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Process capability for a complete electronic product assembly

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Process capability for a complete electronic product assembly

by

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Abstract

There are many studies about the process capability indices which are used to study if a process can meet specification. Unfortunately, there are few studies about the product capability. So, the main aim of this thesis is to present an alternative to determine the product capability. It is important to determine if when the quality characteristics of a product are assembled the final product will still meet the specification. This study proposes an approach to determine the product capability using the C_{pm} to analyze the capability of the quality characteristic. Also, this thesis proposes the use of weight to determine the influence of the quality characteristic in the final quality of the product. This study was divided in four steps, the first one the definition of the product and quality characteristics that will be used. The second one is the simulation study where the estimators used to determine the process capability indices are defined. The third one is the characteristic study which presents the CPI and the yield for the characteristics analyze. The last step is the product capability study which presents the product capability for the product.

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PCI	**Process Capability Index**
LSL	**Lower Specification Limit**
USL	**Upper Specification Limit**
MPPAC	**Multiprocess Performance Analysis Chart**

1. Introduction

Quality is the new wave trend for companies nowadays and for good reason. Every company wants to reduce their costs and increase customer satisfaction. The best way to do this is by increasing the quality management of their products. By doing this, the number of nonconformities and the time spent in inspection will decrease alongside with the increase in reliability of the product and customer satisfaction. Usually, the increase of quality will result in the increase of total cost, so it is extremely important to balance this quality improvement with the final cost of the product.

To achieve this balance, it is necessary to determine the deficiencies of the process, the best yield of the components, and to identify the improvements that will have more of an impact on the quality of the process. Unfortunately, in general, the resources available for improvements are limited. It is difficult to determine the deficiencies of the process because a product is made of different components with different standard quality characteristics (Ouyang, Hsu, & Yang, 2013). Also, a consequence of this process complexity is that it is hard to determine what improvements will have more of an impact on the final product.

In the electronic industry, these difficulties are even worse. The tolerance design of the components is tight, so the stability and reliability of the components will have a high impact on the quality of the product. Even small deviation can cause unpredictable results to the system (Zhai, Zhou, Ye, & Hu, 2013). Besides this, the output of some components will directly impact the output of others, so it is extremity important to analyze all of the connections between the parts and determine which components are more crucial for the whole product.

Due to all of these difficulties, there are many studies about the improvement of quality, how to balance the quality, and the yield of the product with the customer requirement. There are three parameters that have been widely used to measure the ability of the process to meet specification, they are the process yield, process expected loss, and the process capability indices (PCIs) (Chen, Huang, & Li, 2001). The process yield is the percentage of products units that pass the inspection, the process expected quality loos is the cost related with poor quality, and the PCIs are indices used to determine if a process if capable to meet the specifications. The higher the PCIs and process yield, the lower the cost due to poor quality.

There are many studies about PCIs and how they can be used to determine if a product meets the specifications. In this research, several methodologies to determine the process capability of an entire product is presented. Given this background, the best approach to analyze the process capability of electronical products is chosen. The Monte Carlo simulation will be utilized to generate the data that will be used to determine the PCIs values. The report will also examine the impact of the process capability of each characteristic of the whole product.

2. Related Work

2.1. Process Capability

The process capability indices are used to determine if a process is capable of producing products within a specification limit. It is important to remember that the use of the PCIs is recommended just for the process in statistical control, in other words, in any process where special causes of defects were identified and removed (Shewhart, 1939). In general, the PCI will compare the natural variability of the process and can be defined as

$$PCI = \frac{Alawable\ process\ spread}{Actual\ process\ spread} \tag{2.1}$$

In the last twenty years, several process capability indices were proposed. The first generation process capability index is based on the idea that if the process is within specification limits, the quality of the product will be good (Kureková, 2001). There are two first generation PCIs, the C_p and the C_{pk}.

2.1.1. Process Capability Index C_p

The C_p was proposed by Juran (1974) and is defined as

$$C_p = \frac{USL - LSL}{6 * \sigma} = \frac{d}{3 * \sigma} \tag{2.2}$$

Where the USL and LSL are the upper and the lower specification limit respectively, the σ in the process standard deviation and the d is the half specification.

The C_p measures the variability of the process relative to the specification limits, so the bigger its value, the smaller the variability will be. This index doesn’t take into account the deviation of the process mean from the target value and how the data is spread within the specification. In that regard, the use of this index can lead to the wrong acceptance of the process when the process has high variability and out of target. Figure 1 shows five different process samples that present a similar C_p value:

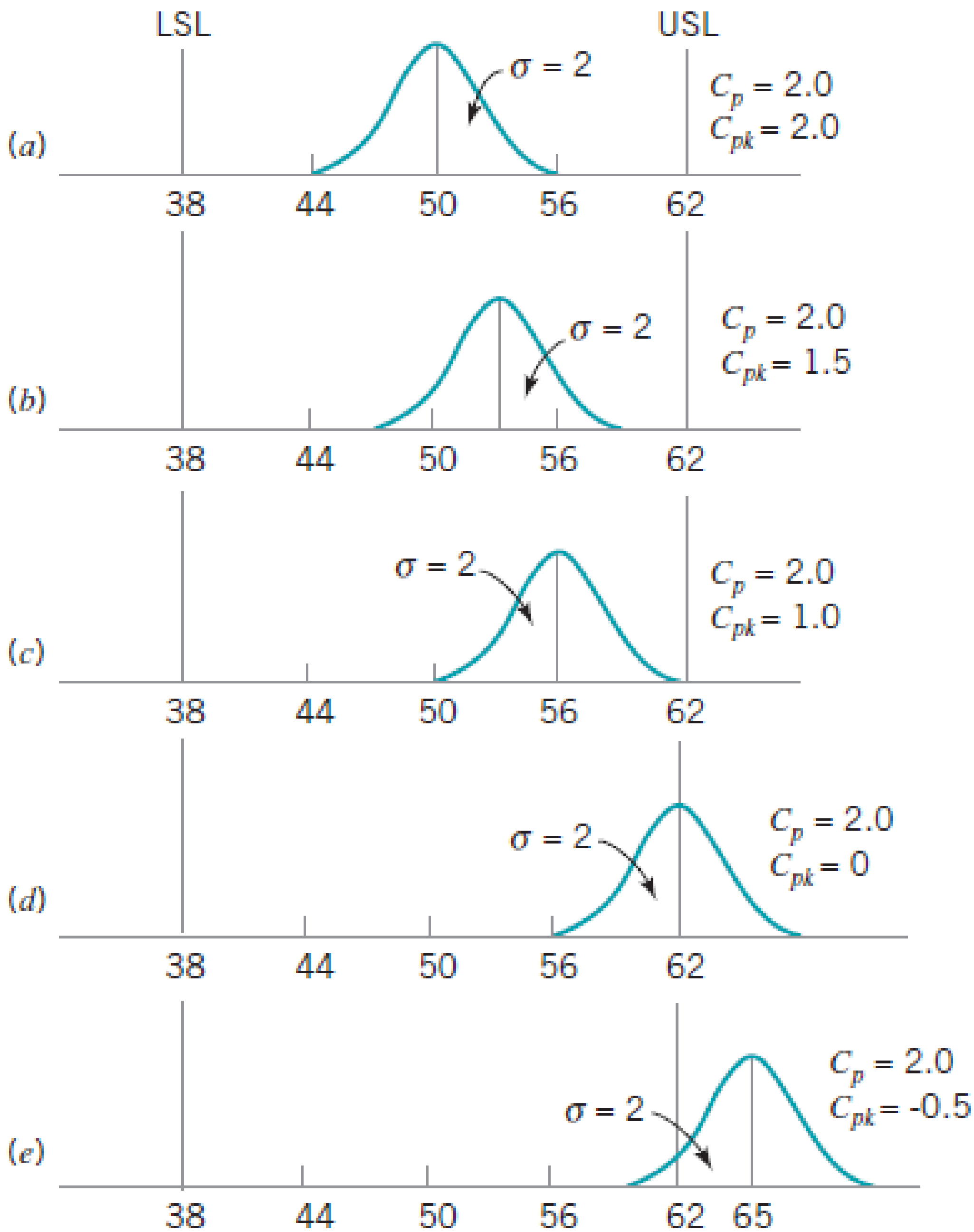

Figure 1 Distribution for five different samples (Montgomery, 2009)

Looking at Figure 1, the five samples present similar standard deviations, so the C_p values are similar. Observing this result, it is possible to assume that all the processes are capable but this assumption is wrong. The processes present data out of the specification limits and the mean for the processes b, c, d and e are off target. Only process "a" is capable.

2.1.2. Process Capability Index C_{pk}

To overcome this problem, Kane (1986) proposed the C_{pk}, this index takes into consideration the deviation of the process mean from the target value. The C_{pk} is defined as

$$C_{pk} = minimum\left\{C_{pu}, C_{pl}\right\} = minimum\left\{\frac{USL-\mu}{3\sigma}, \frac{\mu-LSL}{3\sigma}\right\} \quad (2.3)$$

Where μ is the process mean.

Looking at figure 1 the distribution of the data affects the C_{pk} and also, that of the C_{pk} is equal or smaller than the C_p. These values will be equal when the process is on target and when the data mean is equal to the target process.

The C_p and C_{pk} are independent of the target value, so the use of these indices are recommended for cases where the reduction of the variability and process yield are important (C.-W. Wu, Pearn, & Kotz, 2009). When the target differs from the mean between the upper and lower specifications, these process capabilities indices will lead to the wrong acceptance of the process. In contrary, these indices don't analyze the cost related with the departure from the target.

2.1.3. Process Capability Index C_{pm}

To overcome these limitations, the second generation process capability index (C_{pm}) was proposed. Being within the specification limits alone will not be enough to ensure that the product has high quality, it is also necessary to analyze how the values studied are spread within the specification.

The C_{pm} was proposed by (Chan et al., 1988) and (Hsiang & Taguchi, 1985) independently and it is defined as

$$C_{pm} = \frac{USL - LSL}{6(\sigma^2 + (\mu - T)^2)^{1/2}} \tag{2.4}$$

Where T is the target value of the process. Looking at this equation, it is possible to notice that the minimum value of C_{pm} is 0 and that the maximum value will occur when $\mu - T = 0$ and this value is equal to C_p, so

$$0 \leq C_{pm} \leq \frac{USL - LSL}{6 * \sigma} = C_p \tag{2.5}$$

To apply these PCIs to analyze a process, the sample used in the study must follow a normal distribution in order to calculate the PCIs necessary to use as estimators to replace μ (process mean) and σ (process standard deviation). Instead of using μ and σ, the sample mean ($\bar{X}$) and the sample variance (S) will be used. They are defined as

$$\bar{X} = \sum_{i=1}^{n} \frac{x_i}{n} \tag{2.6}$$

$$S = \sqrt{\sum_{i=1}^{n} \frac{(x_{i-}\bar{x})^2}{n-1}} \tag{2.7}$$

For a sample that follows the normal distribution these estimators will be reliable, but for a different distribution they are not dependable. Other more appropriate and complex PCIs must be used (Pearn & Chen, 1997).

2.2. Process Capability for entire product

In relation to the information presented in the previous section, the process capability indices can be used to analyze one process being capable of measuring the capability of one single product's characteristic. However, it is necessary to determine if when all these processes are put together, if the final product will also be able to meet the specifications required.

To determine the process capability of a final product some approaches were proposed. The first approach to calculate the process capability for an entire product was presented by Bothe (1992). Overall, this method uses the characteristic yield to determine the process capability of the product. Firstly, it is necessary to determine the yield of each characteristic and in order to determine this value it is necessary to calculate the Z_{USL} and Z_{LSL}, that are defined as

$$Z_{USL} = \frac{USL - \mu}{\sigma} \tag{2.8}$$

$$Z_{LSL} = \frac{\mu - LSL}{\sigma} \tag{2.9}$$

Using the Z_{USL} and the Z-table it is possible to determine the probability of the product be below the upper specification limit (Prob. Bad Below) and the probability of the product to be above the upper specification limit (Prob. Bad Above). The yield of the characteristic with bilateral specification is equal to

$$Yield_n = Prob.Good = 1 - (\text{Prob. Bad Below } + \text{ Prob. Bad Above}) \tag{2.10}$$

For unilateral specification, the yield will be determined as

$$Yield_n = Prob.Good = 1 - (\text{Prob. Bad Below or Prob. Bad Above}) \tag{2.11}$$

The yield of the product will be equal to the product of the yield of all characteristics, as shown below

$$Product\ Yield = \prod_{i=1}^{n} Yield_n \tag{2.12}$$

Using this information is possible to determine the C_{pk} of the product using the following equation

$$C_{pk} = \frac{Z_{Score}}{3} \quad (2.13)$$

Where the Z_{Score} is determined using Z-table and the probability (p_{bad}) that the product will not meet specification, this value is defined as

$$p_{bad} = \frac{1 - product\ yield}{2} \quad (2.14)$$

For each C_{pk} value there is a respective yield value for the product. For example, for $C_p = 1$, the yield of the process is equal to 93.30%. Appendix A presents example C_{pk} and its respective yield.

Another method used to analyze the product capability was proposed by Singhal (1990). He presented a visual tool, the Multiprocess Performance Analysis Chart (MPPAC), that presents how the processes behaves in a multi process environment. The chart shows the C_p, C_{pk}, the departure of the process mean from the target, and the variability of the process. One of the limitations of this chart is that it does not present where the process capability must be to ensure the quality of the product, so it is not possible to analyze the performance of the process.

To overcome this problem, Singhal (1992) proposed an improvement to the chart. He added capability zones to it . Figure 1 presents this chart.

This is a very useful visual tool to analyze different processes, but the chart presents some limitations it that it cannot be used to determine the final product quality. Other charts (Chen et al., 2001; Ouyang et al., 2013) were proposed to analyze multi-processes, but they were still not capable of determining the product capability.

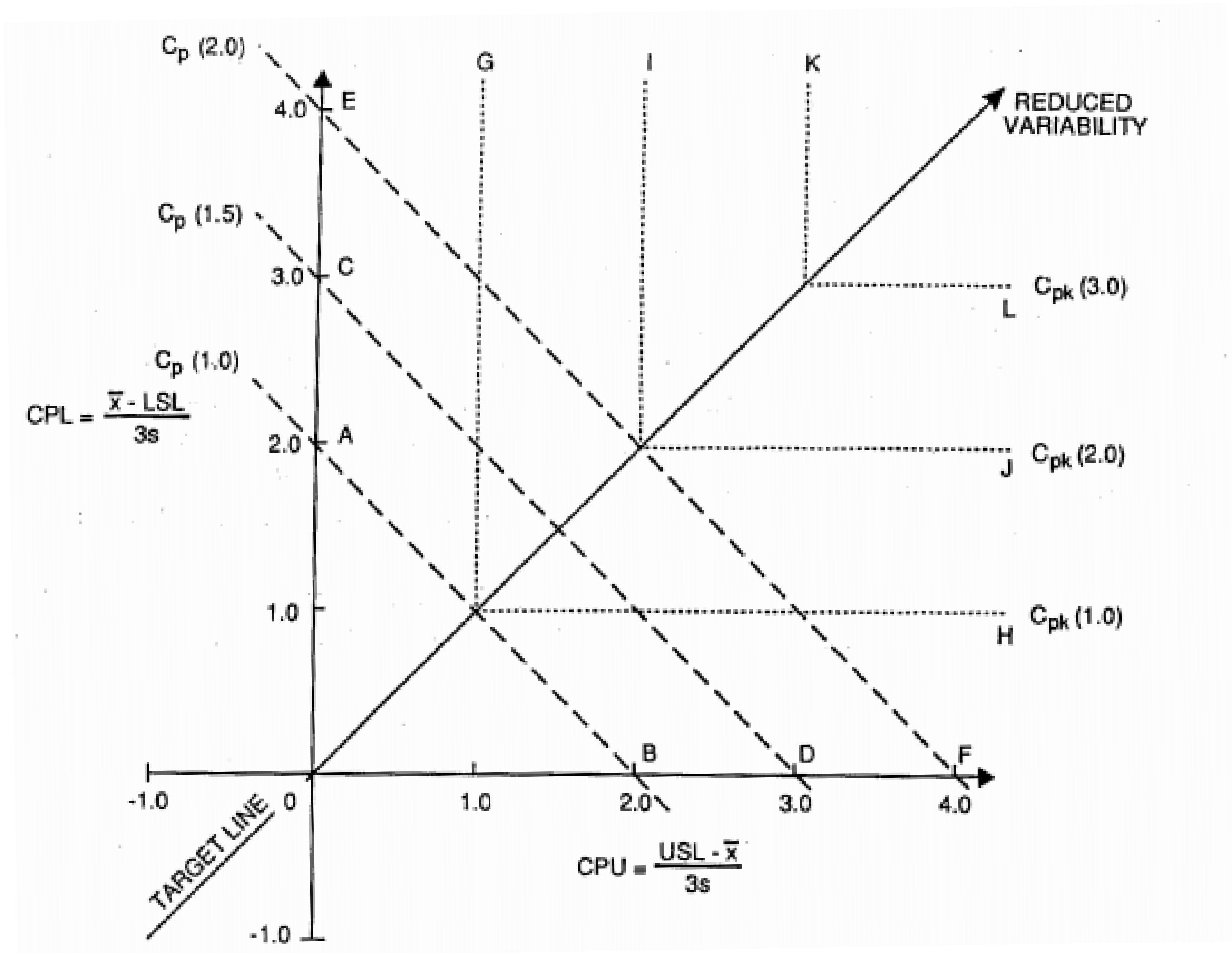

Figure 2 MPPAC with capability zones

Nowadays, there is an approach that studies the product capability and has been widely used (Chen et al., 2001; C. C. Wu, Kuo, & Chen, 2004; Yu, Sheu, & Chen, 2007). Knowing that

$$p = \prod_{i=1}^{n} pi \tag{2.15}$$

This method assumes that for a desired p, the product yield, the characteristic yield must be at least $p_i = p^{1/n}$. To apply this equation, the characteristics must to be independent. This information can be added to the charts presented to help to determine the capability zones. This is a quick way to determine the product capability but this method will result in some loss.

When a minimum value for the yield of the product is fixed, all the characteristics need to meet this requirement. In many cases, some parts of the product don't need to present high quality as the final product and occasionally the part that is critical for the product and this minimum value is not enough to ensure the quality of the final product. The best approach is to look each part individually first and determine the specifications and level of quality of each one looking how it will impact in the final product.

2.3. Sensitivity Analysis

With the data presented, it is possible that it may still be missing some important information to determine the process capability of a product. These approaches don't take into consideration the level of impact of quality of different characteristic in the final product. When the process capability of the product is calculated, it is necessary to take in consideration the weight of the unit (Mu, He, Chang, & Ma, 2009).

Yu, Sheu & Chen (2007) presented one approach to add the influence of importance of the characteristic in the calculation of the process capability for the product. Their proposal integrated capability indices that are defined as

$$PCI^{cm} = \left[\prod_{i=1}^{n}(PCI_i)^{wi}\right]^{\frac{1}{\sum_{i=1}^{n} w_i}} \quad (2.16)$$

Where w_i is an integer number between 1 and 5. The most important characteristic will have $w_i = 5$ and the less important $w_i = 1$.

Another alternative was proposed by Mu at el (2009). In his approach, firstly, the weight is multiplied by the PCI and then all the PCIs are summed. The equation proposed by him is presented bellow

$$PCI^{T2} = \sum_{i=1}^{n} w_i * PCI_i \quad (2.17)$$

Where w_i is a number between 0 and 1.

To determine the value of the weight is important to look the type of process that will be analyzed. For example, for a medical process, the characteristics must to be studied take into account the risk that poor quality will have to the patient. The scale used to the weight must be aligned to the safety of the patient. For a mechanical process, the weight must to be decide following the characteristics that will have a bigger impact in the functionality of the product.

2.4. Process capability in the electronic industry

The electronic products are different than others types of products, due to tighter tolerance and specification limits, so the process yield sensitivity will have a bigger impact in the quality of the product (Huang & Kong, 2010). It is extremely important to determine the best tolerance requirements and specification limits. Spence (1984) presented a parameter space, which is a chart that uses as a input the output of different characteristics and relate these information. He presents an idea of cost and quality balance, so the parameter space will not be the one with higher quality, but the area that presents the best ratio between quality and cost.

It is important to do a sensitivity analysis to determine what parameters will be responsible for causing fluctuation in the output of the final product (Zhai et al., 2013).

3. Methodology

This research aims to present an approach to determine the process capability of electronic products. This research was divided into 4 phases: define the product, simulation study, quality characteristic study and product study.

3.1. Define the product

To start this analysis, it is necessary to determine the product that will be studied and what quality requirements are. Knowing the product, the next step is to determine the quality characteristics that will be used to determine the quality of the final product.

The quality characteristic is a quantitative characteristic of a component that has impact on the quality of the final product. This characteristic can be measured and its data is used to determine if the characteristic does or does not meet the specifications.

For each quality characteristic, it is necessary to determine the specification limits, mean, standard deviation, target, and quality requirements. The data acquired will be used to determine the yield and the process capability indices.

3.2. Simulation Study

To calculate the yield and the process capability indices of the characteristics, it is necessary to calculate their estimator, that is the sample mean ($\bar{X}$) and the sample variance (S^2). To determine these values, a sample of the process data is required. This sample can be obtained in two ways: using real data of the manufacturing process or using a simulation.

To obtain the data using the first option is not easy, it is necessary to find a company record of the data as well as permission to use such information, so the second option was used in this project. In this research, the Monte Carlo Simulation method was used to generate the random data using the sample mean and standard deviation. To run the Monte Carlo simulation, the first step to determine the sample size.

3.2.1. Sample Size Analysis

It is necessary to determine the sample size for each one of the characteristics. To determine this number, the following equation will be used

$$n = \left[\frac{100 * z * S}{E * \bar{x}}\right]^2 \tag{3.1}$$

Where z is the z-score related to the confidence interval required for the data, E is the error percentage acceptable for the mean. The equation presents three unknown variables: n, S and $\bar{x}$. For this case, S and $\bar{x}$ can be replaced by the historical data, σ and μ respectively (Driels & Shin, 2004). How the characteristics differ in mean and variance, the sample size will be different as well. But, how the same product is being analyze, the sample size will be equal to the sample size of the characteristic that has the bigger value.

This research uses electronic products as the subject. The variance and tolerance for this kind of product is tight, so any variance can result in impact of the quality of the product. The confidence interval must to be high and the error must to be low.

Using the sample size, standard deviation, and mean it was possible to create random data for all of the characteristics. It is important to remember that the data must be in statistical control and must follow the normal distribution.

3.3. Characteristic Study

Using the estimators calculated in the section *Simulation Study*, it will be possible to determine the yield of the characteristic and the process capability index. In this project, the C_p, C_{pk} and C_{pm} were calculated and compared. To determine the value of these indices, the following equations were used:

$$C_{pi} = \frac{USL - LSL}{6 * S} = \frac{d}{3 * \sigma} \quad (3.2)$$

$$C_{pki} = minimum\,\{C_{pu}\,, C_{pl}\} = minimum\,\left\{\frac{USL-\bar{x}}{3*S}, \frac{\bar{x}-LSL}{3*S}\right\} \quad (3.3)$$

$$C_{pmi} = \frac{USL - LSL}{6(S^2 + (\bar{x} - T)^2)^{1/2}} \quad (3.4)$$

Where i is a number between 1 and n.

It was also used to calculate the yield of the product using the C_{pk} and C_{pm}. The yield using the C_{pk} is calculated following the equation x

$$pi = \%Yield = \phi(3 * Z_{min}) \quad (3.5)$$

The Appendix A presents a table with the yield value for respective C_{pk}.

To determine the yield using the C_{pm}, it is necessary to use the equation proposed by Chen and Huand (2007). The relationship is presented below

$$pi = \%Yield = \phi\left(\frac{1 + \sqrt{1/(3 * c)^2 - (S/d)^2}}{S/d}\right) + \phi\left(\frac{1 - \sqrt{1/(3 * c)^2 - (S/d)^2}}{S/d}\right) - 1 \quad (3.6)$$

Where $C_{pmi} = c$, $d = (USL - LSL)/2$.

Knowing that

$$0 \leq C_{pm} \leq \frac{USL - LSL}{6 * \sigma} = \frac{d}{3 * \sigma} \quad (3.7)$$

We conclude that σ/d is between zero and $1/3c$. This relationship can be approximated

to

$$\frac{\sigma}{d} = \frac{h}{30 * c} \quad (3.8)$$

Where h is a integer number between 1 and 10. Using this relationship, Chen and Huang (2007) created a table that presents a yield value for some specific C_{pm} and h. The table 1 presents these result. Appendix b presents the table with the yield values for C_{pm} varying between 0 and 1 with 0.01 as increments and h varying between 1 and 10.

For the unilateral specification, the C_{pl} and the C_{pu} will be used as process capability indices. In this case, for the characteristics with lower specification limits, the yield and the C_{pl} will be calculated as

$$C_{pli} = (1/3)\phi^{-1} * (pl)$$

$$pl = \%Yield = P(x > LSL) = \phi\left(3C_{pli}\right)$$

For the characteristics with upper specification limits, the yield and the C_{pu} will be calculated as

$$C_{pui} = (1/3)\phi^{-1} * (pu)$$

$$pu = \%Yield = P(x < USL) = \phi\left(3C_{pui}\right)$$

3.4. Product Study

The last phase of this study is to determine the process capability of the product. To determine the product process capability, two approaches were used: using the process capability indices and process yield. Aside from the PCIs and the yield, it is necessary to know the influence of each characteristic in the quality of the final product. This influence is numeral represented by a weight (w_i).

Table 1 The C_{pm} values and respective yield (Chen and Huang, 2007)

C_{pm}	h = 2	h = 4	h = 6	h = 8	h = 10
0.6	0.9999794334	0.9864026657	0.9522023043	0.9318429007	0.9281393618
0.7	0.9999999893	0.9984553624	0.9848691887	0.9692345598	0.9642711589
0.8	1	0.9998958420	0.9961695712	0.9876871101	0.9836049282
0.9	1	0.9999958773	0.9992290125	0.9956490149	0.9930660524
1.0	1	0.9999999049	0.9998771335	0.9986467043	0.9973002039
1.1	1	0.9999999987	0.9999845457	0.9996303775	0.9990331517
1.2	1	1	0.9999984694	0.9999115067	0.9996817828
1.3	1	1	0.9999998808	0.9999814540	0.9999038073
1.4	1	1	0.9999999927	0.9999966013	0.9999733085
1.5	1	1	0.9999999997	0.9999994559	0.9999932047
1.6	1	1	1	0.9999999239	0.9999984133
1.7	1	1	1	0.9999999907	0.9999996603
1.8	1	1	1	0.9999999990	0.9999999334
1.9	1	1	1	0.9999999999	0.9999999880
2.0	1	1	1	1	0.9999999980

3.4.1. Determining the weight (w_i)

The weight was calculated comparing the influence of the quality characteristic in the final quality of the product. The higher value is given to the most important characteristic and the weight of the other characteristics will be determined based on the most important one. There are many ways to determine the weights; to standardize it, each characteristic was assigned a value k_i between 1 to 10, where 10 is the most important and 1 is the least important quality characteristic.

Using these values, it is possible to determine w_i for any approach. For example, for the approach presented by Yu et al. (2007), w_i is a number between 1 and 5, so it will be equal to

$$w_i = k_i/2 \tag{3.9}$$

For the approach presented by Mu at el (2009), w_i is a number between 0 and 1, so it will be equal to

$$w_i = \frac{k_i}{\sum_{i=1}^{n} k_i} \tag{3.10}$$

3.4.2. How to analyze the product to determine the product capability

To manufacturing a product, many parts must to be assembled together. A company can manufacture all the components need or can get it from outside suppliers. When we try to determine the quality of the product all this parts used to manufacture the product must to be studied such as the quality of the characteristics, the components quality and the quality of the parts provided by the suppliers.

The quality characteristic is the part that is studied individually, it is a part that is independent of all the others parts of the product. The component is built of different quality characteristics, so the quality of this part will depended of the quality of the characteristics. The part provided by the suppliers can be composed of quality characteristics and/or components, but how this part is provided by an outsource, the quality of this part will be analyzed as a quality characteristic.

To determine the quality of the product is necessary to look the interaction between quality characteristics, quality characteristics and components, quality characteristics and parts provided by suppliers, components and parts provided by suppliers, and quality characteristics, components and parts provided by suppliers. The figure 3 presents a flowchart of this interaction.

3.4.3. Process capability using the PCIs

Analyzing everything that was presented in the previous sections, it is possible to notice that the most used PCI to determine the capability of a product is the C_{pk}. In this project, instead of using the C_{pk}, the C_{pm} was used. In addition, the capability of the product will also be calculated using the weight of influence of each characteristic in the final product. Also, it will be presented in an equation to analyze not just components assembling to form the product, but also the subcomponents.

Using the PCI to determine the product capability, the weight will be calculated using the Mu at el (2009) approach. For a component with n quality characteristic the PCI is defined as

$$PCI_i^c = \sum_{i=1}^{n} w_i * PCI_i \tag{3.11}$$

Where the PCI can be replaced by the C_{pk} and C_{pm} and the w_i is the weight for each quality characteristic.

To determine the product capability is necessary to look all the parts that are assembled to manufacture this product. So the product capability is defined as

$$PCI^p = \sum_{i=i}^{g} (w_i^p * PCI_i^g) \tag{3.12}$$

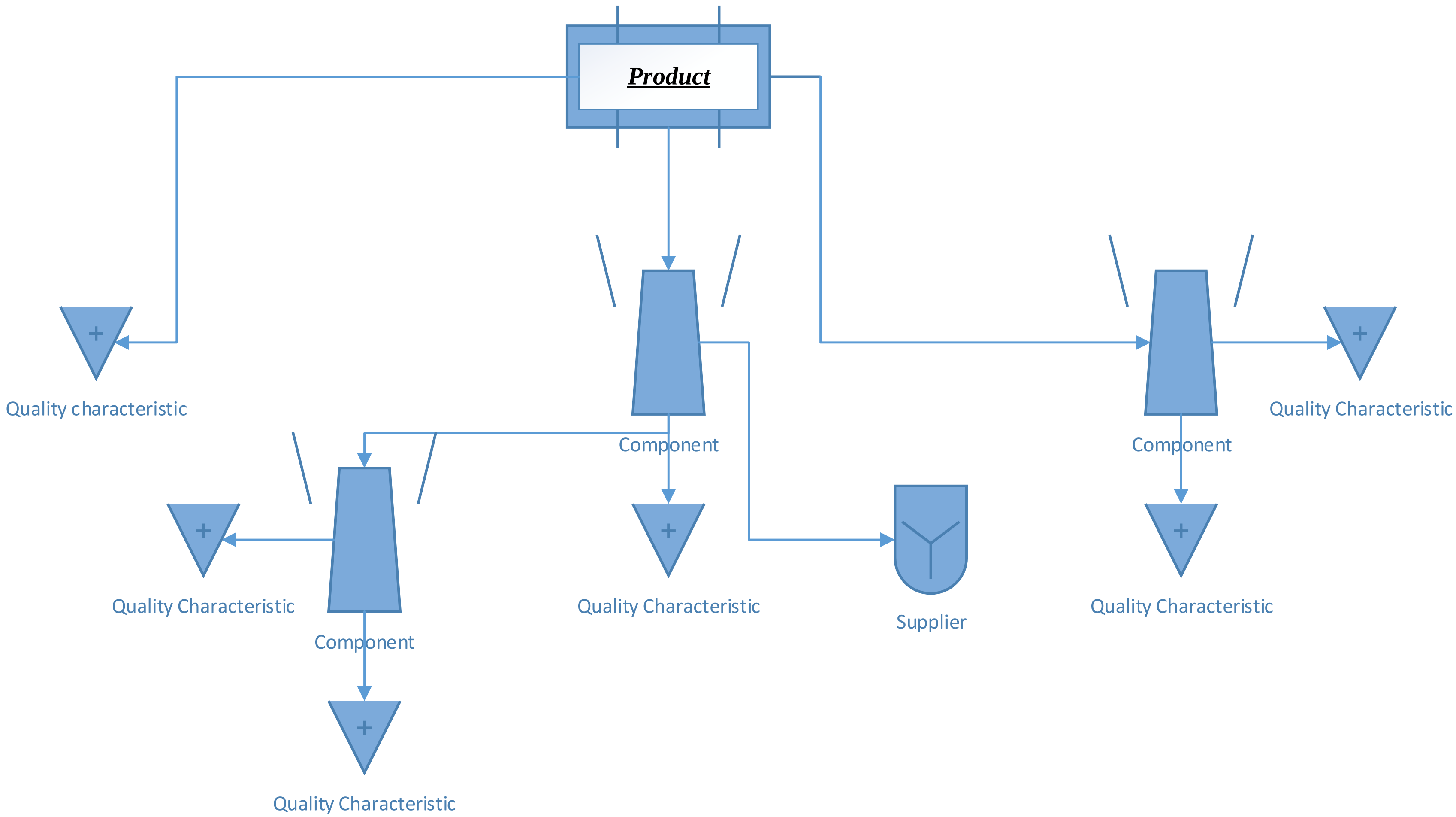

Figure 3 Flowchart for the studied of the product quality

Where w_i^p is the weight for each one of the parts, that can be: quality characteristic, component and part provided by suppliers. The PCI_i^g is the process capability of each part and g is the number of parts.

To compare, the weight will be also calculated using the Yu et al. (2007) approach. The equation below shows how to calculate the capability for a component

$$PCI_i^c = \left[\prod_{i=1}^{n} (PCI_i)^{w_i}\right]^{\frac{1}{\sum_{i=1}^{n} w_i}} \tag{3.13}$$

For a product with g components, the product capability is defined as

$$PCI^p = \left\{\prod_{i=1}^{g} PCI_i^g\right\}^{\frac{1}{\sum_{i=1}^{n} w_i^g}} \tag{3.14}$$

To determine the product capability, different approaches with and without the use of the weights were calculated and compared. Also, instead of using only the C_{pm}, the C_{pk} was also analyzed. Doing so allows comparison of the two process capability indices to determine the advantages of the use of the C_{pm}.

It is important to remember that, the C_{pm} and C_{pk} is used to analyze characteristics with bilateral specification. For characteristics with unilateral specification, the C_{pu} will be used for characteristics with just upper specification limits and C_{pl} for characteristics with just lower specification limit and

3.4.4. Process capability using the yield

Analyzing what was presented in the previous sections, it is known that the PCI can be calculated using the yield. Instead of determining the CPI for each quality characteristic and component, the yield will be used. Using the yield value and the tables presented in Appendix A and B, it is possible to determine the C_{pm} and C_{pk} for the product.

In this case, the yield of the product can be determined using the following equation

$$Y_{pci}^{p} = \prod_{i=1}^{m} Y_{pci}^{c} \tag{3.15}$$

Where the Y_{pci}^{c} is the yield of each quality characteristic and the m is the number of quality characteristics. To compare with the results obtained in the previous section, the yield will be calculated using the C_{pm} and C_{pk}.

3.4.4.1. Process capability for nom-normal samples

Extending the equations presented in the previous section for other types of sample distributions, it will be possible to determine the yield of the product. It is important to remember that anything different from a normal distribution in this case, the yield can't be converted into a process capability index. Table 2 presents how to calculate the yield for different distributions.

The flowchart in figure 4 summarizes all the steps required to determine the product capability.

Table 2 Probability Distributions and their respective yield

Probability Distributions	Yield	Mean	Variance
Binomial Distribution	$p(x) = \binom{n}{x} p^x (1-p)^{n-x}$	$\mu = n * p$	$\sigma^2 = np(1-p)$
Poisson Distribution	$p(x) = \frac{e^{-\lambda}\lambda^x}{x!}$	$\mu = \lambda$	$\sigma^2 = \lambda$
Exponential Distribution	$p(x) = \lambda e^{\lambda x}$	$\mu = \frac{1}{\lambda}$	$\sigma^2 = \frac{1}{\lambda^2}$

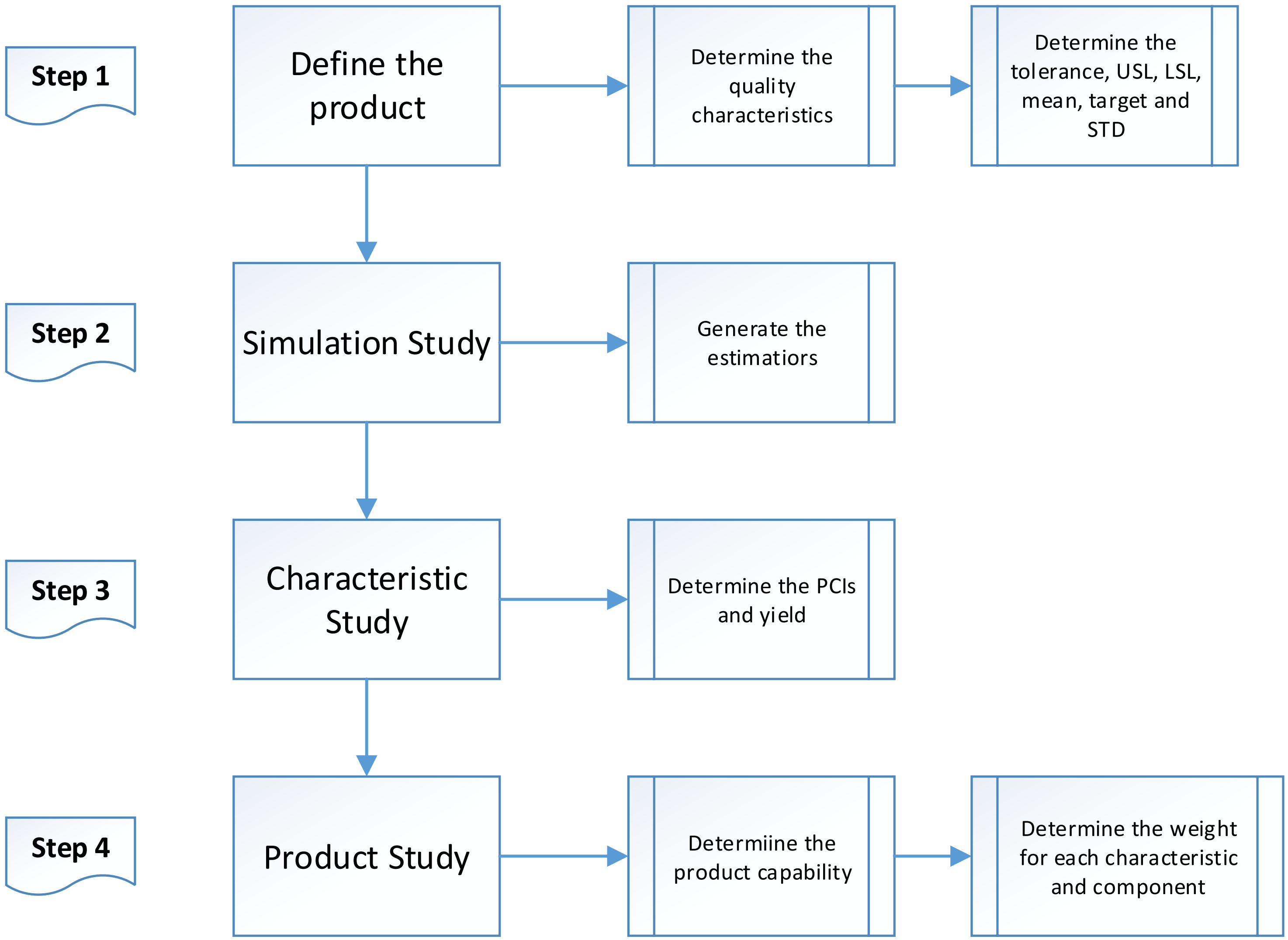

Figure 4 Flowchart methodology

4. Results

The objective of this thesis is to determine the best approach to calculate the process capability of products in the electronic industry. The results obtained will be presented following the steps of the methodology.

4.1. Define the product

The product chosen for this study is the same one used by Ouyang, Hsu & Yang (2013) in their paper. The product is the H-type chip resistor as show in the figure 4.

The characteristics analyzed are length, width, height, upper-width, and lower-width. The mean, standard deviation, upper and lower specification limit, and the target are presented in the table 3.

To apply the ideas presented in this paper, instead of considering the five quality characteristic as part of one component, they will be analyzed in groups. The length, width, and height will be analyzed as a quality characteristic of one component and the upper and lower width as quality characteristic of another component. The final product is composed of these two parts.

4.2. Simulation Study

The first step is to run the simulation; this is to determine the number of samples. In this case, the sample size (n) is 761. The Appendix C presents how these values were calculated. To run the simulation, it was ran using the Excel function NORMINV; this function generates random numbers that follows the normal distribution. Appendix C also presents the steps on how to use this function. Using this data, the estimators were calculated. Table 4 presents these values for each characteristic.

For this study, the data obtained in the simulation must be in statistical control and follow the normal distribution. Appendix D shows the test for these assumptions.

4.3. Characteristic Study

Using the data obtained in the last section, the C_p, C_{pk} and C_{pm} was calculated. Table 5 presents the results.

Using this data, it is also possible to determine the yields of the characteristics. The yield was calculated using the method presented by Bothe (1992). Table 6 presents these results.

The yield was also calculated using the method presented by Chen & Huang, 2007. Table 7 presents the results obtained.

The Appendix E presents the equations that were used in this section.

4.4. Product Study

To calculate the product process capability, the yield and PCIs that were calculated in the previous section were used. Aside from this, it is also necessary to determine the weight of each characteristic. To do this, the first step is to determine what characteristics are more important.

4.4.1. Determining the weight (w_i)

For the quality characteristics of the product, we will assume that for the first component the most important quality characteristic is the length ($k = 10$), followed by the height ($k = 7$) and then the width ($k = 5$). For the second component, the most important is the upper width ($k = 10$), followed by the lower width ($k = 8$). Observing the components, the first ($k = 9$) is less important than the second one ($k = 10$).

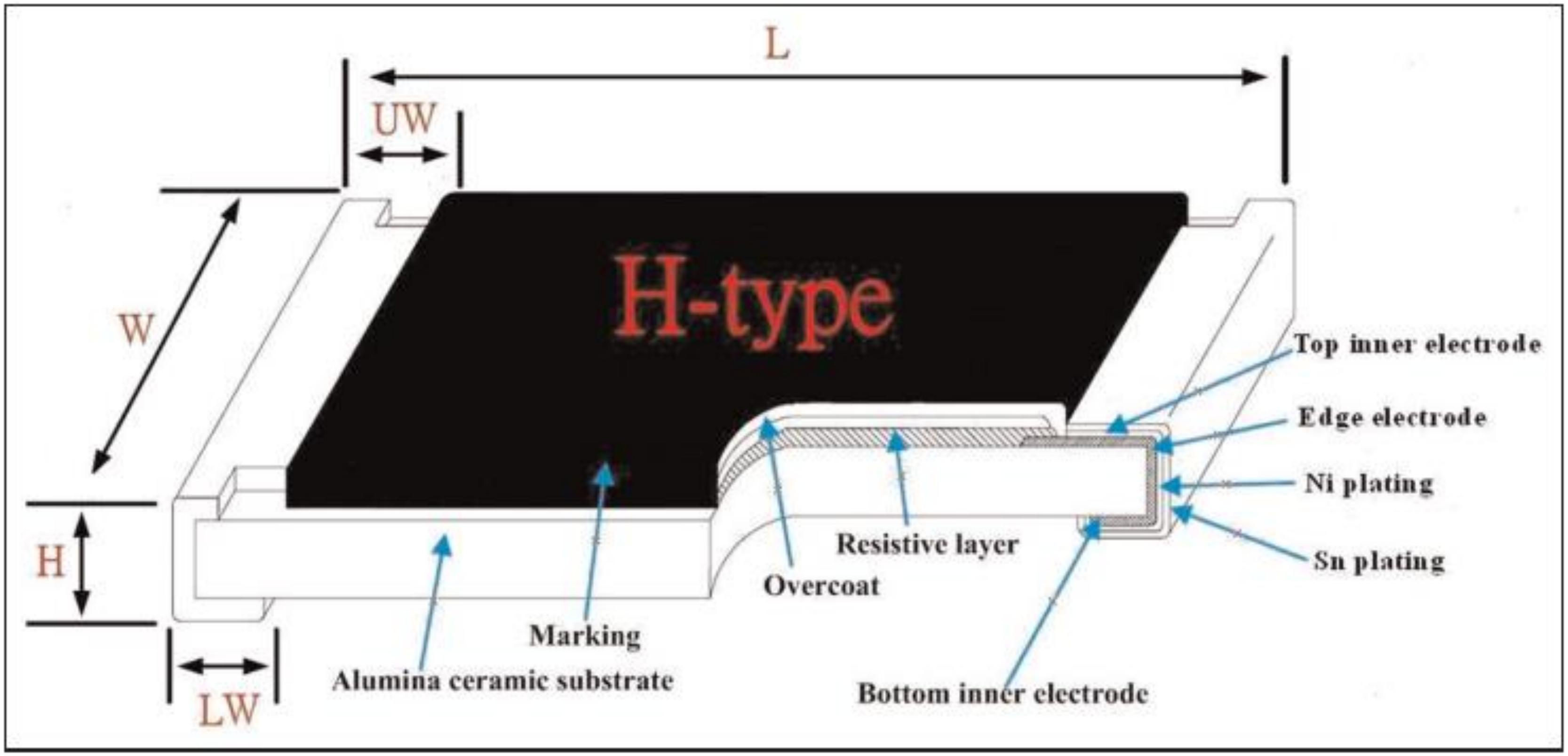

Figure 5: H-type chip resistor (Ouyang et al., 2013)

Table 3 : Specification of the H-type chip resistor(unit: mm) (Ouyang et al., 2013)

Quality characteristic	USL	T	LSL	d	μ	σ
Length (L)	2.10	2.00	1.90	0.10	2.022	0.028
Width (W)	1.35	1.25	1.15	0.10	1.267	0.018
Height (H)	0.65	0.50	0.35	0.15	0.518	0.021
Upper width (UW)	0.55	0.35	0.15	0.20	0.408	0.030
Lower width (LW)	0.50	0.35	0.20	0.15	0.359	0.033

Table 4: Mean and variance for the quality characteristic.

Data Analysis		
Quality characteristic	Mean	STD
Length	2.021661	0.027982
Width	1.266875	0.018106
Height	0.518166	0.020175
Upper width	0.408612	0.028882
Lower width	0.357834	0.031619

Table 5: PCIs for the quality characteristics

Quality characteristic	C_p	C_{pk}	C_{pm}
Length	1.191327	0.933274	0.942026
Width	1.84101	1.53034	1.346768
Height	2.478315	2.178174	1.841731
Upper width	2.308243	1.631789	1.020278
Lower width	1.581328	1.49874	1.534918

Table 6 Yield for the quality characteristics

Quality characteristic	Prod. Good using Bothe approach	Prod. Good using Chen & Huang approach
Length	0.9974366	0.989537
Width	0.9999978	0.999984
Height	1	1
Upper width	0.9999995	0.999998
Lower width	0.9999962	0.999898

4.4.2. Product capability using the PCIs

The first analysis follows the method presented by Mu at el (2009). The weight is between 0 and 1. Table 7 presents the value of the weight of each characteristic as well as the C_{pmi}^{c} and C_{pki}^{c} for each one of the components.

$$C_{pk}^{p} = \frac{9 * C_{pk1}^{c} + 10 * C_{pk2}^{c}}{19} = 1.4938$$

$$C_{pm}^{p} = \frac{9 * C_{pm1}^{c} + 10 * C_{pm2}^{c}}{19} = 1.261455$$

To determine the influence of the use of the weights, it is necessary to repeat this analysis and use the characteristics and components with the same weight. The results are presented in table 8.

Using the results presented in Table 8, it is possible to determine the product capability indices.

$$C_{pk}^{p} = \frac{1 * C_{pk1}^{c} + 1 * C_{pk2}^{c}}{2} = 1.556264$$

$$C_{pm}^{p} = \frac{1 * C_{pm1}^{c} + 1 * C_{pm2}^{c}}{2} = 1.32722$$

The first analysis follows the method presented by Yu et al. (2007). In this case, the weight is between 1 and 5. Table 9 presents the weight for each one of the characteristics and also the C_{pmi}^{c} and C_{pki}^{c} for each one of the components.

$$C_{pk}^{p} = \sqrt[(4.5+5)]{{C^{c}}_{pm1}{}^{4.5} * C_{pm2}{}^{5}} = 1.449035$$

$$C_{pm}^{p} = \sqrt[(4.5+5)]{C_{pk1}{}^{4.5} * {C^{c}}_{pk2}{}^{5}} = 1.22612$$

Table 7 Process capability for the components using the PCIs

Quality characteristic	Weight (w_i)	C_{pki}	$C_{pki} * w_i$	C_{pmi}	$C_{pmi} * w_i$
Length	0.454545	0.933274	0.424215	0.942026	0.428194
Width	0.318182	1.53034	0.486926	1.346768	0.428517
Height	0.227273	2.178174	0.49504	1.841731	0.418575
First Component		$C_{pk1}^{c} =$	1.406181	$C_{pm1}^{c} =$	1.275286
Upper width	0.555556	1.631789	0.90655	1.020278	0.566821
Lower width	0.444444	1.49874	0.666107	1.534918	0.682186
Second Component		$C_{pk2}^{c} =$	1.572656	$C_{pm2}^{c} =$	1.249007

Table 8 Process capability for the components using the PCIs (without weight)

Quality characteristic	Weight (w_i)	C_{pki}	$C_{pki} * w_i$	C_{pmi}	$C_{pmi} * w_i$
Length	0.333333	0.933274	0.311091	0.942026	0.314009
Width	0.333333	1.53034	0.510113	1.346768	0.448923
Height	0.333333	2.178174	0.726058	1.841731	0.61391
First Component		C_{pk1}	1.547263	C_{pm1}	1.376842
Upper width	0.5	1.631789	0.815895	1.020278	0.510139
Lower width	0.5	1.49874	0.74937	1.534918	0.767459
Second Component		C_{pk2}	1.565265	C_{pm2}	1.277598

Table 9 Process capability for the components using the yield

Quality characteristic	Weight (w_i)	C_{pki}	${C_{pki}}^{w_i}$	C_{pmi}	${C_{pmi}}^{w_i}$
Length	5	0.933274	0.70802	0.942026	0.741847
Width	3.5	1.53034	4.433611	1.346768	2.834815
Height	2.5	2.178174	7.002153	1.841731	4.60326
First Component		$C_{pk1}^{c} =$	1.324351	$C_{pm1}^{c} =$	1.229214
Upper width	5	0.933274	0.70802	0.942026	0.741847
Lower width	4	1.53034	4.433611	1.346768	2.834815
Second Component	5	$C_{pk2}^{c} =$	1.571257	$C_{pm2}^{c} =$	1.223342

To determine the influence of the use of the weight, it is necessary to repeat this analysis and use the characteristics and components with the same weight. The results are presented in table 10.

Using the PCI for the first and second components presented in the Table 10, it is possible to determine the product capability.

$$C_{pk}^{p} = \sqrt[2]{{C^{c}}_{pm1}{}^{1} * C_{pm2}{}^{1}} = 1.510935$$

$$C_{pm}^{p} = \sqrt[2]{C_{pk1}{}^{1} * {C^{c}}_{pk2}{}^{1}} = 1.288639$$

4.4.3. Process capability using the yield

Using the yield obtained in the table 6 it is possible to determine the product yield. Using this value and the data presented in the Appendix A and B it is possible to determine the C_{pk}^{p} and the C_{pm}^{p}.

For the C_{pk}^{p}, the yield used is obtained by the product of the data presented in the table 6

$$Y_{Cpk}^{p} = 99.7426$$

Using this value and the data presented in Appendix A, we have that

$$C_{pk}^{p} = 1.45$$

For the C_{pm}^{p}, the yield used is obtained by the product of the data presented in the table 7

$$Y_{Cpk}^{p} = 98.94$$

Using this value and the data presented in the Appendix B, we have that

$$C_{pm}^{p} = 0.95$$

Table 10 Process capability for the components using the yield (without weight)

Quality characteristic	Weight (w_i)	C_{pki}	${C_{pki}}^{w_i}$	C_{pmi}	${C_{pmi}}^{w_i}$
Length	0.33	0.933274	0.966061	0.942026	0.97058
Width	0.33	1.53034	1.237069	1.346768	1.160503
Height	0.33	2.178174	1.475864	1.841731	1.357104
First Component		$C_{pk1}^{c} =$	1.459811	$C_{pm1}^{c} =$	1.326968
Upper width	0.5	1.631789	1.277415	1.020278	1.010088
Lower width	0.5	1.49874	1.22423	1.534918	1.238918
Second Component		$C_{pk2}^{c} =$	1.56385	$C_{pm2}^{c} =$	1.251416

5. Discussion

To better analyze the results, the discussion will be divided into three parts. The first one presents the discussion of the simulation results, the second presents the quality characteristic study results analyses and the third presents the product study results analyses.

5.1. Simulation Analyses

To determine if the data obtained in the simulation is accurate, it is necessary to compare the values obtained for the mean and standard deviation with the historical values. Table 11 presents the results obtained for the μ, σ, $\bar{X}$ and S. Also, the table presents the error for this data.

The results presented in Table 11 of the simulation is accurate. For the mean, all of the errors are less than 1%, so the data generated presents values close to the mean. For the standard deviation, the errors are less than 5%for four characteristics, and the $\boldsymbol{S}$ is smaller than the historical standard deviation.

5.2. Quality Characteristics Analyses

The process capability indices are used to determine if a process is capable of meeting the specification, Table 12 presents C_p, C_{pk} and C_{pm} values. Looking at this table, all of the quality characteristics has the C_p as the biggest value This happen because this CPI doesn't analyze the departure of the mean from the target as well as the distribution of the sample. If the mean is not equal to the target, the C_p will induce a wrong acceptance of the process. Figure 5 shows that all the characteristics presented are off target.

The C_p doesn't analyze how the mean is located in relationship to the specification limits. If the distance between the mean to the upper and lower specification limit is different, the C_p will also create an error.

C_{pk} is more sensitive to how the data is spread and also to the location of the mean relative to the specification limits. The C_{pm} is more sensitive to the mean departure from the target. Table 12 shows the variability between the mean and the target as well as the variability between the mean and the specification limits.

Analyzing table 12 it is possible to conclude that the upper width presents the biggest variance between the mean and the target, due to this, the C_{pm} presents a smaller value. Lower width and the length present similar values for the C_{pm} and C_{pk}; this happens because the variability between the mean and target is small and the distance between the mean and the specifications limits are close.

Even with the variability between the mean and the target (being lower for the width), the mean is not centered between the specifications limits, so the C_{pm} also presents a small value. For the height, the variance between the mean and target and the distance between the mean and the specification limits are high, so, once again the C_{pm} presents the smaller value.

Table 11 Comparison between the real data and the simulation data

Quality Characteristic	μ	$\bar{x}$	error (%)	σ	S	error (%)
Length	2.02200	2.02166	0.01677	0.02800	0.02798	0.07143
Width	1.26700	1.26688	0.00987	0.01800	0.01811	0.58889
Height	0.51800	0.51817	0.03205	0.02100	0.02018	3.92857
Upper width	0.40800	0.40861	0.15000	0.03000	0.02888	3.72667
Lower width	0.35900	0.35783	0.32479	0.03300	0.03162	4.18485

Table 12 Quality characteristics summary results

Quality characteristic	μ	T	$\frac{\mu - T}{\mu} * 100$	USL	USL- μ	LSL	μ-LSL	C_p	C_{pk}	C_{pm}
Length	2.021661	2	1.071446	2.1	0.078339	1.9	0.121661	1.191327	0.933274	0.942026
Width	1.266875	1.25	1.332018	1.35	0.083125	1.15	0.116875	1.84101	1.53034	1.346768
Height	0.518166	0.5	3.505826	0.65	0.131834	0.35	0.168166	2.478315	2.178174	1.841731
Upper width	0.408612	0.35	1.434417	0.55	0.141388	0.15	0.258612	2.308243	1.631789	1.020278
Lower width	0.357834	0.35	2.189283	0.5	0.142166	0.2	0.157834	1.581328	1.49874	1.534918

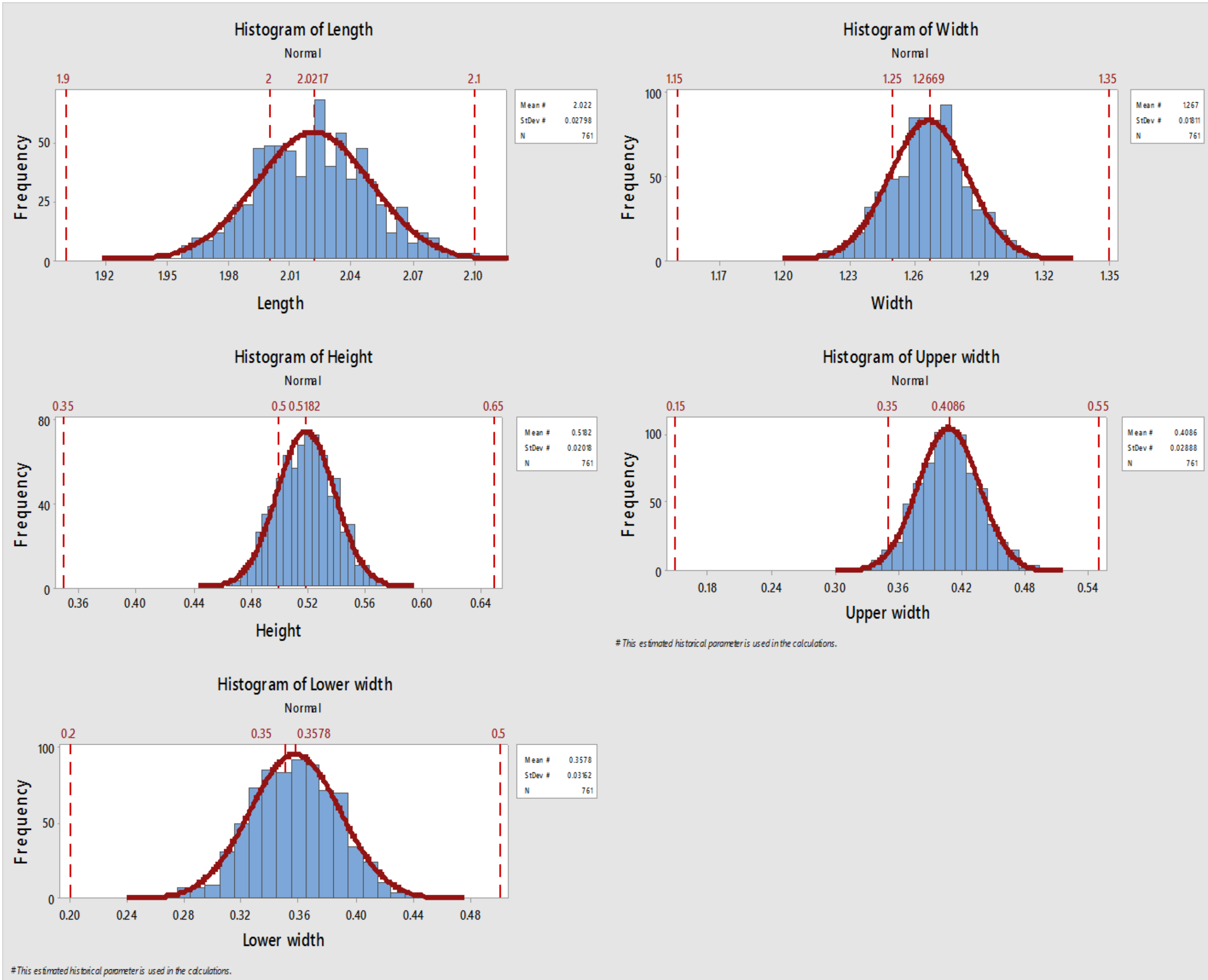

Figure 6 Histogram of the quality characteristics

5.3. Product Capability Analyses

The product capability analyzes the capability of a product to meet the quality standard required. To determine this, it is necessary to analyze the quality characteristics and the components of the product. In this project, the product capability was determined using the PCIs and the yield.

Using the PCIs, the product capability was determined with and without the influence of weight in the quality characteristics. Table 13 summarizes the results obtained in this study.

Observing the results presented in table 13, it is possible to arrive to some conclusions. First, the use of the weight will impact in the product capability. The difference between the PCI with and without the use of the influence of the quality characteristic shows the importance of this study. The use of the weight results in a more accurate PCI value because in a manufacturing process a product is composed of different components and each one will have a different influence in the final quality of the product. Using the weight first will allow to determine if each characteristic is critical. Secondly, it will help in determining where the investment must be done to improve the product quality.

In analyzing the C_{pk} and the C_{pm} values, he C_{pm} is smaller than the C_{pk}. This shows that the use of the C_{pk} to analyze the product capability can lead to a wrong acceptance of the product.

Using the yield to determine the product capability, we have that the C_{pk}^{p} is equal to 1.45 and the C_{pm}^{p} is equal to 0.95. Observing the results presented in the table 14, it is possible to conclude that the use of the yield to determine the C_{pk}^{p} is accurate, but to determine the C_{pm}^{p} this approach will present some errors. This happens because for the C_{pk}^{p}, the value of the yield will depend on the PCI value and the C_{pm}^{p} of the yield depend on the PCI and h.

Table 13 Product capability using the PCI of the characteristics and components

	Chen and Huan approach		Yu et al approach	
	With w_i	Without w_i	With w_i	Without w_i
$\boldsymbol{C_{pk}}$	1.4938	1.556264	1.449035	1.510935
$\boldsymbol{C_{pm}}$	1.261455	1.32722	1.22612	1.288639

6. Conclusion

Observing everything that was presented in this thesis, it is possible to conclude a few things:

- The use of the Monte Carlo simulation to generate data when the real data of the process is not available is accurate. When the deviation of mean and standard deviation value is small, the random data created can be used to determine the estimators of the sample.
- To determine the PCIs values, the data must be in control and must follow the normal distribution, if the data doesn't attend this assumption, the use of the PCIs is not possible.
- Between the process capability indices presented, the C_{pm} will lead to more accurate results because analyzing the variability of the data also presents high sensibility to the mean deviation from the target. The use of the C_p and the C_{pk} for the process of a target will result in the wrong acceptance of the process.
- To determine the capability of the product, it is necessary to firs determine the influence of each quality characteristic in the final quality of the component that composes the product. Lastly, it is then necessary to also determine the weight of the component. Using this weight, the product capability will be more accurate and also, this will help determine the right place as to where to invest in order to result in a higher improvement of quality.
- There are two approaches to apply to the weight of the process capability. One uses the sum and the other uses the product. For this study, any of the approaches can be used to lead to similar results.

- The use of the yield to determine the product capability is accurate when the process capability index analyzed is the C_{pk}, for the C_{pm} . The use of the yield can lead to the wrong results.
- To analyze a process that doesn't follow the normal distribution, the use of the yield is a good option. It's not possible to determine the process capability index, but it is possible to determine the range of parts that meet the specification, which is a good parameter to use to analyze the quality of a product.

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Appendix A: Yield to Process Capability Conversion (C_{pk})

Yield %	Sigma	C_{pk}
6.70%	0	0
15.90%	0.5	0.17
30.90%	1	0.33
50.00%	1.5	0.5
69.10%	2	0.67
84.10%	2.5	0.83
93.30%	3	1
97.70%	3.5	1.17
99.40%	4	1.33
99.87%	4.5	1.5
99.98%	5	1.67
99.9968%	5.5	1.83
99.99966%	6	2

Appendix B: Yield values for different C_{pm}

Com	h									
	1	2	3	4	5	6	7	8	9	10
0,05	0,00000	0,00002	0,00357	0,02383	0,05499	0,08266	0,10163	0,11264	0,11785	0,11924
0,10	0,00000	0,00034	0,01462	0,06045	0,11896	0,16895	0,20335	0,22354	0,23322	0,23582
0,15	0,00000	0,00404	0,04650	0,12143	0,19845	0,26122	0,30481	0,33096	0,34378	0,34729
0,20	0,00004	0,02878	0,11904	0,21431	0,29566	0,35963	0,40500	0,43319	0,44748	0,45149
0,25	0,00715	0,12528	0,24832	0,33858	0,40763	0,46190	0,50219	0,52861	0,54265	0,54675
0,30	0,17109	0,34495	0,42866	0,48353	0,52688	0,56388	0,59413	0,61577	0,62810	0,63188
0,35	0,70888	0,63721	0,62559	0,63070	0,64348	0,66052	0,67845	0,69353	0,70312	0,70628
0,40	0,97982	0,86456	0,79395	0,76075	0,74790	0,74708	0,75306	0,76115	0,76750	0,76986
0,45	0,99981	0,96792	0,90662	0,86075	0,83346	0,82017	0,81656	0,81835	0,82150	0,82298
0,50	1,00000	0,99535	0,96564	0,92768	0,89759	0,87826	0,86843	0,86537	0,86573	0,86639
0,55	1,00000	0,99960	0,98984	0,96665	0,94155	0,92169	0,90901	0,90287	0,90110	0,90106
0,60	1,00000	0,99998	0,99760	0,98640	0,96912	0,95220	0,93941	0,93184	0,92871	0,92814
0,65	1,00000	1,00000	0,99955	0,99511	0,98492	0,97236	0,96119	0,95353	0,94974	0,94882
0,70	1,00000	1,00000	0,99993	0,99846	0,99321	0,98487	0,97611	0,96923	0,96535	0,96427
0,75	1,00000	1,00000	0,99999	0,99957	0,99718	0,99217	0,98587	0,98024	0,97666	0,97555
0,80	1,00000	1,00000	1,00000	0,99990	0,99892	0,99617	0,99198	0,98769	0,98464	0,98360
0,85	1,00000	1,00000	1,00000	0,99998	0,99962	0,99823	0,99564	0,99256	0,99013	0,98923
0,90	1,00000	1,00000	1,00000	1,00000	0,99988	0,99923	0,99772	0,99565	0,99381	0,99307
0,95	1,00000	1,00000	1,00000	1,00000	0,99996	0,99968	0,99886	0,99753	0,99621	0,99563
1,00	1,00000	1,00000	1,00000	1,00000	0,99999	0,99988	0,99945	0,99865	0,99774	0,99730
1,05	1,00000	1,00000	1,00000	1,00000	1,00000	0,99996	0,99975	0,99928	0,99868	0,99837
1,10	1,00000	1,00000	1,00000	1,00000	1,00000	0,99998	0,99989	0,99963	0,99925	0,99903
1,15	1,00000	1,00000	1,00000	1,00000	1,00000	0,99999	0,99995	0,99982	0,99959	0,99944

1,20	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	0,99998	0,99991	0,99978	0,99968
1,25	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	0,99999	0,99996	0,99988	0,99982
1,30	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	0,99998	0,99994	0,99990
1,35	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	0,99999	0,99997	0,99995
1,40	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	0,99999	0,99997
1,45	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	0,99999	0,99999
1,50	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	0,99999
1,55	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000
1,60	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000
1,65	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000
1,70	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000
1,75	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000
1,80	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000
1,85	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000
1,90	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000
1,95	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000
2,00	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000

Appendix C: Simulation Study

To calculate the sample size, it is necessary to know the confidence level required, the error acceptable, the standard deviation, the mean and how the product studied presents a tight tolerance. The confidence level will be 99.75%, which gives a $z = 3$, and the error will be 1% for all the characteristics. The table below presents the data used and the sample size for each characteristic.

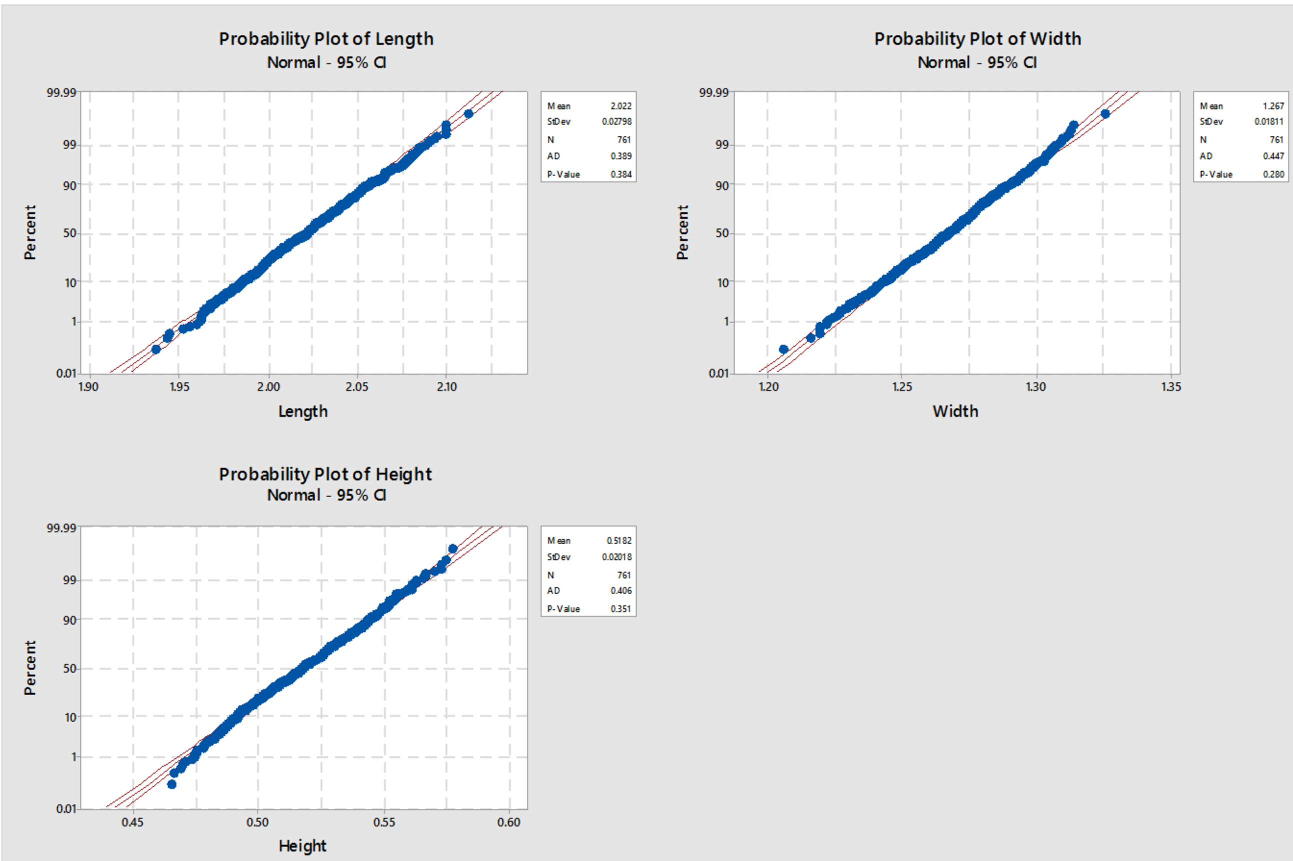
Quality characteristic	z	e	μ	σ	n
Length (L)	3	1	2.022	0.028	18.0
Width (W)	3	1	1.267	0.018	19.0
Height (H)	3	1	0.518	0.021	148.0
Upper width (UW)	3	1	0.408	0.030	487.0
Lower width (LW)	3	1	0.359	0.033	761.0

To use the excel function NORMINV, these steps must to be followed:

- Determine the function input
    - Mean
    - Standard deviation
- Write the function = NORMINV(rand();mean; standard deviation)
- Select the cells that you want to save (the result of the simulation) and the cell with the number of runs
- Go to Data – What If Analysis – Data table
- For the Column/Row input cell: select any empty cell
- Click OK

Appendix D: Data Assumption Test

To use the process capability indices, the data must be in statistical control and must follow the normal distribution. To check the normal distribution assumption, it is necessary to analyze the probability data. Figure D.1 shows these plots for each one of the characteristics.



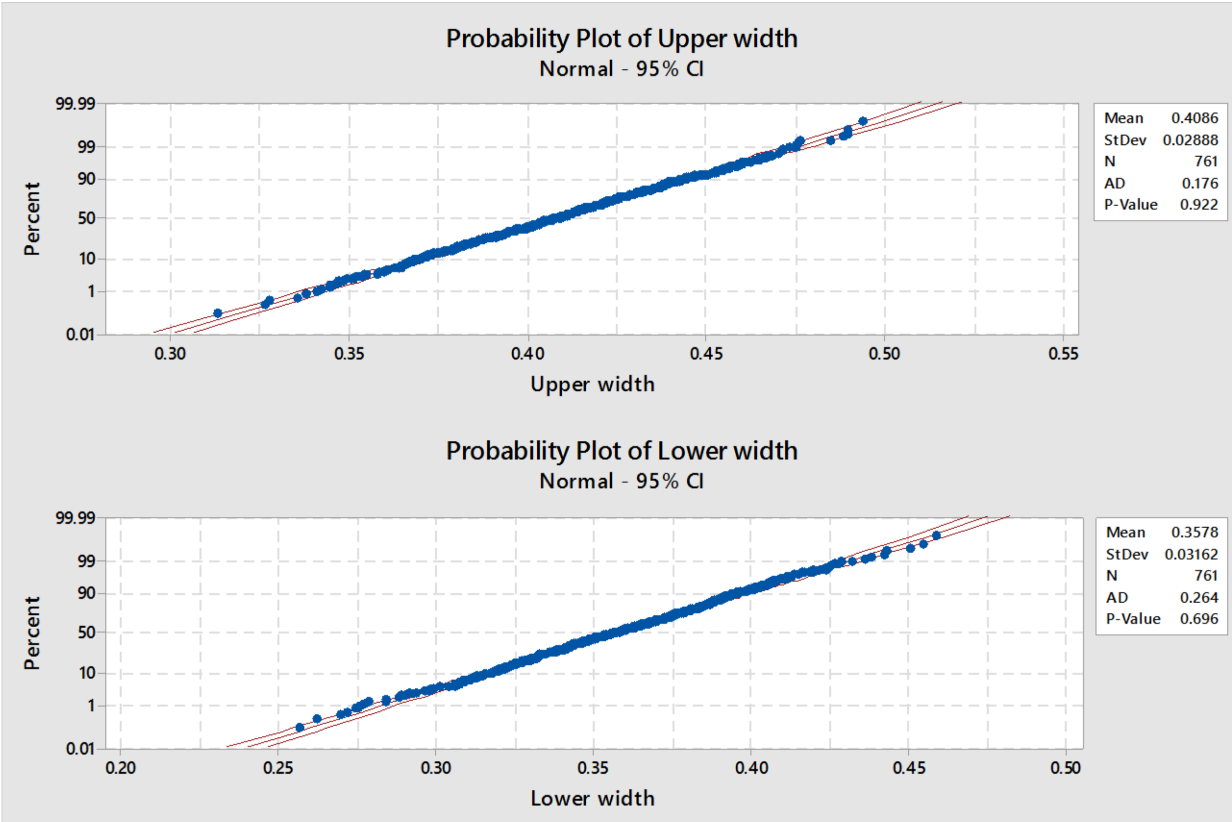


Figure D.1 Normal plot for the quality characteristics

Looking at this figure, it is possible to conclude that the data generated for the simulation follows the normal distribution. All of the points are close to the red line.

To determine if the process is in control, it is necessary to analyze the control chart of the data. Figure D.2 shows the control chart for all of the quality characteristics.

The data is in statistical control. There are some points that are out of the specification limits, but with the sample size being big and the number of data out of the specifications are small, we can assume that these points are outlier.

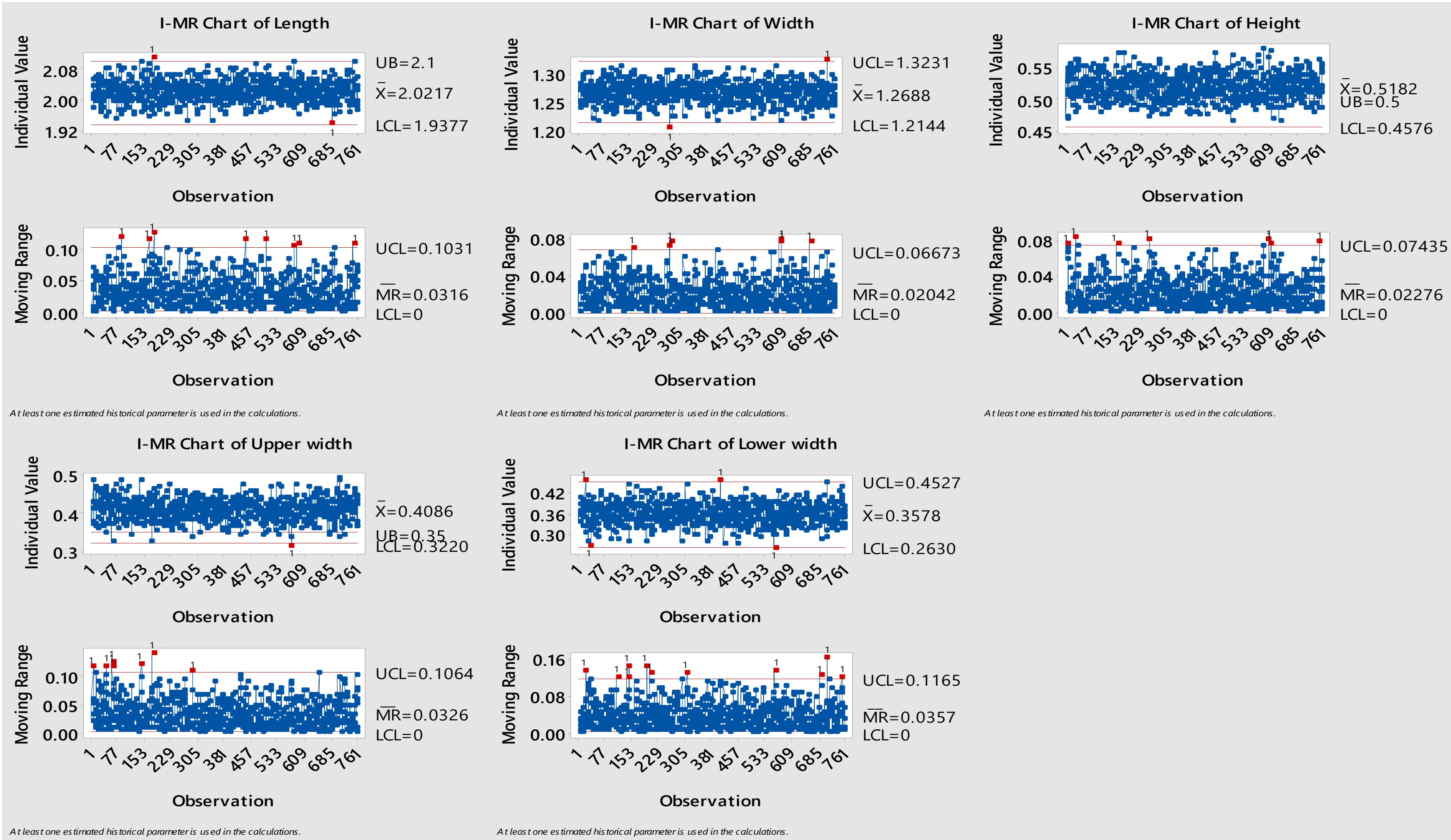


Figure D.2 Control chart for the quality characteristics

Appendix E: Characteristic Study Equations

To determine the yield using the Bothe (1992) method. First, it is necessary to determine the Z_{LSL} and Z_{USL}.

$$Z_{LSL} = \frac{mean - LSL}{std}$$

$$Z_{USL} = \frac{USL - mean}{std}$$

Using the Z_{LSL}. Z_{USL} and the Z-table, it is possible to determine the probability that the characteristic will fall on the outside of the specification limits. Using this data, it is possible to determine the Yield that will be equal to

$$Yield = 1 - Total\ Prob\ Bad = 1 - (Prob\ Bad\ Upper + Prob\ Bar\ Lower)$$

Yield using Bothe approach						
Quality Characteristic	Zlsl	Prob Bad L	Zusl	Prob Bad U	Total Bad	Yield
Length	4.3478	6.8745E-06	2.799621	0.0025581	0.002565	0.997435
Width	6.455	5.4094E-11	4.59102	2.205E-06	2.21E-06	0.9999978
Height	8.3354	0	6.534523	3.191E-11	3.19E-11	1
Upper width	8.9541	0	4.895367	4.906E-07	4.91E-07	0.9999995
Lower width	4.9917	2.9918E-07	4.496221	3.459E-06	3.76E-06	0.9999962

To determine the yield using the Chen & Huang (2007) method. the following equation must to be used

$$pi = \%Yield = \phi\left(\frac{1 + \sqrt{1/(3 * c)^2 - (S/d)^2}}{S/d}\right) + \phi\left(\frac{1 - \sqrt{1/(3 * c)^2 - (S/d)^2}}{S/d}\right) -1$$

Term 1 Term 2

Where $d = \frac{USL-LSL}{2}$ and $c = C_{pm}$

Yield using Chen and Huang approach							
Quality Characteristic	1/((3*c)^2)	s/d	Term 1	Prob Term1	Term 2	Prob Term2	Prob Good
Length	0.125219	0.27982	4.838336	0.99999935	2.309116	0.98953143	0.989531
Width	0.061259	0.18106	6.890015	1	4.156047	0.99998381	0.999984
Height	0.032757	0.1345	8.780589	1	6.0893	1	1
Upper width	0.106738	0.14441	9.187095	1	4.662361	0.99999844	0.999998
Lower width	0.047161	0.210793	5.774219	1	3.713747	0.99989789	0.999898