Attitude Control for Circumnavigating the Sun with Diffractive Solar Sails

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Attitude Control for Circumnavigating the Sun with Diffractive Solar Sails

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ABSTRACT

A solar sail making use of the physics of diffracted light enables the transfer of optical to mechanical momentum for in-space propulsion. In this thesis we describe advantages of diffractive solar sailing for trajectory and attitude control. In particular, a high inclination angle heliophysics mission is examined. A simple roll maneuver of a diffractive sail is described to attain an inclination angle of 60º. A comparison of idealized diffractive and reflective sails for a five-year solar polar orbiter mission, showing higher inclination angles and a smaller orbital radius for the former is performed. As a result, a constellation of diffractive solar sails for heliophysics imaging and data gathering can be envisioned. A series of 14 [kg], 400 [m²] lightsails at various inclination angles could be in place at 0.32 [AU] within six years of launch. Based on our survey of current solar sailing and attitude control systems, the feasibility of performing these maneuvers and the advantages diffractive elements can enable are explored. A theoretical model of the sailcraft is derived and various attitude control systems are numerically modeled. This analysis includes classical control devices such as reaction wheels and novel approaches with electro-optically controlled devices. It is concluded that while a fully electro-optic system is sufficient in the long term, a hybrid system of both small reaction wheels and electrically controlled diffractive elements provides an advantageous solution and could be expanded for other solar sailing applications in the near future.
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NONMENCLATURE

radius [m or AU]
longitude [rad or deg]
latitude ddx[rad or deg]

roll angle [rad or deg]

astronomical unit

radius of Earth to the Sun [m or AU]
velocity of the Earth [AU/year or m/sec]

Newton’s gravitational constant

unit vector normal to sail

angle between and [rad or deg]

angle between the projection of on the plane and [rad or deg]

force of radiation pressure on the lightsail [rad or deg]
solar irradiance [kW/]
sail area []
momentum transfer efficiency vector

mass of sailcraft [kg]
mass to area ratio of the sailcraft [kg/]
lightness number

force coefficient in radial direction

force coefficient in direction

force coefficient in direction

grating momentum vector

vector in the plane of the sail

wavelength of light [m]
diffractive grating period [m]
diffractive angle [rad or degrees]

projection of r on the xy plane [m or AU]

angular velocity of the spacecraft [rad/sec]

angular acceleration of the spacecraft [rad/sec²]

angular momentum [Nms]

external torque [Nm]

inertia matrix of the sailcraft [Nm]

geometric matrix of reaction wheel system

moment of inertia around rotation axis

moment of inertia around rotation axis [kg m²]

radius of reaction wheel [m]

torque [Nm]

distance to center axis of the sail [m]

attitude quaternion

control gain parameter
Subscripts
E    Earth
R    Reflective sail
D    Diffractive sail
e    External
s    Spacecraft
tot  Total
c    Control
e    Error
d    Desired

Other
TRL  Technology Readiness Level
ADCS Attitude Determination and Control System
NASA National Aeronautics and Space Administration
JAXA Japan Aerospace Exploration Agency
RCD  Reflectivity Control Device
NIAC NASA Innovative Advanced Concepts
IKAROS Interplanetary Kite-craft Accelerated by Radiation Of the Sun
AU   Astronomical Unit
U    CubeSat Unit (10 x 10 x 10 cm³)
1.0 PROBLEM INTRODUCTION

Solar sails provide unique advantages over traditional propulsion devices, affording mission opportunities that would otherwise be too impractical owing to cost and complexity. A sail based on the physics of diffracted light provides the potential to be more efficient than the typical reflective sails. Although solar light sails were first proposed in the early 20th century [1,2], in-space demonstration missions have become possible only within the last decade. The dream of "star-sailing" from the late 1970's is finally becoming a reality [3,4]. The draw of using solar sails as a propulsion system is the use of an almost infinite, mass-less propellant. Solar radiation pressure is the main source of thrust for the spacecraft. It may be a relatively small force, even in the space environment, but with large, lightweight areas, this force can be used effectively. Meanwhile, there has been a push for cheaper, faster, solar system missions. A lighter spacecraft tends to yield a lower cost. For solar sailing missions, huge mass savings are gained because there is no additional mass from carrying fuel. The concepts behind solar sailing are not totally novel, but only recently have we been able to start to overcome the technical challenges and efficiency problems of this technology. The first successful use of a solar sail was IKAROS (Interplanetary Kite-craft Accelerated by Radiation Of the Sun), a test mission by the Japanese space agency, JAXA, in 2010 [5]. The spacecraft did not use the sail as a main form of propulsion but showed that it was contributing to the acceleration of the spacecraft. Where IKAROS used a spinning sail to achieve deployment and stability, Nano-Sail D2 launched later that year, used a different deployment mechanism. NASA's goal for Nano-Sail D2 was to demonstrate the deployment of a solar sail without the spinning method. This was completed after some initial launch issues in 2011 [6]. The dynamics and deployment of reflective sails are still being perfected, but are now becoming more reliable with higher TRL’s. With the advancement of meta-materials, NASA has invested in research on using a new type of solar sail. As outlined by the NASA Innovative Advanced Concepts Phase I awarded to Dr. Swartzlander at RIT in 2018, and the Phase II in 2019, these sails use diffractive gratings and the principle law of diffraction instead of the law of reflection [7]. The properties of the diffractive sail may be able to alleviate some of the technological challenges that these sailcraft encounter. Difficult deployment, large complex mechanical steering systems, and theoretical efficiency limits are all obstacles that diffractive sails could overcome. The results may lead to a more efficient type of solar sail, capable of enabling solar polar orbiter missions and constellations. This is not as feasible with common reflective solar sails or typical propulsion methods. The proposed thesis work explores a feasible mission design for a solar polar orbiter using a diffractive solar sail and the attitude control system that would be needed to accomplish the desired trajectory.
2.0 LITERATURE REVIEW

Solar sailing as a form of propulsion for space missions is becoming a favorable alternative with the recent advances in the field. The growth in this technology may stem from the heliophysics opportunities it affords. The successes of IKAROS, Nano-Sail D2 and now LightSail 2 give a hopeful outlook on solar sailing’s applications but there are still challenges as of yet: valid ground testing, reliable deployment methods, sail material properties, and more specifically attitude control of these large structures. Attitude determination and control systems in satellites is a thoroughly researched area, but needs to be applied in very mission specific ways to accomplish its function. Typical satellite systems use momentum wheels or magnetorquers. Solar sailing missions specifically can use a sliding mass table to change attitude of the sailcraft. IKAROS tested the first use of RCD’s (reflectivity control devices) which create a differential solar radiation pressure on the sail to cause a torque. A new concept of diffractive solar sailing can be applied to achieve better results than reflective sails.

2.1 Link Between Heliophysics and Solar Sailing

While information on the heliophysics missions of past, present, and future are well documented, there are only a handful of relevant missions to survey. This contradicts the growing trend of the scientific community who are calling for better solar science. In a 2015 survey of all the prominent major heliophysics missions, there are under thirty listed starting from as early as 1977 [8]. Even so, the survey has critical information on each mission though paling to the amount of Earth science and planetary missions. The author includes proposed missions which exaggerates the actual number of available missions. On top of that, only a handful of the missions surveyed reach beyond Earth orbit. Confusion can also arise from missions changing their name after development begins. In this survey, the recently launched Parker Solar Probe is referred to by its old name: Solar Probe Plus [9]. Furthermore many proposed missions never come to fruition due to lack of interest, feasibility, and funding. This was the case of the Solnechhy Parus Mission proposed by the Russian space program in 2014 [10]. It was slated to use a solar sail to perform inclination cranking of a heliocentric orbit. The Ulysses spacecraft is currently the only mission to have focused on the solar poles, ending in 2009 at 1.3AU [33]. While Ulysses did not hold any imagers, the ESA has stitched together the only image of solar north pole using data from Proba-2 satellite observatory [34]. This will change in the near future, as the recently launched joint NASA-ESA mission will reach a 24 degree inclination in 7 years [35].
The interest in the use of solar sailing by the aerospace community for heliophysics is prominent in the current literature. The 2013 Decadal Survey for Solar and Space Physics [11] advocated for advanced solar sail technology to enable heliophysics missions. This survey of the scientific community listed solar sailing as one of the key technologies needed to advance the exploration of solar sciences. The emphasis on heliophysics missions and the call for the necessity of them may be biased due to the pointed nature of the report, but is cited by journals as a justification for advancing solar sailing technology. In an article analyzing the current state of NASA’s solar sailing abilities, the decadal survey is listed as a source of NASA’s growing interest in the technology. At the time of the aforementioned study, it is estimated that solar sailing technology is at a TRL (Technology Readiness Level) of 6 [12]. In terms of NASA’s risk management this means the technology is acceptable to use in demonstration missions but not in reliable functions. The key to developing the technology further past the proposal phase is to tie solar sailing to enabling a popular science goal. Thus, describing the link between solar sails and heliophysics besides the obvious subject matter relation. Solar sails are contributed to allowing complex trajectories where new types of solar data could be collected. These include polar orbits, artificial Lagrange point orbits, halo orbits, and pole sitting orbits. While solar sailing allows deeper exploration into solar science, the need for a deeper understanding behind our Sun affords opportunity for solar sailing advancement.
2.2 Successful Solar Sailing Missions

While many solar sailing missions are only in the proposal stage, there have been a handful of successfully deployed sails. The first successful solar sailing deployment was IKAROS in 2010. IKAROS was a demonstrator precursor mission to a Jupiter and Trojan asteroid mission. The spacecraft itself was planned to do a flyby of Venus as a demonstration of a small solar sail combined with a main source of ion propulsion. The minimum success of the mission was defined by deployment of the solar sail. The demonstration of attitude control from reflectivity control devices (RCD) and power generation from flexible solar arrays (FSA) may be applicable to potential electronics for a diffraction sail application [5]. Unlike its successors, the deployment of the sail depends on centripetal force from spinning the spacecraft. The deployment took several weeks, while the other demonstration goals happened over the span of half a year [13]. Spinning the sail provided the ability to eliminate the weight of a support structure. It also provided resistance to external disturbance torques due to sail deformation, during attitude adjustments. The benefits came at the cost of a more complicated attitude control thus contributing to NanoSail D2 and Lightsail 2 avoiding spinning their spacecraft. The entire sail was 14 kg, with four 2 kg tip masses. The entire mass of the spacecraft was 310 kg [5]. much more sizable than the solar sailing missions thereafter. Even though IKAROS was not very reliant on its solar sail propulsion, the mission proved to be a success for hybrid solar sail propulsion. In 2010 IKAROS completed its Venus fly by with 100 m/s delta V achieved within 6 months just from solar radiation pressure. The standard opinion is that IKAROS concluded solar sails are successful at allowing spacecrafts to save fuel and generate extra power. Although even with this monumental feat, JAXA recommended solar sailing only for use in a hybrid system.

Fig. (2.2) Picture of IKAROS with solar sail fully extended taken by the deployable camera (DCAM) days after the deployment of the solar sail [36].
NASA came to a different conclusion with the success of NanoSail D2. The mission launched two years after the failed launch of NanoSail D in 2008 [6]. In 2011 the mission demonstrated a successful non-spinning sail deployment on a small satellite scale. The 10 [m$^2$] sail itself was much smaller than the IKAROS sail. Also unlike IKAROS, the sail deployment demonstration was the primary objective. The secondary objective was to provide insight on atmospheric drag perturbations from the sail for deorbiting satellites. There were no other forms of propulsion due to the size of the spacecraft. NanoSail D2 contained a passive attitude control system with magnets because of the tight timeline after the loss of NanoSail D [6]. Although not nearly as complex or massive as IKAROS, NanoSail D2 was successful in paving the road for further solar sailing research and implementation on CubeSats.

Fig. (2.3) NanoSail D ground test of deployment. The sail 10 m$^2$ size compared to engineers conducting the test [37f].

Information on the Planetary Society’s LightSail 2 is easily accessible, though entire journal length studies outlining the successes of the mission will most likely not be completed until the mission itself has completed. The sailcraft successfully launched in June 2019, then successfully deployed after delays in July 2019. Since, there is evidence that the 5.6 meter by 5.6 meter sail changed the eccentricity of the 4 kg CubeSat in a 720 km orbit [14].
Even with outstanding successes, Spencer, Johnson & Long [12] report on the challenges these missions faced and describe the obstacles still present before solar sailing can be used as a flagship mission technology. They summarize and compared the sailcraft parameters of IKAROS, NanoSail D2, LightSail 2, InflateSail 2 and NEA Scout. Ground testing program challenges are largely expanded upon to suggest high fidelity testing would enable advancement in the flight obstacles. Sail areas need to increase 50-500 times relative to the largest deployed sail to date. Reliable, lightweight deployment structures contribute heavily to this challenge. Effects of the harsh space environment on the sail material properties are unknown, but ground testing suggests severe mechanical property degradation due to long periods of radiation exposure. Large sail areas afford large moment of inertias which can be difficult to maneuver by typical attitude control systems. The flexible nature of the sail adds to the attitude control challenge. Spencer, Johnson & Long suggest that solving these obstacles will enable solar polar orbiter and solar storm monitoring missions.

2.3 Attitude Control of Sailcraft

Attitude control of spacecraft is an in depth area of research, since it can be general or be very spacecraft specific. Typical studies suggest the use of reaction wheels to control the satellite orientation. These utilize the control of angular velocities of dense wheels to counteract or add angular momentum into the satellite body coordinate system. Ismail & Varatharajoo [15] compare different configurations of three wheels and then of four wheels to provide three axis
attitude control. Only three wheels are needed to perform effective control in three dimensions, but four wheels can be implemented to lessen the control effort or provide redundancy. The paper provides a detailed summary of all results though fails to clearly conclude the most effective configuration. It is mentioned that all configurations were successful in providing adequate attitude control to their defined parameters. A general control algorithm such as the one derived by Wisniewski & Kulczycki [16] can be applied to a configuration of four reaction wheels integrated with a star camera. Interest in a general derivation stems from the commercial availability and flight heritage of these components. Magnetorquers are implemented for low Earth orbit missions to offload some of the angular momentum build up in the reaction wheels and avoid saturation [17]. Their strength is highly reliant on the strength of the magnetic field of the body the spacecraft is orbiting. They are based on the principle of switching on and off electromagnets to interact with a present magnetic field. Magnetorquers are usually weaker than reaction wheels but provide an effective way to release momentum build up. Use of quaternions in control system models seems prominent in the field as all three different articles derive their systems with respect to quaternions, in order to linearize the system and avoid singularities. While reaction wheels can be implemented on any spacecraft, they require constant power and larger masses. Scholz, Romagnoli, Dachwald, & Theil [18] describe an optimized use for a sliding mass attitude control system on solar sails. This method relies on moving the center of mass of a spacecraft to offset it from the center of pressure provided on the sail. It does not provide attitude control in the roll axis and is a slower form of attitude control, though could provide mass savings on a sailcraft. They conclude that it is possible to provide pitch and yaw attitude adjustment at the same time to further minimize maneuver times. This type of system was later implemented in NEA Scout, a NASA solar sailing mission set to launch in 2020 [19].

The control system needs feedback from sensors to estimate its current attitude. Commonly, a spacecraft will utilize multiple types of sensors in conjunction to fully satisfy noise filtering requirements and accuracy requirements. [47] Much attention to the types of filtering to be performed, biasing, and the commercially available sensors is needed when designing an attitude determination system. Sun sensors are a staple instrument on a sailcraft, both for attitude and characterizing the thrust from the sail. They are low power and compact, so they can be mounted in various locations of the spacecraft. A star tracker may also be implemented to determine the spacecraft’s attitude in reference to external sources. These can be relatively large sensors for CubeSats, so usually one onboard will suffice. In concurrence with the sensors for external reference, gyroscopes are used. These devises measure the angular velocity of a spacecraft from an initial reference. MEMS (microelectromechanical system) gyros are commonly used on smaller spacecraft due to their compact size. This kind of sensor is vulnerable to drift bias that can introduce error into the measurement system [47]. These types of sensors, all used in conjunction are common on CubeSat sized missions.
2.4 Reflectivity Control Devices

IKAROS used diffusion RCD’s, where when the panels were turned on, light hitting the panel undergoes specular reflection much like the rest of the sail [20]. When the power was switched off, the light diffuses within the panel. These panels are made up by a liquid crystal layer sandwiched between two electrodes which can control the orientation of the crystal structure. The device panels change their admittance by 30%. These RCD’s lined the edges of the sail because they are used to provide a rotational torque. While half the RCD’s are turned on, the symmetrical counterparts are turned off, providing an uneven force about the center of mass thus causing the sail to rotate as depicted in Fig. (2.5). There were 72 devices, each 25 cm x 1 m x 70 microns thickness. The RCD experiment ran for 23 hours, tilting the sail less than 1 degree. This method of ACS was determined to be effective but was only a demonstration and not what the spacecraft relied on. The issue with literature on the specific IKAROS RCD’s is the lack of information. There are references to the results of the experiment performed while in transit to Venus [20], but no information on the makeup of the devices themselves. This may be due to the proprietary nature of the devices, yet they have not made a return to space since. Though a NASAFacts publication suggests that NASA Ames Research Center is trying to replicate the devices [21]. The publication is short, but outlines the basic concept of using a polymer-dispersed liquid crystal (PDLC), to change reflectivity of a surface with the application of a voltage.

![Diagram of Reflectivity Control Devices](image)

*Fig. (2.5) Visualization of the Reflectivity Control Devices (RCD’s) causing a rotational torque on the sailcraft. [39f].*
2.5 Diffractive Solar Sailing

Using diffractive grating material in place of reflective material on a solar sail is a very novel area of research. Dr. Grover Swartzlander at Rochester Institute of Technology was awarded a NIAC Phase II in 2019 to pursue this application following the award of a NIAC Phase I in 2018 [7]. Prior to 2017, little if no publications existed about this area of solar sailing. In 2017 Swartzlander describes the optical science of the momentum transfer from the diffractive grating. Most notably, the study shows that an ideal diffractive sail can be designed to provide a tangential force efficiency of unity, while the maximum efficiency for a reflective sail is 0.77. This is due to the change in the angle of the entire sail in relation to the light source, decreasing the effective area. Reflective sails are more effective in providing force in the radial direction of an orbit, as they are able to provide a force efficiency of 2. Useful force on a spacecraft is in the tangential direction of the orbit, therefore even with this weakness diffractive sails are theoretically advantageous. Swartzlander shows that this advantage leads to a 10% faster transfer time in an Earth-Mars trajectory [22]. Switchable optics are mentioned in the end of this study as a potentially beneficial property of diffractive solar sailing. The pitfalls of diffractive solar sailing are explored in a later publication on the optical efficiency of the gratings compared to reflective material [23]. While current reflective materials can use over 90% of the solar spectrum, the optimized diffractive grating described can only use up to 83%. It is concluded that “Photonic and meta-materials research is needed to develop the required diffractive films that provide a high efficiency single diffraction order across the visible and near-infrared region of the solar spectrum” [23]. Although with this pitfall, ideal diffractive gratings are still shown to be more effective in orbit lowering trajectories than reflective sails.
3.0 RESEARCH OBJECTIVES

Solar sailing is becoming a popular suggestion as an alternative to traditional propulsion due to affording opportunities in heliophysics research. IKAROS, Nano-Sail D2 and now LightSail 2 demonstrate success for the technology but also outline obstacles such as valid ground testing, reliable deployment methods, sail material properties, and attitude control of these large structures. Attitude determination and control systems in satellites vary depending on the specific mission. Typical satellite systems use momentum wheels or magnetorquers, though solar sailing missions have the ability to use a sliding mass table to change attitude of the sailcraft. RCD’s may be a favorable alternative but there is little information on their construction. Diffractive solar sailing is a very new area of research that can be applied to achieve better results than reflective sails, though further research into the meta-materials needed to manufacture the sails is suggested. Insight into the attitude control properties a diffractive sail can offer could be a substantial next step in proving the near-term advantages of developing this technology.

The purpose of this research is to advance the concept of diffractive solar sailing using an example heliophysics mission to show feasibility and advantages. We outline the objectives:

1. Derive a model for predicting the trajectory of a sail-craft
2. Analyze and compare trajectories for different parameters
   a. Size a spacecraft for mission
   b. Compare reflective vs. diffractive solar sailing
   c. Potential constellation opportunity
3. Size potential attitude control solutions
   a. Derive a model assuming perfect attitude control
   b. Analyze implementation on tradition reaction wheel system
   c. Analyze implementation on reflectivity control devices
4. Derive complete model for realistic attitude control
   a. Derive control law
   b. Implement non-ideal traditional reaction wheel system
   c. Discuss potential of reflectivity control devices
5. Outline areas for further research
4.0 ORBITAL MODEL OF DIFRACTIVE SOLAR SAILING

4.1 Diffractive Solar Sailing in Spherical Coordinates Derivation

For convenience we express the two-body (Sun and sailcraft) equations of motion in a heliocentric coordinate system depicted in Fig(4.1), where , , and respectively represent the radial distance from the sun, longitude, and latitude of the sailcraft. The ecliptic () lies in -plane.

The time-dependent state vector of the sailcraft is represented by position and velocity components: . It is assumed the initial state vector describes a circular orbit 1 AU from the sun: , where 1 AU, , and is the gravitational constant. It is also assumed the sailcraft is beyond the gravitational influence of other bodies.

![Heliocentric spherical coordinate system for a sail at the point having an outward normal unit vector. The vector is not shown, but is represented by.](image)

In general the solar radiation pressure force on the lightsail is directly proportional to the solar irradiance , sail area , and momentum transfer efficiency vector:

\[ F = \frac{\rho}{(\rho r^2)} \parallel \frac{\Omega}{\Omega} \parallel \]

where is the speed of light. Combining the force of gravitational attraction, the component of force on the lightsail may be expressed
where, is the so-called solar constant, , , , and .

The ratio is called the lightness number. For example, a sheet of water thicker than 1.54 m would have a lightness number less than unity. The NASA NEA-Scout lightsail has been designed for [24]. It is evident from Eq. (2) that the gravitational force dominates the radial dynamics when , whereas small values of and may provide non-negligible solid angle dynamics according to Eq. (4.3) and Eq. (4.4).

The efficiency vector for a sun-facing diffractive sail may be expressed

where is the unit normal vector of the sail surface and is a unit vector in the plane of the sail surface that is assumed to be colinear with the grating momentum vector as depicted in Fig. (4.2.a). The grating vector is a property of the grating, pointing perpendicular to the grating lines of period . The plane of the sail is made tangential to the heliocentric sphere; thus and we set , where the angle is a control parameter described below. The plane containing and (or equivalently, and ) is called the plane of incidence. Rays of incident and diffracted sunlight are naturally confined to the plane of incidence, and are governed by the grating equation, which at normal incidence may be expressed

where is the order diffraction angle, is the wavelength of light, and the plus (minus) sign corresponds to a reflection (transmission) grating. An ideal grating with complete diffraction into a single order (e.g., ) can be assumed below. The efficiency of a sun-facing diffractive sail may thus be expressed

The optimal transverse component of force (perpendicular to the sunline) for a sun-facing diffractive sail occurs when ; the associated radial efficiency is.
Fig. 4.2: Plane of incidence for (left) sun-facing diffractive sail with grating vector and order diffraction angle and (right) tilted reflective sail at angle. The sunline is parallel to the unit vector in both cases.

In contrast, the radiation pressure efficiency vector for an ideal reflective sail may be expressed

$$v \propto \cos \theta$$

where $\theta$ is the angle of incidence subtending the unit vectors and $\hat{n}$. To raise or lower the orbit of a reflective sail, or to change the heliocentric latitude, the sail normal must not be parallel to the sunline ($\hat{n}$). The maximum transverse component of force (perpendicular to $\hat{n}$) for a reflective sail occurs when $\theta$, providing a transverse (radial) efficiency value of $0.77$. In principle an ideal diffractive sail may be 23% more efficient than an ideal reflective sail for orbit and inclination raising maneuvers requiring a large value of $\theta$. We note however that significant advances in grating designs are required to achieve a diffraction angle near $\theta$ across the solar spectrum. However, a non-ideal diffractive sail exceeds the ideal transverse efficiency of a reflective sail (0.77) when the mean diffraction angle exceeds $\theta$, which is feasible.

Let us now establish a forcing law that provides simultaneous inclination cranking and orbit lowering. To lower the orbit from an initial radius of 1 AU, the azimuthal (longitudinal) acceleration must be negative: $\alpha_n < 0$. Inclination cranking may be accomplished by making the latitudinal acceleration positive ($\alpha_\lambda > 0$) when its angular velocity is positive ($\omega_\lambda > 0$), and conversely, making $\alpha_n$ when $\omega_\lambda < 0$. For a fixed sail attitude (for a diffractive sail and for a reflective sail), both conditions may be met by varying the sail force vector in the tangent plane.

The force law equations are defined for a diffractive sail:
where first order diffraction is assumed. The forcing law described above requires $>0$ for all time, for, and for. This rule may be satisfy if the grating vector is designed to satisfy the following equality. That is, we roll the sail to satisfy and: for and for, as depicted in Fig. (4.3). This forcing law causes the sail to cross the ecliptic plane ($\omega$) when and.

![Fig. 4.3: Schematic force law with control parameter (purple line) for a diffractive sail over one orbital period (black line) with (bold red line) and (bold cyan line).](image)

Similarly, the force law equations for a reflective sail are defined:

\[
\text{whereby we fix the angle } \lambda, \text{ and determine the angle } \phi \text{ that provides the desired sign of the acceleration. The forcing law may then be stated: } > 0 \text{ for all time, for, and for. For example, if and, the sail normal vector will project only along the and unit vectors in Fig. (4.1), resulting in radiation pressure in the and directions; hence the sail would likely spiral away from the sun.}
\]

The resulting state space equations with the solar sail incorporated:
where , , and  are described above (see discussion of Eqs. 4.2-4.4): .

4.2 Limitations of the Model

While this model gives an accurate basis for a feasibility study, the compromise for simplicity imposes limitations on analysis. At Eq. (4.13) is undefined due to a division of zero. This term is also present in the base form of the orbit equations without a sail force producing the same result. Any simulation reaching this inclination breaks down as  is undefined at this point. An example is shown in Fig. (4.4).

Fig. 4.4: Example trajectory where the simulation breaks from the singularity at .

Numerous techniques were explored to eliminate the singularity issue. Converting the state space equations of a regular orbit to cartesian coordinates of xyz showed no singularities. However, once the sail forcing laws were converted the singularity returned shown below in Eq. 4.16.

where  as the projection of the radius on the xy plane. When  at the solar poles the force is again undefined.
Although the singularity exists in this mathematical model, in reality the orbit continues as it was and a spacecraft would not suddenly find a drastic change in trajectory. In future studies a different model can be derived owing a more complex sail forcing scheme. It may be useful to convert the entire state space to Cartesian coordinates and perform a rotation about the y axis then convert back to spherical coordinates or use a orbital elements instead. For our purposes this model is still sufficient to analyze the advantages of diffractive solar sailing as the selected mission profile does not achieve an inclination of 90.

5.3 A Solar Polar Orbiter Sailcraft Mission

The 2013 Decadal Survey for Solar and Space Physics [11] advocated for advanced solar sail technology to enable heliophysics missions. This survey calls for a real-time solar weather monitoring network, and the US National Oceanic and Atmospheric Administration (NOAA) is seeking a similar program [25]. To date, the NASA Ulysses 1.3 AU mission is the only spacecraft to retrieve data (but no images) from the solar poles [26]. The European Space Agency Solar Polar Orbiter plans to reach an inclination angle of roughly 25 at 0.3 AU [27]; a journey that will take 3 years and multiple gravity assists from Venus and Earth. As described below, a diffractive solar sail may achieve an inclination of 60 at 0.32 AU, with neither gravity assists nor additional fuel for transfer orbits.

We consider a 6U CubeSat sailcraft, similar to the NASA NEA-Scout design [24] having a 14 [kg] mass, but larger sail area (400 [m] rather than 86 [m]) and lightness number (0.044 vs. 0.010). This lightness number may be achievable using small satellite design principles. The computed trajectories show non-optimized orbits, assuming a diffractive sail having a maximum theoretical tangential efficiency , and a corresponding radial efficiency . this is compared a reflective sail having maximum theoretical efficiencies and .

In Fig. (4.5) these sail types are depicted, showing orbit lowering and inclination raising. The MATLAB function used for these simulations was ode23t, a type of stiff numerical solver. Surprisingly, once the diffractive sail almost reaches 0.3 AU the orbital radius begins to increase again. This is evident at 4 years in Fig. (4.5), but does not occur when the sail is not rolling to increase inclination. The change occurs because of the inclination cranking maneuver; as the tangential force of the orbit is not consistently countering the centripetal force in the orbital plane. Thus, the spacecraft slips away from the Sun. As it is not recommended due to thermal considerations to approach the Sun closer than 0.3 AU, the rolling maneuver can be ceased at the lowest point of the orbital radius to avoid drifting outwards.
Within 5 years the reflective sail reaches 0.42 AU, whereas the diffractive sail reaches 0.32 AU. In this scenario, diffractive sails are 27% more efficient in changing the perigee of the orbit. Comparing inclinations during this period, the reflective sail increases 33 whereas the diffractive sail increases 60, making the diffractive sail 58% more efficient for inclination raising. Based on these ideal comparisons the higher orbital efficiencies of a diffractive sail make it superior to a reflective sail for the proposed heliophysics mission.

Fig. 4.5: Plot of orbital parameter radius vs. time for diffractive ($\theta = 0.022$, $\phi = -0.022$, $\psi = 0.022$) and reflective ($\theta = 0.022$, $\phi = -0.0169$, $\psi = 0.0169$) solar sails.
Fig. 4.6: Plot of orbital parameter phi vs. time for diffractive ($\rho = 0.022$, $\phi = -0.022$, $\phi = 0.022$) and reflective ($\rho = 0.022$, $\phi = -0.0169$, $\phi = 0.0169$) solar sails.
Fig. 4.7: Example trajectory of the diffractive sailcraft described in Section 5.3 ( = 0.022, = -0.022, = 0.022).

5.4 Potential for Solar Monitoring Constellations

The 2013 Decadal Survey for Solar and Space Physics advocated for advanced solar sail technology to enable heliophysics missions [11]. This survey also calls for a real-time solar weather monitoring network and the US National Oceanic and Atmospheric Administration (NOAA) is seeking a similar program [25]. To date, NASA’s Ulysses spacecraft at 1.3 AU is the only spacecraft to retrieve data (but no images) from the solar poles [26]. Learning about our closest star is essential in understanding our solar system and the origin of our Earth. Understanding solar weather patterns and how solar winds interact with our planet could prevent a catastrophic technological failure from a solar flare such as the one in 1859 [30]. An event like this one could disable communication satellites, ground airplanes, and take out the global power grid. The most recent and significant heliophysics mission is the Parker Solar Probe which launched in 2018, costing $1.5 billion. [9, 31]. The use of diffractive solar sailing is particularly advantageous in heliocentric missions and could enable lower cost heliophysics spacecraft.

Let us now imagine placing a constellation of twelve of the foregoing diffractive solar sails at 0.3 AU and at various angular positions around the sun: four at the equator (separated by 90) and one in each octant of the sphere at a 60 inclination from the equator. This distribution is depicted in Fig (4.9). We estimate that the entire constellation could be in place within 6 years of the first launch, assuming three satellites are launched together roughly every three months. If lighter spacecraft, larger sails, or less extreme orbits are desired, this time could be lessened.

Fig. 4.8: Constellation of solar polar orbiters driven by a diffractive solar sail. Trajectory of three simultaneously launched sails.
5.0 PERFECT ATTITUDE CONTROL

5.1 Perfect Attitude Control Tracking Model

We express sailcraft orientation with respect to an origin at the center of the sail. Rotations about the x, y, and z axis are referred to as roll, pitch, and yaw respectively as depicted in Fig. (5.1).

The primary achievement of this analysis is to derive a model in which desired angular velocity [rad/s] and angular acceleration [rad/s²] of the entire spacecraft are inputs into the
system with desired control torque and angular momentum as outputs. In a later analysis, this is performed by a simple control scheme with imperfect tracking, but this method is useful for a feasibility study where the specific control method can be unknown besides angular momentum storage and torque output.

The system to perform the attitude maneuver for the orbital model in Section 4.1 where the sail is always sun facing is derived. The inputs for are respectively:

\[(5.1)\]

where are defined as above in Section 4.1. The term is the time derivative meant to emulate the dynamics of the roll angle as defined in 4.1. is derived as a function of where at , at , and at . The maneuver is described again in Fig. (5.2) and the function is represented in Eq. (5.2).

\[(5.2)\]

Since the function of is not continuous and differentiable at a substitute differentiable function for is utilized to estimate the original function:

\[(5.3)\]

A comparison of the two functions is shown in Fig.(5.3). Respectively, and are then derived:

\[(5.4)\]

\[(5.5)\]
**Fig (5.2):** A simple roll maneuver used to produce the tangential and radial forces in the orbital model. The sail rolls half a rotation, then rolls half a rotation back to the original stating point within one orbit around the Sun.

![Roll Manuever Functions](image)

**Fig (5.3):** A comparison of the actual roll maneuver function vs. the estimated function in this chapter.

The inputs for $\Gamma$ are then respectively:

\[(5.6)\]

Recalling Euler’s moment equation [40] we describe the dynamics of the spacecraft:

\[(5.7)\]

The notation $\mathbf{J}$ and $\mathbf{J}_c$ refer to the total angular momentum of the sailcraft [N m s], including the angular momentum of the control system. Spacecraft angular momentum is the product of the inertia matrix of the satellite [kg m$^2$] and $\omega$. The external torque [N m] refers to any outstanding perturbations. The above Eq. (5.7) can be expanded:

\[(5.8)\]

\[(5.9)\]
where \( \omega \) refers to the control system angular momentum. The cross product of \( \omega \) can be expressed as a matrix operation for simpler implementation:

\[
\begin{align*}
(5.10) \\
\text{Isolating terms to one side and substituting yields:}
\end{align*}
\]

\[
(5.11) \\
(5.12)
\]

In this analysis it is assumed that no other external torques are perturbing the sailcraft. We shall assume an initial state vector similar to the initial state vector of the orbital model: 

\[ \begin{bmatrix}
\end{bmatrix} \]

where \( \theta \) as in Section 4.1.

### 5.2 Perfect Attitude Tracking For Proposed Sailcraft

For the proposed spacecraft in Section 4.2, we find the desired attitude control parameters to perform the proposed solar polar orbiter mission. To estimate the inertia matrix of the spacecraft, a simple CAD model was generated and evaluated. The assembly is depicted in Fig. (5.4).

For the purpose of this analysis the 400 m\(^2\) sail is assumed to be 3 microns thick and includes 2 kg of the overall spacecraft mass. The density of the sail is homogeneous. A 6U box is
placed at the origin of the sail on the non-sun facing side of the sailcraft. This 6U box includes the remaining 12 kg and has a uniform density throughout. The estimated inertia matrix around the center of mass is calculated:

\[
\begin{align*}
\text{(5.13)}
\end{align*}
\]

The values for \( \alpha, \omega, \gamma \) were negligible.

The model was implemented in MATLAB’s Simulink using ode8 (Dormand-Prince) with fixed steps of 5000 seconds.

![Simulink Model](image)

**Fig (5.5): An annotated depiction of the Simulink model used to solve the system. The orange box sections the orbital model derived in Section 4. The green box section formats the outputs from the orbital system into the desired angular velocity and acceleration vectors. The blue section represents the system equations for the perfect control.**

For an ideal diffractive sail with a lightness number of 0.044 the sailcraft will reach 0.3 AU and a 60 inclination within 5 years as shown previously in Section 4.2. To track this trajectory the sailcraft angular velocity around the xyz axis are depicted in Fig (5.6). All the angular velocities are expectedly on the order of \( 10^{-6} \) due to the long nature of the mission. In the first year the spacecraft performs just over one rotation around the z axis. Even as the sailcraft approaches the sun and its orbital period decreases, the maximum rotations for this trajectory is only 5 rotations per year. The maximum angular velocity of the spacecraft conceivably increases as more rotations are performed. Although the sailcraft is undergoing more rotations as it approaches the sun, the yaw velocity does oscillate due to the radial force of the sail and is not consistently increasing. This would hold true if a diffractive sail was designed to eliminate the radial force from the sail. The pitch and roll velocities are understandably oscillatory due to the oscillatory nature of the orbit and the \( \pi \) function. As the orbital period decreases it conforms that the \( \pi \) function’s frequency increases as visualized in Fig. (5.7).
Fig (5.6): The spacecraft angular velocity throughout a 5 year period around all three axis.
We find the overall control system torque is relatively small compared to typical spacecraft and although there is a large moment of inertia of the sail. The angular momentum storage of the control system is therefore also relatively low. Though the deployed state of the sailcraft spreads the moment of inertia, the desired angular velocities are quite small owing the smaller order of magnitude of control torque and angular momentum. The most torque is needed around the $z$ axis to perform the roll maneuver.
Fig (5.8): The control system torque output throughout a 5 year period around all three axis.
Fig (5.9): The control system angular momentum storage throughout a 5 year period around all three axis.

### 5.3 Reaction Wheels in Perfect Control

Let us assume reaction wheels are utilized for attitude control on the sailcraft. The least complex configuration for 3 degrees of freedom of control is one reaction wheel in each principle axis demonstrated in Fig. (5.10). Alternatively, for redundancy and reducing load on each individual wheel, configurations of 4 reaction wheels can be implemented. This system is demonstrated in Fig. (5.11). Conveniently the transformation of configurations can be expressed by a geometric matrix, \( \mathbf{G} \), incorporated into Eq. (5.14):

\[
(5.14)
\]

For the simplest configuration, the geometric matrix is a 3x3 identity matrix, while the configuration in Fig. (5.11) is a 3x4 unique matrix [41]:

\[
(5.14)
\]
The geometric matrix will always have n, rows equal to 3 for the 3 degrees of freedom and m rows equal to the number of wheels in the configuration. Since may not always be a square matrix, the Moore-Penrose Psuedoinverse is utilized when solving Eq.(5.14) for .

Fig (5.10): Configuration of 3 wheels, one on each principle axis of the spacecraft coordinate system.

Fig (5.11): Configuration of 4 wheels forming a tetrahedral. Each wheel is tilted 45 degrees from the xy plane and one wheel lies in each xy quadrant.

The angular velocity of each wheel is simply:
where the moment of inertia around the rotation axis \( \text{[kg m}^2\text{]} \) can be estimated by \( I \), representing the mass of the wheel \([\text{kg}]\) and \( r \) as the radius of wheel \([\text{m}]\).

From the maximum angular momentum and torque values in Fig. (5.8) and Fig. (5.9) an appropriately sized reaction wheel can be selected. With the rise of the SmallSat industry, many companies make CubeSat compatible components. We can assume the mass and size of a feasible reaction wheel based on the commercially available CubeWheel Small Satellite Reaction Wheel \([42]\). These wheels have a mass of 60 grams and are 28 mm in radius for each unit. For a 3 wheel configuration, the reaction wheel system would weigh 180 grams (not including avionics) and could be mounted in less than 1U of space in the CubeSat. The speeds of each individual wheel in both configurations are shown in Fig. (5.12) and Fig. (5.13).

![Reaction Wheel Velocities](image)

**Fig (5.12):** Wheel speeds of a configuration of 3 wheels, one on each principle axis of the spacecraft coordinate system.
Fig (5.13): Wheel speeds of a configuration of 4 wheels forming a tetrahedral. Each wheel is tilted 45 degrees from xy plane.

Conceivably, the maximum wheel velocity value is smaller for the 4 wheel configuration. While this system may be selected for risk mitigation due to redundancy, caution is advised when selecting the configuration due to many reaction wheels owing a minimum accurate RPM. Small CubeSat reaction wheels would be capable to perform the desired maneuvers for the solar polar orbiter mission. Most of these size wheels can provide torques and angular momentum storage on the order of $10^{-3}$, which is substantially above the predicted values in Fig. (5.8) and Fig. (5.9) of the order of $10^{10}$ and $10^{-4}$.

5.4 Optically Controlled Devices in Perfect Control

Unlike a traditional reaction wheel control system, an optically controlled system would provide torque without momentum storage. To analyze this system the Simulink model in Fig. (5.5) is modified to hold the control angular momentum storage at zero. This study shows substantially small values of control torque and angular momentum, thus does not have a large effect on the overall desired control torque as demonstrated in Fig. (5.15). In cases of larger
torques and angular momentum storage this would not hold true. Typical reflectivity control devices need to be tilted out of the plane of the sail to provide torques in all 3 degrees of freedom. Implementing static diffractive elements behind the reflectivity control devices in different orientations could enable control in all three axis while still being mounted as a layer on the sail.

Let us assume that patches of ideal transmissive diffractive elements with ideal reflectivity control devices layered over top of them are placed throughout the sail. The reflectivity control devices can switch completely on and off. An example configuration where the elements are placed at the edges of the sail to provide maximum torque similar to IKAROS [43] can be imagined in Fig. (5.14). Panels of 10cm x 10cm size can be aligned around the edge of the sail. Each of the elements are aligned to cancel out the translational forces its symmetrically placed counterpart. Two sets of elements would be implemented for each axis, one to impart a torque in the clockwise direction, and one to impart a torque in the counterclockwise direction. The elements are placed in an orientation to produce the most torque in the roll axis.

Fig (5.14): Example diffractive element placement. The net components of force throughout each of the element strips are described. The sections are color coded for pitch, roll, and yaw control elements.

The torque from each of the corresponding set of pitch roll and yaw elements from Eq. (4.3) yields:

\begin{align}
\text{(5.16)} \\
\text{(5.17)}
\end{align}
where \( A_1, A_2 \) are the overall areas of the corresponding control elements, \( d_1, d_2 \) are the distance to the center of the element sections to the corresponding axis. The torque in each axis can be controlled by switching off 10 cm x 10 cm panels to manipulate the control area. A negative control area indicates the counterclockwise torque elements. It is assumed the maximum force efficiency of 1 for ideal diffractive elements in both the planar and normal. The theoretical maximum available torque in this configuration is \( T_{\text{max}} \) and \( T_{\text{max}}' \).

Another option for implementing optical control is using beam steering diffractive elements where the diffractive angle can be electro-optically manipulated to control the force efficiency \( \eta \), thus removing the extra layer of RCD panels and saving spacecraft mass. The roll axis elements depend on the force efficiency in the plane of the sail, while the yaw and pitch axis are dependent on the efficiency normal to the sail. Recalling the equations for ideal force efficiency in Section 4.1, \( \eta \) can be written as a function of diffractive angle:

\[(5.18)\]

\[(5.19)\]

\[(5.20)\]
Fig (5.15): The control system torque output throughout a 5 year period around all three axis in a system without angular momentum storage.

The resulting analysis shows a maximum control system area of $2.4 \times 10^{-6}$ which is unachievable in practicality. The proposed individual 10cm x 10cm switchable panels can only achieve a $10^{-2}$ resolution in control area. Alternatively 1cm x 1cm panels would only yield a resolution of $10^{-4} \text{ m}^2$. 
The control area throughout a 5 year period to produce desired torque. The areas for each corresponding set of optical control devices in pitch, yaw, and roll are depicted. The theoretical force efficiencies of the diffractive element layers are static at the maximum.

The resulting maximum change in diffractive angle for a beam steering element is also too small of a resolution to practically achieve. In Fig. (5.17) we find that the total change in angle is only on the order of $10^{-6}$ degrees for the diffractive elements.
**Fig (5.17):** The control diffractive angle over a 5 year period to produce desired torque about the roll, pitch and yaw axis. The control areas are held constant at the maximum available area of the element patches.

If we instead adjust the location of the elements as close to the center of the sail as possible and the available control area is decreased to singular 10cm x 10cm panels then we yield a more achievable diffractive angle resolution, but still an improbable control area system. The resolution shown in Fig. (5.18) is still on the order of $10^{-5}$ and cannot be performed by even 1cm x 1 cm panels.
Fig (5.18): The control area throughout a 5 year period to produce desired torque with a reduced control area. The areas for each corresponding set of optical control devices in pitch, yaw, and roll are depicted. The theoretical force efficiencies of the diffractive element layers are static at the maximum.

However, the maximum difference in control angle has increased in Fig. (5.19) to 0.5 degrees in the roll axis and 1.5 degrees in the pitch and yaw axis. While the resolution of the roll axis may still be improbable, accurate beam steering devises may be able to achieve the resolution for the pitch and yaw rotations.
Fig (5.19): The control diffractive angle over a 5 year period to produce desired torque about the roll axis with a reduced control area and distance. The control areas are held constant at the maximum available area of the element patches.

Overall, we find the torque required for the proposed mission is too small to show an advantage of using opto-electrical devices like RCD’s or beam steering diffractive elements. A larger sailcraft or a mission trajectory requiring faster maneuvers may show benefit from the implementation of these devices over traditional reaction wheel systems.
6.0 IMPERFECT CONTROL MODEL

6.1 Derivation of Attitude Control Model

While the analyses in Section 5 is useful for a general feasibility standpoint, we make large assumptions that may effect a spacecraft in practicality. A perfect sail does not exist as folds and wrinkles in the sail from packaging can cause the center of pressure of the sail to offset from the plane of the center of mass. This would in turn provide a constant substantial external torque onto the sailcraft. The model in Section 5 also assumes that the angular velocity of the spacecraft was correct before the attitude control system started performing and did not require a feedback system. This would most likely not hold true in reality. We derive a new model that implements a simple feedback law into the system to track the desired angular velocity inputs.

Utilizing the same coordinate system as shown in Fig. (5.1) we can recall Eq. (5.10) to describe the overall spacecraft dynamics:

\[
\text{w} \quad \text{(6.1)}
\]

where we can again substitute in \(S\) from Eq. (5.11) and now rearrange to solve for

\[
\text{w} \quad \text{(6.2)}
\]

The standard notation for \(\omega\) and \(\theta\) are still the same from Section 5.

For feedback in this model we will also include the kinematics of the spacecraft in the form of quaternions. Quaternion notation is utilized to avoid gimbal locking and singularity issues. The notation for the feedback quaternion is made up of four states:

\[
\text{q} \quad \text{(6.3)}
\]

And evolves in relation to \(\dot{q}\) as [43]:

\[
\text{q} \quad \text{(6.4)}
\]

which can be simplified to:

\[
\text{q} \quad \text{(6.5)}
\]

The desired angular velocity inputs match the inputs from Section 5.1 and are converted to quaternion inputs using Eq. (6.5). The resulting coordinates represent the attitude quaternion and describes the pointing of the spacecraft. This is useful information for a mission such as this where the orbital forces are reliant on accurate tracking of the Sun.

Although the system is a nonlinear model, for the purpose of simplicity a feedback control law derived from the linearization of the state space equations can be implemented. In a more specific analysis for practicality, an optimized control law for the nonlinear system could
be implemented for better results. The control law could be in the form of sliding mode control, pseudo-linearized, or completely non-linear. This is not required for our study and the linearized simple feedback control law should suffice. For a system linearized around in the form , is the states as a function of time, and is the control torque. We can choose to express as [43]:

\begin{equation}
\begin{split}
(6.6)
\end{split}
\end{equation}

The notations of , are positive 3x3 control gain matrices and , , are the quaternion error and angular velocity error respectively. The quaternion error is represented as a function of the spacecraft attitude quaternion and the desired input quaternion in Eq. (6.7). The angular velocity error is simply .

\begin{equation}
\begin{split}
(6.7)
\end{split}
\end{equation}

It is assumed that the system is receiving perfect feedback from general sensors, not specific types. The model implemented in Simulink is demonstrated in Fig. (6.1) and solved with a stiff variable step solver ode23t.

![Simulink Model](image)

*Fig (6.1): An annotated depiction of the Simulink model used to solve the system. The orange box sections the orbital model derived in Section 4. The green box section formats the outputs from the orbital system into the desired angular velocity and acceleration vectors. The blue section represents the system equations for the perfect control.*

### 6.2 Imperfect Attitude Control with Reaction Wheels

To verify that this model is on par to the results in Section 5.1, we set the same initial conditions and compare the results. The control gains \( K_p \) and \( K_{pd} \) were selected by trial and error. If both \( K_p \) and \( K_{pd} \) are set to identity matrices, we find the results of Fig. (6.2) and Fig. (6.3).
While the angular velocity tracking error is almost acceptable, the attitude quaternion tracking is not.

Fig (6.2): The spacecraft angular velocity throughout a 5 year period around all three axis ($K_p = K_{pd} = I$). There is no external torque implemented on the system. The dashed lines represent the desired spacecraft velocity. The solid lines represent the actual spacecraft velocity.
Fig (6.3): The spacecraft attitude quaternion throughout a 5 year period around all three axis ($K_p = K_{pd} = I$). There is no external torque implemented on the system. The dashed lines represent the actual attitude quaternion. The solid lines represent the actual attitude quaternion.

After numerous attempts at adjusting the gain matrices, we find better results by setting $K_p$ to zero and increasing $K_{pd}$. As $K_{pd}$ the tracking error decreases but yields an increased control effort, namely a higher desired torque for the control system. Let us set $K_{pd}$ to an identity matrix. For no external torques and the same inputs as Section 5, we find the similar results to the previous model as shown in Fig. (6.4) through Fig. (6.7). The spacecraft performs acceptable tracking of the desired attitude quaternion and angular velocity. The angular velocity of the reaction wheels follows the same trend as Fig. (5.6). Besides an initial jump in control torque, the values for the three axis are on the same order of magnitude as the results in Fig. (5.8).
Fig (6.4): The spacecraft angular velocity throughout a 5 year period around all three axis \((K_p = 0, \ K_{pd} = 1)\). There is no external torque implemented on the system. The dashed lines represent the desired spacecraft velocity. The solid lines represent the actual spacecraft velocity.
Fig (6.5): The spacecraft attitude quaternion throughout a 5 year period around all three axis ($K_p =0$, $K_{pd} = I$). There is no external torque implemented on the system. The dashed lines represent the actual attitude quaternion. The solid lines represent the actual attitude quaternion.
Fig (6.6): Control system torque throughout a 5 year period around all three axis ($K_p = 0, K_{pd} = 1$). There is no external torque implemented on the system.
Let us assume that the sail has a center of pressure offset from the center of mass due to imperfections on the sail. This is an important assumption as a solar sail needs to be folded and creased for deployment. The external torque caused by a 1cm offset from the z axis and a 1cm offset from the y axis is calculated in Eq. (6.8)

\[ T = 1.8 \times 10^{-5} \text{ Nm} \]  

(6.8)

where , and . It is assumed this torque is applied in the yaw and pitch axis and that the torque is held constant as the spacecraft approaches the Sun.

To achieve decent tracking with the new external torques implemented into the model, \( K_{pd} \) is the identity matrix multiplied by \( 10^3 \). The resulting control system angular momentum is shown in Fig. (6.8) and the control torque in Fig. (6.9). The required control system angular momentum storage and torque are much larger than without the offset torque. While the torque values could be achieved by the reaction wheels selected in 5.2, the angular momentum storage surpasses the limitation of the components. Each axis is able to provide 1.77mNms angular
momentum storage and 0.23mNm control torque in the 3 wheel configuration in Section 5.2. Implementing these values as saturation limits into the Simulink simulation yields the results in Fig. (6.10) and Fig. (6.11). The wheels become saturated almost immediately, therefore the required control torque cannot be produced. The required angular momentum storage may not be feasible for any commercially available reaction wheel system under 4 kg, which would be far out of the CubeSat class. This suggests in order to still use available reaction wheels as a control method one may implement another method in conjunction. NEA Scout uses a sliding mass table to control the center of pressure center of mass offset and desaturate the reaction wheels [44]. Huang & Zhou also suggest this in their study of attitude control systems for solar sails [45].

Fig (6.8): Control system momentum storage throughout a 5 year period around all three axis ($K_p = 0, K_{pd} = 10^3 I$). There is external torque exerted onto the system.
Fig (6.9): Control system torque throughout a 5 year period around all three axis ($K_p = 0$, $K_{pd} = 10^3 * I$). There is external torque exerted onto the system.
Fig (6.10): Control system momentum storage throughout a 5 year period around all three axes ($K_p = 0$, $K_{pd} = 10^3 I$). There is external torque exerted onto the system and saturation limits for the control system.
Fig (6.11): Control system momentum storage throughout a 5 year period around all three axis ($K_p = 0, K_{pd} = 10^3 I$). There is external torque exerted onto the system and saturation limits for the control system.

6.3 Imperfect Attitude Control with Electro-Optically Controlled Devices

Utilizing the same concepts as Section 5.3, we can characterize an attitude control system consisting of the same two electro-optically controlled methods. If the diffractive elements are placed according to Fig. (5.14), the response can be studied. Just as in Section 5.3, angular momentum of the control system is held constant at zero. The inputs shown previously for a spacecraft with an external torque are utilized. The torque saturation limit was set at $1.8e-4$ Nm for elements with a max efficiency of  and the maximum available area of the configuration.
Fig (6.12): The spacecraft angular velocity throughout a 5 year period around all three axis ($K_p = 0, K_{pd} = 10^3 I$). There is external torque implemented on the system. The dashed lines represent the desired spacecraft velocity. The solid lines represent the actual spacecraft velocity.
Fig (6.13): The spacecraft attitude quaternion throughout a 5 year period around all three axis ($K_p = 0, \ K_{pd} = 10^3 \ I$). There is external torque implemented on the system. The dashed lines represent the actual attitude quaternion. The solid lines represent the actual attitude quaternion.
Fig (6.14): Control system torque throughout a 5 year period around all three axis ($K_p = 0$, $K_{pd} = 10^3 I$). There is external torque exerted onto the system.
Fig (6.15): Control system area with static diffractive elements over a 5 year period around all three axis ($K_p = 0$, $K_{pd} = 10^3 * I$). There is external torque exerted onto the system.

We find that for decent tracking there is an initial jump to the maximum torque. While reaction wheels are not capable of performing the desired maneuver with the small offset, electro-optical devices may be the solution. Although the area required area control is still too small to use 10 cm panels, but is within the realm of 1 cm x 1 cm panels for resolution in the roll and yaw axis. The desired torque in the pitch axis is still too low to be feasible with the available resolution.
Fig (6.16): Control angle with maximum diffractive area held constant over a 5 year period around the roll, pitch and yaw axis ($K_p = 0$, $K_{pd} = 10^3 I$). There is external torque exerted onto the system.

For the angle controlled system the required roll angle and pitch angle still require too small of an resolution to be practically implemented in the near future, but could be solely used in the long term or for larger sail missions. Besides the initial jump to the maximum angle, the overall change in angle is 4 degrees. This resolution could be feasibly achieved. It may be feasible that there is a larger offset than 1cm, namely 5cm or more. Alternatively there may also be a larger center of pressure force due to a larger sail. These elements may be able to accommodate the tracking required while reaction wheels would not.
7.0 CONCLUSIONS

The draw of using solar sails as a propulsion system is the almost infinite, mass-less propellant from solar radiation pressure as the main source of thrust for sailcraft. It may be a relatively small force, even in the space environment, but with the advancement of deployable space structure technology it is feasible to provide the very large, lightweight areas needed to effectively use this force. Meanwhile, there has been a push for cheaper, faster solar system missions. Recently NASA’s budget has fluctuated around $20 billion in 2019 USD dollars. This is 2/3rd of what the agency received in 1989, and 1/3rd of that during the peak of the Apollo era [28]. Of that money, almost 50% goes to human spaceflight rather than unmanned solar system, satellite or deep space missions. While this is only about 0.5% of the entire US government operational budget, many people believe that space is too expensive and that money should be distributed elsewhere. Thus, budget for planetary science, heliophysics, earth science, and interstellar sciences is not proportionate to the needs of the entire scientific community. The current cost estimate for launching a spacecraft into space is $10,000 per pound to orbit [29]. A lighter spacecraft yields a lower cost, therefore enabling more science based missions and useful satellites. For solar sailing missions, even more particularly diffractive solar sailing missions, huge mass savings are gained because there is no additional mass from carrying fuel.

We have numerically demonstrated that an ideal diffractive sail can enable heliocentric missions with an efficient form of infinite low thrust propulsion. Each sail may carry a low mass camera or other instruments for making simultaneous measurements of solar radiations, including high inclination angle measurements. Unlike missions requiring gravitational assists to achieve high inclination angles, solar sails do not impose stringent launch windows. Our proposed example assumes a 6U 14 kg CubeSat having a sail of area 400 m. The ideal force efficiencies assumed have not yet been experimentally demonstrated, and thus the projected time of flight may exceed our estimate of five years for reaching 0.32 AU and a 60 inclination. Compared to an ideal diffractive sail the corresponding values for a reflective sail having a maximum theoretical value of tangential force efficiency (77%) are 27% further from the Sun and at a 58% lower inclination angle. Our numerical model suggests that a diffractive sail having an efficiency of > 80% provides an orbital advantage over a reflective sail. Therefore, while solar sailing is proven to be highly effective, diffractive solar sailing has an advantage over the traditional reflective solar sailing. From another point of view, this analysis suggests that both types of sails can reach a desired orbit within the same time, but with a smaller diffractive sail area. Consequently, sets of satellites can be incrementally launched throughout the year to build up a full constellation. A constellation of 12 satellites can be positioned within 6 years at 0.3 AU. Eight of these satellites could be at various orbits with 60º inclinations. If a diffractive sailcraft of the same size can be manufactured with current small satellite technology, the constellation would be completed with satellites much less massive than spacecraft with typical propulsion systems. Based on our analysis, we find that diffractive solar sails provide a rapid and cost-effective multi-view option for investigating heliophysics.
The difficulty of using solar sails comes from deployment and the attitude control of these large areas. We have analytically demonstrated that even the implementation of diffractive element patches into a system can yield advantages for the attitude control system and thus the sailcraft overall. Small reaction wheels can provide the small torque and angular momentum storage with no external factors on the spacecraft with plenty margin. This system occupies 1U out of the 6U proposed CubeSat, which is a typical ratio. While the maneuver described for a diffractive polar solar orbiter can be performed with traditional CubeSat reaction wheels in an ideal practice, the use of electro-optically controlled devices would be more suitable. These can be aligned in patches on the sail, similar to the RCD’s on the IKAROS mission but include a layer of static diffractive elements. The torque is controlled by manipulating the area of utilized diffractive elements. In an ideal case for the proposed heliophysics mission, the devices are more than able to theoretically provide the desired control torques but cannot be practically implemented with the required resolution. Further research may find that the resolution can be achieved by adjusting the duty cycle of the RCD’s to produce effectively smaller areas. Another method is proposed to remove the RCD layer and the elements’ diffractive angle can be electro-optically tuned similar to a beam steering method [46]. This would yield mass savings for the sail itself, thus the overall spacecraft. For an ideal environment with a fixed area it is found that the resolution in change in angle was too small to practically implement. For larger sailcraft than our proposed 6U mission or missions owing more extreme maneuvering these diffractive systems could show large advantages and be able to perform in application with feasible resolutions.

In practice, it can be assumed that there will be a center of pressure offset from the center of mass due to imperfections in the sail. With the addition of this external torque of even 1cm from the pitch and yaw principle axis, the reaction wheel system becomes almost immediately saturated and cannot perform the mission. Commercially available larger reaction wheel systems also could not perform this maneuver as the large moment of inertia of the deployed sail causes a disadvantage. As larger reaction wheels are implemented to the system, more mass is added. This is counter-productive for solar sails as the characteristic acceleration of the sailcraft is directly proportional to the mass of spacecraft. Sailcraft need a large area to mass ratio to be effective for orbital maneuvers. However, we demonstrated that the electro-optically controlled diffractive systems are easily able to counteract this external torque within a practical resolution. This could be achieved by the implementation of 1cm x 1cm RCD patches over static diffractive elements or by a fixed area element system with a tunable diffractive angle resolution of 4 degrees.

This thesis has shown that a completely electro-optical system is more than able to provide the torque needed to perform attitude control of a solar sail. While in the near future the resolution of the devices may not be able to accurately perform the proposed maneuvers, metamaterials could advance to satisfy those requirements in the long term. The required resolution can be increased by implementation on much larger sails or more extreme mission maneuvers.
Solar sails are on the trajectory to grow larger in area, so the opto-mechanical systems could be the favored method when traditional methods will no longer be feasible.

Our research concludes that a hybrid system of both a traditional reaction wheel system and an optically controlled system would be the most beneficial solution. While this has been suggested and implemented on NEA Scout with reaction wheels and a sliding mass system, this is accompanied by a cold gas thruster assembly to provide torque in the third axis of rotation the sliding mass system cannot produce [44]. A small reaction wheel system can produce the fine adjustments needed for Sun tracking, while either of the electro-optically controlled device methods can be implemented to counteract any large external torques. None of these systems require a limited propellant supply, nor complicated moving mechanisms that are at risk of failure. They can continue as long as they are provided enough power from the spacecraft bus. This contributes to longer mission life capabilities, less massive sailcraft and therefore more cost effective important scientific missions.

A proposed roadmap for the application of diffractive elements into solar sailing is shown in Fig. (7.1). Diffractive elements could immediately begin to be implemented into the attitude control of current reflective sails. The easiest to employ would be the area controlled static diffractive element system. Soon after, beam steering type elements would be feasible affording the thickness of the layers of the elements are decreased. After some advancements from the meta-materials field, entirely diffractive sails could be implemented in the long term.

![Proposed roadmap for diffractive solar sailing at the writing of this thesis (2020).](image)

While we have advanced the potential of diffractive solar sailing in this research, further work is necessary to continue see concepts into fruition. Advanced metamaterials with unique optical properties need to be developed to gain higher force efficiencies across the solar spectrum. We assume the best theoretical efficiency from the law of diffraction but realistic gratings have ranges of wavelengths they perform in. Research into the manufacturing and control of the reflectivity control devices for better performance is suggested. The potential of
advantage of more complex control laws better suited to nonlinear systems and implementation of a hybrid control system could also be explored. Lastly, expanding the use of these systems for different types of missions outside of this scope could advance the potential of diffractive elements in solar sailing further. It would be beneficial to explore non-linear, or pseudo linear control laws for better results of the control effort. Combining these control laws with the logistics of potential measurement systems to define a better resolution would result in realistic prediction of the entire system throughout the whole mission.

Based on the results of our analysis we highlight the potential of implementing diffractive elements into solar sailing for large gains. The addition of diffractive elements into not only the sail, but the attitude control system of a spacecraft enable possibilities that were previously limiting solar sailing in general. These concepts show advantages in mass savings, complexity, and longevity of spacecraft affording cost-effective heliophysics observations in the near-term and far off timeline of space travel.
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APPENDIX

% Orbital Model Code

clear all; close all;

%%% Can use to calculate the alpha, beta, delta values below
% nr = 1; % radial efficiency (based on given angle)
% nt = 1; % tangential efficiency
% sigmacr = 1.54;
%
% sigma = 100; % areal density in g/m^2
% sigmastar = sigmacr/sigma;
%
% alpha = nr*sigmastar/2;
% beta = nt*sigmastar/2;

% insert orbital elements initial conditions

% Parameters normalized in AU
G = 1; % value of mu
alpha = 0.05; % value of force in r direction
beta = -0.05; % value of force in theta direction
delta = 0.05; % value of force in phi direction (if cranking should be same as beta, otherwise 0.
Rearth = 1.0; % radius start value in AU
Vstart = sqrt(G/Rearth^3); % tangential velocity start value in AU/year
% Use Rearth, Vstart when starting at 0 degrees inclination
r0 = [Rearth; 0; 0; 0; Vstart; 0]; % default original state vector, starting at 0 theta, 0 phi

% state vectors including solar sail. [rdot, thetadot, phidot, % rdoubledot, thetadoubledot, phidoubledot]
func = @(t,r) [r(4); r(5); r(6);
    -G/(r(1)^2)+r(1)*(r(5)^2)*(cos(r(3)))^2+r(1)*(r(6))^2+ alpha*G/(r(1)^2);
    -2*r(4)*r(5)/r(1) + 2*r(5)*r(6)*tan(r(3)) +
    abs(sin(r(2)))/cos(r(3))*(beta*G/(r(1)^3));
    (-2*r(4)*r(6)-
    r(1)*(r(5)^2)*sin(r(3))*cos(r(3))+(delta*G/(r(1)^2))*cos(r(2)))/r(1)];

% state vectors without solar sail. [rdot, thetadot, phidot, % rdoubledot, thetadoubledot, phidoubledot]
% func = @(t,r) [r(4); r(5); r(6);
%    -G/(r(1)^2)+r(1)*(r(5)^2)*(cos(r(3)))^2+r(1)*(r(6))^2;
%    -2*r(4)*r(5)/r(1) + 2*r(5)*r(6)*tan(r(3));
%    (-2*r(4)*r(6)-r(1)*(r(5)^2)*sin(r(3))*cos(r(3)))/r(1)];

timespan = linspace(0, 10*pi(), 3000); % creates time vector
options = odeset('RelTol',1e-6,'AbsTol',1e-6); % sets tolerances for solver, this is pretty typical value
[t, state2] = ode23(func, timespan, r0, options); % initiates solver, state2 is the array with all solved values
% state2(:,1)=r, state2(:,2)=theta, state2(:,3)=phi
% converts spherical coordinates to xyz to graph
x = state2(:,1).*cos(state2(:,2)).*cos(state2(:,3));
y = state2(:,1).*cos(state2(:,3)).*sin(state2(:,2));
z = state2(:,1).*sin(state2(:,3));

% graph results
figure(1)
scatter3(0,0,0,500, 'MarkerEdgeColor', [0 0 0],
'MarkerFaceColor', [1 1 0],
'LineWidth', 1.5) % plots the sun
axis([-1 1 -1 1 -0.5 0.5]); % sets axis
set(gca,'FontSize',20)
hold on % creates hold of all points
pp = patch([2 -2 -2 2], [2 2 -2 -2], [0 0 0 0], 'k');
pp.FaceAlpha = .2;

for ii = 1:length(t)
    if state2(ii,2) < 2*pi()
        scatter3(x(ii),y(ii),z(ii), '.', 'r');
    else
        if state2(ii,2) < 4*pi()
            scatter3(x(ii),y(ii),z(ii), '.', 'y');
        else
            if state2(ii,2) < 6*pi()
                scatter3(x(ii),y(ii),z(ii), '.', 'g');
            else
                if state2(ii,2) < 8*pi()
                    scatter3(x(ii),y(ii),z(ii), '.', 'c');
                else
                    if state2(ii,2) < 10*pi()
                        scatter3(x(ii),y(ii),z(ii), '.', 'b');
                    else
                        if state2(ii,2) < 12*pi()
                            scatter3(x(ii),y(ii),z(ii), '.', 'm');
                        else
                            if state2(ii,2) < 14*pi()
                                scatter3(x(ii),y(ii),z(ii), '.', 'k');
                            else
                                if state2(ii,2) < 16*pi()
                                    scatter3(x(ii),y(ii),z(ii), '.', 'r');
                                else
                                    if state2(ii,2) < 18*pi()
                                        scatter3(x(ii),y(ii),z(ii), '.', 'y');
                                    else
                                        if state2(ii,2) < 20*pi()
                                            scatter3(x(ii),y(ii),z(ii), '.', 'g');
                                        else
                                            % further conditions...
                                        end
                                    end
                                end
                            end
                        end
                    end
                end
            end
        end
    end
end
else
    if state2(ii,2) < 22*pi()
        scatter3(x(ii),y(ii),z(ii),
        '.', 'c');
    else
        if state2(ii,2) < 24*pi()
            scatter3(x(ii),y(ii),z(ii),
            '.', 'b');
        else
            scatter3(x(ii),y(ii),z(ii),
            '.', 'm');
        end
    end
end

hold off

figure(2)
hold on
plot(t/(2*pi()),rad2deg(state2(:,3)))
hold off
set(gca,'FontSize',20)
title('Value of Phi Over Time')
xlabel('Time (Years)')
ylabel('Phi (Degrees)')
grid on

figure(3)
hold on
plot(t/(2*pi()),state2(:,1))
hold off
set(gca,'FontSize',15)
title('Value of Radius Over Time')
axis([0 5 0.5 1.5])
xlabel('Time (Years)')
ylabel('Radius (AU)')
grid on