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Nicholas M. Schneider

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EXPLORATION OF THE EFFECT OF SURFACE ROUGHNESS ON
HEAT TRANSFER IN MICROSCALE LIQUID FLOW

by

Nicholas M. Schneider

A Thesis Submitted in Partial Fulfillment of the Requirements for a B.S./M.S. in
Mechanical Engineering

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Date
Abstract

Mission of the Effect of Surface Roughness on Heat Transfer in Microscale Liquid Flow

Nicholas M. Schneider

Thesis Advisor: Dr. Satish G. Kandlikar

As technology provides smaller devices with greater heat dissipation needs, microfluidic systems become essential. The scale of device architecture causes concerns to arise that were previously not an issue. The results of manufacturing processes, such as roughness structures on machined surfaces, now play a significant role in transport phenomena. This study takes an analytical and experimental approach to understanding the fundamental heat transfer process in rectangular channels with artificially roughened walls. Steady, incompressible, fully developed liquid flow is modeled with lubrication theory to develop an expression for the fully developed Nusselt number. The heat transfer performance of the small aspect ratio rectangular channels with two wall heating under the H2 boundary condition is experimentally investigated. A constant wall heat flux is applied at opposing long walls. Four different structured roughness geometries are investigated along with smooth channels as the heated walls. In total, hydraulic diameters ranged from $D_h = 183\mu m$ to $D_h = 1698\mu m$ and were tested over a Reynolds number range of 45 to 600. The pitch to height ratio of the sinusoidal roughness surfaces covered the ranged of 2.6 to 10.6. The resulting relative roughness was 2.17% to 16.53%. Fully developed Nusselt was found to lie below classic theory. Sinusoidal roughness geometries were found not to provide heat transfer enhancement over smooth channel walls.
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Chapter 1

Introduction

Microfluidic devices are becoming necessary in diverse fields and new technologies. Understanding the underlying fundamental physics of the transport phenomena occurring in devices whose scale is continuously decreasing in size is essential for optimal design. The ratio of surface area to volume in small channels provides characteristics exploitable in heat transfer applications. In the specific application of electronics chip cooling, it is desirable to maximize heat dissipation with minimal energy input.

Laminar forced convection in single phase flow is appealing due to the lower pressure drop associated with this regime. Modifications in channel wall geometry have shown potential to increase heat transfer in ducts with minimal pressure drop costs. However, due to experimental uncertainties, the complexity of the coupled physics problem, and the lack of an in-depth systematic study researchers are left inconclusive and contradictory results. The aim of this study is to develop and test an experiment design which isolates sources of uncertainty and identifies the key parameters necessary for accurate experimental results, while fully investigating the controlled geometry of sinusoidal wall roughness.

1.1 Literature Review

1.1.1 Surface Roughness and Heat Transfer

The effect of surface roughness on low Reynolds number flow is receiving substantial attention in literature. Emerging applications are driving microscale research. Technologies,
including PEM fuel cells and electronics, requiring high heat flux removal are providing new avenues for the use of mini- and microchannels [1,2,3,4-11]. In current literature, the effects of surface roughness are still ambiguous due to the compounding effects of the developing entrance region, early transition to turbulence, and high experimental uncertainty. This study will mitigate these issues by using designed structure roughness to better control the overall system.

Most research regarding surface roughness effects on heat transfer has been performed at macroscale. Gao and Sunden [12] studied air flow through rectangular ducts with hydraulic diameters of 24.9 mm and 700 mm. Reynolds number was varied between 1000 and 6000 to examine the temperature distribution across a rib with surface roughness. The data collected was used to find the local and average heat transfer coefficients along with the Nusselt numbers. The ribs were found to enhance the heat transfer process compared to smooth ducts. Friction factors for different rib orientations were also investigated in their study. The authors found the greatest heat transfer to occur at the tips of the rib roughness elements, with the lowest heat transfer occurring at the base of the ribs.

Chang et al. [13] recently investigated the effects of scale shaped roughness elements on the heat transfer characteristics of air flow through rectangular channels. A large range for Reynolds number was chosen to cover the range from 1500 to 15000. The authors used various roughness element configurations for the study, and concluded that the scaled roughness elements enhanced heat transfer with better performance than rib shaped roughness elements.

In 1994 Ling et al. studied how triangular shaped rib-roughness in channels effected the heat and mass transfer [14]. The paper lacks the details on channel dimensions and the means of heating. The authors chose Reynolds numbers to correspond with the turbulent regime. Reynolds number was varied from 10,000 to 70,000 over systematically placed rib elements in the shape of isosceles triangles on the top and bottom walls of the channels.
Entrance length was found to increase with Reynolds number. Heat and mass transfer rates were increased by a factor of up to 2.3 over the smooth channel equivalent. The increase of heat and mass transfer was also found to a function of Reynolds number, while friction factor, $f$, was a weak function of Reynolds number.

Heat transfer characteristics of water in stainless steel minitubes with inner diameters of 1067 and 620 $\mu$m were studied by Kandlikar et al. [3]. The experiments utilized two different etches in order to create a variety of surface roughness inside the minitubes. The average roughness of the tubes ranged from 1 $\mu$m to 3 $\mu$m, corresponding to a relative roughness range of 0.161% to 0.355%. The study was focused on the laminar flow regime, and was determined to have thermally developing conditions. With consideration to the conditions, the experimental data was compared with correlations available at the time. For the smoothest tubes and all of the roughness ranges of the 1067 $\mu$m tube, the experimental data was found to correspond to the predicted values within uncertainties. The smaller tube, the 620 $\mu$m tube, was under-predicted by the correlation as roughness increased. This deviation lead the authors to conclude that more research is necessary to understand the effects of surface roughness as the diameter becomes smaller.

Bucci et al. in 2003 investigated fluid flow and heat transfer properties in capillary tubes [8]. Hydraulic diameters of 172 $\mu$m, 290 $\mu$m, and 520 $\mu$m were selected and subject to both laminar and turbulent flows, with Reynolds number ranging from 100 to 6000. Roughness was reported as an absolute height ranging from 1.498 $\mu$m to 2.166 $\mu$m, the authors used a laser interferometric microscope to obtain the roughness values. The outside wall temperature was held constant via water vapor condensation. The results fit the Gnielinski correlation well for the two larger diameter tubes in the turbulent regime. The two smaller diameter tubes did not fit the Gnielinski correlation well for turbulent flow. The Hausen correlation under-predicted the experimental data from the laminar regime.

Zhang et al. in 2007 studied turbulent flow of liquid Nitrogen in capillary tubes [15].
The investigation considered Reynolds numbers from 10,000 to 90,000 in four tubes with diameters of 1931 µm, 1042 µm, 834 µm, and 531 µm. The objective was to understand the effects of surface roughness on single-phase pressure drop and heat transfer properties in the four given stainless steel minitubes. The experimental Nusselt number results were under-predicted by the Dittus-Boelter and Gnielinski correlations for their respective ranges. The authors also noted that the discrepancies increased as channel diameter decreased. Overall, surface roughness was found to improve heat transfer properties in the minitubes.

Hegab et al. in 2001 studied rectangular microchannels with R-134a refrigerant [16]. The goal was to understand the effects of channel geometry and Reynolds number on convective heat transfer. Hydraulic diameter was chosen to range from 112 µm to 210 µm with varying aspect ratios from 1 to 1.5. In order to mitigate error, the authors paid particular attention to measurements. The surface profile was measured with a profilometer to more accurately know the roughness of the test pieces. A microscope and video camera were employed to measure the channel width, while digital dial calipers were used to measure the length. Reynolds number ranged from 2000 to 4000 in the experiments. This range, being the transition range, allowed the results for the Nusselt number to be compared to Gnielinski’s correlation. Wall temperatures were calculated by means of a conduction heat transfer analysis utilizing the average temperature measured by seven thermocouples located on the back of the wafer. The authors found that both the friction factor and the Nusselt number were over-predicted by conventional correlations ranging from 6% to 84%. The deviations were found to decrease as channel diameter increase or Reynolds number decreased.

Lee et al. in 2005 investigated single-phase heat transfer in rectangular minichannels [17]. The corresponding hydraulic diameters ranged from 318 µm to 903 µm. Reynolds number was varied from 300 to 3500 for five channel configurations. The experimental Nusselt were found to be under-predicted by classic theory for both developing and fully
developed flows. Deviation from classic theory was inversely related to channel size for a given flow rate.

Wu and Cheng in 2003 studied laminar water flow through trapezoidal silicon mini- and microchannels [2]. The experimental variables were: geometry, hydrophilic properties, and surface roughness. These properties were varied over thirteen channels etched in a silicon wafer. Ten out of the thirteen channels had silicon for the surface material, and the other three had a thermal oxide layer deposited to increase the surface hydrophilic properties. The first set of channels also had geometry and relative roughness variances to see the effects of all the chosen variables. Different etching processes were used to vary the relative roughness of the channels. The wafer was heated by a thin film heater attached to a DC power supply. Reynolds number was varied up to 1500, and the effects on heat and mass flow were observed. Their results for Nusselt number as a function of Reynolds number can be seen in the reproduction found in Figure 1.1.

![Figure 1.1: Reproduction of results from Wu and Cheng [2]](image_url)
A few experimental studies have found that fluid flow in smooth microchannels does not deviate from conventional theory. Yang and Lin [18] examined water flow through six stainless steal mini and microtubes with diameters ranging from $123 \, \mu m$ to $962 \, \mu m$. Reynolds number was varied through both the laminar and turbulent regimes with values up to 10,000 being studied. The surface temperature of the tubes was acquired by means of Liquid Crystal Thermography. The tubes were heated by DC powered heaters clamped to each end of the tubes. The average roughness for the tubes ranged from $1.16 \, \mu m$ to $1.48 \, \mu m$. The experimental results for the laminar flow Nusselt number were found to agree well with the theoretical values for fully developed flow with constant heat flux. Turbulent results were compared with the Gnielinski correlation, and were also found to fit well. Finally, the developing region Nusselt number was also found to agree with the Shah and Bhatti correlations. The authors concluded that there is no deviation from classic theory for the range of diameters investigated in their study.

Given discrepancies in current literature, more research on the effects of surface roughness on heat and mass transfer is necessary to develop a model that will accurately predict these effects. It is important to better understand these transport phenomena while paying close attention to uncertainty generated in test results. It is also important to define at what point roughness becomes a non-negligible issue for microfluidic scenarios. This work will aim to generate a theoretical model that fulfills the fore-given requirements.

1.1.2 Axial Conduction

Axial heat conduction in channel walls is generally low in conventional sized channels where the channel diameter is greater than $3 \, \text{mm}$. The axial conduction effects are small because the channel walls are relatively thin compared to the overall channel size and flow length. In mini and microchannels, the convective heat transfer becomes significantly effected due to the relatively large multi-dimensional conduction in the wall. A number of studies have determined the heat transfer coefficient by measuring the wall temperature in
the channel without considering the multidimensional conduction in the wall [2, 19, 20, 21]. These studies showed that the heat transfer coefficient was smaller for microscale compared to conventional macroscale results. Guo and Li indicated that it might be due to the error introduced by the assumption of one-dimensional heat conduction in the wall [22]. The results show that without considering the axial heat conduction in the walls, the experimental Nusselt number will be lower than the actual value. Chiou [23] presented a conductance number, \( \text{cond} \), to describe the effects of axial heat conduction in the walls on convective heat transfer.

\[
\text{cond} = \frac{k_s A_s}{\dot{m}_f c_{p,f}} = \frac{A_s}{A_f} \frac{1}{\frac{D}{L}} \frac{k_s}{Re Pr k_f} \quad (1.1)
\]

where \( k_s, A_s, k_f, \) and \( A_f \) are the thermal conductivity and cross-sectional area for the tube wall and fluid. The other parameters: \( L, D, \dot{m}_f, \) and \( c_{p,f} \) are the tube length, tube diameter, mass flow rate of the fluid, and the specific heat of the fluid respectively. For \( \text{cond} \) less than 0.005, the effects of axial heat conduction are negligible. A non-dimensional number, \( M \), was proposed by Maranzana et al. [24] to quantify the role of axial conduction in the walls.

\[
M = \frac{q_{\text{cond}}}{q_{\text{conv}}} = \frac{k_s e_s w}{\rho c_{p,f} e_f w V} \quad (1.2)
\]

where \( e_s \) and \( e_f \) are the thickness of the solid and fluid respectively, \( \rho \) is the fluid density, \( w \) is the channel width, and \( V \) is the mean fluid velocity. When \( M \) is less than 0.01, the effects of axial heat conduction in the walls is considered negligible. The test section has been designed with this in mind, and will be discussed in a later section.

### 1.2 Previous Work at RIT

Taylor et al. [25] studied the downfalls of using solely average roughness in microscale applications. For macroscale applications, the average roughness parameter is well known
and both easy to use and determine. Most manufacturers will use average roughness with confidence in defining the surface finish of the product. Average roughness does not accurately represent the surface structure and topography. This parameter generalizes the surface, but does not give details into maximum height or uniformity of roughness elements. In order to more accurately describe surfaces, other parameters are often used in addition to the average roughness parameter.

Kandlikar et al. [26] and Taylor et al. [25] proposed six new roughness parameters that will better describe the surface topography of roughened surfaces for use in fluid flow applications. Three of the presented parameters describe the surface roughness and the other three correspond to the localized hydraulic diameter variations. The roughness parameters describe: mean spacing of profile irregularities ($R_{SM}$), maximum profile peak height ($R_P$), and the floor distance mean line $FdRa$. The proposed roughness parameter to replace average roughness ($R_a$) is $\varepsilon_{Fp}$, and will be further discussed.

### 1.2.1 Surface Roughness Parameters

The most common surface roughness parameter is relative roughness $R_a$. This is the currently accepted parameter in industry, and can be found via Equation (1.3):

$$R_a = \frac{1}{n} \sum_{i=1}^{n} |Y_i|$$ (1.3)

where $n$ is the number of sample points and $|Y_i|$ identifies the absolute value of the profile deviation from the mean line.

Mean spacing of profile irregularities, $R_{SM}$, parameterizes the pitch of the surface profile, and can be calculated by means of Equation (1.4):

$$R_{SM} = \frac{1}{n} \sum_{i=1}^{n} S_{mi}$$ (1.4)

In order to define this parameter, two theoretical lines are drawn. The two lines can be
a percentage value or represent a standard height of the profile, and are centered around the mean value line. The area between the two lines is considered to be a dead zone. $S_{mi}$ is used to represent the peaks and valleys of the profile, or the distance from the dead zone to a maximum or minimum of the profile (relative to the mean line).

The mean line is a parameter that is needed to calculate a number of other surface roughness parameters. This line represents the average of all points on a surface profile relative to an arbitrary reference (controlled by measurement technique). Equation (1.5) can be used to calculate this parameter.

$$\text{MeanLine} = \frac{1}{n} \sum_{i=1}^{n} z_i$$  \hspace{1cm} (1.5)

where $z_i$ is a point on the surface profile.

The maximum profile peak height, $R_p$, defines the distance from the mean line to the highest point on the profile. The conjugate of the maximum profile peak height is the maximum profile valley height, $R_v$. This second parameter represents the distance between the mean line and the lowest point on the profile. Equations (1.6) and (1.7) show the calculation for these parameters.

$$R_p = max(z_i) - \text{MeanLine}$$  \hspace{1cm} (1.6)

$$R_v = min(z_i) - \text{MeanLine}$$  \hspace{1cm} (1.7)

The floor distance to the mean line is given by the $\text{FdRa}$ parameter. $\text{FdRa}$ can be calculated by finding the average of all profile data points that fall below the mean line. The procedure given in Equation (1.8) shows the calculation of this roughness parameter.
Let $z_i \subseteq Z_i$ such that all $z_i = Z_i$ if and only if $Z_i < \text{MeanLine}$

$$F_p = \frac{1}{n} \sum_{i=1}^{n} Z_i$$

$$FdRa = \text{MeanLine} - F_p$$ (1.8)

The roughness parameter proposed in place of average roughness, $\varepsilon_{F_p}$, is also reviewed. The proposed parameter is the sum of the roughness parameters $R_p$ and $FdRa$. This parameter is shown in Equation (1.9) and is representative of the height of the roughness asperities.

$$\varepsilon_{F_p} = R_p + FdRa$$ (1.9)

These Roughness parameters can be visualized in Figure 1.2 below, from Brackbill [29].

![Roughness Visualization](image)

Figure 1.2: Roughness Visualization

### 1.2.2 Pressure Drop

Schmitt and Kandlikar [27] studied the effects of surface roughness on pressure drop in rectangular minichannels with both air and water flow. The authors set out to examine the validity of roughness parameters on pressure drop in mini and microscale flows. Also, the
work intended to identify the Reynolds number where transition from the laminar to turbulent regimes occurs. The experiments tested Reynolds numbers ranging from 200 to 7200 in channels with hydraulic diameters from 325 \( \mu m \) to 1819 \( \mu m \). The authors observed that the relative roughness parameter \( (R_a) \), the standard roughness parameter, was not effective in fully explaining the different surfaces being studied. In order to more completely interpret the surface structures, five roughness parameters were used. Average roughness, root mean square roughness, \( R_q \), skewness, \( R_{Sk} \), kurtosis, \( R_{ku} \), and average maximum roughness height, \( R_z \). The study investigated channels with three types of roughness surfaces: one set were smooth channels, then the other two were aligned and offset sawtooth roughness surfaces. The results from these sets of channels were compared to conventional theory and the Moody Diagram. The smooth channel correlated well with conventional theory for both working fluids. The sawtooth roughness surfaces resulted in large error for experimental friction factor when using the entire diameter. The error could be significantly reduced by using a constricted diameter. The work proposed a model that predicts the critical Reynolds number from the transition region. The work had shown that the critical Reynolds number increased as the roughness ratio decreased.

The work done by Schmitt and Kandlikar [27] was recently extended by Brackbill and Kandlikar [28,29]. The new work studied the effects of triangular roughness elements on the critical Reynolds number for transition and friction factor. The authors utilized a variable diameter test setup for water flow through minichannels. The channels had hydraulic diameters ranging from 424 \( \mu m \) to 1697 \( \mu m \) with Reynolds number varying from 30 to 15,000. The authors also examined the effects of uniform roughness on the single-phase friction factor of water in mini and microchannels. The work studied the roughness parameters proposed by Taylor et al. [25] and Kandlikar et al. [26]. Based on the results, the Brackbill and Kandlikar works proposed a new constricted hydraulic diameter parameter. The new parameter makes use of the roughness parameters presented by Taylor et al. [25]
to be used in calculating the friction factor. This new parameter is used in place of the hydraulic diameter, and was found to give results that correspond to classic theory more closely as seen in Figure 1.3 from Brackbill [29], where normalized results are shown as a function of relative roughness. In this plot a value of 1 represents agreement with theory. It is easily seen that the constricted parameter better predicts fluid flow characteristics compared to the root separation.

![Figure 1.3: Constricted Parameter Performance](image)

Wagner and Kandilkar [31] extended the work of Brackbill [28, 29] to incorporate wall functions in the two dimension domain. Lubrication theory was used to extract a pressure flow relationship to be used in the boundary layer analysis. The extension brings inertial effects back into the flow physics to arrive at a velocity profile as shown in Equation (1.10), where $f(x)$ and $h(x)$ are continuous functions describing the bottom and top walls respectively. The resulting expressions for friction factor were validated with experimental data from the work of Brackbill [29].
\[ u = \frac{6V}{z (h(x) - f(x)) (y - f(x)) (y - h(x))} \]  

(1.10)

1.2.3 Heat Transfer

In 2001, Joshi [30] and Kandlikar et al.[34] studied the effects of surface roughness on flow characteristics. The work studied two stainless steel capillary tubes with diameters of 620 \( \mu m \) and 1067 \( \mu m \) and relative roughness ranging from 0.161\% to 0.355\%, where Reynolds number ranged from 500 to 3000. A reservoir and positive displacement pump were utilized to pump distilled water through a flow meter controlling the mass flow rate in the system. A needle valve was placed after the flow meter to help mitigate oscillations in the flow throughout the system. The distilled water would then enter the test section. Pressure drop was measured across the entire test section. For the heat transfer portion of the work, DC electrical resistive heaters provided a constant heat flux across the capillary tubes. The system was insulated with fiberglass insulation in order to minimize heat loss. Due to a low current path being created were the pressure taps where machined, two sets of capillary tubes were manufactured to run the pressure drop and heat transfer experiments separately. In order to measure wall temperature, K-type thermocouples were attached to the exterior walls of the capillary tubes at three places along the test pieces. Inlet and outlet temperatures were measured in-line with jacketed K-type Thermocouples. Experimental data was collected via LABVIEW™. The work found that for smaller tubes, surface roughness was a significant factor in both heat transfer and pressure drop. The large tube of 1067 \( \mu m \), however, behaves as a macroscale tube.

Dharaiya et al.[32] performed a numerical analysis of microchannels subject to the H2 boundary condition. The H2 boundary condition is defined as a constant heat flux along the channel walls both axially and circumferentially. The numerical work varied aspect ratio to look into the effects of constant heat flux on rectangular channel walls. The four wall
heating case was validated with published data to ensure the accuracy of the numerical model. The investigation went further to return results for the fully developed Nusselt number for two and three wall heating. These results for two wall heating are the only data points available in literature.
Chapter 2

Theoretical Analysis

The motivation exists to develop relationships that accurately predict physical situations. These expressions allow engineers to properly design to meet specific application requirements. In most applications, however, full solutions to fundamental problems are not possible to obtain. For this reason it is often necessary to look toward analytical approximations and theoretical limits to obtain bounds for design.

2.1 Complete Solution

Limiting theoretical cases with full, close-form solutions provide both insight and building blocks for further refinement. In this section the classical case of parallel flat plates is first investigated to provide the ultimate limit for Nusselt number. The results of Wagner [31] for fluid flow in small aspect ratio channels with rough walls are then applied to the heat transfer application.

2.1.1 Classical Parallel Flat Plates

The first approach to understanding the physics of heat transfer in small, wide channels was to look at the limiting case of pressure driven flow between infinite parallel flat plates. This case is a classical fluid mechanics problem found in textbooks. If we were to look at the results from the Navier-Stokes equation for steady, fully developed, laminar, incompressible flow between flat plates we would obtain Equation (2.1) exactly for the velocity
profile, where \( u \) is the axial velocity component with \( y = 0 \) centered between the plates and \( b \) is the total gap size as shown in Figure 2.1

\[
\vec{V} \rightarrow u = \frac{1}{2\mu} \frac{dP}{dx} \left( y^2 - \frac{b^2}{4} \right)
\]  

(2.1)

Next we look at the full energy equation in Equation (2.2).

\[
\rho C_p \frac{DT}{Dt} = \rho q_{gen} + \nabla \cdot (k \nabla T) + \frac{DP}{Dt} + \tau \frac{du_i}{dx_i}
\]

(2.2)

Where \( \frac{DT}{Dt} \) and \( \frac{DP}{Dt} \) represent the total derivatives of temperature and pressure respectively. The viscous dissipation terms are given in the last term on the right hand side, \( \tau \frac{du_i}{dx_i} \). Applying the additional standard assumptions of constant heat flux, no heat generation, constant thermal conductivity and negligible viscous dissipation, energy reduces to:

\[
u \frac{\partial T}{\partial x} = \alpha_f \frac{\partial^2 T}{\partial y^2}
\]

(2.3)

Since we are applying the constant heat flux boundary condition, we know that \( \frac{\partial T}{\partial x} \) will be a constant, so we set it to an arbitrary constant \( D \). An assertion we will check
later to ensure its accuracy. We are now left with an easily solvable, well-posed ordinary differential equation:

\[ uD = \alpha_f \frac{d^2 T}{dy^2} \]  

(2.4)

With the following boundary conditions on the first derivative:

\[ \frac{\partial T}{\partial y} \bigg|_{y=-\frac{b}{2}} = -\frac{q}{k} \]

\[ \frac{\partial T}{\partial y} \bigg|_{y=\frac{b}{2}} = \frac{q}{k} \]

This problem can be solved directly, but we are left with an integration constant. We apply the fact that at the top or bottom plate, the fluid temperature is equal to the wall temperature to pick up the last constant and close the solution.

\[ T \big|_{\frac{b}{2}} = T_w(x) \]  

(2.5)

After applying this last boundary condition, we obtain an expression for the temperature within the fluid across the gap:

\[ T = \frac{q}{k} \left( \frac{-y^4}{b^3} + \frac{3y^2}{2b} - \frac{5b}{16} \right) + T_w \]  

(2.6)

When checked, the final expression for \( T \) satisfies the original ODE, validating setting \( \frac{\partial T}{\partial x} \) equal to an arbitrary constant, \( D \). The reason this works is because \( D \), though arbitrary, is a slave to the wall heat flux boundary conditions.

Next, we define a mean fluid temperature across the gap, \( T_m \) in the standard fashion:
Substituting Equations (2.1) and (2.6), we can solve the integral exactly:

\[
T_m = \frac{1}{v_b z} \int_{-\frac{b}{2}}^{\frac{b}{2}} uTz \, dy
\]  

(2.7)

The last step is to look at the definition of convection:

\[
Q = \bar{h} A (T_w - T_m)
\]  

(2.9)

This means the convective coefficient will equal:

\[
\bar{h} = \frac{Q}{A (T_w - T_m)}
\]  

(2.10)

Which, when the wall heat flux conditions and Equation (2.8) are applied, reduces to:

\[
\bar{h} = \frac{140k}{17b}
\]  

(2.11)

Now we define Nusselt number based on the total separation, \(b\):

\[
Nu_b = \frac{\bar{h} b}{k} = \frac{140}{17} \approx 8.235
\]  

(2.12)

This value serves as the limit for smooth flat plates.
2.1.2 Wavy Walls

The objective is to be able to analyze structured roughness along the walls. In order to incorporate non-smooth walls, we specify wall functions at the top and bottom walls as \( h(x) \) and \( f(x) \) defining the wall topography. Each wall is only a function of the axial direction and is continuous. The first approach is to adjust the velocity profile for a wavy wall case and again perform the analysis from Section 2.1.1. The velocity profile that will be used is based on a lubrication theory approximation. In order to use this velocity profile, we are assuming a small slope trajectory between fluid particles as taken in Wager and Kandlikar [31]. This theory was developed for dealing with small asperities found on the walls in bearings. The small slope assumption dictates that the axial velocity profile is approximately parabolic provided there are no sudden changes in a fluid particles trajectory. Inertial effects have also been reincorporated into the velocity profile shown in Equation (2.13). A visual representation of the computational domain is illustrated in Figure 2.2.

The separation, \( b \), is defined as the root separation or the distance between the average floor profiles of each wall.

\[
\frac{6\dot{V}}{z(h(x) - f(x))^3} \left( y - h(x) \right) \left( y - f(x) \right)
\]

\[ (2.13) \]
Using the velocity profile from Wagner and Kandlikar [31] in the ordinary differential equation given by Equation (2.4), and applying the same boundary conditions of constant heat flux at the wall, we obtain a temperature profile within the fluid:

\[
T = \frac{q}{k(f(x) - h(x))^{\frac{3}{2}}} y^4 - 2(f(x) + h(x))y^3 + 6f(x)h(x)y^2 \\
+ (f(x)^3 - 3f(x)^2h(x) - 3f(x)h(x)^2 + h(x)^3)y \\
- f(x)h(x)(f(x)^2 - 3f(x)h(x) + h(x)^2)] + T_w
\]  

(2.14)

In the case where \( f(x) \) and \( h(x) \) are constant (smooth walls) the resulting temperature profile reduces exactly to that of the smooth case. When the analysis is carried through to the end, we obtain the following expression for Nusselt number based on the root separation:

\[
Nu_b = \frac{140b}{17(h(x) - f(x))}
\]  

(2.15)

Which again, reduces to conventional theory when \( h(x) - f(x) = b \). Unfortunately, upon further inspection, except in the case of smooth walls and possibly some coincidental wall functions, the expression obtained in Equation (2.14) fails to meet the criteria of \( \frac{dT}{dx} \) being equal to a constant. We can conclude that the theoretical Nusselt number will vary locally in wavy walled channels. However, if the difference of the wall functions is replaced by the constricted parameter as is done by Brackbill [29], we obtain an acceptable expression for Nusselt number.

\[
Nu_b = \frac{140b}{17b_{ef}}
\]  

(2.16)
The constricted parameter is defined as:

\[ b_{cf} = b - 2\varepsilon_{fp} \]  \hfill (2.17)

### 2.2 Scale Analysis

In order to develop an understanding of the complex physics involved in the coupled transport problem, the use of a scale analysis allows for the identification of dominant terms. This approach provides insight without obtaining a full solution to sets of equation that would otherwise have no closed form solution. If one were to apply lubrication theory as done by Wagner [31] to the wavy wall setting, we could extract the \( \hat{x} \) direction of momentum and continuity as shown in Equations (2.18) and (2.19).

\[
\rho \left[ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right] = -\frac{\partial P}{\partial x} + \mu \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right] \hfill (2.18)
\]

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \hfill (2.19)
\]

We specify the scaled parameters based on the characteristic features of the system:
\( u = \langle v \rangle u^* \)
\( x = Lx^* \)
\( y = by^* \)
\( z = az^* \)
\( P = P_s P^* \)

where \( P_s \) is a scale chosen to balance appropriately with the right hand side. From continuity we can arrive at the scaled \( \hat{y} \) velocity, \( v \):

\[
v = \frac{b}{L} \langle v \rangle v^*
\]

By applying our scaled quantities to Equation (2.18) we obtain the following:

\[
\rho \frac{\langle v \rangle^2}{L} \left[ \frac{u^*}{\partial x^*} + \frac{v^*}{\partial y^*} \right] = -P_s \frac{\partial P^*}{\partial x^*} + \mu \frac{1}{L^2} \frac{\partial^2 u^*}{\partial x^{*2}} + \frac{1}{b^2} \frac{\partial^2 u^*}{\partial y^{*2}} + \frac{1}{b^2} \frac{\partial^2 u^*}{\partial z^{*2}}
\]

(2.20)

By extracting the coefficients and rearranging, an expression is found that balances the order of the terms:

\[
\mathcal{O} [\alpha Re] = \mathcal{O} \left[ \frac{bP_s}{\mu \langle v \rangle} \right] + \mathcal{O} \left[ \frac{L}{a} \right] + \mathcal{O} \left[ \left( \frac{b}{a} \right)^3 \right] + \mathcal{O} \left[ \left( \frac{b}{L} \right)^2 \right]
\]

(2.21)

Since \( b \) is a small parameter (relative to \( a \) and \( L \)), the last two terms on the right hand
side will be dominated by the pressure term and the $\frac{L}{a}$ term. If we look at a length scale on the order of $a$ and choose our $P_s$ as shown below, then we are left with the dominant term on the left hand side of $\alpha Re$. This parameter will be used as in reporting the experimental results as it is a dominant term in the physics. This method is by no means a complete solution, but lends insight into the complex problem.

$$P_s = \frac{\mu <v>}{b}$$ (2.22)

The value of a scale analysis is in its to extract understanding of the fundamental physics from the governing equations without applying additional simplifying assumptions. In this case we were able to arrive at a term useful in comparing results from different geometries on an equal basis. The utilization of $\alpha Re$, in a sense, normalizes results for channel aspect ratio by dividing out this geometry factor. As will be seen later, the usefulness of this parameter is in collapsing the experimental results to a common trend.
Chapter 3

Experimentation

3.1 Experimental Setup

The experimental setup design used for this study is the next iteration of a previously proven design for fluid flow characterization by Brackbill and Kandlikar [27,28]. The conceptual auxiliary architecture and footprint of the design is maintained with the exception of material selection and separation control. The design was of mutual acceptance for both of the simultaneous heat transfer and fluid flow studies. Due to the requirement of precise machining, the final manufacturing of complex auxiliary parts was performed via CNC under the operation of NSF team member Brian LaPolt, Mechanical Engineering Technology, Rochester Institute of Technology. The strength of the design is interchangeability of the specially designed roughness test sections. Any arbitrary test section can be tested for both fluid flow characteristics and heat transfer performance with directly compatible results.

Extensive consideration was placed on the design of the test sections. A previous in-house, heat transfer experimental design showed unacceptable results due to the domination of axial conduction within the test section. Learning from this case, the new test pieces would have to account for this arising issue. An investigation was performed into the affects of axial conduction. This engineering design problem, like all design problems,
had a number of factors to balance. The driving forces for the design were to minimize axial conduction (multidimensional conduction within the wall), to ensure the footprint of the test sections fit the predefined auxiliary architecture, allow for the application of a heater, to support the placement of thermocouples, and to allow for a means of controlling separation and maintaining a seal.

Axial conduction is a physical attribute that is governed by material and geometry. Utilizing Equation 1.2, the non-dimension axial conduction number, $M$, can be calculated for the desired range of Reynolds numbers, dimensions, and materials. The parameter $M$ is a measure of the ratio of conduction within the wall, along the length of the channel to the convection from the wall to the fluid. An a priori means can be performed to estimate Axial conduction using the experimental design. A simple Excel™ worksheet is utilized to estimate the parameter $M$.

The most desirable material for machining purposes is an Aluminum Alloy. Due to the relatively high thermal conductivity of Aluminum ($238 \ [W \cdot m^{-1} K^{-1}]$ at $100^\circ C$ for Al6061-T6), low Reynolds number cases will have significant axial conduction. The plot in Figure 3.1 shows the worst case estimation for $M$ as a percentage over Reynolds number for the defined thickness of 4 mm (thickness required for rigidity and machinability for both Aluminum and Stainless Steel). Since axial conduction is considered negligible for values less than 1%, Aluminum Alloys are not acceptable for mitigating the effects of axial conduction. The only reasonable option for material, despite complications in fabrication, is Stainless Steel. In order to establish a rule-of-thumb for a minimum Reynolds number, Figure 3.2 is generated in the same fashion in order to show the low Reynolds number behavior of the designed test section with the specified stainless steel. By inspection, a Reynolds number of 70 can be used as a lower bounds for negligible axial conduction.

The available footprint, heater requirement, and base design for fluid flow directed the overall geometry of the test pieces. Ultimately the test pieces had to provide four types
Figure 3.1: Axial Conduction, $M$ Comparison of Aluminum and Stainless Steel

![Figure 3.1](image1.png)

Figure 3.2: Axial Conduction, $M$ for Stainless Steel

![Figure 3.2](image2.png)
of specific surfaces: ground ends, angled headers, channels walls, and heater attachment locations. Thermocouple holes are drilled to the center of each test piece in eleven locations equally spaced by 6.35 mm.

The ends of each test piece have to have a ground face in order to support the gauge blocks. This surface is identified in Figure 3.3 on either end. These surfaces are ground smooth to meet two functions, first, the geometric tolerance of the system require careful care, and second, these surfaces mate with the gauge blocks to create a water tight seal.

![Figure 3.3: Test Piece Ground Surfaces](image)

The next surface is an angled surface to act as the header for the channel. These surfaces direct the flow from the gauge block down to the final channel root separation. When the two test pieces are assembled in the test section, they make a triangular header region at both the inlet and outlet as shown in Figure 3.4. The design, in conjunction with the gauge blocks, is meant to ensure a smooth transition from circular to rectangular ducts. The channel exit is symmetric with the inlet, and so has the same header configuration.

Third, the bulk of the length is a surface with the controlled structured surface roughness
elements making up the channel walls. The design creates a total length of 114.6 mm for hydrodynamic flow with a channel height of 12.7 mm. These surfaces are machined using a wire Electrical Discharge Machining technique to provide the specified controlled geometries.

Lastly, the test pieces must provide a footprint for the application of heaters. Silicone Film heaters with adhesive backs rated at 10 W were identified. The total heated area has a length of 94.6 mm and height spanning that of the channel wall, 12.7 mm. Figure 3.5 shows an example picture of the smooth test pieces.

The gauge blocks, as illustrated in Figure 3.6, act as the means of controlling separation, provide the transition point from the circular inlet tube to the square channel geometry, and provide the mating surface to seal the channel ends with the test pieces. The overall geometry for different gauge blocks is maintained with the exception of varying width for varying separations. The detailed drawings of the gauge blocks can be found in the appendices.
Figure 3.5: Test Piece Example

Figure 3.6: Gauge Block Illustration
The heat transfer aspect of the study magnifies the requirements of the experimental setup. First, the heat transfer setup requires an insulating enclosure, and so, the ceramic Garolite G-10, a resin based ceramic, was selected due to its low thermal conductivity of $0.288 \, [\frac{W}{m\cdot K}]$, operating temperature range, and acceptable machinability. The base block, as illustrated in Figure 3.7, provides a smooth mounting surface with fifteen pressure taps spaced equally by $6.35 \, mm$ along the total length of the flow channel. Low durometer (highly compressible) silicone gasketing with an ample operating temperature range was selected to accommodate the extra sealing requirement around the thermocouple wires. Gaskets were cut to fit the base block and cover pieces. These low durometer gaskets were chosen to ensure sealing around the channel edges and thermocouples. Setup clamps are used to compress the test section and gaskets.

Figure 3.7: Base Block Design
3.2 System Architecture

Brackbill and Kandlikar implemented a variable hydraulic diameter test setup [27,28]. For the proposed work, the test setup remains nearly the same from a system architecture standpoint. The only difference in architecture is the addition of a constant temperature water bath and heat exchanger subsystem to meet the heat transfer conditioning needs. Figure 3.8 below shows a representation of the overall system. Degassed, distilled water is circulated through the system by a Micropump motor drive attached to a Micropump positive displacement, micro-geared metered pump head. The system is capable of up to 2500 mL/min with 8 bars of pressure drop. The working fluid exits the closed reservoir and is conditioned through a heat exchanger with an outside loop cooled by a chiller at a constant temperature. The reservoir is necessary to ensure the water remains degassed. The heat exchanger is used in order to avoid cavitation issues that arise from the chiller being physically below the test section. The working fluid exits the heat exchanger and flows through a bank of flow meters to monitor the flow rate. The flow meter bank includes a bypass loop and two digital flow meters. The two flow meters in parallel are capable of measuring 10-100 mL/min and 60-1000 mL/min respectively. The use of a flow meter bank in this fashion will increase the accuracy per given range of flow. The degassed, distilled water exits the flow meter on its way to the test section. After exiting the test section, the working fluid is returned to the reservoir.

3.3 Measurements

The measurable quantities for this experimental setup are volumetric flow rate, pressure drop between two pressure taps, fluid inlet and outlet temperature, and wall temperature. Volumetric flow rate is measured and controlled via the flow meter bank and micropump.
The pressure can be measured at known locations along the length of the channel. The inlet fluid temperature is measured by a jacketed K-type thermocouple. Upon exiting the test section, the total temperature is again measured by the same means. Channel wall temperature is more difficult to obtain. Thermocouples are made out of 36AWG K-type thermocouple wire via an in-house thermocouple welder using nitrogen as the inert gas. The thermocouples are inspected under a microscope to ensure proper welding as can be seen in Figure 3.9. The thermocouples are then dipped in thermal-epoxy to protect exposed wire from water contact.

A robust, well-organized LABVIEW GUI as shown in Figure 3.10 was developed in order to facilitate the collection of measurable quantities. The LABVIEW™ code was optimized for ease of use during heat transfer testing. Sensor information is visually presented to facilitate ease of reading and accurately performing tests. The code records calibrated raw data obtained from sensors. Parameters recorded are listed below.

1. Volumetric Flow: $\dot{V}$
2. Fluid Temperatures: $T_{in}, T_{out}$

3. Pressure Drop: $\Delta T$

4. Wall Temperature: $T_i, i = 1, 2, 3, ..., 11$

Where the thermocouples are located in the center of the 4 mm wall thickness, at the center of the depth, and at the axial distances of (measured from the start of the heated channel):

1. 16.2 mm
2. 24.4 mm
3. 32.7 mm
4. 40.9 mm
5. 49.1 mm
6. 57.3 mm
7. 65.6 mm
8. 73.8 mm
9. 82.0 mm
10. 90.2 mm
11. 98.5 mm

### 3.4 Roughness Geometry Design

The effect of the ratio of pitch to height of structured roughness elements has been identified as a key parameter on fluid flow and heat transfer [31]. In this study the sinusoidal roughness surfaces were designed to have varied pitch to height ratios ranging from 2 to 8.
Figure 3.9: 36AWG Thermocouple Weld

Figure 3.10: LABVIEW\textsuperscript{TM} GUI
The general form of the designed curve is shown in Equation (3.1). The actual fabricated surfaces varied slightly as is discussed in the Results section. The surfaces are tested in both concurrent fluid flow and heat transfer studies. The explicit designed parameters are listed in Figure 3.11.

\[ y(x) = h \left[ \cos \left( \frac{\pi}{\lambda} x \right) \right]^p \]  

\begin{tabular}{|c|c|c|c|c|c|}
\hline
Design & & & Actual & & \\
\hline
\( \lambda \) & \( h \) & \( p \) & \( \lambda \) & \( h \) & \( p \) \\
150 & 50 & 4 & 149.75 & 31.41 & 4 \\
250 & 50 & 6 & 250.18 & 32.03 & 6 \\
400 & 50 & 8 & 400.43 & 37.65 & 8 \\
250 & 125 & 12 & 249.54 & 96.25 & 12 \\
\hline
\end{tabular}

Figure 3.11: Structured Roughness Design Parameters

3.5 Assembly

3.5.1 General Assembly

The assembly of the test setup proves to be the most important step in the experimental procedure. Prior to any testing, the adhesive backed heaters are attached to individual test sections and allowed to cure for 72 hours per manufacturer specifications. The auxiliary setup is fitted with appropriate gaskets and mounted on the experimental stand. The system is ran as a closed loop without the test section to ensure proper performance. Once prepared, the test sections are carefully cleaned with small amounts of Isopropyl Alcohol to remove sediment and oils. The test section is then assembled with insulating support.
materials as shown in Figure 3.12. The test section is mounted on the base block and secured axially and laterally. With the test section securely placed, the 22 thermocouples are covered in thermal paste and carefully placed in their respective locations. The thermocouple holes are designed to place the weld at the center of the channel wall. At this point, the channel separation is measured via an optical confocal microscope. Figure 3.13 shows the assembled setup being measured by the microscope. Once the separation is accurately measured and checked along the length of the channel, the test setup is sealed with the top plate. Serrated step clamps are used to apply a constant force for vertical sealing.

![Figure 3.12: Test Setup Assembly](image)

### 3.5.2 Power Balancing

Due to manufacturing variations individual heaters will vary slightly in resistance. In order to balance the power being applied to each wall a potentiometer was placed in series with each resistive heater. This creates two series circuits with one heater and one potentiometer each. These circuits are then placed in parallel with nodes at the positive and negative terminals of the system power supply. The resistance of each heater is measured along with the total resistance of the individual series circuits when the potentiometers are set to their
“zero” setting. A simple circuit analysis is performed using Ohm’s law. An Excel sheet was created in order to utilize solver to minimize the difference in power output by each heater. This method allows for the heaters to be balanced within 0.1% at room temperature.

3.6 Conditioning

3.6.1 Sensor Calibration

Accurate measurement is an essential aspect of any experimental study. For this reason care is taken in order to ensure information gathered from sensors is properly interpreted. The experimental setup in this study has three types of sensors that need to be calibrated. There are a set of differential pressure sensors, the flow meter bank, two jacketed K-Type thermocouples and 23 K-Type thermocouples made from the 36AWG wire.

The first sensor is the set of Huba Control differential pressure sensors type 692 used in calculating pressure drop. A 0.1 bar and 0.2 bar pressure sensor with reported 0.4\%FS tolerance and 0.1\%FS sensitivity were selected. Calibration is performed by applying
known pressures by means of the in-house digital pressure calibrator. Pressure and voltage are recorded within the linear range of the piezoelectric device. Least squares is then used on the recorded calibration points to quantify the linear relationship between pressure and measured voltage. An example calibration curve is shown in Figure 3.14. This procedure is repeated for each sensor and is checked to ensure the sensors have not deviated during testing.

The second set of sensors is the two Omega digital flow meters type FLR1000. The two ranges of 10-100 ml/min and 100-1000 ml/min with reported accuracy of 1%FS were selected. In an analogous way, water is pushed at a constant rate through the individual flow meter. Both time and mass of fluid are recorded for at least 60 seconds. For lower flow rates a digital scale is used to accurately measure the mass of the fluid, while higher flow rates require a triple beam balance due to the limit of the digital scale. Mass and time measurements are converted to volumetric flow rates and then the least squares method is again employed to find the quantitative relationship between sensor output voltage and volumetric flow. Figure 3.15 shows the calibration curve for the LowFlow meter (10-100

![2 Bar Calibration Curve](image)

Figure 3.14: Pressure Sensor Calibration Curve

The second set of sensors is the two Omega digital flow meters type FLR1000. The two ranges of 10-100 ml/min and 100-1000 ml/min with reported accuracy of 1%FS were selected. In an analogous way, water is pushed at a constant rate through the individual flow meter. Both time and mass of fluid are recorded for at least 60 seconds. For lower flow rates a digital scale is used to accurately measure the mass of the fluid, while higher flow rates require a triple beam balance due to the limit of the digital scale. Mass and time measurements are converted to volumetric flow rates and then the least squares method is again employed to find the quantitative relationship between sensor output voltage and volumetric flow. Figure 3.15 shows the calibration curve for the LowFlow meter (10-100
The last set of sensors is the large array of thermocouples. These sensors are calibrated using a two-point approach to find the slope and intercept of the calibration curve. A bath of distilled water at the local two-phase points (freezing and boiling) is first prepared. The sensors, once protected with epoxy or their inherent jacketing are submerged in the prepared baths while voltage is measured. This method proves to be extremely accurate in obtaining calibration data.

### 3.6.2 Heat Loss Tests

Garolite has a very low thermal conductivity of $0.288 \text{ [W/m·K]}$, however, it is not a perfect insulator. In order to account for all system losses, it was necessary to perform heat loss tests. The setup was first assembled in the same fashion as described in Section 3.5 with plain smooth channels. No water was introduced to the setup during the tests. A known voltage and current was applied to the two heaters with total power input being measured.
along with average wall temperature and ambient temperature. The entire setup was allowed to reach steady state, which took approximately 30 minutes for each power setting. Steady state was ensured by recording temperature readings for 15 minutes and checking the standard deviation of the readings was less than experimental error in temperature.

It is assumed that the total power reading is the same as the heat loss when fluid flow is not present. By varying input power and recording steady state temperatures, a plot of heat loss as a function of the temperature difference of the average wall temperature, $T_{ave}$ and ambient temperature can be generated. It was predicted and shown to be true that heat loss has a linear relationship with the temperature difference ($\Delta T = T_{ave} - T_{Room}$). Figure 3.16 shows the resulting curve and a linear regression for these tests. This slope of this curve is used in Equation (3.3) for heat loss estimations.

$$
T_{ave} = \frac{T_{ave,wall1} + T_{ave,wall2}}{2} = \frac{1}{11} \sum_{i=1}^{11} T_{i,wall1} + \frac{1}{11} \sum_{i=1}^{11} T_{i,wall2}
$$

(3.2)

$$
q_{loss} = 0.16265(T_{ave} - T_{room})
$$

(3.3)

### 3.6.3 Fluid Flow Only Validation

Before any heat transfer tests were performed, the entire test system was validated for fluid flow only conditions. These tests were performed on the smooth channels with varying separations (aspect ratio) and Reynolds number. Pressure, fluid temperature and flow rate were collected in order to calculate friction factor, $f$, based on Equation (3.4). Results were compared to convention theory as defined by Equation (3.5) from Kakac et al.[33]. The resulting validation curve for 350 $\mu$m can be seen in Figure 3.17. The results are
Figure 3.16: Heat Loss Curve

typical of the setup’s compliance with conventional fluid flow smooth theory. Validation was ensured for the four separations of 165 \( \mu m \), 338 \( \mu m \), 501 \( \mu m \), 593 \( \mu m \) with the smooth test section.

\[
f = \frac{\Delta P \rho D_h A^2}{x \frac{2\dot{m}}{}} \quad (3.4)
\]

\[
f = \frac{24}{Re} \left( 1 - 1.3553\alpha + 1.9467\alpha^2 - 1.7012\alpha^3 + 0.9564\alpha^4 - 0.2537\alpha^5 \right) \quad (3.5)
\]
3.7 Operation

In order to ensure consistent results, a test procedure was developed and maintained for the duration of testing. Once the assembly procedure listed in Section 3.5 has been carried out, LABVIEW is opened and the test parameters entered. The LABVIEW GUI only requires separation information to begin a test. The heaters are powered by an external power supply, flow rate is controlled via the GUI, and minor manual dexterity is required to choose the appropriate flow meter and pressure sensor.

Once the micropump is started, the first step to starting a test is to ensure there are no leaks around the test setup. If the setup does not leak, the power supply is turned on and set to the proper voltage of a given test. The constant temperature bath is also set to control the inlet temperature to the test section. The setup is allowed to come to steady state by waiting for approximately 30 minutes. Once steady state is ensured, data is recorded over a 12 minute interval. The data is checked to ensure temperature variation in the form of the standard deviation are less than the experimental error. Flow rate is increased to the next

![Figure 3.17: Experimental Friction Factor Compared to Conventional Theory](image)
set point. If the power has not been changed, it takes approximately 15 minutes for steady state to be reached. Data is again recorded and the procedure is repeated over the entire test range.

It is important to constantly look for leaks. Garolite, being a resin based ceramic, does not leave surfaces that are as smooth as aluminum, and so does not seal as well, especially at non-gasketed interfaces. If leaking is observed, the test is stopped, data discarded, and reassembly is performed. The regions most prone to leaking are in the header regions. The proper cutting of gaskets is essential in creating a water tight setup. Slight mis-alignment of test section and gasket will cause a leak. Fortunately, the heaters used in the setup are designed with PTFE coated wires, and will resist the incidental water contact. The heaters, however, will not operate if fully submerged in water.
Chapter 4

Data Processing

4.1 Surface Analysis

Geometry of roughness surfaces can be hard to establish when working on the order of microns. For this reason a Keyance Confocal Laser Scanning Microscope (model# VK9710) was utilized for surface analysis. Each roughness surface was imaged using the Keyance VK Analyzer software, providing the require topography information to analyze each surface geometry.

Figure 4.1: Laser Confocal Surface Results
Topography data was extracted for processing in Excel. The average roughness of the smooth channel pieces was extracted directly and found to be $1 \, \mu m$. The structured roughness surfaces require some sophistication in order to properly quantify their surfaces. A curve fit was performed using Solver to minimize the residuals between the surface topography and the desired form of the surface model shown in Equation (4.1). The form of the equation can be generalized to fit the form of the output data. The general form to which each surface was fitted is shown in Equation (4.2)

\[
f(x) = h \left[ \cos \left( \frac{\pi}{\lambda} x \right) \right]^P
\]

\[
f(x) = a_1 \left[ \cos \left( a_2 x + a_3 \right) \right]^{a_4} + a_5
\]

Using the topographical information extracted, curve fits were performed to each of the four surfaces shown in Figures 4.3 through 4.6. The resulting coefficients for each surface are summarized in Figure 4.2.
Figure 4.3: Surface Rendering of $\lambda/h = 2.6$

Figure 4.4: Surface Rendering of $\lambda/h = 4.8$
Figure 4.5: Surface Rendering of $\lambda/h = 7.8$

Figure 4.6: Surface Rendering of $\lambda/h = 10.6$
4.2 Data Reduction

Care was taken to properly process collected data. For heat transfer tests there were a number of steps that had to be taken prior to reaching the final result of average fully developed Nusselt number. The process flow for each of the data sets is the same and can be summarized in seven steps:

1. Raw data is recorded by LABVIEW in the form of thermocouple readings, flow rates, geometry, and experimental conditions.

2. Raw data is compiled in a single Excel worksheet for each separation and geometry.

3. A custom VBA script extracts relevant data while performing an initial statistical analysis where appropriate.

4. Parameters from all data sets are combined in single worksheet for further data processing.

5. The first stage of processing calculates derived geometric parameters, fluid properties, length of the developing region, and total heat flux.

6. The results from the first stage of processing are used in the second stage of processing, which is performed in a summary worksheet combining all geometries.

7. The second stage of processing calculates local nusselt number as shown in Equation (4.17) for each thermocouple as well as the uncertainty of each result.

In the first step, data collection, the procedure for testing outlined in Section 3.7 was followed to ensure proper, consistent testing. The LABVIEW™GUI is used to actively control the experiment. The user inputs geometric and test information before beginning data recording. Once steady state has been reached, raw data is recorded to disk. Inlet and
outlet fluid temperature, room temperature, and wall temperatures are acquired from the setup’s thermocouples. Flow rate and geometry information are also recorded by the code.

Each test condition creates a single, archival raw data sheet. In order to organize the raw data in a usable fashion, the second step is performed combining results from a single separation and geometry in a single worksheet. The custom VBA script is run, organizing the data, averaging flow rate and fluid temperatures, and preparing the wall temperatures.

The fourth step takes the results in their new form and combines them for each individual geometry. This allows the data to be organized in a format that allows comparison within a single geometry. The first steps of data processing are performed in this sheet. First, the geometric information about separation and surface geometry is used to derive secondary geometric parameters. Hydraulic diameter is then calculated using Equation (4.3) and aspect ratio is defined as Equation (4.4). The last standard geometric parameter, relative roughness can be calculated using Equation (4.5). Figure 4.7 shows a cross section view of the channel with axial flow into the page.

![Figure 4.7: Channel Orientation](image)
\[ D_h = \frac{4ab}{2(a + b)} \]  
(4.3)

\[ \alpha = \frac{b}{a} \]  
(4.4)

\[ \varepsilon / D = \frac{\varepsilon_{fp}}{D_h} \]  
(4.5)

\[ \Delta T = T_{out} - T_{in} \]  
(4.6)

Next, the constricted parameter can be calculated by Equation (4.7). This parameter can then be used to calculate the constricted hydraulic diameter and aspect ratio by replacing \( b \) with \( b_{cf} \).

\[ b_{cf} = b - 2\varepsilon_{fp} \]  
(4.7)

Fluid temperatures are then used to find the mean fluid temperature to be used in the interpolation of the Prandtl number, viscosity, and thermal conductivity. The total temperature change across the fluid is used to calculate the power input with Equation (4.8).

\[ Q = \dot{m}C_p\Delta T \]  
(4.8)

Reynolds number is now calculated followed by the developing length for a given experimental data set with Equations (4.9) and (4.10) respectively. The developing length can
be used to identify which wall thermocouples are in the fully developed region.

\[ Re = \frac{4\dot{m}l}{2\mu(a + b)} \]  
\[ x_{fd} = 0.05RePrD_h \] (4.9) (4.10)

The last parameters that need to be defined in this stage of the data processing are the two different heat flux calculations. Heat flux is calculated using the projected area (size of the heater), and using an estimate of the actual area including roughness. The actual area is estimated by the arclength of one roughness element over the pitch as found in the curve fitting in Section 4.1.

\[ q_P = \frac{Q}{A_P} \] (4.11) 
\[ q_{actual} = \frac{Q}{A_{actual}} \] (4.12)

At this point the partially processed data can be combined in a summary data sheet along with the average wall temperature at each of the 11 locations. Internal wall temperature, which is in contact with the fluid, can be estimated by a simple wall conduction analysis as seen in Equation (4.14), where all temperatures are in Kelvin [K].

\[ \Delta T = \frac{q}{k_s\Delta x} \] (4.13) 
\[ T_w = T_i - \Delta T = T_i - \frac{q}{16.2(0.00196)} \] (4.14)
The fluid temperature at any location can be estimated assuming a linear axial increase along the heated length (due to the constant heat flux boundary condition).

\[ T_f = \frac{T_{out} - T_{in}}{L_H} x + T_{in} \]  \hspace{1cm} (4.15)

The last two temperature can then be used to find the difference in mean fluid temperature and the internal wall temperature at any thermocouple location. The local heat transfer coefficient can now be defined by Equation (4.16) and calculated for all fully developed locations from a given test set.

\[ \bar{h} = \frac{q}{T_w - T_f} \]  \hspace{1cm} (4.16)

It is important to note that both the wall temperature, \( T_w \), and the local heat transfer coefficient can be calculated using the actual and projected areas. The actual area will result in the true heat transfer coefficient. This aspect is discussed in the Results sections.

Lastly, the local fully developed Nusselt number may be calculated, followed by the average fully developed Nusselt number. The average is found for only the thermocouples in the fully developed region.

\[ Nu_L = \frac{\bar{h} D_h}{k_f} \]  \hspace{1cm} (4.17)

\[ Nu = \frac{1}{11 - i_{fd}} \sum_{i=i_{fd}}^{11} Nu_{L,i} \]  \hspace{1cm} (4.18)
Uncertainty information is also included in the final processed data sheet in order to calculate individual uncertainty values.
Chapter 5

Uncertainty

Uncertainty is manifested in the two forms of bias and precision error. Bias error is the intrinsic error found in the experimental setup and its associated devices. Precision error is the random error naturally introduced in all things. The total uncertainty is given by the combination of the two as shown in Equation (5.1).

\[ U_{T,i} = 2 \sqrt{\left( \frac{Bias}{2} \right)^2 + \left( \frac{\sigma}{\sqrt{N}} \right)^2} \]  

(5.1)

If there were no random error, then this expression would reduce to the bias error.

5.1 Bias Error

Bias error, or systematic error, is error intrinsically encountered due to the experimental design and assembly. Uncertainty caused by bias error in calculated parameters can be arrived at through a simple error propagation.
5.1.1 Propagation Technique

Friction factor, $f$, will be used as an example of this propagation.

\[ f = \frac{\Delta P \rho D_h A^2}{x \dot{m}^2} \]  \hfill (5.2)

To find the error, the propagation of errors in friction factor, $\delta f$, is found by the expansion of errors for each individual parameter via a partial derivative expansion:

\[ \delta f = \frac{\partial f}{\partial \Delta P} \delta \Delta P + \frac{\partial f}{\partial x} \delta x + \frac{\partial f}{\partial D_h} \delta D_h + \frac{\partial f}{\partial A} \delta A + \frac{\partial f}{\partial \dot{m}} \delta \dot{m} \]  \hfill (5.3)

Now, we let the uncertainty in a parameter, $x_i$, be defined as shown in Equation (5.4).

\[ U_{x_i} = \frac{\delta x_i}{x_i} \]  \hfill (5.4)

The error propagation, Equation (5.3), is now divided through by $f$ and rearranged into uncertainties.

\[ \frac{\delta f}{f} = \frac{\Delta P}{f} \frac{\partial f}{\partial \Delta P} \delta \Delta P + \frac{x \delta f}{f \delta x} + \frac{D_h \partial f}{f \partial D_h} \delta D_h + \frac{A \partial f}{f \partial A} \delta A + \frac{\dot{m} \partial f}{f \partial \dot{m}} \delta \dot{m} \]  \hfill (5.5)

Which can be rewritten using the form presented in Equation (5.4):

\[ \frac{\delta f}{f} = \frac{\Delta P}{f} U_{\Delta P} + \frac{x}{f} U_x + \frac{D_h}{f} U_{D_h} + \frac{A}{f} U_A + \frac{\dot{m}}{f} U_{\dot{m}} \]  \hfill (5.6)
The magnitude of the error is found via the standard $L_2$ norm, or the euclidean distance:

$$U_f = \pm \left[ \left( \frac{\Delta P}{f} \frac{\partial f}{\partial \Delta P} U_{\Delta P} \right)^2 + \left( \frac{x}{f} \frac{\partial f}{\partial x} U_x \right)^2 \right.$$

$$\left. + \left( \frac{D_h}{f} \frac{\partial f}{\partial D_h} U_{D_h} \right)^2 + \left( \frac{A}{f} \frac{\partial f}{\partial A} U_A \right)^2 + \left( \frac{\dot{m}}{f} \frac{\partial f}{\partial \dot{m}} U_{\dot{m}} \right)^2 \right]^{\frac{1}{2}} \tag{5.7}$$

Which reduces to the following:

$$U_f = \pm \left[ U_{\Delta P}^2 + U_x^2 + U_{D_h}^2 + 4U_A^2 + 4U_G^2 \right]^{\frac{1}{2}} \tag{5.8}$$

Where:

$$U_{D_h} = \pm \left[ \left( \frac{b}{a+b} U_a \right)^2 + \left( \frac{a}{a+b} U_b \right)^2 \right]^{\frac{1}{2}} \tag{5.9}$$

$$U_A = \pm \left[ U_a^2 + U_b^2 \right]^{\frac{1}{2}} \tag{5.10}$$

Utilizing the same approach, the bias error in Nusselt number can be found to be:

$$U_{Nu} = \pm \left[ 2U_{\dot{m}}^2 + \frac{2(\delta T_{out}^2) + (\delta T_{in}^2)}{(T_{out} - T_{in})^2} + 2U_a^2 + 2U_t^2 + U_{\Delta x}^2 + U_T^2 + U_{T_{in}}^2 \right]^{\frac{1}{2}} \tag{5.11}$$

### 5.2 Experimental Uncertainties

In order to calculate the total uncertainty, the uncertainty of individual devices needs to be addressed. Calibration was performed on every sensor and the worst case error recorded.
Error data is collected for 30 points for each sensor in order to obtain a conservative value for error. Using this method, pressure transducers were found to have error ranging from 1% to 5%, where the 5% error was seen at low flow rates and large separations where pressure drop near the limit of the pressure sensor’s range. Near this limit, the pressure sensor’s sensitivity becomes the dominate form of error, and so is used to arrive at the worst case of 5%. The error in the flow meters was found to be 1.8% at full scale and thermocouples to be accurate within 0.1°C.

Precision error, or random error, was found using the statistical standard error of $\sigma/\sqrt{N}$. Since the experiments were allowed to reach steady state and a large number of samples recorded, the precision error was small. In pressure the precision error was found to be 0.5%. The precision error in the thermocouples was at worst 0.2%, and the precision error in flow rate was found to be 0.3% in the worst case.

When total uncertainty is calculated, Nusselt number was found to have error ranging from 6.7% to 6.9%. The slightly more conservative value of 7% error is used for Nusselt number. The error in Nusselt number varied only slightly due to the limited range of operation. Error in friction factor was found to range from 4.5% to 10.2%, with the worst error at low flow rates.
Chapter 6

Experimental Results

6.1 Smooth Results

Extensive tests were performed using the smooth test section set with a heated length of 94.6 mm and constant height of 12.7 mm. In total, six unique separations having hydraulic diameters ranging from $D_h = 183 \, \mu m$ to $D_h = 1698 \, \mu m$ were tested over a Reynolds number range of approximately 45 to 600. Equations (6.1) and (6.2) show the calculation used for $D_h$ and $Re$. Average fully developed Nusselt number was calculated for each data set. In order to compare individual test results with one another, a normalized Nusselt number, $Nu^*$, is defined for each set of test conditions. Equation (6.3) is used to calculate $Nu^*$ for all experimental cases. The use of this parameter allows for a direct comparison of test results, where a value of 1 represents exact agreement with theory, a value greater than 1 represents a value greater than theory, and a value less than 1 is a result which is less than theory predicts. The theoretical Nusselt number is calculated as a function of aspect ratio ($\alpha = b/a$) based on a linear interpolation of the theoretical flat plate limit ($\alpha = 0$, $Nu = 8.235$) and the results of Dharaiya et al.[32] ($\alpha = 0.1$, $Nu = 6.803$) as shown in Equation (6.4). Figure 6.1 shows the channel orientation with heat flux applied at the two long walls and the short walls being insulated.
Figure 6.1: Channel Orientation

\[ D_h = \frac{4ab}{2(a+b)} \]  
\[ Re = \frac{4\dot{m}}{2\mu(a+b)} \]  
\[ Nu^* = \frac{Nu_{exp}}{Nu_{th}} \]  
\[ Nu_{th}(\alpha) = 6.803 + (\alpha - 0.1) \left( \frac{8.235 - 6.803}{0 - 0.1} \right) \]

The experimental results for \( Nu^* \) are plotted over the entire Reynolds number range tested for the six separations in Figure 6.2.

Upon investigation there appears to be individual trends for each separation as shown in Figure 6.2. Since \( Nu_{th} \) is a strong function of the aspect ratio, it is expected that the results would vary in this fashion. In order to fully compare the individual experimental results, another plot is generated of \( Nu^* \) as a function of \( \alpha Re \) in Figure 6.3. This allows a scaling of the Reynolds number for geometry that places each test in proper proportion with all others, effectively normalizing the entire system.

Figure 6.3 shows that the experimental results all fall below theory and increase as the value of \( \alpha Re \) increases. As either Reynolds number or aspect ratio increases, the results are closer to the theoretical predictions and tend toward a constant value. The summation of the smooth channel results can be found at the end of the chapter. When available,
Figure 6.2: Smooth wall results as a function of $Re$
Figure 6.3: Smooth wall results as a function of $\alpha Re$
friction factor results are also presented in the summary table. Friction factor results are in agreement with theory, but large values of error are present due to the low flow rates and pressure drop in the experiments. Pressure drops within the channel were near the lower limit for the differential pressure sensor, and therefore caused the large errors.

### 6.2 Roughness Results

Four different sinusoidal structured roughness geometries were extensively tested for at least three separations ranging from 92 \( \mu m \) to 680\( \mu m \) as physical geometry would allow. Hydraulic diameters ranging from \( D_h = 205 \ \mu m \) to \( D_h = 1275 \ \mu m \) were tested over a Reynold's number range of approximately 55 to 300. Sinusoidal geometries with pitch to height ratios of 2.6, 4.8, 7.8, and 10.6 created relative roughness ranging from 2.17% to 16.53%. Experimental results are once again normalized using Equation (6.3) in order to ensure proper comparison. The aspect ratio is defined as the ratio of the root separation over the height of the channel.

As an example of roughness performance, experimental results for the pitch to height ratio of 4.8 are presented in Figure 6.4 over the Reynolds number range tested. The results from each of the four separations tested are shown in the form of \( Nu^* \). A value above or below one represents results greater or less than smooth theory predicts for the root separation. A value of one would show exact agreement with smooth theory using the root separation or \( b \). The entire set of the roughness channel results can be found at the end of the chapter. When available, friction factor results are also reported in the summary tables. Friction factor results show an increase over the theoretical friction factor based on root separation, but large values of error are present due to the low flow rates and pressure drop in the experiments. Pressure drops within the channel were near the lower limit for the differential pressure sensor, and therefore caused the large errors.
Figure 6.4: $Nu^*$ vs $Re$ for $\lambda/h = 4.8$ Sinusoidal Roughness
Once again, the results are in a stacked fashion. In order to appropriately assess the results, $Nu^*$ is recast in Figure 6.5 over the scaled Reynolds number based on aspect ratio ($\alpha Re$).

Just as in the smooth case, by plotting the results over $\alpha Re$, we once again see the results collapse to a single trend. This actuality shows the validity of the scale analysis performed in Section 2.2. It is important to note that the actual area was used in calculating the experimental results in this section. If one were to not incorporate the added area of the structured roughness, the resulting values would over-predict the actual values. Figure 6.6 shows the shift in experimental values when the two areas are used. Error bars have not been included for easy of viewing.

The results from each of the geometries tested exhibits the property of collapsing to a single trend when the results are plotted as the normalized Nusselt number over the Reynold’s number scaled by aspect ratio. This property also allows for the different geometries to be authentically compared on a single plot. Figure 6.7 shows each of the four sinusoidal geometries results in a nondiscriminatory way.

Figure 6.7 shows that each of the four geometries act in a similar fashion in terms of the scaled Reynolds number. The range of the results is small and overlaps considerably. The data shows as vertical spread implying the heat transfer is a function of another parameter. In order to investigate this additional functionality the results are plotted as a function of relative roughness. Figure 6.8 shows the normalized Nusselt number plotted for all geometries as a function of the relative roughness, $\varepsilon/D$.

The vertical spread in the results plotted in Figure 6.8 can be attributed to the role of Reynolds number in the experimental results.
Figure 6.5: $Nu^*$ vs $\alpha Re$ for $\lambda/h = 4.8$ Sinusoidal Roughness
Figure 6.6: $\textit{Nu}^*$ vs $\alpha Re$ for $\lambda/h = 4.8$ Sinusoidal Roughness, $A_{\text{actual}}$ Compared to $A_P$
Figure 6.7: \( Nu^* \) vs \( \alpha Re \) for All Sinusoidal Roughness
Figure 6.8: $Nu^*$ over $\varepsilon/D$
6.3 Entrance Region Data

One of the goals of the experimental parameters tested in this work was to minimize the number of thermocouples in the thermal developing region. However, some entrance region data is available. In the entrance region, the heat transfer coefficient it expected to be larger than the fully developed region do to thickness of the boundary layer being small. This trend was found to be valid in within most of the experimental results. An example of this is shown in Figure 6.9. In some of the experimental cases, however, there appear to be end affects near the channel exit. This phenomena can be seen in Figure 6.10.

Figure 6.9: Local Nusselt Number (Smooth, $D_h = 1698 \mu m$, Re = 105)
6.4 Discussion of Results

Four sets of structured roughness in addition to smooth walls were tested for their heat transfer characteristics. The summation of all the results can be seen in Figure 6.13. All of the results are over-predicted by theory with a similar trend that increases toward a constant value as the scaled Reynolds number increases. This trend implies that as the channel separation increases or Reynolds number increases, the heat transfer coefficient measured by the setup begins to approach theory. For large separations the average fully developed Nusselt number agrees with theory. The results show that for the conditions tested, sinusoidal structured roughness does not increase heat transfer. The Nusselt number was found to decrease with increasing relative roughness as shown in Figure 6.8.
6.4.1 Experimental Studies in Agreement with Experimental results

The experimental trends from this work compare well to the work of Hegab et al.[16] where theory over-predicted the Nusselt number. The similar trend where as channel dimensions decrease, Nusselt number decrease was noticed in both works. The results from this study also compare well to Qu et al.[19] who used trapezoidal channels and found that conventional correlations over-predicted Nusselt number. The work of Hetsroni et al.[34] compares most directly out of all literature available. The work investigated heat transfer in a 1.07 mm tube with Reynolds number ranging from 10 to 450. The same trend for Nusselt number of water flow was found as in this study. The results from Hetsroni et al. have been reproduced in Figure 6.12. The work of Wu and Cheng [2] also shows the same trend, their results for Nusselt number as a function of Reynolds number can be seen in the reproduction found in Figure 1.1 in the literature review. Lastly, the work of Qu et al.[19] experimentally showed the same trend for microtubes. The reproduction of their work can be seen in Figure ?? where experimental Nusselt number is plotted as a function of Reynolds number.

6.4.2 Experimental Studies in Disagreement with Experimental results

There are a number of experimental works with which the results of this study do not agree. Several studies utilizing ribbed roughness [7, 9, 12] showed that rib roughened rectangular channels had enhanced heat transfer over smooth ducts. These works, however, focused on transition and turbulent flows, and so a direct quantitative comparison is not appropriate. Wu and Cheng [2] reported that heat transfer increase with increasing relative roughness. Bucci et al.[8] found Nusselt number to be slightly under-predicted by theory for capillary tubes. Finally, Yang and Lin [18] found experimental Nusselt number to agree with theory.
for the constant heat flux boundary condition.

There are several plausible reasons for the difference in results. Aspect ratio plays a strong role in Nusselt number. In this study, considerably small aspect ratios of 0.007 to 0.07 were tested compared to most channels which only reach an aspect ratio of 0.1 [17]. The results found for the smooth channel with the largest aspect ratio and hydraulic diameter ($D_h = 1698 \mu m$), were found to fall only slightly below theory. Also, many rectangular channels tested were macroscale channels [7, 9, 12] on the order of millimeters rather than microns. These differences are probably significant considering the trend of decreasing Nusselt number with smaller aspect ratio.
6.4.3 Fluid Heat Loss Issue

The experimental setup used in this setup, though it does mitigate some experimental issues, has intrinsic characteristics that may cause the results to deviate from theory. One additional source of error is in the measurement of fluid temperature. The jacketed thermocouples used in this study are at a considerable distance (100 mm) from the heated length of the channel. This does not cause an issue for the inlet temperature which is near room temperature, but is a significant issue for the outlet temperature. Any heat loss from the channel exit to this thermocouple will result in an under-prediction of total heat flux. However, the maximum possible loss can be calculated. Using the total possible power input from the power supply, the deviation in measured power was less than 10% of the maximum input power. Some of this loss will be attributed to heat loss of the system as previously investigated, and some will be due to heater resistance fluctuations with temperature.
6.4.4 All Results Summary

The processed results from each individual test geometry and test are plotted in Figure 6.13 and presented in tabular form on the following pages. When available, friction factor is reported for each set of test conditions. It is important to note the large error associated with the friction factor calculation due to the low flow rates tested (leading to pressure drops near the limit of the differential pressure sensors used). In addition, local Nusselt number and entrance region data is also tabulated for each test. The tabular form presents relevant geometry and test conditions for completeness to uniquely identify each set of test data.
Figure 6.13: $Nu^*$ for all $\alpha Re$
<table>
<thead>
<tr>
<th>$f$</th>
<th>$g$</th>
<th>$f/g$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Re</td>
<td>F</td>
<td>Nu</td>
</tr>
<tr>
<td>Rd</td>
<td>Fr</td>
<td>Pr</td>
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<tr>
<td>Sc</td>
<td>Ti</td>
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**Notes:**
- $Re$ = Reynolds number
- $F$ = Froude number
- $Nu$ = Nusselt number
- $Rd$ = Reynolds number
- $Fr$ = Froude number
- $Pr$ = Prandtl number
- $Sc$ = Schmidt number
- $Ti$ = Taylor number
- $Wi$ = Womersley number
- $Pe$ = Péclet number
- $We$ = Weber number
- $Ma$ = Mach number
- $Ca$ = Courant number
- $Ta$ = Taylor number

**References:**
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- Jane Smith
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- Nature Communications

**Year:**
- 2022
- 2023
- 2024
- 2025
- 2026
- 2027
- 2028
- 2029
- 2030
- 2031

**Conference:**
- International Conference on Fluid Dynamics
- Annual Meeting of the American Physical Society
- International Conference on Heat Transfer

**Location:**
- Boston, USA
- New York, USA
- London, UK
- Tokyo, Japan

**Keywords:**
- Numerical Analysis
- Experimental Methods
- Fluid Dynamics
- Heat Transfer
- Turbulence
- Fluid Mechanics
- Heat Exchangers
- Thermal Physics
- Transport Phenomena
- Experimental Techniques
- Fluid Flow
- Heat Transfer
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Chapter 7

Conclusions

7.1 Experimental Conclusions

The heat transfer performance of small aspect ratio rectangular channels with two wall heating under the H2 boundary condition was investigated. A constant wall heat flux was applied at opposing long walls. Four different structured roughness geometries were investigated along with smooth channels as the heated walls. In total, hydraulic diameters ranged from $D_h = 183 \, \mu m$ to $D_h = 1698 \, \mu m$ and were tested over a Reynolds number range of 45 to 600. The pitch to height ratio of the sinusoidal roughness surfaces ranged from 2.593 to 10.635. These conditions created relative roughness ranging from 2.17% to 16.53%. Average fully developed Nusselt was calculated for each unique set of test conditions. The results were analyzed and strong trends within the data found.

1. All smooth and roughness results were self-consistent.

2. Average fully developed Nusselt number was found to be significantly lower than theory for small aspect ratio and/or low Reynolds number.

3. The use of normalized Nusselt number, $Nu^*$, and scaled Reynolds number, $\alpha Re$, was found to provide a means of directly comparing results from different geometries in
an analogous way.

4. A strong relationship between Nusselt number and both geometry and Reynolds number was recognized. As Reynolds number increased, the resulting Nusselt number increased approached a constant. As aspect ratio increased (larger channel separation), the Nusselt number showed a trend toward theory.

5. Nusselt number was observed to be lower in the presence of larger relative roughness.

6. There appears to be no significant heat transfer enhancement due to the roughness structures tested in the experimental range over their smooth wall counterpart.

7.1.1 Summary of Experimental Results

The experimental results of this work were found to be self consistent, but deviated from theoretical expectations significantly. The consistency of the results show that the experimental procedure, data collection method, and interpretation are accurate and reasonable. What is not addressed, however, is any intrinsic flaws within the system design and analysis. Two potential flaws have been identified.

The first potential issue is the use of hydraulic diameter as the characteristic length scale of the channel cross-section. Hydraulic diameter is determined by modeling a non-circular geometry as a circular one. At its root, hydraulic diameter is the circle that is inscribed within the wetted perimeter. In the case of a square, the hydraulic diameter is simply the side length of the square (a simply inscribed circle). For small aspect ratio channels, such as this work, it may be more appropriate to model the system as flat plates rather than a circular geometry. If one were to look at the ratio of the area of the inscribed circle in the limiting case of aspect ratio approaching zero to the actual area of the rectangular channel, they would find the ratio to be small. This implies that the hydraulic radius is
under-representing the area of the channel. This ratio can be seen below.

\[ A = ab \]
\[ A_{Dh} = \frac{\pi}{4} D_h^2 \]
\[ D_h = \frac{4ab}{2a + 2b} = \frac{2b}{1 + \frac{b}{a}} \]
\[ \alpha = \frac{b}{a} \to 0 \]
\[ D_h \to 2b \]
\[ A_{Dh} = \frac{\pi}{4} (2b)^2 \]
\[ \frac{A_{Dh}}{A} = \frac{\pi b^2}{ab} = \pi \frac{b}{a} \to 0 \]

The second potential issue is the \textit{a priori} analysis of axial conduction. By using Reynolds number based on hydraulic diameter, the average velocity within the channel may be over-predicted. If this is so, the assumption that axial conduction is negligible is invalid. This idea is supported by how well the scaled Reynolds number \( \alpha Re \) better represents the results than plain Reynolds number. However, this issue is not limited to the use of hydraulic diameter. Any misinterpretation leading to an invalid negligible axial conduction assumption could explain the behavior of the system.

### 7.2 Theoretical Conclusions

The physics problem of wavy walled channels was investigated using fundamental relationships. Both complete solutions and scaled analyses were utilized in order to build an understanding of the problem at hand. Boundary conditions were taken to mimic the experimental setup as closely as possible, while maintaining as generic of a solution as allowed
by the physics and mathematics (mainly in the form of a generalized wall function).

1. In the limiting case of \( \alpha = 0 \), or parallel flat plates, the fully developed Nusselt number is shown to approach the classical theory result of 8.235 for the constant heat flux boundary condition.

2. A one dimensional approach to wavy walls with a modified velocity profile from lubrication theory does not return consistent results for most wall functions. However, this approach will work with the introduction of the constricted parameter (or any constant separation approximation).

3. A scale analysis shows that Reynolds number scaled by aspect ratio is a dominant term in the flow physics of wavy walls. This is confirmed by the experimental results.
Chapter 8

Recommendations

8.1 Experimental

The experimental results in the performed study were self consistent within reasonable error limits. The results, however, require expanded experimental range before a definitive understanding of the effect of roughness, in general, can be obtained. The controllable factors of interest are as follows:

1. Aspect Ratio: The aspect ratios tested in the current study are considerably smaller than most available in literature. Expanding the tests to larger aspect ratios will lend more insight into the test setup designed for this study.

2. Roughness Geometry: Both different types of structured roughness and testing on homogeneous wall roughness is imperative to a complete understanding of the transport phenomena in microscale channels.

3. Reynolds number: Pushing the limits of the fully developed region within the setup will allow for a slightly higher Reynolds number range. Higher velocities in the wall region may encourage the development of secondary flow to increase the heat transfer characteristics. Increasing Reynolds number will also help to mitigate the
issue of axial conduction even further.

4. Heat Flux: Increased heat flux will provide larger temperature differences between the wall and mean fluid, creating less of a dependence on accurately predicting internal wall temperature.
References


Appendices

Items included:

1. Test Piece Drawing

2. Gauge Block Drawing
NOTE: Overall dimension of 12.7 mm and critical dimension of 4 mm are referenced to the flat surface. The roughness profiles and the blank profiles are additional material on these surfaces. The geometry has been omitted for simplicity. See Lambda-H drawings for correct geometry.

DIMENSIONS ARE IN MM
TOLERANCES:
ONE PLACE DECIMAL 0.8
TWO PLACE DECIMAL 0.13
ANGLES 1DEG

MATERIAL
303 Stainless

FINISH
8 OR BETTER UNLESS NOTED

THE INFORMATION CONTAINED IN THIS DRAWING IS THE SOLE PROPERTY OF <INSERT COMPANY NAME HERE>. ANY REPRODUCTION IN PART OR AS A WHOLE WITHOUT THE WRITTEN PERMISSION OF <INSERT COMPANY NAME HERE> IS PROHIBITED.

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UNLESS OTHERWISE SPECIFIED:
1.0

DO NOT DEBURR KEEP ALL EDGES SHARP.
Max permissible radius for inside corners is .13mm
Not required:
.636mm relief on backside
• .508mm holes x11

4/14/09 LAPOLT B.

303 Stainless