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Recommended Citation

M. Mbonye, Modern Physics Letters A 19, 117 (2004).

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Cosmology with Interacting Dark Energy

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Abstract

The early cosmic inflation, when taken along with the recent observations that the universe is currently dominated by a low density vacuum energy, leads to at least two potential problems which modern cosmology must address. First, there is the old *cosmological constant problem*, with a new twist: the *coincidence problem*. Secondly, cosmology still lacks a model to predict the observed current cosmic acceleration and to determine whether or not there is a future exit out of this state (as previously in the inflationary case). This constitutes (what is called here) a *dynamical problem*. In this article a framework is proposed to address these two problems, based on treating the cosmic background vacuum (dark) energy as both dynamical and interacting. The universe behaves as a vacuum-driven cosmic engine which, in search of equilibrium, always back-reacts to vacuum-induced accelerations by increasing its inertia (internal energy) through vacuum energy dissipation. The process couples cosmic vacuum (dark) energy to matter to produce future-directed increasingly comparable amplitudes in these fields by setting up oscillations in the decaying vacuum energy density and corresponding sympathetic ones in the matter fields. By putting bounds on the relative magnitudes of these coupled oscillations the model offers a natural and conceptually simple channel to discuss the *coincidence problem*, while also suggesting a way to deal with the *dynamical problem*. A result with important observational implications is an equation of state $w(t)$ which specifically predicts a variable, quasi-periodic, acceleration for the current universe. This result can be directly tested by future observational techniques such as SNAP.

1 Introduction

In the last few years, evidence has mounted suggesting that the universe is currently accelerating. Measurements of type Ia supernovae [1] indicate that the evolution of the Hubble parameter departs from that expected for a matter dominated universe, and behaves as if under the influence of a negative pressure due to a smooth and dominant background (dark) energy. Further evidence for a vacuum-dominated universe comes from a combination of observations: large scale structure (LSS) [2] suggests a low matter density universe while the cosmic microwave background (CMB) anisotropy data [3] shows the density of the universe to be virtually critical, consistent with the requirements of the early inflationary scenario [4]. Success of inflation required that the universe be in a vacuum-dominated state with a large associated potential energy. This feature, when taken together with the observations of a currently low density vacuum energy dominating the universe, leads to at least two potential problems that modern cosmology must address. First, the discrepancy between the initial high potential energy state and the current low background dark energy constitutes what has been called [5] the Cosmological Constant Problem. A new twist to

* New address

this problem, and which has been called the *coincidence problem*, relates to the observation [1] that the current background vacuum energy density, $\rho_v^0 \sim 10^{-30} \text{ g cm}^{-3}$ is not only not zero, but is also of the same order of magnitude as the current density ρ_m^0 of the matter fields.

This state of affairs gives rise to yet another complication. The observed current vacuum-dominated state of the universe is not *a priori* predicted by theoretical cosmology, and (as of now) its origin remains mysterious. To this extent, it is not known whether or not there is a future (graceful) exit from the resulting cosmic acceleration (as was the case in the early universe). The situation reflects our current ignorance with regard to the future dynamical evolution of the universe. It constitutes a *dynamical problem*.

Several approaches have been developed to address the “why is Λ small now?” part of the Cosmological Constant Problem. They include dynamical Λ -term models [6, 7], dynamical equation of state models [8], and rolling scalar field models [9]. A common feature in these models is that the vacuum energy takes on a dynamical character, decaying from some initially large value to a small one. One approach, usually favored by particle physicists, is to seek for mechanisms to cancel or at least suppress Λ , as in Coleman’s wormhole approach [10] or by introducing compensating fields in the Lagrangian. In his notable works, Brandenberger has argued (for a review see [11]) that gravitational back-reaction may lead to a dynamical cancellation mechanism for a bare cosmological constant by giving rise to an evolving anti-de Sitter-like field.

Meanwhile, the observation, [1] of a small but non-vanishing background dark energy has made the cosmological constant problem even worse, since it is harder to advance a symmetry argument that justifies an *almost* non-vanishing cosmological constant. In 1998, Caldwell, Dave and Steinhardt [12] proposed the existence of a background field with a dynamical equation of state $w(t)$. They called it *quintessence*. Since then, tracker models based on [12] have evolved to discuss the possible origin of the observed current cosmic acceleration. The issue of tying together the disappearance of the early universe’s cosmological constant with what the source of the current dark energy might be, and why the density of the latter is currently coincidentally close to that of the matter fields, is a challenge for which cosmology still searches for a physically justifiable solution. Some authors have suggested an anthropic principle [13] to explain the observations.

In this article a potential framework for addressing both the Cosmological Constant Problem and the *dynamical problem* is proposed. The framework is based on the premise that the background dark energy is both dynamical and interacting. It is possible that there may be several physical arguments which can be put forward to justify this notion, among which is the following example. According to general relativity a vacuum dominated universe (spacetime) has an asymptotically de Sitter geometry with an asymptotically de Sitter horizon. Quantum considerations indicate (for a review see e.g. [14a,b]) that near the horizon such a spacetime is associated with time asymmetric quantum states (analogous to those in the Unruh vacuum) which lead to a radiation flux. This means that one can treat the horizon (size χ^{-1}) as a radiative surface with a horizon temperature $T = \frac{\Lambda}{2\pi}$ and area $A_\chi = 4\pi\chi^{-2}$ so that the cosmological constant $\Lambda = 3\chi^2$ which is the energy source $\frac{\Lambda}{8\pi G}$ in a de Sitter spacetime also becomes a source of radiation flux, (i.e. particle creation). With this view, one is led to the conclusion that the vacuum energy associated with Λ must (1) decay and (2) create matter in the process. It is in this respect that such cosmic background vacuum (dark) energy is both dynamical and interacting. The issue of a time dependant “cosmological constant” has always been a touchy one in both General Relativity and Quantum Cosmology for seemingly different, but actually, related reasons. In Quantum Cosmology, which is based on Quantum Field Theory, there is no mechanism to change the (de Sitter) vacuum states; while in General Relativity, the Einstein field equations demand that the energy momentum tensor of the de Sitter spacetime can only be proportional to the spacetime metric $g_{\mu\nu}$. On the other hand the foregoing considerations suggest that just as the eternal black hole (and its associated Hartle-Hawking vacuum) in classical General Relativity is an idealization, analogously, a vacuum dominated universe can not stay in an eternally fixed de Sitter state. As pointed out in [14b], the temperature and entropy for a collapsing black hole and those of an asymptotically de Sitter spacetime arise in identical manner due to identical mathematical formalism. It would therefore be surprising if one spacetime (de Sitter) is static while the other one

is non-static. Consequently, in our treatment, Λ must decay. Such a feature of an asymptotically de Sitter spacetime, is among potential tools one can apply to our universe to understand why " Λ is small now but not zero" and also (as we find soon) points to a potential resolution of both the *coincidence problem* and the *dynamical problem*.

This paper discusses only the macroscopic effects (of the above underlying microscopic physics of vacuum matter interactions), namely, the global cosmic dynamics. The approach is reminiscent of what thermodynamics is to statistical mechanics. In this respect the universe can be treated as a vacuum-driven cosmic engine. The cosmic vacuum energy acts as the fuel, doing work on the universe by accelerating it. As is the case with any engine, it is impossible for the universe to convert all its fuel to mechanical work without dissipating some through irreversible processes that increase its internal energy. This is analogous to a statement of the first law of thermodynamics and is suggestive of a first law of cosmic dynamics. In turn, the irreversible dissipations lead to a cosmic form of the second (or entropy creation) law¹, $s^\mu_{;\mu} \geq 0$. Looking at the simplicity of these notions, it is tempting to wonder what new feature the approach brings. In this treatment, the irreversible dissipation process introduced above is, indeed, cardinal to the dynamics of the universe, and to underscore this we make the following *Cosmic Equilibrium Conjecture* (CEC):

Conjecture 1 *The universe will increase its inertia (matter creation) as a back-reaction to any (vacuum energy) influences tending to move it away from (dynamical) equilibrium.*

Facilitating such processes, on a macroscopic scale, is a (bulk viscous) creation pressure π_c , which arises as a back-reaction to the spacetime acceleration. The role of this back-pressure is to build up inertia to oppose the change (acceleration) which creates it, in the first place. It does this by creating matter. This behavior is reminiscent of equilibrium-seeking systems in nature. For example, in electromagnetic induction, Lenz's law [15] predicts a back emf; while in QCD, quark confinement is explained in terms of asymptotic freedom [16]. It is interesting that in this latter process the system will actually create matter, when faced with external influences (forces) tending to shift it from equilibrium.

In the current treatment, the vacuum-dissipation/matter-creation process couples the vacuum to matter through a parameter K , which we constrain. In this scenario, (increasing cosmic inertia) physical modes will be created until the cosmic acceleration is offset and the creation pressure π_c vanishes. As an example, this is how (in this model) the universe is able to get out of inflation and become matter dominated. With no further immediate need for creation, the existing matter fields in a given comoving volume begin to dilute normally, (which is relatively faster). Such redshifting of the matter fields, eventually, leads to a further vacuum domination which in turn commences a new cycle of cosmic acceleration and the consequent creation. As a result, the process sets up oscillations in the decaying vacuum and corresponding sympathetic ones in the matter fields. These coupled oscillations constrain the fields to track each other naturally. As the universe evolves in time, successive periods of acceleration are dominated by ever decreasing (decaying) energy densities. The resulting back-reactions will similarly be increasingly weaker, producing infra-red-shifting dominated modes. Each such acceleration will be weaker and last longer than its predecessors. This would explain why the current cosmic acceleration has lasted much longer than the early cosmic inflation.

These ideas are used in this paper to establish bounds on the relative magnitudes of the evolving fields. It is in this sense that the model offers a natural and conceptually simple channel to discussing the *coincidence problem*, while also suggesting a way to deal with the *dynamical problem*. The approach also independently relates the current vacuum-dominated state of the universe to the early conditions by suggesting inflation as a natural initial condition to the current classical dynamics. At the same time, it provides a rationale for a natural exit to, not only the inflationary scenario but also, the current acceleration. Lastly, the *Cosmic Equilibrium Conjecture* (CEC) suggests a physically motivated ansatz to what is otherwise a (time-old) philosophical question, namely, "why does the universe need and create matter?". The model has significant observational

¹It is interesting that, in cosmology, while the universe is known the second (entropy) law, the notion of a first cosmic law is not developed

consequences. The constructed equation of state $w(a)$ for the interacting dark energy suggests a quasi-periodic acceleration which can be directly tested by future experiments such as SNAP [17]. In this article, the emphasis is to study the interactions between dark energy and matter fields and how, in particular, they lead to a resolution of the Coincidence Problem. To this end, it is sufficient to consider the behavior of the relevant energy-momentum tensor. This leads to the evolution of the fields $\rho_v(a)$ and $\rho_m(a)$ with the scale factor, a on a fixed gravitational background configuration. A derivation of the time evolution of the scale factor $a(t)$, which requires solving the resulting gravitational field equations for the matter fields in the presence of interacting dark energy, will be made in a forthcoming article.

The rest of the article is organized as follows. In Section 2 the working equations for an interacting vacuum energy are set up. Section 3 discusses the evolution of the vacuum energy density in the presence of matter creating processes. Section 4 lays out the framework for the evolution of the matter fields in such an environment. A potential resolution to both the *coincidence problem* and the *dynamical problem* is presented and observational tests are mentioned. Section 5 concludes the article. Throughout the article the terms *cosmic vacuum energy* and *dark energy* are used interchangeably to describe a background cosmic energy that is both dynamical and interacting.

2 Interacting cosmic vacuum energy: working equations

2.1 Features

In modeling an interacting vacuum, we take as the generator of cosmic vacuum energy $\frac{\Lambda}{8\pi G}$, a dynamical cosmological parameter, Λ of the form

$$\Lambda(t) = m_{pl}^4 \left(\frac{a_{pl}}{a(t)} \right)^{\sigma(t)} e^{-\tau H} = \Lambda_{pl} \left(\frac{a_{pl}}{a(t)} \right)^{\sigma(t)} e^{-\tau H}, \quad (2.1)$$

where m_{pl} is the Planck mass, a_{pl} (the fluctuation scale) is the size scale of a causally connected region of space at the Planck time t_{pl} , and τ is of order of the Planck time. Further, H is the Hubble parameter and $a(t)$ is the cosmic scale factor. A distinguishing feature of the model, is that the power index σ is not a fixed constant in time (as in most previous cases reviewed in [6]) but rather depends on time in a manner to be determined in Section 3. This feature, which is a consequence of the *Cosmic Equilibrium Conjecture* allows the background dark energy to couple to matter.

As is apparent from Eq. 2.1, in this scenario, the Λ -field initially ($H^{-1} \rightarrow 0$) appears to tunnel into existence. Thereafter, for $t \lesssim \tau$ the dynamics of the early universe is driven by the growing term $e^{-\tau H}$ and should be dominated by a quantum character. A rigorous discussion of the physics of the period $t \lesssim \tau$ requires a quantum theory of gravity and is, therefore, still beyond the scope of the present treatment. It is, however, worth pointing out that for $t \lesssim \tau$ the field in Eq. 2.1 is growing in density and so one expects its equation of state during this early period to take the form $w(t) < -1$. As the field evolves towards its stationary point, $\frac{d\Lambda(t)}{dt} = 0$, the equation of state $w(t)$ approaches -1 from below, to temporarily mimic a cosmological constant. In the immediate neighborhood of $\frac{d\Lambda(t)}{dt} = 0$, or $\left[\ln \left(\frac{a_{pl}}{a(t)} \right) \frac{d\sigma}{dt} - \sigma(t) H - \tau \frac{dH}{dt} \right] = 0$, Λ is virtually constant with maximum potential energy (possibly in the range $\langle \rho_{vac} \rangle = \frac{\Lambda}{8\pi G} \sim 10^{94} g cm^{-3}$). These conditions give rise to a cosmic inflation, in the early universe. As the Hubble time H^{-1} grows, the quantity $e^{-\tau H}$ quickly approaches saturation $e^{-\tau H} \rightarrow 1$. Subsequently, the dynamics of the universe becomes increasingly classical, being driven by the $a(t)^{-\sigma(t)}$ part of Λ . It is this latter phase that this article discusses.

Assuming successful inflation in the early era, then by the time the universe is about 1 second old, the scale factor is usually considered to have grown by a factor of about 10^{28} . In our case, this means that Λ will have decayed by $10^{-28 < \sigma(t) >}$, in the process (leaving behind a dense field of relativistic particles and radiation). It turns out (as is shown in Section 3), that the time average value of the power index function, in the model, is $\langle \sigma(t) \rangle = 2$. As a result, the cosmic

vacuum energy density does not interfere with the usual early cosmological processes like big bang nucleosynthesis (BBN) [18], but instead allows such processes to proceed as predicted by the standard big bang model. Further, since the inflation era to date, $a(t)$ is known to have evolved by about 10^{60} . The overall result is that $[\Lambda_{now} \sim a_0^{-2}] \approx 10^{-120} \Lambda_{pl}$. This provides a heuristic explanation to the “why is Λ small now” part of the cosmological problem, which is consistent with observations, pending the proof (later in the discussion) that $\langle \sigma(t) \rangle = 2$.

In the remaining part of this article, tools are developed to address both parts of the Cosmological Constant problem and the *dynamical problem*. Throughout the proceeding discussion, we only deal with the late-time evolution of the universe. Here, $H^{-1} \gg \tau$ so that $e^{-\tau H}$ can, justifiably, be set to unity with the result that the effective late-time Λ is controlled by $a(t)^{-\sigma(t)}$ and the universe evolves quasi-classically². The associated vacuum energy density can, thus, be written as

$$\rho_v(t) = \frac{\Lambda(t)}{8\pi G} = \left(\frac{a(t_0)}{a(t)} \right)^{\sigma(t)} \rho_v^{(0)}, \quad (2.2)$$

where, for convenience (in the second equality) the dynamical vacuum energy density is normalized about its observed current value $\rho_v^0 \sim 10^{-30} \text{ g cm}^{-3}$. In our notation the sup/sub-script 0 on a quantity denotes its current value.

2.2 Energy equations and particle creation

Consider the dynamical evolution of a self-gravitating cosmic medium consisting of a two-component perfect fluid. The total energy momentum tensor $T_{\mu\nu}$ for all the fields is given by

$$T_{\mu\nu} = T_{\mu\nu}^{(m)} + T_{\mu\nu}^{(vac)} = [\rho + p] v_\mu v_\nu + p g_{\mu\nu}, \quad (2.3)$$

where $T_{\mu\nu}^{(m)}$ and $T_{\mu\nu}^{(vac)}$ are, respectively, the matter and the vacuum contributions to $T_{\mu\nu}$, while $\rho = \rho_m + \rho_v$, $p = p_m + p_v$ and v_μ is the 4-velocity. Conservation of the total energy, $v_\mu T^{\mu\nu}_{;\nu} = 0$, leads to standard continuity equation

$$[\dot{\rho}_m + (\rho_m + p_m) \theta] + [\dot{\rho}_v + (\rho_v + p_v) \theta] = 0, \quad (2.4)$$

where $\theta = v^\alpha_{;\alpha}$ is the fluid expansion parameter and $\dot{\rho}$ is the derivative taken along the fluid worldline, $\dot{\rho} = v^\alpha \nabla_\alpha \rho$. In this treatment, the total energy in the cosmic fluid is conserved. However, because of the assumed interacting nature of dark energy the individual components are, in general, not conserved. In particular, because of the CEC (and under conditions to be discussed) the vacuum will act as a source of dissipative processes, while the matter component acts as a sink of such processes. This means that one can write

$$v_\mu T^{\mu\nu}_{(vac); \nu} = -v_\mu T^{\mu\nu}_{(m); \nu} = \Psi \quad (2.5)$$

where $\Psi > 0$ is the particle source strength. Note that Eq. 2.5 is still consistent with Eq. 2.4.

In principle, a non-equilibrium system can involve dissipative processes, ranging from scalar fluxes like bulk viscous pressure and particle creation pressure to tensorial shear viscosity stresses and energy transport [19]. For the case under consideration, the latter are globally suppressed in an isotropic and homogeneous universe as assumed here. The contribution to the entropy source, by the remaining scalar processes, is given by [19]

$$S_{;\alpha}^\alpha = -\frac{\Pi \theta}{T} - \frac{\pi_c \theta}{T} - \frac{\mu \Psi}{T}, \quad (2.6)$$

where π_c is the creation pressure, Π is the bulk viscous pressure, μ is the chemical potential and T is the temperature. In general, it can be shown [19, 22] that in the absence of the particle source strength Ψ , the creation pressure and the bulk viscosity become the same process and there is no particle creation.

²A matter creating universe can not be entirely classical.

In this article, we only pay attention to particle creating processes described by the creation pressure, π_c . Then, use of Eq. 2.5 shows Eq. 2.4 to consist of two dissipative equations

$$[\dot{\rho}_v + (\rho_v + p_v) \theta] = \pi_c \theta \quad (2.7)$$

and

$$[\dot{\rho}_m + (\rho_m + p_m) \theta] = -\pi_c \theta. \quad (2.8)$$

We suppose that the interacting dark energy satisfies an effective equation of state of the form

$$p_\Lambda = w \rho_\Lambda, \quad (2.9)$$

where $-1 \leq w \leq 0$. Note the ‘unusual’ upper limit $w \leq 0$ (instead of $w \leq -\frac{1}{3}$) for the interacting dark energy, signifying a matter dominated state, in which the effective pressure of the interacting dark energy hardly contributes to the dynamics of the universe. The quantity w will have an implicit time dependence (through the fields) when particle creating processes are in force. It will be shown (see next sub-section) that $w \rightarrow -1$ in the limit $\rho_m \rightarrow 0$. Thus in this limit, one recovers (in an asymptotically locally inertial frame), the standard Lorentz invariant vacuum. Further, we assume [20] that the newly created particles are virtually in thermal equilibrium with the existing matter fields as soon as they are created. This is reasonable in light of the approach we have adopted above which suppresses non-matter creating bulk viscosity effects. Thus, the only source of entropy is matter creation. As a result, the matter fields satisfy the usual γ -law equation of state

$$p_m = (\gamma - 1) \rho_m, \quad (2.10)$$

where $\gamma = \{1, \frac{4}{3}\}$.

The density fields ρ_v and ρ_m can be determined using any two of the three equations, namely, the energy balance (continuity) equation (Eq. 2.4) and the source-sink equations (Eqs. 2.7 and 2.8). To proceed, however, one requires the functional forms of both the power index function $\sigma(a(t))$ and the effective equation of state $w(a(t))$ for the interacting vacuum. This problem is addressed in the next section.

3 Evolution of vacuum energy density

In this section we develop the functional form of the interacting vacuum energy density ρ_v and the pressure p_v by deriving $\sigma(a)$ and $w(a)$, and study how the evolution of these fields suggests a natural resolution of the two problems set out in Section 1.

3.1 Creation pressure

In a vacuum dominated universe, the total gravitating energy is $\rho + 3p < 0$ (which, as is known, violates the strong energy condition SEC). The excess negative pressure accelerates the universe. In turn, according to the *Cosmic Equilibrium Conjecture*, the universe back-reacts to this non-equilibrium scenario by building inertia (matter modes) through the creation pressure π_c . We assume the local equilibrium hypothesis [21] that non-equilibrium quantities in the model depend locally on similar variables as the equilibrium ones. It follows, then, that the particle creating pressure π_c , which must depend on the available excess negative pressure, will be proportional to the total gravitating energy, $\rho + 3p$, of the universe. Consequently, we can write

$$\pi_c = K [(3\gamma - 2) \rho_m - 2\rho_v], \quad (3.1)$$

where the dimensionless proportionality parameter K is to be constrained. This parameter K (here and henceforth referred to as the vacuum dissipation parameter) couples the cosmic vacuum energy to matter through vacuum dissipation/matter creation processes (Eqs. 2.7 and 2.8). In the (idealized) limit $\rho_m \rightarrow 0$, K would probe the efficiency $\epsilon = 1 - K$ of the universe as a cosmic engine. Our immediate goal is to relate the creation pressure π_c to the dynamical evolution of the density fields ρ_v and ρ_m . To proceed, we start by constructing an effective equation of state for the interacting background dark energy.

3.2 Cosmic Expansion and Field dilution

As the universe expands the densities of the background matter fields are known to dilute as $a(t)^{-3\gamma} = a(t)^{-3(1+\tilde{w})}$, where $\tilde{w} = \{0, \frac{1}{3}\}$. On the other hand, a dynamical interacting vacuum energy (of the functional form in Eq. 2.2) will suffer an energy deficit because it is a source of matter fields $v_\mu T_{(vac)}^{\mu\nu}{}_{;\nu} = \Psi \neq 0$. The vacuum energy in a given comoving volume will appear to redshift with a dilution law $a(t)^{-3(1+w)}$. To preserve the functional behavior of the vacuum energy density, in Eq. 2.2, one must have $3(1+w) = \sigma(a(t))$, where the quantity $\sigma(a(t))$ is to be determined. This gives $w = -(1 - \frac{\sigma}{3})$, which leads to an effective equation of state

$$p_v = -\left(1 - \frac{\sigma}{3}\right) \rho_v. \quad (3.2)$$

It is convenient (for now) to write the time evolution of the vacuum energy as an evolution in the scale factor $a(t)$. In the FRW models, the source of the four-velocity is the Hubble parameter, then $v^\alpha{}_{;\alpha} = \theta = 3\frac{\dot{a}}{a} = 3H$. This, along with use of Eq. 3.2, transforms Eq. 2.7 to

$$a\dot{\rho}_v + \sigma\rho_v = 3\pi_c, \quad (3.3)$$

where $\dot{\rho}_v = \frac{d\rho_v}{da}$ and as before $\rho_v = \rho_v^0 \left(\frac{a_0}{a}\right)^{\sigma(a)}$. Using Eqs. 3.1, 3.2 and 3.3 the effective equation of state of an interacting dark energy can be written as

$$w = -1 + \left\{ \frac{3K[(3\gamma - 2)\rho_m - 2\rho_v] - a\dot{\rho}_v}{3\rho_v} \right\}. \quad (3.4)$$

3.3 Interacting dark energy: a solution

One expects that the asymptotic forms of the relation in Eq. 3.4 should recover the more familiar equations of state of ordinary physical fields. In particular, when the vacuum energy dominates the universe ($\rho_v \gg \rho_m$), one expects $w \rightarrow -1$. From Eq. 3.4, this limit requires that $-2K - \frac{1}{3} \left(a \frac{\dot{\rho}_v}{\rho_v}\right) = 0$, which integrates to

$$\rho_v |_{\rho_m \rightarrow 0} = \rho_v^0 \left(\frac{a_0}{a}\right)^{6K}, \quad (K \geq 0). \quad (3.5)$$

On the other hand, the system evolves so that eventually there is no interaction between the vacuum and matter fields once the creation pressure π_c vanishes. This leads to the limit where effectively $w \rightarrow 0$. Applying this limit on Eq. 3.4 gives $w = -1 - \frac{1}{3} \left(a \frac{\dot{\rho}_v}{\rho_v}\right) = 0$, so that

$$\rho_v |_{\pi_c \rightarrow 0} = \rho_v^0 \left(\frac{a_0}{a}\right)^3. \quad (3.6)$$

The above limits (Eqs. 19 and 20) establish bounds on the power index function $\sigma(t)$ as

$$6K < \sigma \leq 3 \quad (3.7)$$

At $\sigma = 3$, the vacuum (dark) energy has a maximum dilution rate and decouples from matter. The evolution of the dark energy to this state also implies a relative increase (and eventual domination) of the matter fields over the vacuum energy. Here, matter creation is suppressed consistent with the requirements of the *Cosmic Equilibrium Conjecture*. With no more creation, the relative matter dilution grows towards its “normal” rate of $\sim a^{-3\gamma}$. In turn, this increases the density of the dark energy relative to that of the matter fields, eventually setting the former into dominance. The universe then begins to accelerate and, in the process, builds the creation pressure π_c , as a back-reaction to the acceleration. According to Eqs. 3.5 and 3.7 this creation rate will grow towards a maximum, as σ approaches its minimum, $\sigma_{\min} = 6K$. As the matter creating vacuum dumps in more and more matter, the matter fields are evolving with a decreasing dilution rate.

The relative increase in the density of matter fields, makes it less and less favorable for the vacuum to create more matter. As a result, the creation rate is decreasing towards a minimum. Eventually, the system ends up back where it started (with no more creation) as $\rho_m \sim a^{3\gamma}$ and $\pi_c \sim 0$ and a new cycle begins.

It follows, then that, in general, the power index function $\sigma(t)$ will be oscillatory within the bounds $6K < \sigma(t) \leq 3$ as established in Eq. 3.7. Oscillations in $\sigma(t)$ naturally imply oscillations in the decay rate of the density function, $\rho_v \sim a^{-\sigma(t)}$. Here, it should be pointed out that while the power index function (we seek) is oscillatory, the density function $\rho_v \sim a^{-\sigma(t)}$ must be single-valued in $a(t)$ in order to be consistent with the requirement that (globally) matter/entropy creation from the vacuum is an irreversible process. The vacuum energy decays into matter but not vice versa. This concept is explored further in Section 3.4.1. Thus such oscillations will be imprinted on a decaying energy background. As is shown below, this feature is inherent in the model.

The oscillations in the decay rate of ρ_v signify matter creation from the vacuum. This implies that such periodic matter creation will necessarily induce sympathetic oscillations in the matter density fields ρ_m . It is in this sense that the two fields, in time, track each other. To support these assertions we start by studying how the creation pressure will drive the oscillations.

Recall $\rho_v = \left(\frac{a_0}{a}\right)^{\sigma(t)} \rho_v^0$. On taking derivatives with respect to $a(t)$, we have that $\dot{\rho}_v(a) = \left[\dot{\sigma} \ln\left(\frac{a_0}{a}\right) - \frac{\sigma}{a}\right] \rho_v$. Substituting for $\dot{\rho}_v$ in Eq. 3.3 gives

$$\dot{\sigma} a \ln\left(\frac{a_0}{a}\right) = \frac{3\pi_c}{\rho_v}. \quad (3.8)$$

Now, consider $\sigma(\psi)$, where ψ is some function of $a(t)$. Then $\dot{\sigma} \frac{da}{d\psi} = \frac{d\sigma(\psi)}{d\psi}$. Comparing this with Eq. 3.8 one sees that, on setting $\frac{d\psi}{da} = \left[a \ln\left(\frac{a_0}{a}\right)\right]^{-1}$, then

$$d\sigma = \left(\frac{3\pi_c}{\rho_v}\right) d\psi. \quad (3.9)$$

Thus, $\frac{3\pi_c}{\rho_v} d\psi$ is a perfect differential of the power index function σ .

One expects solutions to Eq. 3.9 with certain specific characteristics. First, from the discussion above, such solutions should be oscillatory in ψ and also bounded by Eq. 3.7. Secondly, all the $\psi(a(t))$ dependence in the solution σ should be purely sinusoidal, so as to avoid unphysical solutions of the form $\rho \sim a^{f(a)}$, where $f(a)$ is non-oscillatory. Finally, as is pointed out in the *Cosmic Equilibrium Conjecture*, matter creation (in this model) is an opposite reaction to the vacuum-induced *positive* acceleration of the universe. Thus, the sinusoidal part of $\frac{d\sigma}{d\psi}$ should be negative definite, i.e. of the form $\sim (-\sin^2 \psi)$, at the least.

The simplest solution that satisfies these conditions has the form $\sigma = \sin 2\psi + A$, where A is a positive constant. Only then are the requirements on $\frac{d\sigma}{d\psi}$ satisfied, since now $\left\{\frac{d\sigma}{d\psi} = \frac{3\pi_c}{\rho_v} = 3K \left[\frac{(3\gamma-2)\rho_m - 2\rho_v}{\rho_v}\right]\right\} = -4\sin^2 \psi + 2$. On rearranging out the terms in the last equality of this equation one finds that

$$\sin^2 \psi = \left[\frac{2(1+3K)\rho_v - 3K(3\gamma-2)\rho_m}{4\rho_v}\right]. \quad (3.10)$$

This function is minimum ($\sin^2 \psi = 0$) at

$$\min \rho_v = \left(\frac{3K}{6K+2}\right) (3\gamma-2) \rho_m, \quad (3.11)$$

and maximum ($\sin^2 \psi = 1$) at

$$\max \rho_v = \left(\frac{3K}{6K-2}\right) (3\gamma-2) \rho_m. \quad (3.12)$$

We soon return to these limits in the next section. To formally complete the solution ($\sigma = \sin 2\psi + A$), one notes on applying the limits from Eq. 3.7 that when $\sin 2\psi = 1$, then $1 + A = 3$. With this, we find the solution for the power index function σ to be

$$\sigma(\psi) = 2 + \sin 2\psi, \quad (3.13)$$

where as shown earlier, ψ is given by $\frac{d\psi}{da} = [a \ln(\frac{a_0}{a})]^{-1}$. Using Eq. 3.13 in Eq. 2.2 we finally find that the interacting dark energy density, in this model, evolves with the scale factor as

$$\rho_v(a) = \left[\frac{a_0}{a}\right]^{(2+\sin 2\psi(a))} \rho_v^0. \quad (3.14a)$$

or in terms of the redshift parameter $z = \frac{a_0}{a} - 1$ we have

$$\rho_v(z) = [z + 1]^{(2+\sin 2\psi(z))} \rho_v^0, \quad (3.14b)$$

where now $\frac{d\psi}{dz} = [(z + 1) \ln(z + 1)]^{-1}$. Further, Eqs. 3.2 and 3.13 give the working equation of state for the interacting dark energy in this model

$$p_v = -\frac{1}{3}(1 - \sin 2\psi) \rho_v. \quad (3.15)$$

It is this equation of state that, in future, can be compared with observations like SNAP [17].

3.4 Some features of the ρ_v solution

Eqs. 3.13 and 3.15 give the formal solution for the interacting dark energy in this model. According to this solution, during the evolution of the universe, ρ_v has the highest dilution rate at points characterized by $\sin 2\psi = 1$, where $\psi_n = (\frac{1}{4} + n)\pi$, $\{n = 0, 1, 2, \dots\}$. It is here that the vacuum energy density becomes least interacting, since effectively it decouples from the matter fields as $w \rightarrow 0$. On the other hand, the field has its least dilution rate (and is most interacting) at epochs characterized by $\psi_n = (\frac{3}{4} + n)\pi$, $\{n = 0, 1, 2, \dots\}$.

3.4.1 Mean decay path and why Λ is small now

For $\psi_n = \frac{n}{2}\pi$, $\{n = 0, 1, \dots\}$, $\sin 2\psi = 0$. Since $\psi = \psi(a(t))$, and noting that $\langle \sin 2\psi \rangle = 0$, we see from Eq. 3.13 that $\langle \sigma \rangle = 2$. The implication here is that the interacting vacuum energy density, in this model, decays along a mean evolutionary path of a^{-2} , with oscillations of $\sin 2\psi$ about this decay path. Thus, in an expanding universe, according to this result, the interacting vacuum energy density is inherently a decaying system. This is, indeed, consistent with our premise (in Section 1) that Λ must decay. In Section 2.1, a heuristic argument was given to explain why Λ is small now. It was seen that setting $\sigma = 2$ implies $[\Lambda_{now} \sim a_0^{-2}] \approx 10^{-120} \Lambda_{pl}$. In our treatment, we have recovered this value as the average of the oscillations in the power index, $\langle \sigma \rangle = 2$. Thus our treatment solves the "why is Λ small now" in a way that is consistent with current observations [1].

In the past, several phenomenological models have been developed (see [6] and citations) in which the vacuum energy density ρ_v evolves with the scale factor as a^{-m} , where the index m has a fixed value. In such models ρ_v is not explicitly interacting, as is indicated by the constant nature of the index m . One such set of models that has gained considerable popularity (see citations in [6]) evolves ρ_v as an inverse square power law ($m = 2$), in the scale factor. For the reasons given above, the ($m = 2$) models [6] have in the past offered a satisfactory resolution of why Λ is small now. It is worthy noting that the approach presented in the present work recovers this inverse square law as the mean decay path for an interacting vacuum energy density, $\langle \rho_v \rangle$. Moreover, for $a \gg a_0$, as the universe ages $\sigma \rightarrow 2$ so that (see also Section 4.2) σ can be replaced by its average value, $\langle \sigma \rangle = 2$. In this way, our model can be related to the inverse square, $\Lambda \sim a^{-2}$, models.

The positivity feature of σ (see Eq. 3.13), $\sigma \geq 1 > 0$, ensures that even in the face of the oscillations in the power index function, the energy function ρ_v is still always a decreasing single-valued function of the scale factor $a(t)$. To appreciate the physical significance of this feature, one need only consider its contradiction, where ρ_v evolves as a multi-valued function of $a(t)$. Such a scenario would imply a two-way process where the vacuum (dark) energy and matter alternately turn into each other. Globally, this would be unphysical (at least in our part of the universe) since it would result in periodic global violations of the entropy (evolution) law and suggest a rotating thermodynamic arrow of time. In this model, the creation processes tending to increase cosmic inertia are one-way and lead to entropy growth. Thus the treatment ensures that the evolutionary law of ρ_v is consistent with a causal universe.

3.4.2 The parameter space of K

The preceding discussion depicts a universe undergoing periods of vacuum-driven acceleration, punctuated by periods of matter domination. One can gain some deeper insight in the cosmic dynamics during each of such acceleration periods by paying attention to the associated parameter K . Suppose, for example, at some point in time (not unique) during a given period of cosmic acceleration one can measure the contemporary values of ρ_v and ρ_m or just the ratio $\beta = \frac{\rho_v}{\rho_m}$. Then the maximum value β_{\max} , coinciding with $\max \rho_v$ during such a (local) period can not be lower, i.e. $\beta_{\max} \not\prec \beta$. This argument can be used to put reasonable bounds on K during each such period since from Eq. 3.13 we must have

$$\beta \leq \beta_{\max} = \left(\frac{3K}{6K-2} \right) (3\gamma - 2). \quad (3.16)$$

Thus any measured value of β gives one information on the upper-bound on K for that period (which obviously would be better tightened by knowledge of β_{\max}). For example, as observations indicate, the current value is $\beta \approx 2$. This suggests that for the current (cold $\gamma = 1$) universe, $K \not\prec \frac{4}{9}$. Moreover from Eq. 3.10 one gets a lower bound on K for the interactions to result in particle production. In the (idealized) limit $\rho_m \rightarrow 0$, one finds that $K \rightarrow \frac{1}{3}$.

The two bounds above thus constrain the parameter space of the dissipation parameter K in the current universe to

$$\frac{1}{3} < K \lesssim \frac{4}{9}. \quad (3.17)$$

The one-way character of the dissipation process brings out yet another important feature. Clearly as the universe gets older, the upper bound on K grows as $\beta_{\max} \rightarrow 1$ revealing the source of the current *coincidence problem*. In the next section we put bounds on their relative amplitudes as the fields evolve. The above lower bound $\frac{1}{3} < K$ suggests that, in this model, the parameter space $0 \leq K \leq \frac{1}{3}$ is forbidden with regard to matter creation. One can, however, imagine a time in the very early history of the universe when the vacuum energy therein was very large and in a purely potential (PE) form. Such conditions would imply that, at the time, $K = 0$. Then Eq. 3.7 shows that $\rho_v \sim a^{-(6K)=0}$ and the vacuum energy density is constant in time (albeit temporarily). Only then could the universe as an engine seem to operate at 100% efficiency. Clearly this is the energy in a cosmological constant and it would inflate the universe. In turn, because inflation would tend to strongly and suddenly move the universe away from equilibrium conditions, the *Cosmic Equilibrium Conjecture* implies that the associated backreaction also be strong and sudden. Thus, virtually all the physical (matter) modes are created in the early universe. The space $0 \leq K \leq \frac{1}{3}$ during which matter creation is forbidden seems to provide a small window for inflation before the backreaction sets in as $K \rightarrow 0$ from below. Accordingly, there are two consequences to this. First, it is in immediate reaction to this inflationary scenario that most of the present inertia (matter) and entropy is created. In turn, it is precisely the growth of such inertia (i.e. $K \rightarrow \frac{1}{3}$) that would lead the universe to a graceful exit from inflation. The inflationary scenario would probably last as long as it would take for the dissipation parameter to grow to $K \rightarrow \frac{1}{3}$. Thus, in this model, inflation and its immediate self destruction are natural initial conditions to the current evolution of the universe.

4 Consequences of vacuum decay

4.1 Evolution of the matter fields density, $\rho_m(a)$

The preceding analysis for the evolution of ρ_v has been based on the source equation (Eq. 2.7). In order to discuss the evolution of the matter fields density $\rho_m(a)$ in the presence of a creation pressure π_c , the above results can be introduced either in the energy balance equation (Eq. 2.4) or the sink equation (Eq. 2.8). Choosing the latter and rewriting Eq. 2.4 as a function of $a(t)$ one finds

$$a\dot{\rho}_m + 3\gamma\rho_m + 3\pi_c = 0, \quad (4.1)$$

where we have used Eq. 2.10 and $\theta = 3H$. From Eqs. 3.10,

$$3\pi_c = 2(1 - 2\sin^2\psi)\rho_v, \quad (4.2)$$

where ρ_v is given by Eq. 3.14. Introducing these results in Eq. 4.1 gives

$$a\dot{\rho}_m + 3\gamma\rho_m + 2(1 - 2\sin^2\psi)\left[\bar{\beta}\left(\frac{\alpha}{a}\right)^{(2+\sin 2\psi)}\right] = 0, \quad (4.3)$$

where, as previously established, $\frac{d\psi}{da} = [a \ln(\frac{\alpha}{a})]^{-1}$. Eq. 4.3 governs the evolution of the matter fields density $\rho_m(a)$ in the model.

In this article the main aim has been to build a framework for discussing both the Cosmological Constant Problem and point a way to discussing the *dynamical problem*. In the previous section we have touched on the question of “why Λ is small now”. It still remains to discuss the remaining problems. As it turns out, the results of the preceding section are sufficient for such a discussion. Consequently, we defer a discussion of the solutions to Eq. 4.3 to concentrate on the two remaining problems.

4.2 The coincidence problem and the dynamical problem

Using the results of Eqs. 3.10 to 3.12 one finds that the vacuum dilution rates relate directly to the vacuum to matter ratios β in the universe. In particular, at $\sin^2\psi = 0$, when the vacuum is at its most dilution rate, Eq. 3.11 constrains the minimum density value of the vacuum to $\min\rho_v = \left(\frac{3K}{6K+2}\right)(3\gamma-2)\rho_m$. On the other extreme at $\sin^2\psi = 1$, when the vacuum is at its least dilution rate, Eq. 3.12 constrains the maximum density value of the vacuum to $\max\rho_v = \left(\frac{3K}{6K-2}\right)(3\gamma-2)\rho_m$. These results imply that as the universe evolves, the vacuum and matter fields are coupled through the coupling parameter K (see Eq. 3.16). The vacuum then tracks the matter fields naturally within the bounds given by Eqs. 3.11 and 3.12 as

$$\left(\frac{3K}{6K+2}\right)(3\gamma-2)\rho_m \leq \rho_v \leq \left(\frac{3K}{6K-2}\right)(3\gamma-2)\rho_m. \quad (4.4)$$

In particular, during the radiation era $\gamma = \frac{4}{3}$, the vacuum oscillates between the values $\left(\frac{3K}{3K+1}\right)\rho_m \leq \rho_v \leq \left(\frac{3K}{3K-1}\right)\rho_m$. On the other hand, during the cold matter era $\gamma = 1$, the vacuum oscillates between the values $\left(\frac{3K}{6K+2}\right)\rho_m \leq \rho_v \leq \left(\frac{3K}{6K-2}\right)\rho_m$. Clearly, provided $K > \frac{1}{3}$, the vacuum and matter field densities, ρ_v and ρ_m will track each other naturally. Further, as pointed out in Section. 3 the one-way character of the dissipation process implies that as the universe gets older, the upper bound on K grows so that $\beta_{\max} \rightarrow 1$ in the far future as $t \rightarrow \infty$. Thus, as long as the universe expands, the vacuum energy and matter density fields track each other with decaying amplitudes. This is our explanation of the so-called current *coincidence problem*. Further, as the universe gets older, the field oscillation amplitudes become smaller and also more stretched out since in this case $a \gg a_0 \Rightarrow \sin\psi(a) \rightarrow 0$. As a result the distant future universe creates

conditions under which $[\sigma(\psi) = 2 + \sin 2\psi(a)] \longrightarrow 2$. Under these conditions σ can be replaced by its average value, $\langle \sigma \rangle = 2$.

Finally, since the dynamics of the universe is determined by the behavior of the fields therein, the foregoing results can be used to predict the future evolution of the universe. In this sense, the results also address the *dynamical problem*.

4.3 Observational tests

The model predicts that the universe undergoes periods of variable (quasi-periodic) acceleration. Thus the observed current cosmic acceleration is one of these phases. In this respect the local (in time) dynamical evolution of the universe has two possibilities (Eq. 3.10): the universe is either moving away or towards a local $_{\max}\rho_v$. An increase in acceleration as a function of redshift z , for example, would indicate the universe is moving away from a local maximum of vacuum domination β_{\max} , and vice versa. The effective equation of state, in Eq. 3.15 provides information about this variability and how the resulting motion in the current local phase deviates from that due to a (would-be) constant background Λ vacuum. This behavior can be tested by future experiments like the SNAP project [17]. Further, a signature of such oscillations should be imprinted on CMB. A consideration of these observables is ongoing and will be reported in future.

5 Conclusion

In this paper a framework is proposed for addressing both the Cosmological Constant Problem and the associated *dynamical problem*. The underlying premise is that the background dark energy is both dynamical and interacting. Such a feature gives rise to a universe that behaves as a cosmic thermodynamic engine, with the vacuum (dark) energy as the input fuel. This energy does work by accelerating the universe. However, like any engine, it is impossible for the universe to use all the fuel (vacuum energy) to do work without some of it being dissipated. The rationale for such vacuum energy dissipation and the implied increased inertia (matter creation) is embodied in a *Cosmic Equilibrium Conjecture* we make with regard to the need, on the part of the universe, to always seek for equilibrium conditions. The increase in inertia is facilitated by a creation pressure π_c that arises as a back-reaction to the cosmic acceleration. The treatment, based on *Cosmic Equilibrium Conjecture*, provides a basis for a first law of cosmic dynamics which naturally gives rise to the second (entropy law) of cosmic dynamics.

As pointed out in Section 2, the “why is Λ small now?” part of the Cosmological Constant Problem is addressed by noting that in this model $\langle \sigma \rangle = 2$, so that $\langle \rho_v \rangle$ evolves as $\sim a^{-2}$. Such inverse square behavior was established in Section 3 From the inflation era to date, $a(t)$ has evolved by about 10^{60} . This implies that $\Lambda_{\text{now}} \approx 10^{-60\langle\sigma\rangle}\Lambda_{\text{pl}} = 10^{-120}\Lambda_{\text{pl}}$, a result which is consistent with observations.

It was discussed, in some detail, how the vacuum energy density ρ_v couples to the matter fields ρ_m through matter creation pressure π_c . This coupling is facilitated by a parameter K which to date (for the post-inflationary period) we have constrained to $\frac{1}{3} < K \lesssim \frac{4}{9}$. The process gives rise to a cosmic vacuum energy density which oscillates with a decaying amplitude. In turn, the coupled matter fields oscillate in sympathy. Consequently, the two coupled fields track each other naturally, as they evolve in time. We have put bounds (Eq. 4.4) on the relative evolution of the magnitudes of these fields up to the constrained free parameter K . Because vacuum dissipation is a one-way process, in this treatment, it follows that as the universe grows older the extremum values of the fields $_{\max}\rho_v$ and $_{\min}\rho_m$ become increasingly comparable so that $\beta_{\max} \longrightarrow 1$. It is in this sense that the model addresses the *coincidence problem* and predicts that in future acceleration periods the fields will become even more comparable. The bounds put on the relative magnitudes of these fields (Eq.4.4) also imply that the future evolution of these fields is predictable. Further, since it is these same fields that drive the universe, in the first place, the result is that the future dynamics of the universe becomes equally predictable. In this way the model addresses the *dynamical problem*.

It is pointed out that the coupling parameter K must have, at one time, had to grow from zero to its minimum operative value $\frac{1}{3}$. This growth corresponds to the cosmic vacuum energy changing from a purely potential form at $K = 0$ to a partially dissipated form $K > 0$. As long as $K < \frac{1}{3}$, there would be little or no matter created and the universe, essentially, behaves as a near-perfect engine with $\sim 100\%$ efficiency. The vacuum energy is then, mostly, in the form of a cosmological constant and the universe must inflate. Consequently, our treatment requires inflation as a natural initial condition, both for creation and for the current classical dynamics of the universe. Moreover, the growth of the dissipation parameter K in the early universe (and hence that of the creation pressure) as a back-reaction to the inflationary acceleration, creates a natural graceful exit out of inflation through increase of the universe's internal energy. Thus, in this model, the *Cosmic Equilibrium Conjecture* implies that inflation (through dissipative processes) oversees its own (almost immediate) destruction, to end almost immediately. It is in this sense that the approach predicts both inflation and a graceful exit, while at the same time justifying matter creation. Moreover, using the same mechanism, the universe enters and eventually exits from any subsequent accelerations, including the current one.

The model proposed here makes testable predictions that the dynamics of the universe is predictable with a quasi-periodic character that can be verified by future experiments. There are several issues that the model raises which are under consideration for future report. They include, among others, a discussion of the background global geometry as a solution of the gravitational field equations and a consideration of effects of the model on CMB.

Acknowledgement 2 *This work was made possible by funds from the University of Michigan.*

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