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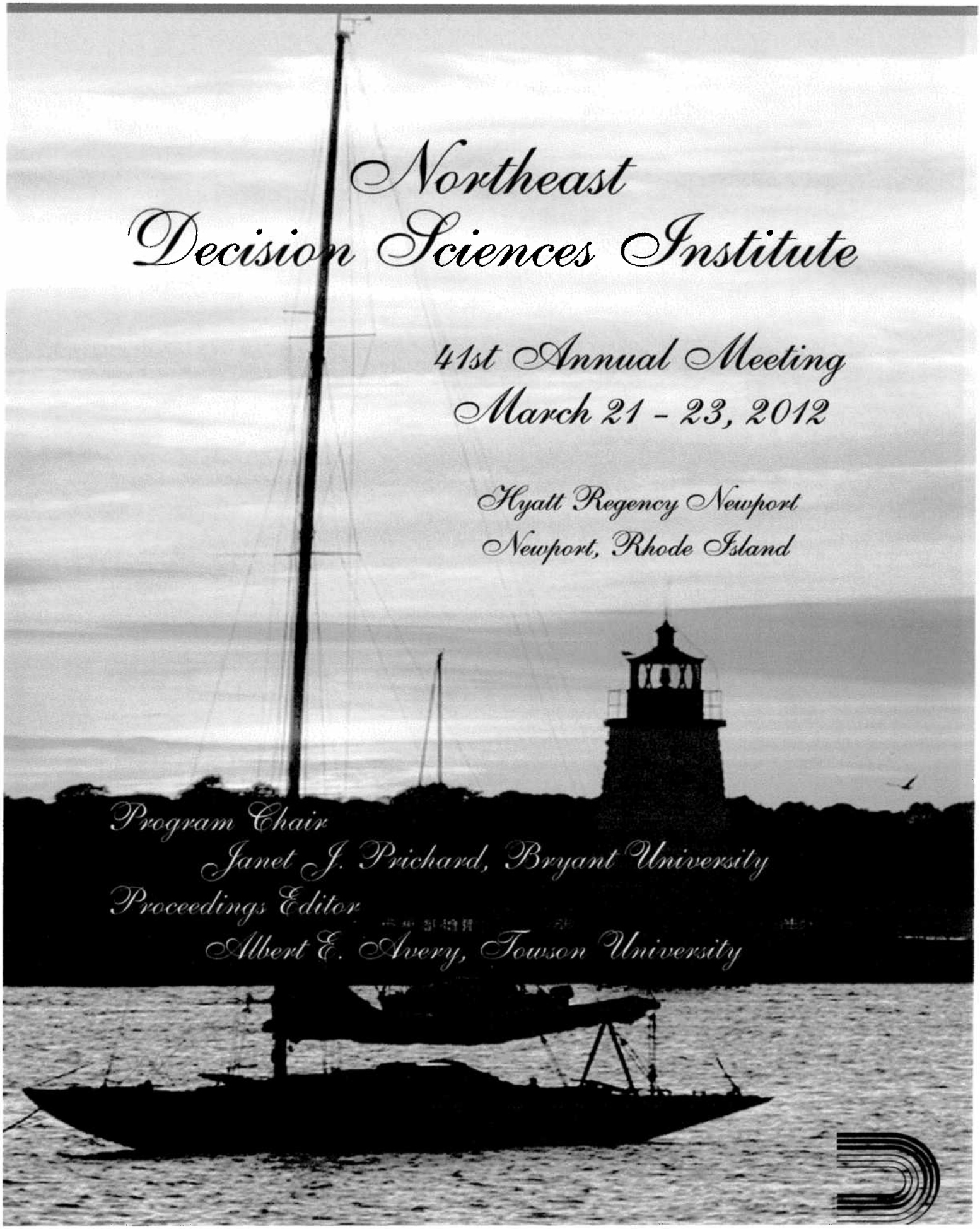

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CONTROL LIMITS VERSUS ACCEPTANCE LIMITS - WHICH LIMITS ARE APPROPRIATE FOR YOUR TASK?

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ABSTRACT

The objective of this paper is to discuss and clarify the meaning and use of the different statistical limits used in managing processes so that mistakes that are commonly seen in industrial practice can be avoided. The difference between "control" and "acceptance" will also be discussed. The idea of this paper was triggered by a recent paper written by Henderson [5] in *Quality Progress* journal.

Key words: Statistical limits, process control, process capability, process acceptance.

DISCUSSION

There are several limits in statistical process control (SPC) and they all serve different purposes depending on the question being asked. In order to ensure that proper action(s) can be taken to rectify and/or improve a process, it is critical for the user to do the following: ask the right question, use the right set of limits, and properly determine the limits in order to come up with the right answer. In this paper, we will discuss the basic questions of process control and acceptance:

- stability (in-control),
- performance (capability), and
- acceptance questions

and also clarify the difference between control and acceptance.

Stability (In-control):
The stability question deals with the stability of the process, e.g., are the process average and width stable over time, does the process distribution stay stable over time, etc. In other words, the question being addressed is, *"Are unexpected things happening in my process that I may capitalize on to improve quality?"* The stability question is critical in process management because the future behavior of the stable processes can be predicted with accuracy and, as a result, operations' planning is easier. Keep in mind that the word 'stability' refers to statistical stability, i.e., the only variation displayed by the process is due to random (common) causes.

Control charts are the most effective way of testing the stability question (see, for example, SPC books by Grant and Leavenworth [3], Montgomery [12] for different types of control charts). Limits on the control charts, i.e., *control chart limits*, define the area in which the variation is accepted as due to random (common) causes. As long as sequentially gathered data remains inside these limits, the process has given no indication of instability. Keep in mind that a stable process does not necessarily produce output that is acceptable to the customer(s). This is one of the common misunderstandings in practice.

A stable (in-control) process only means the process is behaving the way we expected it to behave; i.e., there is nothing unusual going on in the process. Such a process may or may not meet the customer expectations, which is a different question to be tested. This issue was also raised by Grant and Leavenworth [3] in their book:

"The word '*control*' has a special technical meaning in the language of statistical quality control. A process is described as 'in control' when a stable system of chance causes seems to be operating. However, the word is often misused and misinterpreted, particularly by those who have been briefly exposed to the jargon of statistical quality control without having had a chance to learn its principles."

Two common misconceptions among practitioners are: if a process is not meeting the customer specifications, it (the process) will generate data for which the control charts indicate that something unusual is happening; or a process which is meeting the customer specifications will generate data for which the control charts indicate nothing unusual is happening!! These two issues, i.e., stability and meeting the customer specifications, must be kept separate.

In addition, one can also test the stability of the process by comparing the two variance estimators without using the control charts. An alternative estimate for the variance is the one computed using mean square successive differences (MSSD). (See, for example, Neumann, et al. [13], Holmes and Mergen ([6, 10].) The MSSD is defined as

$$\text{MSSD} = \frac{1}{(n-1)} \sum_{i=1}^{n-1} (X_{i+1} - X_i)^2 \tag{1}$$

Using these differences an unbiased estimate for the process variance is defined by Hald [4] as

$$q^2 = \frac{1}{2(n-1)} \sum_{i=1}^{n-1} (X_{i+1} - X_i)^2 \tag{2}$$

and the MSSD standard deviation is determined by taking the square root of the q^2.

The variance estimated through MSSD, q^2, as defined above, looks only at the successive differences (by taking into account the time order of the data) and represents the variation that a process could display if some of the non-random elements, such as trends, cycles, etc., were eliminated.

The significance of the difference between the regular and the MSSD variance estimates can be tested using the test given in Dixon and Massey [1]:

$$z = \frac{1 - \frac{q^2}{s^2}}{\sqrt{\frac{n-2}{(n-1)(n+1)}}} \tag{3}$$

where s^2 is the regular variance estimate and n is the number of observations.

z values between ±3 indicate that the difference between the two estimates is not statistically significant, i.e., the process seems to be stable (i.e., in-control). Values (i.e., z values) bigger than +3 and less than -3 indicate that the two variance estimates are significantly different and thus the process is not stable. Values bigger than +3 imply trend and/or long-term cycles in the process and values less than -3 imply short term cycles in the process.

An often neglected issue in determining the control limits on the chart is whether the subgroups are formed *rationally*. Rational subgroups are those which are formed in such a way that the variation within the subgroups is random so that the non-random process changes that take place between subgroups can be detected faster. Otherwise, inflated variation within the subgroups will hide the process changes; and as a result, it will take longer to detect those changes and react to them. This point was emphasized by Shewhart [14], the originator of control charts, as one of the critical factors for the successful use of control charts. However, there may be cases where the formation of the subgroups does not follow the rational subgroup principles outlined above. In some cases, variation within the subgroups may be inflated intentionally given the nature of the process. One example of the reasons for the inflation of the limits is to minimize the unnecessary "out-of-control" signals given by the control chart, if there are natural (and unavoidable) batch-to-batch variation in the process, such as in some chemical processes.

Performance (Capability):

Another critical question tested by SPC is whether we are producing outputs (goods or services) that meet the customer requirements, i.e., the capability question. This question is different from the stability (in-control) question. Based on the answers for the stability and capability questions, a process can be in one of the four states:

- Process in control and capable of meeting the specification limits
- Process in control but not fully capable of meeting the specification limits
- Process not in control but currently meeting the specification limits
- Process not in control and not meeting the specification limits

The proper way of checking whether the process is fully meeting the specification limits is to compare the *natural tolerance limits* of the process against the specification limits set by the customer. Natural tolerance limits define the area where roughly 99.9% of the process output falls. If the process output follows a Normal distribution, the natural tolerance limits would be:

$$\overline{\overline{X}} \mp 3\sigma_X \tag{4}$$

where $\overline{\overline{X}}$ is the estimated process average and σ_x is the process standard deviation estimate. If the process distribution is significantly different from Normal, then the area under the curve which corresponds to roughly 99.9% should be determined by using the proper distribution which describes the process.

A key issue in process capability is to keep the *center* issue and the *width* issue separate. If there is a target value (nominal) given by the customer, the process average needs to be compared to this value to see how close it is to the target, i.e., the center issue. In addition, the width of the process, determined by the natural tolerance limits, can be compared to the customer specifications to see if the process width is narrow enough to meet the specifications. For example, a process may have a very narrow

width to meet the specifications easily; however, if the process average is too far from the target, the process will still generate some unacceptable output.

In practice, quality control practitioners tend to use capability indices, such as Cp, Cpk, Cpl, Cpu, etc., to do the analysis described above. One needs to be careful using these indices since these indices assume that the process follows a Normal distribution. This is another error that is fairly common in practice, i.e., using these indices when the process distribution is not Normal. For capability indices for non-Normal process distribution see, for example, Holmes and Mergen [7].

When the process is not in-control, the analysis described above should be considered to reflect the current performance of the process, i.e., no prediction should be done for the potential capability of the process. In other words, since the process is not stable, we cannot tell what the future process output will be. To determine the potential capability of the process when the process is not in-control, i.e., not stable, one needs to use a capability standard deviation estimate to calculate the capability indices, such as Cp. For example, one estimate for the capability standard deviation is the one determined using MSSD as described above in equation (2). Another capability standard deviation estimate would be the one that takes into account the "runs" that may exist in the process. The run, defined as successive points above or below the median (see Holmes and Mergen [9] for the details), is another variance estimator which takes into account the time order of the process data. This variance estimate represents what the process variance would be if the runs, as defined above, were eliminated. The smaller of the two capability standard deviation estimates, i.e., the estimate through MSSD and the estimate using runs in the process is one way to determine the C_p index which shows the potential capability of the process. The Cp index is defined as:

$$C_p = \frac{USL - LSL}{6\sigma} \tag{5}$$

where USL and LSL are the upper specification limit and lower specification limit, respectively, and σ is the process standard deviation estimated either using the MSSD or the runs in the process.

Acceptance:
The acceptance issue has not received as much attention as the control and capability issues have in SPC. In some instances, processes, because of their nature, are expected to have unavoidable and natural shifts in their average value but are still able to meet the specifications set by the customer. This may occur when the standard deviation of the process, at the various average values of the process, is very small relative to the difference between the upper and lower specification limits given by the customer. Such a process is not considered in-control by SPC standards but may still be able to meet the specification limits. In a situation like this, the proper question to test should not be "Is the process average in-control?" but rather "Is the process producing output satisfying the customer specifications?" Checking the stability of the process average for such a process would not generate any valuable information for process managers since the answer would be obvious, i.e., "not in-control." In other words, using control chart limits is not the right thing to do in this case. What we want is protection against the case where the process average may deviate so much from its desirable value that it will start producing some non-conforming output, i.e., output that does not lie within customer specification limits.

We could check this through the use of acceptance sampling plans, such as MLT 105 and 414. However, this would be too late. We would like to catch defective output in the process before the quality system audit at the end. In other words, we should move the acceptance procedure on-line with charting similar to control charts, i.e., acceptance charting. Acceptance charting is similar to control charting but acceptance charts address a different question: *Is my process producing defective (nonconforming) output?* The purpose of the acceptance charts is to evaluate a process in terms of whether or not it could be expected to satisfy output specifications. Again, the question that is being tested under acceptance charts is *"Is the process producing acceptable parts?"* <u>not</u> *"Is the process in statistical control?"* We will discuss how to set the maximum/minimum acceptable values for the process average along with the upper and lower acceptance limits for the sample averages using acceptable quality level (AQL) as the design criteria. For more detail on acceptance charts see, for example, Freund [2], Montgomery [12], Holmes and Mergen [8, 11]. The steps for building acceptance chart are as follows:

1. First, decide on the AQL that you think appropriate for the process.
2. Determine a maximum allowable value for the process mean, known as the Upper Acceptable Process Mean (UAPM), at $k_1\sigma_x$ below the upper specification limit (USL), where σ_x is the capability standard deviation of the x's. The capability standard deviation is the standard deviation estimate which is independent of changes in the process average, such as $\bar{R}/d_2$ or $\bar{s}/c_4$. $\bar{R}$ and $\bar{s}$ are the average of the subgroup (sample) ranges and the average of the subgroup standard deviations, respectively. Factors d_2 and c_4 are the correction factors for a given subgroup (sample) size. (The MSSD standard deviation could be another estimate for the capability standard deviation.) For example, if the x values are roughly normally distributed, then using k_1=3 will produce an AQL value of approximately 0.15% for a one-sided specification limit.

$$UAPM = USL - k_1\sigma_x \tag{6}$$

3. Add $k_2\sigma_{\bar{x}}$ to UAPM to arrive at the Upper Acceptance Limit (UAL) for the sample averages, where $\sigma_{\bar{x}}$ is the standard deviation of $\bar{x}$'s, i.e., sample averages. The value of k_2 sets the probability of the acceptance of output that has a quality level of AQL. For example, using k_2 = 3 would set the probability of accepting output that has a quality level of AQL to approximately 0.9985 for a one-sided specification limit.

$$UAL = MAPM + k_2\sigma_{\bar{x}} \tag{7}$$

The acceptance limit for the lower specification limit (LSL) case is done same way as follows.

$$LAPM = LSL + k_1\sigma_x \tag{8}$$

and

$$LAL = LAPM - k_2\sigma_{\bar{x}} \tag{9}$$

where LAPM is the lowest acceptable process mean and the LAL is the lower acceptance limit for

the sample $\bar{x}$.

Note that the standard deviation of the $\bar{x}$'s is the standard deviation of the x's divided by the square root of the sample size, i.e., $\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}}$ where n is the sample size. Thus an $\bar{x}$ value exceeding the UAL or LAL would indicate that the process average may have shifted above/below the level that is tolerated so the process may produce unacceptable output. When the $\bar{x}$ values (i.e., sample averages) stay within the established UAL and LAL, the process would be considered (i.e., accepted) to be producing acceptable output. Note that the acceptance chart can also be used for one-sided specifications; in that case there would be one acceptance limit, either UAL or LAL.

CONCLUSION

In conclusion, in this paper we discussed several key issues in SPC, along with the proper statistical limits to check those issues. It is critical that not only the right kind of limits be used but also that those limits be determined properly given the SPC question that we are trying to answer.

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