore, for ink printed on matte surfaces the densities of the overprinted areas will be greater than if the individual ink films and the overlaps had the same gloss.

2. Multiple Internal Reflections. Light, upon entering an ink film, is expected to go through the ink layer twice, once on the way in and once on the way out. However, a large proportion of the light reflects back and forth between the paper surface and the ink film surface. More light will be absorbed by the ink and substrate due to these multiple internal reflections. This contributes to higher density readings. This factor is less significant for thick ink films than for thin ink films.

3. Opacity. Because the refractive indices of the pigments and vehicles in printing inks are different, the printed ink films will exhibit varying degrees of opacity. Opaque ink layers will have lower reflection densities than transparent ink layers at the same ink film thickness.

4. Ink Transfer. The ink film transferred to a previously printed ink layer can be thinner or thicker than that transferred to the unprinted paper. The result is under or over trapping. The sum of the the ink film thickness of the individual inks will not equal to that of the overprints.

5. Back-transfer Effects. In wet-on-wet printing, some of the first-down ink may be removed. This results in two effects. The first effect is to give lower reflection density than would be expected. The second one is that the first-down ink contaminates the color of the second-down ink.

6. Spectral Characteristics. The spectral response of the densitometer influences the densities of the overprints. Different filters may result in perfect additivity, a lack of additivity, or superadditivity. Perfect additivity, a lack of additivity, or superadditivity means that the density of the overlapping inks is equal to, less than, or more than the sum of the densities of the individual ink. Measuring ink trap with a monochromatic light avoids this type of error.

Ink Trap Measurements

Several methods have been used or proposed for measuring ink trap in the past. These methods can be separated into categories:

optical, gravimetric, and miscellaneous. These methods found in literature³ are summarized below.

Optical Methods

1. Densitometry. A common approach to measuring a single ink layer is to compare its reflection density to some reference, which might be its reflection density on the OK sheet. When we measure higher or lower densities on production sheets, we attribute that to thicker or thinner ink lay down.

To measure overprinted ink layers, the additivity rule is used. Preucil's equation, which will be discussed in Chapter Three, is the most commonly used in the printing industry. The percent ink trapping determined by this equation can be inaccurate due to the factors found in Yule's and Clapper's study⁴, discussed earlier.

Some other concerns about the variations in regard to densitometric measurements were mentioned in Jorgensen's article. Jorgensen states that discontinuities or minute holes in the ink film will cause a lower density reading than the same amount of ink evenly distributed. Furthermore, a lower density reading can also be caused by leakage of infrared light to the densitometer's photocell due to the absorption of organic ink pigments by infrared light.

Hull⁵ suggested that using a polarizing densitometer to eliminate some of the effect of ink gloss will give more accurate density readings.

2. Spectrophotometry. A spectrophotometer is an instrument for measuring the relative intensities of the light in different parts of a spectrum. The advantage over other instruments is that it measures monochromatically; therefore, the rule that density is proportional to the film thickness holds. This eliminates the problems caused in densitometry by wide-band filters and infra-red leakage.

However, additivity failure is still a problem. Some spectrophotometer designs will pick up fluorescent or bronze light leading to error measurements. Further details about these disadvantages are explained in Jorgensen's article⁶.

3. Colorimetry. A colorimeter is an instrument for specifying color. The color is specified as three numbers relating to the color's hue, saturation, and lightness in some given color system. The system might be Hunter's L, a, b, the CIE L, a, b, the CIE L*, a*, b*, or others. Malikho⁷ intended to calculate the ink trap from the L* values, the lightness component in the CIE L*, a*, b*, system, by using the Preucil equation. The result was not valid because it produced a very wide range of yellow-magenta trapping which contradicted the insignificant color differences.

Gravimetric Methods

In this method the amount of ink transfer is measured in terms of ink film thickness or the weight of ink per unit area. The plate is inked and weighed before and after impression to the paper. From the weights of the plate by itself, the plate with inking, and the plate after impression, the weight and percent of ink transferred to the paper can be calculated.⁸ In this study, the gravimetric method will be applied on the IGT (Instituut voor Grafische Techniek) printability tester. Detailed procedures about using this method in this study will be described in Chapter Five.

Miscellaneous Methods

Yelmgren⁹ mentioned mechanical, magnetic, dielectric, and radioactive methods of measuring ink film thickness. Many of these are used on thick film layers, like paint coatings. They are not very suitable for measuring the ink films in lithography where ink film thicknesses are on the order of magnitude of microns.

Saleh¹⁰ provided a method measuring ink film thickness using an X-ray fluorescence spectrometer which counted the amount of heavy metal in the ink and then converted the counting number to the ink film thickness by a calibration chart. However, this is not feasible in

printing since heavy metal must be present in the ink, but absent from the paper.

Factors Affecting Ink Trapping

Factors affecting ink trapping include paper, ink, and press interactions. Some of these are described below.

1. Ink Tack. Ink tack is a measurement of the force required to split liquid films, such as an ink film, positioned between two solid surfaces to which it adheres. Frequently, press operators refer to these properties as the stickiness of an ink. How the ink splits depends on the adhesive force of the two contacting objects and the tack of the ink.

From Carlson and Lindberg's study¹¹ of the effect of the ink tack, the percentages of ink transferred to three types of paper increased about 0.40 - 0.50 percent, when the tack increased by one unit. In wet-on-wet printing, each successive "down" ink should have slightly lower tack than the preceding one. This requirement is to produce a surface that has a greater chance of trapping the second ink.

2. Ink Viscosity. Ink tack and viscosity are two independent ink properties. An ink with high tack can be of low viscosity and vice versa. The viscosity of a liquid is a constant which refers to the force

restricting the liquid to flow. There are various types of viscometers for measuring the viscosity.

In Lindberg's study¹², the best ink transfer for news inks was obtained when the first ink printed had the highest tack and viscosity values. However, for letterpress inks, the best result occurred when the first ink had higher tack, but lower viscosity than the second one.

Karttunen and Oittinen¹³ found that in two-ink printing, the smaller the viscosity difference, the greater the amount of the second-down ink was transferred. When the viscosity of the second ink was raised, the amount of the second ink transferred decreased and more back-trapping occurred.

3. Time Effect. The time interval is the period between transfer of two successive ink lay downs, one on top the other. Tollenaar and Ernst¹⁴ studied the effect of time intervals on ink trapping and found that the shorter the time interval, the less the ink transferred.

4. Ink Film Thickness. In Tollenaar and Ernst's¹⁵ study, the film thickness of the second ink was kept constant while the amount of the first ink was varied. As the amount of film thickness of the first ink increased, a lesser amount of the second ink was transferred.

Carlson and Lindberg¹⁶ in their study found that when the amount of the second ink was increased, the ink transfer was increased. But, the ink transfer did not change when the amount of both inks were equal and equally increased.

5. Ink Temperature. Oittinen and Karttunen¹⁷ investigated this effect and found that the transfer of the first ink increased when the temperature was raised, while the transfer of the second ink was decreased.

6. Ink/water Balance. Ink/water balance influences the rate of the ink flow to the paper as well as the ink trap onto the paper. Neuman and Almendinger¹⁸ found a very large decrease in the reflection density of a single ink layer on paper as the water feed rate increased. Jorgensen¹⁹ observed that in multi-color wet printing, the cumulative water in ink may interfere with the ink transfer onto paper during impression.

FOOTNOTES FOR CHAPTER TWO

1. Yule, J.A.C., and Clapper, F. R., "Additivity of Ink Density in Multicolor Halftone Printing", TAGA Proceedings, 1956, Part B, p. 153-166
2. Billmeyer, Fred W., and Saltzman, Max, Principles of Color Technology, John Wiley & Sons Inc., Second Edition, 1981, p. 10-11
3. Jorgensen, G. W., "Review of the Ink Trap Studies", GATF Research Reports, 1981, p. 49-53
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12. Tollenaar, D., and Ernst, P. A. H., "Tack Experiment for Wet-On-Wet Printing", Eighth International Conference of Printing Research Institute, Helsinki, Finland, 1965, p. 4
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14. Oittinen, Pirkko, and Karttunen, Simo, "Effect of Temperature on Wet-On-Wet Transfer", Graphic Arts in Finland, no. 2, 1972, p. 11-22
15. Tollenaar and Ernst, p. 6
16. Carlson and Lindberg. p. 67
17. Oittinen and Kurttunen, p. 28
18. Neuman, C. P., and Almendinger, F. J., "Experimental Model Building of the Lithography Printing Process, GATF Research Reports, 1980, p. 87-95
19. Jorgensen, G. W., "Control of Ink-Water Balance", GATF Research Reports, No. 113, 1981

CHAPTER THREE

LITERATURE REVIEW

This chapter reviews the literature that discusses the densitometric equations for calculating ink trapping. Densitometric methods for measuring ink trapping are the most widely used in the printing industry. A basic understanding of these equations is necessary for this study.

Preucil's Equation

The most commonly used densitometric ink trap equation is Preucil's equation¹, also known as GATF ink trap formula. It was based on the assumptions that ink density and ink film thickness exhibit a linear relationship, and that the density of combined ink films is equal to the sum of the densities of the individual inks. While being aware that effects, such as a change in gloss between single and two-layer ink films, could influence the trap value, Preucil termed the value given by this equation "apparent trap"². Other effects causing error in apparent trap calculation were discussed in Chapter Two.

The percent apparent trapping is calculated from the reflection densities of three areas: the solid area of the first ink printed on the substrate, the solid area of the second ink on the substrate, and the overlapping area of the two inks. The complementary filter of the second ink on a substrate is used for measuring density. For example, in a cyan-magenta-yellow-black ink down sequence, a magenta-yellow overlap would use a blue filter to measure the magenta solid, yellow solid, and magenta-yellow overlap solid. The equation is shown in below:

$$\% \text{ apparent trap} = \frac{D_{21} - D_1}{D_2} * 100 \quad (3.1)$$

D_1 = the density of the first ink on the paper

D_2 = the density of the second ink on the paper

D_{21} = the density of the overlapping area of the two inks

Jorgensen³ commented that this equation is seldom applied directly to three or more overprint ink layers. This is because the cumulative additivity failure errors usually become too large to be reasonable. Instead, the trap of the third and successive ink are calculated from their two-color overprints using this equation.

Childers' Equation

Childers proposed another ink trap formula in 1980. He states that "the errors occur because the currently used numbers represent

ratios of logarithms. The accurate answer for percent ink trap must be calculated from antilogarithms instead,"⁴ and "discussion of additivity failure would seem to be pure obfuscation. A good reflection densitometer will account for additivity failure automatically, in the remote possibility that it becomes a nontrivial factor in calculations. It is not a consideration with normal process ink densities."⁵ Childers' equation is as follows:

$$\% \text{ antilogarithm trapping} = [10^{D_{21} - D_1 - D_2}] * 100 \quad (3.2)$$

D_{21} = the density of the overlapping area of the two inks

D_1 = the density of the first ink on paper

D_2 = the density of the second ink on paper

Elyjiw^{6,7} criticized that this equation does not compensate for additivity failure. Instead, the calculated percent trap was the density that added to the two-color overprints to make the trapping 100 percent. Elyjiw also pointed out theoretical errors which applied to Childers' equation.

Hamilton⁸ suggested that this equation should be defined as the percentage of the expected reflection by the overlapping inks.

Brunner's Equation

In 1984, Brunner⁹ proposed an ink-trap equation. He said, "The system Brunner trapping formula allows trapping to be expressed as effective dot area coverage, the same terms as used for halftone dot gain, slur, doubling."

$$\% \text{ trap} = \frac{1 - 10^{-(D_{21})}}{1 - 10^{-(D_2 + D_1)}} * 100 \quad (3.3)$$

D_{21} = the density of the overprint of two inks

D_2 = the density of the second ink on paper

D_1 = the density of the first ink on paper

This equation is similar to the Murray-Davies equation. It interprets percent trap as an effective dot area of the two-color patch.

Hamilton¹⁰ commented that this equation should be defined as the percentage of the expected absorption by the overlapping inks.

Hamilton's Equation

In 1986, Hamilton¹¹ derived an equation for calculating ink trap on newsprint, for which he thought Preucil's equation often produces numbers that are much lower than they should be, when judged in comparison to the overprint color. So Hamilton modified Preucil's equation based on Yule's model¹², which calculates the density of a color mixture on paper. Hamilton's equation seems reasonable and

very likely to reduce the error in ink-trap calculation by densitometry. Since this author tends to agree with Hamilton's conjecture, this study will investigate the accuracy of Hamilton's equation. The derivation of Hamilton's equation follows.

Yule's model for the density of overprint area, which already took into account the additivity failure reads as follows:

$$D = K \left[1 - \left(1 - \frac{D_1}{K} \right) \left(1 - \frac{D_2}{K} \right) \dots \left(1 - \frac{D_n}{K} \right) \right] \quad (3.4)$$

D = the density of the overlapping area of inks

K = the density at the point of convergence, which means the maximum printable density for a given paper

$D_1 \dots D_n$ = the densities of the individual ink

Hamilton applied the above model to a two-ink overlap area as:

$$D_{21} = D_2 + D_1 - \frac{D_2 * D_1}{D_m} \quad (3.5)$$

D_{21} = the density of the overlapping area of inks

D_2 = the density of the second ink on paper

D_1 = the density of the first ink on paper

D_m = refer to K as denoted above

The modified Preucil equation--Hamilton's equation is:

$$\% \text{ trap} = \frac{\log \left(1 + \frac{D_{210} - D_{10}}{D_m - D_{210}} \right)}{\log \left(1 + \frac{D_{20} - D_0}{D_m - D_{20}} \right)} * 100 \quad (3.6)$$

D_{210} = the density of the overlapping area on paper

D_{20} = the density of the second ink on paper

D_{10} = the density of the first ink on paper

D_0 = the density of paper

D_m = the maximum printable density of the given paper

Formula(3.5) compensates for the additivity failure when densities, D_1 and D_2 , are high and the printable density range, D_m , is short, which is true for newsprint. For coated paper and some uncoated papers, the densities are relatively small compared to the printable density range. Formula(3.5) compensates less for these cases. As mentioned in Yule's book¹³, the effect of combined optical factors on additivity failure depends mainly on the nature of the paper. Furthermore, several studies, which are stated in the following paragraph, proved that the apparent percent trap calculation for newsprint has more discrepancies than for coated paper. The details of the derivation of Hamilton's equation are presented in his paper.

Related Studies in Ink-trap Measurement

Lawphongpanich¹⁴ compared densitometric methods, which were Preucil's and Childers' equations, with gravimetric measurement. The result showed that neither Childers' equation nor the use of a polarizing densitometer yielded better accuracy than Preucil's equation when compared with the gravimetric method. In spite of a significant difference between the apparent and gravimetric trapping both had similar trapping graphs for coated paper. Lawphongpanich concluded that any ink-trapping formula should take into account the paper characteristic.

Earlier, Chen and Eldred¹⁵ conducted an experiment on a Vandercook proof press for comparing Preucil's equation with gravimetric measurement on different papers. Their conclusion was that densitometry can over-estimate or under-estimate the second-down ink film thickness depending on the substrate.

Field's experiment¹⁶ transferred inks onto clear film substrate on a proof press. This applied the concept of transmission density measurements but measured by reflection densitometry. He found that more absorptive substrates, like newsprint, would probably produce lower percent trap values. Preucil's, Brunner's, and Childers' equations were also theoretically evaluated in his paper.

FOOTNOTES FOR CHAPTER THREE

1. Preucil, F., "Color and Tone Errors of Multicolor Process", TAGA Proceedings, 1958, p. 75-100
2. Ibid, p. 176
3. Jorgensen, G.W., "Review of Ink Trap Studies", GATF Research Reports, 1981, p. 50
4. Childers, Warren, "Experts Shows Math Path to Avoid Ink Trap Trap", Graphic Arts Monthly and the Printing Industry, Dec. 1980, p. 63-64
5. "Final Words on Ink Trap", Letters by Childers, and Elyjiw, Graphic Arts Monthly and the Printing Industry, Sep. 1981, p. 34, 38
6. Ibid, p. 22, 24, 26
7. Elyjiw, Zenon, "More on ink Trap", Graphic Arts Monthly and the Printing Industry, Sep. 1981, p. 34, 38
8. Hamilton, John F., "Ink Trap: The Moving Target", TAGA Proceedings, 1985, p. 397-403
9. "Cromalin Offset Com Guides/System Brunner", Du Pont Publication, 1984, p. 30
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11. Hamilton, John F., " A New Ink-trap Formula for Newsprint", TAGA Proceedings, 1986, p. 158-165

12. Yule, J., Principles of Color Reproduction, John Wiley & Sons, Inc., 1967, p. 231
13. Ibid.
14. Lawphongpanich, Kongsak, "The Comparison of Densitometric Measurement with Gravimetric Measurement of Wet-on-Wet Ink Trapping", Master Thesis in Rochester Institute of Technology, July, 1981
15. Chen, John H., and Eldred, Nelson R., "The Comparison of Densitometric with Gravimetric Measurement of Ink Trapping", GATF Research Reports, 1972, part B, p. 243-257
16. Field, Gary, "Ink Trap Measurement", GATF Research Reports, 1985, p. 383-396

CHAPTER FOUR

HYPOTHESES

The following are the problems to be studied in the form of research questions:

1. What is the percent ink trapping measured by each of four methods, including the Preucil, Brunner, Hamilton equations, and the gravimetric method, for each type of paper?
2. Is the percent ink trapping produced by each of the four methods different from each other?
3. Do Preucil's, Brunner's, and Hamilton's equations based on densitometric measurement predict the percent ink trap as well as the gravimetric method?
4. What are the effects of paper grade, for example, coated, uncoated, and newsprint, on the ink trapping measurements with each of the densitometric methods?

5. Does the thickness of the second ink layer overprinted on the first ink layer affect the ink trapping measurements with each of the densitometric methods?
6. In predicting ink trap, does Hamilton's equation better correlate with gravimetric method than Preucil's equation and Brunner's equation?
7. How different is the calculated D_m value from the generally known value of the maximum printable density on each kind of paper?

It has been proven that Childers' equation does not measure ink trap more accurately than Preucil's equation. It will not be necessary to duplicate this finding in the current study.

Hypotheses:

1. There is a significant difference between the gravimetric method and Preucil's equation. This will be true as comparing Brunner's equation with the gravimetric method.
2. On newsprint, the conventional densitometric equations will produce significantly different ink trapping values when compared with those measured by the gravimetric method.

3. As the ink film thickness of the second ink on the first ink layer gets thicker, the conventional densitometric equations will produce significantly different ink trapping values when compared with those measured by the gravimetric method.

4. The D_m value in Hamilton's equation can be specified or the equation can be further modified to produce an ink trapping value which is closer to the ink trapping value measured gravimetrically than that calculated by Preucil's equation in various trapping conditions and papers.

CHAPTER FIVE

METHODOLOGY

The main purposes of this study were, first, to understand the effectiveness of Preucil's, Brunner's, and Hamilton's equations relative to the gravimetric method in various trapping conditions; and, second, to specify the D_m value for Hamilton's equation in order to predict ink trapping values similar to those obtained gravimetrically. Densitometric ink trapping was calculated by equations with the density readings measured on printed strips. Gravimetric trapping was obtained through several mathematical steps. The independent variables of this experiment were the tack of three inks, the three types of papers, and the three measurement methods. The dependent variables were the percentage of trapping and the calculated D_m values in Hamilton's equation.

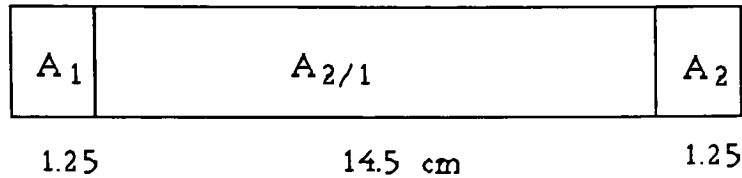
A set of offset inks was used in this experiment. The first task was to determine the specific gravity of the inks. The specific gravity was needed for the calculation of ink film thickness. Ink tack was later measured on a Thwing-Albert inkometer. Tack readings of

each ink over several time intervals are shown in Appendix A. The original yellow ink was mixed with 9.5 percent of the "Les Tack" to make the tack significantly lower than those of the cyan and magenta inks. Appendix B on page 101 shows that the cyan ink had the highest tack, and that the magenta and yellow inks had the medium and the lowest tack respectively. Ink characteristics are shown in Table 3 on page 35.

Two kinds of newsprint and one coated paper strips were printed. The experiment centered on newsprint since Hamilton's equation was introduced to calculate more realistic ink trap values for newsprint. Each paper was subjected to several tests to characterize each paper type. These results are shown on page 35.

The printing process was accomplished on an IGT printability tester in a wet-on-wet printing condition. Each color ink was first printed on each type of paper strip with various amount of ink to obtain the ink transfer curves. One out of nine ink-overlap-sequences was then completed on each paper. The ink-down sequences/ink tack combinations were: 1) yellow-yellow (low-low), 2) yellow-magenta (low-medium), 3) yellow-cyan (low-high), 4) magenta-yellow (medium-low), 5) magenta-magenta (medium-medium), 6) magenta-cyan (medium-high), 7) cyan-yellow (high-low), 8) cyan-magenta (high-medium), 9) cyan-cyan (high-high). The geometry of the printed strip is as diagrammed in Figure 1. A_1 and A_2 , are the areas

on which density readings could be taken for calculating ink trapping densitometrically.



A_1 : the area of the first-down ink
directly printed on paper

A_2 : the area of the second-down
ink directly printed on paper

$A_{2/1}$: the area of two inks overprinted

Figure 1. Printed Paper Strip

For understanding the effectiveness of each ink trapping equation under various trapping conditions, the second-down ink was applied in low, medium, and high amounts onto the first-down ink, while the first-down ink was kept constant. The amount of the second ink applied was compared with SWOP (Specifications for Web-Offset Publications) densities. The density of the medium amount would be approximately SWOP density, and the low or high amount would be under or over SWOP densities.

Each printing condition was repeated three times in order to provide adequate replications. Percentage of ink trapping was calculated by both gravimetric and densitometric methods for each printed strip. The D_m value of Hamilton's equation, which yields the closest trapping values compared with those measured by the gravimetric method, was then defined for each type of paper. The discrepancy between calculated ink trap by densitometry and those gravimetrically was expressed as standard error for comparison.

Tables 1 and 2 on page 33 and 34 show the experimental design.

A variety of graphs were presented to analyze the results. All discussions were based on the average trapping value of three replications in each printing condition for easier interpretation.

A detailed description of all materials and procedures follows.

Table 1. Experimental Design

[illegible]

Table 2. An Enlargement of Part of Table 1

[illegible]

Specification of Materials Used

Table 3. Ink Properties

Ink	Brand	Tack (g/m) (1st min.)	Specific Gravity (g/cm ³)
Cyan	Superior	23.5	1.0524
Magenta	Sinclair & Valentine	14.9	1.0222
Yellow	Marathon	8.4	0.9945

Table 4. Paper Properties

Paper	Basis Weight (lb.)	Thickness (inch)	K & N Absorption	Porosity (c.c.gas/min.)	Smoothness (c.c.gas/min.)
Newsprint A	30	2.5E-3	67.10	88	34
Newsprint B	30	2.5E-3	52.49	10	36
Coated	90	3.9E-3	19.80	0	7

Specification of Equipment Used

K & N Ink; IGT Pipette

Porosimeter: manufactured by Sheffield corp.

Smoothchek: manufactured by Sheffield corp.

Micrometer: made by Testing Machines, Inc., accuracy to 0.0001 inch.

Mettler Analytical Balance: capacity to 160 grams; accuracy to 0.0001 grams.

IGT Tester: model A2, made in Holland by IGT Amsterdam.

IGT inking unit: model AE

Gretag D-186 Densitometer: Filter set 47B according to DIN 16536 without polarization; reproducibility to ± 0.01 D.

Procedures for Generating Ink Transfer Curves

Ink transfer curves, which show the relationship between ink film thickness on the disc and that would be transferred on the paper, were used to estimate the ink film thickness of the second-down ink that printed directly on the paper.

Gravimetric trapping is calculated as the ink film thickness of the second-down ink that is on top of the first-down ink (IFT₂) divided by the film thickness of the second ink transferred directly on paper (IFT₁). IFT₂ was calculated from the experimental data, as explained later in Sample of Calculating Gravimetric Trapping in Appendix D on

page 113. However, IFT_1 needed to be estimated from ink transfer curves.

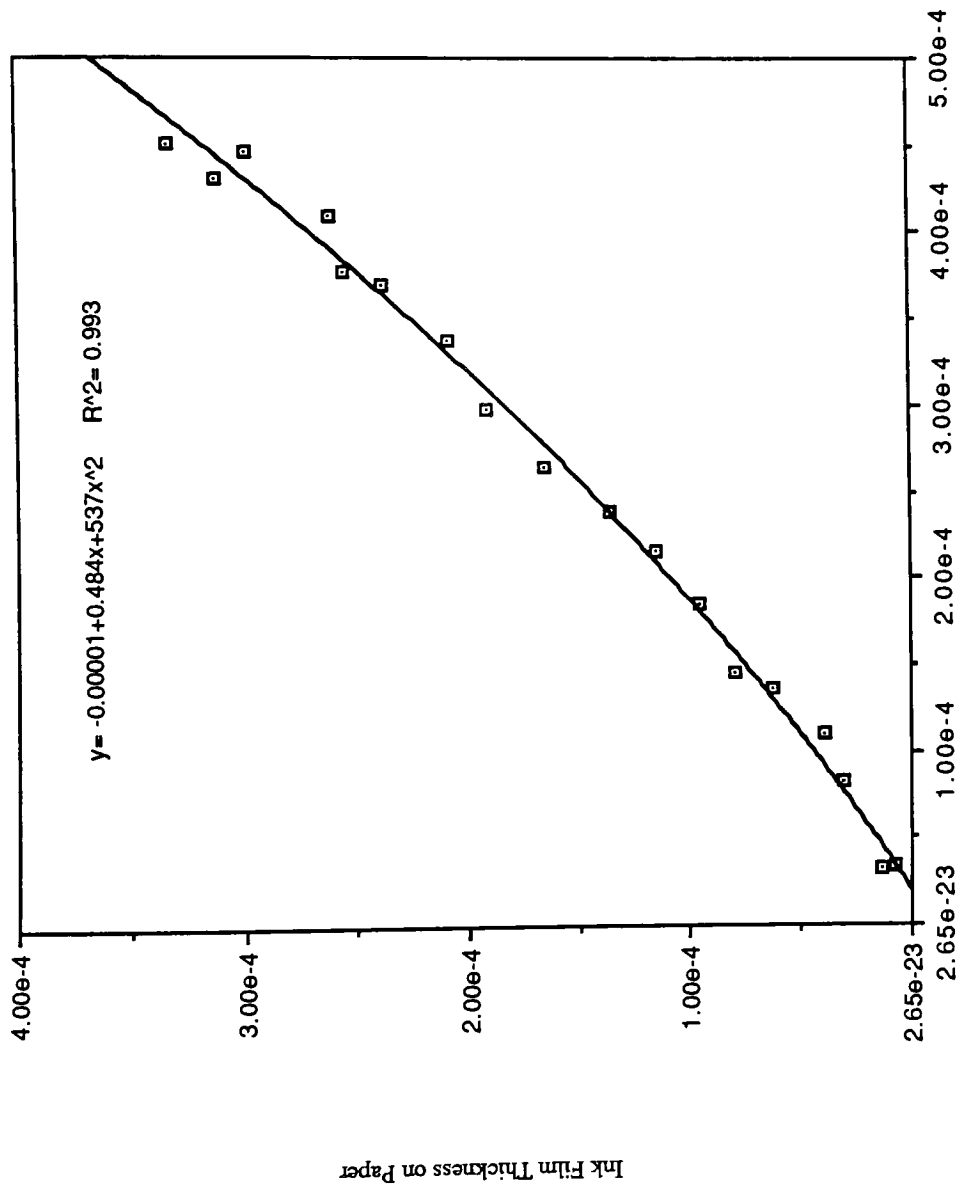
Each color ink was transferred to each type of paper on the IGT tester in incrementally increasing ink film thickness, from a very thin film thickness to a relatively heavy film thickness. Therefore, the range of ink film thickness on paper would cover every possible value of IFT_1 to be estimated.

These curves and regression equations are shown from page 38 to 46. Row data, the sample for calculating ink film thickness, and regression information can be found in Appendices C to E.

Procedures for Overlapping Inks with the IGT Printability Tester

A specific amount of ink was metered onto the inking unit with a pipette which is scaled from zero to two cubic centimeters. The ink was allowed to distribute evenly on the inking unit for seven minutes.

A printing disc was weighed with an analytical balance. A small piece of tape was applied across the disc, covering approximately 5 cm of the disc's circumference. By properly positioning the uninked area of the disc, a printed strip like Figure 1 on page 31 would result. The disc was then inked-up on the inking unit for one and half minutes. The tape was removed and the disc was then weighed



Ink Film Thickness on Disc

Figure 2. Yellow Ink on Newsprint A

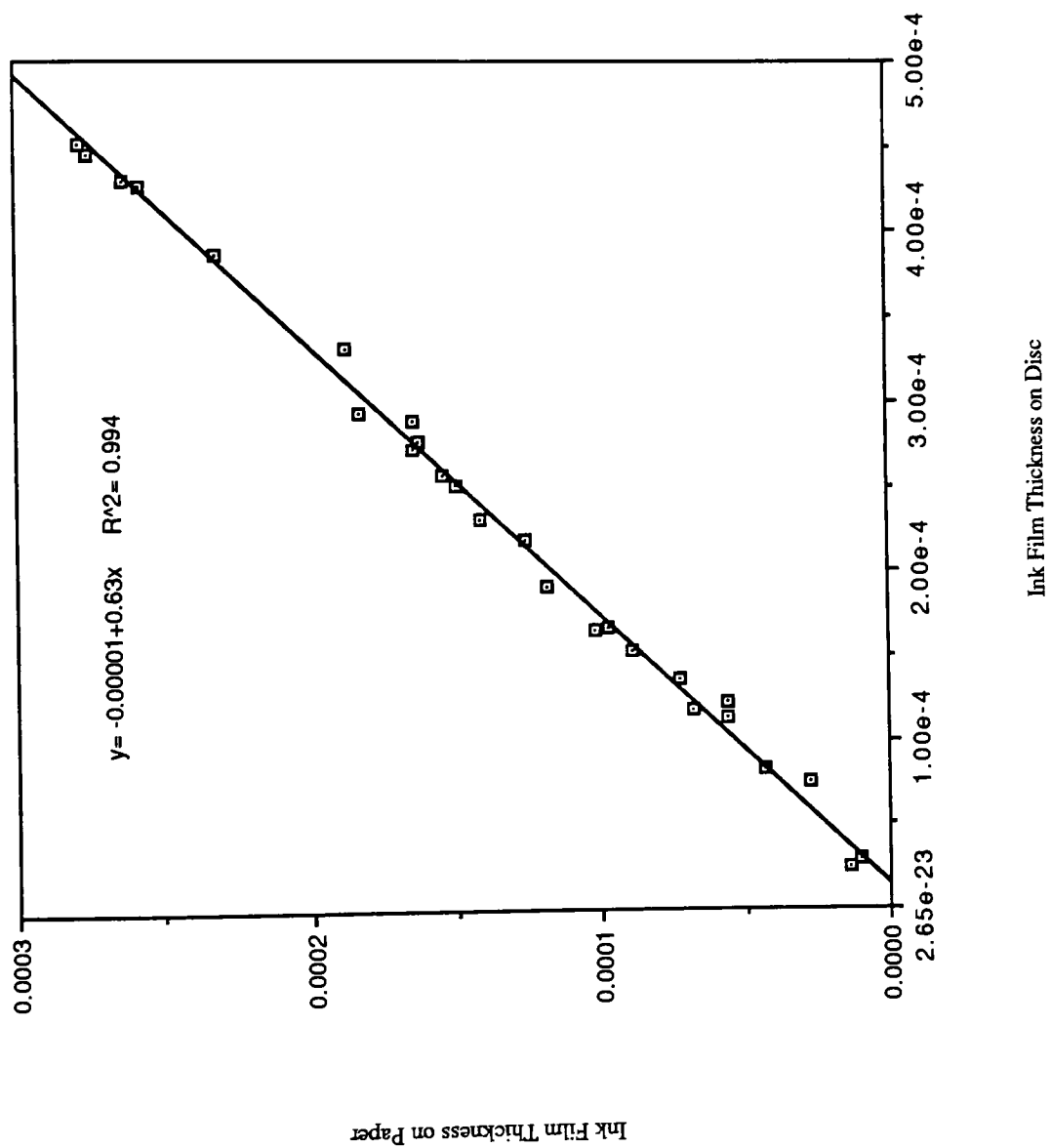


Figure 3. Yellow Ink on Newsprint B

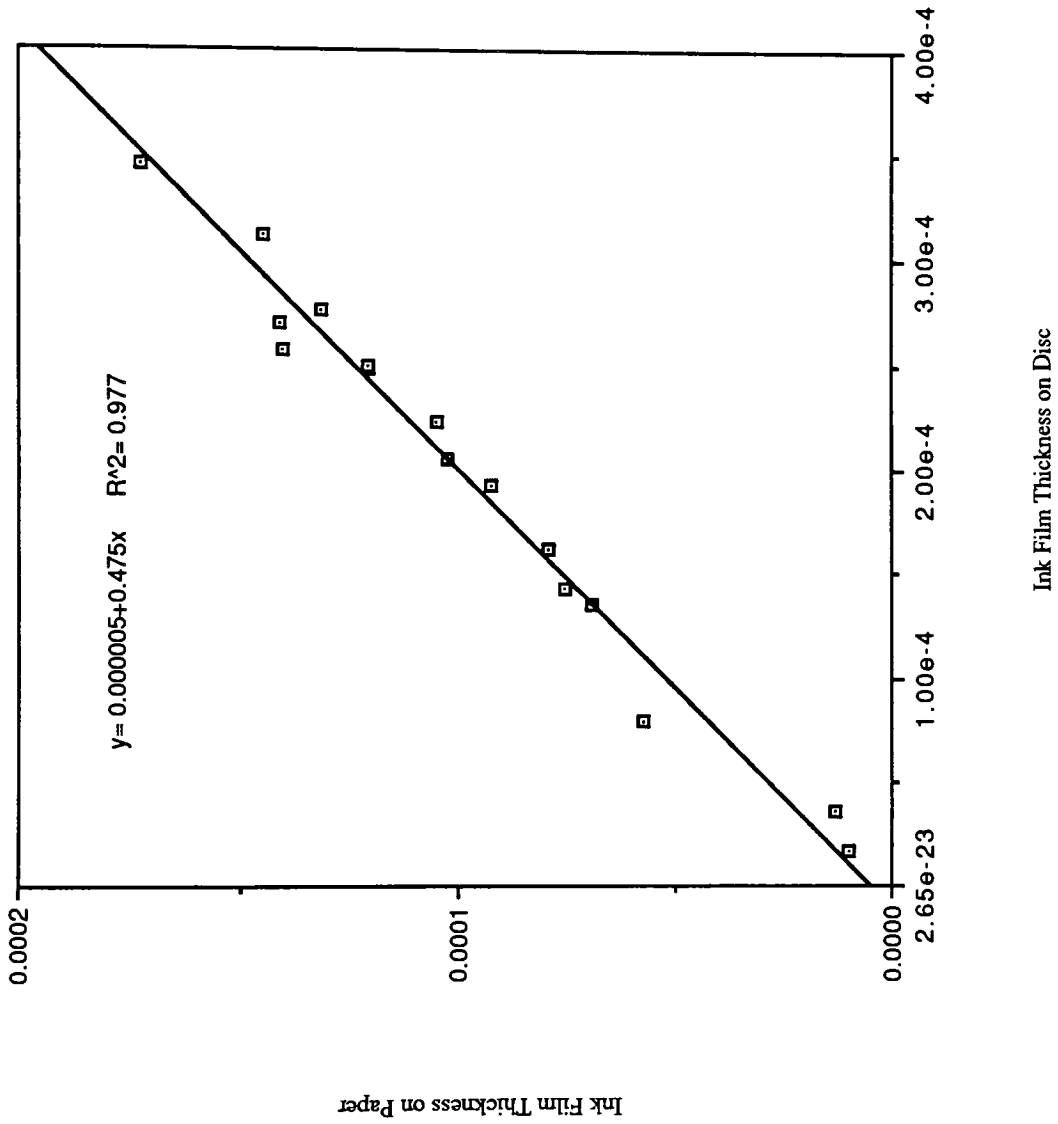


Figure 4. Yellow Ink on Coated Paper

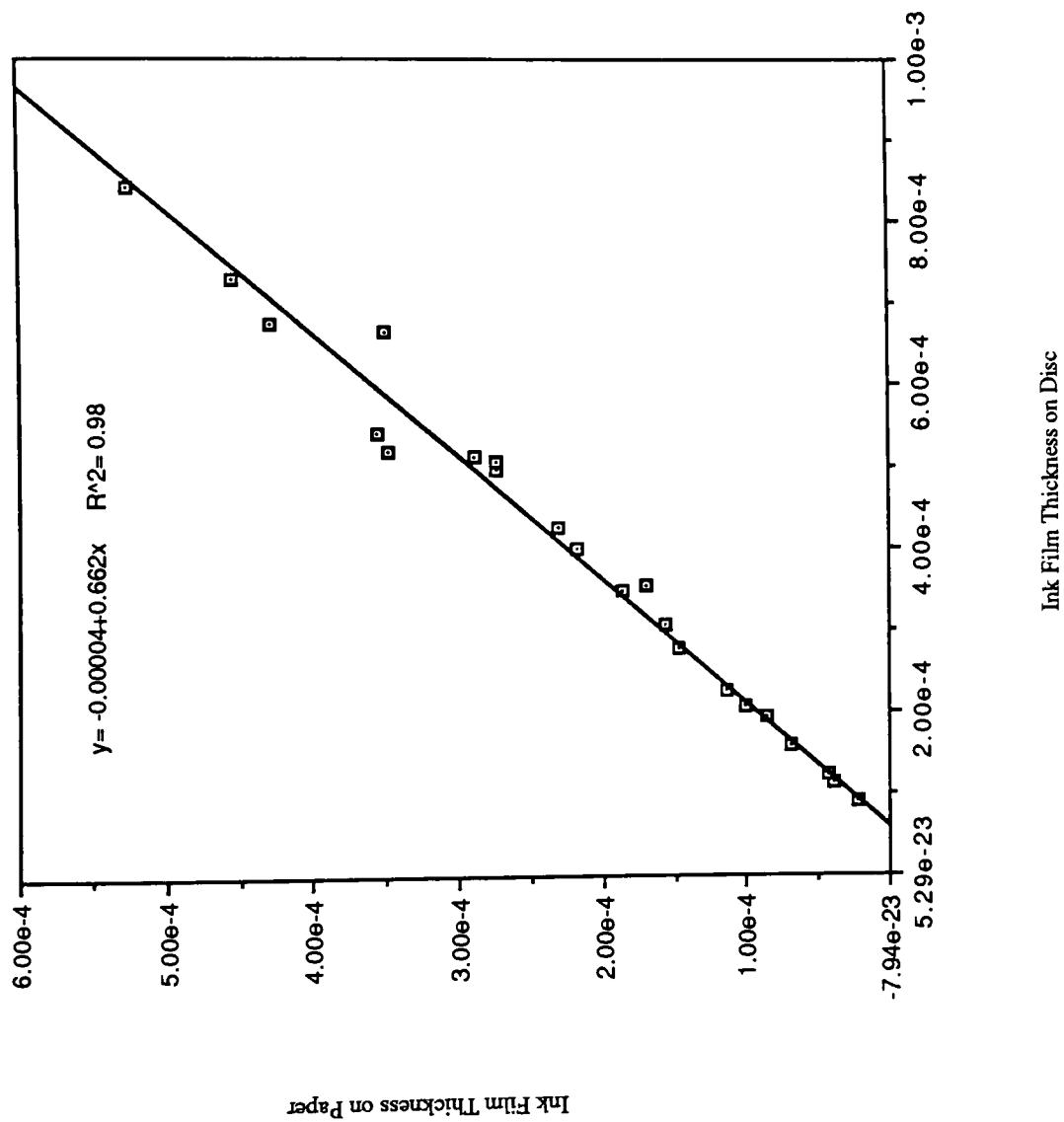


Figure 5. Magenta Ink on Newsprint A

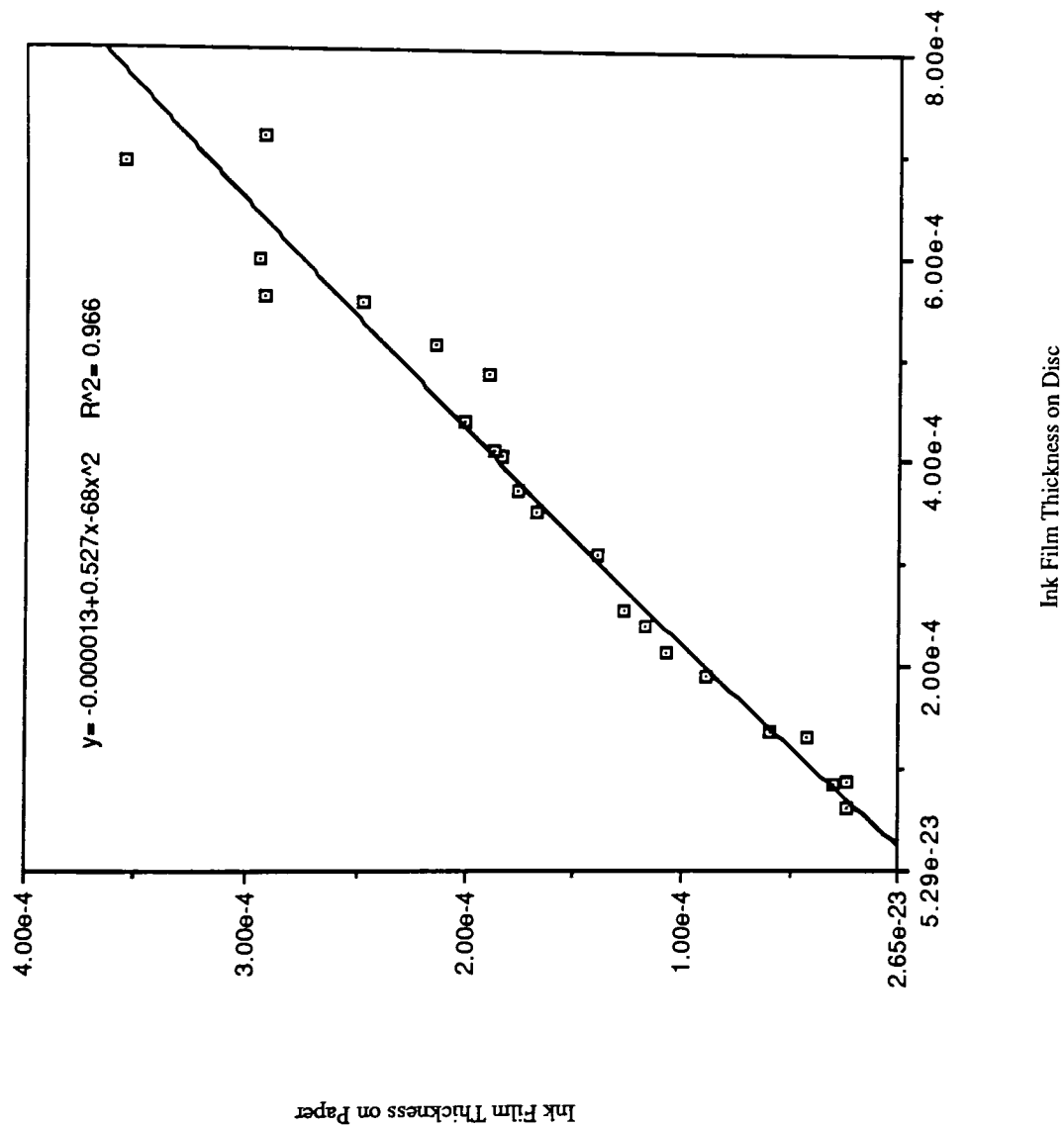


Figure 6. Magenta Ink on Newsprint B

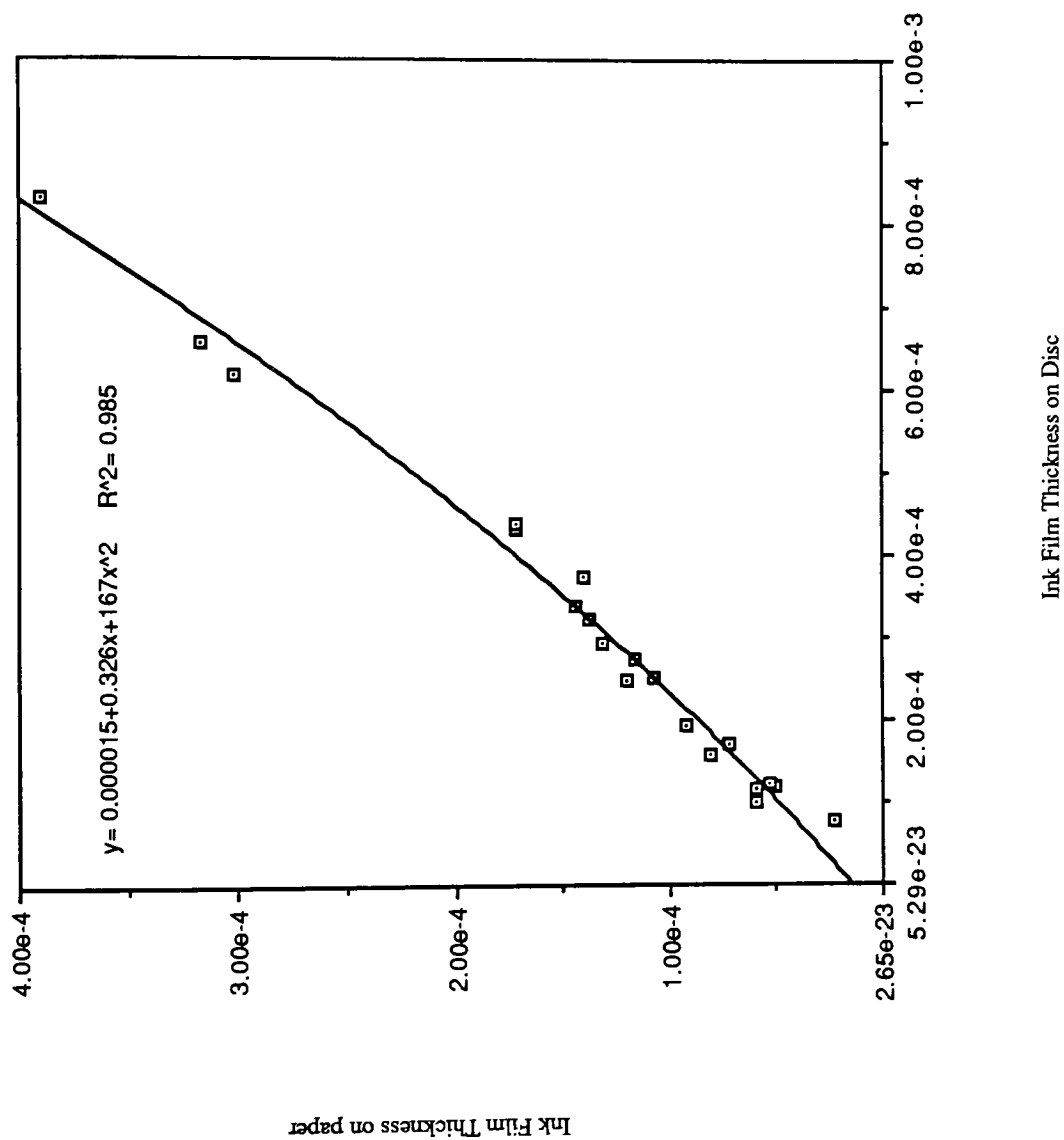
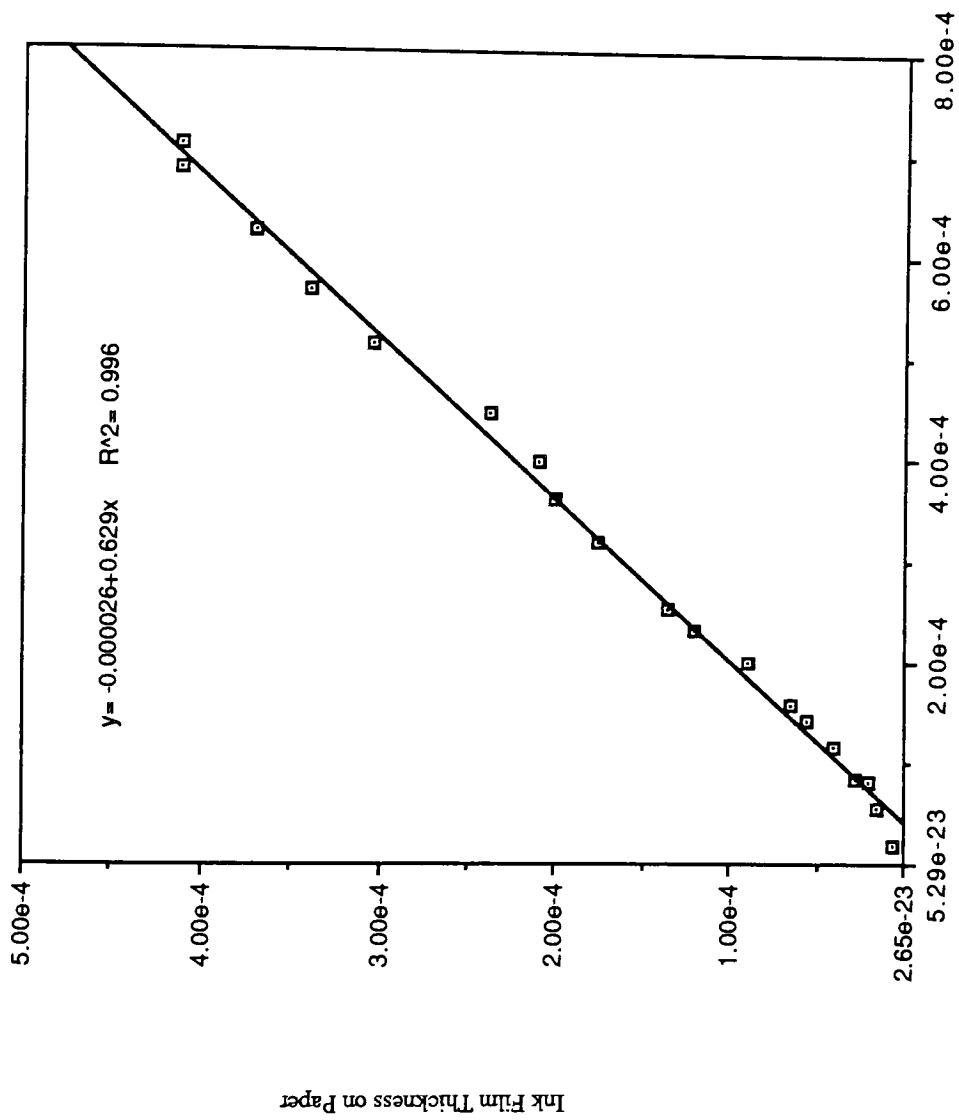


Figure 7. Magenta Ink on Coated Paper



Ink Film Thickness on Disc

Figure 8. Cyan Ink on Newsprint A

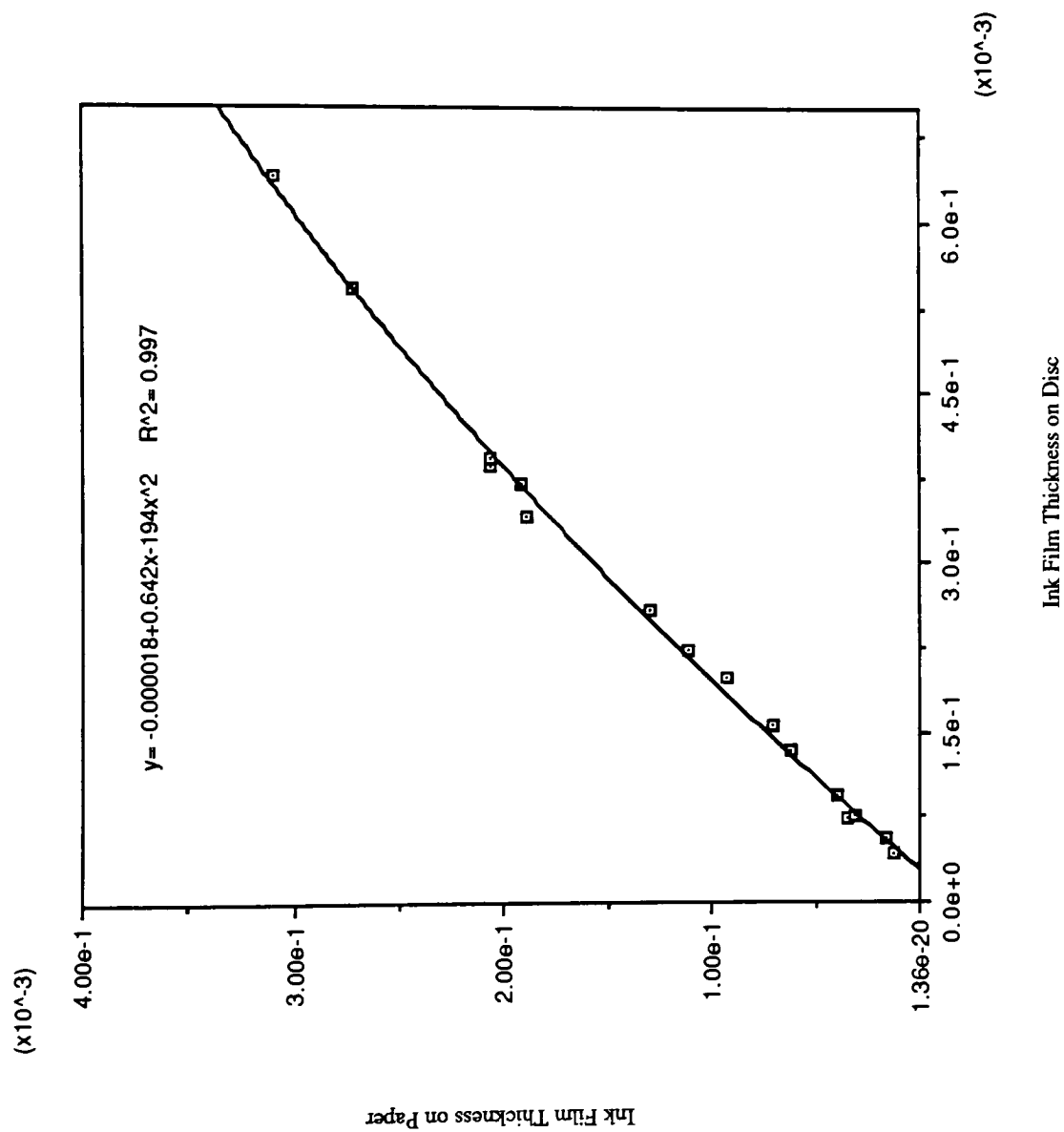


Figure 9. Cyan Ink on Newsprint B

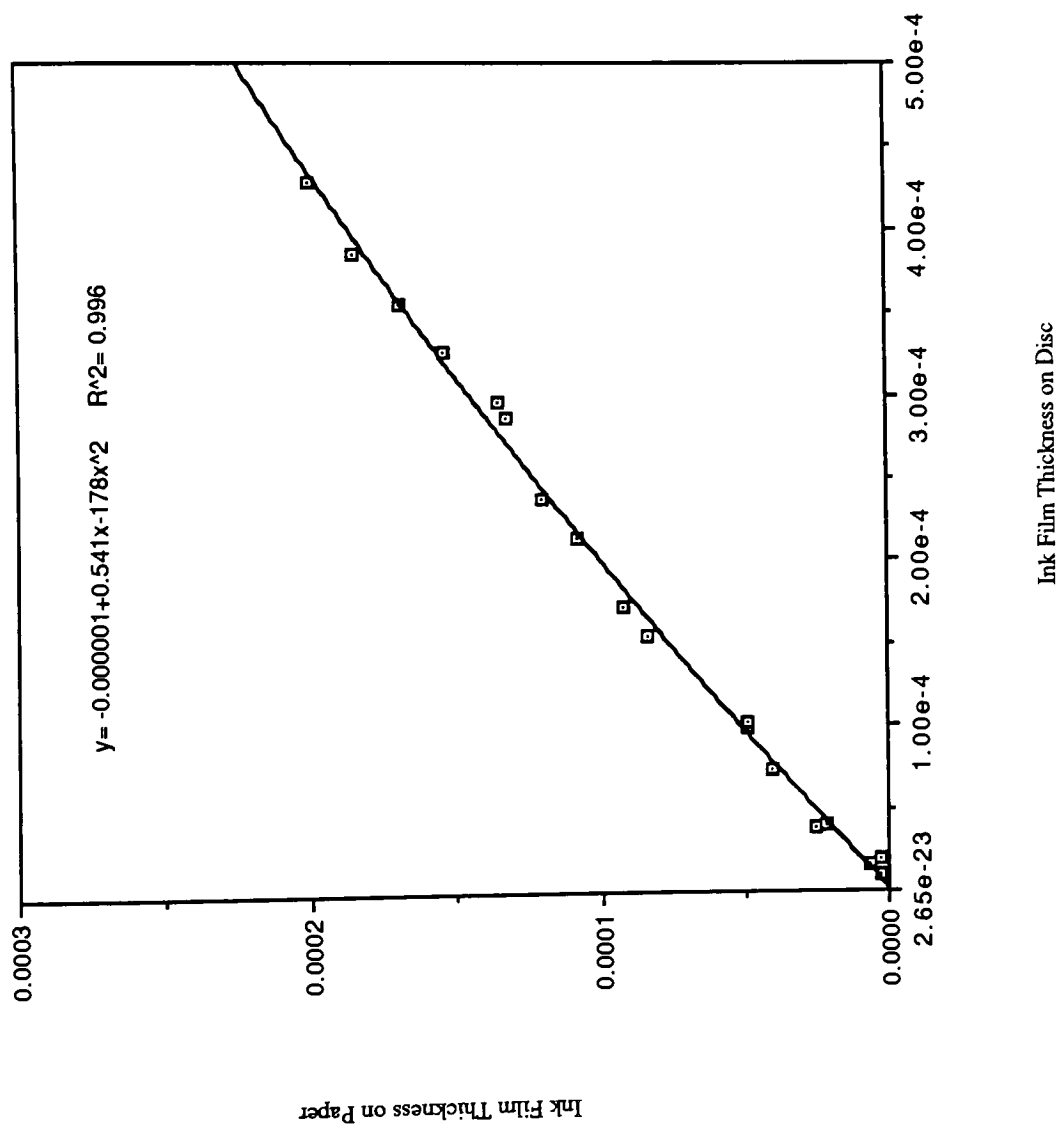


Figure 10. Cyan Ink on Coated Paper

again. The difference between the weight of the disc without ink and that with ink was the weight of the ink on the disc.

A strip of paper was affixed to the IGT tester and the inked disc was mounted in place. The spring tensions were set to 30 kilograms.¹ The ink on the disc was transferred to the strip of paper. The disc was then weighed a third time. The difference in the weight of the disc before and after transfer represented the weight of the ink transferred to the paper.

The second-down ink layers were transferred using the same technique, as was stated above. However, for the purpose of this study, the amount of the second ink inked on the disc was separated to three levels, according to the resulting densities on papers. The densities of medium amount of inks would approximately be those printed in production situations.² The densities of low or high amount of inks would be below or over those references. Although two newsprint samples had the same reference densities, the amount of ink used was different due to different paper properties. The densities of the first ink layer were controlled as a constant, which were slightly below the references. The reference densities for the second ink layer were as follows:

	Newsprint	Coated
Yellow	0.85	1.0
Magenta	0.90	1.35
Cyan	0.90	1.35

The printed paper strips were allowed 24 hours to dry fully, then calculation of ink trapping using densitometric method and gravimetric methods followed.

Calculation of Ink Trapping by Gravimetric Method

Gravimetric trapping was calculated by dividing the ink film thickness of the second ink (IFT_2) that was on top of the first ink by the ink film thickness that printed directly on paper (IFT_1).

The ink film thickness was calculated by dividing the weight of ink by the product of the specific gravity of the ink and the area printed with the ink. In calculating the weight of the second ink on top of the first ink layer, the weight of the small area of the second ink printed directly to paper was subtracted from the total weight of the ink transferred from the disc. It was assumed that the ink split 50/50 when transferred to the paper. As a result, the weight of the ink transferred directly to the paper was one half of the total weight of the ink on the disc, multiplied by a ratio of the area of ink on the paper and the total area of ink on the disc. The actual weight of ink printed on top of the first ink was then converted to IFT_2 .

To calculate IFT_1 , the ink film thickness on the disc was used in the ink transfer curve to derive an estimated ink film thickness directly printed to the paper.

A sample of calculating gravimetric ink trapping is shown in Appendix D on page 115.

Calculation of Ink Trapping by Densitometric Method

A Gretag D-186 densitometer was first calibrated according to the manufacturer's manual. The complementary color filter of the second-down ink was used to measure A_1 , A_2 , and $A_{2/1}$ (refer A_1 , A_2 , and $A_{2/1}$ shown in Figure 1 on page 31). The densitometer was then nulled on paper. Five measurements on A_1 and A_2 and nine measurements on $A_{2/1}$ were then made. The recorded densities were the average of these readings.

With the three average densities of each strip, the percentage of ink trapping was calculated according to the densitometric equations, Preucil's, Brunner's, and Hamilton's, as listed below.

$$\text{Preucil's equation: } \% \text{ trap} = \frac{D_{21} - D_1}{D_2} * 100$$

$$\text{Brunner's equation: } \% \text{ trap} = \frac{1 - 10^{-(D_{21})}}{1 - 10^{-(D_2 + D_1)}} * 100$$

$$\text{Hamilton's equation: \% trap} = \frac{\log \left(1 + \frac{D_{21} - D_1}{D_m - D_{21}} \right)}{\log \left(1 + \frac{D_2}{D_m - D_2} \right)} * 100$$

D_{21} = the density of the overlapping area on paper

D_2 = the density of the second ink on paper

D_1 = the density of the first ink on paper

D_m = the maximum printable density of the paper

Determining the D_m Value in Hamilton's Equation

Hamilton modified Preucil's equation on the basis of Yule's model and derived a new ink trap equation, as explained in Chapter Three on page 20. The D_m value in his equation, however, was not defined either in Hamilton's article or in the literature. As a result, the author replaced D_m with various numbers to find one that yielded the trapping values closest to those measured by the gravimetric method. The best-fit D_m value was determined by graphically comparing the various sets of trapping values with those calculated by gravimetric method and by mathematically calculating the standard error of various D_m values. The standard error was calculated by the following formula:

$$\text{standard error} = \sqrt{\frac{\sum (t_c - t_g)^2}{n}}$$

t_c : ink trapping values calculated by densitometric method

t_g : ink trapping values measured by gravimetric method

n : the number of samples

Scatter diagrams of ink trapping values calculated by each method were plotted and analyzed. Graphs also showed the discrepancy in predicted ink trapping between each densitometric method and the gravimetric method. The percentage ink trapping calculated by Hamilton's equation with the best-fit D_m value were presented along with the ink trapping calculated by gravimetric method. From this analysis, the accuracy of each ink trap equation relative to gravimetric method was characterized.

FOOTNOTES FOR CHAPTER FIVE

1. Consulted Mr. Ching-yih Chen, Senior Technologist, Paper and Ink Laboratory at the Technical and Education Center of the Graphic Arts at RIT
2. Consulted Web-Offset press crew, Technical and Education Center of the Graphic Arts at RIT

CHAPTER SIX

RESULTS AND ANALYSIS

A summary of experimental data is shown on page 54 through 56. Each ink trapping value is the average of three replications. Please refer to Appendices F to G for detailed ink trapping data.

An attempt was made to replicate the trapping conditions within a tight latitude throughout each ink and paper combination. This proved to be somewhat difficult to control since the tack of each ink changed in a different time interval and with a different amount of ink on the disc, thus having an effect on the amount of the second-down ink trapped to the first ink. Although the ink transfers did not follow exactly as planned, the variation among three replications was controlled to the lowest degree.

A series of graphs follows to assist in understanding the results.

Table 5. Average Ink Trap on Newsprint A

		Method	Newsprint A								
			Color of the 2nd Ink								
			Yellow			Magenta			Cyan		
			low	med	high	low	med	high	low	med	high
Color of the 1st Ink	Y	Gravimetric	72.15	117.01	105.91	46.54	87.01	100.72	113.07	70.29	105.53
		Preucil	49.60	83.58	59.38	88.47	72.11	96.07	99.27	95.54	97.69
		Brunner	92.91	94.77	96.10	82.95	88.67	99.43	99.58	98.08	99.70
		Hamilton *	71.48	88.08	83.03	84.75	85.30	94.35	99.85	94.99	93.89
	M	Gravimetric	126.16	100.12	101.96	132.87	115.77	107.34	93.72	86.73	94.82
		Preucil	82.13	70.10	74.64	44.85	44.74	44.02	111.99	94.52	95.58
		Brunner	97.01	95.02	97.44	95.84	96.29	97.45	105.72	95.83	99.44
		Hamilton *	114.61	97.71	107.27	97.07	97.00	104.72	117.98	79.85	94.07
	C	Gravimetric	76.34	93.75	103.34	49.59	91.39	110.67	72.44	101.63	102.64
		Preucil	96.93	95.90	94.04	40.58	83.29	83.23	29.13	49.51	81.14
		Brunner	98.57	98.51	98.99	79.44	92.42	98.24	91.83	95.60	97.84
		Hamilton *	101.31	100.94	99.11	43.12	82.74	96.02	47.55	94.40	113.60

* Note: $D_m = 1.59$ in Hamilton's equation

Table 6. Average Ink Trap on Newsprint B

		Method	Newsprint B								
			Color of the 2nd Ink								
			Yellow			Magenta			Cyan		
			low	med	high	low	med	high	low	med	high
Color of the 1st Ink	Y	Gravimetric	92.68	90.15	96.02	15.45	45.28	108.36	102.66	73.87	97.34
		Preucil	52.24	54.87	59.52	54.11	59.19	96.53	101.51	94.60	97.45
		Brunner	94.48	95.30	97.81	71.09	78.73	99.69	99.80	97.84	99.68
		Hamilton *	74.78	81.79	87.01	52.70	55.40	97.38	102.95	93.89	97.45
	M	Gravimetric	88.88	104.81	82.19	119.39	111.30	112.64	86.03	79.31	88.52
		Preucil	95.84	79.41	71.59	52.41	47.27	51.45	100.00	83.02	77.89
		Brunner	99.28	97.28	98.14	97.83	97.17	98.82	99.88	94.84	96.72
		Hamilton *	125.45	107.84	91.70	115.27	89.63	120.07	103.26	81.19	85.85
	C	Gravimetric	97.77	73.82	105.87	26.65	90.29	116.07	57.78	79.77	98.88
		Preucil	95.91	83.21	95.89	20.63	75.14	83.74	35.85	34.03	51.08
		Brunner	98.58	92.89	99.52	72.58	93.91	98.87	94.87	96.35	98.33
		Hamilton *	100.22	94.88	101.86	22.15	83.78	96.64	59.69	81.80	85.13

*Note: $D_m = 1.99$ in Hamilton's equation

Table 7. Average Ink Trap on Coated Paper

		Method	Coated Paper								
			Color of the 2nd Ink								
			Yellow			Magenta			Cyan		
			low	med	high	low	med	high	low	med	high
Color of the 1st Ink	Y	Gravimetric	88.21	122.01	108.78	49.50	128.46	118.45	-19.30	90.48	95.15
		Preucil	42.06	49.02	36.88	54.16	88.44	97.14	117.81	98.76	96.03
		Brunner	98.20	98.63	99.17	77.24	98.25	99.91	111.87	99.81	99.81
		Hamilton *	77.24	97.57	66.22	51.01	85.82	99.97	110.49	99.31	92.64
	M	Gravimetric	101.63	128.17	108.80	133.30	102.88	118.63	-4.27	97.82	98.77
		Preucil	94.30	73.26	63.28	23.50	24.45	24.96	92.70	90.06	89.62
		Brunner	99.63	98.34	99.20	99.20	98.80	99.48	96.31	98.44	99.54
		Hamilton *	140.99	104.77	82.98	73.05	63.07	65.60	94.55	89.86	80.00
	C	Gravimetric	56.76	130.82	124.80	160.61	162.39	120.84	103.07	120.78	112.79
		Preucil	142.75	130.44	96.41	134.81	114.05	93.75	19.04	45.80	38.34
		Brunner	113.89	106.02	99.85	103.04	100.75	99.87	98.11	99.42	99.65
		Hamilton *	157.57	148.48	104.61	188.12	167.05	143.05	46.30	134.07	130.87

*Note: $D_m = 2.5$ in Hamilton's equation

Effectiveness of Conventional Ink Trap Equations

To understand the differences of ink trapping between those calculated by Preucil's and Brunner's equations and those measured by the gravimetric method, three kinds of graphs are plotted. Figures 11 to 13 show percentage ink trapping measured by the densitometric and gravimetric methods with ink combinations on each type of paper. In Figures 14 to 16, the resulting discrepancy of ink trapping of each densitometric method are further presented. The horizontal line at zero of Y-axis represents gravimetric ink trapping, with which the ink trapping calculated densitometrically is compared. In addition, Figure 17 is a comparison of the ink trapping difference between Preucil's equation and gravimetric method on three papers.

In these Figures and the following, the symbols of l, m, and h along the X-axis represent the low, medium, and high amount of the second ink applied on the disc; y/m means yellow ink was printed on magenta ink, and so on.

The data of ink trapping differences between densitometric methods and the gravimetric method appear in Appendix H.

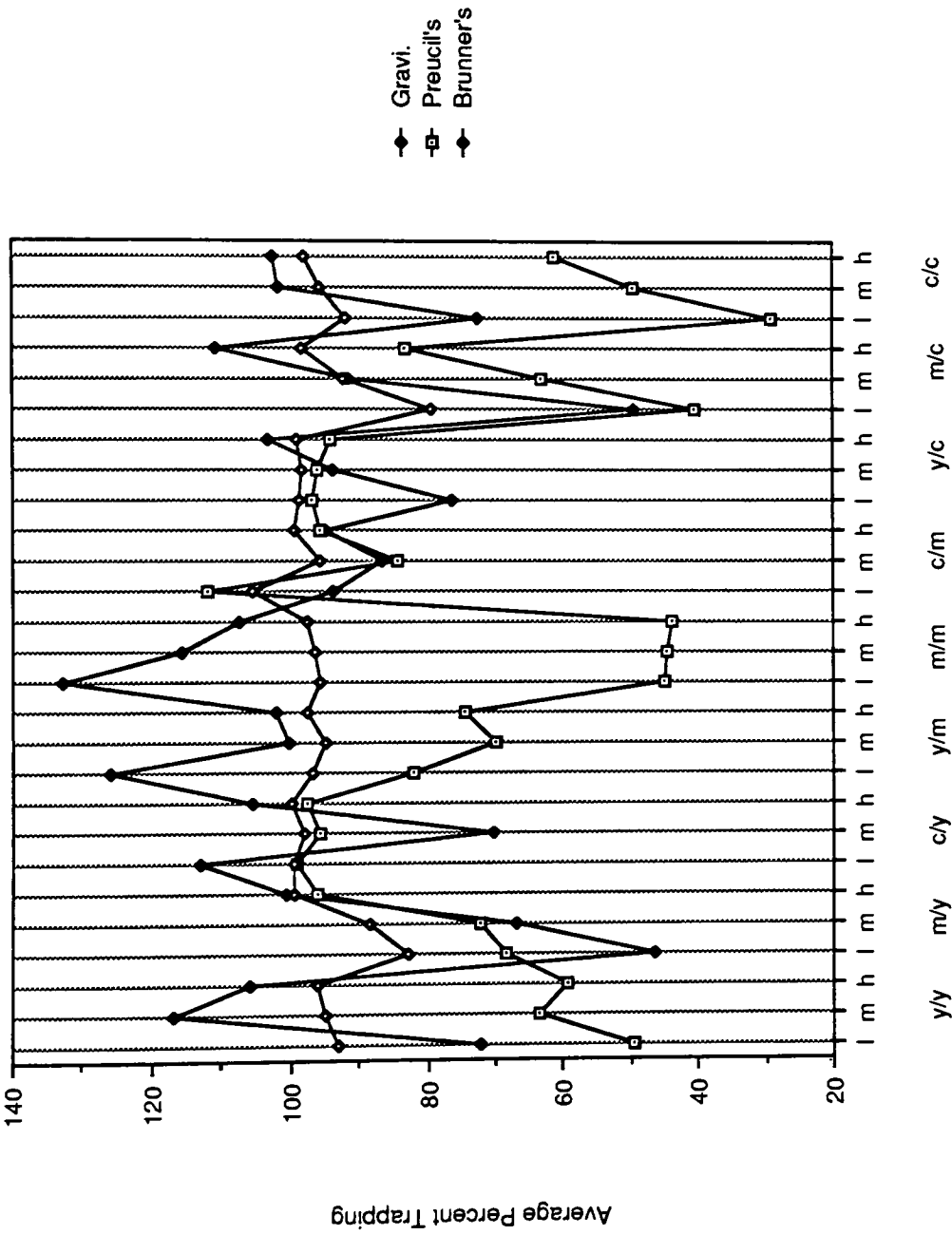


Figure 11. Percent Ink Trapping Measured on Newsprint A

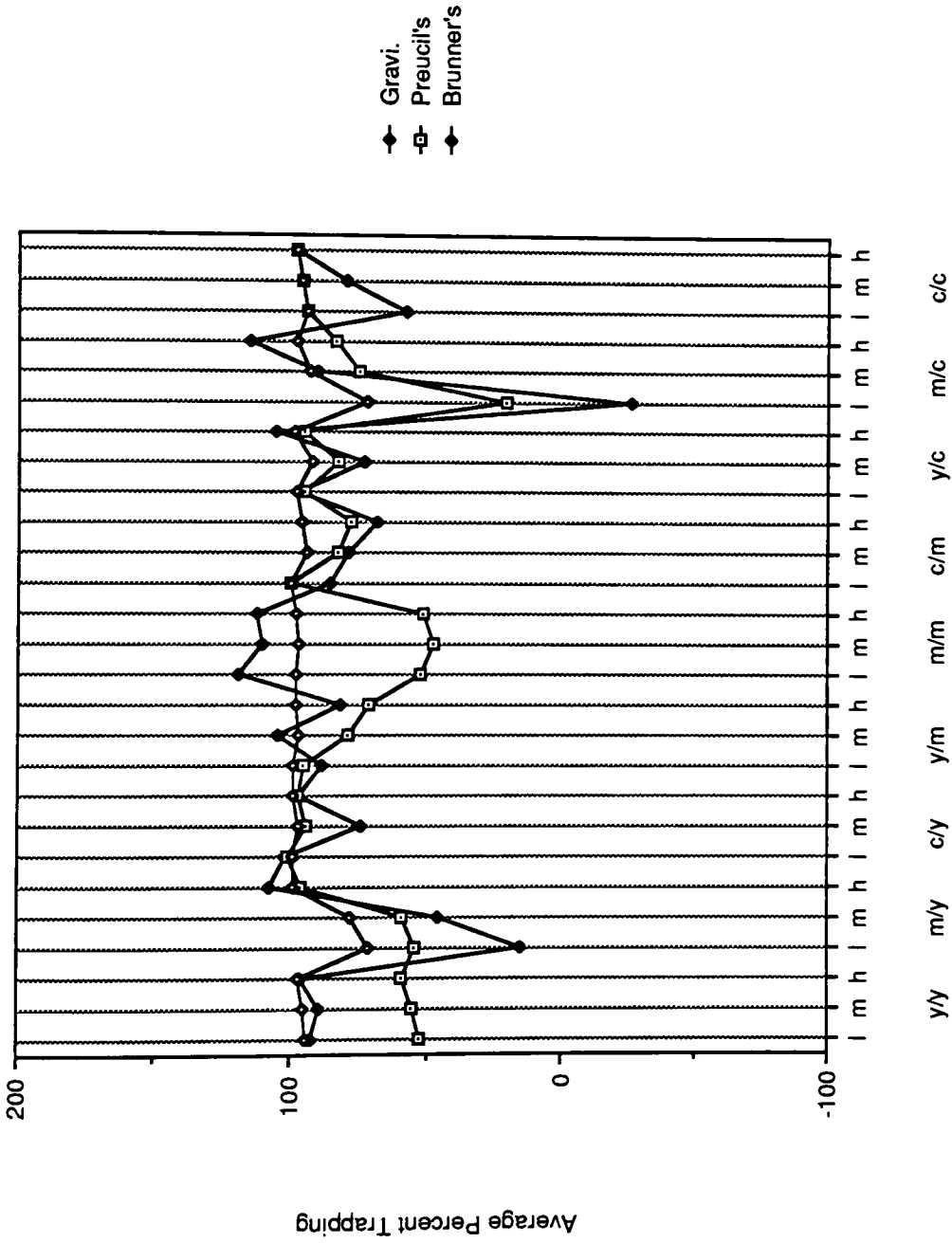


Figure 12. Percent Ink Trapping Measured on Newsprint B

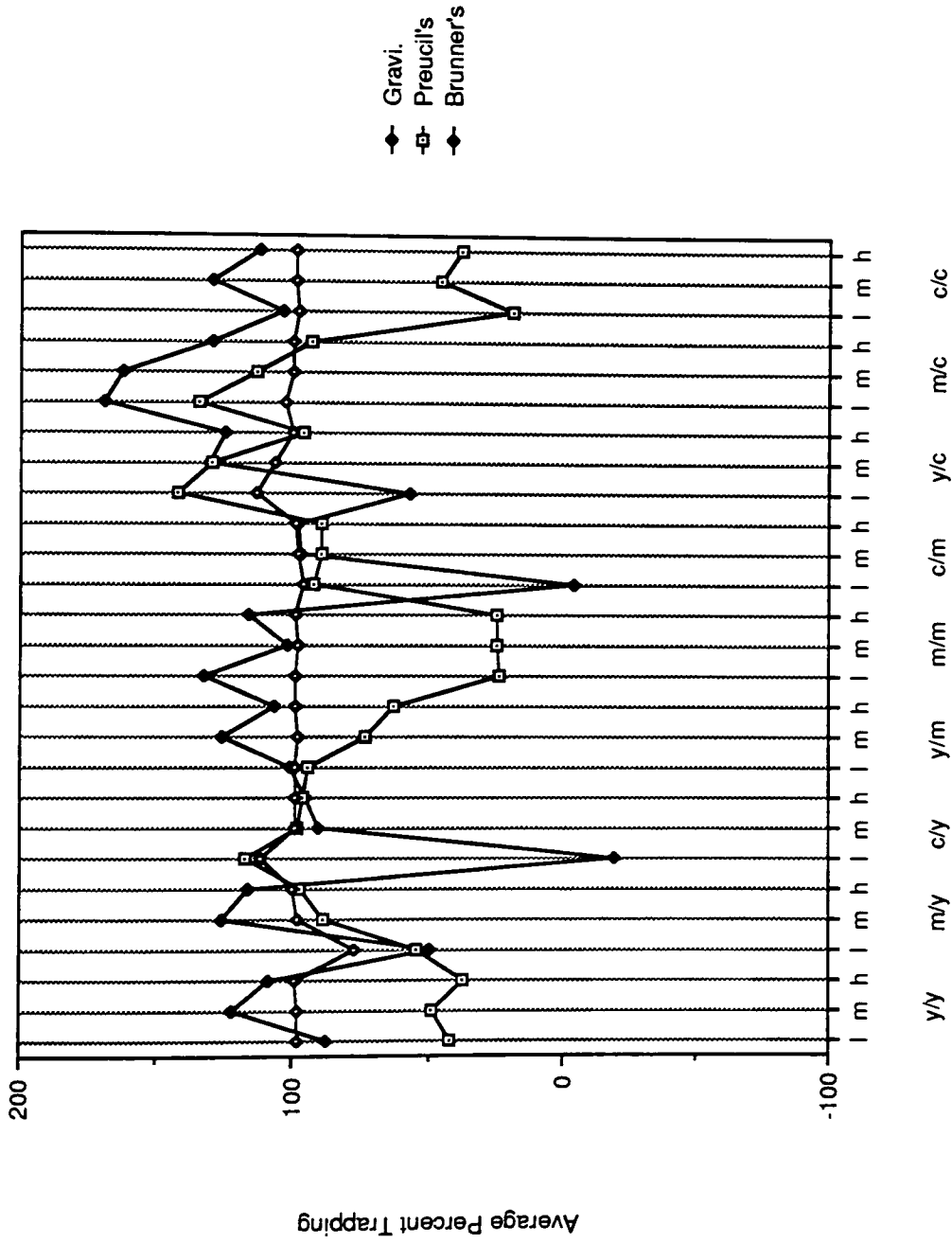


Figure 13. Percent Ink Trapping Measured on Coated Paper

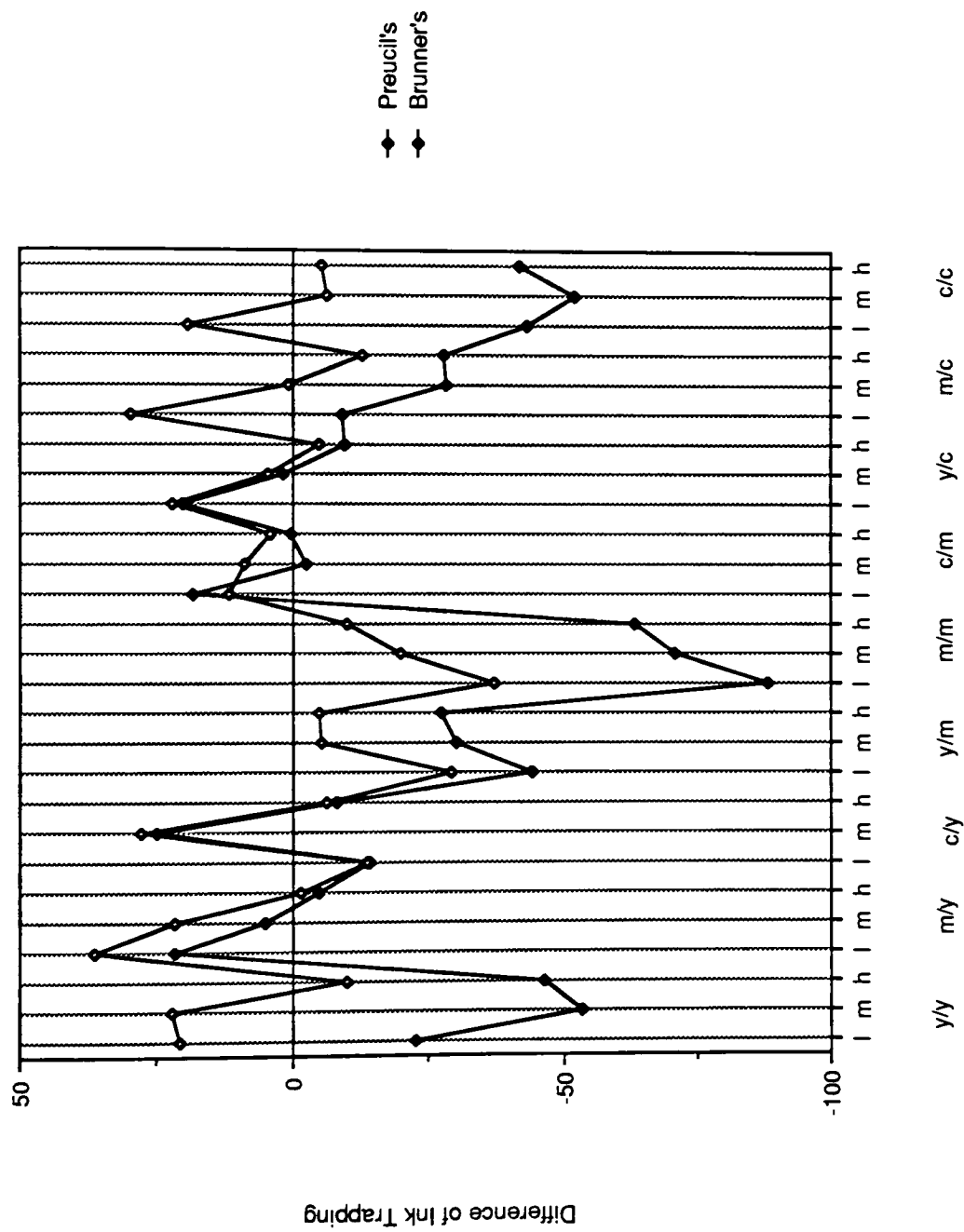


Figure 14. Differences of Percent Ink Trap on Newsprint A

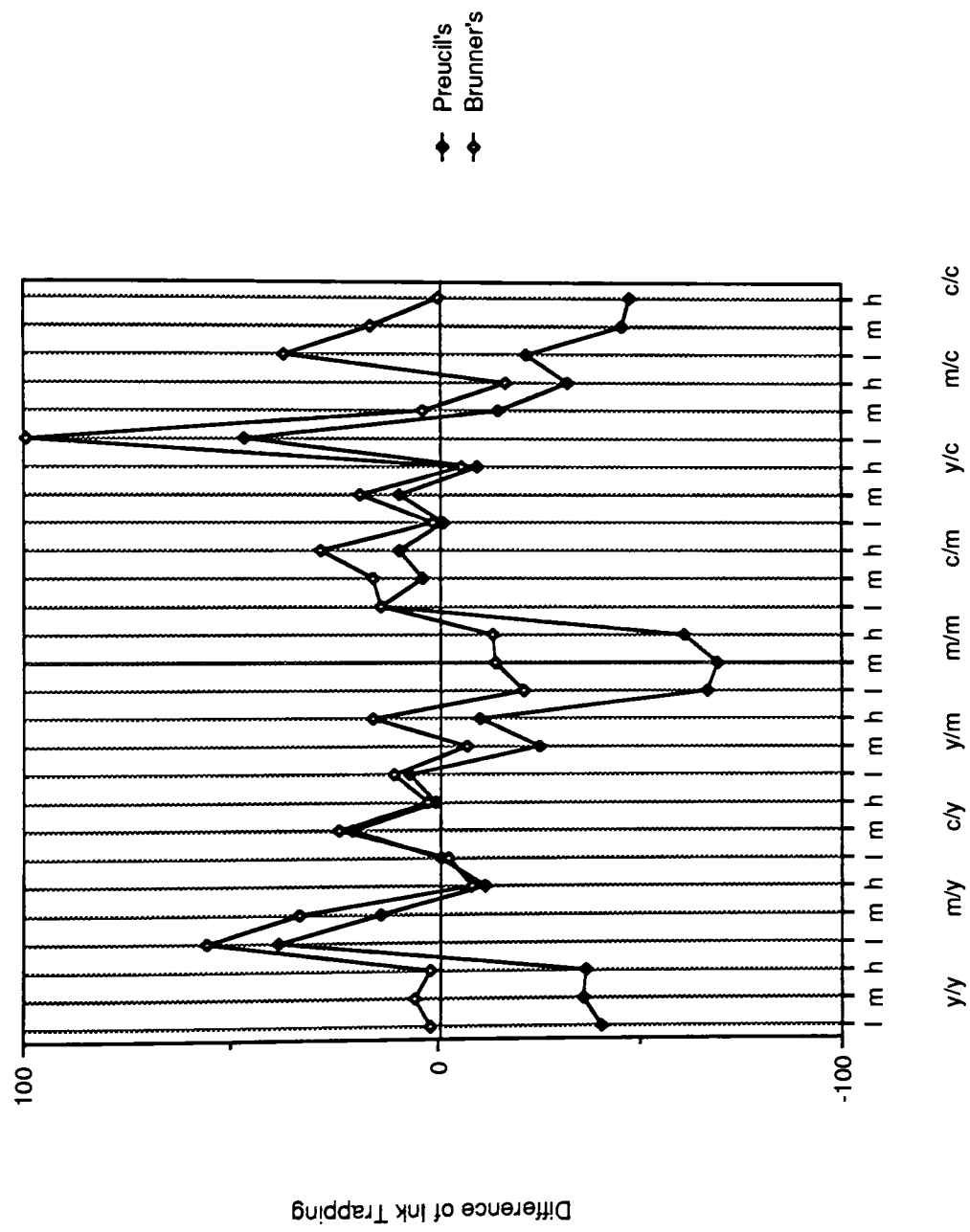


Figure 15. Differences of Percent Ink Trap on Newsprint B

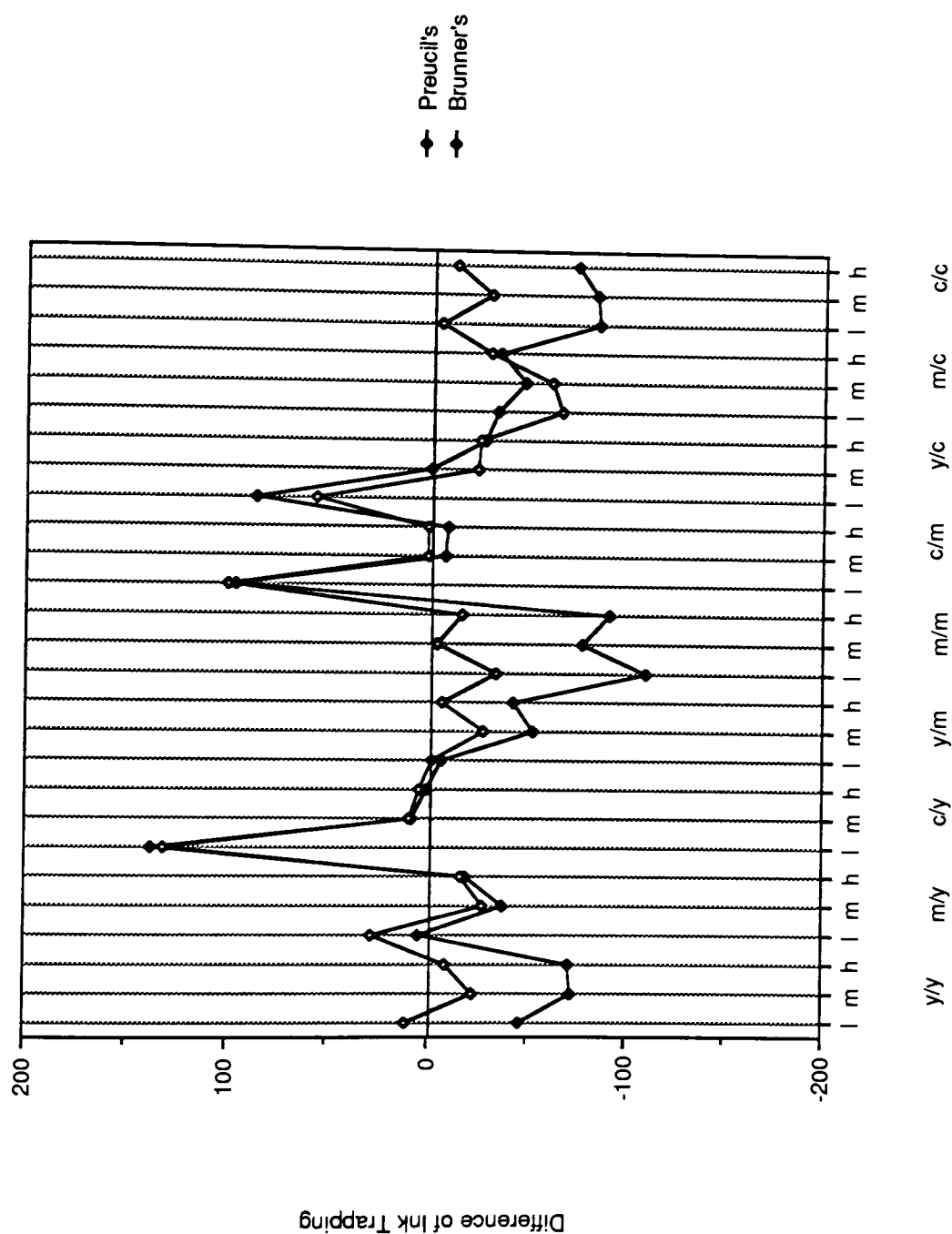


Figure 16. Differences of Percent Ink Trap on Coated Paper

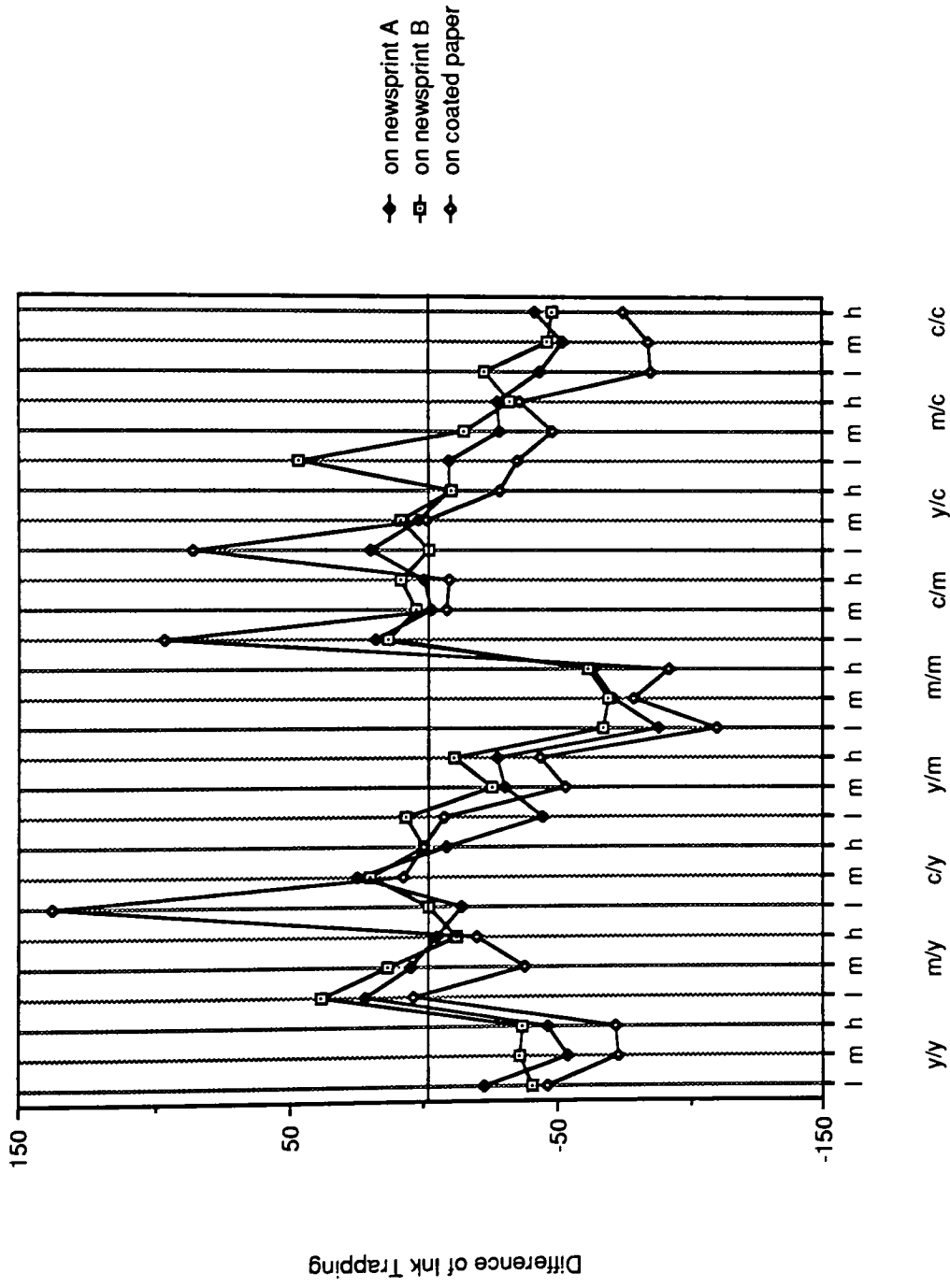


Figure 17. Differences of Percent Ink Trap Calculated by Preucil's Equation

As can be seen from these graphs, Brunner's equation was not able to detect the ink trapping variance correspondingly to those measured gravimetrically. No matter how low or how high the ink trapping was, Brunner's equation constantly predicted ink trapping ranging from 90% to 100% on three papers. The result was that Brunner's equation over-estimated the gravimetric ink trapping, which was below 90% and under-estimated those which were over 100% on each type of paper. It was also noticed that as the percentage of ink trapping measured gravimetrically increased within the same ink combination, the inaccuracy of Brunner's equation increased. This happened with the three papers used in this experiment.

For Preucil's equation, the ink trapping values measured, in most conditions, were below the ink trapping measured gravimetrically. The under-estimation by Preucil's equation in predicting ink trapping occurred on over 60% of the ink combinations on three papers, yet became more critical on newsprint A. Over 80% of the ink trapping calculated by Preucil's equation fell under the corresponding ink trapping measured gravimetrically on newsprint A. This might indicate Preucil's equation has a larger error when applied to very absorptive papers.

Like Brunner's equation, Preucil's equation became more inaccurate when percentage of ink trapping measured gravimetrically increased. This was true on the two newsprint samples and for most

conditions on coated paper. The reason why the larger error in calculating ink trap by Preucil's equation did not always occur on the higher ink trapping, as measured gravimetrically, on coated paper might be associated with several observations, which will be discussed in the last part of this section.

The inaccuracy of the Preucil equation and Brunner equation is not clearly related to the ink combination or the amount of the second-down ink transferred on the disc because the equations only deal with the percentage of ink trapping, yet the resulting ink trapping has no direct relationship with any of those factors. There is neither a specific ink combination nor a particular amount of the second-down ink on the disc that could predict the efficiency of ink trapping. It is true that having the high-tack ink printed first would enhance the percentage of ink trapping; however, the reverse of this statement is not always true.

On newsprint A, which is the most absorptive paper among the tested papers, when the amount of the second-down ink transferred on the disc increased, the percentage of ink trapping measured gravimetrically changed more significantly than on newsprint B. While on coated paper, the medium-amount of the second ink on the disc usually produced a higher percentage of ink trapping, as measured gravimetrically, than the high-amount in each ink

combination. This perhaps implies that coated paper, which is less absorbent, has a limit of how much ink it will take.

Returning to the error of the densitometric equation, the error of the Brunner equation decreased on newsprint A and coated paper as the second-down ink increased on the disc, but no relationship was found on newsprint B. The error of Preucil's equation has no relationship with the amount of the second ink applied on the disc.

In Figures 14, 15, 16, and 17, which show the discrepancy between each equation and the gravimetric method, Preucil's equation has the largest errors in y/y, m/m, and c/c ink combinations on three papers. This suggests that various factors, which affect the ink trapping measurement using a densitometer, may have the largest effect on the same color overprints. While looking at the discrepancy of each equation on different papers, Preucil's equation generated the similar patterns on newsprint A and B, as does Brunner's equation. Also noticed is that the Brunner equation has a pattern similar to the Preucil equation on coated paper and newsprint B. All these show that the densitometric equation would result in similar errors on similar papers, as does absorbency, and that Brunner's equation predicts ink trapping differently from Preucil's equation in many respects.

Several under-trappings, which were negative values measured by the gravimetric method, appeared on newsprint B and coated paper. This may be due to that in this experiment, which was wet-on-wet printing, the first-down ink had difficulty in penetrating into the nonporous papers, newsprint B and coated paper, relative to the highly porous paper, newsprint A, within a short time. Accordingly, some of the first-down ink was removed by the second-down ink.

Finally, several points relating to the Preucil equation did not follow the expected results are stated here:

1. It has the largest overall discrepancy on coated paper. The numerical data is shown in the next section under the standard error on coated paper.
2. Preucil's equation under-estimates the gravimetric trapping on coated paper most significantly.
3. The inaccuracy of the equation is expected to increase as the gravimetric trapping increased in all ink combinations.
4. Some serious back-trapping occurred on coated paper.

In general, the reason that Preucil's equation can not measure ink trapping very accurately is due to the additivity failure and other various factors, such as ink trapping efficiency. The error of Preucil's equation relative to gravimetric method on coated paper should be the smallest, based on the knowledge about the additivity failure.

The following observations made during the experiment may help explain the results that are in opposition to what is known about ink trapping measurements on coated paper:

1. While the inks were distributed on the inking unit for several times, the tackier inks, such as magenta and cyan, became much stickier and resistant to flow¹, due to the increased tack. As these inks were first printed on the paper, their tacks were even higher². During the moment when the paper was impressed with the disc, the first-down ink not only resisted to accept the second ink but was trapped back to the disc since the metallic disc had a stronger pulling force than the paper. Ink contamination might also occur at this moment.
2. Cyan ink overprinted on the lower-tack ink resulted in poor trapping of the second ink on the first ink layer and back trapping of the first ink to the second ink.
3. The above two problems happen to three papers, but became more critical on coated paper because most first-down inks did not penetrate this less-absorbent paper but remained on the surface of the paper.
4. With a low amount of ink transferred to an already printed ink layer, the first two phenomena stated above became worse. This was due to the low ink film which had a much stronger bond with the printing disc than a heavy ink film. Thus, it decreased the trapping efficiency.

Effectiveness of Hamilton's Equation

On the following pages are graphic representations of effectiveness of Hamilton's equation as plotted in three type of graphs. Figures 18 to 20 illustrate the standard error of Hamilton's equation with various D_m values and the standard error of Preucil's equation. The standard error values were determined by comparing the ink trapping values calculated by the equations to those measured by gravimetric method. The ink trapping values calculated by Hamilton's equation were made using a range of D_m values from 1.5 to 6, depending on the type of paper. The formula for calculating the standard error is stated earlier on page 51.

Plotting the standard error against the D_m value produces a characteristic curve for each paper. However, all the regression models, which include simple, polynomial, logarithmic, and exponential models, misinterpret the relationship between the standard error and the D_m value, when the D_m value is below 2.5 or beyond 4.0. Rather, a Stineman interpolation is chosen to represent the variation of the standard error corresponding to the D_m values.

The standard error curves all start from a specific D_m value and either decrease to a point followed by an increase again or nearly straightly increase toward a point as the D_m value increases. This is because the percent ink trapping calculated by Hamilton's equation can not be defined when the D_m value goes below a specific number,

but will approach a constant as the D_m value goes beyond a certain number.

The data of standard error with various D_m values are shown in Appendix J.

The graphs shown in Figures 21 to 23, a smallest difference between the ink trapping values calculated by Hamilton's equation and those measured by the gravimetric method can be found as the D_m is replaced with a specific value. However, the discrepancy will increase again when a smaller D_m value is used in the equation. The symbols shown in X-axis in these graphs, are shown on page 57.

The last type of graph shows the differences of calculated ink trapping, relative to the ink trapping measured gravimetrically, between Preucil's equation and Hamilton's equation, where a specific D_m value is used to produce the least difference. The best-fit D_m value is the number which produces the closest percent ink trapping as compared graphically to gravimetric ink trapping values, and gives the smallest standard error as calculated. These graphs are shown in Figures 24 through 26.

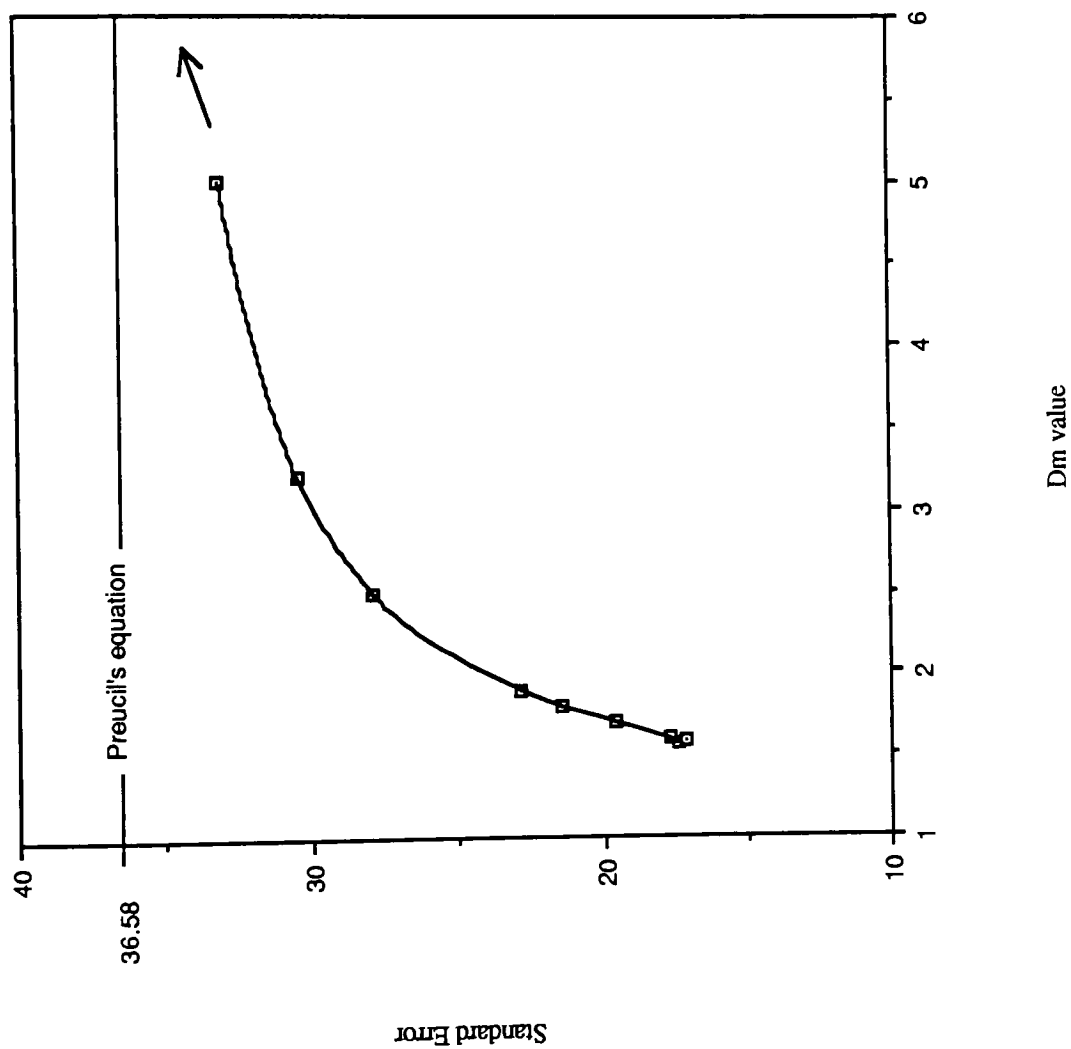


Figure 18. Standard Error of Hamilton's Equation on Newsprint A

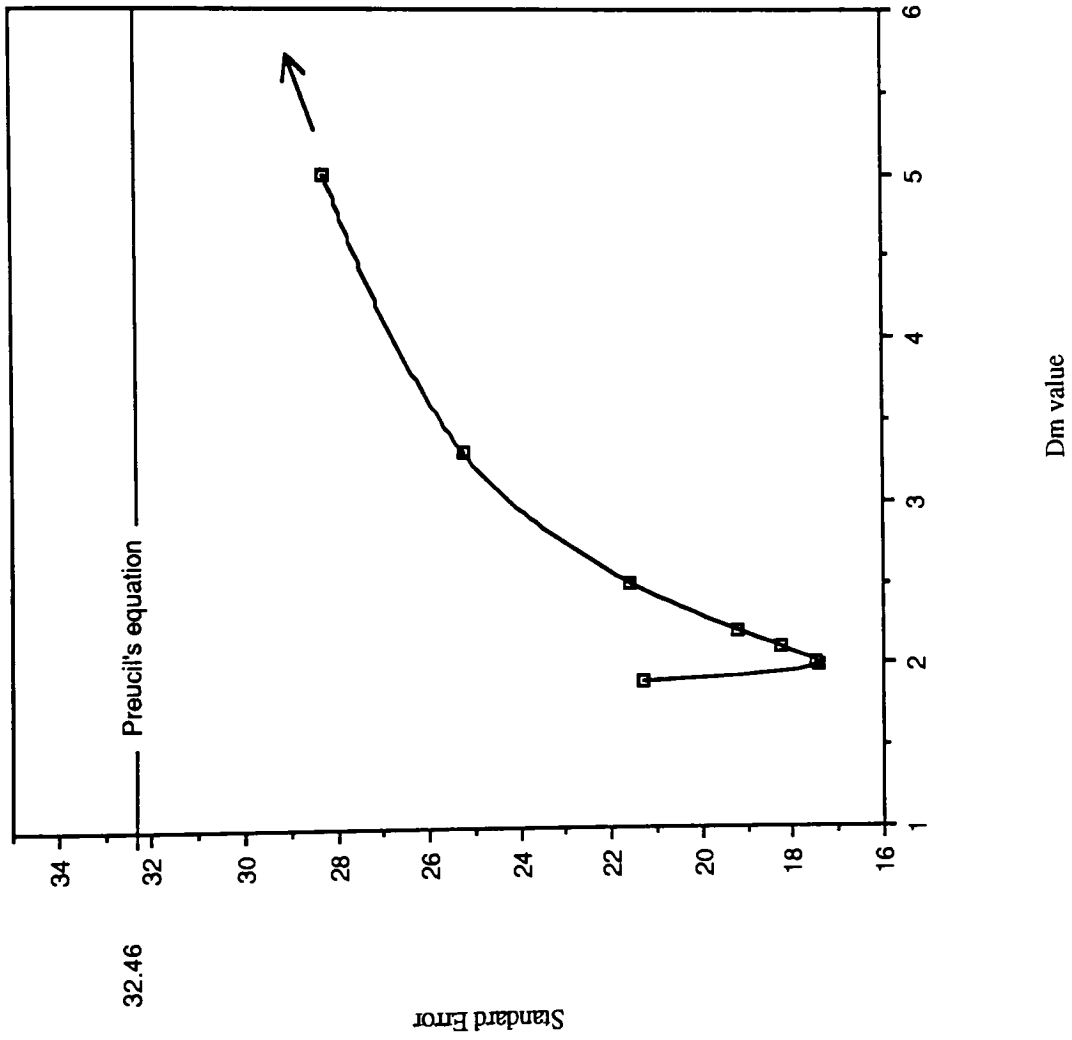


Figure 19. Standard Error of Hamilton's Equation on Newsprint B

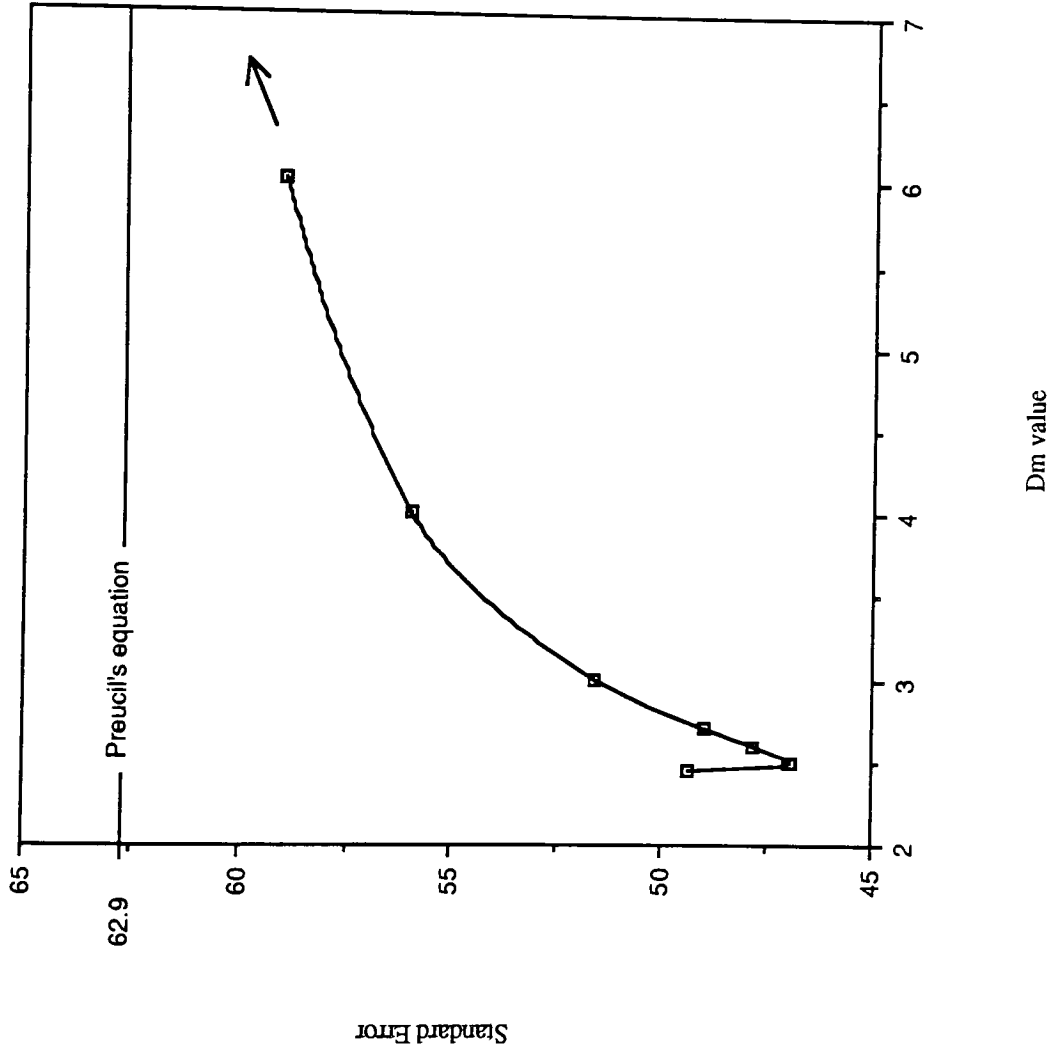


Figure 20. Standard Error of Hamilton's Equation on Coated Paper

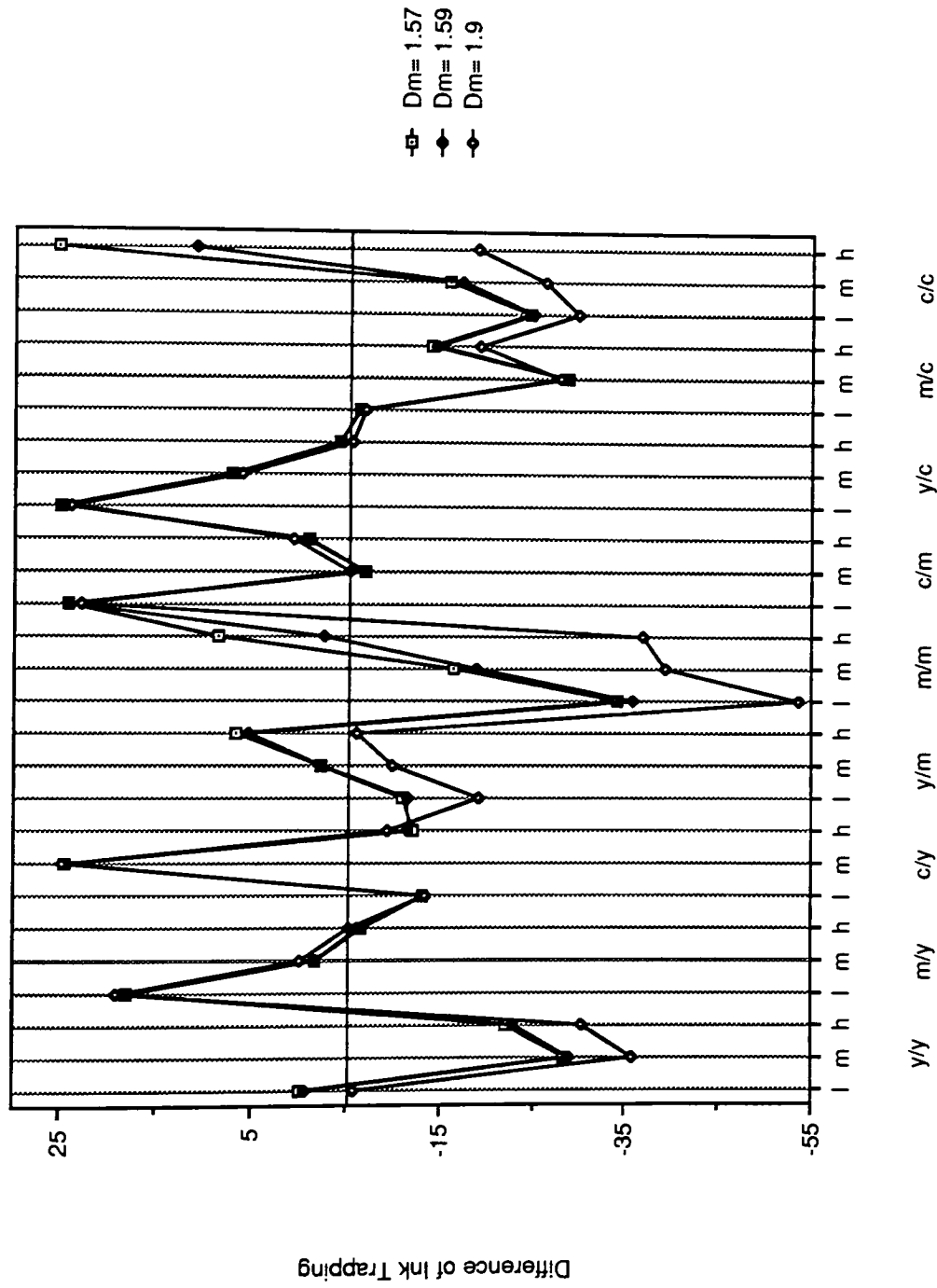


Figure 21. Differences of Ink Trap as Dm Value Varies
in Hamilton's Equation on Newsprint A

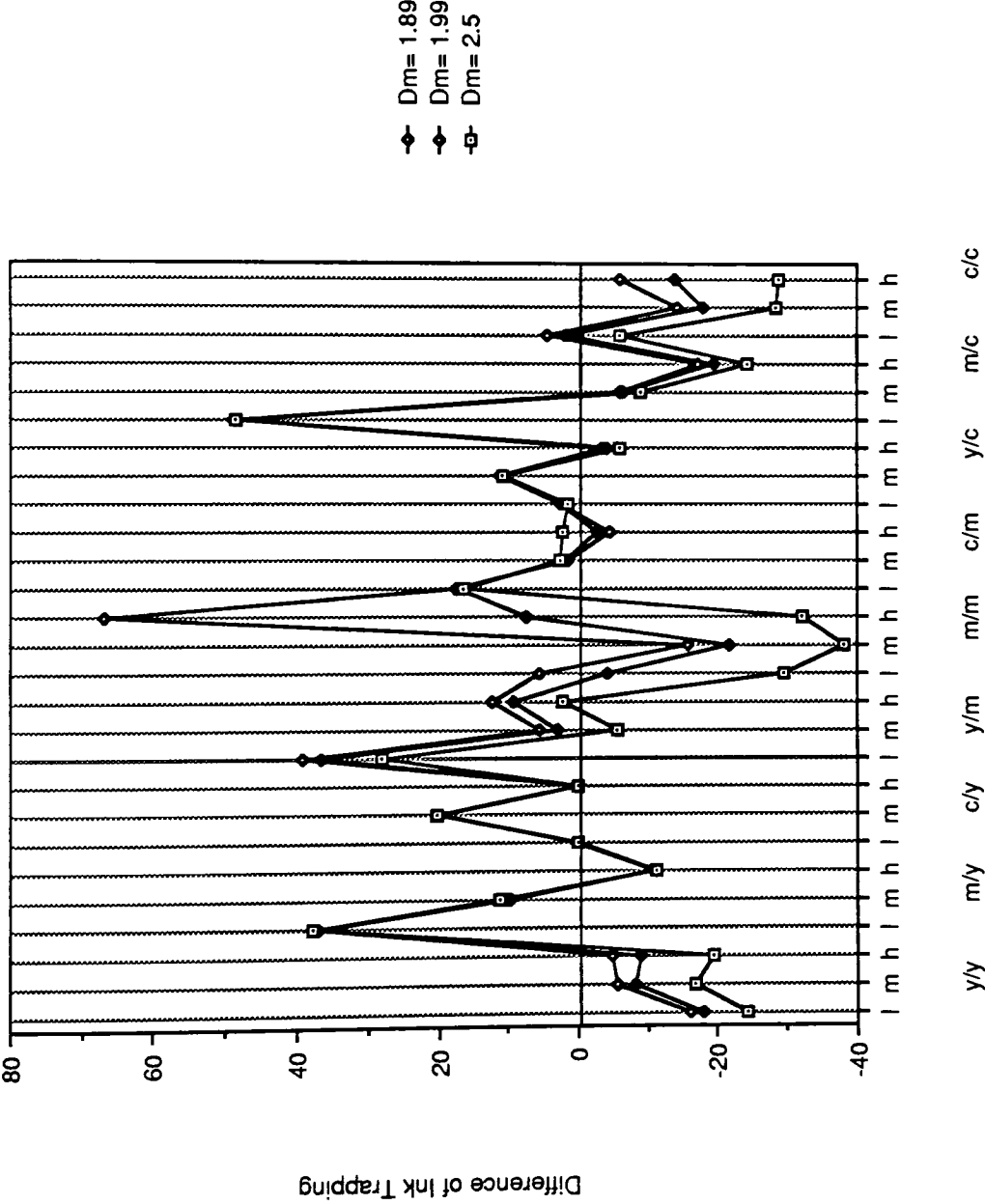


Figure 22. Differences of Ink Trap as Dm Value Varies
in Hamilton's Equation on Newsprint B

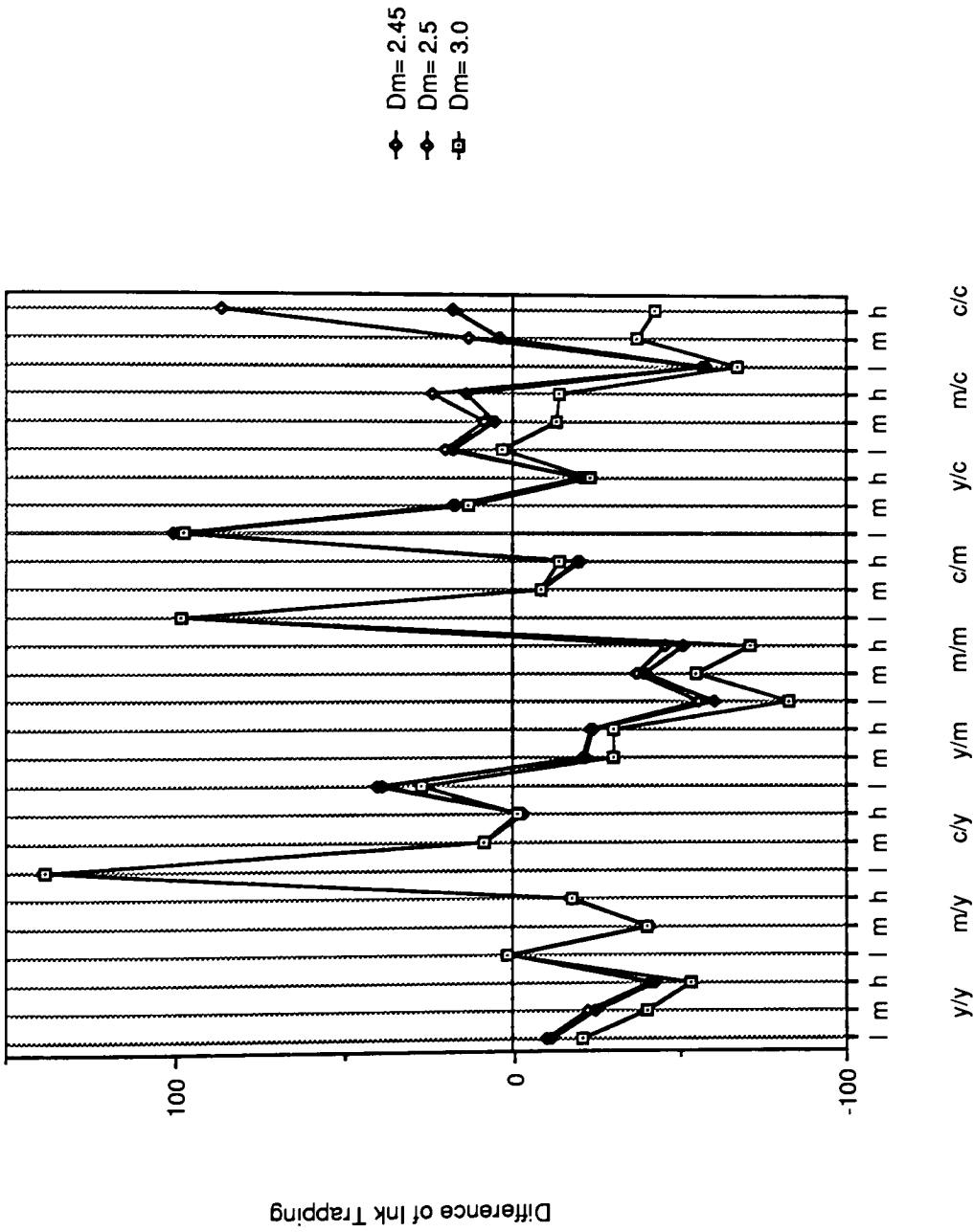


Figure 23. Differences of Ink Trap as D_m Value Varies
in Hamilton's Equation on Coated Paper

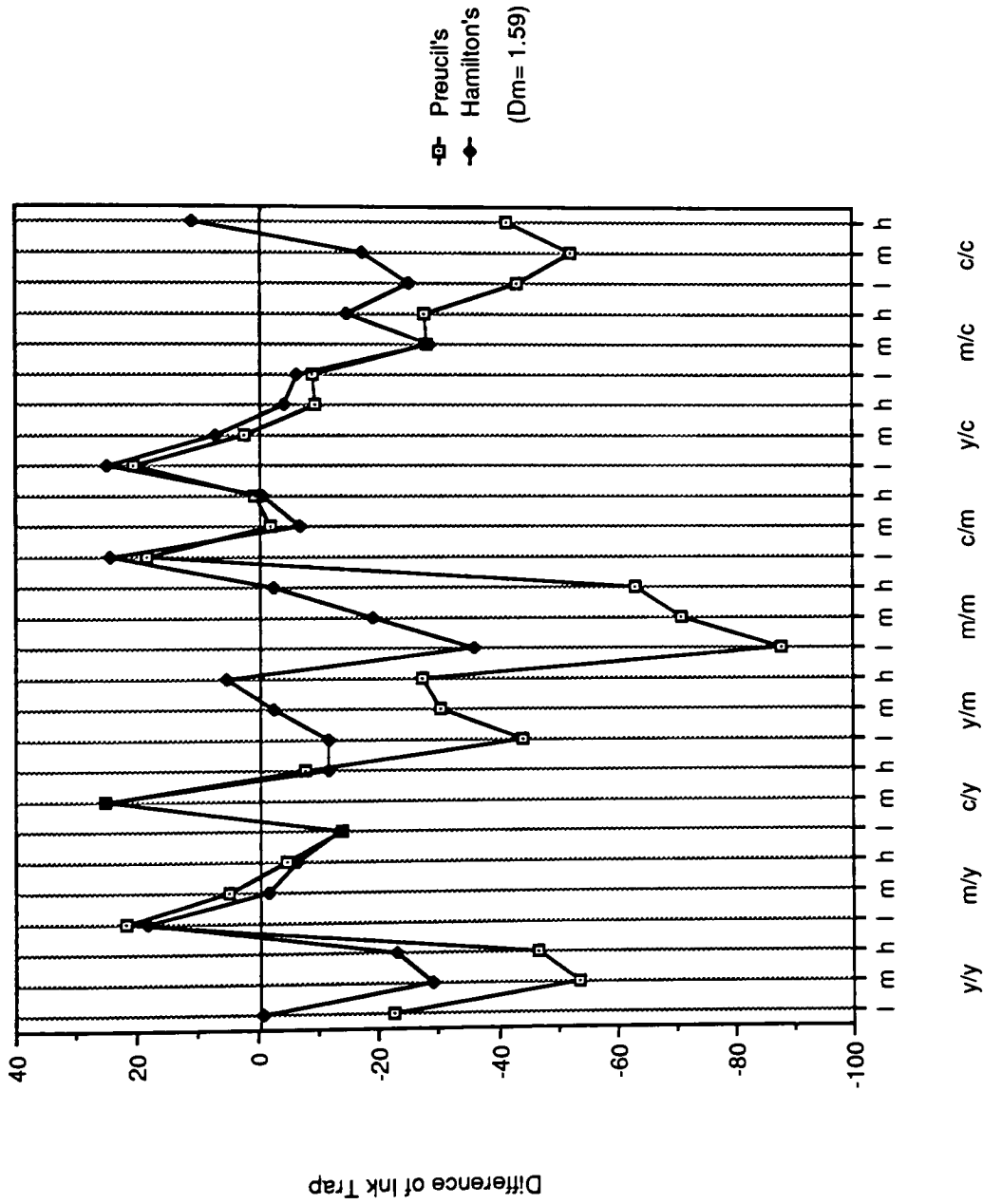


Figure 24. Differences of Calculated Ink Trap on Newsprint A

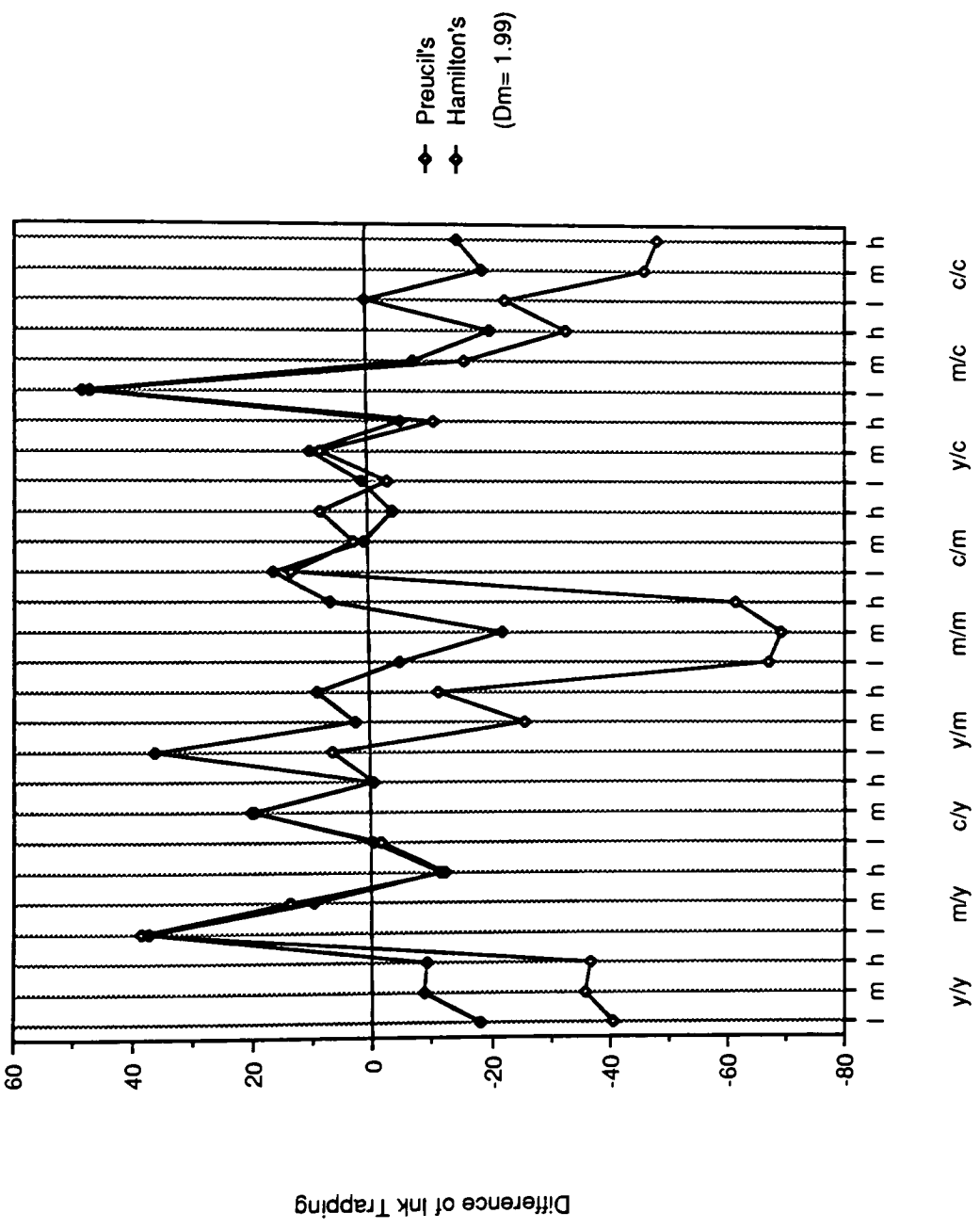


Figure 25. Differences of Calculated Ink Trap on Newsprint B

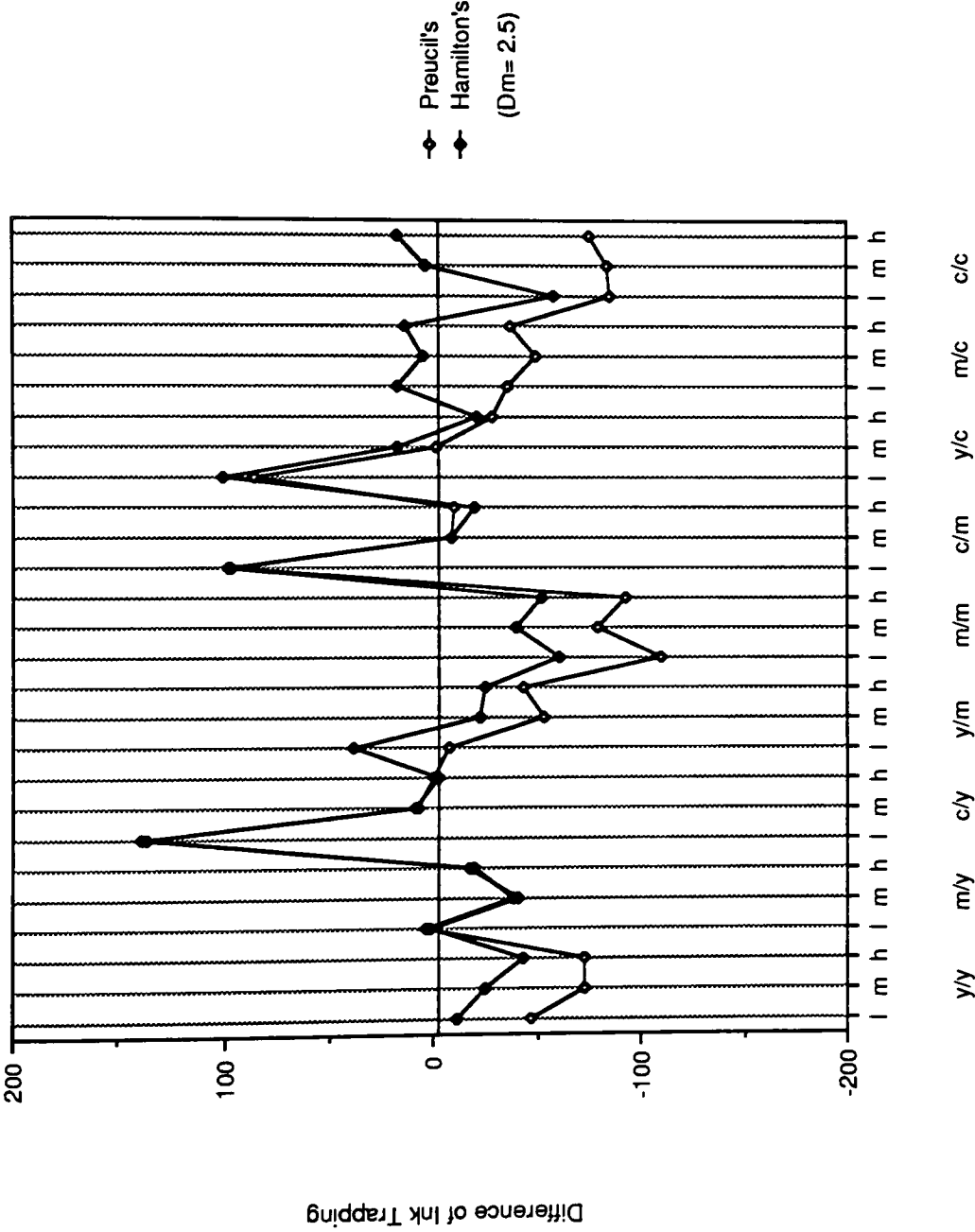


Figure 26. Differences of Calculated Ink Trap on Coated Paper

Figures 18 to 20 show that a smallest standard error is obtained as the D_m value approaches 1.59 on newsprint A, 1.99 on newsprint B, and 2.5 on coated paper. Any value smaller or larger than these will produce a larger standard error on that type of paper. Notice that there is a minimum limit for the D_m value in order to obtain meaningful ink trapping values. The minimum D_m value is the smallest common value for all the ink combinations on the particular type of paper, but it is not necessarily the optimum D_m value for a particular ink combination.

The following are four possibilities for the D_m value, which, when used in Hamilton's equation, will produce undefined ink trapping values. Please refer to Appendix K for mathematical derivation.

1. $D_{21} < D_m < D_1$, or
2. $D_1 < D_m < D_{21}$, or
3. $D_2 < D_m < -1$, or
4. $-1 < D_m < D_2$.

D_{21} = the density of the overlapping area on paper

D_2 = the density of the second ink on paper

D_1 = the density of the first ink on paper

D_m = the maximum printable density of the paper

In brief, there are two ranges of D_m , which are not suitable for Hamilton's equation: (1) D_m is in between D_{21} and D_1 . (2) D_m is in

between -1 and D_2 . However, since D_m , always defined as the maximum printable density on the paper, is always positive, the second condition can be restated as D_m is larger than zero but smaller than D_2 .

On the contrary, as the D_m value gets larger and larger, approaching infinity, the percent ink trapping values calculated by Hamilton's equation are almost the same as those calculated by Preucil's equation. Thus, the standard error of Hamilton's equation with a very large D_m value is equivalent to that of Preucil's equation. It can be expressed as:

$$\lim_{D_m \rightarrow \infty} \left[\frac{\log \left(1 + \frac{D_{21} - D_1}{D_m - D_{21}} \right)}{\log \left(1 + \frac{D_2}{D_m - D_2} \right)} \right] = \frac{D_{21} - D_1}{D_2}$$

The explanation is in Appendix K.

In this study, as shown in Appendix J, the D_m value was replaced with 500 to illustrate the similarity of the two equations. This indicates that using Hamilton's equation with large D_m values is equivalent to using Preucil's equation.

It is interesting to note that on each paper, with ink combinations such as y/y, m/m, and c/c, Preucil's equation becomes more

inaccurate, whereas Hamilton's equation reduces that inaccuracy more. In other words, where the larger additivity failure occurs, Hamilton's equation compensates better. However, in some situations of back-trapping or ink contamination, such as occurred with ly/m, lc/m, ly/c, and mm/c in this experiment, Hamilton's equation did not improve the error of Preucil's equation very significantly. It is suspected that this is due to the experimental error, not the equation itself.

A brief comparison between Preucil's equation and Hamilton's equation in terms of overall standard error follows:

		Standard Error Preucil's	Standard Error Hamilton's	Reduced Standard Error (%) by Hamilton's Equation
Newsprint A		36.58	17.12 (Dm=1.59)	53
Newsprint B		32.46	17.46 (Dm=1.99)	46
Coated paper		62.87	46.94 (Dm=2.5)	25

The standard error of Preucil's equation is improved most significantly on newsprint A, and least on coated paper. The result indicates that Hamilton's equation is especially suitable for very absorbent paper where the additivity failure becomes more critical. A note is directed here that both equations have relatively large standard errors on coated paper as compared to those on newsprint. This does not necessarily imply that the additivity failure is larger on coated paper than on newsprint. Several phenomena observed in the

experiment which may contribute to this were stated earlier on page 69.

Hamilton's equation does predict ink trapping much closer to those measured gravimetrically than Preucil's equation. A question like this now may be asked: Which D_m value should be used for a daily production situation? As can be seen in Figures 14, 15, and 16, any value, which satisfies the range criterion (please refer to page 81) and values up to perhaps 10, will be an improvement of Preucil's equation. A range between 1.5 and 2.5 may be suggested for general printing conditions based on the results of this study. In addition, a precise D_m value is dependent on the densitometer, the ink, and the paper used in that situation. It is difficult, if not impossible, to determine a specific D_m value for the purpose of achieving the greatest accuracy. However, using the D_m value found in this study as a reference, which is 1.59, 1.99, and 2.5 for newsprint A, B, and coated paper respectively, it is reasonable that one could roughly start with a value as the maximum printable density based on the knowledge about that paper.

Theoretically, the reflection density of an ink layer on paper is linearly related with its ink film thickness printed on paper and will increase toward a theoretical density value as the ink film thickness increases. In reality, the measurable maximum printable density of a given paper remains a constant after a certain amount of ink is

transferred to the paper. This maximum density value is much lower than the theoretical density value. Generally, the printer prefers papers with high printable density values to achieve an optimum tone reproduction.

In his book³, Yule concludes that as all the effects of the factors causing the additivity failure are combined, the printable density on the paper is convergent and this point of convergence is 3.5 for cast-coated paper and 1.6 for uncoated paper.

One of the intentions of this study stated in the introduction was to create a regression model based on Hamilton's equation, which, if no appropriate D_m value could be found, is capable of predicting the same ink trap as those measured by gravimetric method. At this point, this is not necessary since Hamilton's equation has been shown to enhance the accuracy of ink trapping measurement significantly. In addition, if any mathematical model could be established, it seems that this model would only be valid for the type of ink and paper used and would be redundant. Such a result is not practical for the production situation.

FOOTNOTES FOR CHAPTER SIX

1. Askew, F.A., Printing Ink Manual, W. Heffer & Sons Ltd., Cambridge, England, 1969, p. 181-193
2. Tollenaar, D., and Ernst, P.A.H., "Tack Experiments for Wet-on-Wet Printing", Eighth International Conference of Printing Research Institute, Helsinki, Finland, 1965, p. 4
3. Yule, John A.C., Principles of Color Reproduction, John Wiley & Sons Inc., 1967, p. 230-231

CHAPTER SEVEN

CONCLUSIONS

In the introduction of this paper, the stated purpose of this study was to compare the densitometric measurements of ink trapping with the gravimetric measurements of ink trapping, and to test the effectiveness of Hamilton's equation. It was also expected that either a specific D_m value could be found for Hamilton's equation, or the equation could be further modified to produce more realistic ink trap values when compared to those measured gravimetrically.

In this study, each densitometric equation was compared with the gravimetric method and an appropriate range of D_m values were found for each paper. The following are the conclusions drawn from the findings presented. Each referred hypothesis is stated again.

As the first hypothesis was stated: There is a significant difference between the gravimetric method and Preucil's equation. This will be true as comparing Brunner's equation with the gravimetric method. Two conclusions can be drawn:

1. Since the results discussed in Chapter Six show that the Preucil equation under-estimates most of the gravimetric ink trapping values, the first hypothesis has been accepted. The results also indicate that the equation is very likely to produce lower ink trapping values than the realistic values when the equation is used to calculate ink trapping.
2. All the ink trapping values calculated by Brunner's equation are in the range from 85% to 100%. There is a large difference in ink trapping measurements between Brunner's equation and the gravimetric method. It is shown that the equation may not be suitable for measuring ink trapping and that the first hypothesis is acceptable.

The second hypothesis was stated as: On newsprint, the conventional densitometric equations will produce significantly different ink trapping values when compared with those measured by the gravimetric method. A conclusion follows:

The under-estimation on the gravimetric ink trapping by Preucil's equation is most critical on newsprint A because newsprint A has the highest absorbency among three paper samples. Since this indicates that the Preucil equation would have a larger error on the very absorbent paper, such as the newsprint, the second hypothesis has been accepted.

The third hypothesis was that as the ink film thickness of the second ink on the first ink layer gets thicker, the conventional densitometric equations will produce significantly different ink trapping values when compared with those measured by the gravimetric method. A conclusion related to it can be drawn as:

The differences between the gravimetric ink trapping values and the densitometric ink trapping values, including those calculated by either Preucil's equation or Brunner's equation, increased as the gravimetric ink trapping values increased. This indicates that both Preucil's equation and Brunner's equation tend to produce more significant errors when they are applied to a situation of a relatively heavier ink film trapped on another ink layer. The results prove that the third hypothesis is acceptable.

The last hypothesis was: The D_m value in Hamilton's equation can be specified or the equation can be further modified to produce an ink trapping value which is closer to the ink trapping value measured gravimetrically than that calculated by Preucil's equation in various trapping conditions and papers. A conclusion is stated as:

Using a small D_m value in Hamilton's equation can result in the reduction of the standard error of Preucil's equation over the conditions tested. In this study, the D_m value of 1.59 for newsprint A, 1.99 for newsprint B, and 2.5 for coated paper resulted in the

greatest improvement over Preucil's equation, close to a factor of 2 on newsprint A and B. A range between 1.5 and 2.5 for the D_m value may be suggested for the conditions similar to those in this study. The findings indicate that with proper D_m values and without further modifications, the Hamilton equation can predict ink trapping more accurately than the Preucil equation. The last hypothesis has been accepted.

Some other comments on this study follows:

1. The discrepancy between the ink trapping values, as calculated by Preucil's equation and Brunner's equation respectively, and those measured by gravimetric method has no direct relationship with the amount of the second-down ink transferred on the disc. With the ink combinations, the discrepancy tends to increase in the situations of same color ink overlappings or the tackier ink printed first. The discrepancy did not prove to be larger on more absorbent papers-newsprint A and B, due to several experimental factors.
2. The discrepancy patterns which Preucil's equation produced on newsprint A and B were very similar. In each ink combination, the variation of ink trapping measured gravimetrically became larger on newsprint A. On coated paper, as the amount of the second-down ink increased from the medium-level to the high-level, the percent ink trapping measured gravimetrically decreased. This indicates if

coated paper can take up more ink, the gravimetric ink trapping will increase accordingly. All these findings may lead to a conclusion: Among paper properties, absorbency is the most crucial factor found in this study for determining the accuracy of ink trapping measurement, see page 35.

3. Poor ink trapping or back-trapping greatly affected the accuracy of both the densitometric method and gravimetric method in ink trapping measurements. Therefore, the tacks of inks should be adequately adjusted to the type of paper to be printed. The type of ink drying should also match the paper property and printing process. And, choosing a proper printing sequence is important.

4. In this study, Hamilton's equation was shown to reduce the error of Preucil's equation, as compared with gravimetric method. This new equation reduced the standard error of Preucil's equation most significantly on newsprint A and in some situations where the additivity failure gets larger. However, Hamilton's equation did not improve the error of Preucil's equation very significantly where back-trapping or ink contamination occurred.

5. While one is using Hamilton's equation, the range criterion of D_m values should be first satisfied. That is, two ranges of D_m values should be avoided. As the D_m value increases, the standard error may decrease toward the lowest first and then increase toward the

standard error as of using Preucil's equation. Using Hamilton's equation with a very large D_m value, for example, over 500, is equivalent to using Preucil's equation.

6. It may be suggested that when one is considering an appropriate D_m value to use, paper absorbency is the key factor.

7. Hamilton's equation is applicable not only to newsprint but to coated paper.

8. Since back-trap and ink contamination affect the ink trap measurements by densitometry, the densitometric method is suggested to use along with the visual evaluation.

Recommendations for Further Study

1. Further investigation with more ink types, papers, and different band-width densitometers like Status-T to find an optimum D_m value for general conditions. Thus, this optimum D_m value will be applicable to the production conditions in the printing industry.

2. Establishing a colorimetric method of calculating ink trap using the color difference(ΔE) value of the overprint relative to a gravimetrically-simulated reference patch. This may compensate the effect of ink contamination, which contributed to the inaccuracy of

measuring ink trapping by densitometry in this study; furthermore, the ink trapping measured in this way may be more closer to the color variation that sensed by visually. Measuring ink trapping directly with a colorimeter may be possible and the colorimeter can detect the ink contamination. However, the additivity failure still applies.

3. Modifying Hamilton's equation based on a variety of simulated back-trap situations to compensate for the back-trap effect. Since the back-trap is suspected part of the reasons that caused Hamilton's equation having errors in some tested conditions, the equation may be modified to compensate the back-trap effect.

4. Finding a theoretical D_m value, with which the realistic D_m value of a given paper can be compared. In this way, the reproduction quality of that paper can be predicted before the printing is conducted. This study only attempts to find the realistic D_m values for three types of papers, no efforts made to discover the theoretical value.

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Appendices

Appendix A

Ink Tack Data

Appendix A

ROCHESTER INSTITUTE OF TECHNOLOGY
GRAPHIC ARTS RESEARCH CENTER
ROCHESTER, NEW YORK 14623

Date: _____

INKOMETER TACK READINGS

Company _____ Formula Number _____

Type _____ Batch Number _____

Purpose _____ Label Date _____

Color Yellow

20 sec. _____	1 min. <u>8.4</u>	6 min. <u>10.4</u>
1 min. _____	2 min. <u>8.8</u>	7 min. <u>10.8</u>
	3 min. <u>9.1</u>	8 min. <u>11.3</u>
	4 min. <u>9.5</u>	9 min. <u>11.8</u>
20 sec. <u>8.3</u>	5 min. <u>9.9</u>	10 min. <u>12.1</u>

Color Magenta

20 sec. _____	1 min. <u>14.9</u>	6 min. <u>17.8</u>
1 min. _____	2 min. <u>15.5</u>	7 min. <u>18.1</u>
	3 min. <u>16.1</u>	8 min. <u>18.7</u>
	4 min. <u>16.6</u>	9 min. <u>19.2</u>
20 sec. <u>14.2</u>	5 min. <u>17.1</u>	10 min. <u>19.6</u>

Color Cyan

20 sec. _____	1 min. <u>23.5</u>	6 min. <u>26.6</u>
1 min. _____	2 min. <u>24.4</u>	7 min. <u>26.8</u>
	3 min. <u>24.8</u>	8 min. <u>27.0</u>
	4 min. <u>25.3</u>	9 min. <u>27.2</u>
20 sec. <u>23.0</u>	5 min. <u>26.1</u>	10 min. <u>27.4</u>

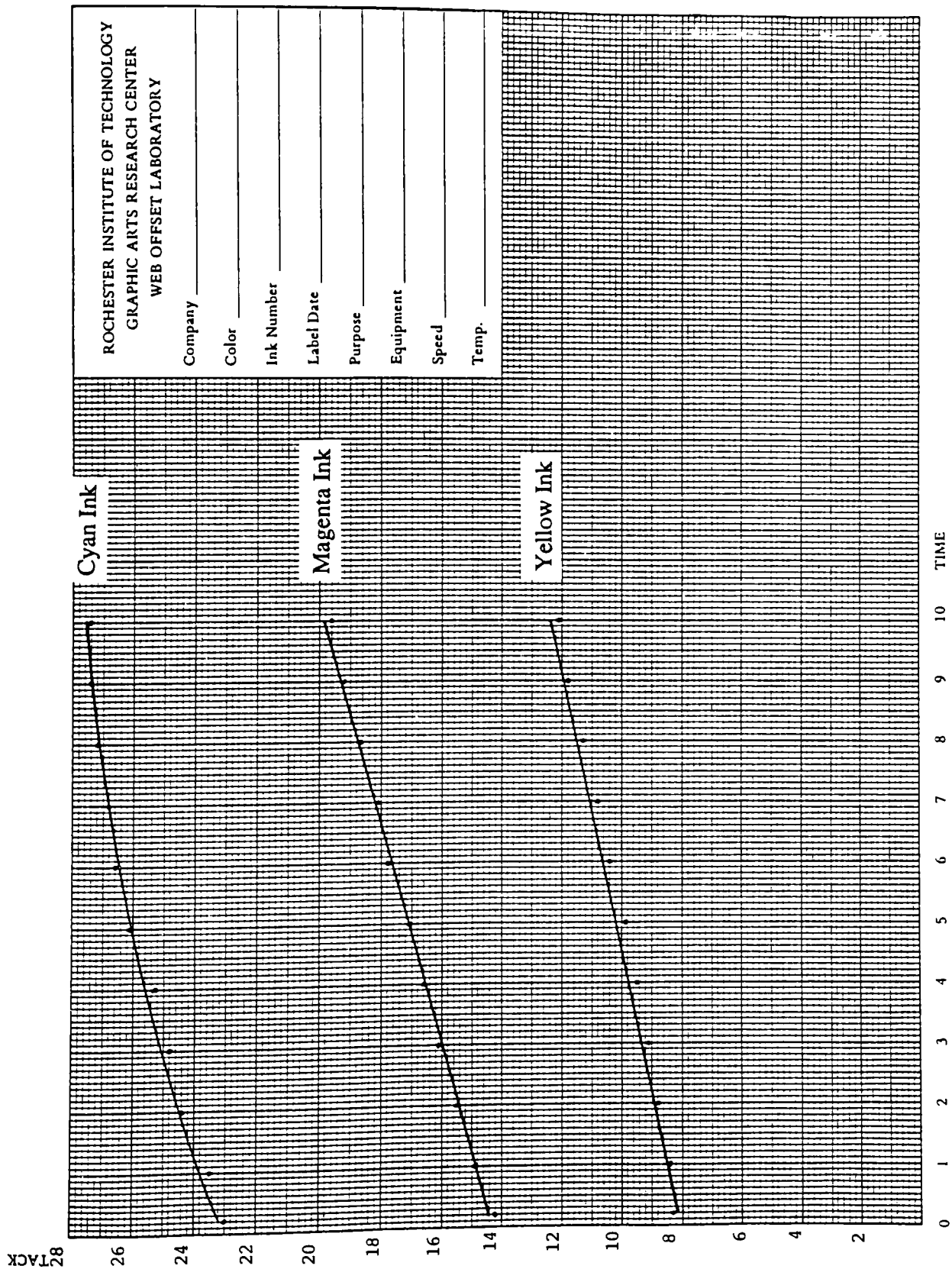
Color _____

20 sec. _____	1 min. _____	6 min. _____
1 min. _____	2 min. _____	7 min. _____
	3 min. _____	8 min. _____
	4 min. _____	9 min. _____
20 sec. _____	5 min. _____	10 min. _____

Appendix B

Ink Tack Curves

Appendix B



Appendix C

Ink Film Thickness Data

Appendix C

----- Yellow Ink on Newsprint A -----

Strip Number	Ink on Disc (g)	Ink Transferred (g)	Area (cm ²)	IFT on Disc (cm)	IFT on Paper (cm)
1	0.0033	0.0012	30.3	0.0001095132	0.000039623
2	0.0041	0.0019	30.4	0.0001758143	0.0000618457
3	0.0044	0.0024	30.4	0.0001455373	0.000079764
4	0.0056	0.0029	30.4	0.0001852293	0.0000959223
5	0.0065	0.0035	30.4	0.0002145993	0.0001157693
6	0.0072	0.0041	30.4	0.0002761519	0.0001358147
7	0.008	0.005	30.4	0.0002646177	0.0001853677
8	0.009	0.0058	30.4	0.0002974699	0.0001919449
9	0.0102	0.0063	30.3	0.0003784954	0.0002078707
10	0.0112	0.0072	30.4	0.0003704536	0.0002231519
11	0.0114	0.0077	30.4	0.0003770739	0.0002546717
12	0.0124	0.0079	30.4	0.0004101366	0.0002613056
13	0.0135	0.009	30.4	0.0004465349	0.0002976899
14	0.001	0.0094	30.7	0.000327534	0.000171014
15	0.001	0.0102	29.5	0.0006740596	0.000603172
16	0.0024	0.0099	29.5	0.0006313059	0.0005064772
17	0.0133	0.0098	29.64	0.0004511795	0.0003824628
18	0.0127	0.0092	29.64	0.0004708447	0.000317166

Appendix C (continued)

 Yellow Ink on Newsprint 8

Strip Number Ink on Disc (g) Ink Transferred (g) Area (cm²) IFT on Disc (cm) IFT on Paper (cm)

1	0.0036	0.0021	30.86	0.000117301	0.0000684256
2	0.0037	0.0017	30.4	0.0001223836	0.0000562303
3	0.0041	0.0022	30.38	0.0001357036	0.0000728166
4	0.0046	0.0027	30.4	0.0001521526	0.000089307
5	0.005	0.0031	30.4	0.0001653833	0.0001025376
6	0.0058	0.0036	30.4	0.0001918446	0.000119076
7	0.0066	0.0038	30.4	0.0002183059	0.0001256913
8	0.007	0.0043	30.5	0.0002307775	0.0001417633
9	0.0078	0.0047	30.6	0.0002563117	0.0001544442
10	0.0083	0.005	30.6	0.0002727419	0.0001643024
11	0.0088	0.005	30.6	0.0002891721	0.0001643024
12	0.0088	0.005	30.6	0.0002891721	0.0001643024
13	0.0101	0.0057	30.6	0.0003318908	0.0001873047
14	0.0051	0.003	30.8	0.0001665002	0.0000979413
15	0.0085	0.005	30.8	0.0002775003	0.0001632355
16	0.0025	0.0013	30.5	0.0000824205	0.0000428587
17	0.0034	0.0017	30.4	0.0001124606	0.0000562303
18	0.0117	0.007	30.4	0.0003869969	0.0002315366
19	0.0009	0.0003	30.6	0.0000295744	0.0000098581
20	0.0077	0.0046	30.8	0.0002513826	0.0001501766
21	0.009	0.0056	30.8	0.0002938238	0.0001828237
22	0.0007	0.0004	29.5	0.00002386	0.0000136343
23	0.0022	0.0008	29.5	0.0000749887	0.0000272686
24	0.0133	0.0082	29.64	0.0004511995	0.0002781832
25	0.0126	0.0076	29.7	0.0004265887	0.0002573074
26	0.0131	0.0081	29.62	0.0004447147	0.0002749762
27	0.0127	0.0078	29.76	0.0004291074	0.0002635463

Appendix C (continued)

 Yellow Ink on Coated Paper

Strip Number	Ink on Disc (g)	Ink Transferred (g)	Area (cm ²)	IFT on Disc (cm)	IFT on Paper (cm)
1	0.0041	0.0021	30.4	0.0001356143	0.000069461
2	0.0043	0.0023	30.4	0.0001422296	0.0000760763
3	0.0049	0.0024	30.4	0.0001620756	0.000079384
4	0.0058	0.0028	30.4	0.0001918446	0.0000926146
5	0.0062	0.0031	30.4	0.0002050753	0.0001025376
6	0.0068	0.0032	30.6	0.0002234512	0.0001051535
7	0.0076	0.0037	30.68	0.0002490884	0.0001212667
8	0.0082	0.0043	30.6	0.0002694559	0.0001413
9	0.0084	0.004	30.6	0.000276028	0.0001314419
10	0.0085	0.0041	30.6	0.000279314	0.0001347279
11	0.0095	0.0044	30.6	0.0003121745	0.0001445861
12	0.0106	0.0053	30.9	0.0003449392	0.0001724696
13	0.0076	0.0037	30.8	0.0002481179	0.0001207942
14	0.0079	0.0043	30.8	0.000257912	0.0001403825
15	0.0011	0.0004	30.6	0.0000361465	0.0000131442
16	0.0005	0.0003	29.5	0.0000170429	0.0000102257
17	0.0023	0.0017	29.5	0.0000783973	0.0000579458

Appendix C (continued)

 Magenta Ink on Newsprint A

Strip Number	Ink on Disc (g)	Ink Transferred (g)	Area (cm ²)	IFT on Disc (cm)	IFT on Paper (cm)
1	0.0027	0.0007	29.86	0.0000884582	0.0000229336
2	0.0034	0.0012	29.86	0.0001113918	0.0000393148
3	0.0037	0.0013	29.95	0.0001208562	0.000042463
4	0.0048	0.0021	29.95	0.0001567865	0.0000685941
5	0.0059	0.0026	29.86	0.0001932975	0.000085182
6	0.0063	0.0031	29.95	0.0002057822	0.0001012579
7	0.0069	0.0035	29.95	0.0002253805	0.0001143235
8	0.0085	0.0045	29.95	0.0002776427	0.0001469873
9	0.0094	0.0048	29.95	0.0003070401	0.0001567865
10	0.0108	0.0052	29.95	0.0003527695	0.000169852
11	0.0106	0.0057	29.95	0.0003462367	0.0001861839
12	0.0126	0.0069	30.9	0.0003989112	0.0002184514
13	0.0134	0.0073	30.8	0.0004256163	0.0002318656
14	0.0161	0.0091	30.8	0.0005113748	0.0002890379
15	0.0159	0.0086	30.8	0.0005050223	0.0002731567
16	0.0156	0.0086	30.8	0.0004954935	0.0002731567
17	0.021	0.011	30.8	0.0006670105	0.0003493865
18	0.0205	0.013	29.64	0.0006766121	0.0004290711
19	0.0221	0.0138	29.64	0.0007294209	0.0004554755
20	0.0157	0.0106	29.76	0.0005160964	0.0003484473
21	0.0164	0.0108	29.7	0.0005401962	0.000355739
22	0.0257	0.016	29.84	0.0008425553	0.0005245481

Appendix C (continued)

Magenta Ink on Newsprint B

Strip Number	Ink on Disc (g)	Ink Transferred (g)	Area (cm ²)	IFT on Disc (cm)	IFT on Paper (cm)
1	0.0041	0.0013	30.7	0.0001306501	0.0000414256
2	0.006	0.0028	30.8	0.0001905744	0.0000889347
3	0.0067	0.0034	30.8	0.0002128081	0.0001079922
4	0.0075	0.0037	30.8	0.0002382181	0.0001175209
5	0.008	0.004	30.8	0.0002540993	0.0001270496
6	0.0095	0.0043	30.2	0.0003077378	0.0001392918
7	0.0115	0.0055	30.4	0.0003700738	0.0001769918
8	0.0109	0.0052	30.4	0.0003507656	0.0001673377
9	0.0127	0.0058	30.76	0.0004039071	0.0001844615
10	0.0128	0.0059	30.7	0.0004078831	0.0001880086
11	0.0137	0.0063	30.7	0.0004365624	0.000200755
12	0.0174	0.0078	30.8	0.0005526659	0.0002477468
13	0.0225	0.0092	30.8	0.0007146542	0.0002922141
14	0.0162	0.0068	30.9	0.0005128858	0.0002152854
15	0.0152	0.006	30.8	0.0004827886	0.0001905744
16	0.0026	0.0007	29.48	0.00008628	0.0000232292
17	0.0019	0.0007	29.52	0.0000629653	0.0000231977
18	0.0026	0.0009	29.56	0.0000860465	0.0000297853
19	0.0041	0.0018	29.56	0.0001356887	0.0000595706
20	0.021	0.0108	29.76	0.0006903201	0.0003550217
21	0.017	0.0089	29.76	0.0005588305	0.0002925642
22	0.0181	0.009	29.78	0.0005945906	0.0002956528

Appendix C (continued)

 Magenta Ink on Coated Paper

Strip Number	Ink on Disc (g)	Ink Transferred (g)	Area (cm ²)	IFT on Disc (cm)	IFT on Paper (cm)
1	0.004	0.0017	32.6	0.0001200346	0.0000510147
2	0.0053	0.0027	32.7	0.0001585595	0.0000807756
3	0.0057	0.0024	32.7	0.0001705262	0.0000718005
4	0.0065	0.0031	32.7	0.0001944598	0.0000927423
5	0.0083	0.004	32.7	0.0002483101	0.0001196675
6	0.0084	0.0036	32.7	0.0002513018	0.0001077008
7	0.0092	0.0039	32.7	0.0002752353	0.0001166759
8	0.0098	0.0044	32.7	0.0002931855	0.0001316343
9	0.0108	0.0046	32.7	0.0003231024	0.0001376177
10	0.0106	0.0045	30.7	0.0003377782	0.0001433964
11	0.0117	0.0044	30.7	0.0003728307	0.0001402098
12	0.0136	0.0054	30.8	0.0004319687	0.000171517
13	0.0138	0.0054	30.8	0.0004383212	0.000171517
14	0.0023	0.0007	29.48	0.0000763246	0.0000232292
15	0.003	0.0018	29.58	0.0000992173	0.0000595304
16	0.0035	0.0018	29.6	0.0001156753	0.0000594901
17	0.0037	0.0016	29.56	0.0001224507	0.0000529517
18	0.02	0.0096	29.68	0.0006592198	0.0003164255
19	0.0189	0.0092	29.79	0.0006206624	0.0003021214
20	0.0254	0.0119	29.9	0.000831049	0.0003893497

Appendix C (continued)

 Cyan Ink on Newsprint A

Strip Number	Ink on Disc (g)	Ink Transferred (g)	Area (cm ²)	IFT on Disc (cm)	IFT on Paper (cm)
1	0.0027	0.0009	30.8	0.0000832975	0.0000277658
2	0.0037	0.0013	30.7	0.0001145203	0.0000402369
3	0.0046	0.0018	30.8	0.0001419143	0.0000555317
4	0.0051	0.0021	30.8	0.0001573398	0.000064787
5	0.0065	0.0029	30.8	0.0002005311	0.0000894677
6	0.0075	0.0039	30.8	0.0002313821	0.0001203187
7	0.0082	0.0044	30.8	0.0002529777	0.0001357441
8	0.0103	0.0057	30.8	0.0003177647	0.0001758504
9	0.0117	0.0065	30.8	0.000360956	0.0002005311
10	0.0129	0.0068	30.8	0.0003979772	0.0002097864
11	0.0144	0.0077	30.8	0.0004442536	0.0002375523
12	0.0006	0.0002	29.24	0.0000194981	0.0000064994
13	0.0017	0.0005	29.6	0.0000545728	0.0000160508
14	0.0025	0.0006	29.5	0.0000805262	0.0000193263
15	0.016	0.0095	29.62	0.0005132797	0.0003047598
16	0.0177	0.0106	29.68	0.0005666678	0.0003393604
17	0.0195	0.0116	29.7	0.0006238746	0.0003711254
18	0.0214	0.0129	29.7	0.0006846624	0.0004127171
19	0.0222	0.0129	29.3	0.0007078739	0.0004113321

Appendix C (continued)

 Cyan Ink on Newsprint B

Strip Number	Ink on disc (g)	Ink Transferred (g)	Area (cm ²)	IFT on Disc (cm)	IFT on Paper (cm)
1	0.0014	0.0004	30.6	0.0000434776	0.000012421
2	0.0024	0.001	30.6	0.0000745262	0.0000310526
3	0.003	0.0013	30.6	0.0000931577	0.0000403684
4	0.0043	0.002	30.6	0.0001335261	0.0000621052
5	0.005	0.0023	30.6	0.0001552629	0.0000714209
6	0.0064	0.003	30.6	0.0001987365	0.0000931577
7	0.0072	0.0036	30.6	0.0002235786	0.0001117893
8	0.0083	0.0042	30.6	0.0002577364	0.0001304208
9	0.011	0.0061	30.6	0.0003415784	0.0001894208
10	0.012	0.0062	30.7	0.0003714172	0.0001918989
11	0.0125	0.0067	30.76	0.0003861383	0.0002069701
12	0.0127	0.0067	30.8	0.000391807	0.0002067013
13	0.0017	0.0005	29.56	0.0000546467	0.0000160725
14	0.0023	0.0011	29.6	0.0000738338	0.0000353118
15	0.0169	0.0085	29.68	0.0005410557	0.0002721286
16	0.02	0.0097	29.7	0.0006398714	0.0003103376

Appendix C (continued)

 Cyan Ink on Coated Paper

Strip Number	Ink on Disc (g)	Ink Transferred (g)	Area (cm ²)	IFT on Disc (cm)	IFT on Paper (cm)
1	0.0013	0.0007	30.8	0.0000401062	0.0000215957
2	0.0024	0.0013	30.8	0.0000740423	0.0000401062
3	0.0032	0.0016	30.8	0.000098723	0.0000493615
4	0.0033	0.0016	30.8	0.0001018081	0.0000493615
5	0.005	0.0027	30.8	0.0001542547	0.0000832975
6	0.0056	0.003	30.8	0.0001727653	0.0000925528
7	0.0069	0.0035	30.8	0.0002128715	0.0001079783
8	0.0077	0.0039	30.8	0.0002375523	0.0001203187
9	0.0093	0.0043	30.8	0.0002869138	0.0001326591
10	0.0096	0.0044	30.8	0.0002961691	0.0001357441
11	0.0106	0.005	30.8	0.00032702	0.0001542547
12	0.0115	0.0055	30.8	0.0003547858	0.0001696802
13	0.0125	0.006	30.8	0.0003856368	0.0001851057
14	0.0139	0.0065	30.8	0.0004288281	0.0002005311
15	0.0005	0.0002	29.4	0.00001616	0.000006464
16	0.0003	0.0001	29.22	0.0000097557	0.0000032519
17	0.0006	0.0001	29.4	0.000019392	0.000003232
18	0.0012	0.0008	29.6	0.000038522	0.0000256813

Appendix D

Sample of Calculating Ink Film Thickness and Ink Trap by Gravimetric Method

Appendix D

Calculating Ink Film Thickness for Plotting Ink Transfer Curve

For yellow ink on newsprint A: (data from strip no.1 on page 104)

Specific gravity: 0.9945 g/cm³

Printed area: 30.3 cm²

weight of disc = 84.5038g

weight of disc + ink = 84.5071 g

weight of ink on disc

before transfer = 84.5071 - 84.5038

= 0.0033 g

ink film thickness on disc = weight / specific gravity / area

= (0.0033 / 0.9945 / 30.3

= 0.0001095132 cm

weight of disc after transfer = 84.5059 g

weight of ink transferred = 0.0012 g

ink film thickness on paper = 0.0012 / 0.9945 / 30.3

= 0.0000398230 cm

Appendix D (continued)

Calculating Ink Trap by Gravimetric Method

For yellow overprinting magenta ink on newsprint A:
(data from Low Y/M replication no. 1 on page 128)

Specific gravity of yellow ink: 0.9945 g

Printed area: $13.85 \times 2 = 27.7 \text{ cm}^2$

"disc2" refers to the printing disc with which the second-down ink (yellow) is printed.

weight of disc2 = 84.4776 g

weight of disc2 + ink = 84.4798 g

weight of ink on disc2 = $84.4798 - 84.4776$

before transfer = 0.0022 g

ink film thickness on disc2 = weight / specific gravity / area
= $0.0022 / 0.9945 / 27.7$
= 0.0000798616 cm

weight of disc2

after transferred = 84.4788 g

weight of ink transferred = $84.4798 - 84.4788$

= 0.0010 g

area of ink transferred

directly to paper (A_2) = 3.1 cm^2

weight of ink transferred

directly to paper

= $0.5 \times (\text{weight of ink on disc}) \times$
(area of coverage on paper / total
area of coverage)

= $0.5 \times (0.0022) \times (3.1/27.7)$

= 0.0001231047 g

ink film thickness of ink2
transferred to paper
as derived from
ink transfer curve $= 0.0000320779 \text{ cm}$

weight of ink2 printed
on top of ink1 $= 0.0022 - 0.0001231047$
 $= 0.000876895 \text{ g}$

area of ink2
overprinted ink1 $= 27.7 - 3.1$
 $= 24.6 \text{ cm}^2$

ink film thickness of ink2
printed on top of ink1 $= 0.000876895 / 0.9945 / 24.6$
 $= 0.0000358433 \text{ cm}$

% gravimetric trapping $= \text{ink film thickness of ink2}$
 $\text{trapped over first-down ink} / \text{ink}$
 $\text{film thickness of ink2 printed}$
 $\text{directly on paper} \times 100$
 $= 0.0000358433 / 0.0000320779$
 $= 111.74\%$

Appendix E

Ink Transfer Curve Data

Appendix E (continued)

Yellow Ink on Newsprint A

MTB > brief 3

MTB > regr c2 on 2, c1 c3

The regression equation is

$$Y = -0.000010 + 0.484 X + 537 X^2$$

Predictor	Coef	Stdev	t-ratio	p
Constant	-0.00000977	0.00000787	-1.24	0.233
X	0.48390	0.07519	6.44	0.000
X2	537.2	148.0	3.63	0.002

s = 0.000009918 R-sq = 99.3% R-sq(adj) = 99.2%

Analysis of Variance

SOURCE	DF	SS	MS	F	P
Regression	2	1.97879E-07	9.89395E-08	1005.82	0.000
Error	15	1.47551E-09	9.83675E-11		
Total	17	1.99355E-07			

SOURCE	DF	SEQ SS
X	1	1.96584E-07
X2	1	1.29515E-09

Obs.	X	Y	Fit	Stdev.Fit	Residual	St.Resid
1	0.000110	0.000040	0.000050	0.000003	-0.000010	-1.06
2	0.000136	0.000063	0.000066	0.000003	-0.000003	-0.31
3	0.000146	0.000079	0.000072	0.000003	0.000007	0.78
4	0.000185	0.000096	0.000098	0.000003	-0.000002	-0.26
5	0.000215	0.000116	0.000119	0.000004	-0.000003	-0.36
6	0.000238	0.000136	0.000136	0.000004	-0.000000	-0.04
7	0.000265	0.000165	0.000156	0.000004	0.000009	1.03
8	0.000298	0.000192	0.000182	0.000004	0.000010	1.07
9	0.000338	0.000209	0.000216	0.000003	-0.000007	-0.69
10	0.000370	0.000238	0.000243	0.000003	-0.000005	-0.54
11	0.000377	0.000255	0.000249	0.000003	0.000006	0.60
12	0.000410	0.000261	0.000279	0.000004	-0.000018	-1.93
13	0.000447	0.000298	0.000313	0.000005	-0.000016	-1.83
14	0.000033	0.000013	0.000007	0.000006	0.000006	0.81
15	0.000034	0.000007	0.000007	0.000006	-0.000001	-0.07
16	0.000082	0.000031	0.000033	0.000004	-0.000003	-0.30
17	0.000451	0.000332	0.000318	0.000005	0.000015	1.72
18	0.000431	0.000312	0.000298	0.000004	0.000014	1.53

Worksheet saved into file: templ.MTW

MTB > stop

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Appendix E (continued)

Yellow Ink on Newsprint B

MTB > BRIEF 3

MTB > REGR C2 ON 1, C1

The regression equation is
 $Y = -0.000010 + 0.630 X$

Predictor	Coef	Stdev	t-ratio	p
Constant	-0.00000965	0.00000249	-3.87	0.001
X	0.629897	0.009516	66.19	0.000

s = 0.000006138 R-sq = 99.4% R-sq(adj) = 99.4%

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	1	1.65047E-07	1.65047E-07	4381.37	0.000
Error	25	9.41753E-10	3.76701E-11		
Total	26	1.65989E-07			

Obs.	X	Y	Fit	Stdev.Fit	Residual	St.Resid
1	0.000117	0.000068	0.000064	0.000002	0.000004	0.71
2	0.000122	0.000056	0.000067	0.000002	-0.000011	-1.89
3	0.000136	0.000073	0.000076	0.000001	-0.000003	-0.51
4	0.000152	0.000089	0.000086	0.000001	0.000003	0.52
5	0.000165	0.000103	0.000095	0.000001	0.000008	1.34
6	0.000192	0.000119	0.000111	0.000001	0.000008	1.31
7	0.000218	0.000126	0.000128	0.000001	-0.000002	-0.36
8	0.000231	0.000142	0.000136	0.000001	0.000006	1.00
9	0.000256	0.000154	0.000152	0.000001	0.000003	0.44
10	0.000273	0.000164	0.000162	0.000001	0.000002	0.36
11	0.000289	0.000164	0.000172	0.000001	-0.000008	-1.37
12	0.000289	0.000164	0.000172	0.000001	-0.000008	-1.37
13	0.000332	0.000187	0.000199	0.000002	-0.000012	-2.04R
14	0.000167	0.000098	0.000095	0.000001	0.000003	0.45
15	0.000277	0.000163	0.000165	0.000001	-0.000002	-0.32
16	0.000082	0.000043	0.000042	0.000002	0.000001	0.10
17	0.000112	0.000056	0.000061	0.000002	-0.000005	-0.84
18	0.000387	0.000232	0.000234	0.000002	-0.000003	-0.44
19	0.000030	0.000010	0.000009	0.000002	0.000001	0.15
20	0.000251	0.000150	0.000149	0.000001	0.000001	0.25
21	0.000294	0.000183	0.000175	0.000001	0.000007	1.23
22	0.000024	0.000014	0.000005	0.000002	0.000008	1.45
23	0.000075	0.000027	0.000038	0.000002	-0.000010	-1.77
24	0.000451	0.000278	0.000275	0.000002	0.000004	0.64
25	0.000427	0.000257	0.000259	0.000002	-0.000002	-0.31
26	0.000445	0.000275	0.000270	0.000002	0.000005	0.79
27	0.000429	0.000264	0.000261	0.000002	0.000003	0.51

R denotes an obs. with a large st. resid.

Appendix E (continued)

Yellow Ink on Coated Paper

MTB > BRIEF 3
 MTB > REGR C2 ON 1, C1

The regression equation is
 $Y = 0.000005 + 0.475 X$

Predictor	Coef	Stdev	t-ratio	p
Constant	0.00000504	0.00000419	1.21	0.247
X	0.47472	0.01889	25.13	0.000

s = 0.000007147 R-sq = 97.7% R-sq(adj) = 97.5%

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	1	3.22680E-08	3.22680E-08	631.72	0.000
Error	15	7.66198E-10	5.10799E-11		
Total	16	3.30342E-08			

Obs.	X	Y	Fit	Stdev.Fit	Residual	St.Resid
1	0.000136	0.000069	0.000069	0.000002	0.000000	0.01
2	0.000142	0.000076	0.000073	0.000002	0.000004	0.51
3	0.000162	0.000079	0.000082	0.000002	-0.000003	-0.38
4	0.000192	0.000093	0.000096	0.000002	-0.000004	-0.51
5	0.000205	0.000103	0.000102	0.000002	0.000000	0.02
6	0.000223	0.000105	0.000111	0.000002	-0.000006	-0.86
7	0.000249	0.000121	0.000123	0.000002	-0.000002	-0.29
8	0.000269	0.000141	0.000133	0.000002	0.000008	1.22
9	0.000276	0.000131	0.000136	0.000002	-0.000005	-0.68
10	0.000279	0.000135	0.000138	0.000002	-0.000003	-0.43
11	0.000312	0.000145	0.000153	0.000003	-0.000009	-1.31
12	0.000345	0.000172	0.000169	0.000003	0.000004	0.58
13	0.000248	0.000121	0.000123	0.000002	-0.000002	-0.30
14	0.000258	0.000140	0.000127	0.000002	0.000013	1.88
15	0.000036	0.000013	0.000022	0.000004	-0.000009	-1.46
16	0.000017	0.000010	0.000013	0.000004	-0.000003	-0.49
17	0.000078	0.000058	0.000042	0.000003	0.000016	2.40R

R denotes an obs. with a large st. resid.

Appendix E (continued)

Magenta Ink on Newsprint A

MTB > BRIEF 3
 MTB > REGR C2 ON 1, C1

The regression equation is
 $Y = -0.000040 + 0.662 X$

Predictor	Coef	Stdev	t-ratio	p
Constant	-0.00003981	0.00000944	-4.22	0.000
X	0.66231	0.02108	31.42	0.000

s = 0.00002081 R-sq = 98.0% R-sq(adj) = 97.9%

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	1	4.27743E-07	4.27743E-07	987.34	0.000
Error	20	8.66456E-09	4.33228E-10		
Total	21	4.36407E-07			

Obs.	X	Y	Fit	Stdev.Fit	Residual	St.Resid
1	0.000088	0.000023	0.000019	0.000008	0.000004	0.22
2	0.000111	0.000039	0.000034	0.000007	0.000005	0.28
3	0.000121	0.000042	0.000040	0.000007	0.000002	0.11
4	0.000157	0.000069	0.000064	0.000007	0.000005	0.23
5	0.000193	0.000085	0.000088	0.000006	-0.000003	-0.15
6	0.000206	0.000101	0.000096	0.000006	0.000005	0.24
7	0.000225	0.000114	0.000109	0.000006	0.000005	0.24
8	0.000278	0.000147	0.000144	0.000005	0.000003	0.14
9	0.000307	0.000157	0.000164	0.000005	-0.000007	-0.33
10	0.000353	0.000170	0.000194	0.000005	-0.000024	-1.18
11	0.000346	0.000186	0.000190	0.000005	-0.000003	-0.16
12	0.000399	0.000218	0.000224	0.000004	-0.000006	-0.29
13	0.000426	0.000232	0.000242	0.000004	-0.000010	-0.50
14	0.000511	0.000289	0.000299	0.000005	-0.000010	-0.49
15	0.000505	0.000273	0.000295	0.000005	-0.000022	-1.06
16	0.000495	0.000273	0.000288	0.000005	-0.000015	-0.75
17	0.000667	0.000349	0.000402	0.000007	-0.000053	-2.69R
18	0.000677	0.000429	0.000408	0.000007	0.000021	1.07
19	0.000729	0.000455	0.000443	0.000008	0.000012	0.64
20	0.000516	0.000348	0.000302	0.000005	0.000046	2.30R
21	0.000540	0.000356	0.000318	0.000005	0.000038	1.88
22	0.000843	0.000525	0.000518	0.000010	0.000006	0.35

R denotes an obs. with a large st. resid.

Worksheet saved into file: TEMP4.MTW

MTB > STOP

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Appendix E (continued)

Magenta Ink on Newsprint B

MTB > BRIEF 3

MTB > REGR C2 ON 2, C1 C3

The regression equation is

$$Y = -0.000013 + 0.527 X - 68 X^2$$

Predictor	Coef	Stdev	t-ratio	p
Constant	-0.00001317	0.00001384	-0.95	0.353
X	0.52688	0.08525	6.18	0.000
X2	-68.4	111.6	-0.61	0.547

s = 0.00001879 R-sq = 96.6% R-sq(adj) = 96.2%

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	2	1.89367E-07	9.46837E-08	268.29	0.000
Error	19	6.70543E-09	3.52918E-10		
Total	21	1.96073E-07			

SOURCE	DF	SEQ SS
X	1	1.89235E-07
X2	1	1.32724E-10

Obs.	X	Y	Fit	Stdev.Fit	Residual	St.Resid
1	0.000131	0.000041	0.000055	0.000006	-0.000013	-0.74
2	0.000191	0.000089	0.000085	0.000005	0.000004	0.23
3	0.000213	0.000108	0.000096	0.000005	0.000012	0.67
4	0.000238	0.000118	0.000108	0.000005	0.000009	0.50
5	0.000254	0.000127	0.000116	0.000005	0.000011	0.60
6	0.000308	0.000139	0.000142	0.000006	-0.000003	-0.18
7	0.000370	0.000177	0.000172	0.000006	0.000005	0.25
8	0.000351	0.000167	0.000163	0.000006	0.000004	0.23
9	0.000404	0.000184	0.000188	0.000006	-0.000004	-0.23
10	0.000408	0.000188	0.000190	0.000006	-0.000002	-0.13
11	0.000437	0.000201	0.000204	0.000006	-0.000003	-0.17
12	0.000553	0.000248	0.000257	0.000006	-0.000009	-0.52
13	0.000715	0.000292	0.000328	0.000012	-0.000036	-2.55RX
14	0.000513	0.000215	0.000239	0.000006	-0.000024	-1.33
15	0.000483	0.000191	0.000225	0.000006	-0.000035	-1.93
16	0.000086	0.000023	0.000032	0.000008	-0.000009	-0.51
17	0.000063	0.000023	0.000020	0.000010	0.000003	0.21
18	0.000086	0.000030	0.000032	0.000008	-0.000002	-0.11
19	0.000136	0.000060	0.000057	0.000006	0.000003	0.14
20	0.000690	0.000355	0.000318	0.000011	0.000037	2.40R
21	0.000559	0.000293	0.000260	0.000006	0.000033	1.83
22	0.000595	0.000296	0.000276	0.000007	0.000020	1.12

R denotes an obs. with a large st. resid.

X denotes an obs. whose X value gives it large influence.

Appendix E (continued)

Magenta Ink on Coated Paper

MTB > BRIEF 3
 MTB > REGR C2 ON 2, C1 C3

The regression equation is
 $Y = 0.000015 + 0.326 X + 167 X^2$

Predictor	Coef	Stdev	t-ratio	p
Constant	0.00001461	0.00000902	1.62	0.124
X	0.32573	0.05300	6.15	0.000
X2	166.63	61.47	2.71	0.015

s = 0.00001238 R-sq = 98.5% R-sq(adj) = 98.3%

Analysis of Variance

SOURCE	DF	SS	MS	F	P
Regression	2	1.73350E-07	8.66750E-08	565.49	0.000
Error	17	2.60567E-09	1.53275E-10		
Total	19	1.75956E-07			

SOURCE	DF	SEQ SS
X	1	1.72224E-07
X2	1	1.12625E-09

Obs.	X	Y	Fit	Stdev.Fit	Residual	St.Resid
1	0.000120	0.000051	0.000056	0.000004	-0.000005	-0.44
2	0.000159	0.000081	0.000070	0.000004	0.000010	0.87
3	0.000171	0.000072	0.000075	0.000003	-0.000003	-0.27
4	0.000194	0.000093	0.000084	0.000003	0.000008	0.71
5	0.000248	0.000120	0.000106	0.000003	0.000014	1.16
6	0.000251	0.000108	0.000107	0.000003	0.000001	0.06
7	0.000275	0.000117	0.000117	0.000003	-0.000000	-0.02
8	0.000293	0.000132	0.000124	0.000004	0.000007	0.61
9	0.000323	0.000138	0.000137	0.000004	0.000000	0.03
10	0.000338	0.000143	0.000144	0.000004	-0.000000	-0.02
11	0.000373	0.000140	0.000159	0.000004	-0.000019	-1.63
12	0.000432	0.000172	0.000186	0.000005	-0.000015	-1.29
13	0.000438	0.000172	0.000189	0.000005	-0.000018	-1.55
14	0.000076	0.000023	0.000040	0.000006	-0.000017	-1.57
15	0.000099	0.000060	0.000049	0.000005	0.000011	0.97
16	0.000116	0.000059	0.000055	0.000005	0.000005	0.43
17	0.000122	0.000053	0.000057	0.000004	-0.000004	-0.35
18	0.000659	0.000316	0.000302	0.000006	0.000015	1.33
19	0.000621	0.000302	0.000281	0.000005	0.000021	1.88
20	0.000831	0.000389	0.000400	0.000011	-0.000011	-1.79 X

X denotes an obs. whose X value gives it large influence.

Worksheet saved into file: TEMP6.MTW

MTB > STOP

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Appendix E (continued)

Cyan Ink on Newsprint A

MTB > BRIEF 3
 MTB > REGR C2 ON 1, C1

The regression equation is
 $Y = -0.000026 + 0.629 X$

Predictor	Coef	Stdev	t-ratio	p
Constant	-0.00002649	0.00000382	-6.93	0.000
X	0.62851	0.01002	62.71	0.000

s = 0.000009496 R-sq = 99.6% R-sq(adj) = 99.5%

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	1	3.54528E-07	3.54528E-07	3931.92	0.000
Error	17	1.53283E-09	9.01667E-11		
Total	18	3.56061E-07			

Obs.	X	Y	Fit	Stdev.Fit	Residual	St.Resid
1	0.000083	0.000028	0.000026	0.000003	0.000002	0.21
2	0.000115	0.000040	0.000045	0.000003	-0.000005	-0.58
3	0.000142	0.000056	0.000063	0.000003	-0.000007	-0.79
4	0.000157	0.000065	0.000072	0.000003	-0.000008	-0.84
5	0.000201	0.000089	0.000100	0.000002	-0.000010	-1.10
6	0.000231	0.000120	0.000119	0.000002	0.000001	0.15
7	0.000253	0.000136	0.000133	0.000002	0.000003	0.35
8	0.000318	0.000176	0.000173	0.000002	0.000003	0.28
9	0.000361	0.000201	0.000200	0.000002	0.000000	0.02
10	0.000398	0.000210	0.000224	0.000002	-0.000014	-1.51
11	0.000444	0.000238	0.000253	0.000003	-0.000015	-1.66
12	0.000019	0.000006	-0.000014	0.000004	0.000021	2.37R
13	0.000055	0.000016	0.000008	0.000003	0.000008	0.93
14	0.000081	0.000019	0.000024	0.000003	-0.000005	-0.54
15	0.000513	0.000305	0.000296	0.000003	0.000009	0.96
16	0.000567	0.000339	0.000330	0.000003	0.000010	1.09
17	0.000624	0.000371	0.000366	0.000004	0.000006	0.63
18	0.000685	0.000413	0.000404	0.000004	0.000009	1.05
19	0.000708	0.000411	0.000418	0.000005	-0.000007	-0.85

R denotes an obs. with a large st. resid.

Worksheet saved into file: TEMP7.MTW

MTB > STOP

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Appendix E (continued)

Cyan Ink on Newsprint B

MTB > BRIEF 3

MTB > REGR C2 ON 2, C1 C3

The regression equation is

Y = -0.000018 + 0.642 X - 194 X2

Predictor	Coef	Stdev	t-ratio	p
Constant	-0.00001781	0.00000358	-4.97	0.000
X	0.64164	0.02829	22.68	0.000
X2	-194.00	43.91	-4.42	0.001

s = 0.000005417 R-sq = 99.7% R-sq(adj) = 99.7%

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	2	1.34230E-07	6.71149E-08	2287.13	0.000
Error	13	3.81480E-10	2.93446E-11		
Total	15	1.34611E-07			

SOURCE	DF	SEQ SS
X	1	1.33657E-07
X2	1	5.72888E-10

Obs.	X	Y	Fit	Stdev.Fit	Residual	St.Resid
1	0.000043	0.000012	0.000010	0.000003	0.000003	0.57
2	0.000075	0.000031	0.000029	0.000002	0.000002	0.42
3	0.000093	0.000040	0.000040	0.000002	0.000000	0.02
4	0.000134	0.000062	0.000064	0.000002	-0.000002	-0.45
5	0.000155	0.000071	0.000077	0.000002	-0.000006	-1.11
6	0.000199	0.000093	0.000102	0.000002	-0.000009	-1.73
7	0.000224	0.000112	0.000116	0.000002	-0.000004	-0.82
8	0.000258	0.000130	0.000135	0.000002	-0.000004	-0.84
9	0.000342	0.000189	0.000179	0.000002	0.000011	2.15R
10	0.000371	0.000192	0.000194	0.000002	-0.000002	-0.37
11	0.000386	0.000207	0.000201	0.000002	0.000006	1.19
12	0.000392	0.000207	0.000204	0.000002	0.000003	0.58
13	0.000055	0.000016	0.000017	0.000002	-0.000001	-0.12
14	0.000074	0.000035	0.000029	0.000002	0.000007	1.37
15	0.000541	0.000272	0.000273	0.000003	-0.000000	-0.09
16	0.000640	0.000310	0.000313	0.000005	-0.000003	-1.08 X

R denotes an obs. with a large st. resid.

X denotes an obs. whose X value gives it large influence.

Worksheet saved into file: TEMP8.MTW

MTB > STOP

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Appendix E (continued)

Cyan Ink on Coated Paper

MTB > BRIEF 3

MTB > REGR C2 ON 2, C1 C3

The regression equation is

$$Y = -0.000001 + 0.541 X - 178 X^2$$

Predictor	Coef	Stdev	t-ratio	P
Constant	-0.00000093	0.00000234	-0.40	0.698
X	0.54124	0.02993	18.08	0.000
X2	-178.24	71.83	-2.48	0.025

s = 0.000004462 R-sq = 99.6% R-sq(adj) = 99.5%

Analysis of Variance

SOURCE	DF	SS	MS	F	P
Regression	2	7.32043E-08	3.66021E-08	1838.44	0.000
Error	15	2.98640E-10	1.99093E-11		
Total	17	7.35029E-08			

SOURCE	DF	SEQ SS
X	1	7.30817E-08
X2	1	1.22579E-10

Obs.	X	Y	Fit	Stdev.Fit	Residual	St.Resid
1	0.000040	0.000022	0.000020	0.000002	0.000001	0.26
2	0.000074	0.000040	0.000038	0.000001	0.000002	0.46
3	0.000099	0.000049	0.000051	0.000001	-0.000001	-0.33
4	0.000102	0.000049	0.000052	0.000001	-0.000003	-0.70
5	0.000154	0.000083	0.000078	0.000002	0.000005	1.20
6	0.000173	0.000093	0.000087	0.000002	0.000005	1.28
7	0.000213	0.000108	0.000106	0.000002	0.000002	0.43
8	0.000238	0.000120	0.000118	0.000002	0.000003	0.66
9	0.000287	0.000133	0.000140	0.000002	-0.000007	-1.68
10	0.000296	0.000136	0.000144	0.000002	-0.000008	-1.91
11	0.000327	0.000154	0.000157	0.000002	-0.000003	-0.66
12	0.000355	0.000170	0.000169	0.000002	0.000001	0.25
13	0.000386	0.000185	0.000181	0.000002	0.000004	0.98
14	0.000429	0.000201	0.000198	0.000003	0.000002	0.69
15	0.000016	0.000006	0.000008	0.000002	-0.000001	-0.33
16	0.000010	0.000003	0.000004	0.000002	-0.000001	-0.28
17	0.000019	0.000003	0.000010	0.000002	-0.000006	-1.56
18	0.000039	0.000026	0.000020	0.000002	0.000006	1.45

X denotes an obs. whose X value gives it large influence.

Worksheet saved into file: TEMP9.MTW

MTB > STOP

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Appendix F

Ink Trap Data Calculated by Gravimetric Method

Appendix F

On Newsprint A

Ink Combination	Amount of 2nd Ink	Replication	Ink	Ink on Disc (g)	Ink Trans. (g)	A 2/1 (cm ²)	A 2 (cm ²)	Gravi. Trap (%)
Y/Y	Low	1	Y	0.0018	0.0005	26.0	2.6	70.99
			Y	0.0034	0.0016	—	—	—
		2	Y	0.0019	0.0004	25.6	2.5	48.81
			Y	0.0030	0.0013	—	—	—
		3	Y	0.0017	0.0006	25.8	2.9	96.65
			Y	0.0032	0.0014	—	—	—
	Med	1	Y	0.0056	0.0037	25.8	2.9	126.08
			Y	0.0028	0.0014	—	—	—
		2	Y	0.0052	0.0030	26.0	2.7	111.00
			Y	0.0031	0.0014	—	—	—
		3	Y	0.0051	0.0030	26.1	2.6	113.94
			Y	0.0031	0.0013	—	—	—
	High	1	Y	0.0123	0.0088	25.9	2.9	106.98
			Y	0.0028	0.0014	—	—	—
		2	Y	0.0122	0.0085	25.9	2.9	104.28
			Y	0.0031	0.0014	—	—	—
		3	Y	0.0119	0.0084	26.1	2.7	106.46
			Y	0.0031	0.0013	—	—	—
Y/M	Low	1	Y	0.0022	0.0010	24.6	3.1	111.74
			M	0.0056	0.0025	—	—	—
		2	Y	0.0020	0.0008	25.0	2.6	100.96
			M	0.0055	0.0026	—	—	—
		3	Y	0.0016	0.0009	25.2	2.5	165.79
			M	0.0056	0.0018	—	—	—
	Med	1	Y	0.0042	0.0023	26.1	2.5	111.42
			M	0.0062	0.0032	—	—	—
		2	Y	0.0041	0.0021	26.0	2.6	104.34
			M	0.0059	0.0026	—	—	—
		3	Y	0.0042	0.0018	25.3	3.3	84.60
			M	0.0055	0.0023	—	—	—
	High	1	Y	0.0124	0.0084	25.8	2.9	100.44
			M	0.0058	0.0027	—	—	—
		2	Y	0.0118	0.0082	25.0	3.7	106.15
			M	0.0058	0.0028	—	—	—
		3	Y	0.0118	0.0078	26.1	2.6	99.30
			M	0.0058	0.0025	—	—	—
Y/C	Low	1	Y	0.0030	0.0011	25.88	2.72	79.10
			C	0.0045	0.0017	—	—	—
		2	Y	0.0019	0.0005	24.88	2.62	63.13
			C	0.0041	0.0016	—	—	—
		3	Y	0.0032	0.0013	25.62	3.0	86.79
			C	0.0050	0.0022	—	—	—
	Med	1	Y	0.0034	0.0016	26.08	2.52	100.69
			C	0.0043	0.0012	—	—	—

Appendix F

On Newsprint A (continued)

Ink Combination	Amount of 2nd Ink	Replication	Ink	Ink on Disc (g)	Ink Trans. (g)	A2/1 (cm ²)	A2 (cm ²)	Gravi. Trap (%)
Y/C	Med	2	Y	0.0041	0.0019	26.0	2.6	93.44
			C	0.0041	0.0015	—	—	—
		3	Y	0.0043	0.0019	25.5	3.1	87.13
			C	0.0052	0.0024	—	—	—
		1	Y	0.0116	0.0080	25.5	3.1	105.04
			C	0.0050	0.0018	—	—	—
	High	2	Y	0.0121	0.0087	25.16	2.54	106.31
			C	0.0053	0.0022	—	—	—
		3	Y	0.0120	0.0080	24.74	2.9	98.66
			C	0.0052	0.0025	—	—	—
M/Y	Low	1	M	0.0050	0.0010	25.6	3.0	38.5
			Y	0.0025	0.0013	—	—	—
		2	M	0.0041	0.0009	25.9	2.6	50.65
			Y	0.0034	0.0011	—	—	—
		3	M	0.0044	0.0010	26.0	2.6	50.48
			Y	0.0036	0.0025	—	—	—
	Med	1	M	0.0061	0.0022	25.36	3.24	72.90
			Y	0.0029	0.0010	—	—	—
		2	M	0.0063	0.0023	25.7	2.9	73.44
			Y	0.0027	0.0009	—	—	—
		3	M	0.0063	0.0018	25.6	3.0	54.70
			Y	0.0028	0.0012	—	—	—
	High	1	M	0.0185	0.0108	25.4	3.2	99.26
			Y	0.0027	0.0012	—	—	—
		2	M	0.0183	0.0109	25.3	3.3	101.67
			Y	0.0028	0.0010	—	—	—
		3	M	0.0182	0.0108	25.4	3.2	101.24
			Y	0.0028	0.0011	—	—	—
M/M	Low	1	M	0.0039	0.0019	25.66	2.9	133.97
			M	0.0064	0.0034	—	—	—
		2	M	0.0042	0.0020	25.86	2.74	123.49
			M	0.0061	0.0032	—	—	—
		3	M	0.0038	0.0019	26.0	2.6	141.14
			M	0.0063	0.0033	—	—	—
	Med	1	M	0.0073	0.0042	25.6	3.0	116.41
			M	0.0069	0.0031	—	—	—
		2	M	0.0071	0.0040	25.8	2.8	114.67
			M	0.0070	0.0033	—	—	—
		3	M	0.0073	0.0042	25.9	2.7	116.22
			M	0.0069	0.0028	—	—	—
	High	1	M	0.0178	0.0119	24.76	3.04	115.23
			M	0.0067	0.0038	—	—	—
		2	M	0.0207	0.0126	25.0	2.8	102.27
			M	0.0067	0.0035	—	—	—

Appendix F

On Newsprint A (continued)

Ink Combination	Amount of 2nd Ink	Replication	Ink	Ink on Disc (g)	Ink Trans. (g)	A2/1 (cm ²)	A2 (cm ²)	Gravi. Trap (%)
M/M	High	3	M	0.0205	0.0127	24.8	3.0	104.52
			M	0.0068	0.0035	--	--	--
M/C	Low	1	M	0.0033	0.0008	23.9	2.6	64.26
			C	0.0044	0.0020	--	--	--
		2	M	0.0036	0.0008	24.08	2.52	53.67
			C	0.0045	0.0018	--	--	--
		3	M	0.0034	0.0005	23.52	2.68	30.80
			C	0.0045	0.0022	--	--	--
	Med	1	M	0.0078	0.0039	24.06	2.54	95.68
			C	0.0047	0.0022	--	--	--
		2	M	0.0071	0.0033	23.6	3.0	90.47
			C	0.0047	0.0021	--	--	--
		3	M	0.0076	0.0035	24.3	2.3	88.03
			C	0.0042	0.0016	--	--	--
	High	1	M	0.0173	0.0114	24.72	2.9	113.56
			C	0.0047	0.0021	--	--	--
		2	M	0.0160	0.0102	25.02	2.6	110.21
			C	0.0045	0.0019	--	--	--
		3	M	0.0164	0.0103	25.0	2.7	108.25
			C	0.0045	0.0016	--	--	--
C/Y	Low	1	C	0.0015	0.0001	25.2	3.4	7.64
			Y	0.0032	0.0014	--	--	--
		2	C	0.0017	0.0003	25.62	2.94	82.30
			Y	0.0029	0.0011	--	--	--
		3	C	0.0014	0.0003	25.4	3.1	249.26
			Y	0.0031	0.0011	--	--	--
	Med	1	C	0.0049	0.0014	25.4	3.3	55.01
			Y	0.0034	0.0014	--	--	--
		2	C	0.0043	0.0011	25.3	3.4	49.96
			Y	0.0033	0.0012	--	--	--
		3	C	0.0067	0.0036	25.5	3.2	105.90
			Y	0.0030	0.0010	--	--	--
	High	1	C	0.0161	0.0096	25.18	3.42	104.99
			Y	0.0032	0.0014	--	--	--
		2	C	0.0156	0.0093	25.4	3.42	105.30
			Y	0.0029	0.0013	--	--	--
		3	C	0.0150	0.0090	25.54	3.26	106.30
			Y	0.0028	0.0013	--	--	--
C/M	Low	1	C	0.0025	0.0007	26.0	2.6	81.65
			M	0.0066	0.0036	--	--	--
		2	C	0.0025	0.0009	25.86	2.84	109.45
			M	0.0060	0.0029	--	--	--
		3	C	0.0022	0.0006	25.60	3.0	90.05
			M	0.0060	0.0029	--	--	--

Appendix F

On Newsprint A (continued)

Ink Combination	Amount of 2nd Ink	Replication	Ink	Ink on Disc (g)	Ink Trans. (g)	A2/1 (cm ²)	A2 (cm ²)	Gravi. Trap (%)
C/M	Med	1	C	0.0069	0.0033	25.8	2.8	92.30
			M	0.0068	0.0040	--	--	--
		2	C	0.0067	0.0027	25.7	2.9	76.54
			M	0.0064	0.0035	--	--	--
		3	C	0.0066	0.0031	25.7	2.9	91.35
			M	0.0069	0.0036	--	--	--
	High	1	C	0.0163	0.0095	25.66	3.1	102.08
			M	0.0075	0.0036	--	--	--
		2	C	0.0157	0.0079	25.9	2.9	87.00
			M	0.0070	0.0033	--	--	--
		3	C	0.0164	0.0090	25.86	2.9	95.39
			M	0.0059	0.0026	--	--	--
C/C	Low	1	C	0.0023	0.0005	24.94	2.66	62.28
			C	0.0043	0.0018	--	--	--
		2	C	0.0024	0.0008	24.7	2.9	99.82
			C	0.0043	0.0017	--	--	--
		3	C	0.0024	0.0005	24.56	2.8	55.22
			C	0.0041	0.0016	--	--	--
	Med	1	C	0.0066	0.0035	24.66	3.04	103.87
			C	0.0046	0.0024	--	--	--
		2	C	0.0063	0.0031	24.6	3.1	96.53
			C	0.0046	0.0021	--	--	--
		3	C	0.0061	0.0032	24.8	2.9	104.50
			C	0.0043	0.0019	--	--	--
	High	1	C	0.0162	0.0096	25.1	2.7	103.52
			C	0.0040	0.0017	--	--	--
		2	C	0.0146	0.0085	24.96	2.84	102.54
			C	0.0041	0.0016	--	--	--
		3	C	0.0150	0.0087	24.96	2.84	101.87
			C	0.0043	0.0017	--	--	--

Appendix F

On Newsprint B

Ink Combination	Amount of 2nd Ink	Replication	Ink	Ink on Disc (g)	Ink Trans. (g)	A2/1 (cm ²)	A2 (cm ²)	Gravi. Trap (%)
Y/Y	Low	1	Y	0.0017	0.0008	25.3	3.2	100.78
			Y	0.0032	0.0017	—	—	—
		2	Y	0.0017	0.0007	25.9	2.7	87.01
			Y	0.0033	0.0016	—	—	—
		3	Y	0.0013	0.0005	25.6	3.0	90.24
			Y	0.0030	0.0014	—	—	—
	Med	1	Y	0.0027	0.0014	25.8	2.8	99.21
			Y	0.0033	0.0017	—	—	—
		2	Y	0.0022	0.0011	25.5	3.1	99.86
			Y	0.0030	0.0016	—	—	—
		3	Y	0.0026	0.0010	25.7	2.9	71.38
			Y	0.0034	0.0020	—	—	—
	High	1	Y	0.0122	0.0072	25.7	3.1	99.10
			Y	0.0037	0.0018	—	—	—
		2	Y	0.0122	0.0069	25.7	3.0	94.50
			Y	0.0031	0.0015	—	—	—
		3	Y	0.0122	0.0069	25.9	2.9	94.46
			Y	0.0037	0.0017	—	—	—
Y/M	Low	1	Y	0.0022	0.0011	24.7	3.0	99.05
			M	0.0058	0.0023	—	—	—
		2	Y	0.0022	0.0010	24.9	2.7	88.99
			M	0.0052	0.0019	—	—	—
		3	Y	0.0020	0.0008	24.5	3.1	78.61
			M	0.0057	0.0021	—	—	—
	Med	1	Y	0.0031	0.0017	26.3	2.3	102.67
			M	0.0055	0.0029	—	—	—
		2	Y	0.0030	0.0017	25.9	2.7	107.18
			M	0.0054	0.0030	—	—	—
		3	Y	0.0029	0.0016	26.3	2.3	104.57
			M	0.0056	0.0030	—	—	—
	High	1	Y	0.0124	0.0065	26.6	2.2	86.7
			M	0.0059	0.0022	—	—	—
		2	Y	0.0121	0.0055	25.8	3.0	74.10
			M	0.0072	0.0034	—	—	—
		3	Y	0.0116	0.0060	25.7	3.0	85.77
			M	0.0066	0.0027	—	—	—
Y/C	Low	1	Y	0.0032	0.0012	25.1	2.9	66.40
			C	0.0046	0.0021	—	—	—
		2	Y	0.0032	0.0020	25.86	2.74	117.95
			C	0.0050	0.0024	—	—	—
		3	Y	0.0037	0.0022	25.7	2.5	108.95
			C	0.0038	0.0015	—	—	—
	Med	1	Y	0.0026	0.0013	24.98	2.62	95.34
			C	0.0037	0.0016	—	—	—

Appendix F

On Newsprint B (continued)

Ink Combination	Amount of 2nd Ink	Replication	Ink	Ink on Disc (g)	Ink Trans. (g)	A2/1 (cm ²)	A2 (cm ²)	Gravi. Trap (%)
Y/C	Med	2	Y	0.0031	0.0012	25.66	2.84	69.55
			C	0.0047	0.0024	—	—	—
		3	Y	0.0028	0.0009	25.6	3.0	56.87
			C	0.0044	0.0019	—	—	—
		1	Y	0.0117	0.0073	24.8	2.84	105.21
			C	0.0045	0.0023	—	—	—
	High	2	Y	0.0117	0.0074	24.72	2.9	106.84
			C	0.0042	0.0021	—	—	—
		3	Y	0.0116	0.0073	25.52	2.1	105.55
			C	0.0044	0.0021	—	—	—
M/Y	Low	1	M	0.0031	0.0004	25.8	2.7	22.69
			Y	0.0034	0.0018	—	—	—
		2	M	0.0031	0.0002	25.2	3.4	1.45
			Y	0.0033	0.0015	—	—	—
		3	M	0.0034	0.0004	26.7	1.9	22.20
			Y	0.0035	0.0017	—	—	—
	Med	1	M	0.0044	0.0011	25.7	2.9	51.53
			Y	0.0028	0.0016	—	—	—
		2	M	0.0047	0.0013	25.5	3.1	57.31
			Y	0.0026	0.0013	—	—	—
		3	M	0.0045	0.0007	25.7	2.9	27.01
			Y	0.0026	0.0013	—	—	—
	High	1	M	0.0177	0.0090	25.5	3.2	109.71
			Y	0.0026	0.0016	—	—	—
		2	M	0.0181	0.0094	25.4	3.4	112.47
			Y	0.0028	0.0014	—	—	—
		3	M	0.0177	0.0085	25.9	2.9	102.91
			Y	0.0030	0.0017	—	—	—
M/M	Low	1	M	0.0036	0.0020	25.7	2.9	136.02
			M	0.0063	0.0040	—	—	—
		2	M	0.0034	0.0016	25.8	2.8	114.75
			M	0.0066	0.0040	—	—	—
		3	M	0.0032	0.0014	25.7	2.9	107.40
			M	0.0063	0.0036	—	—	—
	Med	1	M	0.0051	0.0026	25.34	3.26	115.99
			M	0.0064	0.0028	—	—	—
		2	M	0.0063	0.0031	25.8	2.7	108.64
			M	0.0068	0.0030	—	—	—
		3	M	0.0059	0.0029	25.5	3.1	109.28
			M	0.0066	0.0028	—	—	—
	High	1	M	0.0172	0.0088	24.9	2.8	110.48
			M	0.0064	0.0036	—	—	—
		2	M	0.0163	0.0089	24.76	3.0	118.54
			M	0.0061	0.0029	—	—	—

Appendix F

On Newsprint B (continued)

Ink Combination	Amount of 2nd Ink	Replication	Ink	Ink on Disc (g)	Ink Trans. (g)	A2/1 (cm ²)	A2 (cm ²)	Gravi. Trap (%)
M/M	High	3	M	0.0160	0.0081	24.84	2.9	108.89
M/C	Low	1	M	0.0061	0.0029	--	--	--
			M	0.0029	0.0002	25.6	2.9	5.17
		2	C	0.0043	0.0018	--	--	--
			M	0.0023	-0.0001	25.5	2.9	-29.45
		3	C	0.0044	0.0020	--	--	--
			M	0.0024	-0.0003	25.26	3.14	-55.66
	Med	1	C	0.0044	0.0021	--	--	--
			M	0.0050	0.0017	23.66	2.94	72.13
		2	C	0.0043	0.0018	--	--	--
			M	0.0061	0.0027	23.9	2.7	96.11
		3	C	0.0051	0.0021	--	--	--
			M	0.0052	0.0024	24.2	2.4	102.62
	High	1	C	0.0044	0.0013	--	--	--
			M	0.0141	0.0076	23.8	2.9	116.51
		2	C	0.0047	0.0025	--	--	--
			M	0.0148	0.0078	23.44	3.3	113.87
		3	C	0.0050	0.0024	--	--	--
			M	0.0147	0.0080	23.97	2.8	117.82
C/Y	Low	1	C	0.0049	0.0025	--	--	--
			C	0.0021	0.0006	25.66	2.94	70.49
		2	Y	0.0030	0.0014	--	--	--
			C	0.0023	0.0013	25.48	3.12	146.37
		3	Y	0.0027	0.0011	--	--	--
			C	0.0022	0.0008	25.6	3.0	91.11
	Med	1	Y	0.0026	0.0010	--	--	--
			C	0.0038	0.0017	25.4	3.3	92.84
		2	Y	0.0033	0.0014	--	--	--
			C	0.0036	0.0014	25.6	3.1	80.25
		3	Y	0.0033	0.0015	--	--	--
			C	0.0035	0.0009	25.5	3.2	48.83
	High	1	Y	0.0029	0.0012	--	--	--
			C	0.0098	0.0050	25.6	3.1	97.68
		2	Y	0.0029	0.0010	--	--	--
			C	0.0094	0.0045	25.7	3.0	90.92
		3	Y	0.0028	0.0010	--	--	--
			C	0.0120	0.0064	25.56	3.2	103.43
C/M	Low	1	Y	0.0039	0.0024	--	--	--
			C	0.0027	0.0007	25.8	2.8	54.99
		2	M	0.0063	0.0023	--	--	--
			C	0.0019	0.0006	25.6	3.0	85.37
		3	M	0.0060	0.0026	--	--	--
			C	0.0022	0.0010	25.6	3.0	117.73
			M	0.0062	0.0037	--	--	--

Appendix F

On Newsprint B (continued)

Ink Combination	Amount of 2nd Ink	Replication	Ink	Ink on Disc (g)	Ink Trans. (g)	A2/1 (cm ²)	A2 (cm ²)	Gravi. Trap (%)
C/M	Med	1	C	0.0045	0.0019	25.5	3.0	83.80
			M	0.0061	0.0032	--	--	--
		2	C	0.0042	0.0017	25.74	2.84	81.12
			M	0.0057	0.0029	--	--	--
		3	C	0.0043	0.0016	25.5	3.1	73.02
			M	0.0065	0.0033	--	--	--
	High	1	C	0.0112	0.0041	25.76	2.84	67.37
			M	0.0069	0.0040	--	--	--
		2	C	0.0106	0.0039	25.78	2.84	67.62
			M	0.0066	0.0036	--	--	--
		3	C	0.0118	0.0045	25.6	3.1	70.56
			M	0.0068	0.0038	--	--	--
C/C	Low	1	C	0.0020	0.0004	25.08	2.52	46.25
			C	0.0040	0.0019	--	--	--
		2	C	0.0028	0.0008	24.96	2.66	60.23
			C	0.0044	0.0024	--	--	--
		3	C	0.0024	0.0007	25.32	2.3	66.85
			C	0.0044	0.0024	--	--	--
	Med	1	C	0.0042	0.0018	25.1	2.6	86.11
			C	0.0046	0.0026	--	--	--
		2	C	0.0044	0.0017	24.8	2.9	75.60
			C	0.0046	0.0021	--	--	--
		3	C	0.0041	0.0016	24.84	2.8	77.60
			C	0.0045	0.0024	--	--	--
	High	1	C	0.0113	0.0059	24.8	3.0	100.83
			C	0.0043	0.0020	--	--	--
		2	C	0.0114	0.0060	24.76	3.0	101.79
			C	0.0041	0.0019	--	--	--
		3	C	0.0123	0.0060	24.86	2.9	93.97
			C	0.0040	0.0020	--	--	--

Appendix F

On Coated Paper

Ink Combination	Amount of 2nd Ink	Replication	Ink	Ink on Disc (g)	Ink Trans. (g)	A2/1 (cm ²)	A2 (cm ²)	Gravi. Trap (%)
Y/Y	Low	1	Y	0.0009	0.0005	24.9	3.6	89.11
			Y	0.0032	0.0014	—	—	—
		2	Y	0.0012	0.0006	25.66	2.94	84.24
			Y	0.0027	0.0017	—	—	—
		3	Y	0.0011	0.0006	25.46	2.94	91.27
			Y	0.0028	0.0015	—	—	—
	Med	1	Y	0.0020	0.0013	25.5	3.1	122.36
			Y	0.0034	0.0019	—	—	—
		2	Y	0.0020	0.0012	25.4	3.3	112.20
			Y	0.0029	0.0017	—	—	—
		3	Y	0.0020	0.0014	26.3	2.4	131.46
			Y	0.0028	0.0014	—	—	—
	High	1	Y	0.0074	0.0040	25.5	3.2	110.39
			Y	0.0034	0.0023	—	—	—
		2	Y	0.0073	0.0039	26.0	2.7	108.75
			Y	0.0034	0.0020	—	—	—
		3	Y	0.0076	0.0040	25.8	2.9	107.19
			Y	0.0031	0.0017	—	—	—
Y/M	Low	1	Y	0.0015	0.0009	26.8	2.8	106.51
			M	0.0060	0.0029	—	—	—
		2	Y	0.0009	0.0006	25.1	1.7	108.81
			M	0.0058	0.0030	—	—	—
		3	Y	0.0009	0.0005	24.9	2.5	89.58
			M	0.0057	0.0028	—	—	—
	Med	1	Y	0.0021	0.0015	25.6	3.0	136.24
			M	0.0057	0.0032	—	—	—
		2	Y	0.0019	0.0013	26.8	2.8	127.33
			M	0.0060	0.0028	—	—	—
		3	Y	0.0025	0.0015	25.7	2.9	114.93
			M	0.0052	0.0026	—	—	—
	High	1	Y	0.0085	0.0043	25.2	3.4	103.04
			M	0.0065	0.0030	—	—	—
		2	Y	0.0083	0.0045	26.16	2.54	110.99
			M	0.0063	0.0031	—	—	—
		3	Y	0.0083	0.0043	25.2	3.5	105.77
			M	0.0059	0.0027	—	—	—
Y/C	Low	1	Y	0.0012	0.0006	24.16	2.84	85.2
			C	0.0045	0.0022	—	—	—
		2	Y	0.0013	0.0007	25.86	2.74	92.84
			C	0.0049	0.0020	—	—	—
		3	Y	0.0007	0.0000	25.0	2.6	-7.75
			C	0.0044	0.0020	—	—	—
	Med	1	Y	0.0019	0.0014	25.7	2.9	138.87
			C	0.0044	0.0021	—	—	—

Appendix F

On Coated Paper (continued)

Ink Combination	Amount of 2nd Ink	Replication	Ink	Ink on Disc (g)	Ink Trans. (g)	A2/1 (cm ²)	A2 (cm ²)	Gravi. Trap (%)
Y/C	Med	2	Y	0.0019	0.0014	23.6	2.9	140.71
			C	0.0048	0.0025	—	—	—
		3	Y	0.0018	0.0011	25.36	3.24	112.87
			C	0.0039	0.0019	—	—	—
		1	Y	0.0081	0.0048	24.9	2.7	122.50
			C	0.0047	0.0022	—	—	—
	High	2	Y	0.0078	0.0048	24.92	2.66	127.43
			C	0.0044	0.0022	—	—	—
		3	Y	0.0078	0.0047	25.04	2.6	124.48
			C	0.0045	0.0025	—	—	—
M/Y	Low	1	M	0.0045	0.0018	25.4	3.1	22.69
			Y	0.0035	0.0017	—	—	—
		2	M	0.0045	0.0005	25.4	3.4	13.13
			Y	0.0032	0.0015	—	—	—
		3	M	0.0049	0.0012	25.8	2.8	48.98
			Y	0.0031	0.0018	—	—	—
	Med	1	M	0.0051	0.0028	25.5	3.1	125.81
			Y	0.0025	0.0018	—	—	—
		2	M	0.0047	0.0026	25.7	2.9	125.34
			Y	0.0026	0.0018	—	—	—
		3	M	0.0048	0.0027	25.44	3.1	128.22
			Y	0.0026	0.0019	—	—	—
	High	1	M	0.0161	0.0084	25.4	3.3	117.88
			Y	0.0032	0.0021	—	—	—
		2	M	0.0160	0.0083	25.6	3.1	117.19
			Y	0.0029	0.0018	—	—	—
		3	M	0.0158	0.0080	25.5	3.2	114.27
			Y	0.0032	0.0017	—	—	—
M/M	Low	1	M	0.0041	0.0025	25.4	3.2	136.64
			M	0.0069	0.0036	—	—	—
		2	M	0.0039	0.0023	25.62	2.9	130.28
			M	0.0063	0.0037	—	—	—
		3	M	0.0036	0.0022	25.86	2.74	132.99
			M	0.0065	0.0031	—	—	—
	Med	1	M	0.0046	0.0025	26.7	1.9	122.11
			M	0.0067	0.0028	—	—	—
		2	M	0.0040	0.0016	25.7	2.8	84.92
			M	0.0064	0.0027	—	—	—
	High	3	M	0.0046	0.0021	26.0	2.6	101.02
			M	0.0064	0.0030	—	—	—
		1	M	0.0158	0.0081	27.0	2.74	116.37
			M	0.0065	0.0039	—	—	—
		2	M	0.0150	0.0078	24.84	2.9	117.99
			M	0.0063	0.0034	—	—	—

Appendix F

On Coated Paper (continued)

Ink Combination	Amount of 2nd Ink	Replication	Ink	Ink on Disc (g)	Ink Trans. (g)	A2/1 (cm ²)	A2 (cm ²)	Gravi. Trap (%)
M/M	High	3	M	0.0142	0.0072	24.84	2.9	115.53
M/C	Low	1	M	0.0059	0.0031	--	--	--
			M	0.0032	0.0026	23.6	2.9	180.01
		2	C	0.0042	0.0020	--	--	--
			M	0.0031	0.0025	23.56	2.6	176.99
		3	C	0.0043	0.0019	--	--	--
			M	0.0035	0.0024	24.1	2.3	151.83
	Med	1	C	0.0038	0.0017	--	--	--
			M	0.0046	0.0033	23.96	2.64	167.38
		2	C	0.0049	0.0024	--	--	--
			M	0.0043	0.0030	23.9	2.7	160.98
		3	C	0.0051	0.0023	--	--	--
			M	0.0042	0.0029	23.8	2.8	158.81
	High	1	C	0.0045	0.0021	--	--	--
			M	0.0122	0.0077	25.84	0.86	146.36
		2	C	0.0044	0.0023	--	--	--
			M	0.0114	0.0063	23.4	3.32	129.67
		3	C	0.0042	0.0020	--	--	--
			M	0.0114	0.0056	23.42	3.2	113.49
C/Y	Low	1	C	0.0042	0.0030	--	--	--
			C	0.0009	0.0001	25.38	3.12	12.60
		2	Y	0.0029	0.0017	--	--	--
			C	0.0008	0.0000	25.6	3.0	-11.75
		3	Y	0.0030	0.0018	--	--	--
			C	0.0005	-0.0001	25.6	3.0	-59.02
	Med	1	Y	0.0029	0.0017	--	--	--
			C	0.0031	0.0015	25.72	2.88	93.98
		2	Y	0.0032	0.0016	--	--	--
			C	0.0035	0.0017	25.66	2.94	94.60
		3	Y	0.0028	0.0013	--	--	--
			C	0.0030	0.0013	25.5	3.1	82.85
	High	1	Y	0.0030	0.0014	--	--	--
			C	0.0064	0.0031	25.7	3.0	96.79
		2	Y	0.0032	0.0014	--	--	--
			C	0.0064	0.0032	25.26	3.5	100.27
		3	Y	0.0032	0.0012	--	--	--
			C	0.0065	0.0029	25.86	2.84	88.39
C/M	Low	1	Y	0.0030	0.0009	--	--	--
			C	0.0012	0.0000	25.76	2.8	-10.68
		2	M	0.0063	0.0034	--	--	--
			C	0.0013	0.0000	25.9	2.7	-10.22
		3	M	0.0063	0.0032	--	--	--
			C	0.0011	0.0001	25.4	3.1	8.08
			M	0.0056	0.0029	--	--	--

Appendix F

On Coated Paper (continued)

Ink Combination	Amount of 2nd Ink	Replication	Ink	Ink on Disc (g)	Ink Trans. (g)	A2/1 (cm ²)	A2 (cm ²)	Gravi. Trap (%)
C/M	Med	1	C	0.0034	0.0017	25.48	3.14	97.65
			M	0.0062	0.0033	--	--	--
		2	C	0.0031	0.0018	25.72	2.9	114.97
			M	0.0062	0.0035	--	--	--
		3	C	0.0038	0.0016	25.9	2.7	80.85
			M	0.0057	0.0029	--	--	--
	High	1	C	0.0075	0.0038	25.48	3.12	103.01
			M	0.0069	0.0032	--	--	--
		2	C	0.0080	0.0038	25.78	2.9	96.36
			M	0.0067	0.0032	--	--	--
		3	C	0.0075	0.0036	25.46	3.14	96.93
			M	0.0059	0.0027	--	--	--
C/C	Low	1	C	0.0020	0.0005	24.46	3.14	42.38
			C	0.0045	0.0022	--	--	--
		2	C	0.0016	0.0010	24.76	2.84	124.62
			C	0.0044	0.0023	--	--	--
		3	C	0.0014	0.0010	24.5	3.1	144.90
			C	0.0046	0.0022	--	--	--
	Med	1	C	0.0034	0.0022	24.44	3.2	130.23
			C	0.0043	0.0021	--	--	--
		2	C	0.0030	0.0020	25.0	2.7	133.47
			C	0.0040	0.0024	--	--	--
		3	C	0.0035	0.0022	25.0	2.7	125.65
			C	0.0039	0.0021	--	--	--
	High	1	C	0.0077	0.0042	25.06	2.7	112.25
			C	0.0042	0.0023	--	--	--
		2	C	0.0073	0.0040	25.0	2.76	112.33
			C	0.0043	0.0021	--	--	--
		3	C	0.0076	0.0042	24.96	2.74	113.78
			C	0.0047	0.0024	--	--	--

Appendix G

Ink Trap Data

Calculated by

Preucil's Equation and Brunner's Equation

Appendix G

On Newsprint A

Ink Combination	Amount of 2nd Ink	Replication	D _{2/1}	D ₂	D ₁	Preucil Trap (%)	Brunner Trap (%)
Y/Y	Low	1	0.62	0.25	0.54	32.00	90.73
		2	0.67	0.27	0.50	62.96	94.70
		3	0.64	0.26	0.50	53.85	93.31
	Med	1	1.03	0.82	0.50	64.63	95.23
		2	1.01	0.79	0.49	65.82	95.23
		3	0.96	0.83	0.46	60.24	93.85
	High	1	1.25	1.19	0.58	56.30	96.01
		2	1.21	1.12	0.56	58.04	95.84
		3	1.25	1.16	0.51	63.79	96.44
Y/M	Low	1	0.73	0.36	0.44	80.56	96.71
		2	0.70	0.31	0.45	80.65	96.88
		3	0.64	0.27	0.41	85.19	97.45
	Med	1	0.96	0.64	0.47	76.56	96.53
		2	0.87	0.63	0.46	65.08	94.16
		3	0.87	0.67	0.41	68.66	94.36
	High	1	1.29	1.11	0.46	74.77	97.50
		2	1.30	1.15	0.47	72.17	97.32
		3	1.27	1.13	0.40	76.99	97.51
Y/C	Low	1	0.48	0.44	0.06	95.45	97.82
		2	0.38	0.32	0.06	100.00	100.00
		3	0.49	0.43	0.08	95.35	97.89
	Med	1	0.66	0.59	0.06	101.69	100.66
		2	0.66	0.59	0.08	98.31	99.37
		3	0.66	0.65	0.09	87.69	95.50
	High	1	1.12	1.08	0.07	97.22	99.46
		2	1.17	1.14	0.09	94.74	99.07
		3	1.21	1.22	0.11	90.16	98.44
M/Y	Low	1	0.42	0.55	0.03	70.91	84.10
		2	0.44	0.59	0.03	69.49	83.79
		3	0.42	0.60	0.03	65.00	80.96
	Med	1	0.67	0.89	0.02	73.03	89.65
		2	0.63	0.80	0.03	75.00	89.85
		3	0.59	0.82	0.03	68.29	86.52
	High	1	1.37	1.37	0.03	97.81	99.70
		2	1.29	1.35	0.03	93.33	99.00
		3	1.35	1.36	0.03	97.06	99.59
M/M	Low	1	1.13	0.54	0.89	44.44	96.16
		2	1.14	0.56	0.80	60.71	96.99
		3	1.02	0.51	0.87	29.41	94.38
	Med	1	1.25	0.91	0.88	40.66	95.93
		2	1.31	0.92	0.89	45.65	96.60
		3	1.28	0.96	0.82	47.92	96.35
	High	1	1.49	1.41	0.88	43.26	97.26
		2	1.55	1.42	0.92	44.37	97.63
		3	1.52	1.44	0.88	44.44	97.45

Appendix G

On Newsprint A (continued)

Ink Combination	Amount of 2nd Ink	Replication	D ₂ /1	D ₂	D ₁	Preucil Trap (%)	Brunner Trap (%)
M/C	Low	1	0.52	0.59	0.26	44.07	81.28
		2	0.49	0.54	0.26	42.59	80.38
		3	0.46	0.57	0.26	35.09	76.67
	Med	1	0.96	1.04	0.24	69.23	93.97
		2	0.86	1.07	0.25	57.01	90.53
		3	0.94	1.10	0.24	63.64	92.76
	High	1	1.44	1.36	0.25	87.50	98.79
		2	1.35	1.38	0.24	80.43	97.88
		3	1.36	1.37	0.24	81.75	98.04
C/Y	Low	1	0.33	0.33	0.01	96.97	98.04
		2	0.27	0.27	0.01	96.30	97.43
		3	0.24	0.22	0.01	104.55	103.26
	Med	1	0.70	0.72	0.01	95.83	98.36
		2	0.57	0.61	0.01	91.80	96.15
		3	0.98	0.98	0.01	98.98	99.73
	High	1	1.4	1.46	0.01	95.21	99.39
		2	1.38	1.40	0.01	97.86	99.71
		3	1.39	1.38	0.01	100.00	100.00
C/M	Low	1	0.53	0.41	0.04	119.51	109.25
		2	0.53	0.43	0.04	113.95	106.61
		3	0.45	0.40	0.04	102.50	101.30
	Med	1	0.92	1.05	0.05	82.86	95.57
		2	0.91	1.00	0.05	86.00	96.28
		3	0.87	0.98	0.04	84.69	95.64
	High	1	1.43	1.42	0.04	97.89	99.74
		2	1.38	1.43	0.05	93.01	99.11
		3	1.40	1.42	0.04	95.77	99.47
C/C	Low	1	0.81	0.36	0.69	33.33	92.78
		2	0.76	0.37	0.68	21.62	90.71
		3	0.78	0.37	0.66	32.43	91.99
	Med	1	1.25	1.00	0.76	49.00	96.05
		2	1.15	0.96	0.66	51.04	95.20
		3	1.20	0.99	0.72	48.48	95.55
	High	1	1.54	1.47	0.63	61.90	97.89
		2	1.56	1.48	0.68	59.46	97.92
		3	1.50	1.45	0.60	62.07	97.71

Appendix G
On Newsprint B

Ink Combination	Amount of 2nd Ink	Replication	D ₂ /1	D ₂	D ₁	Preucil Trap (%)	Brunner Trap (%)
Y/Y	Low	1	0.87	0.40	0.66	52.50	94.76
		2	0.83	0.36	0.64	52.78	94.68
		3	0.78	0.35	0.60	51.43	93.94
	Med	1	1.04	0.57	0.71	57.89	95.91
		2	0.98	0.57	0.70	49.12	94.61
		3	0.99	0.57	0.66	57.89	95.38
	High	1	1.53	1.39	0.73	57.55	97.79
		2	1.56	1.40	0.69	62.14	98.04
		3	1.55	1.41	0.72	58.87	97.91
Y/M	Low	1	0.96	0.52	0.43	101.92	100.29
		2	0.85	0.51	0.37	94.12	98.91
		3	0.87	0.47	0.44	91.49	98.65
	Med	1	1.01	0.66	0.50	77.27	96.93
		2	1.02	0.65	0.50	80.00	97.34
		3	1.02	0.63	0.51	80.95	97.51
	High	1	1.50	1.43	0.43	74.83	98.19
		2	1.53	1.41	0.54	70.21	98.15
		3	1.52	1.42	0.53	69.72	98.08
Y/C	Low	1	0.65	0.61	0.09	91.08	96.96
		2	0.79	0.71	0.09	98.59	99.56
		3	0.80	0.75	0.07	97.33	99.16
	Med	1	0.62	0.60	0.06	93.33	97.30
		2	0.53	0.60	0.10	71.67	88.06
		3	0.54	0.52	0.10	84.62	93.62
	High	1	1.42	1.36	0.10	97.06	99.65
		2	1.42	1.38	0.10	95.65	99.49
		3	1.42	1.39	0.10	94.96	99.42
M/Y	Low	1	0.27	0.43	0.05	51.16	69.22
		2	0.26	0.35	0.05	60.00	74.84
		3	0.27	0.43	0.05	51.16	69.22
	Med	1	0.47	0.69	0.04	62.32	81.24
		2	0.43	0.69	0.04	56.52	77.23
		3	0.41	0.63	0.04	58.73	77.71
	High	1	1.65	1.62	0.04	99.38	99.95
		2	1.65	1.64	0.05	97.56	99.80
		3	1.56	1.63	0.05	92.64	99.32
M/M	Low	1	1.44	0.69	1.07	53.62	98.07
		2	1.38	0.63	1.06	50.79	97.83
		3	1.33	0.53	1.05	52.83	97.90
	Med	1	1.36	0.75	0.94	56.00	97.63
		2	1.41	0.91	1.01	43.96	97.28
		3	1.31	0.86	0.95	41.86	96.60
	High	1	1.85	1.58	1.10	47.47	98.79
		2	1.88	1.61	1.03	52.80	98.91
		3	1.82	1.59	0.96	54.09	98.76

Appendix G

On Newsprint B (continued)

Ink Combination	Amount of 2nd Ink	Replication	D ₂ /1	D ₂	D ₁	Preucil Trap (%)	Brunner Trap (%)
M/C	Low	1	0.43	0.49	0.31	24.49	74.68
		2	0.36	0.39	0.29	17.95	71.23
		3	0.35	0.36	0.28	19.44	71.77
	Med	1	0.69	0.83	0.27	50.60	86.45
		2	1.00	0.90	0.28	80.00	96.37
		3	0.95	0.77	0.22	94.81	98.90
	High	1	1.55	1.53	0.30	81.70	98.64
		2	1.60	1.57	0.29	83.44	98.85
		3	1.66	1.58	0.30	86.08	99.12
	Low	1	0.37	0.44	0.01	81.82	88.88
		2	0.64	0.56	0.01	112.50	105.48
		3	0.55	0.49	0.01	110.20	105.03
C/Y	Med	1	0.78	0.81	0.01	95.06	98.28
		2	0.76	0.75	0.01	100.00	100.00
		3	0.64	0.71	0.01	88.73	95.24
	High	1	1.40	1.43	0.01	97.20	99.64
		2	1.35	1.38	0.01	97.10	99.59
		3	1.56	1.54	0.05	98.05	99.81
C/M	Low	1	0.58	0.55	0.05	96.36	98.42
		2	0.49	0.44	0.05	100.00	100.00
		3	0.63	0.55	0.06	103.64	101.46
	Med	1	0.86	0.93	0.06	86.02	96.02
		2	0.82	0.93	0.06	81.72	94.54
		3	0.80	0.91	0.06	81.32	94.25
	High	1	1.20	1.53	0.05	75.16	96.22
		2	1.21	1.47	0.05	78.91	96.76
		3	1.27	1.52	0.06	79.61	97.19
C/C	Low	1	0.98	0.51	0.79	37.25	94.25
		2	1.10	0.64	0.91	29.69	94.73
		3	1.17	0.64	0.91	40.62	95.94
	Med	1	1.32	0.94	1.02	31.91	96.27
		2	1.37	0.95	1.02	36.84	96.77
		3	1.28	0.90	0.98	33.33	96.02
	High	1	1.70	1.57	0.92	49.68	98.32
		2	1.72	1.57	0.89	52.87	98.44
		3	1.67	1.56	0.88	50.64	98.22

Appendix G

On Coated Paper

Ink Combination	Amount of 2nd Ink	Replication	D _{2/1}	D ₂	D ₁	Preucil Trap (%)	Brunner Trap (%)
Y/Y	Low	1	1.32	0.35	1.19	37.14	98.04
		2	1.25	0.31	1.16	29.03	97.69
		3	1.28	0.25	1.13	60.00	98.87
	Med	1	1.69	0.99	1.29	40.40	98.48
		2	1.64	0.76	1.23	53.95	98.72
		3	1.63	0.74	1.24	52.70	98.69
	High	1	2.05	1.87	1.41	34.22	99.16
		2	2.06	1.90	1.34	37.89	99.19
		3	2.05	1.92	1.31	38.54	99.17
	Low	1	1.19	0.41	0.79	97.56	99.84
		2	1.09	0.32	0.78	96.88	99.80
		3	0.99	0.26	0.76	88.64	99.24
	Med	1	1.42	0.90	0.74	75.56	98.45
		2	1.42	0.92	0.77	70.65	98.20
		3	1.47	1.06	0.69	73.58	98.36
Y/M	High	1	1.97	1.94	0.82	59.28	99.10
		2	2.02	1.93	0.79	63.73	99.23
		3	2.02	1.96	0.71	66.84	99.26
	Low	1	0.67	0.34	0.13	158.82	118.91
		2	0.79	0.45	0.14	144.44	112.77
		3	0.42	0.24	0.12	125.00	110.00
	Med	1	1.14	0.90	0.15	110.00	101.83
		2	0.97	0.62	0.14	133.87	108.06
		3	1.01	0.59	0.14	147.46	110.87
	High	1	1.99	1.87	0.13	99.47	99.98
		2	1.98	1.97	0.14	93.40	99.73
		3	1.99	1.92	0.14	96.35	99.85
	Low	1	0.73	0.93	0.06	72.04	90.66
		2	0.30	0.63	0.06	38.10	62.68
		3	0.51	0.86	0.06	52.33	78.54
M/Y	Med	1	1.37	1.55	0.05	85.16	98.20
		2	1.14	1.21	0.05	90.08	98.15
		3	1.23	1.31	0.05	90.08	98.41
	High	1	2.15	2.14	0.07	97.20	99.91
		2	2.15	2.15	0.07	96.74	99.89
		3	2.14	2.12	0.07	97.64	99.92
	Low	1	2.07	1.35	1.80	20.00	99.22
		2	2.05	1.20	1.77	23.33	99.22
		3	1.99	1.03	1.71	27.18	99.16
	Med	1	1.89	0.99	1.61	28.28	98.96
		2	1.68	0.82	1.53	18.29	98.35
		3	1.99	1.27	1.65	26.77	99.10
	High	1	2.27	2.09	1.76	24.40	99.48
		2	2.28	2.09	1.75	25.36	99.49
		3	2.27	2.11	1.74	25.12	99.48
M/M	Low	1	2.07	1.35	1.80	20.00	99.22
		2	2.05	1.20	1.77	23.33	99.22
		3	1.99	1.03	1.71	27.18	99.16
	Med	1	1.89	0.99	1.61	28.28	98.96
		2	1.68	0.82	1.53	18.29	98.35
		3	1.99	1.27	1.65	26.77	99.10
	High	1	2.27	2.09	1.76	24.40	99.48
		2	2.28	2.09	1.75	25.36	99.49
		3	2.27	2.11	1.74	25.12	99.48

Appendix G

On Coated Paper (continued)

Ink Combination	Amount of 2nd Ink	Replication	D _{2/1}	D ₂	D ₁	Preucil Trap (%)	Brunner Trap (%)
M/C	Low	1	1.68	0.99	0.39	130.30	102.17
		2	1.48	0.78	0.37	142.31	104.06
		3	1.52	0.88	0.36	131.82	102.90
	Med	1	2.01	1.48	0.43	106.76	100.26
		2	1.81	1.21	0.42	114.88	100.81
		3	1.81	1.17	0.40	120.51	101.17
	High	1	2.36	2.04	0.40	96.08	99.93
		2	2.35	2.05	0.39	95.61	99.92
		3	2.19	2.01	0.39	89.55	99.75
C/Y	Low	1	0.34	0.30	0.02	106.67	104.13
		2	0.28	0.22	0.02	118.18	111.93
		3	0.11	0.07	0.02	128.57	119.55
	Med	1	1.23	1.26	0.02	96.03	99.32
		2	1.17	1.18	0.02	97.46	99.52
		3	1.13	1.08	0.02	102.78	100.58
	High	1	1.95	2.02	0.01	96.04	99.81
		2	1.91	1.96	0.02	96.43	99.81
		3	1.90	1.96	0.01	96.43	99.81
C/M	Low	1	0.47	0.46	0.06	89.13	94.72
		2	0.42	0.43	0.06	83.72	91.63
		3	0.46	0.38	0.06	105.26	102.57
	Med	1	1.26	1.31	0.08	90.08	98.52
		2	1.18	1.22	0.08	90.16	98.32
		3	1.32	1.39	0.07	89.93	98.63
	High	1	1.93	2.09	0.07	89.00	99.51
		2	2.04	2.23	0.07	88.34	99.59
		3	2.01	2.12	0.07	91.51	99.67
C/C	Low	1	1.45	0.66	1.43	3.03	97.24
		2	1.77	0.66	1.50	40.91	98.99
		3	1.57	0.60	1.49	13.33	98.11
	Med	1	2.20	1.40	1.55	46.43	99.48
		2	2.17	1.31	1.55	47.33	99.46
		3	2.08	1.26	1.53	43.65	99.33
	High	1	2.44	2.21	1.59	38.46	99.65
		2	2.44	2.15	1.56	40.93	99.66
		3	2.44	2.19	1.66	35.62	99.65

Appendix H

Data of Ink Trap Difference between
Calculated by Preucil's Equation and Brunner's Equation
and Measured by the Gravimetric Method

Appendix H

On Newsprint A

Ink	Amount of	Gravi.	Preucil		Brunner	
Combination	2nd Ink	Trap (%)	Trap(%)	Difference	Trap(%)	Difference
Y/Y	Low	72.15	49.60	-22.55	92.91	20.76
	Med	117.01	63.56	-53.45	94.77	22.24
	High	105.91	59.38	-46.53	96.10	-9.81
Y/M	Low	126.16	82.13	-44.03	97.01	-29.15
	Med	100.12	70.10	-30.02	95.02	-5.10
	High	101.96	74.64	-27.32	97.44	-4.52
Y/C	Low	76.34	96.93	20.59	98.57	22.23
	Med	93.75	95.90	2.15	98.51	4.76
	High	103.34	94.04	-9.30	98.99	-4.35
M/Y	Low	46.54	68.47	21.93	82.95	36.41
	Med	67.01	72.11	5.10	88.67	21.66
	High	100.72	96.07	-4.65	99.43	-1.29
M/M	Low	132.87	44.85	-88.02	95.84	-37.03
	Med	115.77	44.74	-71.03	96.29	-19.48
	High	107.34	44.02	-63.32	97.45	-9.89
M/C	Low	49.58	40.58	-9.00	79.44	29.86
	Med	91.39	63.29	-28.10	92.42	1.03
	High	110.67	83.23	-27.44	98.24	-12.43
C/Y	Low	113.07	99.27	-13.80	99.58	-13.49
	Med	70.29	95.54	25.25	98.08	27.79
	High	105.53	97.69	-7.84	99.70	-5.83
C/M	Low	93.72	111.99	18.27	105.72	12.00
	Med	86.73	84.52	-2.21	95.83	9.10
	High	94.82	95.56	0.74	99.44	4.62
C/C	Low	72.44	29.13	-43.31	91.83	19.39
	Med	101.63	49.51	-52.12	95.60	-6.03
	High	102.64	61.14	-41.50	97.84	-4.80

Standard Error

36.5803

Appendix H

On Newsprint B

Ink Combination Y/Y	Amount of 2nd Ink	Gravi. Trap (%)	Preucil		Brunner	
			Trap(%)	Difference	Trap(%)	Difference
Y/Y	Low	92.68	52.24	-40.44	94.46	1.78
	Med	90.15	54.97	-35.18	95.30	5.15
	High	96.02	59.52	-36.50	97.91	1.89
Y/M	Low	88.88	95.84	6.96	99.28	10.40
	Med	104.81	79.41	-25.40	97.26	-7.55
	High	82.19	71.59	-10.60	98.14	15.95
Y/C	Low	97.77	95.91	-1.86	98.56	0.79
	Med	73.92	83.21	9.29	92.99	19.07
	High	105.87	95.89	-9.98	99.52	-6.35
M/Y	Low	15.45	54.11	38.66	71.09	55.64
	Med	45.28	59.19	13.91	78.73	33.45
	High	108.36	96.53	-11.83	99.69	-8.67
M/M	Low	119.39	52.41	-66.98	97.93	-21.46
	Med	111.30	47.27	-69.03	97.17	-14.13
	High	112.64	51.45	-61.19	98.82	-13.82
M/C	Low	-26.65	20.63	47.28	72.56	99.21
	Med	90.29	75.14	-15.15	93.91	3.62
	High	116.07	83.74	-32.33	98.87	-17.20
C/Y	Low	102.66	101.51	-1.15	99.80	-2.86
	Med	73.97	94.60	20.63	97.84	23.87
	High	97.34	97.45	0.11	99.68	2.34
C/M	Low	86.03	100.00	13.97	99.96	13.93
	Med	79.31	83.02	3.71	94.94	15.63
	High	68.52	77.89	9.37	96.72	28.20
C/C	Low	57.78	35.85	-21.93	94.97	37.19
	Med	79.77	34.03	-45.74	96.35	16.58
	High	98.86	51.06	-47.80	98.33	-0.53

Standard Error

32.4612

Appendix H

On Coated Paper

Ink Combination	Amount of 2nd Ink	Gravi. Trap (%)	Preucil		Brunner	
			Trap(%)	Difference	Trap(%)	Difference
Y/Y	Low	88.21	42.06	-46.15	98.20	9.99
	Med	122.01	49.02	-72.99	98.63	-23.38
	High	108.78	36.88	-71.90	99.17	-9.61
Y/M	Low	101.63	94.30	-7.33	99.63	-2.0
	Med	126.17	73.26	-52.91	98.34	-27.83
	High	106.60	63.28	-43.32	99.20	-7.40
Y/C	Low	56.76	142.75	85.99	113.89	57.13
	Med	130.82	130.44	-0.38	106.92	-23.90
	High	124.80	96.41	-28.39	99.85	-24.95
M/Y	Low	49.50	54.16	4.66	77.29	27.79
	Med	126.46	88.44	-38.02	98.25	-28.21
	High	116.45	97.19	-19.26	99.91	-16.54
M/M	Low	133.30	23.50	-109.80	99.20	-34.10
	Med	102.68	24.45	-78.23	98.80	-3.88
	High	116.63	24.96	-91.67	99.48	-17.15
M/C	Low	169.61	134.81	-34.80	103.04	-66.57
	Med	162.39	114.05	-48.34	100.75	-61.64
	High	129.84	93.75	-36.09	99.87	-29.97
C/Y	Low	-19.39	117.81	137.20	111.87	131.26
	Med	90.48	98.76	8.28	99.81	9.33
	High	95.15	96.03	0.88	99.81	4.66
C/M	Low	-4.27	92.70	96.97	96.31	100.58
	Med	97.82	90.06	-7.76	98.49	0.67
	High	98.77	89.62	-9.15	99.59	0.82
C/C	Low	103.97	19.09	-84.88	98.11	-5.86
	Med	129.78	45.80	-83.98	99.42	-30.36
	High	112.79	38.34	-74.45	99.65	-13.14

Standard Error

62.9713

Appendix I

Data of Average Ink Trap Value
Calculated by Hamilton's Equation with Various D_m Value

Appendix I

On Newsprint A

Ink Combination	Amount of 2nd Ink	Trap(%) by Hamilton				
		Dm= 1.57	Dm= 1.59	Dm= 1.6	Dm= 1.7	Dm= 1.8
Y/Y	Low	71.92	71.48	71.26	69.33	67.71
	Med	88.68	88.06	87.76	85.15	83.07
	High	83.79	83.03	82.67	79.68	77.45
Y/M	Low	115.27	114.61	114.30	111.43	109.04
	Med	97.75	97.71	96.81	94.10	91.90
	High	108.57	107.27	106.66	101.79	98.36
Y/C	Low	101.38	101.31	101.28	100.98	100.71
	Med	101.03	100.94	100.90	100.51	100.18
	High	99.24	99.11	99.05	98.53	98.12
M/Y	Low	64.68	64.75	64.78	65.08	65.33
	Med	65.15	65.30	65.37	66.01	66.54
	High	94.18	94.35	94.42	94.96	95.27
M/M	Low	98.91	97.07	91.19	88.85	83.38
	Med	99.40	97.00	95.88	87.01	80.90
	High	115.99	104.72	101.02	83.06	75.32
M/C	Low	43.16	43.12	43.11	42.95	42.82
	Med	62.64	62.74	62.79	63.17	63.42
	High	96.67	96.02	95.74	93.73	92.49
C/Y	Low	99.85	99.85	99.84	99.81	99.77
	Med	94.97	94.99	94.99	95.05	95.09
	High	93.45	93.89	94.08	95.26	95.86
C/M	Low	118.08	117.98	117.93	117.48	117.10
	Med	79.70	79.85	79.92	80.54	81.01
	High	93.81	94.07	94.19	94.90	95.24
C/C	Low	47.98	47.55	47.33	45.46	43.94
	Med	85.66	84.40	83.81	78.90	75.29
	High	128.11	113.60	109.73	93.68	87.50

Appendix I

On Newsprint A (continued)

Ink Combination	Amount of 2nd Ink	Trap(%) by Hamilton				
		Dm= 1.9	Dm= 2.5	Dm=3.2	Dm= 5.0	Dm= 500.0
Y/Y	Low	66.34	61.12	58.05	54.61	49.65
	Med	81.37	75.32	72.02	68.49	63.61
	High	75.71	69.88	66.86	63.71	59.41
Y/M	Low	107.01	99.28	94.72	89.61	82.20
	Med	90.08	83.46	79.76	75.75	70.15
	High	95.79	87.75	83.84	79.90	74.69
Y/C	Low	100.48	99.52	98.91	98.16	96.95
	Med	99.89	98.75	98.05	97.21	95.91
	High	97.79	96.60	95.92	95.17	94.05
M/Y	Low	65.55	66.40	66.93	67.54	68.46
	Med	66.98	68.64	69.59	70.63	72.10
	High	95.46	95.91	96.03	96.09	96.07
M/M	Low	79.13	65.55	59.03	52.70	44.92
	Med	76.39	63.07	57.11	51.51	44.80
	High	70.57	58.70	53.88	49.42	44.07
M/C	Low	42.69	42.17	41.81	41.36	40.59
	Med	63.60	63.95	63.96	63.81	63.30
	High	91.58	88.72	87.23	85.61	83.25
C/Y	Low	99.74	99.63	99.54	99.44	99.27
	Med	95.13	95.27	95.35	95.43	95.54
	High	96.22	97.03	97.29	97.50	97.69
C/M	Low	116.76	115.42	114.58	113.57	112.00
	Med	81.39	82.63	83.24	83.82	84.51
	High	95.44	95.77	95.80	95.75	95.56
C/C	Low	42.68	38.10	35.56	32.86	29.16
	Med	75.52	63.66	59.34	55.05	49.55
	High	83.78	74.29	70.22	66.26	61.19

Appendix I

On Newsprint B

Ink Combination	Amount of 2nd Ink	Trap(%) by Hamilton				
		Dm= 1.89	Dm= 1.99	Dm= 2.0	Dm= 2.1	Dm= 2.2
Y/Y	Low	76.71	74.76	74.58	72.95	71.54
	Med	84.38	81.79	81.55	79.42	77.60
	High	91.27	87.01	86.65	83.63	81.28
Y/M	Low	127.95	125.45	125.22	123.11	121.29
	Med	110.46	107.84	107.60	105.43	103.58
	High	94.47	91.70	91.47	89.46	87.87
Y/C	Low	100.53	100.22	100.19	99.93	99.70
	Med	85.08	84.98	84.97	84.88	84.80
	High	102.61	101.86	101.79	101.23	100.78
M/Y	Low	52.61	52.70	52.71	52.79	52.86
	Med	55.13	55.40	54.42	55.66	55.86
	High	97.43	97.38	97.38	97.35	97.32
M/M	Low	125.14	115.27	114.43	107.15	101.43
	Med	95.76	89.63	89.11	84.51	80.84
	High	179.61	120.07	117.78	102.40	93.71
M/C	Low	22.24	22.15	22.14	22.06	21.98
	Med	84.47	83.78	83.72	83.14	82.64
	High	98.71	96.64	96.47	95.09	94.05
C/Y	Low	103.06	102.95	102.94	102.85	102.77
	Med	93.83	93.89	93.90	93.95	93.99
	High	97.43	97.45	97.45	97.47	97.49
C/M	Low	103.48	103.26	103.24	103.05	102.89
	Med	80.99	81.19	81.21	81.37	81.51
	High	63.90	65.85	66.02	67.44	68.55
C/C	Low	62.33	59.69	59.45	57.33	55.57
	Med	65.67	61.80	61.46	58.53	56.17
	High	92.96	85.13	84.55	79.83	76.45

Appendix I

On Newsprint B (continued)

Ink Combination	Amount of 2nd Ink	Trap(%) by Hamilton			
		Dm= 2.5	Dm= 3.3	Dm= 5.0	Dm= 500.00
Y/Y	Low	68.27	63.28	58.89	52.29
	Med	73.49	67.46	62.37	55.03
	High	76.51	70.43	65.80	59.57
Y/M	Low	117.06	110.56	104.77	95.92
	Med	99.34	93.02	87.56	79.48
	High	84.55	80.15	76.63	71.63
Y/C	Low	99.14	98.23	97.37	95.92
	Med	84.60	84.25	83.88	83.21
	High	99.83	98.53	97.47	95.91
M/Y	Low	53.04	53.33	53.62	54.10
	Med	56.36	57.16	57.93	59.18
	High	97.24	97.08	96.90	96.53
M/M	Low	89.77	75.34	65.03	52.51
	Med	73.23	63.53	56.37	47.34
	High	80.41	67.92	60.26	51.52
M/C	Low	21.81	21.50	21.19	20.63
	Med	81.46	79.61	77.91	75.16
	High	91.95	89.22	87.02	83.77
C/Y	Low	102.58	102.27	101.98	101.51
	Med	94.09	94.25	94.39	94.60
	High	97.52	97.54	97.53	97.45
C/M	Low	102.48	101.81	101.15	100.01
	Med	81.82	82.26	82.60	83.02
	High	70.81	73.57	75.53	77.88
C/C	Low	51.68	46.24	41.87	35.90
	Med	51.25	44.88	40.13	34.08
	High	70.16	62.87	57.69	51.12

Appendix I

On Coated Paper

Ink Combination	Amount of 2nd Ink	Trap(%) by Hamilton				
		Dm= 2.45	Dm= 2.49	Dm= 2.5	Dm= 2.6	Dm= 2.7
Y/Y	Low	78.73	77.57	77.29	74.73	72.52
	Med	100.06	98.04	97.57	93.26	89.66
	High	68.25	66.60	66.22	62.96	60.43
Y/M	Low	142.55	141.29	140.99	138.15	135.63
	Med	105.98	105.00	104.77	102.60	100.72
	High	83.97	83.17	82.98	81.32	79.97
Y/C	Low	157.95	157.65	157.57	156.87	156.23
	Med	149.02	148.59	148.48	147.50	146.61
	High	105.13	104.71	104.61	103.77	103.11
M/Y	Low	50.92	50.99	51.01	51.17	51.31
	Med	85.69	85.79	85.82	86.04	86.22
	High	100.16	100.00	99.97	99.68	99.47
M/M	Low	77.68	73.91	73.05	65.86	60.53
	Med	65.92	63.61	63.07	58.44	54.82
	High	71.12	66.57	65.60	58.24	53.42
M/C	Low	190.36	188.56	188.12	184.16	180.78
	Med	171.10	168.55	167.95	162.68	158.44
	High	154.11	145.60	143.95	132.73	126.31
C/Y	Low	119.52	119.50	119.49	119.42	119.36
	Med	99.32	99.31	99.31	99.28	99.26
	High	92.37	92.63	92.69	93.19	93.56
C/M	Low	94.59	94.56	94.55	94.47	94.40
	Med	89.83	89.86	89.86	89.92	89.96
	High	78.83	79.78	80.00	81.70	82.88
C/C	Low	47.94	46.61	46.30	43.53	41.27
	Med	143.31	135.76	134.07	120.35	110.55
	High	200.09	137.28	130.87	99.90	87.04

Appendix I

On Coated Paper (continued)

Ink Combination	Amount of 2nd Ink	Trap(%) by Hamilton			
		Dm= 3.0	Dm= 4.0	Dm= 6.0	Dm= 500.00
Y/Y	Low	67.39	58.28	51.49	42.15
	Med	81.72	68.88	60.17	49.12
	High	55.27	47.74	42.94	36.94
Y/M	Low	129.57	118.01	108.63	94.44
	Med	96.32	88.34	82.21	73.35
	High	77.02	72.13	68.53	63.33
Y/C	Low	154.62	151.25	148.17	142.82
	Med	144.44	140.14	136.46	130.51
	High	101.76	99.74	98.36	96.43
M/Y	Low	51.67	52.40	53.05	54.14
	Med	86.64	87.35	87.84	88.43
	High	99.04	98.40	97.93	97.20
M/M	Low	50.46	37.74	30.87	23.57
	Med	47.53	37.35	31.33	24.51
	High	45.19	35.64	30.56	25.01
M/C	Low	173.00	159.40	149.26	134.95
	Med	149.43	135.52	126.23	114.16
	High	116.49	106.03	100.44	93.81
C/Y	Low	119.19	118.83	118.48	117.81
	Med	99.20	99.08	98.97	98.76
	High	94.29	95.23	95.75	96.29
C/M	Low	94.21	93.81	93.43	92.71
	Med	90.05	90.15	90.17	90.06
	High	84.99	87.40	88.56	89.61
C/C	Low	36.48	29.21	24.62	19.14
	Med	92.62	70.56	58.68	45.91
	High	70.24	54.41	46.67	38.41

Appendix J

Data of Ink Trap Difference between
Calculated by Hamilton's Equation
and Measured by the Gravimetric Method

Appendix J

On Newsprint A

Ink Combination	Amount of 2nd Ink	Ink Trap Difference (%)				
		Dm= 1.57	Dm= 1.59	Dm= 1.6	Dm= 1.7	Dm= 1.8
Y/Y	Low	-0.23	-0.67	-0.89	-2.82	-4.44
	Med	-28.33	-28.95	-29.25	-31.86	-33.94
	High	-22.12	-22.88	-23.24	-26.23	-28.46
Y/M	Low	-10.89	-11.55	-11.86	-14.73	-17.12
	Med	-2.37	-2.41	-3.31	-6.02	-8.22
	High	6.61	5.31	4.70	-0.17	-3.60
Y/C	Low	25.04	24.97	24.94	24.64	24.37
	Med	7.28	7.19	7.15	6.76	6.43
	High	-4.10	-4.23	-4.29	-4.81	-5.22
M/Y	Low	18.14	18.14	18.24	18.54	18.79
	Med	-1.86	-1.71	-1.64	-1.00	-0.47
	High	-6.54	-6.73	-6.30	-5.76	-5.45
M/M	Low	-33.96	-35.80	-41.68	-44.02	-49.49
	Med	-16.37	-18.77	-19.89	-28.76	-34.87
	High	8.65	-2.62	-6.32	-24.28	-32.02
M/C	Low	-6.42	-6.46	-6.47	-6.63	-6.76
	Med	-28.75	-28.65	-28.60	-28.22	-27.97
	High	-14.0	-14.65	-14.93	-16.94	-18.18
C/Y	Low	-13.22	-13.22	-13.23	-13.26	-13.30
	Med	24.68	24.70	24.70	24.76	24.80
	High	-12.08	-11.64	-11.45	-10.27	-9.67
C/M	Low	24.36	24.26	24.21	23.76	23.38
	Med	-7.03	-6.88	-6.81	-6.19	-5.27
	High	-1.01	-0.75	0.63	0.08	0.42
C/C	Low	-24.46	-24.89	-25.11	-26.98	-28.50
	Med	-15.97	-17.23	-17.82	-22.73	-26.34
	High	25.47	10.96	7.09	-8.96	-15.14
Standard Error		17.3925	17.1150	17.6800	19.5523	21.4734

Appendix J

On Newsprint A (continued)

Ink Combination	Amount of 2nd Ink	Ink Trap Difference (%)				
		Dm= 1.9	Dm= 2.5	Dm=3.2	Dm= 5.0	Dm= 500.0
Y/Y	Low	-5.81	-11.03	-14.10	-17.54	-22.50
	Med	-35.64	-41.69	-44.99	-48.52	-53.40
	High	-30.20	-36.03	-39.05	-42.20	-46.50
Y/M	Low	-19.15	-26.88	-31.44	-36.55	-43.96
	Med	-10.04	-16.66	-20.36	-24.37	-29.97
	High	-6.17	-14.21	-18.12	-22.06	-27.27
Y/C	Low	24.14	23.18	22.57	21.82	20.61
	Med	6.14	5.00	4.30	3.46	2.16
	High	-5.55	-6.74	-7.42	-8.17	-9.29
M/Y	Low	19.01	19.86	20.39	21.00	21.92
	Med	-0.03	1.63	2.58	3.62	5.09
	High	-5.26	-4.81	-4.69	-4.63	-4.65
M/M	Low	-53.74	-67.32	-73.84	-80.17	-87.95
	Med	-39.38	-52.70	-58.66	-64.26	-70.97
	High	-36.77	-48.64	-53.46	-57.92	-63.27
M/C	Low	-6.89	-7.41	-7.77	-8.22	-8.99
	Med	-27.79	-27.44	-27.43	-27.58	-28.09
	High	-19.09	-21.95	-23.44	-25.06	-27.42
C/Y	Low	-13.33	-13.44	-13.53	-13.63	-13.80
	Med	24.84	24.98	25.06	25.14	25.25
	High	-9.31	-8.50	-8.24	-8.03	-7.84
C/M	Low	23.04	21.70	20.86	19.85	18.28
	Med	-5.34	-4.10	-3.49	-2.91	-2.22
	High	0.62	0.95	0.98	0.93	0.74
C/C	Low	-29.76	-34.34	-36.88	-39.58	-43.28
	Med	-26.11	-37.97	-42.29	-46.58	-52.08
	High	-18.86	-28.35	-32.42	-36.38	-41.45
Standard Error		22.8531	27.9833	30.5183	33.1117	36.5491

Appendix J

On Newsprint B

Ink Combination	Amount of 2nd Ink	Ink Trap Difference (%)				
		Dm= 1.89	Dm= 1.99	Dm= 2.0	Dm= 2.1	Dm= 2.2
Y/Y	Low	-15.97	-17.92	-18.10	-19.73	-21.14
	Med	-5.77	-8.36	-8.60	-10.73	-12.55
	High	-4.75	-9.01	-9.37	-12.39	-14.74
Y/M	Low	39.07	36.57	36.34	34.23	32.41
	Med	5.65	3.03	2.79	0.62	-1.23
	High	12.28	9.51	9.28	7.27	5.68
Y/C	Low	2.76	2.45	2.42	2.16	1.93
	Med	11.16	11.06	11.05	10.96	10.88
	High	-3.26	-4.01	-4.08	-4.64	-5.09
M/Y	Low	37.16	37.25	37.26	37.34	37.41
	Med	9.85	10.12	9.14	10.38	10.58
	High	-10.93	-10.98	-10.98	-11.01	-11.04
M/M	Low	5.75	-4.12	-4.96	-12.24	-17.96
	Med	-15.54	-21.67	-22.19	-26.79	-30.46
	High	66.97	7.43	5.14	-10.24	-18.93
M/C	Low	48.89	48.80	48.79	48.71	48.63
	Med	-5.82	-6.51	-6.57	-7.15	-7.65
	High	-17.36	-19.43	-19.60	-20.98	-22.02
C/Y	Low	0.40	0.29	0.28	0.19	0.11
	Med	19.86	19.92	19.93	19.98	20.02
	High	0.09	0.11	0.11	0.13	0.15
C/M	Low	17.45	17.23	17.21	17.02	16.86
	Med	1.68	1.88	1.90	2.06	2.20
	High	-4.62	-2.67	-2.5	-1.08	0.03
C/C	Low	4.55	1.91	1.67	-0.45	-2.21
	Med	-14.10	-17.97	-18.31	-21.24	-23.60
	High	-5.90	-13.73	-14.31	-19.03	-22.41
Standard Error		21.2767	17.4555	17.4677	18.2539	19.2047

Appendix J

On Newsprint B (continued)

Ink Combination	Amount of 2nd Ink	Ink Trap Difference (%)			
		Dm= 2.5	Dm= 3.3	Dm= 5.0	Dm= 500.00
Y/Y	Low	-24.41	-29.40	-33.79	-40.39
	Med	-16.66	-22.69	-27.78	-35.12
	High	-19.51	-25.59	-30.22	-36.45
Y/M	Low	28.18	21.68	15.89	7.04
	Med	-5.47	-11.79	-17.25	-25.33
	High	2.36	-2.04	-5.56	-10.56
Y/C	Low	1.37	0.46	-0.40	-1.85
	Med	10.68	10.33	9.96	9.29
	High	-6.04	-7.34	-8.40	-9.96
M/Y	Low	37.59	37.88	38.17	38.65
	Med	11.08	11.88	12.65	13.90
	High	-11.12	-11.28	-11.46	-11.83
M/M	Low	-29.62	-44.05	-54.36	-66.88
	Med	-38.07	-47.77	-54.93	-63.96
	High	-32.23	-44.72	-52.38	-61.12
M/C	Low	48.46	48.15	47.84	47.28
	Med	-8.83	-10.68	-12.38	-15.13
	High	-24.12	-26.85	-29.05	-32.30
C/Y	Low	-0.08	-0.39	-0.68	-1.15
	Med	20.12	20.28	20.42	20.63
	High	0.18	0.20	0.19	0.11
C/M	Low	16.45	15.78	15.12	13.98
	Med	2.51	2.95	3.29	3.71
	High	2.29	5.05	7.01	9.36
C/C	Low	-6.10	-11.54	-15.91	-21.88
	Med	-28.52	-34.89	-39.64	-45.69
	High	-28.70	-35.99	-41.17	-47.74
Standard Error		21.6285	25.2597	28.2536	32.4255

Appendix J

On Coated Paper

Ink Combination	Amount of 2nd Ink	Ink Trap Difference (%)				
		Dm= 2.45	Dm= 2.49	Dm= 2.5	Dm= 2.6	Dm= 2.7
Y/Y	Low	-9.48	-10.64	-10.92	-13.48	-15.69
	Med	-21.95	-23.97	-24.44	-28.75	-32.35
	High	-40.53	-42.18	-42.56	-45.82	-48.35
Y/M	Low	40.92	39.66	39.36	36.52	34.00
	Med	-20.19	-21.17	-21.40	-23.57	-25.45
	High	-22.63	-23.43	-23.62	-25.28	-26.63
Y/C	Low	101.19	100.89	100.81	100.11	99.47
	Med	18.20	17.77	17.66	16.68	15.79
	High	-19.67	-20.09	-20.19	-21.03	-21.69
M/Y	Low	1.42	1.49	1.51	1.67	1.81
	Med	-40.77	-40.67	-40.64	-40.42	-40.24
	High	-16.29	-16.45	-16.48	-16.77	-16.98
M/M	Low	-55.62	-59.39	-60.25	-67.44	-72.77
	Med	-36.76	-39.07	-39.61	-44.24	-47.86
	High	-45.51	-50.06	-51.03	-58.39	-63.21
M/C	Low	20.75	18.95	18.51	14.55	11.17
	Med	8.71	6.16	5.56	0.29	-3.95
	High	24.27	15.76	14.11	2.89	-3.53
C/Y	Low	138.91	138.89	138.88	138.81	138.75
	Med	8.84	8.83	8.83	8.80	8.78
	High	-2.78	-2.52	-2.46	-1.96	-1.59
C/M	Low	98.86	98.83	98.82	98.74	98.67
	Med	-7.99	-7.96	-7.96	-7.90	-7.86
	High	-19.94	-18.99	-18.77	-17.07	-15.89
C/C	Low	-56.03	-57.36	-57.67	-60.44	-62.70
	Med	13.53	5.98	4.29	-9.43	-19.23
	High	87.3	24.49	18.08	-12.89	-25.75
Standard Error		49.3917	49.9668	46.9437	47.8393	48.9701

Appendix J

On Coated Paper (continued)

Ink Combination	Amount of 2nd Ink	Ink Trap Difference (%)			
		Dm= 3.0	Dm= 4.0	Dm= 6.0	Dm= 500.00
Y/Y	Low	-20.82	-29.93	-36.72	-46.06
	Med	-40.29	-53.13	-61.84	-72.89
	High	-53.51	-61.04	-65.84	-71.84
Y/M	Low	27.94	16.38	7.00	-7.19
	Med	-29.85	-37.83	-43.96	-52.82
	High	-29.58	-34.47	-38.07	-43.27
Y/C	Low	97.86	94.49	91.41	86.06
	Med	13.62	9.32	5.64	-0.31
	High	-23.04	-25.06	-26.44	-28.37
M/Y	Low	2.17	2.90	3.55	4.64
	Med	-39.82	-39.11	-38.62	-38.03
	High	-17.41	-18.05	-18.52	-19.25
M/M	Low	-82.84	-95.56	-102.43	-109.73
	Med	-55.15	-65.33	-71.35	-78.17
	High	-71.44	-80.99	-86.07	-91.62
M/C	Low	3.39	-10.21	-20.35	-34.66
	Med	-12.96	-26.87	-36.16	-48.23
	High	-13.35	-23.81	-29.40	-36.03
C/Y	Low	138.58	138.22	137.87	137.20
	Med	8.72	8.60	8.49	8.28
	High	-0.86	0.08	0.60	1.14
C/M	Low	98.48	98.08	97.70	96.98
	Med	-7.77	-7.67	-7.65	-7.76
	High	-13.78	-11.37	-10.21	-9.16
C/C	Low	-67.49	-74.76	-79.35	-84.83
	Med	-37.16	-59.22	-71.10	-83.87
	High	-42.55	-58.38	-66.12	-74.38
Standard Error		51.6378	56.0034	58.9918	62.9332

Appendix K

Derivation of Range Criteria of
 D_m Value in Hamilton's Equation and
Result of Hamilton's Equation as
 D_m Value Approaching Infinity

Appendix K

The values of D_m , with which Hamilton's equation will produce undefined percent ink trap.

$$\text{Hamilton's equation: \% trap} = \frac{\log \left(1 + \frac{D_{21} - D_1}{D_m - D_{21}} \right)}{\log \left(1 + \frac{D_2}{D_m - D_2} \right)} * 100$$

when

$$1 + \frac{D_{21} - D_1}{D_m - D_{21}} < 0 \quad \text{or} \quad 1 + \frac{D_2}{D_m - D_2} < 0$$

% trap is undefined.

First,

$$1 + \frac{D_{21} - D_1}{D_m - D_{21}} < 0$$

$$(D_{21} - D_1) / (D_m - D_{21}) < -1$$

if $D_m - D_{21} > 0$ (that is $D_m > D_{21}$),

$$D_{21} - D_1 < D_{21} - D_m,$$

$$D_m < D_1,$$

$$\text{which is } D_{21} < D_m < D_1.$$

if $D_m - D_{21} < 0$ (that is $D_m < D_{21}$),

$$D_{21} - D_1 > D_{21} - D_m,$$

$$D_m > D_1,$$

$$\text{which is } D_1 < D_m < D_{21}.$$

Second,

$$1 + \frac{D_2}{D_m - D_2} < 0$$

$$D_2 / (D_m - D_2) < 0$$

Appendix K (continued)

if $D_m - D_2 > 0$ (that is $D_m > D_2$),
 $D_2 < D_2 - D_m$,
 $D_m < -1$,
 which is $D_2 < D_m < -1$.

if $D_m - D_2 < 0$ (that is $D_m < D_2$),
 $D_2 > D_2 - D_m$,
 $D_m > -1$,
 which is $-1 < D_m < D_2$.

However, D_m , as defined as density on paper, is always larger than zero.

In conclusion,

1. $D_{21} < D_m < D_1$, or
2. $D_1 < D_m < D_{21}$, or
3. $0 < D_m < D_2$.

are not suitable for Hamilton's equation.

As $D_m \rightarrow \infty$,

Hamilton's equation:

$$\lim_{D_m \rightarrow \infty} \frac{\log \left[1 + \frac{D_{21} - D_1}{D_m - D_{21}} \right]}{\log \left[1 + \frac{D_2}{D_m - D_2} \right]}$$

can apply Chain rule or Hospital rule, here we apply Chain rule:

Appendix K (continued)

$$\begin{aligned}
& \frac{\frac{d}{dD_m} \left[1 + \frac{D_{21} - D_1}{D_m - D_{21}} \right]}{1 + \frac{D_{21} - D_1}{D_m - D_{21}}} \\
= \lim_{D_m \rightarrow \infty} & \frac{\frac{d}{dD_m} \left[1 + \frac{D_2}{D_m - D_2} \right]}{1 + \frac{D_2}{D_m - D_2}} \\
& \frac{-\frac{(D_{21} - D_1)}{(D_m - D_{21})^2}}{1 + \frac{D_{21} - D_1}{D_m - D_{21}}} \\
= \lim_{D_m \rightarrow \infty} & \frac{-D_2}{\frac{(D_m - D_2)^2}{1 + \frac{D_2}{D_m - D_2}}} \\
& \frac{-\frac{(D_{21} - D_1)}{D_m - D_{21}}}{\frac{D_m - D_{21} + D_{21} - D_1}{-D_2}} \\
= \lim_{D_m \rightarrow \infty} & \frac{\frac{D_m - D_{21}}{D_m - D_{21} + D_2}}{\frac{-\frac{(D_{21} - D_1)}{(D_m - D_{21})(D_m - D_1)}}{-D_2}} \\
& \frac{D_m}{D_m} \left[\frac{D_{21} - D_1}{D_2 \left(1 - \frac{D_1}{D_m} \right)} \right] \\
& \frac{D_m (D_{21} - D_1)}{D_2 (D_m - D_1)} \\
& \frac{D_{21} - D_1}{D_2} \\
& = \text{Preucil's equation}
\end{aligned}$$